Exam STAM
Practice Exam #1

These practice exams should be used during the month prior to your exam.

This practice exam contains **20 questions**, of equal value,
corresponding to about a **2 hour** exam.

Each problem is similar to a problem in my study guides, sold separately.
Solutions to problems are at the end of each practice exam.

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Exam STAM, Practice Exam #1

1. Prior to the application of any deductible, losses follow a Pareto Distribution with $\alpha = 3.7$ and $\theta = 120$.
There is a deductible of 25.
What is the variance of amount paid by the insurer for one loss, including the possibility that the amount paid is zero?
A. less than 3900
B. at least 3900 but less than 4000
C. at least 4000 but less than 4100
D. at least 4100 but less than 4200
E. at least 4200

2. You are given the following information:

	Cumulative Incurred Loss through Development Month (million)			
Accident Year	12	24	36	48
2014	77	98	101	103
2015	82	100	105	106
2016	88	103	107	
2017	92	108		
2018	95			

There is no development past 48 months.
Calculate the indicated Gross IBNR reserve using volume-weighted average loss development factors.
A. less than 28 million
B. at least 28 million but less than 30 million
C. at least 30 million but less than 32 million
D. at least 32 million but less than 34 million
E. at least 34 million

3. You are given:
- The number of mistakes that any particular cashier makes per hour follows a Poisson distribution with mean λ.
- The prior distribution of λ is assumed to follow a Gamma Distribution with $\alpha = 0.8$ and $\theta = 1/40$.
- A particular cashier, Marvelous Marv, is observed for 100 hours and makes 6 errors.

Determine the expected number of errors that Marv will make in his next 100 hours.

A. Less than 3.0
B. At least 3.0, but less than 3.5
C. At least 3.5, but less than 4.0
D. At least 4.0, but less than 4.5
E. At least 4.5

4. You are given:
(i) The distribution of the number of claims per policy during a one-year period for 2000 insurance policies is:

Number of Claims per Policy	Number of Policies
0	1900
1	96
2	4
3 or more	0

(ii) You fit a poisson model using the method of maximum likelihood.
Determine the maximum value of the loglikelihood function.

(A) -350 (B) -400 (C) -450 (D) -500 (E) -550

5. Use the following information:
- Using the Method of Maximum Likelihood, a Pareto Distribution has been fit to data.
- The fitted parameters are $\alpha = 3.0$ and $\theta = 1000$.
- The Inverse of the Information Matrix, with alpha first and theta second, is:

$$\begin{pmatrix} 0.029 & 12 \\ 12 & 5333 \end{pmatrix}$$

Estimate the standard deviation of the maximum likelihood estimate of the Survival Function at 5000, using the delta method.

A. less than 0.0005
B. at least 0.0005 but less than 0.0010
C. at least 0.0010 but less than 0.0015
D. at least 0.0015 but less than 0.0020
E. at least 0.0020

6. Use the following information:
• Claim sizes for any policyholder follow a mixed exponential distribution with density function:

$f(x) = 0.8\lambda e^{-\lambda x} + 0.4\lambda e^{-2\lambda x}$, $0 < x < \infty$.

• The prior distribution of λ is Gamma with $\alpha = 4$ and $\theta = 0.005$.
• A policyholder experiences a claim of size 1000.
Use Bayesian Analysis to determine the expected size of the next claim from this policyholder.
A. 200 B. 225 C. 250 D. 275 E. 300

7. Earned Premiums for Calendar Year 2018: $900,000.
All policies are one-year policies, and are issued uniformly through the year.

Rate Level History	
Effective Date	% Rate Change
July 1, 2015	-4.0%
July 1, 2016	+5.0%
January 1, 2017	+3.0%
April 1, 2017	+4.0%
October 1, 2017	-2.0%
April 1, 2018	+6.0%
July 1, 2018	+2.0%
January 1, 2019	-3.0%
October 1, 2019	-5.0%

Rates are currently at the level set on October 1, 2019.
Calculate the earned premium at the current rate level for CY2018.
A. less than 870,000
B. at least 870,000 but less than 880,000
C. at least 880,000 but less than 890,000
D. at least 890,000 but less than 900,000
E. at least 900,000

Use the following data on two classes, A and B, over three years, for the next three questions:

	Exposures		
Year	A	B	Total
2001	150	90	240
2002	170	100	270
2003	200	80	280
Total	520	270	790

	Pure Premium		
Year	A	B	Total
2001	4.00	4.22	4.08
2002	4.65	4.70	4.67
2003	3.70	5.38	4.18
Total	4.10	4.74	4.32

Assume that the losses in each year have been adjusted to the cost level for the year 2006.

$$\sum_{i=1}^{2} \sum_{j=1}^{3} m_{ij} (X_{ij} - \bar{X}_i)^2 = 142.189.$$

$$\sum_{i=1}^{2} m_i (\bar{X}_i - \bar{X})^2 = 72.796.$$

8. Use nonparametric Empirical Bayes techniques to estimate the Expected Value of the Process Variance.
A. 30 B. 35 C. 40 D. 45 E. 50

9. Use nonparametric Empirical Bayes techniques to estimate the Variance of the Hypothetical Mean Pure Premiums.
A. 0.03 B. 0.05 C. 0.07 D. 0.09 E. 0.11

10. Using the method that preserves total losses so that estimates are in balance, estimate the pure premium in the year 2006 for Class B.
A. 4.44 B. 4.47 C. 4.50 D. 4.53 E. 4.56

11. Donny Brook is a claims adjuster at the regional office.
Claims of size less than 5000 are handled at the branch office.
Claims of size greater than 100,000 are handled at the home office.
Donny has handled the following claims during the past month:

Size of Claim	Number of Claims
5000 to 10,000	50
10,000 to 25,000	20
25,000 to 100,000	10

Let F(x) be the Distribution Function of the size of all claims. S(x) = 1 - F(x).
Determine the likelihood function for Donny's data.

(A) $$\frac{\{F(10,000) - F(5000)\}^{50} \{F(25,000) - F(10,000)\}^{20} \{F(100,000) - F(25,000)\}^{10}}{\{F(100,000) - F(5000)\}^{80}}$$

(B) $$\frac{\{F(10,000) - F(5000)\}^{50} \{F(25,000) - F(10,000)\}^{20} \{F(100,000) - F(25,000)\}^{10}}{S(5000)^{80}}$$

(C) $$\frac{\{F(10,000) - F(5000)\}^{50} \{F(25,000) - F(10,000)\}^{20} \{F(100,000) - F(25,000)\}^{10}}{\{F(100,000)\ F(5000)\}^{80}}$$

(D) $$\frac{\{F(10,000) - F(5000)\}^{50} \{F(25,000) - F(10,000)\}^{20} \{F(100,000) - F(25,000)\}^{10}}{\{S(100,000)\ F(5000)\}^{80}}$$

(E) None of A, B, C, or D.

12. The number of boys in a family with m children is Binomial.
The parameter q varies between the different families via a Beta Distribution with $\alpha = 3$, $\beta = 3$, and $\theta = 1$.
What is the probability that a family with 6 children has 5 boys and 1 girl?
A. less than 11%
B. at least 11% but less than 12%
C. at least 12% but less than 13%
D. at least 13% but less than 14%
E. at least 14%

13. The number of claims, N, made on an insurance portfolio follows the following distribution:

n	Pr(N=n)
2	50%
3	30%
4	20%

If a claim occurs, the benefit is 0 or 10 with probability 0.7 and 0.3, respectively.
The number of claims and the benefit for each claim are independent.
Calculate the probability that aggregate benefits will exceed expected benefits by more than 2.5 standard deviations.
(A) 2.0% (B) 2.5% (C) 3.0% (D) 3.5% (E) 4.0%

14. You are given the following:
Prior to observing any data, you assume that the claim frequency rate per exposure has mean = 0.08 and variance = 0.12.
A full credibility standard is devised that requires the observed sample frequency rate per exposure to be within 10% of the expected population frequency rate per exposure 99% of the time.
You observe 112 claims on 1,000 exposures.
Estimate the number of claims you expect for these 1000 exposures next year.
A. 89 B. 91 C. 93 D. 95 E. 97

15. An insurance company sells a policy with a linearly disappearing deductible such that no payment is made on a claim of 500 or less and full payment is made on a claim of 2000 or more.
There are three losses, of sizes: 400, 1000, and 2500.
Calculate the total payment made by the insurance company for these three losses.
A. less than 3150
B. at least 3150 but less than 3175
C. at least 3175 but less than 3200
D. at least 3200 but less than 3225
E. at least 3225

16. You have the following data from two classes:

Class	Number of Claims	Dollars of Loss
A	120	40,000
B	250	110,000

You assume that the mean claim size for Class B is 1.5 times that for Class A.
You assume the size of claim distribution for each class is Gamma, with $\alpha = 3$.
Estimate the mean claim size for Class A via the method of maximum likelihood applied to the data for both classes.
(A) Less than 310
(B) At least 310, but less than 320
(C) At least 320, but less than 330
(D) At least 330, but less than 340
(E) At least 340

17. You are given:
(i) X_i is the claim count observed for insured i for one year.
(ii) X_i has a Negative Binomial Distribution with parameters $\beta = 0.7$ and r_i.
(iii) The r_i's have a Gamma Distribution with parameters α and θ.
Determine the Buhlmann credibility parameter, K.
(A) 1.19α (B) $1.19\alpha/\theta$ (C) 2.43α (D) $2.43\alpha/\theta$ (E) None of A, B, C, or D

18. William M. Lowe, consulting actuary, works on each assignment in intervals.
The length in hours of these intervals has an Exponential Distribution with a mean of 2.
William bills each work interval at $500 per hour, excluding any fraction of an hour.
So if for example, a work interval lasts 2.7 hours, then the client is only billed $1000.
The number of work intervals per assignment is distributed as
a zero-truncated Geometric Distribution with $\beta = 4$.
Determine the average amount William bills per assignment.
A. Less than $3700
B. At least $3700, but less than $3800
C. At least $3800, but less than $3900
D. At least $3900, but less than $4000
E. At least $4000

19. You are given the following:
- The amount of an individual loss in the year 2000, follows an Exponential Distribution with mean 17,000.
- Between 2000 and 2005, losses will be multiplied by an inflation factor.
- You are uncertain of what the inflation factor between 2000 and 2005 will be, but you estimate that it will be a random draw from an Inverse Gamma Distribution with parameters $\alpha = 3.1$ and $\theta = 2.6$.

Estimate the probability that a loss in 2005 exceeds 80,000.

A. 1% B. 2% C. 3% D. 4% E. 5%

20. You are given the following accident data:

Number of accidents	Number of policies
0	101
1	178
2	282
3	249
4	114
5	54
6	22
7+	0
Total	1000

Which of the following distributions would be the most appropriate model for this data?

(A) Binomial (B) Poisson (C) Negative Binomial, $r \le 1$ (D) Negative Binomial, $r > 1$
(E) None of the Above

END OF PRACTICE EXAM

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Solutions:

1. A. After truncating and shifting from below, one gets another Pareto Distribution with $\alpha = 3.7$ and $\theta = 120 + 25 = 145$.
Thus the nonzero payments are Pareto with $\alpha = 3.7$ and $\theta = 145$.
This has mean: $\theta/(\alpha - 1) = 145/2.7 = 53.7$, second moment: $2\theta^2 / \{(\alpha - 1)(\alpha - 2)\} = 9161$, and variance: $9161 - 53.7^2 = 6277$.
The probability of a nonzero payment is the probability that a loss is greater than the deductible of 25; for the original Pareto, $S(25) = \{120/(120+25)\}^{3.7} = 0.496$.
Thus the payments of the insurer can be thought of as an aggregate distribution, with Bernoulli frequency with mean 0.496 and Pareto severity with $\alpha = 3.7$ and $\theta = 145$.
The variance of this aggregate distribution is:
(Mean Freq.)(Var. Sev.) + (Mean Sev.)^2(Var. Freq.) =
$(0.496)(6277) + (53.7^2) \{(0.496)(1 - 0.496)\} =$ **3834**.
One can also think of this as a two-point mixture between a severity that is always zero and a severity that is the truncated and shifted Pareto, with the former with weight 1 - 0.496 and the latter with weight 0.496. The mean of this mixture is: (0.504)(0) + (0.496)(145/2.7) = 26.64.

The second moment of this mixture is: $(0.504)(0) + (0.496)\ \frac{(2)(145^2)}{(2.7)(1.7)} = 4544$.

The variance of this mixture is: $4544 - 26.64^2 =$ **3834**.
Comment: Similar to Q. 24.46 in "Mahler's Guide to Loss Distributions."
$G(x) = \{F(x+d) - F(d)\} / S(d) = \{S(d) - S(x+d)\} / S(d) = 1 - S(x+d)/S(d) =$
$1 - \{120/(120+x+25)\}^{3.7} / \{120/(120+25)\}^{3.7} = 1 - \{145/(145+x)\}^{3.7}$.
We include the zero payments by introducing a Bernoulli frequency.
0 in the Bernoulli corresponds to a zero payment, while 1 in the Bernoulli corresponds to a non-zero payment (in this case from a truncated and shifted Pareto.)
Thus we can convert this situation into the mathematical equivalent of an aggregate distribution situation.

2. D. The 12-24 development factor: $\frac{98 + 100 + 103 + 108}{77 + 82 + 88 + 92} = 1.206$.

The 24-36 development factor: $\frac{101 + 105 + 107}{98 + 100 + 103} = 1.040$.

The 36-48 development factor: $\frac{103 + 106}{101 + 105} = 1.015$.

AY16 estimated ultimate: (107)(1.015) = 108.6.
AY17 estimated ultimate: (108)(1.015)(1.040) = 114.0.
AY18 estimated ultimate: (95)(1.015)(1.040)(1.206) = 120.9.
Gross IBNR Reserve: (108.6 - 107) + (114.0 - 108) + (120.9 - 95) = **33.5 million**.
Comment: Similar to Q.12.1 in "Mahler's Guide to P&C Ratemaking and Reserving."

3. E. The parameters of the posterior Gamma are $\alpha' = \alpha + C = 0.8 + 6 = 6.8$,
and $1/\theta' = 1/\theta + E = 40 + 100 = 140$.
The posterior mean is: 6.8/140 = 0.0486.
Therefore, the expected number of mistakes that Marv will make in his next 100 hours is:
(0.0486)(100) = **4.86**.
Alternately, $K = 1/\theta = 40$. Z = 100 / (100 + 40) = 71.4%.
Prior mean is 0.02. Observed mean frequency is: 6/100 = 0.06.
The estimated future frequency = (71.4%)(0.06) + (28.6%)(0.02) = 0.0486.
(0.0486)(100) = **4.86**.
Comment: Similar to Q. 4.129 (4B, 5/99, Q. 24) in "Mahler's Guide to Conjugate Priors."

4. B. Maximum likelihood is equal to method of moments. $\hat{\lambda}$ = {96 + (4)(2)} / 2000 = 0.052.
$\ln f(x) = x \ln(\lambda) - \lambda - \ln(x!)$.
The loglikelihood is: $(1900)(-\lambda) + (96)\{\ln(\lambda) - \lambda\} + (4)\{2 \ln(\lambda) - \lambda - \ln(2)\} =$
$-2000\lambda + 104 \ln(\lambda) - 4\ln(2)$.
At $\lambda = 0.052$, this is **-414.25**.
Comment: Similar to Q. 3.61 (4, 11/06, Q.12 & 2009 Sample Q.256)
in "Mahler's Guide to Fitting Frequency Distributions."

5. B. For the Pareto Distribution, $S(x) = \{\theta/(\theta+x)\}^\alpha$.

$$\frac{\partial S(x)}{\partial \alpha} = \ln[\theta/(\theta+x))]\ \{\theta/(\theta+x)\}^\alpha.$$

$$\frac{\partial S(5000)}{\partial \alpha} = \ln[1/(1+5)]\ \{1/(1+5)\}^3 = -0.00830.$$

$$S(x) = \frac{\theta^\alpha}{(\theta+x)^\alpha}.\quad \frac{\partial S(x)}{\partial \theta} = \frac{\frac{\partial\ \theta^\alpha}{\partial \alpha}(\theta+x)^\alpha - \theta^\alpha \frac{\partial\ (\theta+x)^\alpha}{\partial \alpha}}{(\theta+x)^{2\alpha}} = \frac{\alpha\theta^{\alpha-1}\ (\theta+x)^\alpha - \theta^\alpha\ \alpha(\theta+x)^{\alpha-1}}{(\theta+x)^{2\alpha}}$$

$= \alpha\theta^{\alpha-1}\{(\theta+x) - \theta\} / (\theta+x)^{\alpha+1} = \alpha\theta^{\alpha-1}x / (\theta+x)^{\alpha+1}$.

$$\frac{\partial S(5000)}{\partial \theta} = 3(1000^2)(5000) / (6000)^4 = 0.0000116.$$

Thus the gradient vector is: (-0.00830, 0.0000116).
Thus using the delta method, the variance of the estimated value of the Survival Function is:
(transpose of gradient vector) (Inverse of the information matrix) (gradient vector) =

$$(-0.00830,\ 0.0000116)\begin{pmatrix} 0.029 & 12 \\ 12 & 5333 \end{pmatrix}\begin{pmatrix} -0.00830 \\ 0.0000116 \end{pmatrix}$$

= (-0.00830, 0.0000116) (-0.0001015, -0.0378) = 0.00000040.
The standard deviation is: $\sqrt{0.00000040}$ = **0.00064**.

Comment: Similar to Q. 21.44 in "Mahler's Guide to Fitting Loss Distributions."
The given variance-covariance matrix was calculated for 5000 data points.

$S(x) = \{\theta/(\theta+x)\}^\alpha = y^\alpha$, where $y = \theta/(\theta+x)$. $\frac{\partial\ y^\alpha}{\partial \alpha} = \ln[y]\ y^\alpha$.

6. D. The chance of the observation given λ is: $f(1000) = 0.8\lambda e^{-1000\lambda} + 0.4\lambda e^{-2000\lambda}$.
$\pi(\lambda) = \lambda^3 e^{-200\lambda} 0.005^{-4} / \Gamma[4]$.
Therefore, the posterior distribution is proportional to: $2\lambda^4 e^{-1200\lambda} + \lambda^4 e^{-2200\lambda}$.

$$\int_0^\infty 2\lambda^4 e^{-1200\lambda} + \lambda^4 e^{-2200\lambda} \, d\lambda = (2)(1/1200^5)(4!) + (1/2200^5)(4!) = 1.9756 \times 10^{-14}.$$

Thus the posterior distribution of lambda is: $(2\lambda^4 e^{-1200\lambda} + \lambda^4 e^{-2200\lambda}) / (1.9756 \times 10^{-14})$.
The severity distribution is a 80%-20% mixture of Exponentials with means $1/\lambda$ and $1/(2\lambda)$.
Thus the mean given lambda is: $0.8/\lambda + 0.2/(2\lambda) = 0.9 / \lambda$.
Thus the posterior mean severity is:

$$\int_0^\infty (0.9 / \lambda) (2\lambda^4 e^{-1200\lambda} + \lambda^4 e^{-2200\lambda}) \, d\lambda \ / (1.9756 \times 10^{-14}) =$$

$$\frac{0.9}{1.9756 \times 10^{-14}} \int_0^\infty 2\lambda^3 e^{-1200\lambda} + \lambda^3 e^{-2200\lambda} \, d\lambda =$$

$(0.9) \{(2)(1/1200^4)(3!) + (1/2200^4)(3!)\} / (1.9756 \times 10^{-14}) =$ **275.3**.
Comment: Similar to Q. 6.50 in “Mahler’s Guide to Buhlmann Credibility.”
For alpha integer, $\Gamma(\alpha) = (\alpha-1)!$

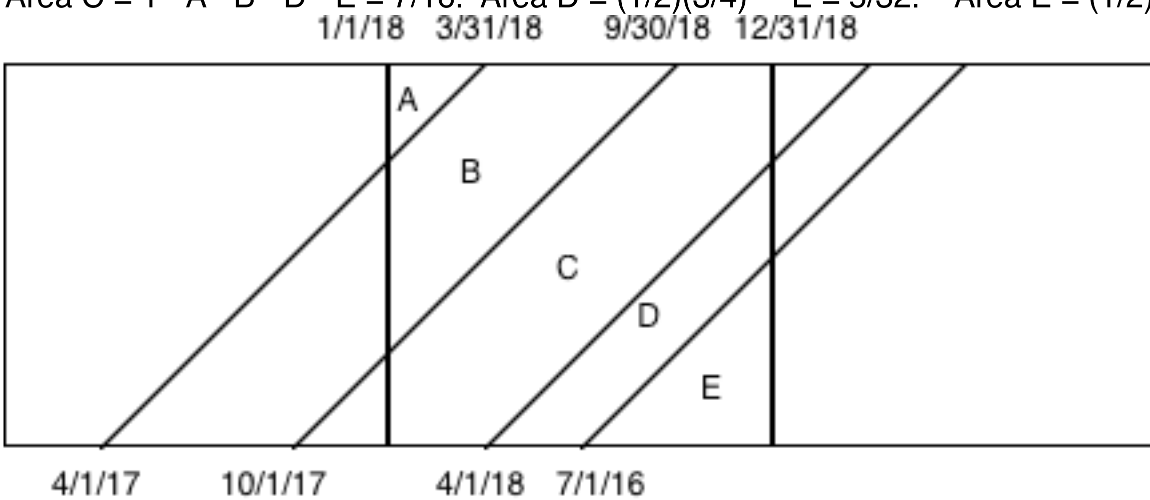
Since the density of a Gamma Distribution must integrate to one: $\int_0^\infty t^{\alpha-1} e^{-t/\theta} \, dt = \Gamma(\alpha)\theta^\alpha$.

The posterior distribution is also proportional to: $0.8\lambda^4 e^{-1200\lambda} + 0.4\lambda^4 e^{-2200\lambda}$;
proceeding in this manner would result in the same posterior distribution.

7. B. The first step is to calculate the rate level index:

Date	Rate Level Change Factor	Rate Level (July 1, 2015 = 1)
July 1, 2015		1.0000
July 1, 2016	1.05	1.0500
January 1, 2017	1.03	1.0815
April 1, 2017	1.04	1.1248
October 1, 2017	0.98	1.1023
April 1, 2018	1.06	1.1684
July 1, 2018	1.02	1.1918
January 1, 2019	0.97	1.1560
October 1, 2019	0.95	1.0982

Area A = $(1/2)(1/4)^2$ = 1/32. Area B = $(1/2)(3/4)^2$ - A = 1/4.
Area C = 1 - A - B - D - E = 7/16. Area D = $(1/2)(3/4)^2$ - E = 5/32. Area E = $(1/2)(1/2)^2$ = 1/8.



Note that for 1 year policies the lines in the diagram have a slope of 1.

Rate Level	Area	Product
1.0815	0.03125	0.0338
1.1248	0.25	0.2812
1.1023	0.4375	0.4822
1.1684	0.15625	0.1826
1.1918	0.125	0.1490
	1	1.1288

On Level Factor: 1.0982/1.1288 = 0.9729.
Earned Premium on current rate level: (0.9729)(900,000) = **$875,610**.
Comment: Similar to Q. 6.52 (STAM Sample Q.314)
in “Mahler’s Guide to P&C Ratemaking and Reserving.”

8. B. Estimated $\text{EPV} = \frac{1}{2}\ \frac{1}{3-1} \sum\sum m_{ij}\,(X_{ij} - \bar{X}_i)^2$ = 142.189 / 4 = **35.55**.

Class	(# of Exposures)(Loss per Exposure - 3 year average)^2				Process
	2001	2002	2003		Variance
A	1.50	51.43	32.00		42.46
B	24.34	0.16	32.77		28.63
Average					**35.55**
	Pure Premium				Weighted Avg.
A	4.00	4.65	3.70		4.10
B	4.22	4.70	5.38		4.74
	Exposures				Sum
A	150	170	200		520
B	90	100	80		270

Comment: Similar to Q. 4.11 in "Mahler's Guide to Nonparametric Credibility."

9. E. Let $\Pi = m - \frac{\sum_{i=1}^{C} m_i^2}{m}$ =

total exposures "adjusted for degrees of freedom" = 790 - $(520^2 + 270^2)$ / 790 = 355.4.

$\bar{X}$ = overall average loss per exposure = 4.32.

estimated VHM = $\frac{\sum_{i=1}^{C} m_i\,(\bar{X}_i - \bar{X})^2 - \text{EPV}\,(C-1)}{\Pi}$ =

$\frac{520(4.10 - 4.32)^2 + 270(4.74 - 4.32)^2 - (2-1)(35.55)}{355.4}$ = **0.105**.

Comment: Similar to Q. 4.12 in "Mahler's Guide to Nonparametric Credibility."
K = EPV/VHM = 35.55/0.105 = 339.
For the Class A, Z = 520/(520 + 339) = 60.5%.
For the Class B, Z = 270/(270 + 339) = 44.3%.

10. D. K = EPV/VHM = 35.55/0.105 = 339.
For the Class A, Z = 520/(520 + 339) = 60.5%.
For the Class B, Z = 270/(270 + 339) = 44.3%.
In order to preserve the total losses, apply the complement of credibility to the credibility weighted average pure premium:
{(0.605)(4.10) + (0.443)(4.74)} / (0.605 + 0.443) = 4.37.
Therefore, the estimated pure premium for Class B is:
(0.443)(4.74) + (1 - 0.443)(4.37) = **4.53**.
Comment: Similar to Q. 4.14 in "Mahler's Guide to Nonparametric Credibility."
The estimated pure premium for Class A is: (0.605)(4.10) + (1 - 0.605)(4.37) = 4.21.
In P&C ratemaking, there is often a 2 or 3 year trend period between the most recent year of data and the future period for which we are making rates.

11. A. Donny's data is truncated from below at 5000 and truncated from above at 100,000.
The remaining probability is: F(100,000) - F(5000).
Thus to get the probabilities for the truncated distribution, we divide by: F(100,000) - F(5000).

For the first interval: $\frac{F(10{,}000) - F(5000)}{F(100{,}000) - F(5000)}$.

For the second interval: $\frac{F(25{,}000) - F(10{,}000)}{F(100{,}000) - F(5000)}$.

For the third interval: $\frac{F(100{,}000) - F(25{,}000)}{F(100{,}000) - F(5000)}$.

Thus the likelihood is:

$$\left(\frac{F(10{,}000) - F(5000)}{F(100{,}000) - F(5000)}\right)^{50} \left(\frac{F(25{,}000) - F(10{,}000)}{F(100{,}000) - F(5000)}\right)^{20} \left(\frac{F(100{,}000) - F(25{,}000)}{F(100{,}000) - F(5000)}\right)^{10} =$$

$$\mathbf{\frac{\{F(10{,}000) - F(5000)\}^{50} \{F(25{,}000) - F(10{,}000)\}^{20} \{F(100{,}000) - F(25{,}000)\}^{10}}{\{F(100{,}000) - F(5000)\}^{80}}}.$$

Comment: Similar to Q. 15.35 (4, 5/01, Q.34) in "Mahler's Guide to Fitting Loss Distributions."
Values below 5000 or above 100,000 are excluded from the data, and thus the data is truncated both from below and above.

12. D. The density of the Beta for $\theta = 1$ is: $f(x) = \frac{(a+b-1)!}{(a-1)!\ (b-1)!} x^{a-1} (1-x)^{b-1}$, $0 \le x \le 1$.

Thus the distribution of q is: $\pi(q) = 30q^2(1-q)^2$.
The density at 5 of a Binomial with m = 6 is: $6\ q^5(1-q)$.
Then we get the density at 5 of the mixed distribution by integrating $f(5 \mid q)\ \pi(q)$.

Prob[family with 6 children has 5 boys] = $\int_0^1 6\ q^5(1-q)\ 30q^2(1-q)^2\ dq = 180 \int_0^1 q^7\ (1-q)^3\ dq$.

Since the density of a Beta Distribution integrates to one over its support:

$\int_0^1 x^{a-1}\ (1-x)^{b-1}\ dx = \frac{(a-1)!\ (b-1)!}{(a+b-1)!}$. $\Rightarrow \int_0^1 q^7\ (1-q)^3\ dq = 7!\ 3!\ /\ 11! = 1/1320$.

Prob[family with 6 children has 5 boys] = $180 \int_0^1 q^7\ (1-q)^3\ dq = 180/1320 =$ **13.64%**.

Comment: Similar to Q. 17.13 in "Mahler's Guide to Frequency."
This mixed distribution is a Beta-Binomial,
as discussed in "Mahler's Guide to Conjugate Priors."
We do not have an observation; we are not using Bayes Theorem nor an updating formula to estimate a value posterior to an observation.
Note that the average probability of a boy is: 3/(3+3) = 1/2.
For a family with q = 1/2 and 6 children, the chance of 5 boys and one girl is:
$(6)(1/2)^5(1 - 1/2) = 9.375\%$.
However, a family with q > 1/2 has higher probability of 5 boys and one girl.
Also the Binomial density at 5 is not linear in q.
Thus we can not just plug the average q into the Binomial density at 5.

13. B. Mean frequency = (2)(0.5) + (3)(0.3) + (4)(0.2) = 2.7.
Second moment of the frequency = $(2^2)(0.5) + (3^2)(0.3) + (4^2)(0.2) = 7.9$.
Variance of the frequency = $7.9 - 2.7^2 = 0.61$.
Mean severity = (0)(0.7) + (10)(0.3) = 3.
Second moment of the severity = $(0^2)(0.7) + (10^2)(0.3) = 30$.
Variance of the severity = $30 - 3^2 = 21$.
Mean aggregate loss = (2.7)(3) = 8.1.
Variance of the aggregate losses = $(2.7)(21) + (3^2)(0.61) = 62.19$.
Mean + 2.5 standard deviations = $8.1 + 2.5\sqrt{62.19} = 27.82$.
The aggregate benefits are greater than 27.82 if and only if there is at least three non-zero claims.
The probability of 3 claims of size 10 is: $(0.3)(0.3^3) = 0.0081$.
If we have have four claims, a 20% probability, then the probability that 3 of them are of size 10 and 1 of them are of size 0 is the density at three of a Binomial Distribution with m = 4 and q = 0.3.
⇒ The probability of 3 claims of size 10 and one claim of size 0 is:
$(0.2)(4)(0.7)(0.3^3) = 0.01512$.
The probability of 4 claims of size 10 is: $(0.2)(0.3^4) = 0.00162$.
Thus the probability of at least 3 non-zero claims is: 0.0081 + 0.01512 + 0.00162 = **2.484%**.
Comment: Similar to Q. 5.99 (3, 11/00, Q.8 & 2009 Sample Q.113)
in “Mahler’s Guide to Aggregate Distributions.”
Note that it did not say to use the Normal Approximation.

14. A. P = 99%. Therefore, y = 2.576, since $\Phi(2.576) = 0.995 = (1+P)/2$. k = 0.10.
Standard For Full Credibility is: $(y / k)^2 (\sigma_f^2/\mu_f) = (2.576/0.1)^2(0.12/0.08) = 995$ claims,
or 995/0.08 = 12,438 exposures.
$Z = \sqrt{1000/12{,}438} = 28.4\%$.
Estimated future frequency is: (28.4%)(112/1000) + (71.6%)(.08) = 8.91%.
Expected number of future claims is: (1000)(8.91%) = **89**.
Alternately, using the expected number of claims of (0.08)(1000) = 80, and the standard for full credibility in terms of expected claims: $Z = \sqrt{80/995} = 28.4\%$. Proceed as before.
Comment: Similar to Q. 6.7 in “Mahler’s Guide to Classical Credibility.”
As stated at page 29 of “Credibility” by Mahler and Dean, when available one generally uses the number of exposures (1000) or the expected number of claims (80) in the square root rule, rather than the observed number of claims (112), since the observed number of claims is subject to random fluctuation.

15. B. There is nothing paid for the loss of size 400.
The whole loss is paid for the loss of size 2500.
The payment for a loss of 500 is 0. The payment for a loss of 2000 is 2000.
Linearly interpolate in order to get the payment for a loss of 1000:

$$\frac{1000 - 500}{2000 - 500}\ (2000) = 667.$$

Total payment is: 0 + 667 + 2500 = **3167**.
Comment: Similar to Q. 20.52 (STAM Sample Q.308)
in "Mahler's Guide to P&C Ratemaking and Reserving.".
For $500 \le X \le 2000$, the payment is: (X - 500) 4/3.

16. A. For a Gamma Distribution with $\alpha = 3$, $f(x) = x^2 e^{-x/\theta} / (2\theta^3)$.
$\ln f(x) = 2\ln(x) - x/\theta - 3\ln(\theta) - \ln(2)$.
Assuming $\theta_B = 1.5\theta_A$, and thus $\ln(\theta_B) = \ln(1.5) + \ln(\theta_A)$, then the loglikelihood is:

$$\sum_A \{2\ln(x_i) - x_i/\theta_A - 3\ln(\theta_A) - \ln(2)\} + \sum_B \{2\ln(x_i) - x_i/(1.5\theta_A) - 3\ln(1.5) - 3\ln(\theta_A) - \ln(2)\}.$$

Setting the partial derivative of the loglikelihood with respect to θ_A equal to zero:

$$0 = \sum_A (x_i/\theta_A^2 - 3/\theta_A) + \sum_B \{x_i/(1.5\theta_A^2) - 3/\theta_A\}.$$

$0 = 40{,}000/\theta_A^2 - (120)(3/\theta_A) + 110{,}000/(1.5\theta_A^2) - (250)(3/\theta_A)$.
$113{,}333/\theta_A^2 = 1110/\theta_A$. $\Rightarrow \theta_A = 102.1$. $\Rightarrow$ Mean for Class A $= \alpha\theta_A = (3)(102.1) =$ **306.3**.
Comment: Similar to Q. 5.27 in "Mahler's Guide to Fitting Loss Distributions."
One could just divide the losses observed for Class B by 1.5, in order to get them to the level of Class A. Then for the Gamma Distribution with α fixed, the method of maximum likelihood equals the method of moments: $3\theta_A = \{40{,}000 + 110{,}000/1.5\} / (120 + 250) = 306.3$.

17. E. The process variance for an individual insured is: $r\beta(1+\beta) = (0.7)(1.7)r = 1.19r$.
EPV = E[1.19r] = 1.19E[r] = $1.19\alpha\theta$.
The mean frequency for an individual insured is: $r\beta = 0.7r$.
VHM = Var[0.7r] = 0.7^2Var[r] = $0.49\alpha\theta^2$.
K = EPV/VHM = $1.19\alpha\theta / (0.49\alpha\theta^2) =$ **2.43/θ**.
Comment: Similar to Q. 10.69 (4, 5/00, Q.37) in "Mahler's Guide to Buhlmann Credibility."

18. C. If William works 0.6 hours he charges nothing.
So he starts charging when $x \ge 1$. If $x < 1$ he charges nothing.
If he works at least one hour, then he charges \$500.
If he works at least two hours, then he charges an additional \$500.
If he works at least three hours, then he charges yet an additional \$500. etc.
Thus, the expected hours billed per work interval is:
$S(1) + S(2) + S(3) + ... = e^{-1/2} + e^{-2/2} + e^{-3/2} + ... = e^{-1/2} / (1 - e^{-1/2}) = 1 / (e^{0.5} - 1) = 1.541$.
Mean number of work intervals per assignment is the mean of
a zero-truncated Geometric distribution: $\frac{\beta}{1 - 1/(1+\beta)} = \frac{4}{1 - 1/5} = 5$.

The average amount William bills per assignment is: (\$500)(5)(1.541) = **\$3853**.
Alternately, the number of work intervals per assignment is 1 + a Geometric Distribution with b = 4. Mean number of work intervals per assignment is: 1 + 4 = 5. Proceed as before.
Comment: Similar to Q. 33.35 (CAS3, 5/06, Q.38) in "Mahler's Guide to Loss Distributions."
The expected hours billed per work interval is what is called the curtate expectation of life at zero: $e_0 = \sum_{t=1}^{\infty} S(t)$. The curtate expectation of life for a person alive at age x is the expected number of complete years remaining to live, the expected number of birthdays that the person will celebrate. This is the discrete analog of the mean residual life (mean excess loss), which is equal to the integral of the survival function.

19. D. Let the inflation factor be y.
Then given y, in the year 2005 the losses have an Exponential Distribution with mean 17,000y.
Let z = 17,000y.
Then since y follows an Inverse Gamma with parameters $\alpha = 3.1$ and scale parameter $\theta = 2.6$,
z follows an Inverse Gamma with parameters $\alpha = 3.1$ and $\theta = (17000)(2.6) = 44{,}200$.
Thus in the year 2005, we have a mixture of Exponentials each with mean z, with z following an Inverse Gamma. This is the (same mathematics as the) Inverse Gamma-Exponential.
For the Inverse Gamma-Exponential the mixed distribution is a Pareto, with
α = shape parameter of the Inverse Gamma, and θ = scale parameter of the Inverse Gamma.
In this case the mixed distribution is a Pareto with $\alpha = 3.1$ and $\theta = 44{,}200$.
For this Pareto, $S(80{,}000) = \{1 + (80{,}000/44{,}200)\}^{-3.1}$ = **4.1**%.
Comment: Similar to Q. 39.12 in "Mahler's Guide to Loss Distributions."

20. E. Calculate (x+1)f(x+1) / f(x).
Since it does not seem to be linear, we do not seem to have a member of the (a, b, 0) class.
Thus the Binomial, Poisson, or Negative Binomial are not appropriate models.

Number of Accident	Observed	Observed Density Function	(x+1) f(x+1)/f(x)
0	101	0.101	1.76
1	178	0.178	3.17
2	282	0.282	2.65
3	249	0.249	1.83
4	114	0.114	2.37
5	54	0.054	2.44
6	22	0.022	
7+	0	0.000	

Comment: Similar to Q. 12.8 (4, 11/03, Q.32 & 2009 Sample Q.25)
in "Mahler's Guide to Frequency Distributions."
$\overline{X}$ = 2.347. The sample variance is: (7.513 - 2.347^2) (1000/999) = 2.007.
If it were a member of the (a, b, 0) class, then since the mean > variance it would be a Binomial.
However, the "accident profile" first tests whether this is a member of the (a, b, 0) class.

While these solutions are believed to be correct, anyone can make a mistake.
If you believe you've found something that may be wrong, send any corrections or comments to:
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