NonParametric Credibility, Missing Solutions to Problems in Section 6:

6.1. A. The Expected Value of the Process Variance = 0.35.

Type of Risk	A Priori Probability	Process Variance	Mean	Square of Mean
A	0.75	0.3	0.3	0.09
B	0.25	0.5	0.5	0.25
Average		0.350	0.350	0.130

The Variance of the Hypothetical Means = $0.130 - 0.35^2 = 0.0075$.
Thus the Bühlmann credibility parameter, K = EPV / VHM = 0.35 / 0.0075 = 46.7.
Therefore, for 3 years of data Z = 3 / (3 + 46.7) = 6.0%.
The estimated future annual frequency for policyholder 3 is:
(6.0%)(4/3) + (1 - 6.0%)(0.350) = **0.409**.

6.2. D. Define the unit of data as one insured over three years.

3 Year Claim Count:	0	1	2	3	4	5 or more
Number of Insureds:	3	2	2	0	1	0

For the observed data the mean frequency (per 3 years) is:
{(3)(0) + (2)(1) + (2)(2) + (1)(4)} / 8 = 1.25.
The second moment is: $\{(3)(0^2) + (2)(1^2) + (2)(2^2) + (1)(4^2)\} / 8 = 3.25$.
Thus the sample variance = $(8/7)(3.25 - 1.25^2) = 1.929$.
EPV = mean = 1.25. VHM = Total Variance - EPV = 1.929 - 1.25 = 0.679.
K = EPV/VHM = 1.25/0.679 = 1.84.
Three years of data has been defined as N = 1, therefore, Z = 1 / (1 + 1.84) = 35.2%.
The observed 3 year frequency for policyholder 3 is 4 and the overall mean is 1.25.
The estimated future 3 year frequency for policyholder 3 is:
(35.2%)(4) + (1 - 35.2%)(1.25) = 2.22.
The estimated future annual frequency is: 2.22/3 = **0.739**.
Alternately, EPV = mean annual frequency = 10/24 = 5/12.
The individual mean annual frequencies are: 2/3, 0, 4/3, 1/3, 2/3, 0, 1/3, 0.
The sample variance of the class means is:
$\{3(0 - 5/12)^2 + 2(1/3 - 5/12)^2 + 2(2/3 - 5/12)^2 + (4/3 - 5/12)^2\} / (8 - 1) = 3/14$.
VHM = (the sample variance of the class means) - EPV / Y = 3/14 - (5/12)/3 = 19/252.
K = EPV/VHM = (5/12) / (19/252) = 105/19.
For 3 years of data, Z = 3/(3 + K) = 57/162 = 35.2%. Proceed as before.
Comment: Over three years, each insureds frequency is Poisson.

6.3. B. EPV = average of the sample variances = 0.5833.
VHM = sample variance of the means - EPV / (# years) = 0.2143 - 0.5833/3 = 0.0199.

Policyholder	Year			Mean	Sample Variance
	2001	2002	2003		
1	1	1	0	0.6667	0.3333
2	0	0	0	0.0000	0.0000
3	1	0	3	1.3333	2.3333
4	1	0	0	0.3333	0.3333
5	0	2	0	0.6667	1.3333
6	0	0	0	0.0000	0.0000
7	0	0	1	0.3333	0.3333
8	0	0	0	0.0000	0.0000
Average				0.4167	0.5833
Sample Variance				0.2143	

K = EPV/VHM = 0.5833/0.0199 = 29.3. For three years of data, Z = 3/(3 + 29.3) = 9.3%.
The observed annual frequency for policyholder 3 is 4/3 and the overall mean is 0.4167.
Estimated future annual freq. for policyholder 3 is: (9.3%)(4/3) + (1 - 9.3%)(0.4167) = **0.502**.

6.4. D. EPV = overall mean = 5/14.
VHM = $\{(1/7 - 5/14)^2 + (4/7 - 5/14)^2\}/(2 - 1)$ - EPV/(# years of data)
= 0.09184 - (5/14)/7 = 0.0408.
K = (5/14)/0.0408 = 8.75. Z = 7/(7 + 8.75) = 0.444
Estimated frequency for insured B: (0.444)(4/7) + (1 - 0.444)(5/14) = 0.452. (7)(0.452) = **3.16**.

6.5. C. Estimated EPV = overall mean = 4/16 = 1/4.

$\Pi = m - \sum m_i^2/m = 16 - (7^2 + 9^2)/16 = 7.875$. $\bar{X}_1 = 3/7$. $\bar{X}_2 = 1/9$.

$$\text{VHM} = \frac{\sum_{i=1}^{C} m_i (\bar{X}_i - \bar{X})^2 - \text{EPV}\,(C - 1)}{\Pi} =$$

$\{7(3/7 - 1/4)^2 + 9(1/9 - 1/4)^2 - (2 - 1)(1/4)\} / 7.875 = 0.01864$.
K = EPV/VHM = 0.25/0.01864 = 13.4. Z_A = 7/(7 + 13.4) = 0.343. Z_B = 9/(9 + 13.4) = **0.402**.
Comment: Similar to Exercise 20 in "Topics in Credibility" by Dean.
The fact that the insureds have different numbers of years of data, does not complicate the calculation. If we had not assumed a Poisson frequency, then the computation of the EPV would have had to take into account the differing number of years of data. .

6.6. A. Since we have assumed a Poisson frequency,
the estimated EPV = $\bar{X}$ = 15/1000 = 0.015.
$\Pi = m - \sum m_i^2 / m = 1000 - (100^2 + 200^2 + 400^2 + 300^2)/1000 = 700$.

$$\text{VHM} = \frac{\sum_{i=1}^{C} m_i (\bar{X}_i - \bar{X})^2 - \text{EPV}\,(C - 1)}{\Pi} =$$

$\{(100)(0.03 - 0.015)^2 + (200)(0.025 - 0.015)^2 + (300)(0.01 - 0.015)^2 + (400)(0.01 - 0.015)^2 - (4 - 1)(0.015)\}/700 = 0.00002143$.
K = EPV/VHM = 0.015/0.00002143 = 700.
Territory D has 300 exposures, and therefore Z = 300/(300 + 700) = **30%**.
Comment: Similar to 4, 11/05, Q.22 (2009 Sample Q.233).

6.7. B. Since we have assumed a Poisson frequency,
the estimated EPV = $\bar{X}$ = 28/1400 = 0.02.
$\Pi = m - \sum m_i^2 / m = 1400 - (400^2 + 300^2 + 500^2 + 200^2)/1400 = 1014$.

$$\text{VHM} = \frac{\sum_{i=1}^{C} m_i (\bar{X}_i - \bar{X})^2 - \text{EPV}\,(C - 1)}{\Pi} =$$

$\{(400)(0.01 - 0.02)^2 + (300)(0.03 - 0.02)^2 + (500)(0.026 - 0.02)^2 + (200)(0.01 - 0.02)^2 - (4 - 1)(0.02)\} / 1014 = 0.00004734$.
K = EPV/VHM = 0.02/0.00004734 = 422.5.
Territory C has 500 exposures, and therefore Z = 500 / (500 + 422.5) = **54.2%**.
Comment: Similar to 4, 11/05, Q.22 (2009 Sample Q.233).

6.8. D. Since the frequency is assumed to be Poisson,
estimated EPV = mean = (0 + 1 + 3)/(4 + 4 + 4) = 4/12 = 1/3.
$\bar{X}_1 = 0$. $\bar{X}_2 = 0.25$. $\bar{X}_3 = 0.75$.

Estimated VHM = $\{(0 - 1/3)^2 + (0.25 - 1/3)^2 + (0.75 - 1/3)^2\}/(3 - 1)$ - EPV/4
= 0.1458 - 1/12 = 0.0625.
K = (1/3)/0.0625 = 5.33. Z_1 = 4/(4 + 5.33) = 42.9%.
Estimated frequency for Insured C is: (42.9%)(0.75) + (1 - 42.9%)(1/3) = **0.512**.

6.9. D. Estimated EPV = overall mean = (9 + 19)/(450 + 600) = 28/1050 = 0.02667.

$\Pi = m - \sum m_i^2/m = 1050 - (450^2 + 600^2)/1050 = 514.3.$

$\bar{X}_1 = 9/450 = 0.02000.$ $\bar{X}_2 = 19/600 = 0.03167.$

$$\text{VHM} = \frac{\sum_{i=1}^{C} m_i (\bar{X}_i - \bar{X})^2 - \text{EPV}\,(C - 1)}{\Pi} =$$

$\{450(0.02000 - 0.02667)^2 + 600(0.03167 - 0.02667)^2 - (2 - 1)(0.02667)\} / 514.3 = 0.00001624.$
K = EPV/VHM = 0.02667/0.00001624 = 1642.
Z_A = 450/(450 + 1642) = 0.215. Z_B = 600/(600 + 1642) = **0.268**.
Comment: Using the method that preserves total claims:
Credibility weighted mean = {(0.215)(0.0200) + (0.268)(0.03167)} / (0.215 + 0.268) = 0.0265.
Estimated frequency for Class A is: (0.215)(0.0200) + (1 - 0.215)(0.0265) = 0.0251.

6.10. E. One can either assume that the number of holes-in-one is Poisson or Bernoulli; since the mean frequency per golfer is so small, there is no practical difference.
For convenience, take the unit of exposure as 1000 golfers.
Then, the overall observed mean frequency is: 33/58 = 56.9% = EPV.
The six observed frequencies are: 75%, 90%, 40%, 50%, 20%, and 100%.

$\Pi = \sum m_i - \sum m_i^2 / \sum m_i = 58 - (8^2 + 10^2 + 20^2 + 12^2 + 5^2 + 3^2) / 58 = 45.207.$

$\sum m_i (\bar{X}_i - \bar{X})^2 = \{(8)(75\% - 56.9\%)^2 + (10)(90\% - 56.9\%)^2 + (20)(40\% - 56.9\%)^2 +$
$(12)(50\% - 56.9\%)^2 + (5)(20\% - 56.9\%)^2 + (3)(100\% - 56.9\%)^2\} = 3.224.$

$$\text{VHM} = \frac{\sum_{i=1}^{C} m_i (\bar{X}_i - \bar{X})^2 - \text{EPV}\,(C - 1)}{\Pi} = \{3.224 - (0.569)(6 - 1)\} / 45.207 = 0.00838.$$

K = EPV/VHM = 0.569/0.00838 = 67.9.
The data for Garden City had 5000 golfers, or 5 exposure units. Z = 5/(5 + 67.9) = 6.9%.
Estimated future frequency for Garden city is: (6.9%)(20%) + (1 - 6.9%)(56.9%) = 54.4%.
The given tournament has 200 golfers, or 0.2 exposures.
The expected number of holes-in-one is: (0.2)(54.4%) = 0.109.
Expected payment is: ($100,000)(0.109) = **$10,900**.
Comment: Here I did not use the method that preserves total claims since I was not told to do so.

6.11. A. Since we have assumed a Poisson frequency, the estimated EPV = $\bar{X}$ = 12/60 = 1/5.

$\Pi = m - \sum m_i^2 / m = 60 - (10^2 + 20^2 + 30^2)/60 = 36.67$.

$$\text{VHM} = \frac{\sum_{i=1}^{C} m_i (\bar{X}_i - \bar{X})^2 - \text{EPV}\,(C-1)}{\Pi} =$$

$\{(10)(0.4 - 0.2)^2 + (20)(0.25 - 0.2)^2 + (30)(0.1 - 0.2)^2 - (0.2)(3 - 1)\} / 36.67 = 0.00954$.
K = EPV/VHM = 0.20/0.00954 = 21.0.
Territory A has 10 exposures, and therefore Z = 10/(10 + 21) = **32.3%**.
Comment: One has to use a combination of semiparametric and empirical bayes estimation.

6.12. C. Since the frequency is assumed to be Poisson,
estimated EPV = mean = (1 + 1 + 0 + 2 + 3)/(2 + 2 + 1 + 3 + 2) = 7/10 = 0.7.
$\bar{X}_1$ = (1 + 1 + 0)/(2 + 2 + 1) = 0.4. $\bar{X}_2$ = (2 + 3)/(3 + 2) = 1.

Estimated VHM = $\{5(0.4 - 0.7)^2 + 5(1 - 0.7)^2 - (2 - 1)(0.7)\} / \{10 - (5^2 + 5^2)/10\} = 0.2/5 = 0.04$.
K = 0.70/0.04 = 17.5. Z_1 = 5/(5 + 17.5) = 0.222.
Estimated frequency for A is: (0.222)(0.4) + (1 - 0.222)(0.7) = **0.633**.
Comment: Notice the Poisson assumption in this past exam question. Thus one uses a combination of semiparametric and empirical bayes estimation.
In this past exam question, we were not told to use the balancing method that uses the credibility weighted average, and the sample solution did not use it.

5, Q.4.6:
A. less than 0.20
B. at least 0.20 but less than 0.22
C. at least 0.22 but less than 0.24
D. at least 0.24 but less than 0.26
E. at least 0.26