# CAS Exam MAS-2

## Seminar Style Slides
## 2026 Edition

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These are slides that I would present at a seminar or weekly class.

Covered is everything on the syllabus of Exam MAS-2.

**Use the bookmarks in the Navigation Panel in order to help you find what you want.**

This provides another way to study the material. Some of you will find it helpful to go through a few sections at a time, either alone or with a someone else, pausing to do each of the problems included. Estimated time to go through all of the slides is about 30 hours.

Some of the problems were written by me and some are past exam questions so labeled.

All the material, problems, and solutions are in my study guides, sold separately.
These slides are a useful supplement to my study guides, but are self-contained.
There are references to page and problem numbers in the latest edition of my study guides, which you can ignore if you do not have my study guides.

Mahler's Guides for Exam MAS-2 are listed below, along with my estimated portion of questions per exam. This is the order in which they appear in these slides.

## Study Guides for Exam MAS-2

| # | Study Guide Name | |
|---|---|---|
| 1 | Mahler's Guide to Classical Credibility | 2% |
| 2 | Mahler's Guide to Buhlmann Credibility & Bayesian Analysis | 10% |
| 3 | Mahler's Guide to Conjugate Priors | 5% |
| 4 | Mahler's Guide to Nonparametric Credibility | 3% |
| 5 | Mahler's Guide to Advanced Statistical Learning | 35% |
| 6 | Mahler's Guide to Advanced GLMs | 10% |
| 7 | Mahler's Guide to Time Series | 20% |
| 8 | Mahler's Guide to Linear Mixed Models | 15% |

In each case, the slides are in the same order as the sections of my study guide.
At the end, there are some additional questions for study.
After the slides, is my section of important ideas and formulas.

Author Biography:

Howard C. Mahler is a Fellow of the Casualty Actuarial Society,
and a Member of the American Academy of Actuaries.

He has published study guides since 1996.
He taught live seminars and/or classes for many different actuarial exams from 1994 to 2017.

He spent over 20 years in the insurance industry, the last 15 as Vice President and Actuary at the Workers' Compensation Rating and Inspection Bureau of Massachusetts.
He has published many major research papers and won the 1987 CAS Dorweiler prize.
He served 12 years on the CAS Examination Committee including three years as head of the whole committee (1990-1993).

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Exam MAS-2

Howard Mahler

# Classical Credibility

The slides are in the same order as the sections of my study guide.

| Section # | Section Name |
|---|---|
| | |
| | Normal Distribution Table |
| | |
| 1 | Introduction |
| 2 | Full Credibility for Frequency |
| 3 | Full Credibility for Severity |
| 4 | *Variance of Pure Premiums & Aggregate Losses* |
| | |
| 5 | **Full Credibility for Pure Premiums & Aggregate Losses** |
| 6 | **Partial Credibility** |
| 7 | Important Formulas and Ideas |

At the end, there are some additional questions for study.
After the slides are my section of important ideas and formulas.

The estimate using credibility = **$Z X$ + $(1 - Z) Y$**,

**where Z is the credibility assigned to the observation X.**

**$0 \le Z \le 1$.**

# Standard Normal Distribution

| $F(z) = \Pr(Z < z)$ | z |
|---|---|
| 0.800 | 0.842 |
| 0.850 | 1.036 |
| 0.900 | 1.282 |
| **0.950** | **1.645** |
| **0.975** | **1.960** |
| 0.990 | 2.326 |
| 0.995 | 2.576 |

# Section 2, Full Credibility for Frequency

**If we have an amount of data ≥**
**the "Standard for Full Credibility",**
**then Z = 1.**

**If we have less data than**
**the Standard for Full Credibility,**
**then Z < 1.**

We will determine the Standard for Full Credibility by requiring that there be a large probability of a small estimation error.

**Assume one desires that the chance of being within ± k of the mean frequency to be at least 1 - α,**

**then for a Poisson Frequency,**

**the Standard for Full Credibility is:**

$$\lambda_F = \left(\frac{z_{1-\alpha/2}}{k}\right)^2,$$

**where $z_{1-\alpha/2}$ is such that $\Phi(z_{1-\alpha/2}) = 1 - \alpha/2$.**

α =10%. ⇔ y = 1.645.

α = 5%. ⇔ y = 1.960.

Assuming frequency is Poisson,
for $\alpha = 5\%$ and for $k = 5\%$,
what is the number of claims required
for Full Credibility for estimating the frequency?

$z_{1-\alpha/2} = 1.960$
since $\Phi(1.960) = 1 - 0.05/2 = 97.5\%$.

$$\lambda_F = \left(\frac{z_{1-\alpha/2}}{k}\right)^2 = (1.96/0.05)^2 = 1537 \text{ claims.}$$

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The **Standard for Full Credibility for Frequency in terms of <u>claims</u> is:**

$$\left(\frac{z_{1-\alpha/2}}{k}\right)^2 \frac{\sigma_f^2}{\mu_f} = \lambda_F \frac{\sigma_f^2}{\mu_f}.$$

Find the number of claims required for full credibility.
Require that there is a 90% chance that the estimate of the frequency is correct within ±2.5%.
The frequency distribution has
a variance twice its mean.

$\alpha$ = 1 - 90% = 10% and $z_{1-\alpha/2}$ = 1.645. k = 2.5%.

$$\lambda_F = \left(\frac{z_{1-\alpha/2}}{k}\right)^2 = (1.645 / 0.025)^2 = 4330 \text{ claims.}$$

We are given that $\frac{\sigma_f^2}{\mu_f} = 2$.

Thus $\lambda_F \frac{\sigma_f^2}{\mu_f} = (4330)(2) = 8660$ claims.

Standards for full credibility can be expressed either in terms of number of (expected) claims or number of exposures (insureds, policies, etc.)

# of claims / mean frequency = # of expos.

(# of expos.) (mean frequency) = # of claims

**2.16. 4B, 11/94, Q.15.** You are given the following:
Y represents the number of independent homogeneous exposures in an insurance portfolio.
The claim frequency rate per exposure is a random variable with mean = 0.025 and variance = 0.0025.
A full credibility standard is devised that requires the observed sample frequency rate per exposure to be within 5% of the expected population frequency rate per exposure 90% of the time.
Determine the value of Y needed to produce full credibility for the portfolio's experience.

**4B, 11/94, Q.15. D.**

$\Phi(1.645) = 0.95 \Rightarrow z_{1-\alpha/2} = 1.645.$

$$\lambda_F = \left(\frac{z_{1-\alpha/2}}{k}\right)^2 = (1.645/0.05)^2 = 1082 \text{ claims.}$$

$$\lambda_F \frac{\sigma_f^2}{\mu_f} = (1082)(0.0025/0.025) = 108.2 \text{ claims.}$$

108.2 claims / 0.025 = **4328 exposures**.

Alternately, the standard for full credibility for frequency in terms of number of exposures is:

$$\left(\frac{z_{1-\alpha/2}}{k}\right)^2 \frac{\sigma_f^2}{\mu_f^2} = (1.645/0.05)^2\ (0.0025\ /0.025^2)$$

= (1082) (4) = **4328 exposures**.

# Section 3

The Standard for Full Credibility for Severity is in terms of claims: ${CV_{Sev}}^2 \lambda_F$.

**3.2.** You are given the following:

- The claim amount distribution is LogNormal, with $\sigma = 1.5$.
- Frequency and severity are independent.

Find the number of claims required for full credibility, if you require that there will be a 95% chance that the estimate of the <u>severity</u> is correct within ±10%.

A. Less than 2900
B. At least 2900, but less than 3000
C. At least 3000, but less than 3100
D. At least 3100, but less than 3200
E. At least 3200

**3.2. E.** $z_{1-\alpha/2} = 1.960$ since $\Phi(1.960) = 0.975$.

$$\lambda_F = \left(\frac{z_{1-\alpha/2}}{k}\right)^2 = (1.960/0.1)^2 = 384.$$

For the LogNormal Distribution:
Mean = $\exp(\mu + 0.5\,\sigma^2)$,
Variance = $\exp(2\mu + \sigma^2)\,\{\exp(\sigma^2) - 1\}$.
$\Rightarrow$ Coefficient of Variation = $\sqrt{\exp(\sigma^2) - 1}$.

For $\sigma = 1.5$, the $CV^2 = \exp(2.25) - 1 = 8.49$.
For severity, the Standard For Full Credibility is:
$\lambda_F\, CV^2 = (384)(8.49) =$ **3260** claims.

Alternately, $1 + CV^2 = E[X^2]/E[X]^2$
$= \exp(2\mu + 2\sigma^2) / \exp(2\mu + \sigma^2) = \exp(\sigma^2)$.
Proceed as before.

## *Section 4, Variance of Pure Premiums and Aggregate Losses*

The **Aggregate Loss** is the total dollars of loss for an insured or set of an insureds. If not stated otherwise, the period of time is one year.

**Aggregate Losses =**
**(Exposures) (Frequency) (Severity) =**
**(Expos.) (# Claims / Expos.) ($ Loss / # Claims).**

If one is not given the frequency per exposure, but is rather just given the frequency for the whole number of exposures, whatever they are for the particular situation, then:

**Aggregate Losses = (Frequency) (Severity).**

# Pure Premium = Aggregate Loss per expos.

Mean pure premium is:
(mean frequency per exposure) (mean severity).

Expected Aggregate Loss =
(Mean Pure Premium) (Exposure).

Frequency has a mean 5% of per exposure
and a variance of 0.06.
Severity has mean of 10 and variance of 200.

Mean pure premium = $\mu_{Freq}\ \mu_{Sev}$
= (5%)(10) = 0.5.

Variance of pure premium =

$\mu_{Freq}\ \sigma_{Sev}^2 + \mu_{Sev}^2\ \sigma_{Freq}^2$ =

$(0.05)\ (200) + (10^2)\ (0.06) = 16.$

There are two types of risks.
For each type of risk, the frequency and severity are independent.

| Type | Frequency Distribution | Severity Distribution |
|---|---|---|
| 1 | Poisson: $\lambda$ = 4% | Gamma: $\alpha = 3$, $\theta = 10$ |
| 2 | Poisson: $\lambda$ = 6% | Gamma: $\alpha = 3$, $\theta = 15$ |

**4.6.** Calculate the Process Variance of the pure premium for Type 1.

$\mu_{freq} = \sigma_{freq}^2 = \lambda = 0.04.$

$\mu_{sev} = \alpha\theta = 30.$ $\sigma_{sev}^2 = \alpha\theta^2 = 300.$

$\sigma_{PP}^2 = \mu_{freq}\ \sigma_{sev}^2 + \mu_{sev}^2\ \sigma_{freq}^2$

$= (0.04)\ (300) + (30^2)\ (0.04) = \mathbf{48}.$

| Type | Frequency Distribution | Severity Distribution |
|---|---|---|
| 1 | Poisson: $\lambda$ = 4% | Gamma: $\alpha = 3$, $\theta = 10$ |
| 2 | Poisson: $\lambda$ = 6% | Gamma: $\alpha = 3$, $\theta = 15$ |

**4.7.** Calculate the Process Variance of the pure premium for Type 2.

$\sigma_{PP}^2 = \lambda$ (second moment of severity)

$= (0.06)\ \{\alpha(\alpha+1)\theta^2\} = (0.06)\ (3)\ (4)\ (15^2) =$ **162**.

Process Variance Type 1: 48
Process Variance Type 2: 162

**4.8.** Assume one has a portfolio made up of 80% risks of Type 1, and 20% risks of Type 2. For this portfolio, what is the expected value of the process variance of the pure premium?

EPV = (80%) (48) + (20%) (162) = **70.8**.

The Expected Value of the Process Variance will be used in Buhlmann Credibility.

# Section 5

The Standard for Full Credibility for Aggregate Loss or Pure Premiums is in terms of claims:

$$(\sigma_{freq}^2 / \mu_{freq} + CV_{Sev}^2)\, \lambda_F.$$

If frequency is Poisson this is: $(1 + CV_{Sev}^2)\ \lambda_F$.

Note: $1 + CV^2 = E[X^2] / E[X]^2$.

Standard for Full Credibility for Aggregate Loss or Pure Premiums in terms of exposures:

$\lambda_F$ (coefficient of variation of the pure premium)$^2$.

**5.65. 4, 11/00, Q.14.**

For an insurance portfolio, you are given:

(i) For each individual insured, the number of claims follows a Poisson distribution.

(ii) The mean claim count varies by insured, and the distribution of mean claim counts follows a gamma distribution.

(iii) For a random sample of 1000 insureds, the observed claim counts are as follows:

| Number Of Claims, n | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| Number Of Insureds, $f_n$ | 512 | 307 | 123 | 41 | 11 | 6 |

$\sum n\ f_n = 750$, $\sum n^2\ f_n = 1494$.

(iv) Claim sizes follow a Pareto distribution with mean 1500 and variance 6,750,000.

(v) Claim sizes and claim counts are independent.

(vi) The full credibility standard is to be within 5% of the expected aggregate loss 95% of the time.

Determine the minimum number of insureds needed for the aggregate loss to be fully credible.

**4, 11/00, Q.14. E.**

The mean frequency is: 750 / 1000 = 0.75.
2nd moment of frequency: 1494 / 1000 = 1.494.
estimated variance: $1494/1000 - 0.75^2 = 0.9315$.

$CV_{Sev}^2 = 6{,}750{,}000 / 1500^2 = 3$.

k = 5%. $\alpha$ = 5%. $z_{1-\alpha/2} = 1.960$.

$$\lambda_F = \left(\frac{z_{1-\alpha/2}}{k}\right)^2 = 1.960^2 / 0.05^2 = 1537 \text{ claims.}$$

Standard for full cred. = $\lambda_F (\sigma_F^2 / \mu_F + CV_{Sev}^2)$ =
(1537) (0.9315 / 0.75 + 3) = 6520 claims,
which corresponds to:
6520 / 0.75 = **8693** exposures.

If you use instead the unbiased estimator of the variance: $(1000 / 999) (1.494 - 0.75^2) = 0.9324$.
Standard for full cred. = (1537) (0.9324 / 0.75 + 3) = 6522 claims. ⇔ 6522 / 0.75 = **8696** exposures.

# Section 6, Partial Credibility

**The estimate using credibility =**
**Z X + (1 - Z) Y, where Z is the credibility assigned to the observation X.**

If we have enough data,
then we assign full credibility; Z = 100%.

Otherwise we assign partial credibility using the square root rule:

$$Z = \sqrt{\frac{\text{amount of data}}{\text{standard for full credibility}}}, \ 0 \le Z \le 1.$$

The numerator and denominator have to be in the same units.

Either claims / claims or exposures / exposures.

**6.45. 4, 11/01, Q.15**

You are given the following information about a general liability book of business comprised of 2500 insureds:

(i) $X_i = \sum_{j=1}^{N_i} Y_{ij}$ is a random variable representing the annual loss of the $i^{th}$ insured.

(ii) $N_1, N_2, \ldots, N_{2500}$ are independent and identically distributed random variables following a negative binomial distribution with parameters r = 2 and $\beta$ = 0.2.

(iii) $Y_{i1}, Y_{i2}, \ldots, Y_{iN_i}$ are independent and identically distributed random variables following a Pareto distribution with $\alpha$ = 3.0 and $\theta$ = 1000.

(iv) The full credibility standard is to be within 5% of the expected aggregate losses 90% of the time.

Using classical credibility theory, determine the partial credibility of the annual loss experience for this book of business.

**4, 11/01, Q.15. C.** k = 0.05. $\alpha$ = 10%.

$z_{1-\alpha/2}$ = 1.645. $\lambda_F = (1.645 / 0.05)^2$ = 1082 claims.

For the Negative Binomial, $\mu_{freq}$ = (2) (0.2) = 0.4.

$\sigma_{freq}^2$ = (2) (0.2) (1.2). $\sigma_{freq}^2 / \mu_{freq}$ = 1.2.

For the Pareto, E[X] = 1000 / (3 - 1) = 500.

$$E[X^2] = \frac{(2)\ 1000^2}{(3 - 1)\ (3 - 2)} = 1{,}000{,}000.$$

$CV_{Sev}^2 = E[X^2] / E[X]^2 - 1 = 1{,}000{,}000 / 500^2 - 1$

= 4 - 1 = 3.

Standard for Full Credibility =

$(\sigma_{freq}^2 / \mu_{freq} + CV_{Sev}^2)\ \lambda_F$ = (1.2 + 3) (1082)

= 4546 claims.

2500 exposures ⇔

(2500) (0.4) = 1000 expected claims.

$Z = \sqrt{1000\ /\ 4546}$ = **47%**.

**6.44. 4, 5/00, Q.26.** You are given:
(i) Claim counts follow a Poisson distribution.
(ii) Claim sizes follow a lognormal distribution with coefficient of variation 3.
(iii) Claim sizes and claim counts are independent.
(iv) The number of claims in the first year was 1000.
(v) The aggregate loss in the first year was 6.75 million.
(vi) In the first year, the provision in the premium in order to pay losses was 5.00 million.
(vii) The exposure in the second year is identical to the exposure in the first year.
(viii) The full credibility standard is to be within 5% of the expected aggregate loss 95% of the time.
Determine the classical credibility estimate of losses (in millions) for the second year.

**4, 5/00, Q.26. A.** $\alpha = 0.05$.

$\Phi(1.960) = 0.975$, so that $z_{1-\alpha/2} = 1.960$.

$$\lambda_F = \left(\frac{z_{1-\alpha/2}}{k}\right)^2 = (1.960 / 0.05)^2 = 1537 \text{ claims.}$$

Standard for full credibility for aggregate loss =

$\lambda_F (1 + CV_{Sev}^2) = 1537 (1 + 3^2) = 15,370$ claims.

$$Z = \sqrt{\frac{1000}{15,370}} = 25.5\%.$$

The prior estimate is given as \$5 million.

The observation is given as \$6.75 million.

Thus the new estimate is:

(25.5%) (6.75) + (1 - 25.5%) (5) = **\$5.45 million**.

# Additional Questions

**5.23** (3 points) You are given the following:

- Claim sizes follow a gamma distribution, with parameters $\alpha = 2.5$ and $\theta$ unknown.
- The number of claims and claim sizes are independent.
- The full credibility standard for <u>frequency</u> has been selected so that the actual number of claims will be within 2.5% of the expected number of claims P of the time.
- The full credibility standard for <u>aggregate loss</u> has been selected so that the actual aggregate losses will be within 2.5% of the expected actual aggregate losses P of the time, using the same P as for the standard for frequency.
- 13,801 expected claims are needed for full credibility for frequency.
- 18,047 expected claims are needed for full credibility for aggregate loss.

Using the methods of Classical credibility, determine the value of P.

A. 80% B. 90% C. 95% D. 98% E. 99%

**5.23. E.** $13{,}800 = (\sigma_f^2 / \mu_f)\, \lambda_F$.

$18{,}050 = (\sigma_f^2 / \mu_f + CV_{Sev}^2)\, \lambda_F$.

Subtracting the first equation from the second:
$CV_{Sev}^2\, \lambda_F = 4246$.

For the Gamma: $CV^2$ = variance / mean$^2$
$= \alpha\theta^2 / (\alpha\theta)^2 = 1/\alpha = 1/2.5 = 0.4$.

$\Rightarrow \lambda_F = 4246/0.4 = 10{,}615 = \left(\frac{z_{1-\alpha/2}}{k}\right)^2$.

$\Rightarrow z_{1-\alpha/2} / k = 103.03$.
$\Rightarrow z_{1-\alpha/2} = (103.03)(2.5\%) = 2.576$.

$\Rightarrow 99.5\% = \Phi[z_{1-\alpha/2}]$.
$\Rightarrow \alpha = 1\%$. P = 1 - 1% = **99%**.

## Important Formulas and Ideas

**The estimate using credibility =**
**ZX + (1-Z)Y, where Z is the credibility assigned to the observation X.**

**new estimate = (observation) (Z) + (old estimate) (1-Z)**
**= (observation) (Z) + (manual rate) (1-Z)**.

Full Credibility (Sections 2, 3, and 5):

**Assume one desires that the chance of being within ± k of the mean frequency to be at least 1 - α, then for a Poisson Frequency, the Standard for Full Credibility is:**

$\lambda_F = \left(\frac{z_{1-\alpha/2}}{k}\right)^2$**, where** $z_{1-\alpha/2}$ **is such that** $\Phi(z_{1-\alpha/2}) = 1 - \alpha/2$..

The Standard for Full Credibility for Frequency is in terms of claims: $\frac{\sigma_f^2}{\mu_f}\ \lambda_F$.

In the Poisson case this is: $\lambda_F$.

The Standard for Full Credibility for Severity is in terms of claims: $CV_{Sev}^2\ \lambda_F$.

**The Standard for Full Credibility for either Pure Premiums or Aggregate Losses is in terms of claims:** $\left(\frac{\sigma_f^2}{\mu_f} + CV_{Sev}^2\right) \lambda_F$ . In the Poisson case this is: $(1 + CV_{Sev}^2)\ \lambda_F$.

$1 + CV^2 = \frac{E[X^2]}{E[X]^2}$ .

The standard can be put in terms of exposures rather than claims by dividing by $\mu_f$.

Variance of Pure Premiums and Aggregate Losses (Section 4):

Aggregate Losses =

$$(\text{\# of Exposures}) \frac{\text{\# of Claims}}{\text{\# of Exposures}} \frac{\text{\$ of Loss}}{\text{\# of Claims}} = (\text{Exposures}) (\text{Frequency}) (\text{Severity}).$$

$$\text{Pure Premiums} = \frac{\text{\$ of Loss}}{\text{\# of Exposures}} = \frac{\text{\# of Claims}}{\text{\# of Exposures}} \frac{\text{\$ of Loss}}{\text{\# of Claims}} = (\text{Frequency})(\text{Severity}).$$

When frequency and severity are independent: $\sigma_{PP}^2 = \mu_{Freq}\ \sigma_{Sev}^2 + \mu_{Sev}^2\ \sigma_{Freq}^2$.

$\sigma_{Agg}^2 = \mu_{Freq}\ \sigma_{Sev}^2 + \mu_{Ssev}^2\ \sigma_{Freq}^2$.

With a Poisson Frequency, the variance of aggregate losses is:

$\lambda$ (2nd moment of severity).

Partial Credibility (Section 6):

**When one has at least the number of claims needed for Full Credibility, then one assigns 100% credibility to the observations.**

**Otherwise use the square root rule:**

$$Z = \sqrt{\frac{\text{number of claims}}{\text{standard for full credibility in terms of claims}}}, \text{ or}$$

$$Z = \sqrt{\frac{\text{number of exposures}}{\text{standard for full credibility in terms of exposures}}}.$$

When available, one generally uses the number of exposures or the expected number of claims in the square root rule, rather than the observed number of claims.

Make sure that in the square root rule you divide comparable quantities; either divide claims by claims or divide exposures by exposures.