

1, p.29, solution to the first exercise:  $f_X(x) S_Y(x) = \lambda_X \exp[-\lambda_X x] \exp[-\lambda_Y x] = \lambda_X \exp[-(\lambda_X + \lambda_Y)x]$ .

1, p.61, solution 2.23: At the next stage, each of two unlinked cities could be connected to each of two linked cities, for a total of 4 possible links. However, we have already eliminated the minimum of the original 6. If we subtract this overall minimum from each of these 4 Exponentials, the remaining variables are Exponential with mean  $\theta$ .

$E[\text{Cost of second link} - \text{cost of first link}] = E[\text{minimum of 4 exponentials}] = \theta/4$ .

$\Rightarrow E[\text{Cost of second link}] = E[\text{cost of first link}] + \theta/4 = \theta/6 + \theta/4 = \theta/12$ .

At the final stage, a single unlinked city can be linked to any of the other 3 cities.

If we subtract the cost of the second link from each of these 3 Exponentials, the remaining variables are Exponential with mean  $\theta$ .

$E[\text{Cost of third link} - \text{cost of 2nd link}] = E[\text{minimum of 3 exponentials}] + E[\text{Cost of second link}] = \theta/3$ .

$\Rightarrow E[\text{Cost of third link}] = E[\text{cost of second link}] + \theta/3 = \theta/12 + \theta/3 = \theta/4$ .

Expected total cost:  $\theta(1/6 + 5/12 + 9/12) = \theta 16/12 = (60)(4/3) = \mathbf{80}$ .

Comment: See Exercise 5.17 in Introduction to Probability Models, by Ross.

When one chooses the minimum of a sample of i.i.d. Exponentials, if one subtracts the minimum from each of the remaining variables, then one gets i.i.d. Exponentials with the original mean. This is a special property of Exponentials.

1, p. 76, solution 2.76:  $P\left[\sum_{i=1}^{100} X_i > 57\right] \cong 1 - \Phi[(57 - \mathbf{50})/5] = 1 - \Phi[1.4] = 0.0808$ .

1, p. 265, sol. 9.13: Alternately,  $5\lambda$  follows a Gamma distribution with  $\alpha = \mathbf{4}$  and  $\theta = (5)(0.1) = 1/2$ .

1, p. 266, sol. 9.14:  $5\lambda$  follows a Gamma distribution with  $\alpha = \mathbf{4}$  and  $\theta = (5)(0.1) = 1/2$ .

1, p. 520, sol. 18.93:  $0.1 + 0.488 - (0.1)(0.488) = 0.5392$ .

1, p. 547, question 19.60:  $q_{35+k}^{Ac} = \mathbf{0.10} + 0.05k$ , for  $k = 0, 1, 2$

1, p. 715 , question 24.10: revise the matrix of transition probabilities

$$\begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{matrix} \begin{pmatrix} 0.6 & 0 & 0 & 0 & 0 & 0.4 \\ 0.1 & 0 & 0 & 0.8 & 0 & 0.1 \\ 0.1 & 0 & 0 & 0 & \mathbf{0.8} & 0.1 \\ 0.1 & 0 & \mathbf{0.8} & 0 & 0 & 0.1 \\ 0.1 & 0.8 & 0 & 0 & 0 & 0.1 \\ 0.3 & 0 & 0 & 0 & 0 & 0.7 \end{pmatrix}$$

2, p. 78, solution to the exercise at the bottom of the page:

Of 130 claims, there are 109 claims greater than 25,000, thus an estimate of  $S(25,000) = 109/130$ .

Of 130 claims, 3 claims are within 5000 of 25,000. Thus this interval covers a probability of  $3/130$ .

**Thus an estimate of  $f(25,000) = (3/130)/5000$ .**

Therefore,  $h(25,000) \cong \{(3/130)/5000\} / (109/130) = 0.0000055$ .

2, pages 78 and 541: For a given age  $x$ , the hazard rate is the density of the deaths, divided by the **probability of still being alive** at age  $x$ .

2, p.650, solution 34.11:

In 1996  $x$  becomes  $1.1x$ .  $z = 1.1x$ .  $x = z/1.1$ .  $\ln(x) = \ln(z/1.1) = \ln(z) - \ln(1.1)$ .

Thus in 1996 the distribution function is  $F(z) = \Gamma[\alpha; \lambda \ln(x)] = \Gamma[\alpha; \lambda \{\ln(z) - \ln(1.1)\}]$ .

This is not of the same form, so the answer is none of the above.

2. page 771 and 784, Q. 36.22: The prior density of  $\theta$  should be  $f(\theta) = e^{-\theta/10}/10$ .

The final numerical solution would then be correct.

$$EPV = \int (\text{Process Variance} | \theta) f(\theta) d\theta = \int_0^{\infty} (3\theta^2 / 4) e^{-\theta/10} / 10 d\theta = (3/4) \int_0^{\infty} \theta^2 e^{-\theta/10} / 10 d\theta =$$

$$(3/4) 10^2 \Gamma(3) = (3/4)(100)(2) = 150.$$

1st Moment of the Hypothetical Means is:

$$\int_0^{\infty} (\theta / 2) f(\theta) d\theta = (1/2) \int_0^{\infty} \theta e^{-\theta/10} / 10 d\theta = (1/2) 10 \Gamma(2) = (1/2)(10)(1) = 5.$$

2nd Moment of the Hypothetical Means is:

$$\int_0^{\infty} (\theta / 2)^2 f(\theta) d\theta = (1/4) \int_0^{\infty} \theta^2 e^{-\theta/10} / 10 d\theta = (1/4) (10^2) \Gamma(3) = (1/4)(100)(2) = 50.$$



4, page 416: The CAS syllabus has added that the formula for VIF at page 101 of An Introduction to Statistical Learning will not be tested. (I believe that this formula is incorrect.) “The definition of variance inflation factor (VIF), as referenced in Knowledge Statement C.2.v, varies across the syllabus readings. Candidates should familiarize themselves with the widely accepted VIF formula found on p.102 of James et al. and p. 101 of Dobson. The VIF formula on p. 101 of James et al. will not be used on the exam.”  
This formula had been used in MAS-1, 11/18, Q.34.

5, page 57: for Binomial,  $b(\theta) = \ln[q/(1-q)]$ .

5, page 364: The question labeled 15.17 should be instead be 15.19.

6, page 47, the denominator should be 5-1 rather than 4-1:

$$\frac{(0.1922-0.1822)^2 + (0.1788-0.1822)^2 + (0.0548-0.1822)^2 + (0.3192-0.1822)^2 + (0.1660-0.1822)^2}{5 - 1}$$

= 0.00884.

6, p. 65, solution 3.2: **B**. The bootstrap estimate of the standard error is:  $\sqrt{4.94} = 2.22$ .

6, p. 70, solution 3.20 : The set {25, 25, 25, 25, **25, 25**} has the smallest possible mean: 25. The set {93, 93, 93, 93, **93, 93**} has the largest possible mean: 93. Final answer okay.

6, p. 124, solution 6.5: The superscripts should be R rather than L.  $\hat{\beta}_1^R / \hat{\beta}_2^R = 1$ .

6, p.129:  $Z = \phi_1 X_1 + \phi_2 X_2 + \phi_3 X_3 + \phi_4 X_4$ .

6, page 139, the equation near the bottom of the page: x-bar should have a subscript j:

$$\tilde{x}_{ij} = \frac{x_{ij}}{\sqrt{\frac{1}{n} \sum_{i=1}^n (x_{ij} - \bar{x}_j)^2}}$$

6, p.152, sol. 7.4:  $Z_2 = \{(4)(-5) + (-2)(3) + (0)(-2) + (-6)(-7) + (-5)(\mathbf{8})\}/9 = -24/9$ .

$\hat{Y} = 5 + (7)(29/9) - (4)(24/9) = \mathbf{38.2}$ .

7, page 4: For a given age  $x$ ,  $h(x)$  is **approximately the number of the deaths at age  $x$** , divided by the number of people still alive.

7, page 6:  $\text{Prob}[t > 80 \mid t > 70] = S(80)/S(70)$ .

7, p. 35: The solutions to 2.17 and 2.18 are reversed.

7, p. 87, sol. 6.15, 2nd line of the end:  ${}_3p_{80}$ . The final answer is okay.

8, p.83, line 3: at time  $t$

8, p.100, line 2: at time  $t$

9, middle of page 95, the summation should go to  $t-2$  rather than  $t-1$ :  $x_t = \sum_{i=0}^{t-2} 0.6^i w_{t-i} + 0.6^{t-1} x_1$ .

More generally,  $x_t = \sum_{i=0}^{t-2} \alpha^i w_{t-i} + \alpha^{t-1} x_1 = \sum_{i=0}^{t-1} \alpha^i w_{t-i} + \alpha^t x_0$ .

9, p.200, sol. 13.4:  $(y_t - y_{t-1}) = -0.2 (y_{t-1} - y_{t-2}) + 0.4 (y_{t-2} - y_{t-3}) + w_t + 0.5 w_{t-1} - 0.1 w_{t-2}$ .  
Final answer is okay.

10, page 146, fourth paragraph:  $G(x) = 1 - e^{-x/(\alpha\theta)}$

10, page 147, below the graph: Let **0.63** and 0.54 be independent random numbers from  $[0, 1]$ .