On January 14, 2014, the SOA added questions 300–306 to its file of sample questions.
I have posted solutions on my webpage www.howardmahler.com/Teaching

1, p. 173, solution 9.7: $P''(z) = 0.6 + \mathbf{2.4}z^3$. Final answer is OK.

1, p. 242, solution 13.23:
As shown in Appendix B of Loss Models, for the zero-truncated Poisson:

$$P(z) = \frac{e^{\lambda z} - 1}{e^{\lambda} - 1} = \frac{e^{3z} - 1}{e^3 - 1}.$$

The p.g.f. for the sum of two independently, identically distributed variables is: $P(z)\,P(z) = P(z)^2$:

$$\mathbf{\left(\frac{e^{3z} - 1}{e^3 - 1}\right)^2}.$$

Comment: The sum of two zero-truncated distributions has a minimum of two events.
Therefore, the sum of two zero-truncated Poissons is not a zero-truncated Poisson.

1, p. 304: $E[(X)(X-1) \ldots (X\mathbf{+1}-n)] = E_\zeta[E[(X)(X-1) \ldots (X\mathbf{+1}-n) \mid \zeta]$.

nth factorial moment of mixed Poisson = $E[(X)(X-1) \ldots (X\mathbf{+1}-n)] = E_\lambda[E[(X)(X-1) \ldots (X\mathbf{+1}-n) \mid \lambda]$ =

2, p. 68: $E[(X - 10000)_+] = \int_{10,000}^{\infty} (x - 10,000)\, f(x)\, dx = \int_{10,000}^{\infty} x\, f(x)\, dx - 10,000 \int_{10,000}^{\infty} f(x)\, dx =$

$$\int_0^{\infty} x\, f(x)\, dx - \left\{ \int_0^{10,000} x\, f(x)\, dx + 10,000\, S(10,000) \right\} = E[X] - E[X \wedge 10000].$$

2, p. 216: 75^{th} percentile $\Leftrightarrow Q_{\mathbf{0.75}} \Leftrightarrow$ 75% quantile $\Leftrightarrow$ third quartile

2, p. 297, Q. 24.99: $(1 + x)^{\mathbf{-4}}$

2, p. 353, Q. 26.7: include the hint, $E[X^k] = \theta^k\, \Gamma[\alpha - k/\tau] / \Gamma[\alpha]$, $k < \alpha\tau$.

2, p. 664, solution 36.35: $\theta = (1.25)(40) = 50$.

3, p. 120, last line of the first paragraph: $\lambda\{E[X^3] - 3\mu_X E[X^2] + 2\mu_X{}^3 + 3\mu_X (E[X^2] - \mu_X{}^2) + \mu_X{}^3\}$

3, p. 167, sol. 5.15: second moment of the uniform distribution is: 3,000,000.
Variance of aggregate loss for N automobiles is: N(0.03)(3,000,000). Final answer is OK.

3, p. 327, sol. 11.32: Var[N] = $9.1667 - 2.5^2$ = **2.9167**.

Var[X] = $1516.67 - 35^2$ = **291.67**.

Var[S] = $(2.5)(291.67) + (35^2)(2.9167) = 4302.1$.

Thus, $E[S^2] = 4302.1 + 87.5^2 = 11{,}958.4$.

$E[(S-40)_+{}^2] = E[S^2] - E[(S \wedge 40)^2] - (2)(40)\{E[S] - E[S \wedge 40]\}$

$= 11{,}958.4 - 1226.3 - (80)(56.026) = 6250.0$.

Thus, $\text{Var}[(S-40)_+] = 6250.0 - 56.026^2$ = **3111**. Letter choices have to change.

4, p.39, sol. 4.5: 8611 + (1510 - 1496)/**0.0050** = **11,411**.

5, p. 74, sol. 3.33: $\frac{\partial \ln[f(x)]}{\partial q} = -\frac{5 - x}{1 - q} + \frac{x}{q}$. $\frac{\partial^2 \ln[f(x)]}{\partial q^2} = -\frac{5 - x}{(1 - q)^2} - \frac{x}{q^2}$.

$E[X] = 5q$. Thus $E[\frac{\partial^2 \ln[f(x)]}{\partial q^2}] = -\frac{5 - 5q}{(1 - q)^2} - \frac{5q}{q^2} = -5\{\frac{1}{(1-q)} + \frac{1}{q}\} = -\frac{5}{q\,(1-q)}$.

6, p. 194, Q. 9.72: estimate for the parameter α.

6, p. 355, Q. 12.27: θ = **5**. The solution is consistent with θ = **5**.

6, p. 456, sol. 15.6: θ_R = (500,000 + 1,250,000/1.2) / (5000 + **10,000**) = 102.78.

6, page 681, comment to sol. 23.6: One could linearly interpolate between 231,919 at the 81.8 percentile and 241,513 at the 90.9 percentile, and instead use **232,130** as the observed 82nd percentile. This would give instead a result of q = -ln(0.18)/ln(2.3213) = 2.04.

6, page 716: get rid of the extra $20^2 - 15^2$

$(3500^2 - 500^2) + (50{,}000^2 - 500^2) + (25{,}000^2 - 1000^2) + (9000^2 - 1000^2) = 3216$ million.

7, p. 68, sol. 2.12: Prob[$15{,}000 \le X \le 50{,}000$] = S(14999.9) - S(50000) = 2/3 - 1/5 = **0.4667**.

7, p. 149-150: Solutions 4.42 & 4.44 are switched. Solution to 4.42 is labled 4.44 and vice-versa.

7, p. 173: 5.5. C. & **5.6**. C. For Group A plus Group B:

7, p. 353: ${}_tq_x^{(T)} = 1 - \prod (1 - {}_t\mathbf{q}_x{}'^{(g)})$.

7, p. 351, sol. 10.49 & 10.50: We also get 8000 lives aged 6**1** when first observed in 201**0**.

9, p. 275, solution 6.38: Final solution is correct.

Thus the posterior mean is: $\int_0^\infty \beta \frac{\beta^x}{(1+\beta)^{x+\alpha+2}} d\beta \ / \int_0^\infty \frac{\beta^x}{(1+\beta)^{x+\alpha+2}} d\beta$.

Let $y = 1/(1+\beta)$. $dy = d\beta/(1+\beta)^2$. $1 - y = \beta/(1+\beta)$.

$$\int_0^\infty \frac{\beta^x}{(1+\beta)^{x+\alpha+2}} d\beta = \int_0^1 y^x (1-y)^\alpha dy = \beta(x+1, \alpha+1) = \Gamma[x+1]\ \Gamma[\alpha+1] / \Gamma[x+2+\alpha].$$

$$\int_0^\infty \beta \frac{\beta^x}{(1+\beta)^{x+\alpha+2}} d\beta = \int_0^1 y^{x+1} (1-y)^{\alpha-1} dy = \beta(x+2, \alpha) = \Gamma[x+2]\ \Gamma[\alpha] / \Gamma[x+2+\alpha].$$

9, p. 288, solution 6.50: $\pi(\lambda) = \lambda^2 e^{-100\lambda}\ \mathbf{100}^3 / \Gamma[3]$. Final answer is OK.

9, p. 291, solution 6.55: $f(20)f(40)S(\mathbf{90})^2 = (\lambda e^{-10\lambda})\ (\lambda e^{-30\lambda})\ (e^{-90\lambda})^2 = \lambda^2 e^{-220\lambda}$.

9, p. 521, solution 10.40:

EPV = $E[\beta + \beta^2] = E[\beta] + E[\beta^2] = E[\beta] + Var[\boldsymbol{\beta}] + E[\beta]^2 = 0.2313 + 0.01589 + 0.2313^2 = 0.3007$.

10, p. 28 , Q. 2.67: A second driver is selected at random from the portfolio.
During five exposure periods, no claims are observed for this second selected driver.
Determine the Buhlmann credibility estimate of the expected number of claims for this second driver during the next exposure period.

10, p. 310: $\mathbf{f(x; \psi) = \frac{p(x)\ e^{r(\psi)x}}{q(\psi)}}$

12, p. 105-107, sol. 5.1-5.7: EPV = $\frac{\sum_{i=1}^{C}\sum_{t=1}^{Y_i} m_{it} (X_{it} - \overline{X}_i)^2}{\sum_{i=1}^{C} (Y_i - 1)}$

13, page 130, 3rd paragraph: m = 100 - 4 = 96.

13, page 163: λ_0 = 2.554 + (-0.5108)(0) = 2.5**5**4.