Solutions to the
CAS Sample Questions for
CAS Exam MAS-1

The CAS has issued 4 Sample Questions for MAS-1.
http://www.casact.org/admissions/MAS-I_Sample_Questions.pdf

The solutions and comments are solely the responsibility of the author.

CAS Exam MAS-1

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1. An actuary is using the inversion method to simulate a gamma random variable with mean and variance both equal to 2. Two random draws from the uniform distribution (0,1) are independently made, and their values are 0.2 and 0.8.

Calculate the value of the simulated gamma random variable.

A. Less than 0.5

B. At least 0.5, but less than 1.0

C. At least 1.0, but less than 1.5

D. At least 1.5, but less than 2.0

E. At least 2.0

1. D. Solve for the parameters of the Gamma Distribution via method of moments:

$\alpha\theta = 2.$ $\alpha\theta^2 = 2.$ $\Rightarrow \alpha = 2. \Rightarrow \theta = 1.$

This Gamma with $\alpha = 2$ is the sum of two independent Exponential variables each with $\theta = 1$.

Setting $F(x) = u. \Rightarrow 1 - e^{-x} = u. \Rightarrow x = -\ln(1-u).$

First simulated Exponential is: -ln(1 - 0.2) = 0.223.

Second simulated Exponential is: -ln(1 - 0.8) = 1.609.

Thus the simulated value of the Gamma is: 0.223 + 1.609 = **1.832**.

Alternately, setting $S(x) = u. \Rightarrow e^{-x} = u. \Rightarrow x = -\ln(u).$

First simulated exponential is: -ln(0.2) = 1.609.

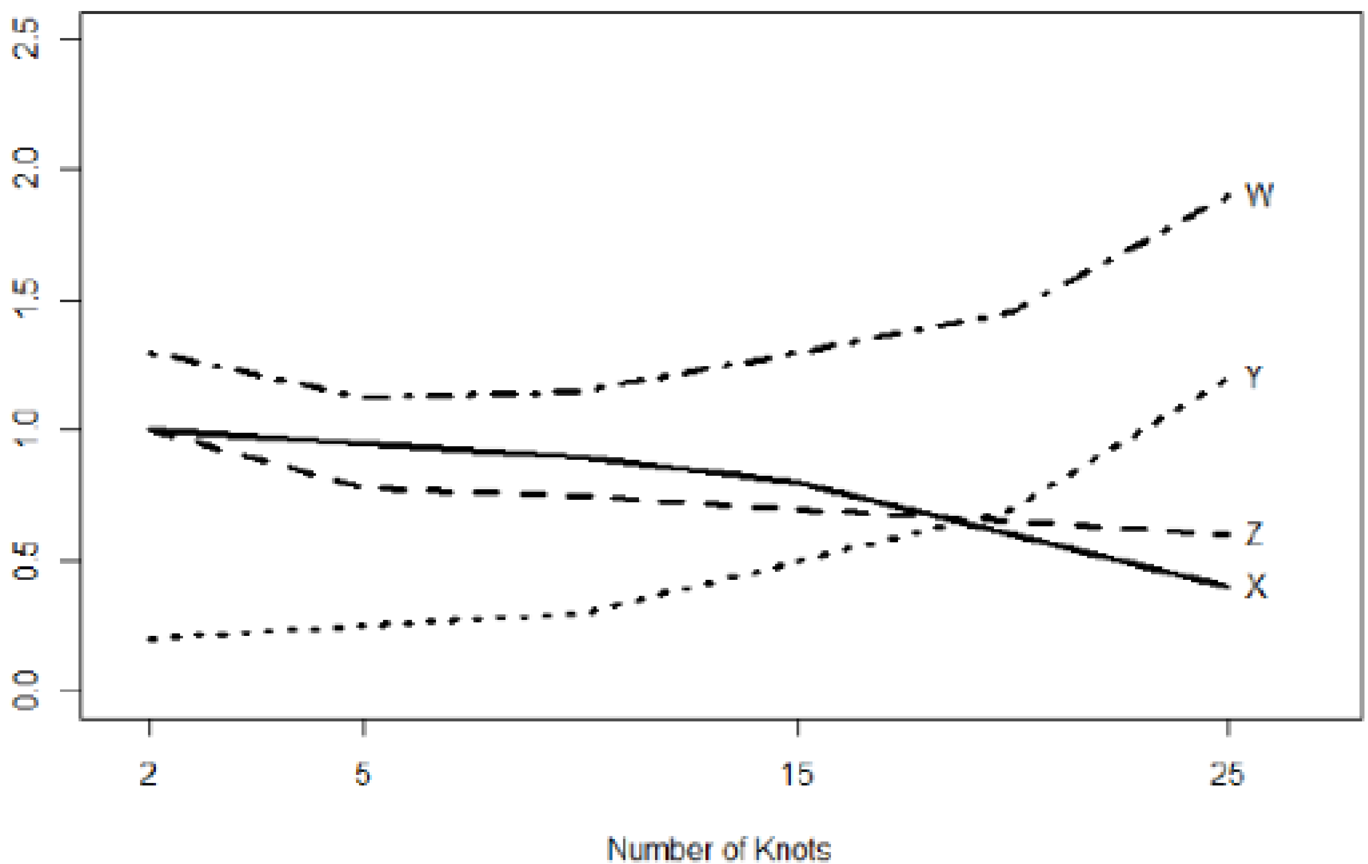
Second simulated exponential is: -ln(0.8) = 0.223.

Thus the simulated value of the Gamma is: 1.609 + 0.223 = **1.832**.

Comment: See Section 9 of "Mahler's Guide to Simulation".

Ross in Example 11.3 of Introduction to Probability Models, states that when simulating exponential random variables using either U or (1-U) as values is valid. However in this question the values are such that the answer is the same either way.

2. You want to fit a cubic spline to a large dataset and need to determine the number of knots to use. Below is a chart of four statistics from this model valued for various numbers of knots:



Determine which set of statistics below best describes each line.

A. W is Test MSE	X is Variance	Y is Squared Bias	Z is Train MSE
B. W is Variance	X is Squared Bias	Y is Test MSE	Z is Train MSE
C. W is Train MSE	X is Test MSE	Y is Variance	Z is Squared Bias
D. W is Test MSE	X is Train MSE	Y is Variance	Z is Squared Bias
E. W is Variance	X is Train MSE	Y is Test MSE	Z is Squared Bias

2. D. All else equal, a spline with more knots is more flexible, and will allow the model to better fit the training data.
Test MSE generally exhibits a U-shaped behavior as per Line W.
Also Test MSE is the sum of Variance + Bias Squared + Irreducible Error, and only Line W can be the sum of a constant plus two of the other values graphed.
Variance increases with flexibility as per Line Y.
Bias squared and Training MSE both decrease with flexibility as per Lines X and Z.
Only answer D meets these conditions.
Comment: See Figure 2.12, Figure 2.9, and Section 7.4 in
An Introduction to Statistical Learning with Applications in R by James, et. al.
See Sections 1 and 12 of "Mahler's Guide to Statistical Learning".
Trying to read values off of the graph:

Number of Knots	W	X	Y	Z	W - Y - Z
2	1.3	1.0	0.2	1.0	0.1
5	1.2	0.95	0.3	0.8	0.1
15	1.3	0.8	0.5	0.7	0.1
25	0.9	0.4	1.2	0.6	0.1

Test MSE - Variance - Bias Squared = Irreducible Error = Var[ε] = constant.
Thus since Test MSE = W and Variance = Y, we can infer that Bias Squared must be Z.
In contrast, W - Y - X is not a constant.

3. You are given the following statements about different resampling methods:
I. Leave-one-out cross-validation (LOOCV) is a special case of k-fold cross-validation
II. k-fold cross-validation has higher variance than LOOCV when $k < n$
III. LOOCV tends to overestimate the test error rate in comparison to validation set approach
Determine which of the above statements are correct.
A. I only
B. II only
C. III only
D. I, II, and III
E. The correct answer isn't given by (A), (B), (C), or (D)

3. A.
I. TRUE: LOOCV is just k-fold cross-validation where $k = n$;
if $k = n$, then we have divided the data into n folds each of size one.
II. FALSE: Because of the higher degree of correlation in the training data sets,
LOOCV has higher variance than k-fold validation, for $k < n$.
III. FALSE: Because of the smaller size of the training data set, the validation set approach tends to overestimate the test error rate more than LOOCV.
This is one of the drawbacks of the validation set approach.
Comment: See Section 5.1 in An Introduction to Statistical Learning with Applications in R by James, et. al.
See Section 2 of "Mahler's Guide to Statistical Learning".
LOOCV has a smaller tendency to overestimate the test error rate than does k-fold cross-validation with $k < n$.
k-fold cross-validation has a smaller tendency to overestimate the test error rate than does the validation set approach
LOOCV has smaller bias than k-fold cross-validation, since LOOCV uses almost all of the data to fit each of the models used to estimate the test MSE. However, k-fold cross validation has a smaller variance than LOOCV.

4. You are given the following models which contain regression splines:

Model	Numbers of Spline Knots	Degree of Regression Spline
A	6	4
B	5	5
C	8	2
D	10	3

Calculate the total number of the regression coefficients in the four models.

A. Less than 5

B. At least 15, but less than 25

C. At least 25, but less than 35

D. At least 35, but less than 45

E. At least 45

4. E. Number of coefficients

= (Degree of Regression Spline) + (Numbers of Spline Knots) + Intercept

= (Degree of Regression Spline) + K + 1.

Number of coefficients for Model A: 6 + 4 + 1 = 11.

Number of coefficients for Model B: 5 + 5 + 1 = 11.

Number of coefficients for Model C: 8 + 2 + 1 = 11.

Number of coefficients for Model D: 10 + 3 + 1 = 14.

$\Rightarrow$ Total number of coefficients: 11 + 11 + 11 + 14 = **47**.

Comment: See Section 12 of “Mahler’s Guide to Statistical Learning”.

See Section 7.4, particularly the top of page 273, in

An Introduction to Statistical Learning with Applications in R by James, et. al.

They mention that a cubic spline has a total of K + 4 degrees of freedom (coefficients).

They do mention linear splines.

They do not mention that with K knots a linear spline has K + 2 degrees of freedom.

They do not mention that more generally:

degrees of freedom = K + 1 + (degree of regression spline).

For a cubic spline, this is: K + 1 + 3 = K + 4.

James, et. al. do not mention using regression splines of degrees greater than 3, nor am I aware of actuaries using such regression splines of higher degree.