5, solution 9.2:

such that $\frac{1}{n}\sum_{i=1}^{n} z_{i1}^2$ is **maximized**,

5, revise Q. 9.4 and solution:
You were given the following six observations with three variables:

Observation	X_1	X_2	X_3
1	21	17	10
2	3	4	14
3	11	20	23
4	33	8	12
5	20	15	6
6	31	6	24

The variables were standardized to have a mean of zero and standard deviation of one.
Then Principal Components Analysis was performed.
The principal component score vectors turned out to be:

PC1	PC2	PC3
-0.87157902	0.6604353	0.3923182
-0.06263353	-0.8361690	-1.8811591
-0.96048588	-1.4958783	1.0176317
0.97745723	1.0816114	0.1072682
-0.93929808	1.0966693	-0.1505856
1.85653927	-0.5066686	0.5145266

Determine the proportion of variance explained by each principal component.

9.4. The variance explained by the m^{th} principal component is: $\frac{1}{n}\sum_{i=1}^{n} z_{im}^2$.

For the first principal component:

$$\frac{(-0.87157902)^2 + (-0.6263353)^2 + (-0.96048588)^2 + 0.97745723^2 + (-0.93929808)^2 + 1.85653927^2}{6}$$

= 1.1618.

For the second principal component:

$$\frac{0.660453^2 + (-0.8361690)^2 + (-1.4958783)^2 + 1.0816114^2 + 1.0966693^2 + (-0.506686)^2}{6}$$

= 1.0004.

For the third principal component:

$$\frac{0.3923182^2 + (-1.8811591^2) + 1.0176371^2 + 0.1072682^2 + (-0.1505856^2) + 0.5145266^2}{6}$$

= 0.8379.

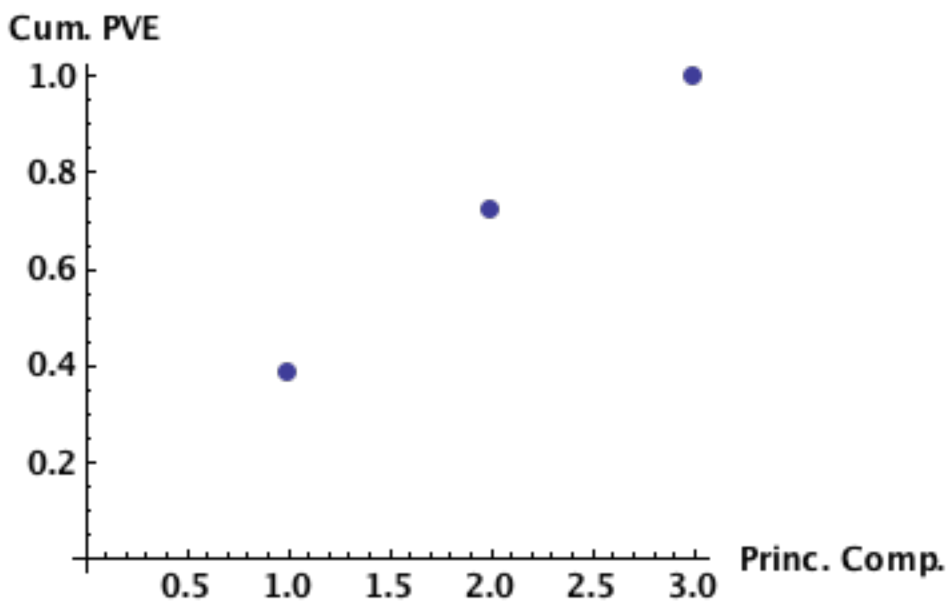
The proportions of variance explained are:

(1.1618, 1.0004, 0.8379) / (1.1618 + 1.0004 + 0.8379) = **38.73%, 33.35%, 27.93%**.

Comment: While Formula 12.9 in the textbook and the R function princomp each divide by n, the R function prcomp divides by n -1.

As long as one is consistent, the resulting proportions of variance explained are the same.

A graph of the cumulative proportions of variance explained:



Since the earlier principal components explain more of the variance than the later ones, such a graph should be concave downwards; however, that is not visually obvious in this case.

(If the original variables were independent, then each principal component would explain an equal amount of the variance.)

6, solution 6.64:

Posterior distribution of θ is proportional to: $\pi(\theta)$ f(11) = 150/(**11** + θ)^4, $5 < \theta < \infty$.

$$\int_5^\infty \frac{150}{(11+\theta)^4} d\theta = -50/(11+\theta)^3 \Big]_{\theta=5}^{\theta=\infty} = 25/2048.$$

Posterior density of θ is: {150/(**11** + θ)^4} / (25/2048) = 12,288/(**11** + θ)^4, $5 < \theta < \infty$.

The posterior probability that θ exceeds 10 is:

$$\int_{10}^\infty \frac{12{,}228}{(11+\theta)^4} d\theta = -4096/(11+\theta)^3 \Big]_{\theta=10}^{\theta=\infty} = 4096/21^3 = 0.442.$$