# Volume One

Mahler's Guide to
# Classical Credibility

## CAS Exam MAS-2

prepared by
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Study Aid 2024-MAS2-**1**

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# Mahler's Guide to Classical Credibility



The concepts in Chapter 6 of Nonlife Actuarial Models: Theory Methods and Evaluation, by Yiu-Kuen Tse.[1]

This material has been on the syllabus for decades, but prior to MAS-2 out of other material.

Information in bold or sections whose title is in bold are more important for passing the exam. Larger bold type indicates it is extremely important. Information presented in italics (and sections whose title is in italics) should not be needed to directly answer exam questions and should be skipped on first reading. It is provided to aid the reader's overall understanding of the subject, and to be useful in practical applications.

Highly Recommended problems are double underlined.
Recommended problems are underlined.

Solutions to the problems in each section are at the end of that section.
Note that problems include both some written by me and some from past exams.[2] The latter are copyright by the Casualty Actuarial Society and the Society of Actuaries and are reproduced here solely to aid students in studying for exams.[3]



[1] Only Sections 6.1-6.3 are on the syllabus.
[2] In some cases I've rewritten these questions in order to match the notation in the current Syllabus.
[3] The solutions and comments are solely the responsibility of the author; the CAS/SOA bear no responsibility for their accuracy. While some of the comments may seem critical of certain questions, this is intended solely to aid you in studying and in no way is intended as a criticism of the many volunteers who work extremely long and hard to produce quality exams.

## Course 4 Exam Questions by Section of this Study Aid[4] [5]

| Section | Sample | 5/00 | 11/00 | 5/01 | 11/01 | 11/02 | 11/03 | 11/04 |
|---|---|---|---|---|---|---|---|---|
| | | | | | | | | |
| 1 | | | | | | | | |
| 2 | | | | | | | | 21 |
| 3 | | | | | | | | |
| 4 | | | | | | 36 | | |
| 5 | 15 | | 14 | | | 14 | 3 | |
| 6 | | 26 | | | 15 | | 35 | |

| Section | 5/05 | 11/05 | 11/06 | 5/07 |
|---|---|---|---|---|
| | | | | |
| 1 | | | | |
| 2 | | | | |
| 3 | | | | |
| 4 | | | | |
| 5 | 2 | 35 | 30 | |
| 6 | | | | |

The CAS/SOA did not release the 5/02, 5/03, 5/04, 5/06, 11/07 and subsequent exams.

## MAS-2 Exam Questions by Section of this Study Aid

| Section | Sample | 11/18 | 5/19 | 11/19 |
|---|---|---|---|---|
| | | | | |
| 1 | | | | |
| 2 | | | | 4 |
| 3 | | | | |
| 4 | | | | |
| 5 | | | 3 | |
| 6 | | 6 | 6 | |

There was no Spring 2020 exam. The CAS did not release the 11/20 and subsequent exams.

[4] Excluding any questions that are no longer on the syllabus.
[5] This material currently on Exam MAS-2 was formerly covered on Exam 4/C,

**Normal Distribution Table**

Entries represent the area under the standardized normal distribution from -∞ to z, Pr(Z < z).
The value of z to the first decimal place is given in the left column.
The second decimal is given in the top row.

| z | 0.00 | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0.0 | 0.5000 | 0.5040 | 0.5080 | 0.5120 | 0.5160 | 0.5199 | 0.5239 | 0.5279 | 0.5319 | 0.5359 |
| 0.1 | 0.5398 | 0.5438 | 0.5478 | 0.5517 | 0.5557 | 0.5596 | 0.5636 | 0.5675 | 0.5714 | 0.5753 |
| 0.2 | 0.5793 | 0.5832 | 0.5871 | 0.5910 | 0.5948 | 0.5987 | 0.6026 | 0.6064 | 0.6103 | 0.6141 |
| 0.3 | 0.6179 | 0.6217 | 0.6255 | 0.6293 | 0.6331 | 0.6368 | 0.6406 | 0.6443 | 0.6480 | 0.6517 |
| 0.4 | 0.6554 | 0.6591 | 0.6628 | 0.6664 | 0.6700 | 0.6736 | 0.6772 | 0.6808 | 0.6844 | 0.6879 |
| 0.5 | 0.6915 | 0.6950 | 0.6985 | 0.7019 | 0.7054 | 0.7088 | 0.7123 | 0.7157 | 0.7190 | 0.7224 |
| 0.6 | 0.7257 | 0.7291 | 0.7324 | 0.7357 | 0.7389 | 0.7422 | 0.7454 | 0.7486 | 0.7517 | 0.7549 |
| 0.7 | 0.7580 | 0.7611 | 0.7642 | 0.7673 | 0.7704 | 0.7734 | 0.7764 | 0.7794 | 0.7823 | 0.7852 |
| 0.8 | 0.7881 | 0.7910 | 0.7939 | 0.7967 | 0.7995 | 0.8023 | 0.8051 | 0.8078 | 0.8106 | 0.8133 |
| 0.9 | 0.8159 | 0.8186 | 0.8212 | 0.8238 | 0.8264 | 0.8289 | 0.8315 | 0.8340 | 0.8365 | 0.8389 |
| 1.0 | 0.8413 | 0.8438 | 0.8461 | 0.8485 | 0.8508 | 0.8531 | 0.8554 | 0.8577 | 0.8599 | 0.8621 |
| 1.1 | 0.8643 | 0.8665 | 0.8686 | 0.8708 | 0.8729 | 0.8749 | 0.8770 | 0.8790 | 0.8810 | 0.8830 |
| 1.2 | 0.8849 | 0.8869 | 0.8888 | 0.8907 | 0.8925 | 0.8944 | 0.8962 | 0.8980 | 0.8997 | 0.9015 |
| 1.3 | 0.9032 | 0.9049 | 0.9066 | 0.9082 | 0.9099 | 0.9115 | 0.9131 | 0.9147 | 0.9162 | 0.9177 |
| 1.4 | 0.9192 | 0.9207 | 0.9222 | 0.9236 | 0.9251 | 0.9265 | 0.9279 | 0.9292 | 0.9306 | 0.9319 |
| 1.5 | 0.9332 | 0.9345 | 0.9357 | 0.9370 | 0.9382 | 0.9394 | 0.9406 | 0.9418 | 0.9429 | 0.9441 |
| 1.6 | 0.9452 | 0.9463 | 0.9474 | 0.9484 | 0.9495 | 0.9505 | 0.9515 | 0.9525 | 0.9535 | 0.9545 |
| 1.7 | 0.9554 | 0.9564 | 0.9573 | 0.9582 | 0.9591 | 0.9599 | 0.9608 | 0.9616 | 0.9625 | 0.9633 |
| 1.8 | 0.9641 | 0.9649 | 0.9656 | 0.9664 | 0.9671 | 0.9678 | 0.9686 | 0.9693 | 0.9699 | 0.9706 |
| 1.9 | 0.9713 | 0.9719 | 0.9726 | 0.9732 | 0.9738 | 0.9744 | 0.9750 | 0.9756 | 0.9761 | 0.9767 |

Table continued on the next page

Entries represent the area under the standardized normal distribution from -∞ to z, Pr(Z < z).
The value of z to the first decimal place is given in the left column.
The second decimal is given in the top row.

| z | 0.00 | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 |
|---|---|---|---|---|---|---|---|---|---|---|
| 2.0 | 0.9772 | 0.9778 | 0.9783 | 0.9788 | 0.9793 | 0.9798 | 0.9803 | 0.9808 | 0.9812 | 0.9817 |
| 2.1 | 0.9821 | 0.9826 | 0.9830 | 0.9834 | 0.9838 | 0.9842 | 0.9846 | 0.9850 | 0.9854 | 0.9857 |
| 2.2 | 0.9861 | 0.9864 | 0.9868 | 0.9871 | 0.9875 | 0.9878 | 0.9881 | 0.9884 | 0.9887 | 0.9890 |
| 2.3 | 0.9893 | 0.9896 | 0.9898 | 0.9901 | 0.9904 | 0.9906 | 0.9909 | 0.9911 | 0.9913 | 0.9916 |
| 2.4 | 0.9918 | 0.9920 | 0.9922 | 0.9925 | 0.9927 | 0.9929 | 0.9931 | 0.9932 | 0.9934 | 0.9936 |
| 2.5 | 0.9938 | 0.9940 | 0.9941 | 0.9943 | 0.9945 | 0.9946 | 0.9948 | 0.9949 | 0.9951 | 0.9952 |
| 2.6 | 0.9953 | 0.9955 | 0.9956 | 0.9957 | 0.9959 | 0.9960 | 0.9961 | 0.9962 | 0.9963 | 0.9964 |
| 2.7 | 0.9965 | 0.9966 | 0.9967 | 0.9968 | 0.9969 | 0.9970 | 0.9971 | 0.9972 | 0.9973 | 0.9974 |
| 2.8 | 0.9974 | 0.9975 | 0.9976 | 0.9977 | 0.9977 | 0.9978 | 0.9979 | 0.9979 | 0.9980 | 0.9981 |
| 2.9 | 0.9981 | 0.9982 | 0.9982 | 0.9983 | 0.9984 | 0.9984 | 0.9985 | 0.9985 | 0.9986 | 0.9986 |
| 3.0 | 0.9987 | 0.9987 | 0.9987 | 0.9988 | 0.9988 | 0.9989 | 0.9989 | 0.9989 | 0.9990 | 0.9990 |
| 3.1 | 0.9990 | 0.9991 | 0.9991 | 0.9991 | 0.9992 | 0.9992 | 0.9992 | 0.9992 | 0.9993 | 0.9993 |
| 3.2 | 0.9993 | 0.9993 | 0.9994 | 0.9994 | 0.9994 | 0.9994 | 0.9994 | 0.9995 | 0.9995 | 0.9995 |
| 3.3 | 0.9995 | 0.9995 | 0.9995 | 0.9995 | 0.9996 | 0.9996 | 0.9996 | 0.9996 | 0.9996 | 0.9997 |
| 3.4 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9998 |
| 3.5 | 0.9998 | 0.9998 | 0.9998 | 0.9998 | 0.9998 | 0.9998 | 0.9998 | 0.9998 | 0.9998 | 0.9998 |
| 3.6 | 0.9998 | 0.9998 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 |
| 3.7 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 |
| 3.8 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 |
| 3.9 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |

| | Values of z for selected values of Pr(Z < z) | | | | | | |
|---|---|---|---|---|---|---|---|
| z | 0.842 | 1.036 | 1.282 | 1.645 | 1.960 | 2.326 | 2.576 |
| Pr(Z < z) | 0.800 | 0.850 | 0.900 | 0.950 | 0.975 | 0.990 | 0.995 |

For Classical Credibility, we will be using the chart at the bottom of the table, showing various percentiles of the Standard Normal Distribution.

**<u>Using the Normal Table</u>**:

Specific instructions from the CAS for using the Normal Table:[6]

"Candidates should not interpolate in the tables unless explicitly instructed to do so in the problem, rather, a candidate should round the result that would be used to enter a given table to the same level of precision as shown in the table and use the result in the table that is nearest to that indicated by rounded result. For example, if a candidate is using the Tables of the Normal Distribution to find a significance level and has a Z value of 1.903, the candidate should round to 1.90 to find cumulative probability in the Normal table."

[6] Hopefully, the letter choice will not depend on whether you interpolate or not.
Unfortunately, I did not go back and always follow this rule in my solutions to questions.

## Section 1, Introduction

Assume Carpenters are currently charged a rate of $10 (per $100 of payroll) for Workers Compensation Insurance.[7] Assume further that the recent experience would indicate a rate of $5. Then an actuary's new estimate of the rate for Carpenters might be $5, $10, or most likely something in between. In other words, the new estimate of the appropriate rate for Carpenters would be a weighted average of the separate $5 and $10 estimates.

If the actuary put more weight on the observation, the new estimate would be closer to the observation of $5. If on the other hand, the actuary put less weight on the observation, then the new estimate would be closer to the current rate of $10. One could write this as:
new estimate = (5)(Z) + (10)(1-Z), where Z is the weight, $0 \le Z \le 1$.

So for example if Z = 20%, then the new estimate is ($5)(0.2) + ($10)(0.8) = $9.
If instead Z = 60%, then the new estimate is ($5)(0.6) + ($10)(0.4) = $7.
The weight Z is generally referred to as the "credibility" assigned to the observed data.

Credibility is commonly used by actuaries in order to weight together two estimates of the same quantity.[8] Let X and Y be two estimates. X might be from a recent observation based on limited data, while Y might be a previous estimate or one obtained from a larger but less specific data set.[9] Then **the estimate using credibility would =**
**ZX + (1 - Z)Y, where Z is the credibility assigned to the observation X.**

1 - Z is sometimes referred to as the complement of credibility.
Confusingly, often instead what I have denoted Y, the item that is given weight 1 - Z, is referred to as the "complement of credibility".
Tse in Nonlife Actuarial Models: Theory Methods and Evaluation does not use the term "complement of credibility". Weight 1 - Z is given to M = manual rate.

*Thus the use of credibility involves a linear estimate of the true expectation derived as a result of a compromise between hypothesis and observations.*

Credibility: A linear estimator by which data external to a particular group or individual are combined with the experience of the group or individual in order to better estimate the expected loss (or any other statistical quantity) for each group or individual.

Credibility or Credibility Factor: Z, the weight given the observation.

The basic formula is: **new estimate = (observation) (Z) + (old estimate) (1-Z).**
**= (observation) (Z) + (manual rate) (1-Z).**

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[7] Assume that there is no change in rates indicated for the Contracting Industry Group in which Carpenters are included. So that in the absence of any specific data for the Carpenter's class, Carpenters would continue to be charged $10.
[8] In some actual applications more than two estimates are weighted together.
[9] For example, Y might be (appropriately adjusted) countrywide data for the Carpenter's class.

Sometimes it is useful to use the equivalent formula:

new estimate = old estimate + Z (observation - old estimate).

This can be solved for the credibility:

$$Z = \frac{\text{new estimate - old estimate}}{\text{observation - old estimate}}.$$

In the example, in order to calculate a new estimate of the appropriate rate for Carpenters, one first has to decide that one will weight together the current observations with the current rate for Carpenters.[10] [11] Generally on the exam when it is relevant to answering the question, it will be clear which two estimates to weight together. Second one has to decide how much credibility to assign to the current observation. On the exam this is generally the crux of the questions asked.

Two manners of determining how much credibility to assign are covered on the Syllabus. The first is called **Classical Credibility** or Limited Fluctuation Credibility and is covered in this Study Aid.[12] The second is referred to as **Buhlmann Credibility**, **Least Squares Credibility**, or Greatest Accuracy Credibility and is covered in another Study Aid.[13]

Either form of credibility can be applied to various actuarial issues such as: Classification and/or Territorial Ratemaking, Experience Rating (Individual Risk Rating), Loss Reserving, Trend, etc. On the exam, credibility questions will usually involve experience rating[14] or perhaps classification ratemaking[15], unless they deal with urns, dies, spinners, etc., that are used to model probability and credibility theory situations.[16]

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[10] One would have to decide what period of time to use, for example the most recently available 3 years. *Also one would adjust the data for law changes, trend, development, etc.*

[11] In actual applications, various adjustments would be made to the current rate for Carpenters before using it to estimate the proposed rate for Carpenters.

[12] *Classical Credibility was developed in the U.S. in the first third of the 20th century by early members of the CAS such as Albert Mowbray and Francis Perryman.*

[13] *Greatest Accuracy Credibility was developed in the late 1940s by Arthur Bailey, FCAS, based on earlier work by Albert Whitney and other members of the CAS.*

[14] Experience Rating refers to the use of the experience of an individual policyholder in order to help determine his premium. This can be for either Commercial Insureds (e.g. Workers Compensation) or Personal Lines Insureds (e.g. Private Passenger Automobile.)
See Basic Ratemaking by Werner and Modlin.

[15] For example making the rates for the Workers Compensation class of Carpenters. Similar situations occur when making the rates for the territories of a state or for the classes and territories in a state.

[16] The reason you are given problems involving urns, etc. is that one can then ask questions that do not require the knowledge of the specific situation. For example, in order to ask a question involving an actual application to Workers Compensation Classification Ratemaking would require knowledge many students do not have and which can not be covered on the syllabus for this exam. *Also, the questions involving urns, etc., illustrate the importance of modeling. In actual applications, someone has to propose a model of the underlying process, so that one can properly apply Credibility Theory.* Urn models, etc. allow one to determine which features are important and how they are likely to affect real world situations. *A good example is Philbrick's target shooting example.*

In general, all other things being equal, one would assign more credibility to a larger volume of data. In Classical Credibility, one determines how much data one needs before one will assign to it 100% credibility. This amount of data is referred to as the **Full Credibility Criterion or the Standard for Full Credibility**. If one has this much data or more, then Z = 100%; if one has observed less than this amount of data then one has $0 \le Z < 1$.

For example, if I observed 1000 full time Carpenters, then I might assign 100% credibility to their data.[17] Then if I observed 2000 full time Carpenters I would also assign them 100% credibility. I might assign 100 full time Carpenters 32% credibility. In this case we say we have assigned the observation **partial credibility**, i.e., less than full credibility. Exactly how to determine the amount of credibility assigned to different amounts of data is discussed in the following sections.

There are five basic concepts from Classical Credibility you need to know how to apply in order to answer exam questions:

1. How to determine the Criterion for Full Credibility when estimating frequencies.

2. How to determine the Criterion for Full Credibility when estimating severities.

3. How to determine the Criterion for Full Credibility when estimating pure premiums or aggregate losses.

4. How to determine the amount of partial credibility to assign when one has less data than is needed for full credibility.

5. How to use credibility to estimate the future, by combining the observation and the old estimate.

[17] For Workers Compensation that data would be dollars of loss and dollars of payroll.

Problems:

**1.1** (1 point) The observed claim frequency is 120. The credibility given to this data is 25%. The complement of credibility is given to the prior estimate of 200.
What is the new estimate of the claim frequency?
A. Less than 165
B. At least 165 but less than 175
C. At least 175 but less than 185
D. At least 185 but less than 195
E. At least 195

**1.2** (1 point) The prior estimate was 100 and after an observation of 800 the new estimate is 150. How much credibility was assigned to the data?
A. Less than 4%
B. At least 4% but less than 5%
C. At least 5% but less than 6%
D. At least 6% but less than 7%
E. At least 7%

Solutions to Problems:

**1.1. C.** (25%)(120) + (75%)(200) = **180**.

**1.2. E.** New estimate = old estimate + Z (observation - old estimate).
⇒ Z = (new estimate - old estimate) / (observation - old estimate)
$= \frac{150 - 100}{800 - 100}$ = 50/700 = **7.1%**.

## Section 2, Full Credibility for Frequency[18]

The most common uses of Classical Credibility, assume that the frequency is (approximately) Poisson. Also, this is the only case covered in the parts of the textbook on the syllabus of this exam. Thus we'll deal with that case first.

**Poisson Case**:

Assume we have a Poisson process for claim frequency, with an average of 500 claims per year. Then if we observe the numbers of claims, they will vary from year to year around the mean of 500. The variance of a Poisson process is equal to its mean of 500. We can approximate this Poisson Process by a Normal Distribution with a mean of 500 and a variance of 500.

We can use this Normal Approximation to estimate how often we will observe results far from the mean. For example, how often can one expect to observe more than 550 claims? The standard deviation is: $\sqrt{500}$ = 22.36. So 550 claims corresponds to about 50 / 22.36 = 2.24 standard deviations greater than average. Since $\Phi(2.24) = 0.9875$, there is approximately a 1.25% chance of observing more than 550 claims.

Thus there is about a 1.25% chance of observing more than 10% greater than the expected number of claims. Similarly, we can calculate the chance of observing fewer than 450 claims as approximately 1.25%. Thus the chance of observing outside ±10% from the mean number of claims is about 2.5%. In other words, the chance of observing within ±10% of the expected number of claims is 97.5% in this case.[19]

If we had a mean of 1000 claims instead of 500 claims, then there would be a greater chance of observing within ±10% of the expected number of claims. This is given by the Normal approximation as: $\Phi[\frac{(10\%)\ (1000)}{\sqrt{1000}}] - \Phi[-\frac{(10\%)\ (1000)}{\sqrt{1000}}] = \Phi[3.162] - \Phi[-3.162] =$

$1 - (2)\{1 - \Phi[3.162]\} = 2\Phi[3.162] - 1 = (2)(0.9992) - 1 = 99.84\%$.

Exercise: Compute the Probability of being within ± 5% of the mean, for 100 expected claims.
[Solution: $2\Phi[\frac{(5\%)\ (100)}{\sqrt{100}}] - 1 = 38.29\%$.]

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[18] A subsequent section deals with estimating Aggregate Loss or Pure Premiums rather than Frequencies. As will be seen in order to calculate a Standard for Full Credibility for Aggregate Loss or Pure Premium generally one first calculates a Standard for Full Credibility for the Frequency. Thus questions about the former also test whether one knows how to do the latter.

[19] Note that here we have ignored the "continuity correction." Including the continuity correction, the probability of more than 550 claims is approximately: $1 - \Phi[(550.5\text{-}500)/\sqrt{500}] = 1 - \Phi(2.258) = 1 - 0.9880 = 1.20\%$.

In general, let (1 - $\alpha$) be the chance of being within ±k of the mean, given an expected number of claims equal to n. Then $1 - \alpha = 2\Phi[k \sqrt{n}] - 1. \Rightarrow \Phi[k \sqrt{n}] = 1 - \alpha/2$.

Here is a table showing the probability of being within ±k of the mean, for k = 10%, 5%, 2.5%, 1%, and 0.5%, and for 10, 50, 100, 500, 1000, 5000, and 10,000 claims:

| | Probability of Being Within ± k of the Mean | | | | |
|---|---|---|---|---|---|
| Expected # of Claims | k=10% | k=5% | k=2.5% | k=1% | k=0.5% |
| 10 | 24.82% | 12.56% | 6.30% | 2.52% | 1.26% |
| 50 | 52.05% | 27.63% | 14.03% | 5.64% | 2.82% |
| 100 | 68.27% | 38.29% | 19.74% | 7.97% | 3.99% |
| 500 | 97.47% | 73.64% | 42.39% | 17.69% | 8.90% |
| 1000 | 99.84% | 88.62% | 57.08% | 24.82% | 12.56% |
| 5000 | 100.00% | 99.96% | 92.29% | 52.05% | 27.63% |
| 10000 | 100.00% | 100.00% | 98.76% | 68.27% | 38.29% |

Turning things around, given values of $\alpha$ and k, then one can compute the number of expected claims $\lambda_F$ such that the chance of being within ±k of the mean is 1 - $\alpha$.
For example, if $\alpha$ = 10% and k = 2.5%, then based on the above table $\lambda_F$ is somewhat less than 5000 claims. More precisely, $\alpha = 2\{1 - \Phi[k \sqrt{\lambda_F}]\}$, and therefore for $\alpha$ = 0.1 and k = 2.5%,
$0.1 = 2\{1 - \Phi[k \sqrt{\lambda_F}]\}$.

Thus we want $\Phi[2.5\% \sqrt{\lambda_F}] = 0.95$. Let $z_{1-\alpha/2}$ be such that $\Phi(z_{1-\alpha/2}) = 1 - \alpha/2 = 0.95$.
Consulting the Standard Normal Table, $z_{1-\alpha/2}$ = 1.645. Then we want $z_{1-\alpha/2} = 0.025 \sqrt{\lambda_F}$.
Thus $\lambda_F = z_{1-\alpha/2}{}^2 / k^2 = 1.645^2 / 0.025^2 = 4330$ claims.

Having taken $\alpha$ = 10%, with k = 2.5%, we would refer to 4330 as the Standard for Full Credibility for estimating frequencies.

In general, **assume one desires that the chance of being within ± k of the mean frequency to be at least 1 - α, then for a Poisson Frequency, the Standard for Full Credibility is:**[20]

$$\lambda_F = \left(\frac{z_{1-\alpha/2}}{k}\right)^2$$, **where $z_{1-\alpha/2}$ is such that $\Phi(z_{1-\alpha/2}) = 1 - \alpha/2$.**

[20] See Equations 6.7 in NonLife Actuarial Models: Theory, Methods and Evaluation.

Exercise: Assuming frequency is Poisson, for $\alpha$ = 5% and for k = 5%, what is the number of claims required for Full Credibility for estimating the frequency?
[Solution: $z_{1-\alpha/2}$ = 1.960 since $\Phi(1.960) = 1 - 0.05/2 = 97.5\%$.

Therefore, $\lambda_F = \left(\frac{z_{1-\alpha/2}}{k}\right)^2 = (1.96/0.05)^2 = 1537$ claims.]

Here are values of $z_{1-\alpha/2}$ corresponding to various values of $\alpha$:

| $\alpha$ | $1 - \alpha/2$ | $z_{1-\alpha/2}$ |
|---|---|---|
| | | |
| *40.00%* | *80.00%* | *0.842* |
| *30.00%* | *85.00%* | *1.036* |
| 20.00% | *90.00%* | 1.282 |
| **10.00%** | *95.00%* | **1.645** |
| **5.00%** | *97.50%* | **1.960** |
| 2.00% | *99.00%* | 2.326 |
| 1.00% | *99.50%* | 2.576 |

The relevant values are shown in the lower portion of the Normal table attached to the exam.

Here is a table of values for the Standard for Full Credibility for the Frequency $\lambda_F$, given various values of $\alpha$ and k:[21]

| | Standards for Full Credibility for Frequency (Claims) | | | | | | |
|---|---|---|---|---|---|---|---|
| $\alpha$ | k = 30% | k = 20% | k = 10% | k = 7.5% | k = 5% | k = 2.5% | k = 1% |
| 20.00% | 18 | 41 | 164 | 292 | 657 | 2,628 | 16,424 |
| 10.00% | 30 | 68 | 271 | 481 | **1,082** | 4,329 | 27,055 |
| 5.00% | 43 | 96 | 384 | **683** | 1,537 | 6,146 | 38,415 |
| 4.00% | 47 | 105 | 422 | 750 | 1,687 | 6,749 | 42,179 |
| 3.00% | 52 | 118 | 471 | 837 | 1,884 | 7,535 | 47,093 |
| 2.00% | 60 | 135 | 541 | 962 | 2,165 | 8,659 | 54,119 |
| 1.00% | 74 | 166 | 664 | 1,180 | 2,654 | 10,616 | 66,349 |
| 0.10% | 120 | 271 | 1,083 | 1,925 | 4,331 | 17,324 | 108,276 |
| 0.01% | 168 | 378 | 1,514 | 2,691 | 6,055 | 24,219 | 151,367 |

The Standard of 1082 claims corresponding to $\alpha$ = 10% and k = 5% is the most commonly used, followed by the Standard of 683 claims corresponding to $\alpha$ = 5% and k = 7.5%.

[21] See Longley-Cook's "An Introduction to Credibility Theory" PCAS 1962, or "Some Notes on Credibility" by Perryman, PCAS 1932.

You should on several different occasions verify that you can calculate quickly and accurately a randomly selected value from this table. The value 1082 claims corresponding to $\alpha$ = 10% and k = 5% is commonly used in applications. For $\alpha$ = 10%, we want to have a 90% chance of being within ±k of the mean, so we are willing to have a 5% probability outside on either tail, for a total of 10% probability of being outside the error bars. Thus $\Phi(z_{1-\alpha/2}) = 0.95$ or $z_{1-\alpha/2} = 1.645$.

Thus $\lambda_F = \left(\frac{z_{1-\alpha/2}}{k}\right)^2 = (1.645 / 0.05)^2 = 1082$ claims.

<u>Variations from the Poisson Assumption</u>:[22]

Assume one desires that the chance of being within ±k of the mean frequency to be at least

1 - $\alpha$, then the Standard for Full Credibility is $\lambda_F = \left(\frac{z_{1-\alpha/2}}{k}\right)^2$, where $z_{1-\alpha/2}$ is such that

$\Phi(z_{1-\alpha/2}) = 1 - \alpha/2$.

However, this depended on the following assumptions:[23]

1. One is trying to Estimate Frequency
2. Frequency is given by a Poisson Process (so that the variance is equal to the mean)
3. There are enough expected claims to use the Normal Approximation.

If any of these assumptions do <u>not</u> hold then one should not apply the above technique.

One can also deal with situations where the frequency is not assumed to be Poisson.

If a Binomial, Negative Binomial, or other frequency distribution is substituted for a Poisson distribution, then the difference in the derivation is that the variance is not equal to the mean.

For example, assume one has a Binomial Distribution with parameters m = 1000 and q = 0.3. The mean is 300 and the variance is (1000)(0.3)(0.7) = 210. So the chance of being within ±5% of the expected value is approximately: $\Phi[\frac{(5\%)\ (300)}{\sqrt{210}}] - \Phi[-\frac{(5\%)\ (300)}{\sqrt{210}}] =$

$\Phi(1.035) - \Phi(-1.035) = 0.8496 - 0.1504 = 69.9\%$. So in the case of a Binomial with parameter 0.3, the "Standard for Full Credibility" with $\alpha$ = 30% and k = ±5% is about 1000 exposures or 300 expected claims.

If instead a Negative Binomial Distribution had been assumed, then the variance would have been greater than the mean. This would have resulted in a standard for Full Credibility greater than in the Poisson situation.

One can derive a more general formula when the Poisson assumption does not apply.

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[22] See Section 6.4 of <u>NonLife Actuarial Models: Theory, Methods and Evaluation</u>, not on the syllabus. See also Exercises 6.13, 6.16, and 6.18.

[23] Unlike Buhlmann Credibility, in Classical Credibility the weight given to the prior mean does not depend on the actuary's view of its accuracy.

Standard for Full Credibility for Frequency, General Case:[24]

**Standard for Full Credibility for Frequency in terms of claims is:**[25]

$$\left(\frac{z_{1-\alpha/2}}{k}\right)^2 \frac{\sigma_f^2}{\mu_f} = \lambda_F \frac{\sigma_f^2}{\mu_f}.$$

which reduces to the Poisson case when $\frac{\sigma_f^2}{\mu_f} = 1$.

Exercise: Find the number of claims required for full credibility. Require that there is a 90% chance that the estimate of the frequency is correct within ±2.5%. The frequency distribution has a variance twice its mean.
[Solution: $\alpha$ = 1 - 90% = 10% and $z_{1-\alpha/2}$ = 1.645. k = 2.5%.

$\lambda_F = \left(\frac{z_{1-\alpha/2}}{k}\right)^2 = (1.645 / 0.025)^2$ = 4330 claims.

We are given that $\sigma_f^2 / \mu_f = 2$. Thus $\lambda_F (\sigma_f^2 / \mu_f)$ = (4330)(2) = 8660 claims.]

Exercise: Find the number of claims required for full credibility. Require that there is a 99% chance that the estimate of the frequency is correct within ±10%. Assume the frequency distribution is Negative Binomial, with parameters $\beta$ = 1.5 and r unknown.
[Solution: $\alpha$ = 1 - 99% = 1%. $\Phi(2.576)$ = 0.995. Thus $z_{1-\alpha/2}$ = 2.576.

k = 10%. $\lambda_F = \left(\frac{z_{1-\alpha/2}}{k}\right)^2 = (2.576 / 0.10)^2$ = 664 claims. We are given that the frequency is

Negative Binomial with mean $\mu_f = r\beta$ and variance $\sigma_f^2 = r\beta(1+\beta)$. Thus $\frac{\sigma_f^2}{\mu_f} = 1+\beta = 2.5$.

Thus $\lambda_F \frac{\sigma_f^2}{\mu_f}$ = (664)(2.5) = 1660 claims.

Comment: This is larger than the standard of 664 for a Poisson frequency, since the Negative Binomial has a variance greater than its mean. In this case the variance is 2.5 times the mean. Thus the standard of 1660 claims is 2.5 times 664.]

[24] See CAS MAS-2, 11/19, Q.4.
[25] See for example, Equation 2.2.6 in Credibility by Mahler and Dean, not on the syllabus.

*Derivation of the Standard for Full Credibility for Frequency*:

Require that the observed frequency should be within 100k% of the expected pure premium with probability 1 - $\alpha$. Use the following notation:

$\mu_f$ = mean frequency. $\sigma_f^2$ = variance of frequency.

Let $z_{1-\alpha/2}$ be such that $\Phi(z_{1-\alpha/2}) = 1 - \alpha/2$.

Using the Normal Approximation what is a formula for the number of claims needed for full credibility of the frequency?

Assume there are N claims expected and therefore $N/\mu_f$ exposures.

The mean frequency is $\mu_f$. The variance of the frequency for a single exposure is: $\sigma_f^2$.

A key idea is that if one adds up for example 3 independent, identically distributed variables, one gets 3 times the variance. In this case we are assumed to have $N/\mu_f$ independent exposures.

Therefore, the variance of the number of claims observed for $N/\mu_f$ independent exposures is: $(N/\mu_f)\ \sigma_f^2$.

The observed frequency is the number of claims divided by the number of exposures, $N/\mu_f$.

When one divides by a constant, the variance is divided by that constant squared.

Therefore, the variance of the observed frequency is the variance of the number of claims, $(N / \mu_f)\ \sigma_f^2$, divided by $(N/\mu_f)^2$, which is: $\mu_f\ \sigma_f^2 / N$.

Thus the standard deviation of the observed claim frequency is: $\sigma = \sigma_f \sqrt{\mu_f/N}$.

We desire that $\text{Prob}(\mu_f - k\mu_f \le X \le \mu_f + k\mu_f) \ge 1 - \alpha$.

Using the Normal Approximation this is true provided: $k\mu_f = z_{1-\alpha/2}\ \sigma = z_{1-\alpha/2}\ \sigma_f \sqrt{\mu_f/N}$.

Solving for $N = \left(\frac{z_{1-\alpha/2}}{k}\right)^2 \frac{\sigma_f^2}{\mu_f}$.

<u>Exposures vs. Claims</u>:

Standards for Full Credibility have been calculated so far in terms of the expected number of claims. It is common to translate these into a number of exposures by dividing by the (approximate) expected claim frequency. So for example, if the Standard for Full Credibility is 1082 claims ($\alpha$ = 10%, k = 5%) and the expected claim frequency in Homeowners Insurance were 0.04 claims per house-year, then 1082 / 0.04 ≅ 27,000 house-years would be a corresponding Standard for Full Credibility in terms of exposures. In general, one can divide the Standard for Full Credibility in terms of claims by $\mu_f$, in order to get it in terms of exposures.

Thus in general, the Standard for Full Credibility for Frequency in terms of <u>exposures</u> is:

$$\left(\frac{z_{1-\alpha/2}}{k}\right)^2 \frac{\sigma_f^2}{\mu_f^2} = \lambda_F \frac{\sigma_f^2}{\mu_f^2}.$$

Exercise: Find the number of exposures required for full credibility. Require that there is a 99% chance that the estimate of the frequency is correct within ±10%. Assume the frequency distribution is Negative Binomial, with parameters $\beta$ = 1.5 and r = 4.

[Solution: $\alpha$ = 1% and thus we want $\Phi(z_{1-\alpha/2}) = 0.995$. Thus $z_{1-\alpha/2} = 2.576$.

k = 10%. $\lambda_F = \left(\frac{z_{1-\alpha/2}}{k}\right)^2 = (2.576 / 0.10)^2 = 664$ claims. We are given that the frequency is

Negative Binomial with mean $\mu_f = r\beta$ and variance $\sigma_f^2 = r\beta(1+\beta)$.

Thus $\frac{\sigma_f^2}{\mu_f^2} = \frac{1+\beta}{r\,\beta} = \frac{2.5}{(4)\,(1.5)} = 0.4167$.

Thus $\lambda_F \frac{\sigma_f^2}{\mu_f^2} = (664)(0.4167) = 277$ exposures.

<u>Comment</u>: Note the assumed mean frequency is: (4)(1.5) = 6. Thus 277 exposures correspond to about (277)(6) = 1660 expected claims, as found in a previous exercise. ]

*The Choice of alpha and k:*

On the exam one will be given $\alpha$ and k.[26]

In practical applications, appropriate values of $\alpha$ and k have to be selected.[27] While there is clearly some judgment involved in the choice of $\alpha$ and k, the Standards for Full Credibility for a given application are generally chosen by actuaries within a similar range.[28]

This same type of judgment is involved in the choice of error bars around an estimate of a quantity such as the loss elimination ratio at $10,000. Often ±2 standard deviations (corresponding to about a 95% confidence interval) will be chosen, but that is not necessarily better than choosing ±1.5 or ±2.5 standard deviations. Similarly one has to decide at what significance level to reject or accept $H_0$ when doing hypothesis testing. Should one use 5%, 1%, or some other significance level?

So while Classical Credibility also involves somewhat arbitrary judgments, that has not stood in the way of it being very useful for many decades in many applications.

[26] One might be given the probability 1 - $\alpha$.
[27] For situations that come up repeatedly, the choice of $\alpha$ and k may have been made several decades ago, but nevertheless the choice was made at some point in time. 1082 claims corresponding to $\alpha$ = 10% and k = 5% is the single most commonly used value.
[28] For example, if an actuary were estimating frequency for private passenger automobile insurance, he would probably pick values of $\alpha$ and k that have been used before by other actuaries. These practical applications are beyond the syllabus of this exam.

Problems:

**2.1** (1 point) Assume frequency is Poisson.
How many claims are required for Full Credibility if one requires that there be a 98% chance of the estimated frequency being within ±2.5% of the true value?
A. Less than 8,000
B. At least 8,000 but less than 9,000
C. At least 9,000 but less than 10,000
D. At least 10,000 but less than 11,000
E. At least 11,000

**2.2** (3 points) Y represents the number of independent homogeneous exposures in an insurance portfolio. The claim frequency rate per exposure is a random variable with mean = 0.10 and variance = 0.25.
A full credibility standard is devised that requires the observed sample frequency rate per exposure to be within 4% of the expected population frequency rate per exposure 95% of the time.
Determine the value of Y needed to produce full credibility for the portfolio's experience.
A. 50,000 B. 60,000 C. 70,000 D. 80,000 E. 90,000

**2.3** (1 point) Let A be the number of claims needed for full credibility, if the estimate is to be within ±3% of the true value with a 80% probability. Let B be the similar number using 8% rather than 3%. What is the ratio of A divided by B?
A. 3 B. 4 C. 5 D. 6 E. 7

**2.4** (2 points) Assume you are conducting a poll relating to a single question and that each respondent will answer either yes or no. You pick a random sample of respondents out of a very large population. Assume that the true percentage of yes responses in the total population is between 20% and 80%. How many respondents do you need, in order to require that there be a 90% chance that the results of the poll are within ±8% of the true answer?
A. Less than 1,000
B. At least 1,000 but less than 2,000
C. At least 2,000 but less than 3,000
D. At least 3,000 but less than 4,000
E. At least 4,000

**2.5** (1 point) Assume frequency is Poisson. The full credibility standard for a company is set so that the total number of claims is to be within 8% of the true value with probability 1 - $\alpha$.
This full credibility standard is calculated to be 625 claims. What is the value of $\alpha$?
A. Less than 4%
B. At least 4% but less than 5%
C. At least 5% but less than 6%
D. At least 6% but less than 7%
E. 7% or more

**2.6** (1 point) Find the number of claims required for full credibility.
Require that there is a 95% chance that the estimate of the frequency is correct within ±10%.
The frequency distribution has a variance 3 times its mean.
A. Less than 1,000
B. At least 1,000, but less than 1,100
C. At least 1,100, but less than 1,200
D. At least 1,200, but less than 1,300
E. 1,300 or more

**2.7** (2 points) A Standard for Full Credibility in terms of claims has been established for frequency assuming that the frequency is Poisson. If instead the frequency is assumed to follow a Negative Binomial with parameters $r = 12$ and $\beta = 0.5$, what is the ratio of the revised Standard for Full Credibility to the original one?
A. Less than 1.0
B. At least 1.0 but less than 1.2
C. At least 1.2 but less than 1.4
D. At least 1.4 but less than 1.6
E. At least 1.6

**2.8** (1 point) Assume frequency is Poisson. How many claims are required for Full Credibility if one requires that there be a 95% chance of being within ±10% of the true frequency?
A. Less than 250
B. At least 250 but less than 300
C. At least 300 but less than 350
D. At least 350 but less than 400
E. 400 or more

**2.9** (1 point) The total number of claims for a group of insureds is Poisson distributed with a mean of m. Using the Normal approximation, calculate the value of m such that the observed number of claims will be within 6% of m with a probability of 0.98.
A. Less than 1,000
B. At least 1,000, but less than 1,500
C. At least 1,500, but less than 2,000
D. At least 2,000, but less than 2,500
E. 2,500 or more

**2.10** (1 point) Assume frequency is Poisson.
How many claims are required for Full Credibility if one requires that there be a 99% chance of the estimated frequency being within ±7.5% of the true value?
A. Less than 800
B. At least 800 but less than 900
C. At least 900 but less than 1000
D. At least 1000 but less than 1100
E. At least 1100

**2.11** (2 points) Frequency is assumed to follow a Binomial with parameters q = 0.4 and m.
How many claims are required for Full Credibility if one requires that there be a 90% chance of the estimated frequency being within ±5% of the true value?
(A) 650 (B) 700 (C) 750 (D) 800 (E) 850

**2.12 (CAS Part 2 Exam, 1965, Q.39)** (1.5 points) If you wish to estimate the proportion of Democrats in a certain district, and want your estimate to be correct within 0.02 units of the true proportion with a probability of 0.95, how large a sample should you take if, on the basis of preliminary samples, you have estimated the true proportion to be near 4/10?

**2.13 (4, 5/86, Q.34)** (1 point) Let X be the number of claims needed for full credibility, if the estimate is to be within 5% of the true value with a 90% probability.
Let Y be the similar number using 10% rather than 5%.
What is the ratio of X divided by Y?
A. 1/4 B. 1/2 C. 1 D. 2 E. 4

**2.14 (4, 5/87, Q.46)** (2 points) The "Classical" approach to credibility optimizes which of the following error measures?
A. least squares error criterion
B. variance of the hypothetical means
C. normal approximation for skewness
D. coefficient of variation
E. None of the above

**2.15 (4, 5/89, Q.29)** (1 point) The total number of claims for a group of insureds is Poisson distributed with a mean of m. Calculate the value of m such that the observed number of claims will be within 3% of m with a probability of 0.975 using the normal approximation.
A. Less than 5,000
B. At least 5,000, but less than 5,500
C. At least 5,500, but less than 6,000
D. At least 6,000, but less than 6,500
E. 6,500 or more

**2.16 (4B, 11/94, Q.15)** (3 points) You are given the following:
Y represents the number of independent homogeneous exposures in an insurance portfolio.
The claim frequency rate per exposure is a random variable with mean = 0.025 and variance = 0.0025.
A full credibility standard is devised that requires the observed sample frequency rate per exposure to be within 5% of the expected population frequency rate per exposure 90% of the time. Determine the value of Y needed to produce full credibility for the portfolio's experience.
A. Less than 900
B. At least 900, but less than 1,500
C. At least 1,500, but less than 3,000
D. At least 3000, but less than 4,500
E. At least 4,500

**2.17 (4B, 5/96, Q.13)** (1 point) Using the methods of Classical credibility, a full credibility standard of 1,000 expected claims has been established such that the observed frequency will be within 5% of the underlying frequency, with probability P.
Determine the number of expected claims that would be required for full credibility if 5% were changed to 1%.
A. 40 B. 200 C. 1,000 D. 5,000 E. 25,000

**2.18 (4, 11/04, Q.21)** (2.5 points) You are given:
(i) The number of claims has probability function:

$$p(x) = \binom{m}{x} q^x (1-q)^{m-x}, \ x = 0, 1, 2, \ldots, m$$

(ii) The actual number of claims must be within 1% of the expected number of claims with probability 0.95.
(iii) The expected number of claims for full credibility is 34,574.
Determine q.
(A) 0.05 (B) 0.10 (C) 0.20 (D) 0.40 (E) 0.80

**2.19 (CAS MAS-2, 11/19, Q.4)** (2.5 points) You are given the following parameters.
- Assume the full-credibility standard using limited-fluctuation credibility is based on $\alpha = 0.05$ and $k = 0.02$.
- The expected claim frequency per exposure unit is 0.03.
- W is the full-credibility standard for claim frequency in exposure units assuming Poisson claim frequency.
- V is the full-credibility standard for claim frequency in exposure units assuming binomial claim frequency.

Calculate $|W - V|$.

A. Less than 4,000
B. At least 4,000 but less than 6,000
C. At least 6,000 but less than 8,000
D. At least 8,000 but less than 10,000
E. At least 10,000

Solutions to Problems:

**2.1. B.** $\alpha$ = 1 - 0.98 = 0.02. $\Phi$(2.326) = 1 - 0.02/2 = 0.99. so that $z_{1-\alpha/2}$ = 2.326.

$\lambda_F = \left(\frac{z_{1-\alpha/2}}{k}\right)^2 = (2.326 / 0.025)^2 = \mathbf{8656}$.

**2.2. B.** k = 0.04, $\alpha$ = 1 - 95% = 5%, $z_{1-\alpha/2}$ = 1.960, $\mu_f$ = 0.10, $\sigma_f^2$ = 0.25, and $\left(\frac{z_{1-\alpha/2}}{k}\right)^2 \frac{\sigma_f^2}{\mu_f}$

= $(1.960/0.04)^2$ (0.25/0.10) = 6002.5 claims. ⇔ 6002.5/0.10 = **60,025** exposures.

**2.3. E.** $\lambda_F = \left(\frac{z_{1-\alpha/2}}{k}\right)^2$ and thus for a given $\alpha$ the standard for full credibility is inversely proportional to the square of k. Thus A/B = $8^2 / 3^2$ = **7.11**.

Comment: The standard for full credibility is larger the smaller k; being within ±3% is more stringent a requirement which requires more claims than than being within ±8%.

**2.4. B.** Let m be the number of respondents and let q be the true percentage of yes respondents in the total population. The number of yes responses in the sample is given by a Binomial Distribution with parameters q and m, with variance mq(1-q).

The percentage of yes responses is N/m, with variance: $mq(1-q) / m^2 = q(1-q) / m$.

Using the Normal Approximation 90% probability corresponds to ±1.645 standard deviations of the mean of q. Thus we want: (0.08)(q) = (1.645) $\sqrt{q(1-q)/m}$ .

$\sqrt{m}$ = (1.645) $\sqrt{(1-q)/q}$ / 0.08. m = 423 {(1/q) - 1}. The desired m is a decreasing function of q.

However, we assume q ≥ 0.2, so that m ≤ 423(5 - 1) = **1692**.

Alternately, for each respondent, which can be thought of as an exposure, we have a Bernoulli distribution, with $\sigma_f^2/\mu_f^2 = (1-q)q / q^2 = 1/q - 1$.

The standard for full credibility is in terms of exposures:

$$\frac{\sigma_f^2}{\mu_f} \left(\frac{z_{1-\alpha/2}}{k}\right)^2 / \mu_f = \frac{\sigma_f^2}{\mu_f^2} \left(\frac{z_{1-\alpha/2}}{k}\right)^2 = (1.645/0.08)^2 (1/q - 1) = 423 (1/q - 1).$$

For 0.2 ≤ q ≤ 0.8, this is maximized when q = 0.2, and is then: 423(5 - 1) = **1692** exposures.

Comment: The number of exposures needed for full credibility depends on q. We want a standard for full credibility that will be enough exposures to satisfy the criterion regardless of q, so we pick the maximum over q from 20% to 80%.

**2.5. B.** $\lambda_F = \left(\frac{z_{1-\alpha/2}}{k}\right)^2$. Therefore $z_{1-\alpha/2} = k\sqrt{\lambda_F}$ = 0.08 $\sqrt{625}$ = 2.00. $\Phi$(0.9772) = 2.00.

⇒ 1 - $\alpha$/2 = 0.9772. ⇒ $\alpha$ = **0.0456**.

**2.6. C.** $\alpha$ = 1 - 95% = 5%.. $\Phi$(1.960) = 1 - $\alpha$/2 = 0.975, so that $z_{1-\alpha/2}$ = 1.960.

$\frac{\sigma_f^2}{\mu_f}\left(\frac{z_{1-\alpha/2}}{k}\right)^2$ = (3)(1.960 / 0.10)$^2$ = (3)(384) = **1152** claims.

**2.7. D.** For frequency, the general formula for the Standard for Full Credibility in terms of claims is: $\frac{\sigma_f^2}{\mu_f}\left(\frac{z_{1-\alpha/2}}{k}\right)^2$ Assuming $\alpha$ and k are fixed, then the Standard for Full Credibility is proportional to the ratio of the variance to the mean. For the Poisson this ratio is one. For the Negative Binomial this ratio is: {r$\beta$(1+$\beta$)} / (r$\beta$) = 1 + $\beta$.
Thus the second Standard is 1+$\beta$ = 1.5 times the first Standard.
Comment: The Negative Binomial has more random fluctuation than the Poisson, and therefore the standard for Full Credibility is larger.

**2.8. D.** $\Phi$(1.960) = 0.975, so that $z_{1-\alpha/2}$ = 1.960. $\lambda_F = \left(\frac{z_{1-\alpha/2}}{k}\right)^2$ = (1.960 / 0.10)$^2$ = **384**.

**2.9. C.** $\Phi$(2.326) = 0.99, so that $z_{1-\alpha/2}$ = 2.326.

$\lambda_F = \left(\frac{z_{1-\alpha/2}}{k}\right)^2$ = (2.326 / 0.06)$^2$ = **1503** claims.

**2.10**. **E.** $\Phi$(2.576) = 0.995, so that $z_{1-\alpha/2}$ = 2.576.

$\lambda_F = \left(\frac{z_{1-\alpha/2}}{k}\right)^2$ = (2.576 / 0.075)$^2$ = **1180** claims.

**2.11. A.** $\Phi$(1.645) = 0.95, so that $z_{1-\alpha/2}$ = 1.645.

$\lambda_F = \left(\frac{z_{1-\alpha/2}}{k}\right)^2$ = (1.645 / 0.05)$^2$ = 1082 claims. $\sigma_f^2/\mu_f$ = (0.4)(0.6)m/(0.4m) = 0.6.

$\lambda_F \frac{\sigma_f^2}{\mu_f}$ = (1082)(0.6) = **650 claims**.

**2.12.** Let N be the sample size. Variance is: N (0.4)(1 - 0.4) = 0.24 N.
We want: (0.02) (0.4N) = 1.960 $\sqrt{0.24N}$. $\Rightarrow$ N = **14,406**.
Alternately, we have a Bernoulli frequency, with variance / mean = (0.4)(0.6) / 0.4 = 0.6.

$\left(\frac{z_{1-\alpha/2}}{k}\right)^2$ = (1.960/0.02)$^2$ = 9604. Standard for full credibility is: (0.6)(9604) = 5762.4 claims.

Standard for Full Credibility in terms of exposures is: 5762.4/0.4 = **14,406**.

**2.13. E.** Since the full credibility standard is inversely proportional to the square of k:
$\lambda_F = \left(\frac{z_{1-\alpha/2}}{k}\right)^2$, X/Y = (10%/5%)$^2$ = 4. Alternately, one can compute the values of X and Y assuming one is dealing with the standard for frequency and that the frequency is Poisson.
(The answer to this question does not depend on these assumptions.)
For k = 5% and $\alpha$ = 1 - 90% = 10%: $\Phi$(1.645) = 0.95, so that $z_{1-\alpha/2}$ = 1.645,

$\lambda_F = \left(\frac{z_{1-\alpha/2}}{k}\right)^2$ = (1.645 / 0.05)$^2$ = 1082 = X.

For k = 10% and $\alpha$ = 1 - 90% = 10%: $\Phi$(1.645) = 0.95, so that $z_{1-\alpha/2}$ = 1.645,

$\lambda_F = \left(\frac{z_{1-\alpha/2}}{k}\right)^2$ = (1.645 / 0.10)$^2$ = 271 = Y.

Thus X/Y = 1082 / 271 = **4**.
Comment: As the requirement gets less strict, for example k = 10% rather than 5%, the number of claims needed for Full Credibility decreases.

**2.14. E.** The classical approach to credibility attempts to limit the probability of “large” errors. What is considered a “large” error is determined by the choice of k. The classical approach to credibility does not optimize any particular error measure. The Buhlmann or “greatest accuracy” approach, optimizes the least squares error criterion.

**2.15. C.** Classical Credibility for frequency with k = 0.03 and $\alpha$ = 1 - 0.975 = 0.025.
$z_{1-\alpha/2}$ = 2.24, since $\Phi$(2.24) = 0.9875.

$\lambda_F = \left(\frac{z_{1-\alpha/2}}{k}\right)^2$ = (2.24/0.03)$^2$ = **5575** claims.

**2.16. D.** $\Phi$(1.645) = 0.95 $\Rightarrow$ $z_{1-\alpha/2}$ = 1.645. $\lambda_F = \left(\frac{z_{1-\alpha/2}}{k}\right)^2$ = (1.645/0.05)$^2$ = 1082 claims.

$\lambda_F \frac{\sigma_f^2}{\mu_f}$ = (1082)(0.0025/0.025) = 108.2 claims. 108.2 claims / 0.025 = **4328 exposures**.

Alternately, the standard for full credibility for frequency in terms of number of exposures is:
$\left(\frac{z_{1-\alpha/2}}{k}\right)^2$ $(\sigma_f^2 / \mu_f^2)$ = (1.645/0.05)$^2$ (0.0025 /0.025$^2$) = (1082) (4) = **4328 exposures**.

**2.17. E.** The Standard for Full Credibility (whether it is for frequency, severity, or pure premiums) is inversely proportional to $k^2$.
Thus the revised Standard is: (0.05/0.01)$^2$ (1000) = **25,000**.

**2.18. B.** k = 1%. $\alpha$ = 5%. $\Rightarrow$ $z_{1-\alpha/2}$ = 1.960. $\lambda_F = \left(\frac{z_{1-\alpha/2}}{k}\right)^2$ = 38,416 claims.

Binomial Frequency.

$\frac{\sigma_f^2}{\mu_f}$ = mq(1-q)/(mq) = 1 - q. 34,574 = $\lambda_F \frac{\sigma_f^2}{\mu_f}$ = 38,416(1 - q). $\Rightarrow$ q = **0.100**.

**2.19. D.** $\alpha$ = 5%. $\Phi$[1.960] = 0.975. $\Rightarrow$ $z_{1-\alpha/2}$ = 1.960. k = 0.02.

$\lambda_F$ = (1.960/0.02)$^2$ = 9604 claims.

9604 claims. $\Leftrightarrow$ 9604/0.03 = 320,133 exposures.

Assuming Poisson claim frequency: W = 320,133 exposures.

For a Binomial: $\frac{\text{variance of frequency}}{\text{mean fequency}} = \frac{mq(1-q)}{mq}$ = 1 - q = 1 - 0.03 = 0.97.

Standard for Full Credibility: (0.97)(9604) = 9316 claims.

$\Leftrightarrow$ 9316/0.03 = 310,533 exposures = V.

$|W - V| = |\ 320,133 - 310,533\ |$ = **9600** exposures.

## Section 3, Full Credibility for Severity

You are less likely to be asked a question on the exam involving applying Classical Credibility to estimating future severities. However, the same ideas easily apply as they did to frequencies.

Assume we have 5 claims each independently drawn from an Exponential Distribution:
$F(x) = 1 - e^{-x/100}$.
Then since the variance of an Exponential is $\theta^2$, the variance of a single claim is: $100^2 = 10{,}000$.
Thus the variance of the total cost of five independent claims is (5)(10,000) = 50,000.
The observed severity is the total observed cost divided by the number of claims, in this case 5.
Thus the variance of the observed severity is $(1/5)^2$ (50000) = 2000.

When one has N claims, the variance of the observed severity is $(N10000) / N^2 = 10{,}000 / N$.
In general, the variance of the observed severity =
(process variance of the severity) / (number of claims) = $\sigma_{Sev}^2 / N$.
Therefore, the standard deviation for the observed severity is $\sigma_{Sev} / \sqrt{N}$.

Assume we wish to have a chance of $1 - \alpha$ that the observed severity will be within ±k of the true average severity. As before with credibility for the frequency, use the Normal Approximation, with $z_{1-\alpha/2}$ such that $\Phi(z_{1-\alpha/2}) = 1 - \alpha/2$.
Then within $\pm z_{1-\alpha/2}$(standard deviations of observed severity) of the mean covers probability of $1 - \alpha$ on the Normal Distribution. Therefore, in order to have $1 - \alpha$ probability of differing from the mean severity by less than $\pm k\mu_S$, we want $z_{1-\alpha/2}\ (\sigma_{Sev} / \sqrt{N}) = k\ \mu_S$.

Solving: $N = \left(\frac{z_{1-\alpha/2}}{k}\right)^2 \frac{\sigma_S^2}{\mu_S^2} = \lambda_F\ CV_{Sev}^2$.

The Standard for Full Credibility for the Severity in terms of number of expected claims is:

$$\boldsymbol{\left(\frac{z_{1-\alpha/2}}{k}\right)^2 \frac{\sigma_S^2}{\mu_S^2} = \lambda_F\ CV_{Sev}^2},$$

where $CV_{Sev}$ is the coefficient of variation of the severity = standard deviation / mean.[29]

Note that no assumption was made about the distributional form of the frequency.
The Standard for Full Credibility for severity does not depend on whether the frequency is Poisson, Negative Binomial, etc. However, we have assumed that frequency and severity are independent and that all of the claims are drawn from the same size of loss distribution.

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[29] Equation 6.11 in NonLife Actuarial Models: Theory, Methods and Evaluation.

Exercise: Let $\alpha$ = 10% and k = 5%.
If the coefficient of variation of the severity is 3, then what is the Standard for Full Credibility for the <u>severity</u> in terms of expected claims?
[Solution: $\lambda_F = (1.645/0.05)^2$ = 1082 claims.
Then the Standard for Full Credibility for the severity is: $(1082)(3^2)$ = 9738 expected claims.]

<u>Exposures vs. Claims</u>:

Standards for Full Credibility have been calculated so far in terms of the expected number of claims. It is common to translate these into a number of exposures by dividing by the (approximate) expected claim frequency. So for example, if the Standard for Full Credibility is 9738 claims and the expected claim frequency in Homeowners Insurance were 0.04 claims per house-year, then 9738 / 0.04 = 243,000 house-years would be a corresponding Standard for Full Credibility in terms of exposures.

In general, one can divide the Standard for Full Credibility in terms of claims by $\mu_f$, in order to get it in terms of exposures.

Thus in general, the Standard for Full Credibility for the Severity in terms of number of exposures is: $\left(\frac{z_{1-\alpha/2}}{k}\right)^2 \frac{\sigma_s^2}{\mu_s^2} \frac{1}{\mu_f} = \lambda_F \frac{CV_{Sev}^2}{\mu_f}$,
where $CV_{Sev}$ is the coefficient of variation of the severity.

Problems:

**3.1** (2 points) You are given the following:
● The claim amount distribution has mean 500, variance 5,000,000.
● Frequency and severity are independent.
Find the number of claims required for full credibility, if you require that there will be a 80% chance that the estimate of the severity is correct within ±2%.
A. Less than 60,000
B. At least 60,000 but less than 70,000
C. At least 70,000 but less than 80,000
D. At least 80,000 but less than 90,000
E. At least 90,000

**3.2** (3 points) You are given the following:
● The claim amount distribution is LogNormal, with $\sigma$ = 1.5.
● Frequency and severity are independent.
Find the number of claims required for full credibility, if you require that there will be a 95% chance that the estimate of the severity is correct within ±10%.
A. Less than 2900
B. At least 2900, but less than 3000
C. At least 3000, but less than 3100
D. At least 3100, but less than 3200
E. At least 3200

**3.3** (3 points) You are given the following:
● The claim amount distribution is Pareto, with $\alpha$ = 2.3.
● Frequency and severity are independent.
Find the number of claims required for full credibility, if you require that there will be a 90% chance that the estimate of the severity is correct within ±7.5%.
(A) 2900 (B) 3100 (C) 3300 (D) 3500 (E) 3700

**3.4** (2 points) You require that there will be a 99% chance that the estimate of the severity is correct within ±5%. 17,000 claims are required for full credibility.
Determine the coefficient of variation of the size of loss distribution.
A. Less than 1
B. At least 1, but less than 2
C. At least 2, but less than 3
D. At least 3, but less than 4
E. At least 4

**3.5** (2 points) You are given the following:
● The estimated claim frequency is 4%.
● Number of claims and claim severity are independent.
● Claim severity has the following distribution:

| Claim Size | Probability |
|---|---|
| 10 | 0.50 |
| 20 | 0.30 |
| 50 | 0 20 |

Determine the number of <u>exposures</u> needed so that the estimated average size of claim is within 2% of the expected size with 95% probability.
(A) 95,000 (B) 105,000 (C) 115,000 (D) 125,000 (E) 135,000

<u>**3.6**</u> (3 points) An actuary is determining the number of claims needed for full credibility in three different situations:
(1) Assuming claim severity is Pareto, the estimated claim severity is to be within r of the true value with probability p.
(2) Assuming claim frequency is Binomial, the estimated claim frequency is to be within r of the true value with probability p.
(3) Assuming claim severity is Exponential, the estimated claim severity is to be within r of the true value with probability p.
The same values of r and p are chosen for each situation.
Rank these three limited fluctuation full credibility standards from smallest to largest.
(A) 1, 2, 3 (B) 2, 1, 3 (C) 3, 1, 2 (D) 2, 3, 1 (E) None of A, B, C, or D

**3.7** (4 points) Claim severity has a mean of 200 and a standard deviation of 500.
A sample of 400 claims is observed.
Assume that the sample mean is approximately normally distributed.
(a) (0.5 points) What is the coefficient of variation of the severity distribution?
(b) (0.5 points) What is the coefficient of variation of the sample mean of the claim severity?
(c) (2 points) What is the probability that the sample mean is within 5% of the true mean?
(d) (1 point) Within what percentage of the true mean will the sample mean be 98% of the time?

Solutions to Problems:

**3.1. D.** $z_{1-\alpha/2}$ = 1.282 since $\Phi$(1.282) = 0.90. $\lambda_F = \left(\frac{z_{1-\alpha/2}}{k}\right)^2$ = $(1.282/0.02)^2$ = 4109.

For severity, the Standard For Full Credibility is:
$\lambda_F$ $CV^2$ = (4109) (5,000,000/$500^2$) = (4109)(20) = **82,180 claims**.

**3.2. E.** $z_{1-\alpha/2}$ = 1.960 since $\Phi$(1.960) = 0.975. $\lambda_F = \left(\frac{z_{1-\alpha/2}}{k}\right)^2$ = $(1.960/0.1)^2$ = 384.

For the LogNormal Distribution: Mean = $\exp(\mu + 0.5\,\sigma^2)$,
Variance = $\exp(2\mu + \sigma^2)\,\{\exp(\sigma^2) - 1\}$, and therefore, the Coefficient of Variation = $\sqrt{\exp(\sigma^2) - 1}$.

For $\sigma$ = 1.5, the $CV^2$ = exp(2.25) - 1 = 8.49.
For severity, the Standard For Full Credibility is: $\lambda_F$ $CV^2$ = (384)(8.49) = **3260** claims.

Alternately, 1 + $CV^2$ = $E[X^2]/E[X]^2$ = $\exp(2\mu + 2\sigma^2)$ / $\exp(2\mu + \sigma^2)$ = $\exp(\sigma^2)$. Proceed as before.

**3.3. E.** $\Phi$(1.645) = 0.95, so that $z_{1-\alpha/2}$ = 1.645. $\lambda_F = \left(\frac{z_{1-\alpha/2}}{k}\right)^2$ = $(1.645/0.075)^2$ = 481.

Using the formulas for the moments: $CV^2 = E[X^2] / E[X]^2 - 1 = \dfrac{2\theta^2 / \{(\alpha-1)(\alpha-2)\}}{(\theta/(\alpha-1))^2} - 1 =$

$2(\alpha-1)/(\alpha-2) - 1 = \alpha/(\alpha-2)$. For $\alpha$ = 2.3, $CV^2$ = 2.3 / 0.3 = 7.667.
Therefore $\lambda_F$ ($CV^2$) = (481)(7.667) = **3688** claims.

Comment: The smaller the shape parameter of the Pareto Distribution $\alpha$, the heavier-tailed the Pareto Distribution, making it harder to limit fluctuations in the estimated severity, since a single large clam can affect the observed average severity. Therefore, the smaller $\alpha$, the larger the Standard for Full Credibility.

**3.4. C.** $\Phi$(2.576) = 0.995, so that $z_{1-\alpha/2}$ = 2.576. $\lambda_F = \left(\frac{z_{1-\alpha/2}}{k}\right)^2$ = $(2.576/0.05)^2$ = 2654.

17,000 = $\lambda_F\, CV_{Sev}^2$. $\Rightarrow$ $CV_{Sev} = \sqrt{\frac{17,000}{2654}}$ = **2.53**.

**3.5. D.** We have $z_{1-\alpha/2}$ = 1.960 since $\Phi$(1.960) = 0.975. Therefore $\lambda_F = \left(\frac{z_{1-\alpha/2}}{k}\right)^2$ =

$(1.960/0.02)^2$ = 9604. The mean severity is: (10)(0.5) + (20)(0.3) + (50)(0.2) = 21.
The variance of the severity is: $(11^2)(0.5) + (1^2)(0.3) + (29^2)(0.2) = 229$.
Thus the coefficient of variation squared = 229 / $21^2$ = 0.519.
$\lambda_F$ ($CV^2$) = (9604) (0.519) = 4984 claims.
This corresponds to: 4984 / 0.04 = **124,600** exposures.

**3.6. D.** (1) The coefficient of variation for the Pareto is greater than 1 (or infinite).
Thus the Standard for Full Credibility for Severity is: $CV_{Sev}^2 \lambda_F > 1^2 \lambda_F = \lambda_F$.
(2) For the Binomial, $\frac{\text{variance}}{\text{mean}} = \frac{m\ q\ (1 - q)}{m\ q}$ = 1 - q.
Standard for Full Credibility for Frequency is: $(1 - q)\ \lambda_F < \lambda_F$.
(3) The CV for the Exponential is 1.
Thus the Standard for Full Credibility for Severity is: $CV_{Sev}^2 \lambda_F = 1^2 \lambda_F = \lambda_F$.
Thus ranking the standards from smallest to largest: 2, 3, 1.
Comment: Since it is heavier tailed than the Exponential, when it is finite, the CV of the Pareto is greater than that of the Exponential.
From its mean and second moment, one can determine that for a Pareto Distribution:
Coefficient of Variation = $\sqrt{\frac{\alpha}{\alpha - 2}}$, $\alpha > 2$.

**3.7.** (a) 500/200 = **2.5**.
(b) $\frac{500 / \sqrt{400}}{200}$ = 2.5/20 = **0.125**.
(c) We want $\bar{X}$ within 190 to 210.
$\bar{X}$ is approximately normally distributed with mean 200 and standard deviation: $500/\sqrt{400}$ = 25.
Probability = $\Phi[\frac{210 - 200}{25}] - \Phi[\frac{190 - 200}{25}]$ = $\Phi$[0.4] - $\Phi$[-0.4] = 0.6554 - (1 - 0.6554) = **31.08%**.
(d) We want 1% on either side. $z_{99\%}$ = 2.326.
So we are within: ±(2.326)(25) = ±58.15. 58.15/200 = **29.08%**.
Comment: Similar to Exercise 6.1 in Nonlife Actuarial Models: Theory, Methods and Evaluation.
The standard for full credibility for severity, in order to be within 29.08% of the true mean 98% of the time is: $(2.5^2)(2.326/0.2908)^2$ = 400 claims.

## *Section 4, Variance of Pure Premiums and Aggregate Losses*[30] [31]

The same formulas can be used to calculate the process variance of pure premiums, aggregate losses, or loss ratios. The loss ratio is defined as losses divided by premiums.

Exercise: XYZ Insurance insures 123,000 automobiles for one year.
Total premiums are $57 million. Total loss payments are $48 million.
What are the pure premium, aggregate annual loss, and loss ratio?
[Solution: The aggregate loss is $48 million. Pure premium = $48 million/123,000 car years = $390/ car year. Loss Ratio = $48 million/ $57 million = 84.2%.]

Aggregate Loss:

The Aggregate Loss is the total dollars of loss for an insured or set of an insureds. If not stated otherwise, the period of time is one year.

For example, during 1999 the MT Trucking Company may have had $952,000 in aggregate losses on its commercial automobile collision insurance policy. All of the trucking firms insured by the Fly-by-Night Insurance Company may have had $15.1 million dollars in aggregate losses for collision. The dollars of aggregate losses are determined by how many losses there are and the severity of each one.

Exercise: During 1998 MT Trucking suffered three collision losses for $8,000, $13,500, and $22,000. What are its aggregate losses?
[Solution: $8,000 + $13,500 + $22,000 = $43,500.]

**Aggregate Losses =**

$$\textbf{(\# of Exposures)} \frac{\text{\# of Claims}}{\text{\# of Exposures}} \frac{\text{\$ of Loss}}{\text{\# of Claims}} = \textbf{(Exposures) (Frequency) (Severity)}.$$

If one is not given the frequency per exposure, but is rather just given the frequency for the whole number of exposures,[32] whatever they are for the particular situation, then
**Aggregate Losses = (Frequency) (Severity)**.

Similarly, the Aggregate Payment is the total dollars paid by an insurer on an insurance policy or set of insurance policies. If not stated otherwise, the period of time is one year.

Exercise: During 1998 MT Trucking suffered three collision losses for $8,000, $13,500, and $22,000. MT Trucking has a $10,000 per claim deductible on its policy with the Fly-by-Night Insurance Company. What are the aggregate payments by Fly-by-Night?
[Solution: $0 + $3,500 + $12,000 = $15,500.]

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[30] See Equation 3.27 in NonLife Actuarial Models: Theory, Methods and Evaluation, not on the syllabus.
[31] These ideas underlie the formulas in the following two sections.
[32] For example, the expected number claims from a large commercial insured is 27.3 per year or the expected number of Homeowner's claims expected by XYZ Insurer in the State of Florida is 12,310.

Pure Premium:

**Pure Premium = Aggregate Loss per exposure.**

The mean pure premium is: (mean frequency per exposure)(mean severity).

Expected Aggregate Loss = (Mean Pure Premium)(Exposure)

Estimated expected pure premiums serve as a starting point for pricing insurance.[33]

Process Variance:

Random fluctuation occurs when one rolls dice, spins spinners, picks balls from urns, etc. The observed result varies from time period to time period due to random chance. This is also true for the pure premium observed for a collection of insureds.[34] The variance of the observation for a given risk that occurs due to random fluctuation is referred to as the **process variance**. That is what will be discussed here.[35]

Since pure premiums depend on both the number of claims and the size of claims, pure premiums have more reasons to vary than do either frequency or severity individually.

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[33] *One would have to load for loss adjustment expenses, expenses, taxes, profits, etc.*
[34] In fact this is the fundamental reason for the existence of insurance.
[35] The process variance is distinguished from the variance of the hypothetical pure premiums as discussed in Buhlmann Credibility.

**Independent Frequency and Severity**:

You are given the following:
- For a given risk, the number of claims for a single exposure period is given by a Binomial Distribution with q = 0.3 and m = 2.
- The size of a claim will be 50, with probability 80%, or 100, with probability 20%.
- Frequency and severity are independent.

Exercise: Determine the variance of the pure premium for this risk.
[Solution: List the possibilities and compute the first two moments:

| Situation | Probability | Pure Premium | Square of P.P. |
|---|---|---|---|
| 0 claims | 49.00% | 0 | 0 |
| 1 claim @ 50 | 33.60% | 50 | 2500 |
| 1 claim @ 100 | 8.40% | 100 | 10000 |
| 2 claims @ 50 each | 5.76% | 100 | 10000 |
| 2 claims: 1 @ 50 & 1 @ 100 | 2.88% | 150 | 22500 |
| 2 claims @ 100 each | 0.36% | 200 | 40000 |
| | | | |
| Overall | 100.0% | 36 | 3048 |

For example, the probability of 2 claims is: $0.3^2 = 9\%$. We split this 9% among the possible claim sizes: 50 and 50 @ (0.8)(0.8) = 64%, 50 and 100 @ (0.8)(0.2) = 16%,
100 and 50 @ (0.2)(0.8) = 16%, 100 and 100 @ (0.2)(0.2) = 4%.
(9%)(64%) = 5.76%, (9%)(16% + 16%) = 2.88%, (9%)(4%) = 0.36%.
One takes the weighted average over all the possibilities. The average Pure Premium is 36.
The second moment of the Pure Premium is 3048.
Therefore, the variance of the pure premium is: $3048 - 36^2 = 1752$.]

In this case since **frequency and severity are independent** one can make use of the following formula:

**Process Variance of Pure Premium =**
**(Mean Frequency) (Variance of Severity) + (Mean Severity)$^2$ (Variance of Frequency)**

$$\sigma_{PP}^2 = \mu_{Freq}\, \sigma_{Sev}^2 + \mu_{Sev}^2\, \sigma_{Freq}^2.$$

Memorize this formula! Note that each of the two terms has a mean and a variance, one from frequency and one from severity. Each term is in dollars squared; that is one way to remember that the mean severity (which is in dollars) enters as a square while that for mean frequency (which is not in dollars) does not.

In the above example, the mean frequency is mq = 0.6 and the variance of the frequency is: mq(1 - q) = (2)(0.3)(0.7) = 0.42. The average severity is 60 and the variance of the severity is: $(0.8)(10^2) + (0.2)(40^2) = 400$. Thus, the process variance of the pure premium is: $(0.6)(400) + (60^2)(0.42) = 1752$, which matches the result calculated previously.

**This same formula can also be used to compute the process variance of the aggregate losses, when frequency and severity are independent.**

$$\sigma_{Agg}^2 = \mu_{Freq}\, \sigma_{Sev}^2 + \mu_{Sev}^2\, \sigma_{Freq}^2.$$

The sum of losses is just the product of the pure premium and the number of exposures. Provided the risk processes for the individual exposures are independent and identical, then both $\mu_F$ and $\sigma_F^2$ are multiplied by the number of exposures as is $\sigma_{PP}^2$.
In the above example, the process variance of the sum of the losses from 10 exposures is: $(6)(400) + (60^2)(4.2) = 17,520 = (10)(1752)$.

*Dependent Frequency and Severity:*

While frequency and severity are almost always independent, if they are dependent one can use a more general technique.[36] The first and second moments can be calculated by listing the pure premiums for all the possible outcomes and taking the weighted average, applying the probabilities as weights to either the pure premium or its square. In continuous cases, this will involve taking integrals, rather than sums. Then one can calculate the variance of the pure premium as: second moment - (first moment)$^2$.

Aggregate Losses Versus Pure Premiums:

Exercise: Assume frequency is Poisson with mean 5% for one exposure.
Severity is Exponential, with mean 100. What is the mean and variance of the pure premium?
[Solution: The mean pure premium = $\mu_{Freq}\, \mu_{Sev}$ = (5%)(100) = 5.
Variance of pure premium = $\mu_{Freq}\, \sigma_{Sev}^2 + \mu_{Sev}^2\, \sigma_{Freq}^2$ = $(5\%)(100^2) + (100^2)(5\%) = 1000$.]

Exercise: If we insure 1000 independent, identically distributed exposures, what is the mean and variance of the aggregate loss?
[Solution: Overall frequency is Poisson with mean: (5%)(1000) = 50.
Mean aggregate: (50)(100) = 5000. Variance of aggregate: $(50)(100^2) + (100^2)(50)$ = 1 million.]

[36] See 4B, 5/95, Q.14 and 4, 11/02, Q.36 for examples where frequency and severity are dependent.

So we can use basically the same formula for the mean and variance when working with either the aggregate losses or pure premiums.[37] When working with pure premiums, we used 5% as the mean frequency and 5% as the variance of the frequency, the mean and variance of the frequency distribution for a single exposure. However, when working with the aggregate losses, we used 50 as the mean frequency and 50 as the variance of the frequency, the mean and variance of the frequency distribution of the whole portfolio.

Note that when we add up 1000 independent, identically distributed exposures, we get 1000 times the mean and 1000 times the variance for a single exposure.
In general, when we have N identical, independent exposures:
Mean aggregate loss = (N)(mean pure premium).
Variance of aggregate loss = (N)(variance of pure premium).

*Derivation of the formula for the Process Variance of the Pure Premium:*

The above formula for the process variance of the pure premium for independent frequency and severity is a special case of the formula that also underlies analysis of variance:
$\text{Var}(Y) = E_X[\text{VAR}_Y(Y|X)] + \text{VAR}_X(E_Y[Y|X])$, where X and Y are any random variables.

Letting Y be the pure premium PP and X be the number of claims N in the above formula gives:
$\text{Var}(PP) = E_N[\text{VAR}_{PP}(PP|N)] + \text{VAR}_N(E_{PP}[PP \mid N]) = E_N[N\sigma_{Sev}^2] + \text{VAR}_N(\mu_{Sev}N) =$
$E_N[N]\sigma_{Sev}^2 + \mu_{Sev}^2\text{VAR}_N(N) = \mu_{Freq}\ \sigma_{Sev}^2 + \mu_{Sev}^2\ \sigma_{Freq}^2$.

Where I have used the assumption that the frequency and severity are independent and the facts:

- For a fixed number of claims N, the variance of the pure premium is the variance of the sum of N independent identically distributed variables each with variance $\sigma_{Sev}^2$. (Since frequency and severity are assumed independent, $\sigma_{Sev}^2$ is the same for each value of N.) Such variances add so that $\text{VAR}_{PP}(PP \mid N) = N\ \sigma_{Sev}^2$.
- For a fixed number of claims N, for frequency and severity independent the expected value of the pure premium is N times the mean severity: $E_{PP}[PP \mid N] = \mu_{Sev}\ N$.
- Since with respect to N the variance of the severity acts as a constant:
  $E_N[N\sigma_{Sev}^2] = \sigma_{Sev}^2 E_N[N] = \mu_{Freq}\ \sigma_{Sev}^2$.
- Since with respect to N the mean of the severity acts as a constant:
  $\text{VAR}_N(\mu_{Sev}N) = \mu_{Sev}^2\ \text{VAR}_N(N) = \mu_{Sev}^2\ \sigma_{Freq}^2$.

[37] One can define the whole portfolio as one “exposure”; then the Aggregate Loss is mathematically just a special case of the Pure Premium.

Let's apply this derivation to a previous example. You were given the following:
- For a given risk, the number of claims for a single exposure period is given by a Binomial Distribution with q = 0.3 and m = 2.
- The size of the claim will be 50, with probability 80%, or 100, with probability 20%.
- Frequency and severity are independent.

There are only three possible values of N: N = 0, N = 1 or N = 2. If N = 0, then PP = 0. If N = 1, then either PP = 50 with 80% chance or PP = 100 with 20% chance. If N = 2, then PP = 100 with 64% chance, PP =150 with with 32% chance or PP = 200 with 4% chance.

We then get:

| N | Probability | Mean PP Given N | Square of Mean PP Given N | Second Moment of of PP Given N | Var of PP Given N |
|---|---|---|---|---|---|
| 0 | 49% | 0 | 0 | 0 | 0 |
| 1 | 42% | 60 | 3600 | 4000 | 400 |
| 2 | 9% | 120 | 14400 | 15200 | 800 |
| | | | | | |
| Mean | | 36 | 2808 | | 240 |

For example given two claims the second moment of the pure premium =
$(64\%)(100^2) + (32\%)(150^2) + (4\%)(200^2) = 15,200$.
Thus given two claims the variance of the pure premium is: $15,200 - 120^2 = 800$.

Thus $E_N[VAR_{PP}(PPIN)] = 240$, and $VAR_N(E_{PP}[PPIN]) = 2808 - 36^2 = 1512$. Thus the variance of the pure premium is $E_N[VAR_{PP}(PPIN)] + VAR_N(E_{PP}[PPIN]) = 240 + 1512 = 1752$, which matches the result calculated above. The (total) process variance of the pure premium has been split into two pieces. The first piece calculated as 240, is the expected value over the possible numbers of claims of the process variance of the pure premium for fixed N. The second piece calculated as 1512, is the variance over the possible numbers of the claims of the mean pure premium for fixed N.

<u>Expected Value of the Process Variance</u>:

In order to solve questions involving Greatest Accuracy/Buhlmann Credibility and Pure Premiums or Aggregate Losses one has to compute the Expected Value of the Process Variance of the Pure Premium or Aggregate Losses.[38] This involves being able to compute the process variance for each specific type of risk and then averaging over the different types of risks possible. This may involve taking a weighted average or performing an integral.

[38] "See "Mahler's Guide to Buhlmann Credibility."

**Poisson Frequency**:

Assume you are given the following:
- For a given risk, the number of claims for a single exposure period is Poisson with mean 7.
- The size of the claim will be 50, with probability 80%, or 100, with probability 20%.
- Frequency and severity are independent.

Exercise: Determine the variance of the pure premium for this risk.
[Solution: $\mu_{Freq} = \sigma_{Freq}^2 = 7$. $\mu_{Sev} = 60$. $\sigma_{Sev}^2 = 400$.
$\sigma_{PP}^2 = \mu_{Freq}\ \sigma_{Sev}^2 + \mu_{Sev}^2\ \sigma_{Freq}^2 = (7)(60^2) + (400)(7) = 28{,}000$.]

In the case of a Poisson Frequency with independent frequency and severity the formula for the process variance of the pure premium simplifies. Since $\mu_{Freq} = \sigma_{Freq}^2$ :
$\sigma_{PP}^2 = \mu_{Freq}\ \sigma_{Sev}^2 + \mu_{Sev}^2\ \sigma_{Freq}^2 = \mu_{Freq}(\sigma_{Sev}^2 + \mu_{Sev}^2) = \mu_{Freq}$(2nd moment of severity).

When there is a **Poisson Frequency, the variance of aggregate losses is:**
**$\lambda$ (2nd moment of severity).**

In the example above, the second moment of the severity is: $(0.8)(50^2) + (0.2)(100^2) = 4000$.
Thus $\sigma_{PP}^2 = \lambda$ (2nd moment of the severity) = (7)(4000) = 28,000. If instead we have 20 independent exposures and take the sum of the losses, then the variance of these aggregate losses is (140)(4000) = (20)(28,000) = 560,000.

As another example, assume you are given the following:
- For a given risk, the number of claims for a single exposure period is Poisson with mean 3645.
- The severity distribution is LogNormal, with parameters $\mu = 5$ and $\sigma = 1.5$.
- Frequency and severity are independent

Exercise: Determine the variance of the pure premium for this risk.
[Solution: The second moment of the severity = $\exp(2\mu + 2\sigma^2) = \exp(14.5) = 1{,}982{,}759.264$.
Thus $\sigma_{PP}^2 = \lambda$ (2nd moment of the severity) = (3645)(1,982,759) = $7.22716 \times 10^9$.]

Normal Approximation:

For large numbers of expected claims, the observed pure premiums are approximately Normally Distributed.[39] For example, continuing the example above,
mean severity = $\exp(\mu + 0.5\sigma^2)$ = exp(6.125) = 457.14.
Thus the mean pure premium is (3645)(457.14) = 1,666,292.
One could ask what the chance of the observed pure premiums being between 1.4997 million and 1.8329 million.
Since the variance is $7.22716 \times 10^9$, the standard deviation of the pure premium is 85,013.
Thus the probability of the observed pure premiums being within ±10% of 1.6663 million is approximately:

$\Phi$[(1.8329 million - 1.6663 million) / 85,013] - $\Phi$[(1.4997 - 1.6663 million) / 85,013] =

$\Phi$[1.96] - $\Phi$[-1.96] = 0.975 - (1 - 0.975) = 95%.

Thus in this case with an expected number of claims equal to 3645, there is about a 95% chance that the observed pure premium will be within ±10% of the expected value. One could turn this around and ask how many claims would one need in order to have a 95% chance that the observed pure premium will be within ±10% of the expected value. The answer of 3645 claims could be taken as a Standard for Full Credibility for the Pure Premium.[40]

[39] The more skewed the severity distribution, the higher the expected frequency has to be for the Normal Approximation to produce worthwhile results.
[40] As discussed in the next section.

Policies of Different Types:

Let us assume we have a portfolio consisting of two types of policies:

| Type | Number of Policies | Mean Aggregate Loss per Policy | Variance of Aggregate Loss per Policy |
|---|---|---|---|
| A | 10 | 6 | 3 |
| B | 20 | 9 | 4 |

Assuming the results of each policy are independent, then the mean aggregate loss for the portfolio is: (10)(6) + (20)(9) = 240.
The variance of aggregate loss for the portfolio is: (10)(3) + (20)(4) = 110.

For independent policies, the means and variances add.
Note that as we have more policies, all other things being equal, the coefficient of variation goes down.

Exercise: Compare the coefficient of variation of aggregate losses in the above example to that if one had instead 100 policies of Type A and 200 policies of type B.
[Solution: For the original example, CV = $\sqrt{110}$ / 240 = 0.043.
For the new example, CV = $\sqrt{1100}$ / 2400 = 0.0138.]

Exercise: For each of the two cases in the previous exercise, using the Normal Approximation estimate the probability that the aggregate losses will be at least 5% more than their mean.
[Solution: For the original example, Prob[Agg. > 252] ⇔ 1 - $\Phi[(252 - 240)/\sqrt{110}]$ = 1 - $\Phi[1.144]$ = 12.6%. For the new example, Prob[Agg. > 2520] ⇔ 1 - $\Phi[(2520 - 2400)/\sqrt{1100}]$ = 1 - $\Phi[3.618]$ = 0.015%.]

For a larger portfolio, all else being equal, there is less chance of an extreme outcome in a given year measured as a percentage of the mean.

Problems:

Use the following information for the next five questions:
- The number of claims for a single year is Poisson with mean 6200.
- The severity distribution is LogNormal, with parameters $\mu = 5$ and $\sigma = 0.6$.
- Frequency and severity are independent.

**4.1** (1 point) Determine the expected annual aggregate losses.
A. Less than 0.8 million
B. At least 0.8 million but less than 0.9 million
C. At least 0.9 million but less than 1.0 million
D. At least 1.0 million but less than 1.1 million
E. At least 1.1 million

**4.2** (2 points) Determine the variance of the annual aggregate losses.
A. Less than 270 million
B. At least 270 million but less than 275 million
C. At least 275 million but less than 280 million
D. At least 280 million but less than 285 million
E. At least 285 million

**4.3** (2 points) Determine the chance that the observed annual aggregate losses will be more than 1.130 million. (Use the Normal Approximation.)
A. Less than 4%
B. At least 4%, but less than 5%
C. At least 5%, but less than 6%
D. At least 6%, but less than 7%
E. At least 7%

**4.4** (2 points) Determine the chance that the observed annual aggregate losses will be less than 1.075 million. (Use the Normal Approximation.)
A. Less than 4%
B. At least 4%, but less than 5%
C. At least 5%, but less than 6%
D. At least 6%, but less than 7%
E. At least 7%

**4.5** (1 point) Determine the chance that the observed annual aggregate losses will be within ±2.5% of its expected value. (Use the Normal Approximation.)
A. 86% B. 88% C. 90% D. 92% E. 94%

Use the following information for the next three questions:
There are two types of risks.
The risks are independent of each other.
For each type of risk, the frequency and severity are independent.

| Type | Frequency Distribution | Severity Distribution |
|---|---|---|
| 1 | Poisson: $\lambda$ = 4% | Gamma: $\alpha = 3$, $\theta = 10$ |
| 2 | Poisson: $\lambda$ = 6% | Gamma: $\alpha = 3$, $\theta = 15$ |

**4.6** (1 point) Calculate the process variance of the pure premium for Type 1.
A. 48 B. 50 C. 52 D. 54 E. 56

**4.7** (1 point) Calculate the process variance of the pure premium for Type 2.
A. 150 B. 156 C. 162 D. 168 E. 174

**4.8** (1 point) Assume one has a portfolio made up of 80% risks of Type 1,
and 20% risks of Type 2.
For this portfolio, what is the expected value of the process variance of the pure premium?
A. 65 B. 67 C. 69 D. 71 E. 73

Use the following information for the next 3 questions:
- Number of claims for a single insured follows a Negative Binomial distribution, with parameters $r = 30$ and $\beta = 2/3$.
- The amount of a single claim has a Gamma distribution with $\alpha = 4$ and $\theta = 1000$.
- Number of claims and claim severity distributions are independent.

**4.9** (2 points) Determine $E_N[VAR_{PP}(PP \mid N)]$, the expected value over the number of possible claims of the variance of the pure premium for a given number of claims.
A. 50 million B. 60 million C. 70 million D. 80 million E. 90 million

**4.10** (2 points) Determine $VAR_N(E_{PP}[PP \mid N])$, the variance over the number of claims of the expected value of the pure premium for a given number of claims.
A. Less than 400 million
B. At least 400 million but less than 450 million
C. At least 450 million but less than 500 million
D. At least 500 million but less than 550 million
E. At least 550 million

**4.11** (2 points) Determine the pure premium's process variance for a single insured.
A. 575 million B. 585 million C. 595 million D. 605 million E. 615 million

Use the following information for the next four questions:
There are three types of risks.
For each type of risk, the frequency and severity are independent.

| Type | Frequency Distribution | Severity Distribution |
|---|---|---|
| I | Binomial: m =10, q = 0.3 | Pareto: $\alpha$ = 3, $\theta$ = 500 |
| II | Poisson: $\lambda$ = 5 | LogNormal: $\mu$ = 6, $\sigma$ = 0.8 |
| III | Negative Binomial: r = 2.7, $\beta$ = 7/3 | Gamma: $\alpha$ = 2, $\theta$ = 250 |

**4.12** ( 2 points) For a risk of Type **I**, what is the process variance of the pure premium?
A. Less than 0.5 million
B. At least 0.5 million but less than 0.6 million
C. At least 0.6 million but less than 0.7 million
D. At least 0.7 million but less than 0.8 million
E. At least 0.8 million

**4.13** ( 2 points) For a risk of Type **II**, what is the process variance of the pure premium?
A. Less than 2.7 million
B. At least 2.7 million but less than 2.8 million
C. At least 2.8 million but less than 2.9 million
D. At least 2.9 million but less than 3.0 million
E. At least 3 million

**4.14** ( 2 points) For a risk of Type **III**, what is the process variance of the pure premium?
A. Less than 5.7 million
B. At least 5.7 million but less than 5.8 million
C. At least 5.8 million but less than 5.9 million
D. At least 5.9 million but less than 6.0 million
E. At least 6.0 million

**4.15** ( 2 points) Assume one has a portfolio made up of 55% risks of Type **I**, 35% risks of Type **II**, and 10% risks of Type **III**.
For this portfolio, what is the expected value of the process variance of the pure premium?
A. Less than 1.7 million
B. At least 1.7 million but less than 1.8 million
C. At least 1.8 million but less than 1.9 million
D. At least 1.9 million but less than 2.0 million
E. At least 2.0 million

**4.16** (2 points) The aggregate loss distribution follows a compound distribution with the claim frequency distributed as a Poisson with mean 400.
The claim severity is distributed with mean 100 and standard deviation 150.
Assume that the sample mean of the aggregate loss is approximately normally distributed.
Calculate the probability that the observed aggregate loss is within 10% of the mean aggregate loss.
A. Less than 70%
B. At least 70% but less than 75%
C. At least 75% but less than 80%
D. At least 80% but less than 85%
E. At least 85%

**4.17 (4, 5/89, Q.35)** (1 point) For a given risk situation, the frequency distribution follows the Poisson process with mean 0.5. The second moment about the origin for the severity distribution is 1,000. Frequency and severity are independent of each other.
What is the process variance of the aggregate claim amount?
A. 500 B. $0.5^2$ C. $\sqrt{1000}$ D. $0.5\sqrt{1000}$
E. Cannot be determined from the information given

**4.18 (4, 5/90, Q.43)** (2 points) Let N be a random variable for the claim count with:
Pr{N = 4} = 1/4 Pr{N = 5} = 1/2 Pr{N = 6} = 1/4
Let X be a random variable for claim severity with probability density function
$f(x) = 3x^{-4}$, for $1 \le x < \infty$.
Find the coefficient of variation, R, of the aggregate loss distribution, assuming that claim severity and frequency are independent.
A. $R < 0.35$ B. $0.35 \le R < 0.50$ C. $0.50 \le R < 0.65$
D. $0.65 \le R < 0.70$ E. $0.70 \le R$

**4.19 (4, 5/91, Q.26)** (2 points)
The probability function of claims per year for an individual risk is Poisson with a mean of 0.10.
There are four types of claims.
The number of claims has a Poisson distribution for each type of claim.
The table below describes the characteristics of the four types of claims.

| Type of Claim | Mean Frequency | Severity | |
|---|---|---|---|
| | | Mean | Variance |
| W | 0.02 | 200 | 2,500 |
| X | 0.03 | 1,000 | 1,000,000 |
| Y | 0.04 | 100 | 0 |
| Z | 0.01 | 1,500 | 2,000,000 |

Calculate the variance of the pure premium.
A. Less than 70,000
B. At least 70,000 but less than 80,000
C. At least 80,000 but less than 90,000
D. At least 90,000 but less than 100,000
E. At least 100,000

**4.20 (4B, 5/92, Q.31)** (2 points)
You are given that N and X are independent random variables where:
- N is the number of claims, and has a binomial distribution with parameters m = 3 and q = 1/6.
- X is the size of claim and has the following distribution:

P[X=100] = 2/3    P[X=1100] = 1/6    P[X=2100] = 1/6

Determine the coefficient of variation of the aggregate loss distribution.
A. Less than 1.5
B. At least 1.5 but less than 2.5
C. At least 2.5 but less than 3.5
D. At least 3.5 but less than 4.5
E. At least 4.5

**4.21 (5A, 5/94, Q.22)** (1 point) The probability of a particular automobile's being in an accident in a given time period is 0.05. The probability of more than one accident in the time period is zero. The damage to the automobile is assumed to be uniformly distributed over the interval from 0 to 2000. What is the variance of the pure premium?
A. Less than 40,000
B. At least 40,000, but less than 50,000
C. At least 50,000, but less than 60,000
D. At least 60,000, but less than 70,000
E. 70,000 or more

**4.22 (5A, 5/94, Q.35)** (2 points) Your company plans to sell a certain type of policy that is expected to have a claim frequency per policy of 0.15, and a claim size distribution with a mean of 1200 and a standard deviation of 2000.
Management believes that 40,000 of these policies can be written this year.
Assume that for the portfolio of policies, the number of claims is Poisson distributed.
Assume that the premium for each policy is 105% of expected losses. Ignore expenses.
What is the amount of surplus that must be held for this portfolio such that the probability that the surplus will be exhausted is 0.005?

**4.23 (5A, 5/94, Q.39)** (2 points) Your company plans to sell a certain policy but will not commit any surplus to support it. You have determined that the policy will have a mean frequency per policy of 0.045, and a claim size distribution with a mean of 750 and a second moment about the origin of 60,000,000. The price that is suggested is 105% of expected losses.
Management will allow the policy to be written only if the probability that losses will exceed premiums is less than 1%. Ignore expenses and assume that for the portfolio of policies, the number of claims is Poisson distributed. What is the smallest number of policies that must be sold in order to satisfy management's requirement?

**4.24 (5A, 11/94, Q.22)** (1 point) Assume S is a compound Poisson distribution of aggregate claims with a Poisson parameter of 3. Individual claims are uniformly distributed with integer values from 1 to 6. What is the variance of S?
A. Less than 30
B. At least 30, but less than 40
C. At least 40, but less than 50
D. At least 50, but less than 60
E. Greater than or equal to 60

**4.25 (5A, 11/94, Q.38)** (3 points) Your company's automobile liability portfolio consists of three tiers. You have determined that the aggregate claim distribution for each tier is compound Poisson, characterized by the following:

| | Tier 1 | Tier 2 | Tier 3 |
|---|---|---|---|
| Poisson parameter | 2.3 | 3.0 | 1.9 |

| | Pr[X = $x_i$ I a claim has occurred] | | |
|---|---|---|---|
| Claim Amount | Tier 1 | Tier 2 | Tier 3 |
| $x_1$ = 1,000 | 0.60 | 0.70 | 0.80 |
| $x_2$ = 5,000 | 0.30 | 0.20 | 0.15 |
| $x_3$ = 10,000 | 0.10 | 0.10 | 0.05 |

What are the mean and variance of the aggregate claim distribution for the entire automobile portfolio?

**4.26 (4B, 5/95, Q.14)** (3 points) You are given the following:
- For a given risk, the number of claims for a single exposure period will be 1, with probability 3/4; or 2, with probability 1/4.
- If only one claim is incurred, the size of the claim will be 80, with probability 2/3; or 160, with probability 1/3.
- If two claims are incurred, the size of each claim, independent of the other, will be 80, with probability 1/2; or 160, with probability 1/2.

Determine the variance of the pure premium for this risk.
A. Less than 3,600
B. At least 3,600, but less than 4,300
C. At least 4,300, but less than 5,000
D. At least 5,000, but less than 5,700
E. At least 5,700

**4.27 (5A, 5/95, Q.20)** (1 point)
Assume S is compound Poisson with a mean number of claims = 4.
Individual claims will be of amounts 100, 200, and 500 with probabilities 0.4, 0.5, and 0.1, respectively. What is the variance of S?
A. Less than 150,000
B. At least 150,000, but less than 175,000
C. At least 175,000, but less than 200,000
D. At least 200,000 but less than 225,000
E. Greater than or equal to 225,000

**4.28 (4B, 5/96, Q.7)** (3 points) You are given the following:
- The number of claims follows a negative binomial distribution with mean 800 and variance 3,200.
- Claim sizes follow a transformed gamma distribution with mean 3,000 and variance 36,000,000.
- The number of claims and claim sizes are independent.

Using the Central Limit Theorem, determine the approximate probability that the aggregate losses will exceed 3,000,000.
A. Less than 0.005
B. At least 0.005, but less than 0.01
C. At least 0.01, but less than 0.1
D. At least 0.1, but less than 0.5
E. At least 0.5

**4.29 (4B, 5/96, Q.18)** (2 points) Two dice, A and B, are used to determine the number of claims. The faces of each die are marked with either a 1 or a 2, where 1 represents 1 claim and 2 represents 2 claims. The probabilities for each die are:

| Die | Probability of 1 Claim | Probability of 2 Claims |
|---|---|---|
| A | 2/3 | 1/3 |
| B | 1/3 | 2/3 |

In addition, there are two spinners, X and Y, which are used to determine claim size.
Each spinner has two areas marked 2 and 5. The probabilities for each spinner are:

| Spinner | Probability that Claim Size = 2 | Probability that Claim Size = 5 |
|---|---|---|
| X | 2/3 | 1/3 |
| Y | 1/3 | 2/3 |

For the first trial, a die is randomly selected from A and B and rolled. If 1 claim occurs, spinner X is spun. If 2 claims occur, both spinner X and spinner Y are spun. For the second trial, the same die selected in the first trial is rolled again. If 1 claim occurs, spinner X is spun. If 2 claims occur, both spinner X and spinner Y are spun.
Determine the expected amount of total losses for the first trial.
A. Less than 4.8
B. At least 4.8, but less than 5.1
C. At least 5.1, but less than 5.4
D. At least 5.4, but less than 5.7
E. At least 5.7

**4.30** (3 points) In the previous question, 4B, 5/96, Q.18, determine the variance of the distribution of total losses for the first trial.
A. 4 B. 5 C. 6 D. 7 E. 8

**4.31 (5A, 5/96, Q.37)** (2.5 points) Given the following information regarding a single commercial property exposure:
The probability of claim in a policy period is 0.2.
Each risk has at most one claim per period.
The distribution of individual claim amounts is LogNormal with parameters
$\mu = 7.54$ and $\sigma = 1.14$.
Assume that all exposures are independent and identically distributed.
Using the normal approximation, how many exposures must an insurer write to be 95% sure that the total loss does not exceed twice the expected loss?

**4.32 (5A, 11/96, Q.38)** (2 points) A portfolio of insurance policies is assumed to follow a compound Poisson claims process with 100 claims expected. The claim amount distribution is assumed to have an expected value of 1,000 and variance of 1,000,000. These insureds would like to self insure their risk provided that there is no more than a 5% chance of insolvency in the first year. If the premium equals the expected loss, and if there are no other risks, then how much capital must the insureds possess in order to meet their solvency requirement?

**4.33 (5A, 11/98, Q.22)** (1 point) Assume S is compound Poisson with mean number of claims (N) equal to 3. Individual claim amounts follow a distribution with E[X] = 560 and Var[X] = 194,400. What is the variance of S?
A. Less than 1,500,000
B. At least 1,500,000, but less than 1,750,000
C. At least 1,750,000, but less than 2,000,000
D. At least 2,000,000, but less than 2,250,000
E. At least 2,250,000

**4.34 (5A, 11/98, Q.36)** (2 points) Assume the following:
i. $S = X_1 + X_2 + X_3 + ... + X_N$ where $X_1, X_2, X_3, ... X_N$ are identically distributed and $N, X_1, X_2, X_3, \ldots, X_N$ are mutually independent random variables.
ii. N follows a Poisson distribution with $\lambda = 4$.
iii. Expected value of the variance of S given N, E[Var(S I N)] = 1,344.
iv. $\text{Var}(N)\,[E(X)]^2 = 4{,}096$.
Calculate $E[X^2]$.

**4.35 (5A, 5/99, Q.23)** (1 point) Let S be the aggregate amount of claims. The number of claims, N, has the following probability function: Pr(N=0) = 0.25, Pr(N = 1) = 0.25, and Pr(N=2) = 0.50. Each claim size is independent and is uniformly distributed over the interval (2, 6). The number of claims and the claim sizes are mutually independent. What is Var(S)?
A. Less than 6
B. At least 6, but less than 9
C. At least 9, but less than 12
D. At least 12, but less than 15
E. At least 15

**4.36 (5A, 5/99, Q.36)** (2 points) In a given time period, the probability that a particular automobile insurance policyholder will have a physical damage claim is 0.05.
Assume that the policyholder can have at most one claim during the given time period.
If a physical damage claim is made, the cost of the damages is uniformly distributed over the interval (0, 5000). Calculate the mean and variance of aggregate policy losses within the given time period.

Use the following information for the next two questions:
- The number of claims per year follows a Poisson distribution with mean 300.
- Claim sizes follow a Generalized Pareto distribution, as per Loss Models, with parameters $\theta = 1{,}000$, $\alpha = 3$, and $\tau = 2$.
- The nth moment of a Generalized Pareto Distribution is:

$$E[X^n] = \frac{\theta^n\, \Gamma(\alpha - n)\, \Gamma(\tau + n)}{\Gamma(\alpha)\, \Gamma(\tau)}, \text{ for } \alpha > n.$$

- The number of claims and claim sizes are independent.

**4.37 (4B, 11/99, Q.12)** (2 points) Using the Normal Approximation, determine the probability that annual aggregate losses will exceed 360,000.
A. Less than 0.01
B. At least 0.01, but less than 0.03
C. At least 0.03, but less than 0.05
D. At least 0.05, but less than 0.07
E. At least 0.07

**4.38 (4B, 11/99, Q.13)** (2 points) After a number of years, the number of claims per year still follows a Poisson distribution, but the expected number of claims per year has been cut in half. Claim sizes have increased uniformly by a factor of two. Using the Normal Approximation, determine the probability that annual aggregate losses will exceed 360,000.
A. Less than 0.01
B. At least 0.01, but less than 0.03
C. At least 0.03, but less than 0.05
D. At least 0.05, but less than 0.07
E. At least 0.07

**4.39 (Course 151 Sample Exam #1, Q.4)** (0.8 points) For an insurance portfolio:
(i) the number of claims has the probability distribution

| n | p(n) |
|---|---|
| 0 | 0.4 |
| 1 | 0.3 |
| 2 | 0.2 |
| 3 | 0.1 |

(ii) each claim amount has a Poisson distribution with mean 4
(iii) the number of claims and claim amounts are mutually independent.
Determine the variance of aggregate claims.
(A) 8 (B) 12 (C) 16 (D) 20 (E) 24

**4.40 (Course 151 Sample Exam #2, Q.4)** (0.8 points)
You are given $S = S_1 + S_2$, where $S_1$ and $S_2$ are independent and have compound Poisson distributions with the following characteristics:
(i) $\lambda_1 = 2$ and $\lambda_2 = 3$
(ii)

| x | $p_1(x)$ | $p_2(x)$ |
|---|---|---|
| 1 | 0.6 | 0.1 |
| 2 | 0.4 | 0.3 |
| 3 | 0.0 | 0.5 |
| 4 | 0.0 | 0.1 |

Determine the variance of S.
(A) 15.1 (B) 18.6 (C) 22.1 (D) 26.6 (E) 30.1

**4.41 (Course 151 Sample Exam #3, Q.1)** (0.8 points)
For a portfolio of insurance, you are given the distribution of number of claims:

| n | Pr(N=n) |
|---|---|
| 0 | 0.40 |
| 5 | 0.10 |
| 10 | 0.50 |

and the distribution of the claim amounts:

| x | p(x) |
|---|---|
| 1 | 0.90 |
| 2 | 0.10 |

Individual claim amounts and the number of claims are mutually independent.
Determine the variance of aggregate claims.
(A) 22.3 (B) 24.1 (C) 25.0 (D) 26.9 (E) 27.4

**4.42 (Course 151 Sample Exam #3, Q.13)** (1.7 points) You are given:
- The number of claims is given by a mixed Poisson with an Inverse Gaussian mixing distribution, with $\mu = 500$ and $\theta = 5000$.
- The number and amount of claims are independent.
- The mean aggregate loss is 1000.
- The variance of aggregate losses is 150,000.

Determine the variance of the claim amount distribution.
(A) 88 (B) 92 (C) 96 (D) 100 (E) 104

**4.43 (4, 11/02, Q.36)** (2.5 points) You are given:

| Number of Claims | Probability | Claim Size | Probability |
|---|---|---|---|
| 0 | 1/5 | | |
| 1 | 3/5 | 25<br>150 | 1/3<br>2/3 |
| 2 | 1/5 | 50<br>200 | 2/3<br>1/3 |

Claim sizes are independent.
Determine the variance of the aggregate loss.
(A) 4,050 (B) 8,100 (C) 10,500 (D) 12,510 (E) 15,612

Solutions to Problems:

**4.1. E.** The mean severity = $\exp(\mu + 0.5\sigma^2) = \exp(5.18) = 177.6828$.
Thus the mean aggregate losses are: (6200)(177.6828) = **1,101,633**.

**4.2. D.** The second moment of the severity = $\exp(2\mu + 2\sigma^2) = \exp(10.72) = 45{,}252$.
Thus since the frequency is Poisson and independent of the severity:
$\sigma_{PP}^2 = \lambda$ (2nd moment of the severity) = (6200)(45252) = **280.56 million**.

**4.3. B.** Since the variance is 280.56 million, the standard deviation of the aggregate losses is 16750. Thus the probability of the observed aggregate losses being more than 1130 thousand is approximately: $1 - \Phi[(1130 - 1101.63) / 16.75] = 1 - \Phi[1.69] = 1 - 0.9545 =$ **4.55%**.

**4.4. C.** Prob[aggregate losses < 1075 thousand] $\cong \Phi[(1075 - 1101.63) / 16.75] =$
$\Phi(-1.59) = 1 - 0.9441 =$ **5.59%**.

**4.5. C.** Using the solutions to the prior two questions: 1 - 4.51% - 5.59% = **89.9%**.
Comment: If one were asked for the Full Credibility criterion for Aggregate Losses corresponding to a 90% chance of being within ±2.5% of the expected aggregate losses, in the case of a Poisson frequency, as explained in the next section the answer would be:
$(y/k)^2 (1 + CV^2) = (1.645/0.025)^2 \exp(\sigma^2) = 4330(1.4333) = 6206$ claims. Note that for the LogNormal Distribution: $1 + CV^2 = \exp(\sigma^2) = \exp(0.8^2) = 1.4333$. That is just another way of saying there is about a 90% chance of being within ±2.5% of the expected aggregate losses when one has about 6200 expected claims.

**4.6. A.** $\mu_{freq} = \sigma_{freq}^2 = \lambda = 0.04$. $\mu_{sev} = \alpha\theta = 30$. $\sigma_{sev}^2 = \alpha\theta^2 = 300$.
$\sigma_{PP}^2 = \mu_{freq}\sigma_{sev}^2 + \mu_{sev}^2\sigma_{req}^2 = (0.04)(300) + (30^2)(0.04) =$ **48**.

**4.7. C.** $\sigma_{PP}^2 = \lambda$(second moment of severity) $= (0.06)\{\alpha(\alpha+1)\theta^2\} = (0.06)(3)(4)(15^2) =$ **162**.

**4.8. D.** EPV = (80%)(48) + (20%)(162) = **70.8**.

**4.9. D.** For a fixed number of claims N, the variance of the pure premium is the variance of the sum of N independent identically distributed variables each with variance $\sigma_{Sev}^2$. (Since frequency and severity are assumed independent, $\sigma_{Sev}^2$ is the same for each value of N.)
Such variances add so that $VAR_{PP}(PP \mid N) = N\sigma_{Sev}^2$.
$E_N[VAR_{PP}(PP \mid N)] = E_N[N\sigma_{Sev}^2] = \sigma_{Sev}^2 E_N[N] = \sigma_{Sev}^2 \mu_{Freq.}$
For the Negative Binomial Distribution: mean = $r\beta$ = (30)(2/3) = 20.
For the Gamma the variance = $\alpha\theta^2 = 4(1000^2) = 4{,}000{,}000$.
Thus $E_N[VAR_{PP}(PP|N)] = \sigma_{Sev}^2 \mu_{Freq}$ = (20)(4 million) = **80 million**.

**4.10. D.** For a fixed number of claims N, with frequency and severity independent, the expected value of the pure premium is N times the mean severity: $E_{PP}[PP|N] = \mu_{Sev}N$.
$VAR_N(E_{PP}[PP \mid N]) = VAR_N(\mu_{Sev}\ N) = \mu_{Sev}{}^2 VAR_N(N) = \mu_{Sev}{}^2\sigma_{Freq}{}^2$.
For the Negative Binomial: variance = $r\beta(1+\beta)$ = (30)(2/3)(5/3) = 33.33.
For the Gamma the mean is: $\alpha\theta$ = 4(1000) = 4000.
Therefore, $VAR_N(E_{PP}[PP \mid N]) = \mu_{Sev}{}^2\ \sigma_{Freq}{}^2 = (4000)^2(33.33)$ = **533.3 million**.

**4.11. E.** For the Negative Binomial Distribution: mean = 20, variance = 33.33.
For the Gamma the mean 4000, variance = 4,000,000.
Thus $\sigma_{PP}{}^2 = \mu_{Freq}\ \sigma_{Sev}{}^2 + \mu_{Sev}{}^2\ \sigma_{Freq}{}^2$ =
(20)(4 million) + $(4000)^2$(33.33) = 80 million + 533.3 million = **613.3 million**.
Comment: Note that the process variance is also the sum of the answers to the two previous questions: 80 million + 533.3 million = 613.3 million. This is the analysis of variance that is used in the derivation of the formula used to solve this problem.

**4.12. C.** For the Binomial frequency: mean = mq = 3, variance = mq(1-q) = (10)(0.3)(0.7) = 2.1.
For the Pareto severity: mean = $\theta$ / ($\alpha$-1) = 500 / 2 = 250,

$$\text{variance} = \frac{\alpha\theta^2}{(\alpha-1)^2(\alpha-2)} = \frac{(3)\ (500^2)}{(3-1)^2\ (3-2)} = 187{,}500.$$

Since the frequency and severity are independent:
$\sigma_{PP}{}^2 = \mu_{Freq}\ \sigma_{Sev}{}^2 + \mu_{Sev}{}^2\ \sigma_{Freq}{}^2 = (3)(187{,}500) + (250^2)(2.1)$ = **693,750**.

**4.13. D.** For the Poisson frequency: mean = variance = $\lambda$ = 5.
For the LogNormal severity: Mean = $\exp(\mu + 0.5\ \sigma^2) = \exp[6 + (0.5)(0.8^2)]$ = 555.573,
Variance = $\exp(2\mu + \sigma^2)\ \{\exp(\sigma^2) - 1\} = \exp[(2)(6) + (0.8^2)]\ (\exp[0.8^2] - 1)$ =
(308,661.3) (1.89648 - 1) = 276,709.
Since the frequency and severity are independent:
$\sigma_{PP}{}^2 = \mu_{Freq}\ \sigma_{Sev}{}^2 + \mu_{Sev}{}^2\ \sigma_{Freq}{}^2 = (5)(276{,}709) + (555.573^2)(5)$ = **2,926,852**.
Alternately, since the frequency is Poisson and the frequency and severity are independent:
$\sigma_{PP}{}^2$ = (mean frequency)(2nd moment of the severity).
The 2nd moment of a LogNormal Distribution is:
$\exp(2\mu + 2\sigma^2) = \exp[2(6) + 2(0.8^2)] = \exp(13.28)$ = 585,370.3. Therefore,
$\sigma_{PP}{}^2$ = (mean frequency)(2nd moment of the severity) = (5)(585,370.3) = **2,926,852**.

**4.14. E.** For the Negative Binomial frequency: mean = $r\beta$ = (2.7)(7/3)= 6.3,
variance = $r\beta(1+\beta)$ = (2.7)(7/3)(10/3) = 21.
For the Gamma severity: mean = $\alpha\theta$ = 2(250) = 500, variance = $\alpha\theta^2 = 2\ (250)^2 = 125000$.
Since the frequency and severity are independent:
$\sigma_{PP}{}^2 = \mu_{Freq}\ \sigma_{Sev}{}^2 + \mu_{Sev}{}^2\ \sigma_{Freq}{}^2 = (6.3)(125000) + (500^2)(21)$ = **6,037,500**.

**4.15. E.** (55%)(693,750) + (35%)(2,926,852) + (10%) (6,037,500) = **2,009,711**.

**4.16. B.** The distribution of aggregate loses has a mean of (400)(100) = 40,000,
and a variance of: $(400)(150^2 + 100^2) = 13{,}000{,}000$.
We want to be within 36,000 and 44,000.

$$\text{Probability} = \Phi\left[\frac{44{,}000 - 40{,}000}{\sqrt{13{,}000{,}000}}\right] - \Phi\left[\frac{36{,}000 - 40{,}000}{\sqrt{13{,}000{,}000}}\right] = \Phi[1.11] - \Phi[-1.11]$$

= 0.8665 - (1 - 0.8665) = **73.3%**.

<u>Comment</u>: Similar to Exercise 6.2b in
<u>Nonlife Actuarial Models: Theory, Methods and Evaluation</u>, Yiu-Kuen Tse.
The standard for full credibility for aggregate loss, in order to be within 10% of the true mean 73.3% of the time is: $\{1 + (150/100)^2\}\,(1.11/0.10)^2 = 400$ claims.

**4.17. A.** For a Poisson frequency, $\sigma_{PP}^2 = \lambda$ (2nd moment of the severity) = (0.5)(1000) = **500**.

**4.18. A.** The mean frequency = (1/4)(4) + (1/2)(5) + (1/4)(6) = 5. 2nd moment of frequency = $(1/4)(4^2) + (1/2)(5^2) + (1/4)(6^2) = 25.5$. The variance of the frequency = $25.5 - 5^2 = 0.5$.

$$\text{mean severity} = \int_1^\infty x\, f(x)\, dx = \int_1^\infty x \frac{3}{x^4}\, dx = -3x^{-2}/2 \Big]_{x=1}^{x=\infty} = 3/2.$$

$$\text{second moment} = \int_1^\infty x^2 f(x)\, dx = \int_1^\infty x^2 \frac{3}{x^4}\, dx = -3x^{-1}/1 \Big]_{x=1}^{x=\infty} = 3.$$

Thus the variance of the severity is: $3 - (3/2)^2 = 3/4$.
For independent frequency and severity, the variance of the pure premiums =
(mean frequency)(variance of severity) + (mean severity)$^2$(variance of frequency) =
$(5)(3/4) + (3/2)^2(0.5) = 4.875$.
The mean of the pure premium is: (mean frequency)(mean severity) = (5)(3/2) = 7.5.

$$\text{The coefficient of variation of the pure premium} = \frac{\sqrt{\text{variance of P.P.}}}{\text{mean of P.P.}} = \frac{\sqrt{4.875}}{7.5} = \mathbf{0.294}.$$

<u>Comment</u>: The severity distribution is a Single Parameter Pareto with $\theta = 1$ and $\alpha = 3$.

The mean = $\frac{\alpha\theta}{\alpha - 1} = 3/2$. The variance = $\frac{\alpha\theta^2}{(\alpha-1)^2(\alpha-2)} = 3/4$.

**4.19. E.** Since we have a Poisson Frequency, the Process Variance for each type of claim is given by the mean frequency times the second moment of the severity.
For example, for Claim Type Z, the process variance of the pure premium is:
(0.01)(2,250,000 + 2,000,000) = 42,500.
Then the process variances for each type of claim add to get the total variance, 103,570.

| Type of Claim | Mean Frequency | Mean Severity | Square of Mean Severity | Variance of Severity | Process Variance of P.P. |
|---|---|---|---|---|---|
| W | 0.02 | 200 | 40,000 | 2,500 | 850 |
| X | 0.03 | 1000 | 1,000,000 | 1,000,000 | 60,000 |
| Y | 0.04 | 100 | 10,000 | 0 | 400 |
| Z | 0.01 | 1500 | 2,250,000 | 2,000,000 | 42,500 |
| | | | | | |
| SUM | | | | | **103,750** |

Comment: This is like adding up four independent die rolls; the variances add. For example this could be a nonrealistic model of homeowners insurance with the four types of claims being: Fire, Liability, Theft and Windstorm.

**4.20. B.** The mean frequency is mq = 1/2, while the variance of the frequency is mq(1-q) = (3)(1/6)(5/6) = 5/12.
The mean severity is: (2/3)(100) + (1/6)(1100) + (1/6)(2100) = 600.
The second moment of the severity is: $(2/3)(100^2) + (1/6)(1100^2) + (1/6)(2100^2) = 943,333$.
Thus the variance of the severity is: $943,333 - 600^2 = 583,333$.
The variance of the pure premium = (variance of frequency)(mean severity)$^2$ + (variance of severity)(mean frequency) = $(5/12)(600)^2 + (1/2)(583,333) = 441,667$.
The mean pure premium is (1/2)(600) = 300. Therefore, the coefficient of variation is:
standard deviation / mean = $\sqrt{441,667}$ / 300 = **2.2**.

**4.21. D.** The variance of the frequency is: (0.05)(0.95) = 0.0475. The mean damage is: 1000.
The variance of the damage is: $(2000 - 0)^2 / 12 = 333,333$.
The variance of the pure premium = $(1000^2)(0.0475) + (0.05)(333,333)$ = **64,167**.

**4.22.** The mean aggregate loss is: (40000)(0.15)(1200) = 7.2 million.
Since frequency is Poisson with mean: (40000)(0.15) = 6000, the variance of aggregate losses is: (mean frequency)(2nd moment of severity) = $(6000)\ (2000^2 + 1200^2)$ = 32,640 million.
The standard deviation of aggregate losses is: $\sqrt{32,640 \text{ million}}$ = 180,665.
Premium = (1.05)(expected losses) = (1.05)(7.2 million) = 7.56 million.
We want the Premiums + Surplus ≥ Actual Losses.
⇔ Surplus ≥ Actual Losses - Premiums = Actual Losses - 7.56 million.
$\Phi(2.756) = 0.995$. ⇔ the 99.5$^{th}$ percentile of the Standard Normal Distribution is 2.576.
Therefore, 99.5% of the time actual losses are less than or equal to:
7.2 million + (2.576)(180,665) = 7.665 million.
Therefore, we want surplus of at least: 7.665 million - 7.56 million = **105 thousand**.
Comment: 100% - 99.5% = 0.5% of the time, actual losses will be greater than 7.665 million, and a surplus of 105 thousand would be exhausted.

**4.23.** Let N be the number of policies written.
The mean aggregate loss = N(0.045)(750) and the variance of aggregate losses = N(0.045)(60,000,000). Thus premiums are: 1.05N(0.045)(750).
The 99th percentile of the Unit Normal Distribution is 2.326. Thus we want
Premiums - Expected Losses = 2.326(standard deviation of aggregate losses).
$(0.05)N(0.045)(750) = 2.326 \sqrt{N\ (0.045)\ (60{,}000{,}000)}$.

Therefore, $N = (60{,}000{,}000/750^2)\ (2.326/0.05)^2 / 0.045 =$ **5,129,743**.

**4.24. C.** Second moment of the severity = $(1^2 + 2^2 + 3^2 + 4^2 + 5^2 + 6^2)/6 = 15.167$.
Since the frequency is Poisson, the variance of aggregate losses =
(mean frequency)(second moment of the severity) = (3)(15.167) = **45.5**.

**4.25.** For each tier, the mean aggregate loss = (mean frequency)(mean severity) and since frequency is Poisson, the variance of aggregate loss = (mean frequency)(second moment of the severity). The means and variances of the tiers add to get an overall mean of: **19,125**,
and an overall variance of: **106,875,000.**

| | Tier 1 | Tier 2 | Tier 3 | | Overall |
|---|---|---|---|---|---|
| | | | | | |
| 1000 | 0.6 | 0.7 | 0.8 | | |
| 5000 | 0.3 | 0.2 | 0.15 | | |
| 10000 | 0.1 | 0.1 | 0.05 | | |
| | | | | | |
| Mean Severity | 3100 | 2700 | 2050 | | |
| 2nd Moment of Severity | 18,100,000 | 15,700,000 | 9,550,000 | | |
| | | | | | |
| Poisson Parameter | 2.3 | 3 | 1.9 | | |
| | | | | | |
| Mean Aggregate | 7,130 | 8,100 | 3,895 | | **19,125** |
| Variance of Aggregate | 41,630,000 | 47,100,000 | 18,145,000 | | **106,875,000** |

Alternately, the tiers, each of which is compound Poisson, add to get a new compound Poisson with mean frequency: 2.3 + 3 + 1.9 = 7.2. The mean severity overall is a weighted average of the means for the individual tiers:
{(2.3)(3100) + (3)(2700) + (1.9)(2050)} / 7.2 = 2556.25.
Thus the mean aggregate loss is: (2556.25)(7.2) = 19,125.
The second moment of the severity overall is a weighted average of the second moments for the individual tiers:
{(2.3)(18.1 million) + (3)(15.7 million) + (1.9)(9.55 million)} / 7.2 = 14.844 million.
Thus the variance of aggregate losses is: (7.2)(14.844 million) = 106.9 million.

**4.26. D.** For example, the chance of 2 claims of size 80 each is the chance of having two claims times the chance given two claims that they will each be 80 = $(1/4)(1/2)^2 = 1/16$. In that case the pure premium is 80 + 80 = 160. One takes the weighted average over all the possibilities. The average Pure Premium is 140. The second moment of the Pure Premium is 24800. Therefore, the variance = $24800 - 140^2$ = **5200**.

| Situation | Probability | Pure Premium | Square of P.P. |
|---|---|---|---|
| 1 claim @ 80 | 0.5000 | 80 | 6400 |
| 1 claim @ 160 | 0.2500 | 160 | 25600 |
| 2 claims @ 80 each | 0.0625 | 160 | 25600 |
| 2 claims: 1 @ 80 & 1 @ 160 | 0.1250 | 240 | 57600 |
| 2 claims @ 160 each | 0.0625 | 320 | 102400 |
| | | | |
| Overall | 1.0000 | 140 | 24800 |

Comment: Note that the frequency and severity are not independent.

**4.27. C.** Since the frequency is Poisson, the variance of aggregate losses =
(mean frequency) (second moment of the severity) =
$(4) \{(0.4)(100^2) + (0.5)(200^2) + (0.1)(500^2)\}$ = **196,000**.

**4.28. B.** The mean pure premium is (3000)(800) = 2.4 million. Since frequency and severity are independent, the (process) variance of the aggregate losses is: $\mu_{Freq}\ \sigma_{Sev}^2 + \mu_{Sev}^2\ \sigma_{Freq}^2$ =
$(800)(36 \text{ million}) + (3000)^2(3200)$ = 57.6 billion.
Thus the standard deviation of the pure premiums is: $\sqrt{57.6 \text{ billion}}$ = 240,000.
To apply the Normal Approximation we subtract the mean and divide by the standard deviation.
The probability that the total losses will exceed 3 million is approximately:
$1 - \Phi[(3 \text{ million} - 2.4 \text{ million})/ 240{,}000] = 1 - \Phi(2.5) = 1 - 0.9938$ = **0.0062**.
Comment: One makes no specific use of the information that the frequency is given by a Negative Binomial, nor that the severity is given by a Transformed Gamma Distribution.

**4.29. B.** Since Die A and Die B are equally likely, the chance of 1 claim is: (1/2)(2/3) + (1/2)(1/3) = 1/2, while the chance of 2 claims is: (1/2)(1/3) + (1/2)(2/3) = 1/2.
The mean of Spinner X is: (2/3)(2) + (1/3)(5) = 3,
while the mean of Spinner Y is: (1/3)(2) + (2/3)(5) = 4.
If we have one claim the mean loss is E[X] = 3. If we have two claims, then the mean loss is:
E[X+Y] = E[X] + E[Y] = 3 + 4 = 7. The overall mean pure premium is:
(chance of 1 claim)(mean loss if 1 claim) + (chance of 2 claims)(mean loss if 2 claims)
= (1/2)(3) + (1/2)(7) = **5**.
Comment: In this problem frequency and severity are not independent.

**4.30. D.** Since Die A and Die B are equally likely, the chance of 1 claim is: (1/2)(2/3) + (1/2)(1/3) = 1/2, while the chance of 2 claims is: (1/2)(1/3) + (1/2)(2/3) = 1/2.
If we have one claim, then spinner X is spun and the loss is either:
2 with probability 2/3 or 5 with probability 1/3.
If we have 2 claims, then spinners X and Y are spun and the loss is either:
4 with probability 2/9, 7 with probability 5/9, or 10 with probability 2/9.
Thus the distribution of losses is:
2 @ 1/3, 5 @ 1/6, 4 @ 1/9, 7 @ 5/18, and 10 @ 1/9.
Mean loss is: (2)(1/3) + (5)(1/6) + (4)(1/9) + (7)(5/18) + (10)(1/9) = 5.
Second moment is: $(2^2)(1/3) + (5^2)(1/6) + (4^2)(1/9) + (7^2)(5/18) + (10^2)(1/9) = 32$.
Variance = $32 - 5^2 =$ **7**.
Alternately, this is a 50-50 mixture of two situations one claim or two claims.
The mean of Spinner X is: (2/3)(2) + (1/3)(5) = 3.
The variance of Spinner X is: $(2/3)(2 - 3)^2 + (1/3)(5 - 3)^2 = 2$.
The mean of Spinner Y is: (1/3)(2) + (2/3)(5) = 4.
The variance of Spinner Y is: $(1/3)(2 - 4)^2 + (2/3)(5 - 4)^2 = 2$.
If we have one claim the mean loss is E[X] = 3.
If we have two claims, then the mean loss is: E[X+Y] = E[X] + E[Y] = 3 + 4 = 7.
The overall mean is: (1/2)(3) + (1/2)(7) = 5.
If we have one claim the second moment is from spinner X: $2 + 3^2 = 11$.
If we have two claims the variance is the sum of those for X and Y: 2 + 2 = 4.
Thus if we have two claims the second moment: $4 + 7^2 = 53$.
Thus the second moment of the mixture is: (1/2)(11) + (1/2)(53) = 32.
Therefore, the variance of the mixture is: $32 - 5^2 =$ **7**.
Alternately, take the two types as 1 or 2 claims, equally likely.
The hypothetical means for 1 and 2 claims are: 3 and 7.
Therefore, the variance of the hypothetical means is: $(1/2)(3 - 5)^2 + (1/2)(7 - 5)^2 = 4$.
When there is one claims, the process variance is that of spinner X: 2.
When there are 2 claims, the process variance is the sum of those for spinners X and Y:
2 + 2 = 4.
Expected Value of the process variance is: (1/2)(2) + (1/2)(4) = 3.
Total variance is: EPV + VHM = 3 + 4 = **7**.
Alternately, take the two types as Die A and B, equally likely.
The hypothetical mean if Die A is: (2/3)(3) + (1/3)(7) = 13/3.
The hypothetical mean if Die B is: (1/3)(3) + (2/3)(7) = 17/3.
Therefore, the variance of the hypothetical means is: $(1/2)(13/3 - 5)^2 + (1/2)(17/3 - 5)^2 = 4/9$.
When there is one claim, the second moment of pure premium is: $2 + 3^2 = 11$.
When there is two claims, the second moment of pure premium is: $4 + 7^2 = 53$.
Therefore, if one has die A, the second moment of pure premium is: (2/3)(11) + (1/3)(53) = 25.
Thus the process variance when die A is: $25 - (13/3)^2 = 56/9$.
Therefore, if one has die B, the second moment of pure premium is: (1/3)(11) + (2/3)(53) = 39.
Thus the process variance when die B is: $39 - (17/3)^2 = 62/9$.
Expected Value of the process variance is: (1/2)(56/9) + (1/2)(62/9) = 59/9.
Total variance is: EPV + VHM = 59/9 + 4/9 = **7**.
<u>Comment</u>: In this problem frequency and severity are <u>not</u> independent.

**4.31.** For the LogNormal Distribution, $E[X] = \exp[\mu + \sigma^2/2] = \exp[7.54 + 1.14^2/2] = 3604$.
$E[X^2] = \exp[2\mu + 2\sigma^2] = \exp[(2)(7.54) + (2)(1.14^2)] = 47{,}640{,}795$.
$Var[X] = 47{,}640{,}795 - 3604^2 = 34{,}651{,}979$.
Mean aggregate loss (per exposure) = (0.2)(3604) = 721.
Variance of Aggregate Losses (per exposure) is:
$(0.2)(34{,}651{,}979) + (0.2)(0.8)(3604^2) = 9{,}008{,}606$.
Thus if we write N exposures, mean loss = 721N,
and standard deviation of the aggregate loss is $3001\sqrt{N}$.
We want N such that Prob(Aggregate Loss > (2)(mean)) ≤ 5%.
Prob((Aggregate Loss - mean) > 721N) ≤ 5%.
Using the Normal Approximation, we want: (1.645)(stddev) < 721N.
$\Rightarrow 1.645(3001\sqrt{N}) < 721N. \Rightarrow N >$ **46.9**.

**4.32.** The variance of aggregate losses is: $(100)(1{,}000{,}000 + 1000^2) = 200{,}000{,}000$.
The 95th percentile of the aggregate losses exceeds the mean by about 1.645 standard deviations: $(1.645)\sqrt{200{,}000{,}000}$ = **23,264**.
With at least this much capital, there is no more than a 5% chance of insolvency in the first year.

**4.33. B.** For a compound Poisson, variance of aggregate losses =
(mean frequency)(second moment of severity) = $(3)(194400 + 560^2)$ = **1,524,000**.

**4.34.** $1344 = E_N[Var(Sev \mid N)] = E_N[Var(X_1 + X_2 + X_3 + ... + X_N)] = E_N[N\ Var[X]] = E[N]\ Var[X] = 4Var[X]. \Rightarrow Var[X] = 1344/4 = 336$. $4{,}096 = Var(N)[E(X)]^2 = 4[E(X)]^2. \Rightarrow [E(X)]^2 = 1024$.
$E[X^2] = Var[X] + [E(X)]^2 = 336 + 1024 =$ **1360**.
<u>Comment</u>: E[Var(Sev I N)] given in the question is the expected value over N of the variance of the aggregate losses conditional on N.

**4.35. D.** Mean Frequency = (0.25)(0) + (0.25)(1) + (0.50)(2) = 1.25.
Second Moment of the Frequency = $(0.25)(0^2) + (0.25)(1^2) + (0.50)(2^2) = 2.25$.
Variance of the Frequency = $2.25 - 1.25^2 = 0.6875$.
Mean Severity = (2 + 6)/2 = 4. Variance of the severity = $(6-2)^2/12 = 4/3$.
Variance of the Aggregate Losses = $(4^2)(0.6875) + (4/3)(1.25)$ = **12.67**.

**4.36.** Mean frequency = 0.05. Variance of frequency = (0.05)(0.95) = 0.0475.
Mean severity = (0 + 5000)/2 = 2500. Variance of severity = $(5000 - 0)^2/12 = 2{,}083{,}333$.
Mean aggregate loss = (0.05)(2500) = 125.
Variance of aggregate losses = $(0.05)(2083333) + (2500^2)(0.0475)$ = **401,042**.

**4.37. B.** The mean and variance of the frequency is 300.
The mean of the Generalized Pareto severity is: $\theta\tau/(\alpha-1) = (1000)(2)/(3-1) = 1000$.
The 2nd moment of the Generalized Pareto severity is:

$$\frac{\theta^2\tau(\tau+1)}{(\alpha-1)(\alpha-2)} = \frac{(1000^2)(2)(3)}{(3-1)(3-2)} = 3 \text{ million.}$$

Mean aggregate losses = (300)(1000) = 300,000.
Variance of Aggregate Losses = (mean of Poison)(2nd moment of severity) =
(300)(3 million) = 900 million.
Standard Deviation of Aggregate Losses = 30,000.
Using the Normal Approximation, the chance that the aggregate losses are greater than 360,000 is approximately: $1 - \Phi[(360{,}000 - 300{,}000)/30{,}000] = 1 - \Phi[2] = 1 - 0.9772 =$ **0.0228**.

**4.38. E.** The mean and variance of the frequency is 150.
The mean of the Generalized Pareto severity is twice what it was or 2000.
The 2nd moment of the Generalized Pareto severity is four times what it was or 12 million.
Mean aggregate losses = (150)(2000) = 300,000.
Variance of Aggregate Losses = (mean of Poison)(2nd moment of severity) =
(150)(12 million) = 1800 million.
Standard Deviation of Aggregate Losses = 42,426.
Using the Normal Approximation, the chance that the aggregate losses are greater than 360,000 is approximately: $1 - \Phi[(360{,}000 - 300{,}000)/42{,}426] = 1 - \Phi[1.41] = 1 - 0.9207 =$ **0.0793**.
Comment: The second moment is always multiplied by the square of the inflation factor under uniform inflation. Alternately, one can instead use the behavior under uniform inflation of the Generalized Pareto Distribution; the new severity distribution is also a Generalized Pareto, but with parameters $\theta = 2000$, $\alpha = 3$ and $\tau = 2$. Its mean and second moment are as I've stated. In general, when one halves the frequency and uniformly doubles the claim size, while the expected aggregate losses remain the same, the variance of the aggregate losses increases. (Given a Poisson Frequency, the variance of the aggregate losses doubles.) Therefore, there is a larger chance for an unusual year. *High Severity/Low Frequency lines of insurance are more volatile than High Frequency/Low Severity lines of insurance.*

**4.39. D.** Mean Frequency = (0.4)(0) + (0.3)(1) + (0.2)(2) + (0.1)(3) = 1.
2nd moment of Frequency = $(0.4)(0^2) + (0.3)(1^2) + (0.2)(2^2) + (0.1)(3^2) = 2$.
Variance of Frequency = $2 - 1^2 = 1$.
Mean Severity = Variance of Severity = 4.
Variance of aggregate claims = $(4)(1) + (4^2)(1) =$ **20**.

**4.40. D.** The second moment of severity $p_1$ is: $(0.6)(1^2) + (0.4)(2^2) = 2.2$.
The second moment of severity $p_2$ is: $(0.1)(1^2) + (0.3)(2^2) + (0.5)(3^2) + (0.1)(4^2) = 7.4$.
Var[S] = (2)(2.2) + (3)(7.4) = **26.6**.

**4.41. E.** Mean frequency is: (0)(0.4) + (5)(0.1) + (10)(0.5) = 5.5.
The 2nd moment of the frequency is: $(0^2)(0.4) + (5^2)(0.1) + (10^2)(0.5) = 52.5$.
Variance of the frequency is: $52.5 - 5.25^2 = 22.25$.
Mean severity is: (0.9)(1) + (0.1)(2) = 1.1.
2nd moment of the severity is: $(0.9)(1^2) + (0.1)(2^2) = 1.3$.
Variance of the severity is: $1.3 - 1.1^2 = 0.09$.
Variance of aggregate losses = $(1.1^2)(22.25) + (5.5)(0.09) =$ **27.4**.

**4.42. C.** The mean frequency = mean of the Inverse Gaussian = $\mu$ = 500.
Variance of frequency = mean of Inverse Gaussian + variance of Inverse Gaussian =
$\mu + \mu^3/\theta = 500 + 500^3/5000 = 25{,}500$.
Let X be the severity distribution. Then we are given that:
1000 = Mean aggregate loss = 500E[X].
150,000 = Variance of aggregate losses = 500 Var[X] + 25,500 $E[X]^2$.
Therefore, E[X] = 1000 / 500 = 2 and Var[X] = $\{150{,}000 - 25{,}500(2^2)\} / 500 =$ **96**.
<u>Comment</u>: In general when one has a mixture of Poissons,
Mean frequency = $E[\lambda]$ = mean of mixing distribution, and
Second moment of the frequency =
$E_\lambda$[second moment of Poisson | $\lambda$] = $E[\lambda + \lambda^2]$ =
mean of mixing distribution + second moment of mixing distribution.
Variance of frequency =
mean of mixing distribution + second moment of mixing - (mean of mixing distribution)$^2$
= mean of mixing distribution + variance of mixing distribution.

**4.43. B.** List the different possible situations and their probabilities:

| Situation | Probability | Aggregate Loss | Square of the Aggregate Loss |
|---|---|---|---|
| no claims | 20.00% | 0 | 0 |
| 1 claim @25 | 20.00% | 25 | 625 |
| 1 claim @ 150 | 40.00% | 150 | 22500 |
| 2 claims each @50 | 8.89% | 100 | 10000 |
| 1 claim @ 50 and 1 claim @ 200 | 8.89% | 250 | 62500 |
| 2 claims @ 200 | 2.22% | 400 | 160000 |
| | | | |
| Weighted Average | | 105 | 19125 |

Mean = 105. Second Moment = 19,125. Variance = $19{,}125 - 105^2 =$ **8100**.

## Section 5, Full Credibility for Pure Premiums & Aggregate Losses

A single standard for full credibility applies when one wishes to estimate either pure premiums, aggregate losses, or loss ratios.

$$\text{Pure Premium} = \frac{\text{\$ of Loss}}{\text{\# of Exposures}} = \frac{\text{\# of Claims}}{\text{\# of Exposures}} \frac{\text{\$ of Loss}}{\text{\# of Claims}} = \text{(Frequency) (Severity)}.$$

$$\text{Loss Ratio} = \frac{\text{\$Loss}}{\text{\$Premium}}.$$

Since they depend on both the number of claims and the size of claims, pure premiums and aggregate losses have more reasons to vary than do either frequency or severity. Since pure premiums are more difficult to estimate than frequencies, all other things being equal the Standard for Full Credibility for Pure Premiums is larger than that for Frequencies.

Poisson Frequency Example:

For example, assume frequency is Poisson distributed with a mean of 9 (and a variance of 9) and every claim is of size 10. Then since the severity is constant, it does not increase the random fluctuations. Since $Var[cX] = c^2 Var[X]$, the variance of the pure premium for a single exposure is: (variance of the frequency)($10^2$) = 900.

Exercise: In the above situation, what is the Standard for Full Credibility (in terms of expected number of claims), so that the estimated pure premium will have a 90% chance of being within ±5% of the true value?
[Solution: We wish to have a 90% probability, so we are to be within ±1.645 standard deviations, since $\Phi(1.645) = 0.95$.
For X exposures the variance of the sum of the pure premiums for each exposure is X900.
The variance of the average pure premium per exposure is this divided by $X^2$.
Thus we have a variance of 900 / X and a standard deviation of $30 / \sqrt{X}$.
The mean pure premium is (9)(10) = 90.
We wish to be within ±5% of this or ±4.5.
Setting this equal to ±1.645 standard deviations we have:
$4.5 = 1.645(30 / \sqrt{X})$, or $X = \{(1.645)(30/4.5)\}^2 = 120.26$ exposures.
The expected number of claims is: (120.26)(9) = 1082.
Comment: Since severity is constant, the Standard for Full Credibility is the same as that for estimating the frequency, with Poisson frequency, $\alpha = 10\%$, and $k = 5\%$: 1082 claims.]

If the severity is not constant but instead varies, then the variance of the pure premium is greater than 900. Specifically assume that the severity is given by a Gamma Distribution, with $\alpha = 3$ and $\theta = 10$. This distribution has a mean of: $\alpha\theta = 30$, and a variance of: $\alpha\theta^2 = 300$.

Then if we assume frequency and severity are independent we can use the formula developed for the variance of the pure premium in terms of that of the frequency and severity:
$\sigma_{pp}^2 = \mu_{Freq}\, \sigma_{Sev}^2 + \mu_{Sev}^2\, \sigma_{Freq}^2$.
In this case $\mu_{sev} = 30$, $\sigma_{sev}^2 = 300$, $\mu_{freq} = \sigma_{freq}^2 = 9$, $\sigma_{pp}^2 = 10{,}800$.

Assume we wish the Standard for Full Credibility (in terms of expected number of claims), to be such that the estimated pure premium will have a 90% chance of being with ±5% of the true value. We wish to have a 90% chance, so we want to be within ±1.645 standard deviations since $\Phi(1.645) = 0.95$.

For X exposures the variance of the sum of the pure premiums for each exposure is 10,800X. The variance of the average pure premium per exposure is this divided by $X^2$.
Thus we have a variance of 10,800 / X and a standard deviation of $103.9 / \sqrt{X}$.
The mean pure premium is: (9)(30) = 270.

We wish to be within ±5% of this or ±13.5. Setting this equal to ±1.645 standard deviations we gave: $13.5 = 1.645(103.9 / \sqrt{X})$, or $X = \left(1.645\ \frac{103.9}{13.5}\right)^2 = 160.3$ exposures.
The expected number of claims is: (160.3)(9) = 1443.

Note that this is greater than the 1082 claims needed for Full Credibility of the frequency when $\alpha = 10\%$ and $k = 5\%$. In fact the ratio is: $1443/1082 = 1 + 1/3 = 1 + CV^2$, where $CV^2$ is the square of the coefficient of variation of the severity distribution, which for the Gamma is $1/\alpha = 1/3$.

It turns out in general, when frequency is Poisson, that the Standard for Full Credibility for the Pure Premium is: the Standard for Full Credibility for the Frequency times (1 + square of coefficient of variation of the severity):[41]

$$\lambda_F\, (1 + CV_{Sev}^2) = \left(\frac{z_{1-\alpha/2}}{k}\right)^2 (1 + CV_{Sev}^2).$$

[41] Equation 6.18 in NonLife Actuarial Models: Theory, Methods and Evaluation.

*Derivation of the Standard for Full Credibility for Pure Premiums, Poisson Case:*

The derivation follows that of the particular case above.

Let $\mu_{Sev}$ be the mean of the severity distribution while $\sigma_{Sev}^2$ is the variance. Assume that the frequency is Poisson and therefore $\mu_{Freq} = \sigma_{Freq}^2$. Assuming the frequency and severity are independent, the variance of the Pure Premium for one exposure unit is:
$\sigma_{pp}^2 = \mu_{Freq}\ \sigma_{Sev}^2 + \mu_{Sev}^2\ \sigma_{Freq}^2 = \mu_{Freq}\ (\mu_{Sev}^2 + \sigma_{Sev}^2)$.
For X exposure units, the variance of the estimated average pure premium is this divided by X.
We wish to be within ±y standard deviations, where as usual $z_{1-\alpha/2}$ is such that
$\Phi(z_{1-\alpha/2}) = 1 - \alpha/2$.
For a mean pure premium of $\mu_{Freq}\ \mu_{Sev}$ we wish to be within $\pm k\mu_f\ \mu_{Sev}$.
Setting the two expressions for the error bars equal:
$k\ \mu_{Freq}\ \ \mu_{Sev} = z_{1-\alpha/2}\ \sqrt{\mu_{Freq}\ (\mu_{Sev}^2 + \sigma_{Sev}^2)/X}$ .

Solving for X, $X = \left(\frac{z_{1-\alpha/2}}{k}\right)^2\ (\mu_{Sev}^2 + \sigma_{Sev}^2)\ /\ (\mu_{Freq}\ \ \mu_{Sev}^2)$.

The expected number of claims needed for Full Credibility is:
$\mu_{Freq}\ X = \left(\frac{z_{1-\alpha/2}}{k}\right)^2\ (1 + \sigma_{Sev}^2/\mu_{Sev}^2) = \lambda_F\ (1 + CV_{Sev}^2)$.

<u>A Formula for the Square of the Coefficient of Variation</u>:

The following formula for unity plus the square of the coefficient of variation follows directly from the definition of the Coefficient of Variation.
$CV^2$ = Variance / $E[X]^2 = (E[X^2] - E[X]^2)\ /\ E[X]^2 = (E[X^2]\ /\ E[X]^2) - 1$.
Thus, $\mathbf{1 + CV^2} = \frac{\mathbf{E[X^2]}}{\mathbf{E[X]^2}}$ = 2nd moment divided by the square of the mean.
This formula is useful for Classical credibility problems involving the Pure Premium.

For example, assume one has a Pareto Distribution. Then using the formulas for the moments:
$1 + CV^2 = E[X^2]\ /\ E[X]^2 = \frac{2\theta^2\ /\ \{(\alpha-1)(\alpha-2)\}}{\{\theta\ /\ (\alpha-1)\}^2} = 2(\alpha-1)\ /\ (\alpha-2)$.
For example if $\alpha = 5$, then $(1 + CV^2) = 2(4)/3 = 8/3$.

Exercise: Assume frequency and severity are independent and frequency is Poisson.
For $\alpha$ = 10% and k = 5%, and if severity follows a Pareto Distribution with $\alpha = 5$,
what is the Standard for Full Credibility for the Pure Premium in terms of claims?
[Solution: $\lambda_F\ (1 + CV^2) = (1082)(8/3) = 2885$ claims.]

In general the Standard for Full Credibility for the pure premium is the sum of those for frequency and severity. $\lambda_F(1 + CV^2) = \lambda_F + \lambda_F\, CV^2$. In this case: 1082 + 1803 = 2885.

General Case, if Frequency is Not Poisson:

As with the Standard for Full Credibility for frequency, one can derive a more general formula when the Poisson assumption does not apply. **The Standard for Full Credibility for estimating either pure premiums or aggregate losses is**:[42]

$$\left(\frac{z_{1-\alpha/2}}{k}\right)^2 \left(\frac{\sigma_{Freq}^2}{\mu_{Freq}} + \frac{\sigma_{Sev}^2}{\mu_{Sev}^2}\right) = \lambda_F \left(\frac{\sigma_{Freq}^2}{\mu_{Freq}} + CV_{sev}^2\right).$$

which reduces to the Poisson case when $\sigma_{freq}^2 / \mu_{freq} = 1$.
Note that if every claim is of size one, then the variance of the severity is zero and the standard for full credibility reduces to that for frequency: $\lambda_F \frac{\sigma_{freq}^2}{\mu_{freq}}$.

Exercise: Frequency is Negative Binomial with r = 0.1 and $\beta$ = 0.5. Severity has a coefficient of variation of 3. The number of claims and claim sizes are independent.
The observed aggregate loss should be within 5% of the expected aggregate loss 90% of the time. Determine the expected number of claims needed for full credibility.
[Solution: $\alpha$ = 10%. $z_{1-\alpha/2}$ = 1.645. k = 0.05. $\frac{\sigma_{freq}^2}{\mu_{freq}} = r\beta(1 + \beta)/(r\beta) = 1 + \beta = 1.5$.

$$\left(\frac{z_{1-\alpha/2}}{k}\right)^2 \left(\frac{\sigma_{freq}^2}{\mu_{freq}} + CV_{sev}^2\right) = \left(\frac{1.645}{0.05}\right)^2 (1.5 + 3^2) = 11{,}365 \text{ claims.]}$$

Note that a Negative Binomial has $\frac{\sigma_{freq}^2}{\mu_{freq}} > 1$, so the standard for full credibility is larger than if one assumed a Poisson frequency. Note that if one limits the size of claims, then the coefficient of variation is smaller. Therefore, the criterion for full credibility for basic limits losses is less than that for total losses.

In general the Standard for Full Credibility for the pure premium is the sum of those for frequency and severity: $\lambda_F\left(\frac{\sigma_{freq}^2}{\mu_{freq}} + CV_{Sev}^2\right) = \lambda_F \frac{\sigma_{freq}^2}{\mu_{freq}} + \lambda_F\, CV_{Sev}^2$.

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[42] See Equation 2.5.5 in Credibility by Mahler and Dean.
See "The Credibility of the Pure Premium," by Mayerson, Jones, and Bowers, PCAS 1968.

*Derivation of the Standard for Full Credibility for Pure Premiums or Aggregate Loss:*

Let $\mu_{Freq}$ be the mean of the frequency distribution while $\sigma_{Freq}^2$ is its variance.
Let $\mu_{Sev}$ be the mean of the severity distribution while $\sigma_{Sev}^2$ is its variance.
Assuming that frequency and severity are independent, the variance of the Pure Premium for one exposure unit is: $\sigma_{pp}^2 = \mu_{Freq}\ \sigma_{Sev}^2 + \mu_{Sev}^2\ \sigma_{Freq}^2$.

We assume the number of exposures is known; it is not random.[43]
For X exposure units, the variance of the estimated average pure premium is this divided by X.

We wish to be within ±y standard deviations, where as usual $z_{1-\alpha/2}$ is such that $\Phi(z_{1-\alpha/2}) = 1 - \alpha/2$.
For a mean pure premium of $\mu_{Freq}\ \mu_{Sev}$ we wish to be within $\pm k\ \mu_{Freq}\ \mu_{Sev}$.

Setting the two expressions for the error bars equal yields:
$k\ \mu_{Freq}\ \mu_{Sev} = z_{1-\alpha/2}\ \sqrt{(\mu_{Freq}\ \sigma_{Sev}^2 + \mu_{Sev}^2\ \sigma_{Freq}^2)/X}$ .
Solving for X, the full credibility standard in exposures:

$$X = \left(\frac{z_{1-\alpha/2}}{k}\right)^2\ (\mu_{Sev}^2\sigma_{Freq}^2 + \mu_{Freq}\sigma_{Sev}^2)\ /\ (\mu_{Freq}^2\ \mu_{Sev}^2).$$

The expected number of claims needed for Full Credibility is:

$$\mu_{Freq}\ X = \left(\frac{z_{1-\alpha/2}}{k}\right)^2\ (\sigma_{Freq}^2/\mu_{Freq} + \sigma_{Sev}^2/\mu_{Sev}^2) = \lambda_F\ (\sigma_{Freq}^2/\mu_{Freq} + CV_{Sev}^2).$$

Exposures vs. Claims:

Standards for Full Credibility are calculated in terms of the expected number of claims. It is common to translate these into a number of exposures by dividing by the (approximate) expected claim frequency. So for example, if the Standard for Full Credibility is 2885 claims and the expected claim frequency in Auto Insurance were 0.07 claims per car-year, then 2885 / 0.07 ≅ 41,214 car-years would be a corresponding Standard for Full Credibility in terms of exposures.

**The Standard for Full Credibility in terms of claims, can be converted to exposures by dividing by $\mu_f$, the mean claim frequency**.

Standard for Full Credibility in terms of exposures = (number of claims for full credibility) / $\mu_{Freq}$ = $\lambda_F\ (\sigma_{Freq}^2/\mu_{Freq} + \sigma_{Sev}^2 / \mu_{Sev}^2)/\ \mu_{Freq} = \lambda_F\ (\mu_{Sev}^2\sigma_{Freq}^2 + \mu_{Freq}\sigma_{Sev}^2)\ /\ (\mu_{Freq}\mu_{Sev})^2 =$
$\lambda_F$ (variance of pure premium) / (mean pure premium)$^2$ = $\lambda_F$ (CV of the Pure Premium)$^2$.

[43] While we will solve for that number of exposures which satisfies the criterion for full credibility, in any given application of the credibility technique the number of exposures is known.

When asked for the number of exposures needed for Full Credibility for Pure Premiums one can directly use this formula:[44]
Standard for Full Credibility for <u>Pure Premiums</u> in terms of <u>exposures</u> is:

$\lambda_F$ (Coefficient of Variation of the Pure Premium)$^2$ = $\left(\frac{z_{1-\alpha/2}}{k}\right)^2$ $(CV_{PP})^2$.

Exercise: The variance of pure premiums is 100,000. The mean pure premium is 40. Frequency is Poisson. We require that the estimated pure premiums be within 2.5% of the true value 90% of the time. How many exposures are need for full credibility?
[Solution: The square of the Coefficient of Variation of the Pure Premium is $100{,}000/40^2 = 62.5$.

$z_{1-\alpha/2} = 1.645$. $k = 0.025$. $\lambda_F = \left(\frac{z_{1-\alpha/2}}{k}\right)^2 = 4330$.

$\lambda_F$(Coefficient of Variation of the Pure Premium)$^2$ = 4330(62.5) = 270,625 exposures.
Alternately, let m be the mean frequency. Then since the frequency is assumed to be Poisson, variance of pure premium = m(second moment of severity).
Thus $E[X^2] = 100{,}000 / m$. $E[X] = 40 / m$. Standard for Full Credibility in terms of claims is:
$\lambda_F (1 + CV^2) = \lambda_F E[X^2] / E[X]^2 = 4330 (100{,}000 / 40^2)$ m = 270,625 m claims.
To convert to exposures divide by m, to get 270,625 exposures.]

<u>Assumptions</u>:

The formula: number of claims needed for full credibility = $\lambda_F \left(\frac{\sigma_{freq}^2}{\mu_{freq}} + CV_{sev}^2\right)$, assumes:

1. Frequency and Severity are independent.
2. The claims are drawn from the same distribution or at least from distributions with the same finite mean and variance.[45]
3. The pure premium or aggregate loss is approximately Normally Distributed (the Central Limit Theorem applies.)[46]
4. The number of exposures is known; it is not stochastic.

The pure premiums are often approximately Normal; generally the greater the expected number of claims or the shorter tailed the frequency and severity distributions, the better the Normal Approximation. It is assumed that one has enough claims that the aggregate losses approximate a Normal Distribution.

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[44] In the formula for the standard for full credibility the CV is calculated for <u>one</u> exposure. The CV would go down as the number of exposures increased, since the mean increases as a factor of N, while the standard deviation increases as a factor of square root of N. This is precisely why when we have a lot of exposures, we get a good estimate of the pure premium by relying solely on the data. In other words, this is why there is a standard for full credibility.
[45] The claim sizes can follow any distribution with a finite mean and variance, so that one can compute the coefficient of variation.
[46] While it is possible to derive formulas that don't depend on the Normal Approximation, they are not on the Syllabus. See for example Appendix 1 of "Classical Partial Credibility with Application to Trend" by Gary Venter, PCAS 1986.

*Other Notations:*

NonLife Actuarial Models: Theory, Methods and Evaluation does not use the same notation as many casualty actuarial papers and other textbooks. Here is a sample of other notations to help you if you read other material on Classical Credibility.

| Non-Life Actuarial Models | Mahler-Dean | Loss Models | |
|---|---|---|---|
| | | | |
| $1 - \alpha$ | P | p | probability level |
| | | | |
| k | k | r | range parameter |
| | | | |
| $z_{1-\alpha/2}$ | y | $y_p$ | such that the mean ±y standard deviations covers |
| | | | |
| $\lambda_F$ | $n_0$ | $\lambda_0$ | Standard for Full Credibility for Poisson Frequency |
| | | | |
| | $n_F$ | | Standard for Full Credibility for Pure Premium |

Classical Credibility is also referred to as Limited Fluctuation Credibility.

Problems:

**5.1** (2 points) You are given the following information:
● The number of claims is Poisson.
● The severity distribution is LogNormal, with parameters $\mu = 6$ and $\sigma = 1.2$.
● Frequency and severity are independent
● Full credibility is defined as having a 95% probability of being within plus or minus 10% of the true pure premium.
What is the minimum number of expected claims that will be given full credibility?
A. Less than 1600
B. At least 1600 but less than 1700
C. At least 1700 but less than 1800
D. At least 1800 but less than 1900
E. At least 1900

**5.2** (2 points) The number of claims is Poisson.
Mean claim frequency = 7%. Mean claim severity = $500.
Variance of the claim severity = 1 million. Full credibility is defined as having a 80% probability of being within plus or minus 5% of the true pure premium.
What is the minimum number of policies that will be given full credibility?
A. 47,000 B. 48,000 C. 49,000 D. 50,000 E. 51,000

**5.3** (3 points) The number of claims is Poisson. The full credibility standard for a company is set so that the total number of claims is to be within 5% of the true value with probability P. This full credibility standard is calculated to be 5000 claims. The standard is altered so that the total cost of claims is to be within 10% of the true value with probability P. The claim frequency has a Poisson distribution and the claim severity has the following distribution:

$$f(x) = 0.000008 (500 - x),\ 0 \le x \le 500$$

What is the expected number of claims necessary to obtain full credibility under the new standard?
A. 1825 B. 1850 C. 1875 D. 1900 E. 1925

**5.4** (2 points) You are given the following information:
● A standard for full credibility of 3,000 claims has been selected so that the actual pure premium would be within 5% of the expected pure premium 98% of the time.
● The number of claims follows a Poisson distribution, and is independent of the severity distribution.
Using the concepts of classical credibility, determine the coefficient of variation of the severity distribution underlying the full credibility standard.
A. Less than 0.6
B. At least 0.6 but less than 0.7
C. At least 0.7 but less than 0.8
D. At least 0.8 but less than 0.9
E. At least 0.9

**5.5** (2 points) You are given the following:
● The number of claims is Poisson distributed.
● Number of claims and claim severity are independent.
● Claim severity has the following distribution:

| Claim Size | Probability |
|---|---|
| 1 | 0.50 |
| 5 | 0.30 |
| 10 | 0 20 |

Determine the number of claims needed so that the total cost of claims is within 3% of the expected cost with 90% probability.
A. Less than 5000
B. At least 5000 but less than 5100
C. At least 5100 but less than 5200
D. At least 5200 but less than 5300
E. At least 5300

**5.6.** (2 points) Frequency is Poisson, and severity is Pareto with $\alpha = 4$.
The standard for full credibility is that actual aggregate losses be within 10% of expected aggregate losses 99% of the time.
50,000 exposures are needed for full credibility.
Determine the expected number of claims per exposure.
A. 2% B. 3% C. 4% D. 5% E. 6%

**5.7** (2 points) The distribution of pure premium has a coefficient of variation of 5.
The full credibility standard has been selected so that actual aggregate losses will be within 5% of expected aggregate losses 90% of the time.
Using limited fluctuation credibility, determine the number of exposures required for full credibility.
(A) 23,000 (B) 24,000 (C) 25,000 (D) 26,000 (E) 27,000

**5.8** (3 points) Require that the estimated pure premium should be within 100k% of the expected pure premium with probability 1 - $\alpha$. Assume frequency and severity are independent.
Use the following notation:

$\mu_f$ = mean frequency $\sigma_f^2$ = variance of frequency

$\mu_S$ = mean severity $\sigma_S^2$ = variance of severity

Let $z_{1-\alpha/2}$ be such that $\Phi(z_{1-\alpha/2}) = 1 - \alpha/2$.

Using the Normal Approximation, which of the following is a formula for the number of claims needed for full credibility of the pure premium?

A. $\left(\frac{\sigma_f^2}{\mu_f} + \frac{\sigma_s}{\mu_s}\right)\left(\frac{z_{1-\alpha/2}}{k}\right)^2$ B. $\left(\frac{\sigma_f^2}{\mu_f^2} + \frac{\sigma_s}{\mu_s}\right)\left(\frac{z_{1-\alpha/2}}{k}\right)^2$

C. $\left(\frac{\sigma_f^2}{\mu_f} + \frac{\sigma_s^2}{\mu_s^2}\right)\left(\frac{z_{1-\alpha/2}}{k}\right)^2$ D. $\left(\frac{\sigma_f^2}{\mu_f^2} + \frac{\sigma_s^2}{\mu_s^2}\right)\left(\frac{z_{1-\alpha/2}}{k}\right)^2$

E. None of the above.

**5.9** (2 points) Using the formula derived in the previous question, find the number of claims required for full credibility. Require that there is a 90% chance that the estimate of the pure premium is correct within ±7.5%.
The frequency distribution has a variance 2.5 times its mean.
The claim amount distribution is a Pareto with $\alpha = 2.3$.
A. Less than 4500
B. At least 4500 but less than 4600
C. At least 4600 but less than 4700
D. At least 4700 but less than 4800
E. At least 4800

**5.10** (2 points) The number of claims is Poisson. The expected number of claims needed to produce a selected standard for full credibility for the pure premium is 1500. If the severity were constant, the same selected standard for full credibility would require 850 claims.
Given the information below, what is the variance of the severity in the first situation?
Average Claim Frequency = 200 Average Claim Severity = 500.
A. less than 190,000
B. at least 190,000 but less than 200,000
C. at least 200,000 but less than 210,000
D. at least 210,000 but less than 220,000
E. at least 220,000

**5.11** (1 point) The expected number of claims needed to produce full credibility for the claim frequency is 700. Let:

Average claim frequency = 100 Average claim cost = 400
Variance of claim frequency = 100 Variance of claim cost = 280,000

What is the expected number of claims required to produce full credibility for the pure premium?

A. Less than 1,750
B. At least 1,750, but less than 1,850
C. At least 1,850, but less than 1,950
D. At least 1,950, but less than 2,050
E. 2,050 or more

**5.12** (2 points) A full credibility standard is determined so that the total number of claims is within 5% of the expected number with probability 99%. If the same expected number of claims for full credibility is applied to the total cost of claims, the actual total cost would be within 100k% of the expected cost with 95% probability. The coefficient of variation of the severity is 2.5. The frequency is Poisson. Frequency and severity are independent. Using the normal approximation of the aggregate loss distribution, determine k.

A. 4% B. 6% C. 8% D. 10% E. 12%

**5.13** (1 point) Which of the following are true regarding Standards for Full Credibility?
1. A Standard for Full Credibility should be adjusted for inflation.
2. All other things being equal, if severity is not constant, a Standard for Full Credibility for pure premiums is larger than that for frequency.
3. All other things being equal, a Standard for Full Credibility for pure premiums is larger as applied to losses limited by a policy limit than when applied to unlimited losses.

A. None of 1, 2 or 3 B. 1 C. 2 D. 3 E. None of A, B, C or D

**5.14** (2 points) You are given the following:
- The frequency distribution is Poisson
- The claim amount distribution has mean 1000, variance 4,000,000.
- Frequency and severity are independent.

Find the number of claims required for full credibility, if you require that there will be a 80% chance that the estimate of the pure premium is correct within 10%.

A. Less than 750
B. At least 750 but less than 800
C. At least 800 but less than 850
D. At least 850 but less than 900
E. At least 900

**5.15** (3 points) Standards for full credibility for aggregate losses are being determined for three situations.
The only thing that differs among the situations is the assumed size of loss distribution:
1. Exponential.
2. Weibull, $\tau = 1/2$.
3. LogNormal, $\sigma = 0.8$.

Rank the resulting standards for full credibility from smallest to largest.

A. 1, 2, 3 B. 1, 3, 2 C. 2, 1, 3 D. 2, 3, 1 E. none of A, B, C, or D

**5.16** (2 points) You are given the following:
- The total losses for one risk within a class of homogeneous risks equals T.
- $E[\{T - E(T)\}^2] = 40,000$.
- The average amount of each claim = 100.
- The frequency for each insured is Poisson.
- The average number of claims for each risk = 2.

Find the number of claims required for full credibility, if you require that there will be a 90% chance that the estimate of the pure premium is correct within 5%.
A. Less than 1,000
B. At least 1,000 but less than 1,500
C. At least 1,500 but less than 2,000
D. At least 2,000 but less than 2,500
E. 2,500 or more

**5.17** (1 point) You are given the following:
- You require that the estimated frequency should be
  within 100k% of the expected frequency with probability P.
- The standard for full credibility for frequency is 800 claims.
- You require that the estimated pure premium should be
  within 100k% of the expected pure premium with probability P.
- The standard for full credibility for pure premiums is 2000 claims.
- You require that the estimated severity should be
  within 100k% of the expected severity with probability P.

What is the standard for full credibility for the <u>severity</u>, in terms of the number of claims?
A. 900 B. 1000 C. 1100 D. 1200 E. 1300

**5.18** (3 points) You are given the following:
- The number of claims follows a Poisson distribution.
- Claim sizes follow a Burr distribution, with parameters $\theta$ (unknown), $\alpha = 9$, and $\gamma = 0.25$.
- The number of claims and claim sizes are independent.
- The full credibility standard has been selected so that actual aggregate claim
  costs will be within 10% of expected aggregate claim costs 85% of the time.

Using the methods of classical credibility, determine the expected number of claims needed for full credibility.
A. Less than 1000
B. At least 1000, but less than 10,000
C. At least 10,000, but less than 100,000
D. At least 100,000, but less than 1,000,000
E. At least 1,000,000

**5.19** (2 points) You are given the following:
- The number of claims follows a Poisson distribution.
- The variance of the number of claims is 20.
- The variance of the claim size distribution is 35.
- The variance of aggregate claim costs is 1300.
- The number of claims and claim sizes are independent.
- The full credibility standard has been selected so that actual aggregate claim costs will be within 7.5% of expected aggregate claim costs 98% of the time.

Using the methods of classical credibility, determine the expected number of claims required for full credibility.

A. Less than 2,000
B. At least 2,000, but less than 2,100
C. At least 2,100, but less than 2,200
D. At least 2,200, but less than 2,300
E. At least 2,300

**5.20** (3 points) Determine the number of claims needed for full credibility in three situations. In each case, there will be a 90% chance that the estimate is correct within 10%.
1. Estimating frequency. Frequency is assumed to be Negative Binomial with $\beta = 0.3$.
2. Estimating severity. Severity is assumed to be Pareto with $\alpha = 5$.
3. Estimating aggregate losses. Frequency is assumed to be Poisson.
   Severity is assumed to be Gamma with $\alpha = 2$.

Rank the resulting standards for full credibility from smallest to largest.
A. 1, 2, 3  B. 1, 3, 2  C. 2, 1, 3  D. 2, 3, 1  E. none of A, B, C, or D

**5.21** (2 points)
A company has determined that the limited fluctuation full credibility standard is 16,000 claims if:
(i) The total cost of claims is to be within r% of the true value with probability p.
(ii) The number of claims follows a Geometric distribution with $\beta = 0.4$.
(iii) The severity distribution is Exponential.
The standard is changed so that the total cost of claims is to be within 3r% of the true value with probability p, where claim severity is Gamma with $\alpha = 2$.
Using limited fluctuation credibility, determine the expected number of claims necessary to obtain full credibility under the new standard.
A. 1100  B. 1200  C. 1300  D. 1400  E. 1500

***5.22** (2 points) You are given the following information about a book of business:*
*(i) Each insured's claim count has a Poisson distribution with mean $\lambda$, where $\lambda$ has a gamma distribution with $\alpha = 4$ and $\theta = 0.5$.*
*(ii) Individual claim size amounts are independent and uniformly distributed from 0 to 500.*
*(iii) The full credibility standard is for aggregate losses to be within 10% of the expected with probability 0.98.*
*Using classical credibility, determine the expected number of claims required for full credibility.*
*(A) 600  (B) 700  (C) 800  (D) 900  (E) 1000*

**5.23** (3 points) You are given the following:
● Claim sizes follow a gamma distribution, with parameters $\alpha$ = 2.5 and $\theta$ unknown.
● The number of claims and claim sizes are independent.
● The full credibility standard for frequency has been selected so that the actual number of claims will be within 2.5% of the expected number of claims P of the time.
● The full credibility standard for aggregate loss has been selected so that the actual aggregate losses will be within 2.5% of the expected actual aggregate losses P of the time, using the same P as for the standard for frequency.
● 13,801 expected claims are needed for full credibility for frequency.
● 18,047 expected claims are needed for full credibility for aggregate loss.
Using the methods of classical credibility, determine the value of P.
A. 80% B. 90% C. 95% D. 98% E. 99%

**5.24** (2 points) You are given the following information about a book of business:
(i) The claim count distribution has a mean of 3 and variance of 11.
(ii) The distribution of individual claim size amounts has a coefficient of variation is of 4.
Where S is the aggregate losses, determine the minimum number of expected claims such that:
Prob[ 0.9E[S] < S < 1.1E[S] ] = 0.95.
A. Less than 4000
B. At least 4000 but less than 5000
C. At least 5000 but less than 6000
D. At least 6000 but less than 7000
E. At least 7000

**5.25** (2 points) You are given the following information about an auto liability book of business:
(i) The distribution of claim counts has a variance 1.5 times its mean.
(ii) Individual claim size amounts are independent of each other and also independent of the number of claims.
(iii) The distribution of claim sizes has mean 5000 and variance 700 million.
(iv) The full credibility standard is for aggregate losses to be within 10% of the expected with probability 0.95.
Using classical credibility, determine the expected aggregate losses required for full credibility.
A. Less than 20 million
B. At least 20 million but less than 30 million
C. At least 30 million but less than 40 million
D. At least 40 million but less than 50 million
E. At least 50 million

**5.26** (2 points) You are using classical credibility in order to estimate future aggregate losses.
Frequency is Negative Binomial with $\beta = 0.25$.
If severity is assumed to be constant, then a sample size of 2500 claims is necessary for full credibility.
What is the number of claims needed for full credibility if instead severity is assumed to be Gamma with $\alpha = 1/2$?
(A) Less than 3000
(B) At least 3000, but less than 4000
(C) At least 4000, but less than 5000
(D) At least 5000, but less than 6000
(E) At least 6000

**5.27** (3 points) You are given:
(i) The number of claims and claim sizes are independent.
(ii) The full credibility standard has been selected so that actual aggregate losses will be within 3% of expected aggregate losses 90% of the time.
(iii) $n_i$ is the number of claims observed for policy i.
(iv) $\sum_{i=1}^{5000} n_i = 200$.
(v) $\sum_{i=1}^{5000} n_i^2 = 250$.
(vi) $x_j$ is the size of claim number j.
(vii) $\sum_{j=1}^{200} x_j = 600{,}000$.
(viii) $\sum_{j=1}^{200} x_j^2 = 27{,}000$ million.

Using limited fluctuation (classical) credibility, determine the expected number of claims required for full credibility.
(A) Less than 40,000
(B) At least 40,000, but less than 45,000
(C) At least 45,000, but less than 50,000
(D) At least 50,000, but less than 55,000
(E) At least 55,000

**5.28** (2 points) You are given the following:
● Frequency is Poisson.
● The number of claims and claim sizes are independent.
● The full credibility standard for aggregate loss has been selected so that the actual aggregate losses will be within 5% of the expected actual aggregate losses 90% of the time,
● 15,000 expected claims are needed for full credibility for aggregate loss.
Determine the coefficient of variation of the severity distribution.
A. 3.6 B. 3.8 C. 4.0% D. 4.2 E. 4.4

**5.29** (3 points) You are given the following information:
● The number of claims is Poisson.
● The unlimited severity distribution is LogNormal, with parameters $\mu = 9$ and $\sigma = 1.3$.
● The maximum claim amount is capped at 100,000.
● Frequency and severity are independent.
● The full credibility standard for frequency is 750 claims.
What is the full credibility standard for aggregate loss in terms of number of claims?
A. Less than 1800
B. At least 1800 but less than 1900
C. At least 1900 but less than 2000
D. At least 2000 but less than 2100
E. At least 2100

**5.30** (2 points) You are given the following information:
● The number of claims is Poisson.
● Frequency and severity are independent.
● Claim severity has a mean of 1000 and a standard deviation of 1400.
● The limited fluctuation standard for full credibility for severity is G claims.
● Using the same k and $\alpha$, the limited fluctuation standard for full credibility for aggregate loss is H claims.
Determine G/H.

**5.31 (4, 11/82, Q.47)** (3 points) You are given the following:
● The frequency distribution is Negative Binomial with variance equal to twice its mean.
● The claim amount distribution is LogNormal with mean 100, variance 25,000.
● Frequency and severity are independent.
Find the number of claims required for full credibility, if you require that there will be a 90% chance that the estimate of the pure premium is correct within 5%.
Use the Normal Approximation.
A. Less than 4500
B. At least 4500 but less than 4600
C. At least 4600 but less than 4700
D. At least 4700 but less than 4800
E. At least 4800

**5.32 (4, 5/83, Q.36)** (1 point) The number of claims is Poisson. Assume that claim severity has mean equal to 100 and standard deviation equal to 200. Which of the following is closest to the factor which would need to be applied to the full credibility standard based on frequency only, in order to approximate the full credibility standard for the pure premium?

A. 0.5 B. 1.0 C. 1.5 D. 2.0 E. 5.0

**5.33 (4, 5/83, Q.46)** (3 points) Chebyshev's inequality says that for a probability distribution X, with mean m and standard deviation σ, for any constant "a": $\text{Prob}(|X - m| \geq a\sigma) \leq 1/a^2$.

Using Chebyshev's inequality (rather than the Normal Approximation) derive a formula for the number of claims needed for full credibility of the pure premium.

Assume frequency and severity are independent. Require that the observed pure premium should be within 100k% of the expected pure premium with probability 1 - α.

Use the following notation:

$\mu_f$ = mean frequency $\sigma_f^2$ = variance of frequency

$\mu_s$ = mean severity $\sigma_s^2$ = variance of severity

A. $\dfrac{\dfrac{\sigma_f^2}{\mu_f} + \dfrac{\sigma_s^2}{\mu_s^2}}{k^2\,\alpha}$ B. $\left(\dfrac{\sigma_f^2}{\mu_f^2} + \dfrac{\sigma_s^2}{\mu_s^2}\right) k^2\,\alpha$

C. $\left(\dfrac{\sigma_f^2}{\mu_f} + \dfrac{\sigma_s^2}{\mu_s^2}\right) \dfrac{k^2}{\alpha}$ D. $\left(\dfrac{\sigma_f^2}{\mu_f^2} + \dfrac{\sigma_s^2}{\mu_s^2}\right) \dfrac{k^2}{\alpha}$

E. None of the above.

**5.34** (2 points) Using the formula derived in the previous question, find the number of claims required for full credibility.

Require that there is a 90% chance that the estimate of the pure premium is correct within 7.5%.

The frequency distribution has a variance 2.5 times its mean.

The claim amount distribution is a Pareto with α = 2.3.

A. Less than 15,000

B. At least 15,000 but less than 16,000

C. At least 16,000 but less than 17,000

D. At least 17,000 but less than 18,000

E. At least 18,000

**5.35 (4, 5/85, Q.32)** (1 point) The expected number of claims needed to produce a selected level of credibility for the claim frequency is 1200. Let:

Average claim frequency = 200 Average claim cost = 400

Variance of claim frequency = 200 Variance of claim cost = 80,000

What is the expected number of claims required to produce the same level of credibility for the pure premium? (Use Classical Credibility.)

A. Less than 1,750

B. At least 1,750, but less than 1,850

C. At least 1,850, but less than 1,950

D. At least 1,950, but less than 2,050

E. 2,050 or more

**5.36 (4, 5/85, Q.33)** (2 points) How many claims are necessary for full credibility if the standard for full credibility is to have the estimated pure premium be within 8% of the true pure premium 90% of the time? Assume the average claim severity is $1000 and the standard deviation of the claim severity is 4000. Assume the variance of the number of claims is 1.5 times the mean number of claims. Assume frequency and severity are independent.

A. Less than 7,150
B. At least 7,150, but less than 7,250
C. At least 7,250, but less than 7,350
D. At least 7,350, but less than 7,450
E. 7,450 or more

**5.37 (4, 5/87, Q.34)** (1 point) The expected number of claims needed to produce a selected standard for full credibility for the pure premium is 1800. If the claim size were constant, the same selected standard for full credibility would require 1200 claims.
Given the information below, what is the variance of the claim cost in the first situation?

● The number of claims is Poisson.
● Average Claim Frequency = 200.
● Average Claim Cost = 400.

A. 20,000 B. 40,000 C. 80,000 D. 120,000 E. 160,000

**5.38 (4, 5/87, Q.35)** (2 points) The number of claims for a company's major line of business is Poisson distributed, and during the past year, the following claim size distribution was observed:

| | |
|---|---|
| $ 0 - 400 | 20 |
| 400 - 800 | 240 |
| 800 - 1200 | 320 |
| 1200 - 1600 | 210 |
| 1600 - 2000 | 100 |
| 2000 - 2400 | 60 |
| 2400 - 2800 | 30 |
| 2800 - 3200 | 10 |
| 3200 - 3600 | 10 |
| | |
| Total | 1000 |

The mean of this claim size distribution is $1216 and the standard deviation is $\sqrt{\$362{,}944}$ .

You need to select the number of claims needed to ensure that the estimate of losses is within 8% of the actual value 90% of the time. How many claims are needed for full credibility if the claim size distribution is considered?

A. Less than 450 claims
B. At least 450, but less than 500 claims
C. At least 500, but less than 550 claims
D. At least 550, but less than 600 claims
E. 600 claims or more

**5.39 (4, 5/90, Q.29)** (2 points) The ABC Insurance Company has decided to establish its full credibility requirements for an individual state rate filing using Classical Credibility. The full credibility standard is to be set so that the observed total cost of claims underlying the rate filing should be within 5% of the true value with probability 0.95. The claim frequency follows a Poisson distribution and the claim severity is distributed according to the following distribution:

$f(x) = \frac{1}{100,000}$, for $0 \le x \le 100,000$.

What is the expected number of claims, $n_F$ necessary to obtain full credibility.

A. $n_F < 1500$

B. $1500 \le n_F < 1800$

C. $1800 \le n_F < 2100$

D. $2100 \le n_F < 2400$

E. $2400 \le n_F$

**5.40 (4, 5/91, Q.22)** (1 point) The average claim size for a group of insureds is $1,500 with standard deviation $7,500. Assuming a Poisson claim count distribution, calculate the expected number of claims so that the total loss will be within 6% of the expected total loss with probability 90%.

A. Less than 10,000
B. At least 10,000 but less than 15,000
C. At least 15,000 but less than 20,000
D. At least 20,000 but less than 25,000
E. At least 25,000

**5.41 (4, 5/91, Q.39)** (3 points) The full credibility standard for a company is set so that the total <u>number</u> of claims is to be <u>within 5%</u> of the true value with probability P. This full credibility standard is calculated to be 800 claims. The standard is altered so that the total <u>cost</u> of claims is to be <u>within 10%</u> of the true value with probability P. The claim frequency has a Poisson distribution and the claim severity has the following distribution.

$f(x) = (0.0002)(100 - x)$, $0 \le x \le 100$.

What is the expected number of claims necessary to obtain full credibility under the new standard?

A. Less than 250
B. At least 250 but less than 500
C. At least 500 but less than 750
D. At least 750 but less than 1000
E. At least 1000

**5.42 (4B, 5/92, Q.1)** (2 points) You are given the following information:
- A standard for full credibility of 1,000 claims has been selected so that the actual pure premium would be within 10% of the expected pure premium 95% of the time.
- The number of claims follows a Poisson distribution, and is independent of the severity distribution.

Using the concepts from Classical Credibility determine the coefficient of variation of the severity distribution underlying the full credibility standard.
A. Less than 1.20
B. At least 1.20 but less than 1.35
C. At least 1.35 but less than 1.50
D. At least 1.50 but less than 1.65
E. At least 1.65

**5.43 (4B, 5/92, Q.16)** (2 points) You are given the following information:
- The number of claims follows a Poisson distribution.
- Claim severity is independent of the number of claims and has the following distribution:
  $f(x) = (5/2)\ x^{-7/2}$ , x > 1.

A full credibility standard is determined so that the total <u>number</u> of claims is within 5% of the expected number with probability 98%. If the same expected number of claims for full credibility is applied to the total <u>cost</u> of claims, the actual total cost would be within 100K% of the expected cost with 95% probability.
Using the normal approximation of the aggregate loss distribution, determine K.
A. Less than 0.04
B. At least 0.04 but less than 0.05
C. At least 0.05 but less than 0.06
D. At least 0.06 but less than 0.07
E. At least 0.07

**5.44 (4B, 11/92, Q.1)** (2 points) You are given the following:
- The number of claims is Poisson distributed.
- Number of claims and claim severity are independent.
- Claim severity has the following distribution:

| Claim Size | Probability |
|---|---|
| 1 | 0.50 |
| 2 | 0.30 |
| 10 | 0.20 |

Determine the number of claims needed so that the total cost of claims is within 10% of the expected cost with 90% probability.
A. Less than 625
B. At least 625 but less than 825
C. At least 825 but less than 1,025
D. At least 1,025 but less than 1,225
E. At least 1,225

**5.45 (4B, 11/92, Q.10)** (2 points) You are given the following:
- A full credibility standard of 3,025 claims has been determined using classical credibility concepts.
- The full credibility standard was determined so that the actual pure premium is within 10% of the expected pure premium 95% of the time.
- Number of claims is Poisson distributed.

Determine the coefficient of variation for the severity distribution.
A. Less than 2.25
B. At least 2.25 but less than 2.75
C. At least 2.75 but less than 3.25
D. At least 3.25 but less than 3.75
E. At least 3.75

**5.46 (4B, 11/92, Q.15)** (2 points) You are given the following:
- X is the random variable for claim size.
- N is the random variable for number of claims and has a Poisson distribution.
- X and N are independent.
- $n_0$ is the standard for full credibility based only on number of claims.
- $n_F$ is the standard for full credibility based on total cost of claims.
- n is the observed number of claims.
- C is the random variable for total cost of claims.
- Z is the amount of credibility to be assigned to total cost of claims.

According to the Classical credibility concepts, which of the following are true?
1. Var(C) = E(N) Var(X) + E(X) Var(N)
2. $n_F = n_0 \frac{E(X)^2 + Var(X)}{E(X^2)}$
3. $Z = \sqrt{\frac{n}{n_F}}$

A. 1 only B. 2 only C. 1, 3 only D. 2, 3 only E. 1, 2, 3

**5.47 (4B, 5/93, Q.10)** (2 points) You are given the following:
- The number of claims for a single insured follows a Poisson distribution.
- The coefficient of variation of the severity distribution is 2.
- The number of claims and claim severity distributions are independent.
- Claim size amounts are independent and identically distributed.
- Based on Classical credibility, the standard for full credibility is 3415 claims.

With this standard, the observed pure premium will be within k% of the expected pure premium 95% of the time.
Determine k.
A. Less than 5.75%
B. At least 5.75% but less than 6.25%
C. At least 6.25% but less than 6.75%
D. At least 6.75% but less than 7.25%
E. At least 7.25%

**5.48 (4B, 11/93, Q.11)** (3 points) You are given the following:
- Number of claims follows a Poisson distribution.
- Claim severity is independent of the number of claims and has the following probability density distribution
  $f(x) = 5x^{-6}$, $x > 1$.

A full credibility standard has been determined so that the total cost of claims is within 5% of the expected cost with a probability of 90%. If the same number of claims for full credibility of total cost is applied to frequency only, the actual number of claims would be within 100k% of the expected number of claims with a probability of 95%.
Using the normal approximation of the aggregate loss distribution, determine k.
A. Less than 0.0545
B. At least 0.0545, but less than 0.0565
C. At least 0.0565, but less than 0.0585
D. At least 0.0585, but less than 0.0605
E. At least 0.0605

**5.49 (4B, 5/94, Q.13)** (2 points) You are given the following:
- 120,000 exposures are needed for full credibility.
- The 120,000 exposures standard was selected so that the actual total cost of claims is within 5% of the expected total 95% of the time.
- The number of claims per exposure follows a Poisson distribution with mean m.
- m was estimated from the following observed data using the method of moments:

| Year | Exposures | Claims |
|---|---|---|
| 1 | 18,467 | 1,293 |
| 2 | 26,531 | 1,592 |
| 3 | 20,002 | 1,418 |

If mean claim severity is $5,000, determine the standard deviation of the claim severity distribution.
A. Less than $9,000
B. At least $9,000, but less than $12,000
C. At least $12,000, but less than $15,000
D. At least $15,000, but less than $18,000
E. At least $18,000

**5.50 (4B, 11/94, Q.11)** (3 points) You are given the following:
Number of claims follows a Poisson distribution with mean m.
X is the random variable for claim severity, and has a Pareto distribution with parameters $\alpha = 3.0$ and $\theta = 6000$.
A standard for full credibility was developed so that the observed pure premium is within 10% of the expected pure premium 98% of the time.
Number of claims and claims severity are independent.
Using Classical credibility concepts, determine the number of claims needed for full credibility for estimates of the pure premium.
A. Less than 600
B. At least 600, but less than 1200
C. At least 1200, but less than 1800
D. At least 1800, but less than 2400
E. At least 2400

**5.51 (4B, 5/95, Q.10)** (1 point) You are given the following:
● The number of claims follows a Poisson distribution.
● The distribution of claim sizes has a mean of 5 and variance of 10.
● The number of claims and claim sizes are independent.
How many expected claims are needed to be 90% certain that actual claim costs will be within 10% of the expected claim costs?
A. Less than 100
B. At least 100, but less than 300
C. At least 300, but less than 500
D. At least 500, but less than 700
E. At least 700

**5.52 (4B, 5/95, Q.26)** (3 points) You are given the following:
● 40,000 exposures are needed for full credibility.
● The 40,000 exposures standard was selected so that the actual total cost of claims is within 7.5% of the expected total 95% of the time.
● The number of claims per exposure follows a Poisson distribution with mean m.
● The claim size distribution is lognormal with parameters $\mu$ (unknown) and $\sigma = 1.5$.
● The lognormal distribution has the following moments:
mean: $\exp(\mu + \sigma^2/2)$ variance: $\exp(2\mu + \sigma^2)\ \{\exp(\sigma^2) - 1\}$.
● The number of claims per exposure and claim sizes are independent.
Using the methods of classical credibility, determine the value of m.
A. Less than 0.05
B. At least 0.05, but less than 0.10
C. At least 0.10, but less than 0.15
D. At least 0.15, but less than 0.20
E. At least 0.20

**5.53 (4B, 11/95 Q.11)** (2 points) You are given the following:
- The number of claims follows a Poisson distribution.
- Claim sizes follow a Pareto distribution, with parameters $\theta = 3000$ and $\alpha = 4$.
- The number of claims and claim sizes are independent.
- 2000 expected claims are needed for full credibility.
- The full credibility standard has been selected so that actual claim costs will be within 5% of expected claim costs P% of the time.

Using the methods of Classical credibility, determine the value of P.
A. Less than 82.5
B. At least 82.5, but less than 87.5
C. At least 87.5, but less than 92.5
D. At least 92.5, but less than 97.5
E. At least 97.5

**5.54 (4B, 5/96, Q.27)** (2 points) You are given the following:
- The number of claims follows a Poisson distribution.
- Claim sizes follow a gamma distribution, with parameters $\alpha = 1$ and $\theta$ (unknown).
- The number of claims and claim sizes are independent.

The full credibility standard has been selected so that actual claim costs will be within 5% of expected claim costs 90% of the time. Using the methods of Classical credibility, determine the expected number of claims required for full credibility.
A. Less than 1,000
B. At least 1,000, but less than 2,000
C. At least 2,000, but less than 3,000
D. At least 3,000
E. Cannot be determined from the given information.

**5.55 (4B, 11/96, Q.2)** (1 point) Using the methods of Classical credibility, a full credibility standard of 1,000 expected claims has been established so that actual claim costs will be within 100c% of expected claim costs 90% of the time. Determine the number of expected claims that would be required for full credibility if actual claim costs were to be within 100c% of expected claim costs 95% of the time.
A. Less than 1,100
B. At least 1,100, but less than 1,300
C. At least 1,300, but less than 1,500
D. At least 1,500, but less than 1,700
E. At least 1,700

**5.56 (4B, 11/96, Q.28)** (2 points) You are given the following:
- The number of claims follows a Poisson distribution.
- Claim sizes are discrete and follow a Poisson distribution with mean 4.
- The number of claims and claim sizes are independent.

The full credibility standard has been selected so that actual claim costs will be within 10% of expected claim costs 95% of the time. Using the methods of Classical credibility, determine the expected number of claims required for full credibility.
A. Less than 400
B. At least 400, but less than 600
C. At least 600, but less than 800
D. At least 800, but less than 1,000
E. At least 1,000

**5.57 (4B, 5/97, Q.2)** (2 points) The number of claims follows a Poisson distribution.
Using the methods of Classical credibility, a full credibility standard of 1,200 expected claims has been established for aggregate claim costs. Determine the number of expected claims that would be required for full credibility if the coefficient of variation of the claim size distribution were changed from 2 to 4 and the range parameter, k, were doubled.
A. 500 B. 1,000 C. 1,020 D. 1,200 E. 2,040

**5.58 (4B, 11/97, Q.24 & Course 4 Sample Exam 2000, Q.15)** (3 points)
You are given the following:
- The number of claims per exposure follows a Poisson distribution with mean 0.01.
- Claim sizes follow a lognormal distribution, with parameters $\mu$ (unknown) and $\sigma = 1$.
- The number of claims per exposure and claim sizes are independent.
- The full credibility standard has been selected so that actual aggregate claim costs will be within 10% of expected aggregate claim costs 95% of the time.

Using the methods of Classical credibility, determine the number of **exposures** required for full credibility.
A. Less than 25,000
B. At least 25,000, but less than 50,000
C. At least 50,000, but less than 75,000
D. At least 75,000, but less than 100,000
E. At least 100,000

Use the following information for the next two questions:

You are given the following:
- The number of claims follows a Poisson distribution.
- Claim sizes follow a inverse gamma distribution, with parameters $\alpha = 4$ and $\theta$ unknown.
- The number of claims and claim sizes are independent
- The full credibility standard has been selected so that the actual aggregate claim costs will be within 5% of expected aggregate claim costs 95% of the time.

**5.59 (4B, 5/98 Q.18)** (2 points) Using the methods of Classical credibility, determine the expected number of claims required for full credibility.
A. Less than 1,600
B. At least 1,600, but less than 18,00
C. At least 1,800, but less than 2,000
D. At least 2,000, but less than 2,200
E. At least 2,200

**5.60 (4B, 5/98 Q.19)** (1 point) If the number of claims were to follow a negative binomial distribution instead of a Poisson distribution, determine which of the following statements would be true about the expected number of claims required for full credibility.
A. The expected number of claims required for full credibility would be smaller.
B. The expected number of claims required for full credibility would be the same.
C. The expected number of claims required for full credibility would be larger.
D. The expected number of claims required for full credibility would be either the same or smaller, depending on the parameters of the negative binomial distribution.
E. The expected number of claims required for full credibility would be either smaller or larger, depending on the parameters of the negative binomial distribution.

**5.61 (4B, 11/98, Q.5)** (2 points) You are given the following:
- The number of claims follows a Poisson distribution.
- The variance of the number of claims is 10.
- The variance of the claim size distribution is 10.
- The variance of aggregate claim costs is 500.
- The number of claims and claim sizes are independent.
- The full credibility standard has been selected so that actual aggregate claim costs will be within 5% of expected aggregate claim costs 95% of the time.

Using the methods of Classical credibility, determine the expected number of claims required for full credibility.
A. Less than 2,000
B. At least 2,000, but less than 4,000
C. At least 4,000, but less than 6,000
D. At least 6,000, but less than 8,000
E. At least 8,000

**5.62 (4B, 11/98, Q.29)** (3 points) You are given the following:
- The number of claims follows a Poisson distribution.
- Claim sizes follow a Burr distribution, with parameters $\theta$ (unknown), $\alpha = 6$, and $\gamma = 0.5$ .
- The number of claims and claim sizes are independent.
- 6,000 expected claims are needed for full credibility.
- The full credibility standard has been selected so that actual aggregate claim costs will be within 10% of expected aggregate claim costs P% of the time.

Using the methods of Classical credibility, determine the value of P.

Hint: For the Burr Distribution $E[X^n] = \frac{\theta^n \, \Gamma(1 + n/\gamma)\, \Gamma(\alpha - n/\gamma)}{\Gamma(\alpha)}$ .

A. Less than 80
B. At least 80, but less than 85
C. At least 85, but less than 90
D. At least 90, but less than 95
E. At least 95

**5.63 (4B, 5/99, Q.19)** (1 point) You are given the following:
- The number of claims follows a Poisson distribution.
- The coefficient of variation of the claim size distribution is 2.
- The number of claims and claim sizes are independent.
- 1,000 expected claims are needed for full credibility.
- The full credibility standard has been selected so that the actual number of claims will be within k% of the expected number of claims P% of the time.

Using the methods of Classical credibility, determine the number of expected claims that would be needed for full credibility if the full credibility standard were selected so that actual aggregate claim costs will be within k% of expected aggregate claim costs P% of the time.
A. 1,000  B. 1,250  C. 2,000  D. 2,500  E. 5,000

**5.64 (4B, 11/99, Q.2)** (2 points) You are given the following:
- The number of claims follows a Poisson distribution.
- Claim sizes follow a lognormal distribution, with parameters $\mu$ and $\sigma$.
- The number of claims and claim sizes are independent.
- 13,000 expected claims are needed for full credibility.
- The full credibility standard has been selected so that actual aggregate claim costs will be within 5% of expected aggregate claim costs 90% of the time.

Determine $\sigma$.
A. Less than 1.2
B. At least 1.2, but less than 1.4
C. At least 1.4, but less than 1.6
D. At least 1.6, but less than 1.8
E. At least 1.8

**5.65 (4, 11/00, Q.14)** (2.5 points) For an insurance portfolio, you are given:
(i) For each individual insured, the number of claims follows a Poisson distribution.
(ii) The mean claim count varies by insured, and the distribution of mean claim counts follows a gamma distribution.
(iii) For a random sample of 1000 insureds, the observed claim counts are as follows:

| Number Of Claims, n | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| Number Of Insureds, $f_n$ | 512 | 307 | 123 | 41 | 11 | 6 |

$\sum n\, f_n = 750$, $\sum n^2\, f_n = 1494$.

(iv) Claim sizes follow a Pareto distribution with mean 1500 and variance 6,750,000.
(v) Claim sizes and claim counts are independent.
(vi) The full credibility standard is to be within 5% of the expected aggregate loss 95% of the time.
Determine the minimum number of insureds needed for the aggregate loss to be fully credible.
(A) Less than 8300
(B) At least 8300, but less than 8400
(C) At least 8400, but less than 8500
(D) At least 8500, but less than 8600
(E) At least 8600

***5.66 (4, 11/02, Q.14 )** (2.5 points) You are given the following information about a commercial auto liability book of business:*
*(i) Each insured's claim count has a Poisson distribution with mean $\lambda$, where $\lambda$ has a gamma distribution with $\alpha = 15$ and $\theta = 0.2$.*
*(ii) Individual claim size amounts are independent and exponentially distributed with mean 5000.*
*(iii) The full credibility standard is for aggregate losses to be within 5% of the expected with probability 0.90.*
*Using classical credibility, determine the expected number of claims required for full credibility.*
*(A) 2165 (B) 2381 (C) 3514 (D) 7216 (E) 7938*

**5.67 (4, 11/03, Q.3)** (2.5 points) You are given:
(i) The number of claims has a Poisson distribution.
(ii) Claim sizes have a Pareto distribution with parameters $\theta = 0.5$ and $\alpha = 6$.
(iii) The number of claims and claim sizes are independent.
(iv) The observed pure premium should be within 2% of the expected pure premium 90% of the time.
Determine the expected number of claims needed for full credibility.
(A) Less than 7,000
(B) At least 7,000, but less than 10,000
(C) At least 10,000, but less than 13,000
(D) At least 13,000, but less than 16,000
(E) At least 16,000

**5.68 (4, 5/05, Q.2)** (2.9 points) You are given:
(i) The number of claims follows a negative binomial distribution with parameters r and $\beta$ = 3.
(ii) Claim severity has the following distribution:

| Claim Size | Probability |
|---|---|
| 1 | 0.4 |
| 10 | 0.4 |
| 100 | 0.2 |

(iii) The number of claims is independent of the severity of claims.
Determine the expected number of claims needed for aggregate losses to be within 10% of expected aggregate losses with 95% probability.
(A) Less than 1200
(B) At least 1200, but less than 1600
(C) At least 1600, but less than 2000
(D) At least 2000, but less than 2400
(E) At least 2400

**5.69 (4, 11/05, Q.35)** (2.9 points) You are given:
(i) The number of claims follows a Poisson distribution.
(ii) Claim sizes follow a gamma distribution with parameters $\alpha$ (unknown) and $\theta$ = 10,000.
(iii) The number of claims and claim sizes are independent.
(iv) The full credibility standard has been selected so that actual aggregate losses will be within 10% of expected aggregate losses 95% of the time.
Using limited fluctuation (classical) credibility, determine the expected number of claims required for full credibility.
(A) Less than 400
(B) At least 400, but less than 450
(C) At least 450, but less than 500
(D) At least 500
(E) The expected number of claims required for full credibility cannot be determined from the information given.

**5.70 (4, 11/06, Q.30)** (2.9 points)
A company has determined that the limited fluctuation full credibility standard is 2000 claims if:
(i) The total number of claims is to be within 3% of the true value with probability p.
(ii) The number of claims follows a Poisson distribution.
The standard is changed so that the total cost of claims is to be within 5% of the true value with probability p, where claim severity has probability density function:

$$f(x) = \frac{1}{10{,}000},\ 0 \le x \le 10{,}000.$$

Using limited fluctuation credibility, determine the expected number of claims necessary to obtain full credibility under the new standard.
(A) 720 (B) 960 (C) 2160 (D) 2667 (E) 2880

**5.71 (CAS MAS-2, 5/19, Q.3)** (2.5 points)
An insurance company is determining limited-fluctuation credibility standards for its automobile losses. You are given the following information:
- The company selects all of its credibility standards to be the number of claims at which there is a 99% probability that the observed amount is within 10% of the mean.
- The standard for full credibility for aggregate loss is 4,800 claims.
- Claim frequency follows a Poisson distribution.
- Claim frequency and claim severity are independent.

Calculate the limited-fluctuation credibility standard for claim severity.
A. Less than 4,100
B. At least 4,100, but less than 4,300
C. At least 4,300, but less than 4,500
D. At least 4,500, but less than 4,700
E. At least 4,700

Solutions to Problems:

**5.1. B.** The mean severity = $\exp[\mu + 0.5\sigma^2] = \exp(6.72) = 828.82$. The second moment of the severity = $\exp[2\mu + 2\sigma^2] = \exp(14.88) = 2{,}899{,}358$. Thus $1 + CV^2 = E[X^2]/E[X]^2 = 2{,}899{,}358 / 828.82^2 = 4.221$. $z_{1-\alpha/2} = 1.960$ since $\Phi(1.960) = 0.975$. Therefore

$\lambda_F = \left(\frac{z_{1-\alpha/2}}{k}\right)^2 = (1.96/0.1)^2 = 384$. Therefore, $\lambda_F\ (1+CV^2) = (384)(4.221) =$ **1621 claims**.

**5.2. A.** Square of Coefficient of Variation = (1 million)/($500^2$) = 4.
$z_{1-\alpha/2} = 1.282$ since $\Phi(1.282) = 0.9$. k = 5%.
Therefore in terms of number of claims the full credibility standard is:
$\left(\frac{z_{1-\alpha/2}}{k}\right)^2 (1 + CV^2) = (1.282/0.05)^2(1 + 4) = 3287$ claims.
This is equivalent to: 3287 / 0.07 = **46,958 policies**.

**5.3. C.** The severity has a mean of 166.7, and a second moment of 41,667:

$$\int_0^{500} x\ f(x)\ dx = 0.000008 \int_0^{500} (500x - x^2)\ dx = (0.000008)\ (250x^2 - x^3/3)\Big]_{x=0}^{x=500} = 166.7.$$

$$\int_0^{500} x^2\ f(x)\ dx = 0.000008 \int_0^{500} (500x^2 - x^3)\ dx = (0.000008)\ (500x^3/3 - x^4/4)\Big]_{x=0}^{x=500} = 41{,}667.$$

$1 + CV^2 = E[X^2] / E[X]^2 = 41667 / 166.7^2 = 1.5$.
The standard for Full Credibility for the pure premiums for k = 5% is therefore
$\lambda_F\ (1+CV^2) = (5000)(1.5) = 7500$. For k = 10% we need to multiply by: $(5\%/10\%)^2 = 1/4$ since the full credibility standard is inversely proportional to $k^2$. 7500/4 = **1875**.

**5.4. B.** We have $z_{1-\alpha/2} = 2.326$ since $\Phi(2.326) = 0.99$

Therefore, $\lambda_F = \left(\frac{z_{1-\alpha/2}}{k}\right)^2 = (2.326 / 0.05)^2 = 2164$.

$3000 = \lambda_F\ (1+CV^2)$. Therefore CV = $\sqrt{\frac{3000}{2164} - 1}$ = **0.62**.

**5.5. D.** We have $z_{1-\alpha/2}$ = 1.645 since $\Phi(1.645) = 0.95$.

Therefore, $\lambda_F = \left(\frac{z_{1-\alpha/2}}{k}\right)^2 = (1.645 / 0.03)^2 = 3007$.

The mean severity is (1)(0.5) + (5)(0.3) + (10)(0.2) = 4.
The 2nd moment of the severity is: $(1^2)(0.5) + (5^2)(0.3) + (10^2)(0.2) = 28$.
$1 + CV^2 = E[X^2]/E[X]^2 = 28 / 4^2 = 1.75$. $\lambda_F (1+CV^2) = (3007)(1.75) =$ **5262**.

**5.6. C.** $\alpha = 1\% \Rightarrow z_{1-\alpha/2} = 2.576. \Rightarrow \lambda_F = (2.576/0.1)^2 = 664$ claims.

For the Pareto severity: $1 + CV^2 = E[X^2] / E[X]^2 = \frac{2\,\theta^2}{(4 - 1)(4-2)} \Big/ \left(\frac{\theta}{4 - 1}\right)^2 = 3$.

Thus the standard for full credibility is: (664)(3) = 1992 claims.
Thus, 1992 claims. ⇔ 50,000 exposures. ⇒ $\lambda$ = 1992 / 50,000 = **3.98%.**

**5.7. E.** $\alpha = 10\%. \Rightarrow z_{1-\alpha/2} = 1.645. \Rightarrow \lambda_F = (1.645/0.05)^2 = 1082$.

Standard for full credibility is: $\lambda_F\, CV_{PP}^2 = (1082)(5^2) =$ **27,050** exposures.

**5.8. C.** Assume there are N claims expected and therefore $N/\mu_f$ exposures.
The mean pure premium is $m = N\mu_S$.
For frequency and severity independent, the variance of the pure premium for a single exposure is: $\mu_f\, \sigma_S^2 + \mu_S^2\, \sigma_f^2$.
The variance of the aggregate loss for $N/\mu_f$ independent exposures is:
$\sigma^2 = (N / \mu_f)(\mu_f\, \sigma_S^2 + \mu_S^2\, \sigma_f^2) = N\,(\sigma_S^2 + \mu_S^2\, \sigma_f^2/\mu_f)$.
We desire that Prob[m - km ≤ X ≤ m + km] ≥ 1 - $\alpha$.
Using the Normal Approximation this is true provided $km = z_{1-\alpha/2}\,\sigma$
Therefore, $k^2 m^2 = z_{1-\alpha/2}^{\ 2}\, \sigma^2$. Thus, $k^2 N^2 \mu_S^2 = z_{1-\alpha/2}^{\ 2}\, N\,(\sigma_S^2 + \mu_S^2 \sigma_f^2/\mu_f)$.

Solving, $N = z_{1-\alpha/2}^{\ 2}\,(\sigma_S^2 + \mu_S^2\sigma_f^2/\mu_f)/(k^2\mu_S^2) = \boldsymbol{(\sigma_f^2/\mu_f + \sigma_S^2 / \mu_S^2)}\left(\frac{z_{1-\alpha/2}}{k}\right)^2$

$= \lambda_F\,(\sigma_f^2/\mu_f + CV_{Sev}^2)$.
Comment: *See Mayerson, Jones and Bowers "The Credibility of the Pure Premium", PCAS 1968*. Note that if one assumes a Poisson Frequency Distribution, then $\sigma_f^2/\mu_f = 1$ and the formula becomes: $(1 + CV^2)\left(\frac{z_{1-\alpha/2}}{k}\right)^2$.

**5.9. E.** $\Phi(1.645) = 0.95$ so that $z_{1-\alpha/2} = 1.645$. $\lambda_F = \left(\frac{z_{1-\alpha/2}}{k}\right)^2 = (1.645/0.075)^2 = 481$.

Using the formulas for the moments:

$$CV^2 = E[X^2] / E^2[X] - 1 = \frac{2\theta^2 / \{(\alpha-1)(\alpha-2)\}}{\left(\theta / (\alpha-1)\right)^2} - 1 = 2(\alpha-1)/(\alpha-2) - 1$$

$= \alpha/(\alpha-2)$. For $\alpha = 2.3$, $CV^2 = 2.3 / 0.3 = 7.667$.
Therefore, $\lambda_F (\sigma_f^2/\mu_f + CV^2) = (481)(2.5 + 7.667) =$ **4890**.

**5.10. B.** standard for full credibility is: $\lambda_F (1 + CV^2)$.
Therefore $CV^2 = \{(1500 / 850) - 1\} = 0.7647$.
Variance of severity = $CV^2$ mean$^2$ = $(0.7647)(500)^2$ = **191,175**.

**5.11. C.** $CV^2 = 280{,}000 / 400^2 = 1.75$. $\lambda_F (1 + CV^2) = (700)(1 + 1.75) =$ **1925**.

**5.12. D.** $\Phi(2.576) = 0.995$, so $z_{1-\alpha/2} = 2.576$.
For frequency the standard for full credibility is: $(2.576/0.05)^2 = 2654$.
$\Phi(1.960) = 0.975$, so $z_{1-\alpha/2} = 1.960$ for the Standard for Full Credibility for pure premium.
Thus $2654 = (z_{1-\alpha/2}{}^2 / k^2)(1 + CV^2) = \{1.96^2 / k^2\}(1 + 2.5^2) = 27.85 / k^2$.
Thus $k = \sqrt{\frac{27.85}{2654}}$ = **0.102**.

**5.13. C.** 1. F. The formula for the Standard for Full Credibility for either severity or the Pure Premium involves the severity distribution via the coefficient of variation, which is not affected by (uniform) inflation. (The Standard for Full Credibility for frequency doesn't involve severity at all, and is thus also unaffected.)

2. T. $\left(\frac{z_{1-\alpha/2}}{k}\right)^2 (1 + CV^2) \geq \left(\frac{z_{1-\alpha/2}}{k}\right)^2$.

3. F. Limited (basic limits) losses have a smaller coefficient of variation than do unlimited (total limits) losses. Therefore, the Standard for Full Credibility for Basic Limits losses is less.

**5.14. C.** $\lambda_F = \left(\frac{z_{1-\alpha/2}}{k}\right)^2 = (1.282/0.10)^2 = 164$. For the Pure Premium, the Standard For Full Credibility is: $\lambda_F (1 + CV^2) = (164)(1 + 4000000/1000^2) = (164)(5) =$ **820**.

**5.15. E.** The standard for full credibility is $\lambda_F(\sigma_f^2 / \mu_f + CV_{sev}^2)$. Since the only thing that differs is the severity distribution, the ranking depends on $CV_{sev}$, the coefficient of variation of the severity distribution. For the Exponential, CV = 1.

For the Weibull, $1 + CV^2 = E[X^2]/E[X]^2 = \frac{\theta^2\Gamma(1 + 2/\tau)}{\{\theta\Gamma(1 + 1/\tau)\}^2} = \frac{\Gamma(1 + 2/\tau)}{\Gamma(1 + 1/\tau)^2} = \frac{\Gamma(5)}{\Gamma(3)^2} = \frac{4!}{(3!)^2} = 6.$

$CV = \sqrt{5} = 2.236.$

For the Lognormal, $1 + CV^2 = E[X^2]/E[X]^2 = \exp[2\mu + 2\sigma^2] / \exp[\mu + \sigma^2/2]^2 = \exp[\sigma^2] =$ $\exp[0.64] = 1.896$. $CV = \sqrt{0.896} = 0.95$. From smallest to largest: **3, 1, 2**.

**5.16. D.** k = 5% and $\alpha$ = 10%. We have $z_{1-\alpha/2} = 1.645$ since $\Phi(1.645) = 0.95 = 1 - \alpha/2$.

Therefore, $\lambda_F = \left(\frac{z_{1-\alpha/2}}{k}\right)^2 = (1.645/0.05)^2 = 1082.$

For Poisson frequency, the variance of the total losses is:
(mean frequency)$(\mu_S^2 + \sigma_S^2)$ = (mean frequency)(mean severity)$^2(1 + CV_{sev}^2)$.

Thus $40,000 = (2)(100^2)(1 + CV_{sev}^2)$. $\Rightarrow (1 + CV^2) = 2$.

$\lambda_F\ (1+CV_{sev}^2) = (1082)(2) =$ **2164 claims**.

**5.17. D.** The Standard for Full Credibility for the pure premium is the sum of those for frequency and severity. Thus in this case, the standard for full credibility for the <u>severity</u> is:
2000 - 800 = **1200** claims.

**5.18. E.** For the Burr Distribution $E[X^n] = \lambda^{n/\tau}\Gamma(\alpha - n/\tau)\Gamma(1+ n/\tau)/\Gamma(\alpha)$.

For $\alpha = 9$ and $\tau = 0.25$, $E[X] = \lambda^4\ \Gamma(9\text{-}4)\Gamma(1+ 4)/\Gamma(9) = \lambda^4 (4!)(4!)/8! = \lambda^4/70$.

$E[X^2] = \lambda^8\ \Gamma(9\text{-}8)\Gamma(1+ 8)/\Gamma(9) = \lambda^8 (1)(8!)/8! = \lambda^8$.

$(1+CV^2) = E[X^2] / E^2[X] = (\lambda^8)/(\lambda^4/70)^2 = 4900$.

We have $z_{1-\alpha/2} = 1.439$ since $\Phi(1.439) = 0.925$. k = 0.10.

Therefore, standard for full credibility is:

$\lambda_F\ (1+CV^2) = \left(\frac{z_{1-\alpha/2}}{k}\right)^2 (1+CV^2) = (1.439/0.10\}^2\ (4900) =$ **1.015 million** claims.

<u>Comment</u>: For $\tau = 0.25$ one gets a very heavy-tailed Burr Distribution and therefore a very large Standard for Full Credibility.

**5.19. B.** Frequency is Poisson and therefore $\mu_f = \sigma_f^2$.
$\sigma_{pp}^2 = \mu_{sev}^2\sigma_{freq}^2 + \mu_{freq}\sigma_{sev}^2 = \mu_{freq}(\mu_{sev}^2 + \sigma_{sev}^2)$.
Thus $1300 = 20(\mu_{sev}^2 + 35)$. Therefore, $\mu_{sev}^2 = 30$.
$CV_{sev}^2 = \sigma_{sev}^2 / \mu_{sev}^2 = 35/30 = 1.167$. k = 0.075.
$\Phi(z_{1-\alpha/2}) = 0.99$. Thus $z_{1-\alpha/2} = 2.326$.

$\lambda_F = \left(\frac{z_{1-\alpha/2}}{k}\right)^2 = (2.326/0.075)^2 = 962$. $\lambda_F\,(1+CV^2) = (962)\,(1 + 1.167) =$ **2085**.

**5.20. B.** For situation #1: $\lambda_F(\sigma_f^2/\mu_f) = \lambda_F\, r\beta(1 + \beta)/(r\beta) = \lambda_F(1 + \beta) = 1.3\,\lambda_F$.
For situation #2: $\lambda_F(CV^2) = \lambda_F(E[X^2]/E[X]^2 - 1) = \lambda_F(\{2\theta^2/(\alpha-1)(\alpha-2)\}/\{\theta/(\alpha-1)\}^2 - 1) =$
$\{2(\alpha-1)/(\alpha-2) - 1\}\,\lambda_F = (\alpha/(\alpha - 2))\,\lambda_F = (5/3)\,\lambda_F = 1.67\,\lambda_F$.
For situation #3: $\lambda_F(1 + CV^2) = \lambda_F(1 + \alpha\theta^2/(\alpha\theta)^2) = \lambda_F(1 + 1/\alpha) = 1.5\,\lambda_F$.
From smallest to largest: **1, 3, 2**.

**5.21. D.** For the Geometric Distribution: variance / mean = $\beta(1+\beta) / \beta = 1 + \beta = 1.4$.
For the Exponential distribution: CV = 1.
For the Gamma Distribution, $CV^2 = \alpha\theta^2 / (\alpha\theta)^2 = 1/\alpha = 1/2$.

Old standard: $16{,}000 = \left(\frac{z_{1-\alpha/2}}{r}\right)^2 (1.4 + 1^2)$. $\Rightarrow \left(\frac{z_{1-\alpha/2}}{r}\right)^2 = 16{,}000/2.4$.

New standard: $\left(\frac{z_{1-\alpha/2}}{3r}\right)^2 (1.4 + 1/2) = (1/9)\left(\frac{z_{1-\alpha/2}}{r}\right)^2 (1.9) = (1.9/9)\,(16{,}000/2.4) =$ **1407**.

**5.22. E.** For the Gamma-Poisson, the mixed distribution is Negative Binomial,
with $r = \alpha = 4$ and $\beta = \theta = 0.5$. Therefore, for frequency $\sigma_f^2 / \mu_f = r\beta(1+\beta)/(r\beta) = 1 + \beta = 1.5$.
For the Uniform Distribution from 0 to 500, $\sigma_{sev}^2 / \mu_{sev}^2 = \{(500^2)/12\}/250^2 = 1/3$.
For $\alpha = 0.02$, $z_{1-\alpha/2} = 2.326$. k = 0.1.

$\left(\frac{z_{1-\alpha/2}}{k}\right)^2 (\sigma_f^2 / \mu_f + \sigma_S^2 / \mu_S^2) = (2.326/0.1)^2(1.5 + 0.333) =$ **992 claims**.

<u>Comment</u>: Similar to 4, 11/02, Q.14.

**5.23. E.** $13{,}800 = \left(\frac{z_{1-\alpha/2}}{k}\right)^2 \lambda_F$.

$18{,}050 = (\sigma_f^2 / \mu_f + CV_{Sev}^2)\,\lambda_F$.

Subtracting the first equation from the second: $CV_{Sev}^2\, \lambda_F = 4246$.

For the Gamma, $CV^2$ = variance/mean$^2$ = $\alpha\theta^2/(\alpha\theta)^2 = 1/\alpha = 1/2.5 = 0.4$.

Therefore, $\lambda_F = 4246/0.4 = 10{,}615$. $\Rightarrow \left(\frac{z_{1-\alpha/2}}{k}\right)^2 = 10{,}615$. $\Rightarrow z_{1-\alpha/2}/k = 103.03$.

$\Rightarrow z_{1-\alpha/2} = (103.03)(2.5\%) = 2.576$.

$\Rightarrow 99.5\% = \Phi[z_{1-\alpha/2}]$. $\Rightarrow \alpha = 1\%$. P = 1 - 1% = **99%.**

**5.24. E.** This is the standard for full credibility for aggregate losses.
$k = 0.1$. $\alpha = 5\%$. $\Rightarrow z_{1-\alpha/2} = 1.960$.

Standard for Full Credibility: $(1.960/0.1)^2\,(11/3 + 4^2)$ = **7555** claims.

**5.25. E.** $\alpha = 5\%$ and thus $z_{1-\alpha/2} = 1.960$.

Standard for full credibility is: $(1.960/0.10)^2\,\{1.5 + (700 \text{ million}) / 5000^2\}$ = 11,333 claims.
This corresponds to expected aggregate losses of: (11,333)(5000) = **56.66 million**.

**5.26. E.** For the Negative Binomial, variance / mean = $1 + \beta = 1.25$.

For constant severity, full credibility is: $1.25\,\lambda_F$. $\Rightarrow \lambda_F = 2500/1.25 = 2000$.

For the Gamma, $1 + CV^2 = E[X^2] / E[X]^2 = \alpha(\alpha+1)\theta^2 / (\alpha\theta)^2 = 1 + 1/\alpha$. $\Rightarrow CV^2 = 1/\alpha = 2$.

Thus for Gamma severity full credibility is: $(1.25 + 2)\,\lambda_F = (3.25)(2000) =$ **6500**.

**5.27. C.** Using the frequency data, the sample mean is: 200/5000 = 0.04,
and the estimated variance is: $250/5000 - 0.04^2 = 0.0484$.
Using the severity data, the sample mean is: 600,000/200 = 3000,
and the estimated variance is: $27{,}000 \text{ million} / 200 - 3000^2$ = 126 million.
For $\alpha = 0.10$, $z_{1-\alpha/2} = 1.645$. $k = 0.03$.

$$\left(\frac{z_{1-\alpha/2}}{k}\right)^2 (\sigma_{freq}^2/\mu_{freq} + \sigma_{sev}^2/\mu_{sev}^2) = (1.645/0.03)^2\,(0.0484/0.04 + 126 \text{ million} / 3000^2)$$

= **45,732 claims**.

**5.28. A.** $\lambda_F = \left(\frac{z_{1-\alpha/2}}{k}\right)^2 = (1.645 / 0.05)^2 = 1082$ claims.

Thus we have that: $15{,}000 = (1 + CV^2)(1082)$. $\Rightarrow$ CV = **3.59**.

**5.29. D.** The first limited moment of the LogNormal is:

$E[X \wedge 100{,}000] = \exp[9 + 1.3^2/2]\ \Phi[\frac{\ln[100{,}000] - 9 - 1.3^2}{1.3}] + (100{,}000)\ (1 - \Phi[\frac{\ln[100{,}000] - 9}{1.3}])$

= (18,864) $\Phi$[0.63] + (100,000) (1 - $\Phi$[1.93]) = (18,864) (0.7357) + (100,000)(1 - 0.9732) = 16,558.

The second limited moment of the LogNormal is: $E[(X \wedge 100{,}000)^2]$ =

$\exp[(2)(9) + (2)(1.3^2)]\ \Phi[\frac{\ln[100{,}000] - 9 - (2)(1.3^2)}{1.3}] + (100{,}000)^2\ (1 - \Phi[\frac{\ln[100{,}000] - 9}{1.3}])$

= (1,928,483,925) $\Phi$[-0.67] + (10,000,000,000) (1 - $\Phi$[1.93])

= (1,928,483,925) (1 - 0.7486) + (10,000,000,000)(1 - 0.9732) = 752,820,859.

$1 + CV^2$ is: $752{,}820{,}859 / 16{,}558^2 = 2.746$.

Thus, the standard for full credibility for aggregate loss is: (2.746)(750 claims) = **2060** claims.

**5.30.** $G = (1400/1000)^2\ \lambda_F$. $H = \{1 + (1400/1000)^2\}\ \lambda_F$.

$G/H = 1400^2 / (1000^2 + 1400^2)$ = **0.662**.

**5.31. E.** $\lambda_F = \left(\frac{z_{1-\alpha/2}}{k}\right)^2 = (1.645/0.05)^2 = 1082$. For the Pure Premium, when we have a general frequency distribution (not necessarily Poisson), the Standard For Full Credibility is:

$\lambda_F\ (\sigma_f^2 / \mu_f + CV_{sev}^2) = (1082)\ (2 + 25000/100^2) = (1082)(4.5) =$ **4869**.

**5.32. E.** The coefficient of variation = standard deviation / mean = 200 /100 = 2.

$\lambda_F\ (1 + CV_{sev}^2) = \lambda_F\ (1 + 2^2) = 5\,\lambda_F$.

**5.33. A.** Assume there are N claims expected and therefore $N/\mu_f$ exposures. The mean pure premium is $m = N\mu_S$. For frequency and severity independent, the variance of the pure premium for a single exposure is: $\mu_f \sigma_S^2 + \mu_S^2 \sigma_f^2$. The variance of the aggregate loss for $N/\mu_f$ independent exposures $= \sigma^2 = (N/\mu_f)(\mu_f \sigma_S^2 + \mu_S^2 \sigma_f^2) = N (\sigma_S^2 + \mu_S^2\sigma_f^2/\mu_f)$.
We desire that: $\text{Prob}[|X-m| \geq km] \leq \alpha$.
This is in the form of Chebyshev's inequality provided we take $1/a^2 = \alpha$, and $km = a\sigma$.
Thus $a = 1 / \sqrt{\alpha}$ and $km = \sigma / \sqrt{\alpha}$. Therefore $k^2m^2\alpha = \sigma^2$.
Thus, $k^2 N^2 \mu_S^2 \alpha = N (\sigma_S^2 + \mu_S^2 \sigma_f^2 / \mu_f)$. Solving for
$N = (\sigma_S^2 + \mu_S^2 \sigma_f^2 / \mu_f) / \{k^2 \mu_S^2 \alpha\} =$ $\mathbf{(\sigma_f^2/\mu_f + \sigma_S^2/\mu_S^2) / \{k^2\alpha\}}$.
Comment: *See Dale Nelson's review in PCAS 1969 of Mayerson, Jones and Bowers "The Credibility of the Pure Premium".* Note that this formula resembles that derived from the normal approximation, but with $z_{1-\alpha/2}$ replaced by $1/\sqrt{\alpha}$.
For example for $\alpha = 5\%$, $z_{1-\alpha/2}{}^2 = 1.96^2 = 3.84$, while $1/\alpha = 1/0.05 = 20$.
Thus while Chebyshev's inequality holds regardless of the form of the distribution, it is very conservative if the distribution is approximately Normal. For $\alpha = 5\%$ it results in a standard for full credibility 5.2 times as large.

**5.34. E.** Using the formulas for the moments: $CV^2 = E[X^2] / E^2[X] - 1 =$

$$\frac{2\theta^2 / \{(\alpha-1)(\alpha-2)\}}{\left(\theta / (\alpha-1)\right)^2} - 1 = 2(\alpha-1)/(\alpha-2) - 1 = \alpha / (\alpha-2).$$

For $\alpha = 2.3$, $CV^2 = 2.3 / 0.3 = 7.667$.
Therefore, $(\sigma_f^2/\mu_f + \sigma_S^2/\mu_S^2) / \{k^2(1-P)\} = (2.5 + 7.667) / \{(0.075^2)(1 - 0.9)\} =$ **18,075**.
Comment: Note how much larger the Standard for Full Credibility is than when using the Normal Approximation as in a previous question.

**5.35. B.** $\lambda_F (1+CV^2) = (1200)(1 + 80000/400^2) = (1200)(1.5) =$ **1800** claims.

**5.36. D.** $k = 8\%$, $\alpha = 10\%$. Therefore $z_{1-\alpha/2} = 1.645$, since $\Phi(1.645) = 0.95$.

$$\lambda_F = \left(\frac{z_{1-\alpha/2}}{k}\right)^2 = (1.645/0.08)^2 = 423.$$ CV = standard deviation / mean = 4000/1000 = 4.

$$(\sigma_f^2 / \mu_f + \sigma_{sev}^2/\mu_{sev}^2) \left(\frac{z_{1-\alpha/2}}{k}\right)^2 = \lambda_F (\sigma_f^2 / \mu_f + CV_{sev}^2) = (423)(1.5 + 4^2) = \mathbf{7403}.$$

**5.37. C.** For a Poisson frequency, the standard for full credibility for the pure premium is $\lambda_F(1 + CV^2)$, where CV is the coefficient of variation of the severity and $\lambda_F$ is the standard for full credibility for frequency. Therefore in this case, $1800 = 1200(1 + CV^2)$.
Therefore $CV^2 = (1800/1200) - 1 = 0.5$. But the square of the coefficient of variation = variance / mean$^2$. Therefore variance of severity = $(0.5)(400^2)$ = **80,000**.
Comment: Given an output, you are asked to solve for the missing input.
Note that one makes no use of the given average frequency.

**5.38. C.** k = 0.08 and $\alpha = 0.10$. $z_{1-\alpha/2} = 1.645$ , since $\Phi(1.645) = 0.95$.

$$\lambda_F = \left(\frac{z_{1-\alpha/2}}{k}\right)^2 = (1.645/0.08)^2 = 423.$$

The coefficient of variation of the severity = standard deviation / mean.
$CV^2 = 362,944 / 1216^2 = 0.245$.
Thus $\lambda_F(1 + CV^2) = (423)(1.245) =$ **527**.

**5.39. C.** The mean of the severity distribution is 100,000/2 = 50,000.
The Second Moment of the Severity Distribution is the integral from 0 to 100,000 of $x^2f(x)$, which is $100,000^3 / \{3\ (100,000)\}$. Thus the variance is: $100,000^2 / 3 - 50,000^2 = 833,333,333$.
Thus the square of the coefficient of variation is $833,333,333 / 50,000^2 = 1/3$.
k = 5% (within ±5%) and since $\alpha = 0.05$, $z_{1-\alpha/2} = 1.960$ since $\Phi(1.960) = 0.975$.

The Standard for Full Credibility Pure Premium is: $\left(\frac{z_{1-\alpha/2}}{k}\right)^2 (1+CV^2) =$

$(1.96/0.05)^2\ (1 + 1/3) = (1537)\ (4/3) =$ **2049** claims.
Comment: For the uniform distribution on the interval (a,b) the coefficient of variation is:

$$\frac{b-a}{(b+a)\sqrt{3}}. \text{ Thus the } CV^2 = \frac{(b-a)^2}{(b+a)^2\ (3)} = \frac{(100,000 - 0)^2}{(100,000 + 0)^2\ (3)} = 1/3.$$

Note that the $CV^2$ is 1/3 whenever a = 0.

**5.40. C.** k = 6% (within 6% of the expected total cost). $\Phi(1.645) = 0.95$, so that

$z_{1-\alpha/2} = 1.645$. Standard for full credibility for frequency = $z_{1-\alpha/2} = \left(\frac{z_{1-\alpha/2}}{k}\right)^2 = (1.645 / 0.06)^2$

= 751.7. Coefficient of Variation of the severity = 7500 /1500 = 5.
Standard for full credibility for pure premium = $z_{1-\alpha/2}(1+CV^2) = (751.7)(1 + 5^2) =$ **19,544** claims.

**5.41. B.** For the given severity distribution the mean is:

$$\int_0^{100} x\, f(x)\, dx = (0.0002) \int_0^{100} x\,(100 - x)\, dx = (0.0002)\ (50x^2 - x^3/3)\Big]_{x=0}^{x=100} = 33.33$$

For the given severity distribution the second moment is:

$$\int_0^{100} x^2 f(x)\, dx = (0.0002) \int_0^{100} x^2 (100 - x)\, dx = (0.0002)\ \{(100/3)x^3 - x^4/4\}\Big]_{x=0}^{x=100} = 1666.67$$

Thus the variance of the severity is: $1666.67 - 33.33^2 = 555.8$.
Coefficient of variation squared = $CV^2 = 555.8 / 33.33^2 = 0.50$.

For the given standard for full credibility for frequency, $800 = \left(\frac{z_{1-\alpha/2}}{k}\right)^2 = z_{1-\alpha/2}{}^2 / 0.05^2$.

$\Rightarrow z_{1-\alpha/2}{}^2 = (800)(0.05^2) = 2$.

Now for the same $\alpha$ value that produced this $z_{1-\alpha/2}$ value, we want a standard for full credibility for pure premiums, with k = 0.10:

$\left(\frac{z_{1-\alpha/2}}{k}\right)^2 (1+CV^2) = \{2 / 0.1^2\}\ (1 + 0.50) = (200)(1.5) =$ **300 claims**.

**5.42. B.** k = 10% (within 10% of the expected pure premium). $\Phi(1.960) = 0.975$,
so that $z_{1-\alpha/2} = 1.960$.

Standard for full credibility for frequency = $\lambda_F = \left(\frac{z_{1-\alpha/2}}{k}\right)^2 = (1.960 / 0.10)^2 = 384$ claims.

Standard for full credibility for pure premium = $\lambda_F\ (1+CV^2)$.
Therefore $CV^2 = (1000/ 384) - 1 = 1.604$.
Thus CV = **1.27**.

**5.43. C.** k = 5% (within 5% of the expected frequency). $\Phi(2.327) = 0.99$, so that

$z_{1-\alpha/2}$ = 2.327. Standard for full credibility for frequency = $\lambda_F = \left(\frac{z_{1-\alpha/2}}{k}\right)^2$ = $(2.327 / 0.05)^2$ =

2166 claims. Now one has to start fresh and write down the formula for a standard for full credibility for the pure premium, with a new $z_{1-\alpha/2}$ and k. Since $\Phi(1.960) = 0.975$,

the new $z_{1-\alpha/2}$ = 1.960.

The standard for full credibility for pure premium = $\lambda_F\ (1+CV^2)$.

The mean severity is: $\int_1^\infty x\ f(x)\ dx = \int_1^\infty x\ (5/2)\ x^{-7/2}\ dx = \{(5/2)/(-3/2)\}\ x^{-3/2} \Big]_{x=1}^{x=\infty} = 5/3.$

The 2nd moment is: $\int_1^\infty x^2\ (5/2)\ x^{-7/2}\ dx = \{(5/2)/(-1/2)\}\ x^{-1/2} \Big]_{x=1}^{x=\infty} = 5.$

Thus the variance = $5 - (5/3)^2 = 2.22$.

The coefficient of variation is the standard deviation divided by the mean: $\sqrt{2.22}$ / (5/3) = 0.894.

We are given that this standard for full credibility for pure premium is equal to the previously calculated standard for full credibility for frequency; thus

$2166 = (1.960^2/ k^2)\ (1 + 0.894^2)$. Solving, the new k = **0.056.**

Comment: The severity is a Single Parameter Pareto Distribution, with $\alpha$ = 2.5 and $\theta$ = 1.

**5.44. A.** We are given k = 10%, $\alpha$ = 10%. $\Phi(z_{1-\alpha/2})$ = 95%. The Normal distribution has a 95%

chance of being less than 1.645. Thus $z_{1-\alpha/2}$ = 1.645. $\lambda_F = \left(\frac{z_{1-\alpha/2}}{k}\right)^2$ = 271.

The mean severity is 3.1 and the variance of the severity = $21.7 - 3.1^2 = 12.09$.

| | Claim Size | Probability | Square of Claim Size |
|---|---|---|---|
| | 1 | 0.5 | 1 |
| | 2 | 0.3 | 4 |
| | 10 | 0.2 | 100 |
| | | | |
| Average | 3.1 | | 21.7 |

Therefore the Square of the Coefficient of Variation = variance / mean$^2$ = 12.09 / 3.1$^2$ = 1.258.

Therefore the full credibility standard is = $\lambda_F(1 + CV^2)$ = (271)(1 + 1.258) = **612 claims**.

**5.45. B.** $\lambda_F(1+CV^2) = \left(\frac{z_{1-\alpha/2}}{k}\right)^2 (1 + CV^2)$. k =10%.

$z_{1-\alpha/2}$ = 1.960 since $\Phi(1.960) = 0.975$.

Thus $3025 = (1.96/0.1)^2\ (1 + CV^2)$. Therefore: $7.87 = 1 + CV^2$. $\Rightarrow$ CV = **2.62**.

**5.46. D.** 1. False. The correct formula contains the square of the mean severity:
Var(C) = E(N) Var(X) + $E(X)^2$ Var(N).
2. True. Using the fact that the Coefficient of Variation is the mean over the standard deviation:
$n_0 \{E(X)^2 + Var(X)\}/E(X)^2 = n_0 \{1 + Var(X)/E(X)^2\} = n_0(1 + CV^2) = n_F$.
3. True. The "square root rule" for partial credibility used in Classical Credibility.
<u>Comment</u>: Statement 3 is only true for $n \le n_F$. For $n \ge n_F$, Z = 1.

**5.47. E.** The Normal distribution has a 97.5% chance of being less than 1.960.
Thus $z_{1-\alpha/2}$ = 1.960. Therefore in terms of number of claims the full credibility standard is:

$$\left(\frac{z_{1-\alpha/2}}{k}\right)^2 (1 + CV^2) = (1.96^2)(1 + 4) / k^2 = 3415 \text{ claims.}$$

Therefore k = (1.96) $\sqrt{5/3415}$ = **0.075**.
<u>Comment</u>: You are given the output, 3415 claims, and asked to solve for the missing input, k.

**5.48. C.** We are given k = 5%. $\Phi(z_{1-\alpha/2})$ = 0.95, therefore $z_{1-\alpha/2}$ = 1.645.

$$\lambda_F = \left(\frac{z_{1-\alpha/2}}{k}\right)^2 = 1.645^2 / 0.05^2 = 1082 \text{ claims.}$$

The given severity distribution is a Single Parameter Pareto, with $\alpha$ = 5 and $\theta$ = 1.
First moment is: $\alpha\theta / (\alpha-1)$ = 5/4.
Second moment is: $\alpha\theta^2 / (\alpha-2)$ = 5/3.
$1 + CV^2 = E[X^2] / E[X]^2 = (5/3) / (5/4)^2 = 16/15$.
Therefore, the standard for full credibility for the Pure Premium is:
$\lambda_F (1 + CV^2)$ = (1082)(16/15) = 1154 claims.
Next the problem states that this is also a full credibility standard for frequency.
In this case, $\Phi(z_{1-\alpha/2})$ = 0.975, therefore $z_{1-\alpha/2}$ = 1.960.

Thus setting 1154 claims = $\left(\frac{z_{1-\alpha/2}}{k}\right)^2 = 1.96^2 / k^2$, one solves for k = **0.0577**.

**5.49. B.** Let the standard deviation of the severity distribution, for which we will solve, be $\sigma$.
The Classical Credibility Standard for the Pure Premium is given by:

$\lambda_F(1 + CV^2)$. $CV^2 = \sigma^2 / 5000^2$. $\lambda_F = \left(\frac{z_{1-\alpha/2}}{k}\right)^2 = (1.96/0.05)^2 = 1537$ claims.

One must now translate $\lambda_F$ into exposures, since that is the manner in which the full credibility criterion is stated in this problem. One does so by dividing by the expected frequency, which is the fitted Poisson parameter m.
Using the method of moments, m = (observed # of claims) / (observed number of exposures) = (1293 + 1592 + 1418) / (18,567 + 26,531 + 20,002) = 4303 / 65,000 = 0.0662.
Thus $\lambda_F$ in terms of exposures is:
1537 claims / (0.0662 claims / exposure) = 23,218 exposures.
Now one sets the given criterion for full credibility equal to its calculated value:
$120{,}000 = 23{,}218(1 + \sigma^2/5000^2)$. Solving, $\sigma$ = **$10,208**.
<u>Comment</u>: Assuming a Poisson frequency with parameter 0.0662, a severity distribution with a mean of $5000 and a standard deviation of $10,208, how many exposures are needed for full credibility if we want the actual total cost of claims to be within ±5% of the expected total 95% of the time? The solution to this alternate question is:
(1537 claims) $\{1 + (10208/5000)^2\}$ / (0.0662 claims per exposure) ≅ 120,000 exposures.

**5.50. D.** $\lambda_F$\{1 + square of coefficient of variation of severity\}.

$\lambda_F = \left(\frac{z_{1-\alpha/2}}{k}\right)^2$. k = 10%. $\alpha$ = 2%. $\Rightarrow z_{1-\alpha/2} = 2.326$

For the Pareto, mean = $\theta/(\alpha-1)$ = 3000, and the second moment = $\frac{2\theta^2}{(\alpha-1)(\alpha-2)}$ = 36 million.

$1 + CV^2 = E[X^2] / E[X]^2$ = 36 million / $3000^2$ = 4. $(2.326 / 0.1)^2 (4)$ = **2164**.

**5.51. C.** k = 0.10 (“within 10% of the expected”)
$z_{1-\alpha/2}$ = 1.645 since $\Phi(1.645) = 0.95$ (“to be 90% certain”, allow 5% outside on each tail.)

$\lambda_F = \left(\frac{z_{1-\alpha/2}}{k}\right)^2 = 1.645^2/0.1^2 = 271$.

Coefficient of Variation$^2$ = Variance / mean$^2$ = 10 / 25 = 0.4.
$\lambda_F (1+CV^2)$ = (271)(1.4) = **379** claims.

**5.52. D.** k = 0.075 ("within 7.5% of the expected")
$z_{1-\alpha/2}$ = 1.960 since $\Phi(z_{1-\alpha/2})$ = 0.975 ("to be 95% certain", allow 2.5% outside on each tail.)

$\lambda_F = \left(\frac{z_{1-\alpha/2}}{k}\right)^2$ = $1.960^2/0.075^2$ = 683. 1 + $CV^2$ = second moment / $mean^2$ =

$\exp(2\mu + 2\sigma^2) / \exp(2\mu + \sigma^2) = \exp(\sigma^2) = e^{2.25} = 9.49$.
$\lambda_F (1 + CV^2)$ = (683)(9.49) = 6482 claims.
But we are given that the full credibility criterion with respect to exposures is 40,000.
To convert to claims we multiply by the mean claim frequency.
Therefore standard for full credibility = 40,000m. Therefore m = 6442 / 40,000 = **16.2%.**

**5.53. A.** $\lambda_F = \left(\frac{z_{1-\alpha/2}}{k}\right)^2 = z_{1-\alpha/2}^2 / 0.05^2 = 400\ z_{1-\alpha/2}^2$. For the full credibility standard for pure premiums ("claims costs") we need to compute the coefficient of variation.
For a Pareto with θ = 3000 and α = 4, the second moment is: $2(3000)^2 / \{(4-1)(4-2)\}$,
while the mean is: 3000 / (4-1).
Thus 1 + $CV^2$ = second moment / $mean^2$ = 2(3) / 2 = 3.
Therefore, the standard for full credibility is: $\lambda_F (1 + CV^2) = 400\ z_{1-\alpha/2}^2 (3) = 1200\ z_{1-\alpha/2}^2$.

Setting this equal to the given 2000 claims we solve for $z_{1-\alpha/2}$: $z_{1-\alpha/2} = \sqrt{\frac{2000}{1200}}$ = 1.291.

One then needs to compute how much probability is within ± 1.291 standard deviations on the Normal Distribution. $\Phi(1.291)$ = 0.9017.
Therefore, P = 1 - (2)(1 - 0.9017) = **0.803**. (9.83% is outside on each tail.)
Comment: If one had been given 1 - α = 80.3% and were asked to solve for the standard for full credibility, then we would want 0.0985 outside on either tail, so we want $\Phi(z_{1-\alpha/2})$ = 0.9015.
Thus $z_{1-\alpha/2} \cong 1.29$ and the standard for full credibility ≅ (3) $(1.29^2)/(0.05^2)$ ≅ 2000.

**5.54. C.** For the Gamma Distribution the Coefficient of Variation = $1 / \sqrt{\alpha}$ = 1.
We are given k = 5% and α = 10%. $\Phi(1.645) = 0.95$. ⇒ $z_{1-\alpha/2}$ = 1.645.

Therefore, $\lambda_F = \left(\frac{z_{1-\alpha/2}}{k}\right)^2 = (1.645/0.05)^2 = 1082$. $\lambda_F (1 + CV^2) = 1082(1 + 1^2)$ = **2164**.

Comment: The Gamma Distribution for α = 1 is an Exponential Distribution,
with Coefficient of Variation of 1 (and Skewness of 2.)

**5.55. C.** standard for full credibility = $\left(\frac{z_{1-\alpha/2}}{k}\right)^2$ $(1 + CV^2)$, is proportional to $z_{1-\alpha/2}{}^2$.

For P = 90%, $\Phi(1.645) = 0.95$. ⇒ $z_{1-\alpha/2} = 1.645$.

For P = 95%, $z_{1-\alpha/2} = 1.960$, since $\Phi(1.960) = 0.975$.

Thus the new criterion for full credibility = $(1000)(1.960/1.645)^2$ =**1420**.

**5.56. B.** $\alpha = 0.05$ and $k = 0.1$. $\Phi(1.960) = 0.975$, so that $z_{1-\alpha/2} = 1.960$.

$\lambda_F = \left(\frac{z_{1-\alpha/2}}{k}\right)^2 = (1.960 / 0.10)^2 = 384$.

The mean severity is 4 and so is the variance, since it follows a Poisson Distribution. Thus the square of the coefficient of variation of the severity = $CV^2$ = variance / mean$^2$ = $4 / 4^2 = 1/4$.
$\lambda_F\ (1+CV^2) = (384)(1 + 1/4) =$ **480**.
Comment: It's unusual to have severity follow a Poisson Distribution. This situation is mathematically equivalent to a Poisson-Poisson compound frequency distribution.

**5.57. C.** Standard for Full Credibility = $\left(\frac{z_{1-\alpha/2}}{k}\right)^2$ $(1 + CV^2)$. If the CV goes from 2 to 4, and k doubles then the Standard for Full Credibility is multiplied by: $\{(1+4^2) / (1+2^2)\} / 2^2 = (17/5) / 4$.
Thus the Standard for Full Credibility is altered to: (1200)(17/5) / 4 = **1020**.
Comment: If k doubled and the CV stayed the same, then the Standard for Full Credibility would be altered to: 1200 / 4 = 300. If k stayed the same and the CV went from 2 to 4, then the Standard for Full Credibility would be altered to: $(1200)\ \{(1 + 4^2) / (1 + 2^2)\} = 4080$.

**5.58. E.** For the Lognormal, Mean = $\exp(\mu + \sigma^2/2)$, 2nd Moment = $\exp(2\mu + 2\sigma^2)$,
1 + square of coefficient of variation = 2nd moment / mean$^2$ = $\exp(\sigma^2) = e^1 = 2.718$.
k = 0.1. $\alpha = 0.05$, so that $z_{1-\alpha/2} = 1.96$ since $\Phi(1.96) = 0.975$.

Thus $\lambda_F = \left(\frac{z_{1-\alpha/2}}{k}\right)^2 = 384$. Thus the number of claims needed for full credibility of the pure premium is: $\lambda_F(1 + CV^2) = 384(2.718) = 1044$ claims.
To convert to the full credibility standard to exposures, divide by the expected frequency of 0.01:
1044/0.01 = **104.4 thousand exposures**.

**5.59. E.** k = 5% (within 5%), $\alpha$ = 5% (95% of the time). $\Phi(z_{1-\alpha/2}) = 0.975$, thus $z_{1-\alpha/2} = 1.960$.

The Inverse Gamma has: $E[X] = \theta/(\alpha-1)$. $E[X^2] = \frac{\theta^2}{(\alpha-1)\,(\alpha-2)}$.

$\lambda_F\,(1 + CV^2) = \left(\frac{z_{1-\alpha/2}}{k}\right)^2 (E[X^2]/E^2[X]) = (1.96/0.05)^2 \{(\alpha-1)/(\alpha-2)\} = (1537)(3/2) \cong$ **2306** claims.

Comment: In this case, $CV^2 = 1/2$. For the Inverse Gamma Distribution, $CV^2 = 1/(\alpha-2)$.

**5.60. C.** The Negative Binomial has a larger variance than the Poisson, so there is more random fluctuation, and therefore the standard for Full Credibility is larger. Specifically, one can derive a more general formula than when the Poisson assumption does not apply. The Standard for Full Credibility is: $\left(\frac{z_{1-\alpha/2}}{k}\right)^2 (\sigma_f^2/\mu_f + \sigma_{sev}^2 / \mu_{sev}^2)$, which reduces to the Poisson case when $\sigma_f^2 / \mu_f = 1$.

For the Negative Binomial the variance is greater than the mean, so $\sigma_f^2/\mu_f > 1$. Thus for the Negative Binomial the standard for Full Credibility is larger than the Poisson case, all else equal.

**5.61. A.** Frequency is Poisson and therefore $\mu_f = \sigma_f^2$.

$\sigma_{pp}^2 = \mu_{sev}^2\sigma_{freq}^2 + \mu_{freq}\sigma_{sev}^2 = \mu_{freq}\,(\mu_{sev}^2 + \sigma_{sev}^2)$. Thus $500 = 10(\mu_{sev}^2 + 10)$.

Therefore, $\mu_{sev}^2 = 40$. $\Rightarrow CV_{sev}^2 = \sigma_{sev}^2 / \mu_{sev}^2 = 10/40 = 0.25$.

k = 0.05. $\Phi(z_{1-\alpha/2}) = 0.975$. Thus $z_{1-\alpha/2} = 1.96$.

$\lambda_F = \left(\frac{z_{1-\alpha/2}}{k}\right)^2 = (1.96/0.05)^2 = 1537$.

$\lambda_F\,(1 + CV^2) = 1537(1 + 0.25) =$ **1921**.

**5.62. D.** For the Burr Distribution $E[X^n] = \frac{\theta^n\,\Gamma(1 + n/\gamma)\,\Gamma(\alpha - n/\gamma)}{\Gamma(\alpha)}$.

For $\alpha = 6$ and $\gamma = 0.5$, $E[X] = \theta\,\Gamma(1+2)\,\Gamma(6-2) / \Gamma(6) = \theta(2!)(3!) / 5! = \theta/10$.

$E[X^2] = \theta^2\Gamma(1+1)\Gamma(6-1)/\Gamma(6) = \theta^2(1!)(4!) / 5! = \theta^2/5$.

$(1+CV^2) = E[X^2] / E^2[X] = (\theta^2/5)/(\theta/10)^2 = 100/5 = 20$.

k = 0.10. 6000 = standard for full credibility $= \lambda_F\,(1 + CV^2) = \left(\frac{z_{1-\alpha/2}}{k}\right)^2 (1 + CV^2)$.

$6000 = (z_{1-\alpha/2}/0.1)^2\,(20). \Rightarrow z_{1-\alpha/2} = 0.1\sqrt{\frac{6000}{20}} = 0.1\sqrt{300} = \sqrt{3} = 1.732$.

$\Phi(z_{1-\alpha/2}) = \Phi(1.732) = 0.9584$. Thus P = (2)(1 - 0.9584) = **0.917**.

**5.63. E.** $\lambda_F\,(1 + CV^2) = (1000)(1 + 2^2) =$ **5000**.

**5.64. C.** For the LogNormal Distribution, $1 + CV^2$ = (2nd moment)/mean$^2$ =
$\exp(2\mu + 2\sigma^2) / \exp(\mu + 0.5\sigma^2)^2 = \exp(\sigma^2)$.
k = 5%, $\alpha$ = 10%. We have $z_{1-\alpha/2}$ = 1.645, since $\Phi(1.645)$ = 0.95.

Therefore, $\lambda_F = \left(\frac{z_{1-\alpha/2}}{k}\right)^2 = (1.645 / 0.05)^2 = 1082$.

We are given 13,000 = 1082(1+CV$^2$). 1 + CV$^2$ = 12.01.
Therefore, $12.01 = \exp(\sigma^2)$. $\Rightarrow \sigma = \sqrt{\ln(12.01)}$ = **1.577**.

**5.65. E.** The mean frequency is: 750/1000 = 0.75.
The variance of the frequency is: 1494/1000 - $0.75^2$ = 0.9315.
$CV_{Sev}^2 = 6{,}750{,}000/1500^2 = 3$. k = 5%. $\alpha$ = 5%. $z_{1-\alpha/2}$ = 1.960.

$\lambda_F = \left(\frac{z_{1-\alpha/2}}{k}\right)^2 = 1.960^2/0.05^2 = 1537$.

Standard for full credibility = $\lambda_F(\sigma_F^2/\mu_F + CV_{Sev}^2)$ = (1537)(0.9315/0.75 + 3) = 6520 claims.
6520 claims corresponds to 6520/0.75 = **8693** exposures.
Alternately, Standard for full credibility in terms of exposures =
$\lambda_F$ (coefficient of variation of the pure premium)$^2$ =
(1537)(variance of the pure premium)/(mean pure premium)$^2$ =
(1537){(0.75)(6,750,000) + (1500$^2$)(0.9315)} / {(0.75)(1500)}$^2$ =
(1537)(7.1584 million)/1125$^2$ = 8693 exposures.
Comment: Items (i) and (ii) are not needed to answer the question, although they do imply that the frequency for the whole portfolio is Negative Binomial. Therefore the factor $\sigma_F^2/\mu_F$ should be greater than 1. That the severity is Pareto is also not used to answer the question, although one can infer that $\alpha$ = 3 and $\theta$ = 3000.

**5.66. B.** For the Gamma-Poisson, the mixed distribution is Negative Binomial, with $r = \alpha = 15$ and $\beta = \theta = 0.2$. Therefore, for frequency, $\sigma_f^2/\mu_f = r\beta(1 + \beta) / (r\beta) = 1 + \beta = 1.2$.
For the Exponential Distribution, $\sigma_{sev}^2 / \mu_{sev}^2 = \theta^2/\theta^2 = 1$.
For $\alpha$ = 0.10, $z_{1-\alpha/2}$ = 1.645. k = 0.05.

$\left(\frac{z_{1-\alpha/2}}{k}\right)^2 (\sigma_f^2/\mu_f + \sigma_{sev}^2/\mu_{sev}^2) = (1.645/0.05)^2(1.2 + 1) =$ **2381 claims**.

Comment: We use the Negative Binomial Distribution for the whole portfolio of insureds, in order to compute the standard for full credibility, thereby taking into account the larger random fluctuation of results due to the heterogeneity of the portfolio.

**5.67. E.** k = 0.02. $\alpha$ = 10% $\Rightarrow$ $z_{1-\alpha/2}$ = 1.645. $\lambda_F = \left(\frac{z_{1-\alpha/2}}{k}\right)^2$ = $(1.645/0.02)^2$ = 6765 claims.

For the Pareto, E[X] = $\theta/(\alpha-1)$ = 0.5/5 = 0.1, $E[X^2] = \frac{2\theta^2}{(\alpha-1)\ (\alpha-2)} = \frac{(2)\ (0.5^2)}{(6-1)\ (6-2)}$ = 0.025,

and $1 + CV^2 = E[X^2]/E[X]^2 = 0.025/0.1^2 = 2.5$.
Standard for Full Credibility for pure premium when frequency is Poisson =
$\lambda_F(1 + CV_{Sev}^2)$ = (6765)(2.5) = **16,913 claims**.
Comment: For the Pareto Distribution, $CV^2 = \alpha/(\alpha-2)$ = 6/4 = 1.5.

**5.68. E.** $\sigma_f^2/\mu_f = r\beta(1 + \beta)/(r\beta) = 1 + \beta = 4$.
E[X] = 24.4. $E[X^2]$ = 2040.4. $CV_{sev}^2 = 2040.4/24.4^2 - 1 = 2.427$.
k = 10%. $\alpha$ = 5%. $z_{1-\alpha/2}$ = 1.960. $\lambda_F = (1.960/0.1)^2 = 384$.
The standard for full credibility for aggregate losses is: (4 + 2.427)(384) = **2468** claims.

**5.69. E.** Since we have a Poisson frequency, the standard for full credibility is
$\lambda_F(1 + CV_{Sev}^2)$. Thus we need to determine the coefficient of variation of severity.

Mean = $\alpha\theta$. Variance = $\alpha\theta^2$. $CV^2 = \frac{\alpha\theta^2}{(\alpha\theta)^2} = 1/\alpha$, **can not be determined**.

Comment: One need only know $\alpha$ in order to determine the coefficient of variation of the Gamma Distribution, as in 4B, 5/96, Q.27. $\alpha$ = 5%. $z_{1-\alpha/2}$ = 1.960. $\lambda_F$ = 384.

**5.70. B.** From the standard for frequency, 2000 = $z_{1-\alpha/2}^2/0.03^2$. $\Rightarrow$ $z_{1-\alpha/2}^2$ = 1.8.
For the uniform severity: $CV^2$ = variance/mean$^2$ = $(10000^2/12)/(10000/2)^2$ = 1/3.
Standard for Aggregate Losses is: $\lambda_F(1 + CV^2) = (1.8/0.05^2)(1 + 1/3)$ = **960** claims.

**5.71. B.** $\alpha$ = 1%. $\Phi[2.576]$ = 0.995. $\Rightarrow$ $z_{1-\alpha/2}$ = 2.576. k =10%.
$\Rightarrow$ $\lambda_F = (2.576/0.1)^2$ = 664 claims.
4800 - 664 = **4136** claims.
Alternately, we are given: 4800 = $(1 + CV_S^2)$ 664. $\Rightarrow$ $CV_S^2$ = 6.229.
The full credibility standard for claim severity is: (6.229)(664) = **4136** claims.

## Section 6, Partial Credibility[47]

**When one has at least the number of claims needed for Full Credibility, then one assigns 100% credibility to the observed data.** However, when one has less data then is needed for full credibility, one assigns an amount of Credibility less than 100%.

If the Standard for Full Credibility is 683 claims and one has only 300 claims, then one assigns less than full credibility to this data. How much less is determined via the "square root rule."

**Let $\lambda_N$ be the (expected) number of claims for the volume of data, and $\lambda_F$ be the standard for Full Credibility for the pure premium or aggregate losses. Then the partial credibility assigned is $Z = \sqrt{\frac{\lambda_N}{\lambda_F}}$.** [48]

**When dealing with frequency or severity a similar formula applies.**

Unless stated otherwise assume that for Classical Credibility **the partial credibility is given by this square root rule**.[49] Use the square root rule for partial credibility for either frequency, severity, pure premiums, or aggregate losses.

For example if 1000 claims are needed for full credibility for frequency, then the following credibilities would be assigned:

| Expected # of Claims | Credibility |
|---|---|
| 1 | 3% |
| 10 | 10% |
| 25 | 16% |
| 50 | 22% |
| 100 | 32% |
| 200 | 45% |
| 300 | 55% |
| 400 | 63% |
| 500 | 71% |
| 600 | 77% |
| 700 | 84% |
| 800 | 89% |
| 900 | 95% |
| 1000 | 100% |
| 1500 | 100% |

[47] See Section 6.3 of NonLife Actuarial Models: Theory, Methods and Evaluation.
[48] See Equation 6.24 in NonLife Actuarial Models: Theory, Methods and Evaluation.
[49] In contrast for Buhlmann/ Greatest Accuracy Credibility, Z = N / (N+K) for K equal to the Buhlmann Credibility parameter. There is no Standard for Full Credibility for Buhlmann Credibility.

Exercise: The Standard for Full Credibility is 683 claims and one has observed 300 claims.
How much credibility is assigned to this data?

[Solution: $\sqrt{\frac{300}{683}}$ = 66.3%.]

Exercise: The Standard for Full Credibility is 683 claims and one has observed 2000 claims.
How much credibility is assigned to this data?
[Solution: 100%. When the volume of data is greater than (or equal to) Standard for Full Credibility, one assigns 100% credibility to the data.]

**When available, one generally uses the number of exposures or the expected number of claims in the square root rule, rather than the observed number of claims.**[50]

**Make sure that in the square root rule you divide comparable quantities**:

$$Z = \sqrt{\frac{\text{number of claims}}{\text{standard for full credibility in terms of claims}}}, \text{ or}$$

$$Z = \sqrt{\frac{\text{number of exposures}}{\text{standard for full credibility in terms of exposures}}}.$$

Exercise: Prior to observing any data, you assume that the claim frequency rate per exposure has mean = 0.25. The Standard for Full Credibility for frequency is 683 claims.
One has observed 300 claims on 1000 exposures.
Estimate the number of claims you expect for these 1000 exposures next year.
[Solution: The expected number of claims on 1000 exposures is: (1000)(0.25) = 250.

$Z = \sqrt{\frac{250}{683}}$ = 60.5%.

Alternately, a standard of 683 claims corresponds to 683/0.25 = 2732 exposures.

$Z = \sqrt{\frac{1000}{2732}}$ = 60.5%.

In either case, the estimated future frequency = (60.5%)(0.30) + (1 - 60.5%)(0.25) = 0.280.
(1000)(0.280) = 280 claims.]

[50] See Credibility by Mahler and Dean, page 29.

*Limiting Fluctuations*:

For example, assume that the mean frequency per exposure is 2%, and we have 50,000 exposures. Then the expected number of claims is: (2%)(50,000) = 1000.

If frequency is Poisson, then the variance of the number of claims from a single exposure is 2%.
The variance of the average frequency for the portfolio of 50,000 exposures is: $\frac{2\%}{50,000}$.[51]

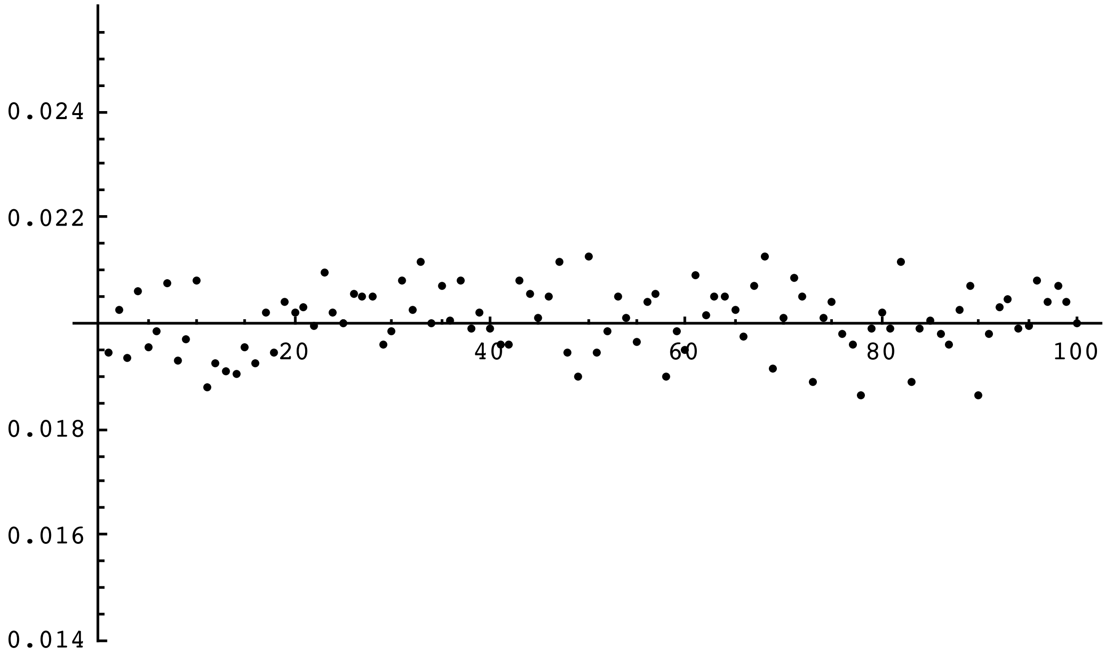
The standard deviation of the observed claim frequency is: $\frac{\sqrt{2\%}}{\sqrt{50,000}}$ = 0.000632.

If instead of 50,000 exposures one had only 5000 exposures, then the expected number of claims is: (2%)(5,000) = 100.
The standard deviation of the estimated frequency is: $\frac{\sqrt{2\%}}{\sqrt{5000}}$ = 0.002.

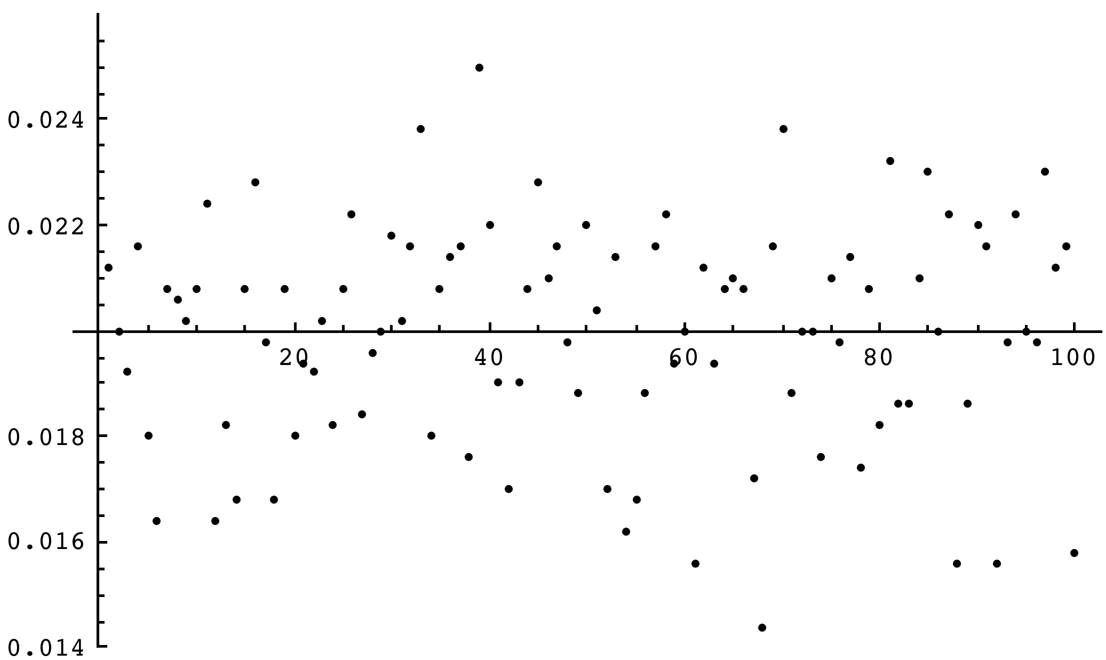
With only 5000 rather than 50,000 exposures, there would be considerably more fluctuation in the observed claim frequency.

[51] The variance of an average is the variance of a single draw divided by the number of items being averaged.

Below are shown 100 random simulations of the claim frequency for 50,000 exposures with a Poisson parameter $\lambda = 0.02$, for 1000 expected claims:




Below are shown 100 random simulations of the claim frequency for 5,000 exposures with a Poisson parameter $\lambda = 0.02$, for 100 expected claims:




With only 100 expected claims, there is much more random fluctuation in the observed claim frequency, than with 1000 expected claims.

Let us now assume that the standard for full credibility for estimating frequency is chosen as 1000 expected claims.[52] Then if we had 50,000 exposures and 1000 expected claims, we would give the observed frequency a credibility of one; we would rely totally on the observed frequency to estimate the future frequency. As discussed previously, for this amount of data the standard deviation of the observed claim frequency is: 0.000632. This is also the standard deviation of the estimated claim frequency. Thus the chosen standard for full credibility results in a standard deviation of the estimated claim frequency of 0.000632.[53]

If we had only 5000 exposures and 100 expected claims, then as discussed previously, the standard deviation of the observed claim frequency is: 0.002. If we were to rely totally on the observed frequency to estimate the future frequency, then the standard deviation of that estimate would be much larger than desired.

However, with only 100 expected claims, in estimating the future frequency we multiply the observation by $Z = \sqrt{\frac{100}{1000}} = 31.6\%$. The standard deviation of Z times the observation is: (0.316)(0.002) = 0.000632. This is the same standard deviation as when we had full credibility. Therefore, using credibility, the fluctuation in the estimated frequency due to the fluctuations in the data will be the same.

To reiterate, the standard deviation of the observed claim frequency is larger for 100 expected claims than for 1000 claims. If one uses 1000 claims as the Standard for Full Credibility, then the credibility assigned to 100 expected claims is the ratio of the standard deviation with 1000 expected claims to the standard deviation with 100 expected claims.

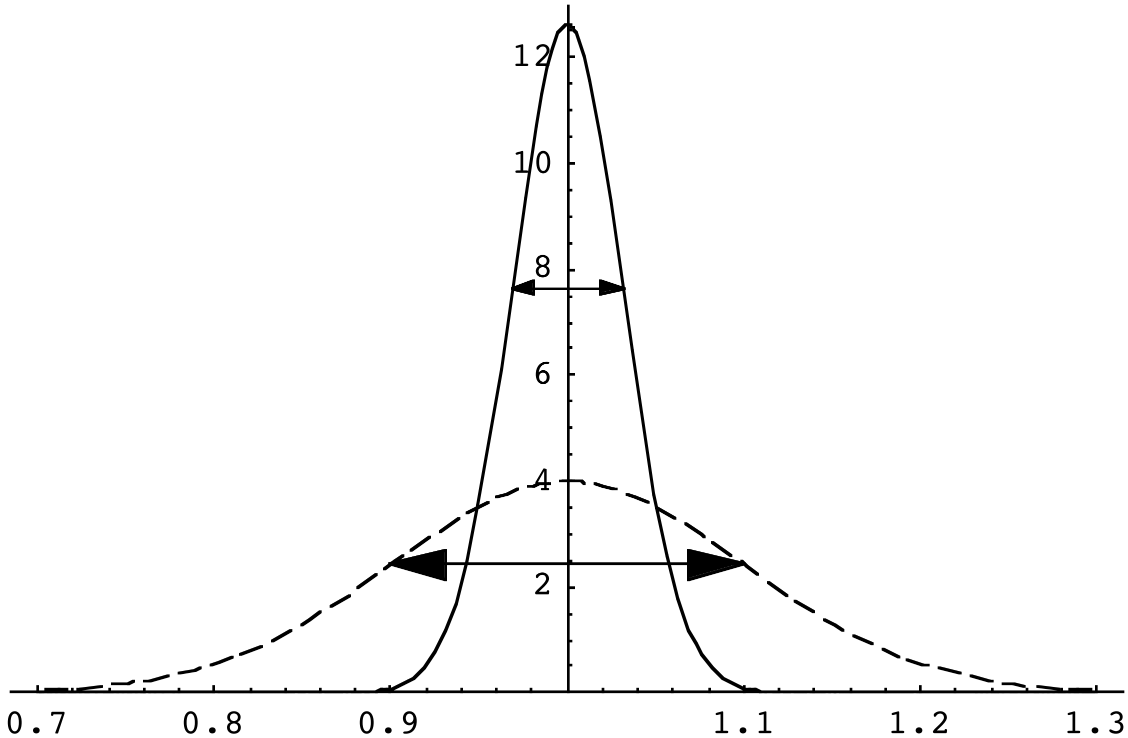
In this case, $Z = 0.000632 / 0.002 = 31.6\% = \sqrt{\frac{100}{1000}}$.

This concept is shown below for two Normal Distributions, approximating Poisson frequency Distributions, one with mean 1000 and variance 1000 (solid curve) and the other with mean 100 and variance 100 (dotted curve).

[52] This would have been based on some choice of $\alpha$ and k, as discussed previously.

[53] If we had more than 50,000 exposures, the standard deviation of the estimated claim frequency would be less.

The x-axis is the number of claims / mean number of claims.




Each arrow is plus or minus one coefficient of variation, since each curve has been scaled in terms of its mean number of claims. With a full credibility standard of 1000 claims, the partial credibility for 100 expected claims is the ratio of the lengths of the arrows:
Z = 0.0316/0.100 = 31.6%.

The credibilities are inversely proportional to the standard deviations of the observed frequencies.

In general, the partial credibility assigned to $\lambda_N$ claims for $\lambda_N \le \lambda_F$ will be the ratio of the standard deviation with $\lambda_F$ expected claims to the standard deviation with $\lambda_N$ expected claims.

This ratio will be such that $Z = \sqrt{\frac{\lambda_N}{\lambda_F}}$.

The standard deviation of Z times the observation will be that for full credibility:

$\text{VAR}[(Z)(\text{observation})] = Z^2\ \text{VAR}[\text{observation}] = \frac{\lambda_N}{\lambda_F}\ \frac{\mu}{\lambda_N} = \frac{\mu}{\lambda_F}$. Thus the random fluctuation in the estimate that is due to the contribution of Z times the observation has been limited to that which was deemed acceptable when the Standard for Full Credibility was determined. This is why the term “Limited Fluctuation Credibility” is sometimes used to describe Classical Credibility.

The square root rule for partial credibility is designed so that when one has less data than the standard for the full credibility, the weight given the observation is such that the standard deviation of the estimate of the future has the same value it would have had if instead we had an amount of data equal to the standard for full credibility.

*Deriving the Square Root Rule*:[54]

Let $\lambda_F$ be such that when the observed pure premium $X_{full}$ is based on $\lambda_F$ claims:

$1 - \alpha = \text{Prob}[\mu - k\mu \le X_{full} \le \mu + k\mu]$. $\Leftrightarrow$ $1 - \alpha = \text{Prob}[-k\mu/\sigma_{full} \le (X_{full} - \mu)/\sigma_{full} \le k\mu\sigma_{full}]$.

In this case, our estimate = $X_{full}$.

Let $X_{partial}$ be the observed pure premium based on $\lambda_N$ claims, with $\lambda_N < \lambda_F$.

In this case, our estimate = $ZX_{partial} + (1-Z)Y$, where Y is other information.

We desire to limit the fluctuation in this estimate due to the term $ZX_{partial}$.

We desire $ZX_{partial}$ to have a large probability of being close to $Z\mu$:

$1 - \alpha = \text{Prob}[Z\mu - k\mu \le ZX_{partial} \le Z\mu + k\mu] \Leftrightarrow$

$1 - \alpha = \text{Prob}[-k\mu/(Z\sigma_{partial}) \le (X_{partial} - \mu) / \sigma_{partial} \le k\mu/(Z\sigma_{partial})]$.

Assuming both $(X_{full} - \mu)/\sigma_{full}$ and $(X_{partial} - \mu)/\sigma_{partial}$ are approximately Standard Normals, comparing the two requirements, in order to make both probabilities $1 - \alpha$, we require that

$$\frac{k\,\mu}{\sigma_{full}} = \frac{k\,\mu}{Z\sigma_{partial}}. \Rightarrow Z = \frac{\sigma_{full}}{\sigma_{partial}}.$$

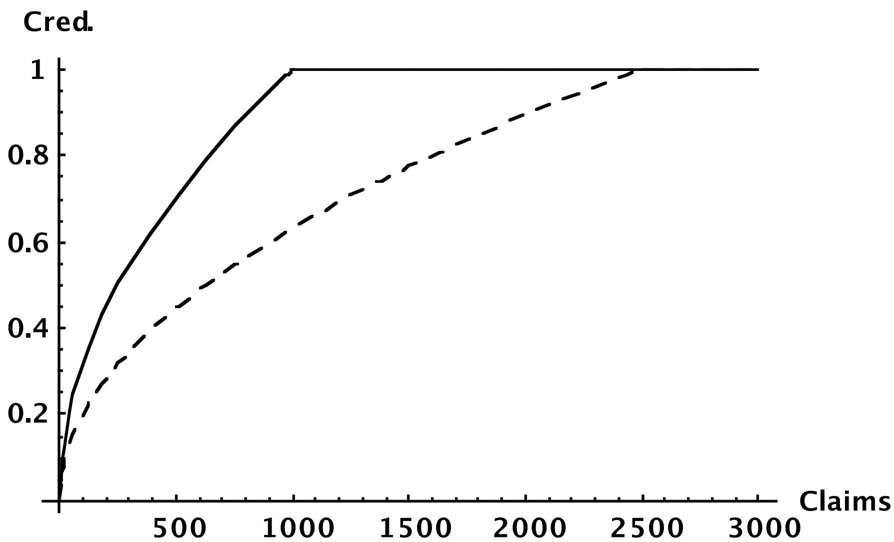
However, the standard deviation of an average goes down as the inverse of the amount of data.

Therefore, $$\frac{\sigma_{full}}{\sigma_{partial}} = \frac{1/\sqrt{\lambda_F}}{1/\sqrt{\lambda_N}} = \sqrt{\frac{\lambda_N}{\lambda_F}}. \Rightarrow Z = \sqrt{\frac{\lambda_N}{\lambda_F}}.$$

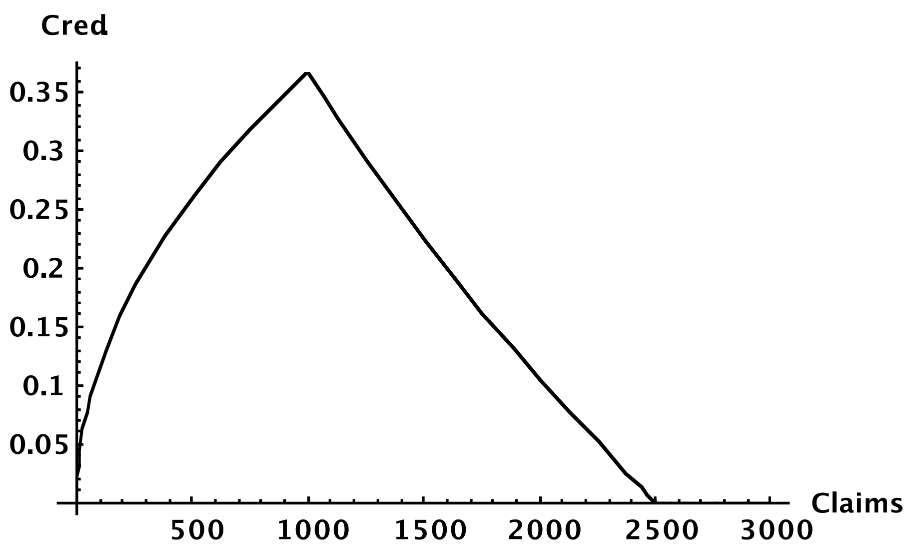
[54] See pages 514-515 of Credibility by Mahler and Dean.

<u>Comparing Different Standards for Full Credibility</u>:

The credibilities assigned to various numbers of claims under either a Standard for Full Credibility of 2500 claims (dashed) or 1000 claims (solid) are shown below.




For large volumes of data the credibility is 100% under either Standard. For smaller volumes of data, more credibility is assigned when using a Standard for Full Credibility of 1000 claims rather than 2500 claims. The differences in the amount of credibility assigned using these two different Standards for Full Credibility of 1000 and 2500 claims are:




For smaller volumes of data there is as much as a 37% difference in the credibilities depending on the Standard for Full Credibility. Nevertheless, even for the criteria differing by a factor of 2.5, the credibilities assigned to most volumes of data are not that dissimilar.[55]

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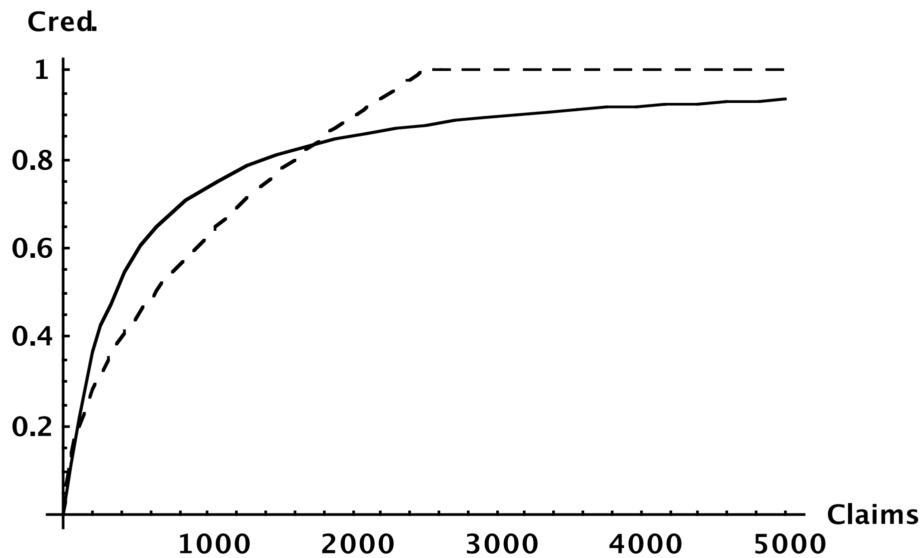
[55] Rounding Standards for Full Credibility to a whole number of claims should be more than sufficient.

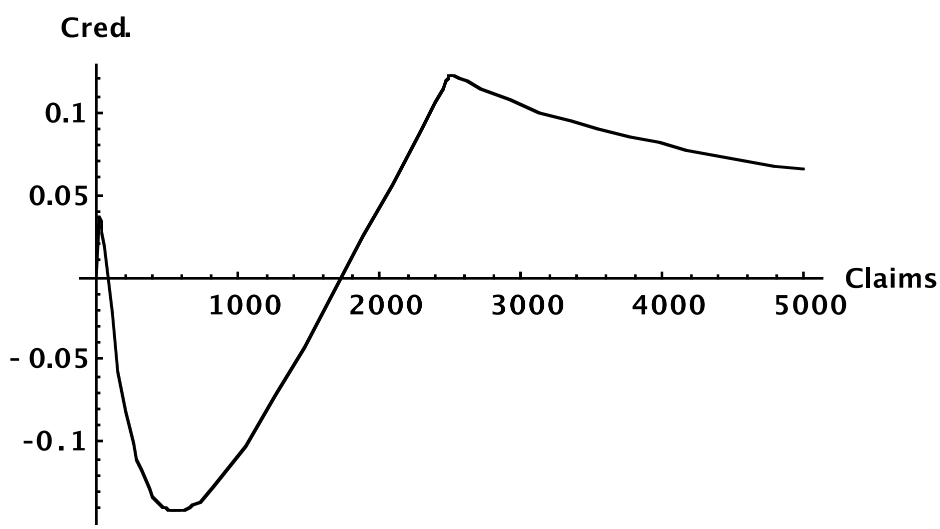
<u>Classical Credibility vs. Buhlmann Credibility</u>:

Below the Classical Credibility formula for credibility with 2500 claims for Full Credibility (dashed curve) is compared to one from Buhlmann Credibility (solid curve): Z = N/(N + 350).[56]




One important distinction is that as the volume of data increases the Buhlmann Credibility approaches but never quite attains 100% credibility.[57]

Here is the difference in the credibilities produced by these two formulas:




[56] Z = N/(N+K) for K equal to the "Buhlmann Credibility parameter". In this example, K = 350. See "Mahler's Guide to Buhlmann Credibility."

[57] *However, the credibilities produced by these two formula are relatively similar. Generally this will be true provided the Standard for Full Credibility is about 7 or 8 times the Buhlmann Credibility Parameter. See "An Actuarial Note on Credibility Parameters", by Howard Mahler, PCAS 1986.*

Problems:

**6.1** (1 point) The Standard for Full Credibility is 1500 claims.
How much credibility is assigned to 200 claims?
A. less than 0.2
B. at least 0.2 but less than 0.3
C. at least 0.3 but less than 0.4
D. at least 0.4 but less than 0.5
E. at least 0.5

**6.2** (1 point) The 1996 pure premium underlying the rate equals $1,000. The loss experience is such that the actual pure premium for that year equals $1,200 and the number of claims equals 400. If 8000 claims are needed for full credibility and the square root rule for partial credibility is used, estimate the pure premium underlying the rate in 1997.
(Assume no change in the pure premium due to inflation.)
A. Less than $1,020
B. At least $1,020, but less than $1,030
C. At least $1,030, but less than $1,040
D. At least $1,040, but less than $1,050
E. $1,050 or more

**6.3** (1 point) Using the square root rule for partial credibility a certain volume of data is assigned credibility of 0.26.
How much credibility would be assigned to 20 times that volume of data?
A. less than 0.5
B. at least 0.5 but less than 0.7
C. at least 0.7 but less than 0.9
D. at least 0.9 but less than 1.1
E. at least 1.1

**6.4** (2 points) Assume a Standard for Full Credibility for severity of 2000 claims.
Assume that for the class of Plumbers one has observed 513 claims totaling $4,771,000.
Assume the average cost per claim for all similar classes is $10,300.
What is the estimated average cost per claim for the Plumbers class?
A. less than 9600
B. at least 9600 but less than 9650
C. at least 9650 but less than 9700
D. at least 9700 but less than 9750
E. at least 9750

**6.5** (1 point) The Standard for Full Credibility is 4500 claims. The expected claim frequency is 4% per house-year. How much credibility is assigned to 5000 house-years of data?
A. less than 0.2
B. at least 0.2 but less than 0.3
C. at least 0.3 but less than 0.4
D. at least 0.4 but less than 0.5
E. at least 0.5

**6.6** (2 points) You are given the following information:
- Frequency is Poisson
- Severity follows a Gamma Distribution with $\alpha = 2.5$.
- Frequency and Severity are Independent.
- Full credibility is defined as having a 98% probability of being within plus or minus 6% of the true pure premium.

What credibility is assigned to 200 claims?
A. less than 0.32
B. at least 0.32 but less than 0.34
C. at least 0.34 but less than 0.36
D. at least 0.36 but less than 0.38
E. at least 0.38

**6.7** (3 points) You are given the following:
Prior to observing any data, you assume that the claim frequency rate per exposure has mean = 0.05 and variance = 0.15.
A full credibility standard is devised that requires the observed sample frequency rate per exposure to be within 3% of the expected population frequency rate per exposure 98% of the time.
You observe 9021 claims on 200,000 exposures.
Estimate the number of claims you expect for these 200,000 exposures next year.
A. less than 9200
B. at least 9200 but less than 9300
C. at least 9300 but less than 9400
D. at least 9400 but less than 9500
E. at least 9500

**6.8** (1 point) The standard for full credibility is 1000 exposures.
For how many exposures would Z = 40%?

**6.9** (3 points) You are given the following:
- The number of claims follows a Poisson distribution.
- The variance of the pure premium distribution is 100.
- The a priori estimate of the mean pure premium is 6.
- The full credibility standard has been selected so that the estimated pure premiums will be within 2.5% of their expected value 80% of the time.
- You observe $3,200 of losses for 800 exposures.

Using the methods of classical credibility, estimate the future pure premium.
A. Less than 5.0
B. At least 5.0, but less than 5.2
C. At least 5.2, but less than 5.4
D. At least 5.4, but less than 5.6
E. At least 5.6

Use the following information for the next two questions:
- The number of claims follows a Poisson distribution.
- Claim sizes follow an exponential distribution.
- The number of claims and claim sizes are independent.
- Credibility is assigned to the observed data using the concepts of classical credibility.

**6.10** (2 points) If one were estimating the future frequency, the volume of data observed would be assigned 60% credibility. Assume the same value of k and $\alpha$ are used to determine the Full Credibility Criterion for frequency and pure premiums. How much credibility would be assigned to this same volume of data for estimating the future pure premium?
A. Less than 45%
B. At least 45%, but less than 50%
C. At least 50%, but less than 55%
D. At least 55%
E. Cannot be determined from the given information.

**6.11** (2 points) If one were estimating the future frequency, the volume of data observed would be assigned 100% credibility. Assume the same value of k and $\alpha$ are used to determine the Full Credibility Criterion for frequency and pure premiums. How much credibility would be assigned to this same volume of data for estimating the future pure premium?
A. Less than 85%
B. At least 85%, but less than 90%
C. At least 90%, but less than 95%
D. At least 95%
E. Cannot be determined from the given information.

**6.12** (3 points) You are given the following:
- The number of claims follows a Poisson distribution.
- The number of claims and claim sizes are independent.
- Credibility is assigned to the observed data using the concepts of classical credibility.
- The estimated pure premium is to be within 10% of its expected value 95% of the time.
- You observe the following data:

| Year: | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| Dollars of Loss: | 200 | 150 | 230 | 180 |

- Estimate the coefficient of variation of the pure premium by using the sample variance.
- There is no inflation.
- There is no change in exposure.
- The current manual premium contains a provision for losses of 210.

Estimate the future annual losses.
A. Less than 197
B. At least 197, but less than 199
C. At least 199, but less than 201
D. At least 201, but less than 203
E. At least 203

<u>**6.13**</u> (3 points) You are given:
- Claim counts follow a Poisson distribution.
- Claim sizes have a coefficient of variation squared of 1/2.
- Claim sizes and claim counts are independent.
- The number of claims in 2001 was 810.
- The aggregate loss in 2001 was $1,134,000.
- The manual premium for 2001 was $1.6 million.
- The expected loss ratio underlying the manual rates is 80%.
  (The expected aggregate losses are 80% of manual premiums.)
- The exposure in 2002 is 12% more than the exposure in 2001.
- The full credibility standard is to be within 2.5% of the expected aggregate loss 90% of the time.

Estimate the aggregate losses (in millions) for 2002.
(A) Less than 1.25
(B) At least 1.25, but less than 1.30
(C) At least 1.30, but less than 1.35
(D) At least 1.35, but less than 1.40
(E) At least 1.40

**6.14** (2 points) So far this baseball season, the Houston Astros baseball team has won 35 games and lost 72 games. Using a simulation, a website has predicted that for the entire season the Houston Astros are expected to win 55.4 games and lose 106.6 games.
Sanford Beech is an actuarial student.
Sandy notices that using classical credibility, giving weight 1 - Z to a 50% winning percentage, he can get the same estimate.
How many games would if take for Sandy to give full credibility?
A. Less than 190
B. At least 190 but less than 195
C. At least 195 but less than 200
D. At least 200 but less than 205
E. At least 205

**6.15** (3 points) You are given the following:
● The number of losses is Poisson distributed with mean 500.
● Number of losses and loss severity are independent.
● Loss severity has the following distribution:

| Loss Size | Probability |
|---|---|
| 100 | 0.30 |
| 1000 | 0.40 |
| 10,000 | 0.20 |
| 100,000 | 0.10 |

● There is a 1000 deductible and maximum covered loss of 25,000.
How much credibility would be assigned so that the estimated total cost of claim payments is within 10% of the expected cost with 90% probability?
A. Less than 55%
B. At least 55% but less than 60%
C. At least 60% but less than 65%
D. At least 65% but less than 70%
E. At least 70%

**6.16** (2 points) Prior to the beginning of the baseball season you expected the New York Yankees to win 100 of 162 games. The Yankees have won 8 of their first 19 games this season. Using a standard for full credibility of 1000 games, predict how many games in total the Yankees will win this season.
A. 88 B. 90 C. 92 D. 94 E. 96

**6.17** (3 points) You are given the following information:
● Claim counts follow a Poisson distribution.
● Claim sizes follow a Gamma Distribution.
● Claim sizes and claim counts are independent.
● The full credibility standard is to be within 5% of the expected aggregate loss 90% of the time.
● The number of claims in 2007 was 77.
● The average size of claims in 2007 was 6861.
● In 2007, the manual rate was 400,000.
● The exposure in 2008 is identical to the exposure in the 2007.
● There is 4% inflation between 2007 and 2008.
If the estimate of aggregate losses in 2008 is 447,900,
what is the value of the a parameter for the Gamma distribution of severity?
(A) 2 (B) 3 (C) 4 (D) 5 (E) 6

**6.18** (2 points) For Workers Compensation Insurance for Hazard Group D you are given the following information on lost times claims:

| | State of Con Island | Countrywide |
|---|---|---|
| Number of Claims | 7,363 | 442,124 |
| Dollars of Loss | 218 million | 23,868 million |

The full credibility standard has been selected so that actual severity will be within 7.5% of expected severity 99% of the time.
The coefficient of variation of the size of loss distribution is 4.
What is the estimated average severity for Hazard Group D in the state of Con Island?
A. 37,000 B. 39,000 C. 41,000 D. 43,000 E. 45,000

**6.19** (3 points) The average baseball player has a batting average of 0.260.
In his first six at bats, Reginald Mantle gets 3 hits, for a batting average of 0.500.
In his 3000 at bats, Willie Mays Hayes has gotten 900 hits, for a batting average of 0.300.
Which of these two players would you expect to have a better batting average in the future?
Use Classical Credibility to discuss why.

**6.20** (2 points) You are given the following information:
● Claim counts follow a Negative Binomial distribution.
● Claim sizes follow a Gamma Distribution with $\alpha = 0.8$.
● Claim sizes and claim counts are independent.
● The full credibility standard is to be within k of the expected value with a certain probability.
● When estimating frequency, a certain number of claims is given credibility of 70%.
● When estimating aggregate losses, the same number of claims is given credibility of 60%.
Determine the $\beta$ parameter of the Negative Binomial Distribution.
(A) 1.5 (B) 2.0 (C) 2.5 (D) 3.0 (E) 3.5

**6.21** (2 points) X is a random variable.
In a limited fluctuation credibility analysis of X, the estimate of X which is not based on the current data is 500.
Credibility is determined based on the "2.5% closeness" and "90% probability" criteria.
A sample of 400 observations have a sum of 240,000.
The estimated future severity is 510.
Determine Var[X] / $E[X]^2$.
A. 8 B. 9 C. 10 D. 11 E. 12

<u>**6.22**</u> (2 points) You are given the following:
- The number of claims follows a Poisson distribution.
- Claim sizes follow a distribution with a coefficient of variation of 2.3.
- The number of claims and claim sizes are independent.

The full credibility standard has been selected so that the actual aggregate losses will be within 5% of the expected aggregate losses 90% of the time.
Using the methods of limited fluctuation credibility, determine the expected number of claims needed for 60% credibility.
A. Less than 1500
B. At least 1500, but less than 1750
C. At least 1750, but less than 2000
D. At least 2000, but less than 2250
E. At least 2250

**6.23** (2 points) You are given the following:
- Industrywide data on claim amounts has a mean of 12,000 and variance of 500 million.
- For your insurance company you observe 200 claims totaling 2.1 million.
- The full-credibility standard is to be set so that the severity will be within 5% of the true value with probability 99%.

Using limited fluctuation credibility, estimate the mean severity for your insurance company.
A. 11,400 B. 11,500 C. 11,600 D. 11,700 E. 11,800

**6.24** (1 point) The prior estimate of total losses is $500,000.
The expected number of claims per year is 200.
The minimum number of claims for full credibility is 700. $650,000 in losses are observed.
Using partial credibility, estimate the total losses next year.

**6.25 (4, 5/84, Q.35)** (2 points) Frequency is Poisson. Three years of data are used to calculate the pure premium. In the case of an average annual claim count of 36 claims, 20% credibility is assigned to the observed <u>pure premium</u>. The standard for full credibility was chosen so as to achieve a 90% probability of departing no more than 5% from the expected value. What is the ratio of the standard deviation to the mean for the claim severity distribution?
A. Less than 1.1
B. At least 1.1, but less than 1.4
C. At least 1.4, but less than 1.7
D. At least 1.7, but less than 2.0
E. 2.0 or more

**6.26 (4, 5/85, Q.30)** (1 point) The 1984 pure premium underlying the rate equals $1,000.
The loss experience is such that the actual pure premium for that year equals $1,200 and the number of claims equals 600.
If 5400 claims are needed for full credibility and the square root rule for partial credibility is used, estimate the pure premium underlying the rate in 1985.
(Assume no change in the pure premium due to inflation.)
A. Less than $1,025
B. At least $1,025, but less than $1,075
C. At least $1,075, but less than $1,125
D. At least $1,125, but less than $1,175
E. $1,175 or more

**6.27 (4, 5/86, Q.35)** (1 point) You are in the process of revising rates.
The premiums currently being used reflect a loss cost per insured of $100.
The loss costs experienced during the two year period used in the rate review averaged $130 per insured.
The average frequency during the two year review period was 250 claims per year.
Using a full credibility standard of 2,500 claims and assigning partial credibility, what loss cost per insured should be reflected in the new rates?
(Assume that there is no inflation.)
A. Less than $105
B. At least $105, but less than $110
C. At least $110, but less than $115
D. At least $115, but less than $120
E. $120 or more

**6.28 (4, 5/87, Q.36**) (2 points) The actuary for XYZ Insurance Company has just developed a new rate for a particular class of insureds. The new rate has a loss cost provision of $125. In doing so, he used the partial credibility approach of classical credibility. In the experience period used, there were 10,000 insureds with an average claim frequency of 0.0210. If the loss cost in the old rate was $100 and the loss cost in the experience period was $200, what was the actuary's standard for full credibility? (Assume zero inflation.)
A. Less than 3,000
B. At least 3,000, but less than 3,200
C. At least 3,200, but less than 3,400
D. At least 3,400, but less than 3,600
E. 3,600 or more.

**6.29 (4, 5/88, Q.34)** (2 points) Assume the random variable N, representing the number of claims for a given insurance portfolio during a one year period, has a Poisson distribution with a mean of n. Also assume $X_1$, $X_2$..., $X_N$ are N independent, identically distributed random variables with $X_i$ representing the size of the $i^{th}$ claim. Let $C = X_1 + X_2 + ... X_n$ represent the total cost of claims during a year. We want to use the observed value of C as an estimate of future costs. Using Classical credibility procedures, we are willing to assign full credibility to C provided it is within 10.0% of its expected value with probability 0.96. Frequency is Poisson.
If the claim size distribution has a coefficient of variation of 0.60, what credibility should we assign to the experience if 213 claims occur?
A. Less than 0.60
B. At least 0.60, but less than 0.625
C. At least 0.625, but less than 0.650
D. At least 0.650, but less than 0.675
E. 0.675 or more

**6.30 (4, 5/88, Q.35)** (2 points) The High Risk Insurance Company is revising its rates, based on its experience during the past two years. The company experienced an average of 1,250 claims annually over these two years. The loss costs underlying the current rates average $500 per insured. The Actuary is proposing that this loss costs provision be revised upward to $550, based on the average loss costs of $700 experienced over the two year experience period. The Actuary is using the Classical credibility approach. The expected number of claims necessary for full credibility is determined by the requirement that the observed total cost of claims should be within 100k% of the true value 100P% of the time. What is the probability that a fully credible estimate of the loss costs (for a sample whose expected number of claims is equal to the full credibility standard) would be within 5% of the true value?
Assume that frequency is Poisson, the average claim size is $700, and the variance of the claim size distribution is 17,640,000.
A. Less than 0.775
B. At least 0.775, but less than 0.825
C. At least 0.825, but less than 0.875
D. At least 0.875, but less than 0.925
E. 0.925 or more

**6.31 (4, 5/89, Q.30)** (2 points) The Slippery Rock Insurance Company is reviewing their rates. In order to calculate the credibility of the most recent loss experience they have decided to use Classical credibility.
The expected number of claims necessary for full credibility is to be determined so that the observed total cost of claims should be within 5% of the true value 90% of the time. Based on independent studies, they have estimated that individual claims are independent and identically distributed as follows: $f(x) = \frac{1}{200{,}000}$, $0 \le x \le 200{,}000$.
Assume that the number of claims follows a Poisson distribution.
What is the credibility Z to be assigned to the most recent experience given that it contains 1,082 claims? Use a normal approximation.
A. $Z \le 0.800$
B. $0.800 < Z \le 0.825$
C. $0.825 < Z \le 0.850$
D. $0.850 < Z \le 0.875$
E. $0.875 < Z$

**6.32 (4, 5/91, Q.23)** (2 points) The average claim size for a group of insureds is $1,500 with standard deviation $7,500. Assuming a Poisson claim count distribution, use as your standard for full credibility, the expected number of claims so that the total loss will be within 6% of the expected total loss with probability 90%. We observe 6,000 claims and a total loss of $15,600,000 for a group of insureds. If our prior estimate of the total loss is 16,500,000, find the Classical credibility estimate of the total loss for this group of insureds.
A. Less than 15,780,000
B. At least 15,780,000 but less than 15,870,000
C. At least 15,870,000 but less than 15,960,000
D. At least 15,960,000 but less than 16,050,000
E. At least 16,050,000

**6.33 (4B, 5/92, Q.6)** (1 point)
You are given the following information for a group of insureds:

| | |
|---|---|
| Prior estimate of expected total losses | $20,000,000 |
| Observed total losses | $25,000,000 |
| Observed number of claims | 10,000 |
| Required number of claims for full credibility | 17,500 |

Using the partial credibility as in Classical credibility, determine the estimate for the group's expected total losses based upon the latest observation.
A. Less than $21,000,000
B. At least $21,000,000 but less than $22,000,000
C. At least $22,000,000 but less than $23,000,000
D. At least $23,000,000 but less than $24,000,000
E. At least $24,000,000

**6.34 (4B, 11/93, Q.20)** (2 points) You are given the following:
- P = Prior estimate of pure premium for a particular class of business.
- O = Observed pure premium during latest experience period for same class of business.
- R = Revised estimate of pure premium for same class following observations.
- F = Number of claims required for full credibility of pure premium.

Based on the concepts of Classical credibility, determine the number of claims used as the basis for determining R.

A. $\frac{F\ (R - P)}{O - P}$ B. $\frac{F\ (R - P)^2}{(O - P)^2}$ C. $\frac{\sqrt{F}\ (R - P)}{O - P}$

D. $\frac{\sqrt{F}\ (R - P)^2}{(O - P)^2}$ E. $\frac{F^2\ (R - P)}{O - P}$

**6.35 (4B, 11/95, Q.12)** (1 point) 2000 expected claims are needed for full credibility.
Determine the number of expected claims needed for 60% credibility.
A. Less than 700
B. At least 700, but less than 900
C. At least 900, but less than 1100
D. At least 1100, but less than 1300
E. At least 1300

**6.36 (4B, 5/96, Q.28)** (1 point) The full credibility standard has been selected so that the actual **number of claims** will be within 5% of the expected **number of claims** 90% of the time.
Frequency is Poisson.
Using the methods of Classical credibility, determine the credibility to be given to the experience if 500 claims are expected.
A. Less than 0.2
B. At least 0.2, but less than 0.4
C. At least 0.4, but less than 0.6
D. At least 0.6, but less than 0.8
E. At least 0.8

**6.37 (4B, 11/96, Q.29)** (1 point) You are given the following:
- The number of claims follows a Poisson distribution.
- Claim sizes are discrete and follow a Poisson distribution with mean 4.
- The number of claims and claim sizes are independent.

The full credibility standard has been selected so that the actual **number of claims** will be within 10% of the expected **number of claims** 95% of the time. Using the methods of Classical credibility, determine the expected number of claims needed for 40% credibility.
A. Less than 100
B. At least 100, but less than 200
C. At least 200, but less than 300
D. At least 300, but less than 400
E. At least 400

**6.38 (4B, 5/99, Q.18)** (1 point) You are given the following:
● The number of claims follows a Poisson distribution.
● The coefficient of variation of the claim size distribution is 2.
● The number of claims and claim sizes are independent.
● 1,000 expected claims are needed for full credibility.
● The full credibility standard has been selected so that the actual number of claims will be within k% of the expected number of claims P% of the time.
Using the methods of Classical credibility, determine the number of expected claims needed for 50% credibility.
A. Less than 200
B. At least 200, but less than 400
C. At least 400, but less than 600
D. At least 600, but less than 800
E. At least 800

**6.39 (4B, 11/99, Q.18)** (2 points) You are given the following:
● Partial Credibility Formula A is based on the methods of classical credibility, with 1,600 expected claims needed for full credibility.
● Partial Credibility Formula B is based on Buhlmann's credibility formula with a Buhlmann Credibility Parameter of K = 391.
● One claim is expected during each period of observation.
Determine the largest number of periods of observation for which Partial Credibility Formula B yields a larger credibility value than Partial Credibility Formula A.
A. Less than 400
B. At least 400, but less than 800
C. At least 800, but less than 1,200
D. At least 1,200, but less than 1,600
E. At least 1,600

**6.40 (4, 5/00, Q.26)** (2.5 points) You are given:
(i) Claim counts follow a Poisson distribution.
(ii) Claim sizes follow a lognormal distribution with coefficient of variation 3.
(iii) Claim sizes and claim counts are independent.
(iv) The number of claims in the first year was 1000.
(v) The aggregate loss in the first year was 6.75 million.
(vi) In the first year, the provision in the premium in order to pay losses was 5.00 million.
(vii) The exposure in the second year is identical to the exposure in the first year.
(viii) The full credibility standard is to be within 5% of the expected aggregate loss 95% of the time.
Determine the classical credibility estimate of losses (in millions) for the second year.
(A) Less than 5.5
(B) At least 5.5, but less than 5.7
(C) At least 5.7, but less than 5.9
(D) At least 5.9, but less than 6.1
(E) At least 6.1
<u>Note</u>: I have reworded bullet vi in the original exam question.

**6.41 (4, 11/01, Q.15)** (2.5 points) You are given the following information about a general liability book of business comprised of 2500 insureds:

(i) $X_i = \sum_{j=1}^{N_i} Y_{ij}$ is a random variable representing the annual loss of the $i^{th}$ insured.

(ii) $N_1, N_2, \ldots, N_{2500}$ are independent and identically distributed random variables following a negative binomial distribution with parameters r = 2 and $\beta$ = 0.2.

(iii) $Y_{i1}, Y_{i2}, \ldots, Y_{iN_i}$ are independent and identically distributed random variables following a Pareto distribution with $\alpha$ = 3.0 and $\theta$ = 1000.

(iv) The full credibility standard is to be within 5% of the expected aggregate losses 90% of the time.

Using classical credibility theory, determine the partial credibility of the annual loss experience for this book of business.

(A) 0.34 (B) 0.42 (C) 0.47 (D) 0.50 (E) 0.53

**6.42 (4, 11/03, Q.35)** (2.5 points) You are given:

(i) $X_{partial}$ = pure premium calculated from partially credible data

(ii) $\mu = E[X_{partial}]$

(iii) Fluctuations are limited to $\pm k\,\mu$ of the mean with probability $1 - \alpha$

(iv) Z = credibility factor

Which of the following is equal to $1 - \alpha$?

(A) $\Pr[\mu - k\mu \le X_{partial} \le \mu + k\mu]$

(B) $\Pr[Z\mu - k \le Z\,X_{partial} \le Z\mu + k]$

(C) $\Pr[Z\mu - \mu \le Z\,X_{partial} \le Z\mu + \mu]$

(D) $\Pr[1 - k \le Z\,X_{partial} + (1-Z)\mu \le 1 + k]$

(E) $\Pr[\mu - k\mu \le Z\,X_{partial} + (1-Z)\mu \le \mu + k\mu]$

**6.43 (CAS MAS-2, 11/18, Q.6)** (2.5 points) You are given the following information:
- A block of insurance policies had 1,384 claims this period.
- The claims had a mean loss of 55 and variance of loss of 6,010.
- Frequency is Poisson.
- The mean frequency of these claims is 0.085 per policy.
- The block has 21,000 policies.
- Full credibility is based on a coverage probability of 98% for a range of within 5% deviation from the true mean.

You calculate the partial-credibility factor for severity, $Z_x$, and the partial-credibility factor for pure premium, $Z_p$, using the limited-fluctuation credibility method.
Calculate the absolute difference between $Z_x$ and $Z_p$.
A. Less than 0.05
B. At least 0.05, but less than 0.15
C. At least 0.15, but less than 0.25
D. At least 0.25, but less than 0.35
E. At least 0.35
Note: I have slightly written this exam question.

**6.44 (CAS MAS-2, 5/19, Q.6)** (2.5 points) An insurance company is currently using a limited-fluctuation credibility approach for a line of business with the following assumptions:
- The claim frequency follows a Poisson distribution.
- The mean of the claim frequency is large enough to justify the normal approximation to the Poisson.
- The square root rule is used to determine partial credibility.
- The standard for full credibility is the number of claims at which there is a 99% probability that the observed aggregate loss is within 5% of the mean.

You are given the following information about a block of 10,000 policies:
- The mean claim frequency is 0.12.
- The mean claim seventy is 100.
- The variance of claim severity is 14,400.

Calculate the credibility for this block of policies using the partial credibility method for aggregate loss.
A. Less than 0.45
B. At least 0.45, but less than 0.55
C. At least 0.55, but less than 0.65
D. At least 0.65, but less than 0.75
E. At least 0.75

Solutions to Problems:

**6.1. C.** $Z = \sqrt{\frac{200}{1500}}$ = **36.5%**.

**6.2. D.** $Z = \sqrt{\frac{400}{8000}}$ = 22.4%.

Estimated Pure Premium = (22.4%)(1200) + (77.6%)(1000) = **$1045**.

**6.3. D.** Since the credibility is proportional to the square root of the number of claims, we get (26%) $\sqrt{20}$ = 116%. However, the credibility is limited to **100%**.

**6.4. E.** $Z = \sqrt{\frac{513}{2000}}$ = 0.506. Observed average cost per claim is: 4,771,000 / 513 = 9300.

Thus the estimated severity = (0.506)(9300) + (1 - 0.506)(10,300) = **$9794**.

**6.5. B.** The expected number of claims is (0.04)(5000) = 200.

$Z = \sqrt{\frac{200}{4500}}$ = **21.1%**.

**6.6. A.** $\Phi(2.326) = 0.99$, so that $z_{1-\alpha/2} = 2.326$. $\lambda_F = \left(\frac{z_{1-\alpha/2}}{k}\right)^2 = (2.326 / 0.06)^2 = 1503$.

For the Gamma Distribution, the mean is $\alpha\theta$, while the variance is $\alpha\theta^2$.

Thus $CV^2 = \frac{\alpha\theta^2}{(\alpha\theta)^2} = 1 / \alpha = 1/2.5 = 0.4$. $\lambda_F(1 + CV^2) = (1503)(1.4) = 2104$.

$Z = \sqrt{\frac{200}{2104}}$ = **30.8%**.

**6.7. B.** $\alpha$ = 2%. Therefore $z_{1-\alpha/2}$ = 2.326, since $\Phi$(2.326) = 0.99. k = 0.03.

Standard For Full Credibility is: $\left(\frac{z_{1-\alpha/2}}{k}\right)^2 (\sigma_f^2/\mu_f)$ = (2.326/0.03)$^2$(0.15/0.05) = 18,034 claims,

or 18,034/0.05 = 360,680 exposures. Z = $\sqrt{\frac{200,000}{360,680}}$ = 74.5%.

Estimated future frequency is: (74.5%)(9021/200000) + (25.5%)(0.05) = 4.635%.
Expected number of future claims is: (200,000)(4.635%) = **9270**.
Comment: When available, one generally uses the number of exposures or the expected number of claims in the square root rule, rather than the observed number of claims.

Using the expected number of claims, Z = $\sqrt{\frac{10,000}{18,034}}$ = 74.5%.

**6.8.** $\sqrt{\frac{x}{1000}}$ = 0.4. ⇒ x = (0.4$^2$) (1000) = **160** exposures.

**6.9. C.** CV$^2$ of the Pure Premium is: 100/6$^2$ = 2.778. $z_{1-\alpha/2}$ = 1.282. k = 0.025.

$\lambda_F = \left(\frac{z_{1-\alpha/2}}{k}\right)^2$ = 2630.

Standard for Full Credibility for P.P. = $\lambda_F$ (Coefficient of Variation of the P.P.)$^2$ = (2630)(2.778) =

7306 exposures. Z = $\sqrt{\frac{800}{7306}}$ = 33.1%. Observation = 3200/800 = 4.

New Estimate = (4)(33.1%) + (6)(66.9%) = **5.34**.
Alternately, let m be the mean frequency. Then since the frequency is assumed to be Poisson, variance of pure premium = m(second moment of severity). Thus E[X$^2$] = 100 / m. E[X] = 6 / m.
Standard for Full Credibility in terms of claims is: $\lambda_F$ (1 + CV$^2$) = $\lambda_F$ E[X$^2$] / E[X]$^2$ =

(2630) (100 / 6$^2$)m = 7306m claims. Expected number of claims = 800m.

Z = $\sqrt{\frac{800m}{7306m}}$ = 33.1%. Proceed as before.

Comment: You are given the number of exposures and not the number of claims, so that it may be easier to get a standard for full credibility in terms of exposures. When computing Z, make sure the ratio you use is either claims/claims or exposures/exposures. The numerator and the standard for full credibility in the denominator should be in the same units.

**6.10. A.** The Exponential Distribution has a coefficient of variation of 1. For a Poisson frequency, standard for full credibility for pure premium = $\lambda_F$ (1 + CV$^2$) = $\lambda_F$ (1 + 1$^2$) =

2 $\lambda_F$ = twice standard for full credibility for frequencies. Since the credibility is inversely proportional to the square root of the standard for full credibility, the credibility for pure premiums is that for frequency divided by $\sqrt{2}$ : 60%/$\sqrt{2}$ = **42.4**%.

**6.11. E.** We know we have an amount of data at least equal to the full credibility criterion for frequency. If we have a lot more data, we would also assign 100% credibility for estimating pure premiums. If we have just enough data to assign 100% credibility for estimating frequencies, then we would assign 100%/$\sqrt{2}$ = 70.7% credibility for estimating pure premiums.
Thus we cannot determine the answer from the given information.
Comment: One could proceed as in the previous question and calculate 100%/$\sqrt{2}$ = 70.7%.
However, this assumes that we have just enough data to assign 100% credibility for estimating frequencies. In fact we may have much more data than this. For example, if the full credibility criteria for frequency is 1082 claims, we might have either 1082 or 100,000 claims in our data.

**6.12. B.** $z_{1-\alpha/2}$ = 1.960. k = 0.10. $\lambda_F = \left(\frac{z_{1-\alpha/2}}{k}\right)^2$ = 384.

Estimated annual pure premium is: (200 + 150 + 230 + 180) / 4 = 190.
Estimated variance of the pure premium is :
$\{(200 - 190)^2 + (150 - 190)^2 + (230 - 190)^2 + (180 - 190)^2\} / (4-1) = 1133$.
Using the formula: Standard for Full Credibility for P.P. in exposures =
$\lambda_F$ (Coefficient of Variation of the Pure Premium)$^2$ = (384)(1133/190$^2$) = 12.1 exposures.
Since we have 4 exposures (we have counted each year as one exposure,)
$Z = \sqrt{\frac{4}{12.1}}$ = 57.5%. Observation = 190. Prior estimate is 210.
Therefore, estimated P.P. = (190)(57.5%) + (210)(42.5%) = **198.5.**
Alternately, let $\mu_F$ be the mean frequency, $\sigma_F$ be the standard deviation of the frequency, $\mu_{Sev}$ be the mean frequency, and $\sigma_{Sev}$ be the standard deviation of the severity. Then in terms of claims, the Standard for Full Credibility for P.P. is: $\lambda_F (\sigma_F^2/\mu_F + CV^2) = \lambda_F (\sigma_F^2/\mu_F + \mu_{Sev}^2/\mu_{Sev}^2)$.
Thus in terms of exposures, the Standard for Full Credibility for P.P. is:
$\lambda_F (\sigma_F^2/\mu_F + \sigma_{Sev}^2/\mu_{Sev}^2)/\mu_F = \lambda_F (\sigma_F^2\mu_{Sev}^2 + \mu_F\sigma_{Sev}^2) / (\mu_F^2\ \mu_{Sev}^2)$ =
$\lambda_F$ (variance of P.P.)/(mean of P.P.)$^2$ = $\lambda_F$ (CV of the Pure Premium)$^2$. Proceed as above.

**6.13. D.** $\alpha$ = 0.10 and k = 0.025. $\Phi(1.645) = 0.95$, so that $z_{1-\alpha/2}$ = 1.645.
$\lambda_F = (1.645 / 0.025)^2 = 4330$. $CV_{Sev}^2 = 1/2$.
$\lambda_F (1 + CV_{Sev}^2)$ = (4330)(1 + 1/2) = 6495 claims. $Z = \sqrt{810 / 6495}$ = 35.3%.
The prior estimate of aggregate losses is: (80%)($1.6 million) = $1.28 million.
The observation of aggregate losses is $1.134 million.
Thus the new estimate is: (0.353)(1.134) + (1 - 0.353)(1.28) = 1.228 million.
Since exposures have increased by 12%, the estimate of aggregate losses for 2002 is:
(1.12)(1.228) = **$1.38 million**.

**6.14. B.** The observed winning percentage is: 35/107.
The predicted winning percentage for the remainder of the season is:
(55.4 - 35) / (162 - 107) = 20.4/55.
Z 35/107 + (1 - Z) (0.5) = 20.4/55. ⇒ Z = 0.7466.
$\sqrt{107/\lambda_F}$ = 0.7466. ⇒ $\lambda_F$ = **192 games**.
Comment: Information taken from www.coolstandings.com, as of August 4, 2012.

**6.15. D.** We are given k = 10%, $\alpha$ = 10%. Thus $z_{1-\alpha/2}$ = 1.645. $\lambda_F = \left(\frac{z_{1-\alpha/2}}{k}\right)^2$ = 271.

For losses of size 100 and 1000 the insurer makes no payment.
In the case of 10,000, the insurer pays 10000 - 1000 = 9000.
In the case of 100,000, the insurer pays 25000 - 1000 = 24000.
The distribution of the size of nonzero payments is: 9000 @2/3 and 24,000 @1/3.
This has mean of: (2/3)(9000) + (1/3)(24000) = 14,000.
This has second moment of: (2/3)($9000^2$) + (1/3)($24000^2$) = 246,000,000.
1 + $CV^2$ = 246,000,000/$14,000^2$ = 1.255.
We expect 500 losses, and (0.3)(500) = 150 nonzero payments.
Number of claims (nonzero payments) needed for full credibility is: (271)(1.255) = 340.
$Z = \sqrt{\frac{150}{340}}$ = **66.4%**.
Alternately, the distribution of amounts paid is: 0 @0.7, 9000 @0.2 and 24,000 @0.1.
This has mean of: (0.2)(9000) + (0.1)(24000) = 4,200.
This has second moment of: (0.2)($9000^2$) + (0.1)($24,000^2$) = 73,800,000.
1 + $CV^2$ = 73,800,000/$4200^2$ = 4.184.
Number of losses needed for full credibility is: (271)(4.184) = 1134.
$Z = \sqrt{\frac{500}{1134}}$ = **66.4%**.
Comment: The expected total payments are: (150)(14000) = 2,100,000 = (500)(4200).
If for example, we observed 2,500,000 in total payments this year, we would estimate total payments next year of: (66.4%)(2,500,000) + (33.6%)(2,100,000) = 2,365,600.

**6.16. C.** $Z = \sqrt{\frac{19}{1000}}$ = 13.8%. Observed frequency = 8/19 = 0.421.
Prior estimate of frequency = 100/162 = 0.617.
Estimated future frequency = (13.8%)(0.421) + (1 - 13.8%)(0.617) = 0.590.
Estimated number of games won rest of season = (0.590)(162 - 19) = 84.4.
Estimated total number of games won = 8 + 84.4 = **92.4**.

**6.17. C.** Prior to taking into account inflation, the estimate of aggregate losses in 2008 must have been: 447,900/1.04 = 430,673.
The observed aggregate loss is: (77)(6861) = 528,297.
Z 528,297 + (1 - Z)(400,000) = 430,673. ⇒ Z = 23.9%.

$\alpha = 10\%$. $z_{1-\alpha/2} = 1.645$. $\Rightarrow \lambda_F = \left(\frac{z_{1-\alpha/2}}{k}\right)^2 = (1.645/0.05)^2 = 1082$ claims.

For a Gamma, $CV^2 = (\alpha\theta^2) / (\alpha\theta)^2 = 1/\alpha$.
Standard for Full Credibility is: $(1082)(1 + 1/\alpha)$.

$23.9\% = Z = \sqrt{\frac{77}{(1082)\ (1 + 1/\alpha)}}$. $\Rightarrow 0.239^2 \{(1082)(1 + 1/\alpha)\} = 77. \Rightarrow \alpha =$ **4.07**.

Comment: Do not confuse the shape parameter of the Gamma Distribution with the notation used for the Classical Credibility criterion.

**6.18. B.** Average severity for the State = \$218 million / 7,363 = \$29,608.
Average severity for countrywide = \$23,868 million / 442,124 = \$53,985.
$\alpha = 1\%. \Rightarrow z_{1-\alpha/2} = 2.576$. $\lambda_F = (2.576/0.075)^2 = 1180$ claims.

Standard for full credibility for severity is: $CV^2\ \lambda_F = (4^2)(1180) = 18,880$ claims.

$Z = \sqrt{\frac{7,363}{18,880}} = 62.4\%$.

Estimated state severity is: (62.4%)(\$29,608) + (1 - 62.4%)(\$53,985) = **\$38,774**.
Comment: You are not responsible for knowing the details of any specific line of insurance.
*A simplified portion of the calculation of State/Hazard Group Relativities for Workers Compensation Insurance.*

**6.19.** I would expect Willie to have a better batting average in the future than Reginald.
While Reginald has a batting average of 0.500, there is too little data to have much credibility.
Thus the estimated future batting average of Reginald is probably only slightly higher than the overall mean of 0.260.
On the other hand, Willie has a considerable amount of data.
His estimated future batting average is close to or equal to his observed 0.300.
For example, let us assume a Binomial Model.

Then for q = 0.26, the ratio of the variance to the mean frequency is: $\frac{m\,q\,(1-q)}{m\,q} = 1 - q = 0.74$.

If for example, we were to take $\alpha$ = 10% and k = 5%, then $\lambda_F$ = 1082 claims.
The Standard for Full Credibility for frequency would be: (0.74)(1082) = 801 claims.
This is equivalent to: 801 / 0.26 = 3081 exposures (at bats).

Then for Reginald's data, $Z = \sqrt{\frac{6}{3081}} = 4.4\%$.

Reginald's estimated future batting average is: (4.4%)(0.5) + (1 - 4.4%)(0.26) = 0.271.

For Willie's data, $Z = \sqrt{\frac{3000}{3081}} = 98.7\%$.

Willie's estimated future batting average is: (98.7%)(0.3) + (1 - 98.7%)(0.26) = 0.299.
<u>Comment</u>: <u>Not</u> the style of question you will get on your exam.
Other reasonable choices for $\alpha$ and k would produce somewhat different credibilities.
With additional information besides the results of their batting, one could make better estimates.

**6.20. C.** For the Negative Binomial Distribution, variance divided by mean is: $1 + \beta$.
Thus the full credibility standard in claims for frequency is: $\lambda_F (1 + \beta)$.
For the Gamma Distribution, $CV^2 = (\alpha\theta^2) / (\alpha\theta)^2 = 1/\alpha = 1/0.8 = 1.25$.
Thus the full credibility standard in claims for aggregate loss is: $\lambda_F (1.25 + 1 + \beta) = \lambda_F (2.25 + \beta)$.
Let us assume we observe n claims. Then we are given that:

$\sqrt{\frac{n}{\lambda_F (1+\beta)}} = 0.7$, and $\sqrt{\frac{n}{\lambda_F (2.25+\beta)}} = 0.6$.

Dividing the two equations: $\sqrt{\frac{2.25+\beta}{1+\beta}} = 7/6$. $\Rightarrow$ $81 + 36\beta = 49 + 49\beta$. $\Rightarrow$ $\beta$ = **2.46**.

**6.21. B.** The observed mean is: 240,000 / 400 = 600.
Therefore, 510 = Z 600 + (1 - Z) 500. $\Rightarrow$ Z = 10%.
k = 0.025. $\alpha$ = 10%. $\Rightarrow$ $z_{1-\alpha/2} = 1.645$. $\Rightarrow$ $\lambda_F = (1.645/0.025)^2 = 4330$.
The standard for full credibility (for severity) is: 4330 $CV^2$.

$10\% = Z = \sqrt{\frac{400}{4330\, CV^2}}$. $\Rightarrow$ Var[X] / E[X]$^2$ = $CV^2$ = **9.24**.

**6.22. E.** $\alpha$ = 10%, so $z_{1-\alpha/2}$ = 1.645. $\Rightarrow \lambda_F = (1.645/0.05)^2$ = 1082 claims.
Frequency is Poisson and CV of severity is 2.3.
Thus the standard for full credibility for aggregate losses is: $(1 + 2.3^2)$ (1082) = 6806 claims.
Let x be the unknown number of expected claims.
Then in order to have 60% credibility: $0.6 = \sqrt{x/6086}$ . $\Rightarrow x = (0.6^2)$ (6086) = **2450** claims.

**6.23. E.** Based on the industrywide data, $CV^2$ = (500 million) / $(12{,}000^2)$ = 3.472.

$\alpha$ = 1%, so $z_{1-\alpha/2}$ = 2.576. $\lambda_F = \left(\frac{z_{1-\alpha/2}}{k}\right)^2 = (2.756/0.05)^2$ = 2654 claims.

Standard for Full Credibility is: (3.472)(2654 claims) = 9215 claims.

$Z = \sqrt{\frac{200}{9215}}$ = 14.7%.

Estimated severity is: (14.7%)(2.1 million / 200) + (1 - 14.7%)(12,000) = **11,780**.

**6.24.** $Z = \sqrt{200/700}$ = 0.535.
Estimate is: (0.535)(650,000) + (1 - 0.535)(500,000) = **$580,250**.

**6.25. B.** k = 5% and $\alpha$ = 10%. We have $z_{1-\alpha/2}$ = 1.645, since $\Phi(1.645)$ = 0.95.

Therefore, $\lambda_F = \left(\frac{z_{1-\alpha/2}}{k}\right)^2 = (1.645/0.05)^2$ = 1082.

Let x = Standard for Full Credibility for pure premium.
When we have 36 claims per year for three years we assign 20% credibility;
therefore $0.20 = \sqrt{\frac{108}{x}}$ . Thus x = 2700.

But $x = \lambda_F (1 + CV^2)$. Thus $2700 = 1082(1 + CV^2)$. $\Rightarrow$ CV = **1.22**.

**6.26. B.** The credibility $Z = \sqrt{\frac{600}{5400}}$ = 1/3.

Thus the new estimate is: (1/3)(1200) + (1- 1/3)(1000) = **$1067**.

**6.27. C.** The credibility assigned to (2)(250) = 500 claims, $Z = \sqrt{\frac{500}{2500}}$ = 0.447.

The new estimate is (0.447)(130) + (1 - 0.447)(100) = **$113**.

**6.28. C.** The credibility assigned was: $\frac{\text{change in loss cost}}{\text{difference between observation and prior estimate}}$ =

(125 - 100) / (200 - 100) = 25%. The expected number of claims was (10,000)(0.0210) = 210.
Let x = Standard for Full Credibility for pure premium.

$Z = \sqrt{\frac{210}{x}}$. Therefore x = 210 / $0.25^2$ = **3360**.

Comment: We expect 210 claims, and $Z = \sqrt{\frac{210}{3360}}$ = 0.25.

Then the new estimate of the loss costs is: ($200)(0.25) + ($100)(1 - 0.25) = $125.

**6.29. B.** $z_{1-\alpha/2}$ = 2.054, since $\Phi(2.054)$ = 0.98.

The standard for full credibility is: $\left(\frac{z_{1-\alpha/2}}{k}\right)^2$ $(1+CV^2) = (2.054/0.10)^2\ (1+0.6^2)$ = 574 claims.

Thus we assign credibility of $Z = \sqrt{\frac{213}{574}}$ = **60.9**%.

**6.30. D.** $CV^2$ = variance / $\text{mean}^2$ = 17,640,000 / $700^2$ = 36. k = 0.05, while $\alpha$ (and $z_{1-\alpha/2}$) are to be solved for. The credibility being applied to the observation is:
Z = (change in estimate) / (observation - prior estimate) = (550-500) / (700-500) = 0.25.
We expect: (2)(1250) = 2500 claims. Thus since 2500 claims are given 0.25 credibility, the full credibility standard is: 2500/$0.25^2$ = 40,000 claims. However, that should equal

$\left(\frac{z_{1-\alpha/2}}{k}\right)^2$ $(1+CV^2) = (z_{1-\alpha/2}{}^2/0.05^2)\ (1 + 36)$. Thus: $z_{1-\alpha/2} = (0.05)\sqrt{\frac{40{,}000}{37}}$ = 1.644.

1 - $\alpha$ = P = $2\Phi(1.644)$ - 1 = (2)(0.9499) - 1 = **0.90**.

**6.31. D.** k = 0.05 and $\alpha$ = 0.10. $z_{1-\alpha/2}$ = 1.645 , since $\Phi(1.645)$ = 0.95.

$\lambda_F = \left(\frac{z_{1-\alpha/2}}{k}\right)^2$ = $(1.645/0.05)^2$ = 1082. The mean of the severity distribution is 100,000.

The second moment of the severity is the integral of $x^2$/200,000 from 0 to 200,000, which is $200{,}000^2/3$.
Thus the variance is 3,333,333,333. The square of the coefficient of variation is variance / $\text{mean}^2$ = 3,333,333,333 / $100{,}000^2$ = 0.3333. $\lambda_F(1+CV^2)$ = (1082)(1.333) = 1443.

For 1082 claims, $Z = \sqrt{\frac{1082}{1443}} = \sqrt{\frac{3}{4}}$ = **0.866**.

Comment: For the uniform distribution on [a,b], the CV = $\frac{b-a}{(b+a)\sqrt{3}}$. For a = 0, $CV^2$ = 1/3.

**6.32. D.** k = 6%. $\Phi(1.645) = 0.95$, so that $z_{1-\alpha/2} = 1.645$.

Standard for full credibility for frequency = $\lambda_F = \left(\frac{z_{1-\alpha/2}}{k}\right)^2 = (1.645 / 0.06)^2 = 756$.

Coefficient of Variation of the severity = 7500 /1500 = 5.
Standard for full credibility for pure premium = $\lambda_F$ $(1 + CV^2) = 756(1+5^2) = 19,656$ claims.

$Z = \sqrt{\frac{6000}{19,656}} = 0.552$. The prior estimate is given as $16.5 million.

The observation is given as $15.6 million. Thus the new estimate is:
(0.552)(15.6) + (1 - 0.552)(16.5) = **$16.00 million**.

**6.33. D.** $Z = \sqrt{\frac{10,000}{17,500}} = 75.6\%$.

Thus the new estimate = (25 million)(0.756) + (20 million)(1 - 0.756) = **$23.78 million**.

**6.34. B.** $Z = \sqrt{\frac{N}{F}}$. Thus, $R = O \sqrt{\frac{N}{F}} + P\{1 - \sqrt{\frac{N}{F}}\}$.

Solving for N, $\mathbf{N = \frac{F\,(R - P)^2}{(O - P)^2}}$.

<u>Comment</u>: Writing the revised estimate as R = P + Z(O-P) can be useful in general and allows a slightly quicker solution of the problem. This can also be written as
Z = (R - P) / (O - P); i.e., the credibility is the ratio of the revision of the estimate from the prior estimate to the deviation of the observation from the prior estimate.

**6.35. B.** $0.6 = Z = \sqrt{\frac{n}{2000}}$. Therefore, n = $(0.6^2)(2000)$ = **720** claims.

**6.36. D.** We are given k = 5% and $\alpha$ = 1 - 90% = 10%,
therefore we have $z_{1-\alpha/2} = 1.645$ since $\Phi(1.645) = 0.95$.

Therefore, $\lambda_F = \left(\frac{z_{1-\alpha/2}}{k}\right)^2 = (1.645/.05)^2 = 1082$.

The partial credibility is given by the square root rule: $Z = \sqrt{\frac{500}{1082}}$ = **0.68.**

**6.37. A.** $\alpha$ = 0.05 and k = 0.1. $\Phi(1.960) = 0.975$, so that $z_{1-\alpha/2} = 1.960$.

$\lambda_F = \left(\frac{z_{1-\alpha/2}}{k}\right)^2 = (1.960 / 0.10)^2 = 384$. $Z = \sqrt{\frac{n}{384}} = 0.4$. Thus n = $(384)(0.4^2)$ = **61.4**.

**6.38. B.** $\sqrt{\frac{n}{1000}}$ = 0.5. Thus n = (1000) $(0.5)^2$ = **250**.

**6.39. B.** For N observations, Classical Credibility = $\sqrt{\frac{N}{1600}} = \frac{\sqrt{N}}{40}$, for N ≤ 1600.

For N observations, Greatest Accuracy/ Buhlmann Credibility = N/(N + K) = N / (N + 391).

We want: $\frac{N}{N + 391} > \frac{\sqrt{N}}{40}$. ⇒ N - 40 $\sqrt{N}$ + 391 > 0.

N - 40 $\sqrt{N}$ + 391 = 0. ⇒ $\sqrt{N} = \frac{40 \pm \sqrt{40^2 - (4)(1)(391)}}{2}$ = 17 or 23.

For N between $17^2$ = 289 and $23^2$ = **529**, the Buhlmann Credibility is greater than the Classical Credibility.

<u>Comment</u>: The two formulas for K = 391 and $\lambda_F$ = 1600 produce very similar credibilities.

| N | 0 | 100 | 200 | 300 | 400 | 500 | 529 | 600 | 1000 |
|---|---|---|---|---|---|---|---|---|---|
| Classical Cred. | 0.0% | 25.0% | 35.4% | 43.3% | 50.0% | 55.9% | 57.5% | 61.2% | 79.1% |
| Buhlmann Cred. | 0.0% | 20.4% | 33.8% | 43.4% | 50.6% | 56.1% | 57.5% | 60.5% | 71.9% |

*See "An Actuarial Note on Credibility Parameters," by Howard Mahler, PCAS 1986.*

**6.40. A.** P = 0.95 and $\alpha$ = 0.05. $\Phi(1.960)$ = 0.975, so that $z_{1-\alpha/2}$ = 1.960.

$\lambda_F = \left(\frac{z_{1-\alpha/2}}{k}\right)^2$ = $(1.960 / 0.05)^2$ = 1537. Standard for full credibility for pure premium =

$\lambda_F$ (1 + $CV^2$) = 1537(1 + $3^2$) = 15,370 claims. Z = $\sqrt{\frac{1000}{15,370}}$ = 25.5%.

The prior estimate is given as \$5 million. The observation is given as \$6.75 million.
Thus the new estimate is: (25.5%)(6.75) + (1 - 25.5%)(5) = **\$5.45 million**.

**6.41. C.** k = 0.05. $\alpha$ = 10%. $z_{1-\alpha/2}$ = 1.645. $\lambda_F = (1.645/0.05)^2$ = 1082 claims.

For the Negative Binomial, $\mu_f$ = (2)(0.2) = 0.4. $\sigma_f^2$ = (2)(0.2)(1.2). $\sigma_f^2/\mu_f$ = 1.2.

For the Pareto, E[X] = 1000 / (3-1) = 500. $E[X^2] = \frac{(2)\,(1000^2)}{(3-1)\,(3-2)}$ = 1,000,000.

$CV^2 = E[X^2]/E[X]^2 - 1$ = 1,000,000/$500^2$ - 1 = 4 - 1 = 3.
Standard for Full Credibility = $(\sigma_f^2/\mu_f + CV_{Sev}^2)\,\lambda_F$ = (1.2 + 3)(1082) = 4546 claims.

2500 exposures. ⇔ (2500)(0.4) = 1000 expected claims. Z = $\sqrt{\frac{1000}{4546}}$ = **47%**.

**6.42. E.** The estimate using classical credibility is: $Z\ X_{partial} + (1-Z)\mu$.
We want this estimate to be within ±kμ of μ, with probability P ⇔
$P = Pr[\mu - k\mu \le Z\ X_{partial} + (1-Z)\mu \le \mu + k\mu]$.
Comment: $P = Pr[- k\mu \le Z\ X_{partial} - Z\mu \le k\mu]$ ⇔
$P = Pr[Z\mu - k\mu \le Z\ X_{partial} \le Z\mu + k\mu]$ ⇔
$P = Pr[(1-Z)\mu + Z\mu - k\mu \le (1-Z)\mu + Z\ X_{partial} \le (1-Z)\mu + Z\mu + k\mu]$ ⇔
$P = Pr[\mu - k\mu \le Z\ X_{partial} + (1-Z)\mu \le \mu + k\mu]$.

**6.43. A.** $\Phi(2.326) = 0.99$. ⇒ $z_{1-\alpha/2} = 2.326$. ⇒ $\lambda_F = (2.326/0.05)^2 = 2164$ claims.
Standard for full credibility for Severity is: $(2164)\ (6010/55^2) = 4299$ claims.
There are 1384 claims. $Z_x = \sqrt{\frac{1384}{4299}} = 56.7\%$.
Standard for full credibility for Pure Premium is:
$(2164)(1 + 6010/55^2)/0.085 = 76{,}040$ exposures.
$Z_p = \sqrt{\frac{21{,}000}{76{,}040}} = 52.6\%$. $|56.7\% - 52.6\%| =$ **4.1%.**
Alternately, standard for full credibility for Pure Premium is:
$(2164)(1 + 6010/55^2) = 6463$ claims.
The expected number of claims is: $(0.085)(21{,}000) = 1785$. $Z_p = \sqrt{\frac{1785}{6463}} = 52.6\%$.
$|56.7\% - 52.6\%| =$ **4.1%.**
Comment: In the case of severity, the number of claims is in the denominator.
So the observed number of claims measures the amount data, and is used to determine the partial credibility.
In the case of pure premium, the number of exposures is in the denominator.
So the observed number of exposures measures the amount data, and is used to determine the partial credibility. This is equivalent to using the expected number of claims together with a standard for full credibility in terms of number of claims.

**6.44. A.** $\alpha = 1\%$. $\Phi[2.576] = 0.995$. ⇒ $z_{1-\alpha/2} = 2.576$. $k = 5\%$.
⇒ $\lambda_F = (2.576/0.05)^2 = 2654$ claims.
Standard for Full Credibility for aggregate loss is:
$(1 + CV_S^2)\ \lambda_F = (1 + 14{,}400/100^2)\ (2654) = 6476$ claims.
Expected number of claims is: $(0.12)(10{,}000) = 1200$. $Z = \sqrt{\frac{1200}{6476}} =$ **0.430**.
Alternately, the full credibility standard in terms of exposures is: $6476/0.12 = 53{,}967$.
$Z = \sqrt{\frac{10{,}000}{53{,}967}} =$ **0.430**.

## Section 7, Important Formulas and Ideas

**The estimate using credibility =**
**ZX + (1-Z)Y, where Z is the credibility assigned to the observation X.**

**new estimate = (observation) (Z) + (old estimate) (1-Z)**
**= (observation) (Z) + (manual rate) (1-Z)**.

Full Credibility (Sections 2, 3, and 5):

**Assume one desires that the chance of being within ± k of the mean frequency to be at least 1 - α, then for a Poisson Frequency, the Standard for Full Credibility is:**

$\lambda_F = \left(\frac{z_{1-\alpha/2}}{k}\right)^2$, **where** $z_{1-\alpha/2}$ **is such that** $\Phi(z_{1-\alpha/2}) = 1 - \alpha/2$..

The Standard for Full Credibility for Frequency is in terms of claims: $\frac{\sigma_f^2}{\mu_f}\ \lambda_F$.

In the Poisson case this is: $\lambda_F$.

The Standard for Full Credibility for Severity is in terms of claims: $CV_{Sev}^2\ \lambda_F$.

**The Standard for Full Credibility for either Pure Premiums or Aggregate Losses is in terms of claims:** $\left(\frac{\sigma_f^2}{\mu_f} + CV_{Sev}^2\right) \lambda_F$ . In the Poisson case this is: $(1 + CV_{Sev}^2)\ \lambda_F$.

$1 + CV^2 = \frac{E[X^2]}{E[X]^2}$.

The standard can be put in terms of exposures rather than claims by dividing by $\mu_f$.

Variance of Pure Premiums and Aggregate Losses (Section 4):

Aggregate Losses =

$$(\text{\# of Exposures}) \frac{\text{\# of Claims}}{\text{\# of Exposures}} \frac{\text{\$ of Loss}}{\text{\# of Claims}} = (\text{Exposures})\ (\text{Frequency})\ (\text{Severity}).$$

$$\text{Pure Premiums} = \frac{\text{\$ of Loss}}{\text{\# of Exposures}} = \frac{\text{\# of Claims}}{\text{\# of Exposures}} \frac{\text{\$ of Loss}}{\text{\# of Claims}} = (\text{Frequency})(\text{Severity}).$$

When frequency and severity are independent: $\sigma_{PP}^2 = \mu_{Freq}\ \sigma_{Sev}^2 + \mu_{Sev}^2\ \sigma_{Freq}^2$.

$\sigma_{Agg}^2 = \mu_{Freq}\ \sigma_{Sev}^2 + \mu_{Ssev}^2\ \sigma_{Freq}^2$.

With a Poisson Frequency, the variance of aggregate losses is:

$\lambda$ (2nd moment of severity).

Partial Credibility (Section 6):

**When one has at least the number of claims needed for Full Credibility, then one assigns 100% credibility to the observations.**

**Otherwise use the square root rule:**

$$\textbf{Z} = \sqrt{\frac{\text{number of claims}}{\text{standard for full credibility in terms of claims}}}\text{, or}$$

$$\textbf{Z} = \sqrt{\frac{\text{number of exposures}}{\text{standard for full credibility in terms of exposures}}}.$$

When available, one generally uses the number of exposures or the expected number of claims in the square root rule, rather than the observed number of claims.

Make sure that in the square root rule you divide comparable quantities; either divide claims by claims or divide exposures by exposures.