

Read the portion of my second study guide on Bayes Analysis prior to the McElreath textbook.

1, page 6:

1 - Z is sometimes referred to as the “complement of credibility”.

Confusingly, often instead what I have denoted Y, the item that is given weight 1 - Z, is referred to as the “complement of credibility”.

Tse in Nonlife Actuarial Models: Theory Methods and Evaluation does not use the term “complement of credibility”. Weight 1 - Z is given to M = manual rate.

2, page 208, last line: = **0.4259**.

2, page 306, 5th line from the bottom:  $(300)^2(0.21)$

3, solution 5.1:  $f(x) = \{(a+b-1)! / (a-1)! (b-1)!\} (x/\theta)^{a-1} \{1 - (x/\theta)\}^{b-1} / \theta =$

3, page 275, solutions 8.16 and 8.17 are missing:

**8.16. E.** A Gamma-Exponential with prior Gamma with  $\alpha = 1$  and  $\theta = 1/100$ .

Thus the posterior Gamma has parameters  $\alpha = 1 + 2 = 3$ , and  $1/\theta = 100 + 40 + 80 = 220$ .

$E[1/\lambda]$  = negative first moment of the posterior Gamma =  $220/(3-1) = \mathbf{110}$ .

Alternately, the posterior distribution is proportional to:  $e^{-100\lambda} \lambda e^{-40\lambda} \lambda e^{-80\lambda} = \lambda^2 e^{-220\lambda}$ .

$E[X | \lambda] = 1/\lambda$ . Therefore, the expected size of the next claim is:

$$\frac{\int_0^{\infty} (1/\lambda) \lambda^2 e^{-220\lambda} d\lambda}{\int_0^{\infty} \lambda^2 e^{-220\lambda} d\lambda} = \frac{220^2 \Gamma(2)}{220^3 \Gamma(3)} = 220/2 = \mathbf{110}.$$

Comment: Since  $\alpha = 1 \leq 2$ , one can not apply Buhlmann Credibility.

The posterior mixed distribution is Pareto with  $\alpha = 3$  and  $\theta = 220$ , with mean  $220/(3-1) = 110$ .

**8.17. D.** A Gamma-Exponential.

Thus the posterior Gamma has parameters  $\alpha = 4 + 3 = 7$ ,

and  $1/\theta = 1000 + 100 + 200 + 500 = 1800$ .

$E[1/\lambda]$  = negative first moment of the posterior Gamma =  $1800/(7-1) = \mathbf{300}$ .

Alternately,  $K = \alpha - 1 = 4 - 1 = 3$ .  $Z = (3/(3+K)) = 3/(3+3) = 50\%$ .

The prior mean is the negative first moment of the prior Gamma =  $1000/(4-1) = 1000/3$ .

Estimate of the future severity is:  $(50\%)(800/3) + (50\%)(1000/3) = \mathbf{300}$ .

Comment: The posterior mixed distribution is Pareto with  $\alpha = 7$  and  $\theta = 1800$ , with mean  $1800/(7-1) = 300$ .

5, Q. 2.10: Model III: K-Nearest Neighbors with  $K = 5$ .

5, solution. 8.16: Final solution is okay.

$Z_1$  and  $Z_3$  are uncorrelated. It turns out that constraining  $Z_3$  to be uncorrelated with  $Z_1$  is equivalent to constraining the direction  $\phi_3$  to be orthogonal (perpendicular)

to the direction  $\phi_1$ . Thus,  $\sum_{j=1}^5 \phi_{j1} \phi_{j3} = 0$ . Thus Statement III is true.

Comment:  $Z_1$ ,  $Z_2$ , and  $Z_3$  are each orthogonal to the others.