Mahler's Guide to
"Basics of Reinsurance Pricing," by David R. Clark

CAS Exam 9

prepared by
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Mahler's Guide to "Basics of Reinsurance Pricing," by Clark[1 2]

Information in bold or sections whose title is in bold are more important for passing the exam. Larger bold type indicates it is extremely important. Information presented in italics (including subsections whose titles are in italics) should rarely be needed to directly answer exam questions and should be skipped on first reading. It is provided to aid the reader's overall understanding of the subject, and to be useful in practical applications.

I have doubled underlined highly recommended questions to do on your first pass through the material, underlined recommended questions to do on your second pass, and starred additional questions to do on a third pass through the material.[3] No questions were labeled from the 2011 exam or later, in order to allow you to use them as practice exams.

Solutions to problems are at the end.[4]

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He has published dozens of major research papers and won the 1987 CAS Dorweiler prize.
He served 12 years on the CAS Examination Committee including three years as head of the whole committee (1990-1993).

Please send me any suspected errors by Email.

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[1] "Basics of Reinsurance Pricing," by David R. Clark, CAS Study Note, first version 1996, updated 2014.
Section 6 of the paper was added to the syllabus for Fall 2021.
[2] Prior to 2024, Clark's study note was on Exam 8.
The current material was part of my study guides for Exam 8.
Prior to 2011, Clark's study note was on Exam 6.
[3] Obviously feel free to do whatever questions you want. This is just a guide for those who find it helpful.
[4] Note that problems include both some written by me and some from past exams. The latter are copyright by the Casualty Actuarial Society and are reproduced here solely to aid students in studying for exams. The solutions and comments are solely the responsibility of the author; the CAS bears no responsibility for their accuracy. While some of the comments may seem critical of certain questions, this is intended solely to aid you in studying and in no way is intended as a criticism of the many volunteers who work extremely long and hard to produce quality exams. There are also some past exam questions copyright by the Society of Actuaries.

This CAS study note covers the basic pricing of reinsurance. Unfortunately it assumes you know a little about reinsurance.[5] Insurers can buy various types of insurance-like protection from a reinsurer for various different purposes.

In order to make rates for reinsurance, actuaries face many of the same challenges as making rates for primary insurance. However, there are additional challenges, for example there is often a much smaller volume of data to use.

“A major difference between reinsurance and primary insurance is that a reinsurance program is generally tailored more closely to the buyer; there is no such thing as the average reinsured or the average reinsurance price. Each contract must be individually priced to meet the particular needs and risk level of the reinsured.
This leads to what might be called the pricing paradox: If the reinsurer can precisely price a given contract, the ceding company will not want to buy it. That is to say, if the historical experience is stable enough to provide data to make a precise expected loss estimate, then the reinsured would be willing to retain that risk.”[6]

A reinsurance treaty may be on a “losses occurring” basis; the treaty covers losses that occur during the treaty period.[7] So for example, a treaty on a losses occurring basis that was effective from January 1, 2015 until December 31, 2015, would cover losses that occurred during 2015.

Instead, a reinsurance treaty may be on a “risks attaching” basis; the treaty on a risks attaching basis covers losses on policies written during the treaty period. So for example, a treaty that was effective from January 1, 2015 until December 31, 2015, would cover losses on policies written by the primary insurer during 2015.

Losses Occurring Basis ⇔ Accident Year.

Risks Attaching Basis ⇔ Policy Year.

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[5] When written, the study note by Clark was one of several readings on reinsurance on the same exam.
Currently, reinsurance is one of the assignments for the first CAS online course which is part of the Syllabus.
If you took this online course, it would not be a bad idea to briefly review your notes on reinsurance.
“Reinsurance,” by Gary S. Patrik, Chapter 7, of Foundations of Casualty Actuarial Science (Fourth Edition), Casualty Actuarial Society, 2001, also contains some background information at pages 343-357.
Following that, Patrik discusses pricing reinsurance.
http://www.casact.org/library/studynotes/ch7.pdf

[6] Quoted from page 2 of Clark.

[7] See page 4 of Clark.

Outline of Clark:

Introduction

1. Proportional Treaties
    - 1A. Basic Tools
    - 1B. Special Features of Proportional Treaties
        - a) Sliding Scale Commission
        - *b) Profit Commission*
        - c) Loss Corridors

2. Property Per Risk Excess Treaties
    - 2A. Experience and Exposure Rating Models
        - a) Experience Rating
        - b) Exposure Rating
    - 2B. Other Issues on Property Per Risk Treaties
        - a) Free Cover
        - b) Credibility
        - c) Inuring Reinsurance

3. Casualty Per Occurrence Excess Treaties
    - 3A. Experience and Exposure Rating Models
        - a) Experience Rating
        - b) Exposure Rating
    - 3B. Special Problems on Casualty Excess Treaties
        - a) Including Umbrella Policies
        - b) Loss Sensitive Features
        - *c) Workers Compensation Experience Rating*

4. Aggregate Distribution Models
    - *a) Empirical Distribution*
    - b) Single Distribution Model
    - c) Recursive Calculation of Aggregate Distribution
    - *d) Other Collective Risk Models*

5. Property Catastrophe Covers
    - 5A. Traditional Products and Methods
    - *5B. Alternative Risk Products*

6. Calculating the Final Price

Functions of Reinsurance:[8]

- Capital Relief:
  By reducing risk, reinsurance can reduce the capital the insurer is required to hold.
- Increasing Underwriting Capacity:
  An insurer can write risks that were too large to write on its own.
- Catastrophe Protection
- Stabilize Loss Experience
  Reinsurance can help stabilize the cedant's underwriting and financial results over time and help protect the cedant's surplus against shocks from large, unpredictable losses.
- Financial Results Management:
  Reinsurance can alter the timing of income, enhance statutory and/or GAAP surplus, and improve various financial ratios by which insurers are judged.
- Risk Concentration:
  Reinsurance can lessen the effects of the insurer writing a lot of risks of the same type or in the same geographical area.
- Technical Expertise:
  The reinsurer has more experience pricing higher limits, handling large claims, etc.
  The reinsurer may also aid an insurer in entering a new line of insurance or market.
  Many professional reinsurers have the knowledge and ability to provide an informal consulting service for their cedants. This service can include advice and assistance on underwriting, marketing, pricing, loss prevention, claims handling, reserving, actuarial, investment, and personnel issues.
- Withdrawal:
  An insurer withdrawing from a market may cede its run-off business to a reinsurer.

Higher Variability of Reinsurance Results:[9]

Among the challenges faced by the actuary pricing reinsurance is that underwriting results are usually significantly more variable than for primary insurance. Reasons include:

- Non-homogeneous risks; each reinsurance contract may have different terms.
- Lower frequency and higher severity.
- Longer time delay between occurrence and the reinsurer finding out about the incident.
- Longer average time between reporting and settlement.
- Greater impact of inflation over a retention, and thus greater uncertainty from inflationary trends.
- The longer tail results in more uncertainty in case reserves and IBNR.

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[8] Adapted from Section 5.5.1 of Introduction to Ratemaking and Loss Reserving for Property Casualty Insurance, by Robert L. Brown and W. Scott Lennox, not on the syllabus.
See also "Reinsurance" by Gary S. Patrik, in Foundations of Casualty Actuarial Science.
Different forms of reinsurance chiefly perform different subsets of these functions.

[9] Adapted from Section 5.5.4 of Introduction to Ratemaking and Loss Reserving for Property Casualty Insurance, by Robert L. Brown and W. Scott Lennox, not on the syllabus.

How Reinsurance Affects the Direct Insurer:[10]

The reinsurer adds value in many ways to the services a direct insurer provides to his clients.
- The reinsurer reduces the probability of the direct insurer's ruin by assuming his catastrophe risks.
- He stabilizes the direct insurer's balance sheet by taking on a part of his risk of random fluctuation, risk of change, and risk of error.
- He improves the balance of the direct insurer's portfolio by covering large sums insured and highly exposed risks.
- He enlarges the direct insurer's underwriting capacity by accepting a proportional share of the risks and by providing part of the necessary reserves.
- He increases the amount of capital effectively available to the direct insurer by freeing equity that was tied up to cover risks.
- He enhances the effectiveness of the direct insurer's operations by providing many kinds of services: for example, by:
    - compiling and presenting underwriting data from sources around the world;
    - assessing and evaluating special risks;
    - offering consultation in loss prevention;
    - providing loss adjustment support;
    - performing actuarial work;
    - training members of the cedant's staff; and
    - helping ceding companies to invest their capital, to recruit managerial staff, find cooperation partners, arrange mergers, etc.

Commutations:[11]

A commutation is a settlement agreement reached between a reinsured and a reinsurer by which the reinsurance obligation is terminated. A commutation agreement specifies how to value settle and discharge all obligations between parties to a reinsurance agreement.

The reinsurer agrees to pay funds at present value that are not yet due under the reinsurance agreement. A commutation allows the reinsured to receive cash now to invest for the payment of claims that will come due in the future.[12] The reinsurer's obligations for future payments are terminated and the reinsurance contract is terminated.

Pro-Rata or Proportional Reinsurance:

Under proportional reinsurance, both the primary insurer and the reinsurer proportionately share the amounts of insurance, the policy premiums, and the insured claims. Clark discusses pricing the two types of proportional reinsurance: quota share or surplus share. The reinsurer pays a ceding commission to the primary insurer to compensate the primary insurer for policy acquisition expenses that were incurred when the policy was sold.

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[10] Quoted from "An introduction to Reinsurance," by SwissRe.
[11] See for example, "Reinsurance Commutation," by Jim Klann, a CAS Study Note not on the syllabus of this exam.
[12] Actuaries are usually involved in helping to determine the amount of money that will be paid to commute a reinsurance agreement, See for example, "Commutation Pricing in the Post Tax-reform Era," by Vincent P. Connor and Richard Alan Olsen, PCAS 1991, not on the syllabus.

Quota Share Reinsurance:

Minime Insurance writes fire insurance. Minime could buy a 60% quota share treaty from Powers Reinsurance. Then Powers would pay 60% of every loss, while Minime paid 40% of every loss that Minime suffers. In addition, Powers would pay 60% of the allocated loss adjustment expenses.

In exchange, Powers would also get 60% of each premium dollar Minime collects, less a ceding commission. The amount of the ceding commission would be agreed upon in advance. In theory, the ceding commission should allow Minime to cover its expenses. For example, the ceding commission might be 30%.[13] Then Powers would cede back to Minime Insurance 30% of the premiums it gets; Powers would keep (70%)(60%) = 42% of the total premiums.

Exercise: During January, Minime Insurance collected $20 million in premium and paid $11 million in losses plus ALAE. What happens due to the quota share treaty?
[Solution Minime sends (60%)($20 million) = $12 million of premium to Powers; Powers in turn cedes back 30% of this or (30%)($12 million) = $3.6 million to Minime.
In other words, net of ceding commissions, Powers gets from Minime:
12 million - 3.6 million = $8.4 million = (42%)(20 million).
In addition, Powers pays Minime for losses and ALAE: (60%)($11 million) = $6.6 million.
In total, Minime pays to Powers: 8.4 million - 6.6 million = $1.8 million.
Comment: During a good month with a low loss ratio, on a net basis Minime pays money to Powers. During a bad month with a high loss ratio, on a net basis Powers pays money to Minime.]

Under a X% Quota Share Reinsurance Treaty, the primary insurer cedes X% of every loss, and also cedes X% of each dollar of premium less a ceding commission.
In theory, the ceding commission allows the primary insurer to cover its expenses.

Under a Quota Share treaty, the retained losses and ceded losses are each a constant percentage of the gross losses. Multiplying by a constant does not change the coefficient of variation. Thus the CV is the same for gross, retained, and ceded losses.

Pricing Quota Share Reinsurance:

The basic question for the reinsurer is whether given the expected experience of the primary insurer, the ceding commission, and the reinsurer’s expenses, the reinsurer would expect the treaty to be profitable. If it is expected to be sufficiently profitable, then the reinsurer would write the treaty.[14] If the treaty is not expected to be sufficiently profitable, then the reinsurer can decline to write the treaty or can try to negotiate a lower ceding commission.

Combined ratio = loss ratio + expense ratio.
Reinsurer’s technical ratio = loss ratio + ceding commission ratio.

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[13] The final ceding commission could be on a sliding scale and depend on the loss experience under the treaty.
[14] This is similar to the decision of a primary insurer whether or not to write a policy. There are other considerations. For example, reinsurers prefer to maintain longterm relationships with primary insurers, reinsurers have to deal with competitive market conditions, and reinsurers have a limited total capacity to write business.

Exercise: A primary insurer will have an expected loss and ALAE ratio of 60%.
The ceding commission will be 29%.
The reinsurer's expenses including brokerage fees are 7% of ceded premiums.
Determine the reinsurer's expected combined ratio.
Determine the reinsurer's expected technical ratio.
[Solution: Combined ratio = 60% + 29% + 7% = 96%.
Technical ratio = loss ratio + commission ratio = 60% + 29% = 89%.]

The reinsurer would have to decide whether a 96% combined ratio is sufficiently profitable based on the line(s) of insurance being reinsured, market conditions, etc.[15]

The difficult part is to estimate the primary insurer's expected loss ratio. This is similar to the main part of an overall rate indication for this primary insurer.[16]

The steps as listed by Clark are:[17]

1. Compile the historical experience on the treaty.[18]
   Clark uses five accident years of premiums and losses & ALAE from the primary insurer.

2. Exclude catastrophe and shock losses.[19]

3. Adjust experience to ultimate level and project to future period.

4. Select the expected non-catastrophe loss ratio for the treaty.
   If there enough credibility, use the ratio of losses to premiums from step 3.

5. Load the expected non-catastrophe loss ratio for catastrophes (and shock losses).[20]
   For property reinsurance, the most common approach is to use the output of a catastrophe computer model.[21] [22]

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[15] This is not something you would be asked to decide on your exam
[16] See the example at pages 6 to 9 of Clark. See also 6, 11/09, Q.29.
[17] See pages 4 to 9 of Clark for the steps and his example.
[18] If the treaty is on a losses occurring basis, calendar year earned premium and accident year losses & ALAE should be used. For risks attaching treaties, written premium and the losses & ALAE covered by those policies are used.
[19] "Catastrophe losses are due to a single event, such as a hurricane or earthquake, which may affect a large number of risks. Shock losses are any other losses, usually affecting a single policy, which may distort the overall results. For property contracts, catastrophes are generally defined on a per occurrence (multiple risk) basis, whereas shock losses are large losses due to a single risk. For casualty contracts, catastrophes may include certain types of claims impacting many insureds (e.g. environmental liability), whereas shock losses would represent a single large settlement on a single policy."
[20] On your exam, if you need it to answer a question, you should be given a load for catastrophe losses as an expected percent of non-catastrophe losses.
[21] For casualty proportional treaties, a loading may still be needed to reflect the potential for large losses not reflected in the historical experience.
[22] Discussed subsequently in Clark as well as Grossi and Kunreuther on the syllabus of this exam.

<u>Surplus Share Reinsurance</u>:

Another type of proportional reinsurance is Surplus Share.[23] Unlike Quota Share, the percent of each risk ceded to the reinsurer by the insurer depends on the policy limit for that risk. As with a quota share, the appropriate percent of loss and ALAE is ceded, while the appropriate percent of premiums less ceding commissions is ceded.

Minime Insurance writes fire insurance. Minime buys a surplus share treaty from Powers Reinsurance. Let us assume that the treaty has four-lines with a net line of $100,000.

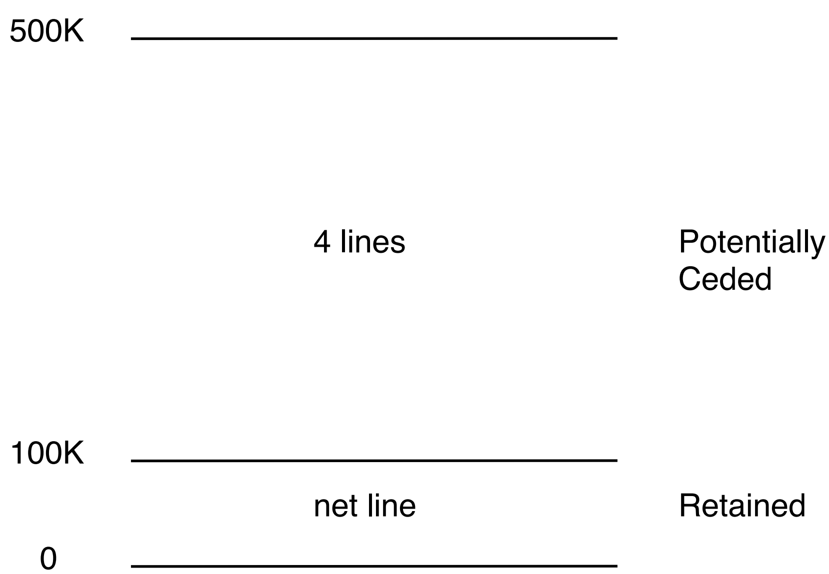
That means that Minime would retain 100% of a small risk, one with an insured value of $100,000 or less. Minime would cede a portion of any big risk.

For example, for a property with an insured value of $250,000, the percent ceded would be: ($250,000 - $100,000) / $250,000 = 60%.

Since the treaty has four lines, for a property with an insured value greater than: (1 + 4)($100,000) = $500,000, the percent ceded would be: $400,000 / Value.

Surplus Share Treaty

four-lines with a net line of $100,000



[23] Surplus share treaties are common for Homeowners Insurance and Commercial Property Insurance.

Exercise: Minime writes four properties:

Policy Number	Policy Limit	Loss & ALAE
1	$80,000	$50,000
2	$120,000	$30,000
3	$350,000	$140,000
4	$700,000	$280,000

Determine Minime's loss & ALAE net of reinsurance.
[Solution:

Policy Number	Policy Limit	Percent Ceded	Net Loss & ALAE
1	$80,000	0	(100%)(50,000) = 50,000
2	$120,000	20/120	(5/6)(30,000) = 25,000
3	$350,000	250/350	(2/7)(140,000) = 40,000
4	$700,000	400/700	(3/7)(280,000) = 120,000

Loss & ALAE net of reinsurance is: 50,000 + 25,000 + 40,000 + 120,000 = $235,000.
Comment: Minime would have written many more than 4 properties covered by this treaty, most of which would have had no loss.
Having bought this surplus treaty with 4 lines, Minime probably was expecting to write very few if any properties with a value more than $500,000, and none with a value significantly more than $500,000.]

Exercise: Minime writes four properties:

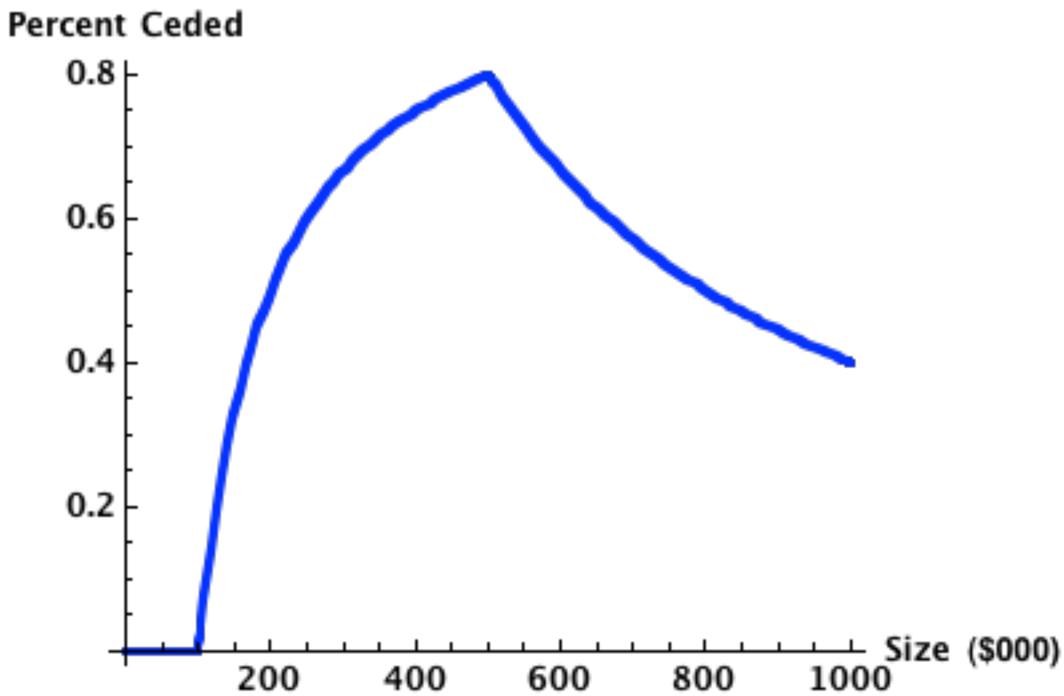
Policy Number	Policy Limit	Premium
1	$80,000	$900
2	$120,000	$1200
3	$350,000	$2800
4	$700,000	$5600

There is a 30% ceding commission. Determine the premium ceded net of ceding commissions.
[Solution:

Policy Number	Policy Limit	Percent Ceded	Ceded Premium
1	$80,000	0	(0)(900) = 0
2	$120,000	20/120	(1/6)(1200) = 200
3	$350,000	250/350	(5/7)(2800) = 2000
4	$700,000	400/700	(4/7)(5600) = 3200

Premium ceded net of ceding commission is: (70%) (0 + 200 + 2000 + 3200) = 3780.
Comment: The insurer gets to keep premiums of: 10,500 - 3780 = $6720]

For this example, for this surplus share treaty with a retained line of $100,000 and a maximum of 4 ceded lines, here is graph of the percent ceded as function of the insured property limit:



For this example of a surplus share treaty, the amount ceded is:

$$\begin{cases} 0, \text{ for value} < 100{,}000 \\ (\text{value} - 100{,}000) / \text{value}, \text{ for } 100{,}000 \leq \text{value} \leq 500{,}000 \\ 400{,}000 / \text{value}, \text{ for value} > 500{,}000 \end{cases}$$

Pricing a surplus share treaty is basically the same as pricing a quota share treaty.[24]

<u>Distributions of Loss Ratios</u>:

As will be discussed, in order to price features like Sliding Scale Commissions, Profit Commissions, and Loss Corridors, one will need a distribution of the loss ratios.

For quota share, the same percent is ceded of each size property, and thus the distribution of loss ratios for the ceded business is that of the primary insurer for the book of business being reinsured on a direct basis, in other words prior to reinsurance.

However, for a surplus share treaty, none of the small properties are ceded, and varying percentages of the larger properties are ceded. Thus if the distribution of loss ratios differs by size of of property, then the loss ratio distribution of the ceded business may differ from that of the direct business.[25]

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[24] Provided that the reinsurer believes that the primary insurer's expected loss ratio does not vary by the size of property insured, so that it would not matter that the insurer is ceding different percentages of different sized risks.
[25] There is usually no reason to assume that the expected loss ratio differs by size of property, but the variance of the distribution of loss ratios could differ by size of property.

Sliding Scale Commission:[26]

A sliding scale commission is a percent of premium paid by the reinsurer to the ceding company which "slides" with the actual loss experience, subject to set minimum and maximum amounts. The reinsurer pays a preliminary ceding commission. Then if the experience under the treaty is good, the ceding commission is increased, while if the experience under the treaty is bad, the ceding commission is reduced.[27] [28]

For example, assume the following for a proportional reinsurance treaty:[29]
- Provisional commission of 28%.
- Minimum commission of 20% at a 70% loss ratio.
- Sliding 1:1 to 40% at a 50% loss ratio.
- Sliding 0.5:1 to a maximum of 50% at a 30% loss ratio.

If the loss ratio turns out to be 75%, then the final ceding commission is 20%.
If the loss ratio turns out to be 60%, then the final ceding commission is 30%.
If the loss ratio turns out to be 40%, then the final ceding commission is 45%.
If the loss ratio turns out to be 20%, then the final ceding commission is 50%.

To calculate the expected ceding commission, we need a distribution of the loss ratios for this book of business, based on historical data for this treaty or a model of aggregate losses.
For example:

Loss Ratio Range	Average in Range	Probability in Range
$x \leq 30\%$	24%	0.20
$30\% < x \leq 50\%$	42%	0.20
$50\% < x \leq 70\%$	58%	0.30
$x > 70\%$	92%	0.30

Exercise: Calculate the expected ceding commission for this book of business.
[Solution: Commission at a 42% loss ratio is: 40% + 8%/2 = 44%.
Commission at a 58% loss ratio is: 40% - 8%/1 = 32%.
Average commission is: (20%)(50%) + (20%)(44%) + (30%)(32%) + (30%)(20%) = 34.4%.
Comment: Initially a 28% ceding commission would be paid by the reinsurer as a percent of the ceded premiums; however this would be adjusted once the experience on the proportional treaty is known, as per the schedule of sliding scale commissions. If the insurer's experience was good, then the reinsurer would owe additional ceding commissions, while if the experience was bad, then the insurer would have to return some of the ceding commissions.]

[26] See pages 9 to 12 of Clark. See also for example 6, 11/08, Q.38.
[27] This is conceptually like retrospective rating.
[28] With a sliding scale, rarely does it turn out that the average commission equals either the preliminary commission or the commission that corresponds to the average loss ratio.
[29] Note that on each of two intervals, the commission is a linear function of the loss ratio.

<u>Carryforward Provision</u>:

The volatility of annual results of using a sliding scale of commissions depends on the distribution of annual loss ratios as well as the details of the commission structure: the maximum commission, the minimum commission, and the slopes. One way to smooth out the volatility of results is via a carryforward provision.

A carryforward provision allows that if the past loss ratios have been above the loss ratio corresponding to the minimum commission, then the excess loss amount can be included with the current year's loss in the estimate of the current year's commission.[30] In this example, if the loss ratio in 2015 was 80%, then the minimum commission of 20% would still be paid. For 2016 one could carryforward: 80% - 70% = 10% loss ratio. Thus if for example the loss ratio in 2016 were 35%, then the ceding commission would instead be based on a loss ratio of: 35% + 10% = 45%.

Clark discusses two ways of estimating the expected effect of carryforward provisions on sliding scale ceding commissions; each method has shortcomings. In the first method, one includes any carryforward from past years and estimates the impact on the current year only. This amounts to shifting the slide by the amount of the carryforward.

Exercise: Assume a 10% carryforward from previous years.

Loss Ratio Range	Average in Range	Probability in Range
x ≤ 20%	18%	0.05
20% < x ≤ 40%	34%	0.25
40% < x ≤ 60%	52%	0.30
x > 60%	83%	0.40

Calculate the expected ceding commission for this book of business.
[Solution: Commission at a 34% loss ratio in 2016 is the same as that at a 44% loss ratio on the schedule: 40% + 6%/2 = 43%.
Commission at a 52% loss ratio in 2016 is the same as that at a 62% loss ratio on the schedule: 40% - 12%/1 = 28%.
Average commission is: (5%)(50%) + (25%)(43%) + (30%)(28%) + (40%)(20%) = 29.65%.
<u>Comment</u>: The average commission is smaller than without the 10% carryforward.]

The problem with this approach is that it ignores the potential for carryforward beyond the current year. For example, in the first year of the program we would calculate the expected commission for the current year as though the program would be cancelled at the end of the year.

The second approach is to look at the "long run" of the contract. The sliding scale is modeled as applying to a longer block of years rather than just the single current year. The variance of the aggregate distribution would be reduced on the assumption that individual bad years would be smoothed by good experience on other years. The variance of the average loss ratio for a block of years should be significantly less that the variance of the loss ratio for a single year.

[30] Carryforward provisions could be from several past years. A carryforward provision may also allow similar consideration of years in which the loss ratio was below that corresponding to the maximum commission.

The first problem with this approach is that the method for reducing the variance is not obvious. A second problem is that it ignores the fact that the contract may not renew the following year, potentially leaving the reinsurer with no carryforward benefit.

Previously, we assumed a distribution of loss ratios for a single year:

Loss Ratio Range	Average in Range	Probability in Range
x ≤ 30%	24%	0.20
30% < x ≤ 50%	42%	0.20
50% < x ≤ 70%	58%	0.30
x > 70%	92%	0.30

The average loss ratio is: (0.2)(24%) + (0.2)(42%) + (0.3)(58%) + (0.3)(92%) = 58.2%.
We could judgmentally come up with a distribution of the average loss ratios over several years that had the same mean, but a smaller variance. For example:

Loss Ratio Range	Average in Range	Probability in Range
x ≤ 30%	27%	0.10
30% < x ≤ 50%	45%	0.30
50% < x ≤ 70%	61%	0.40
x > 70%	88%	0.20

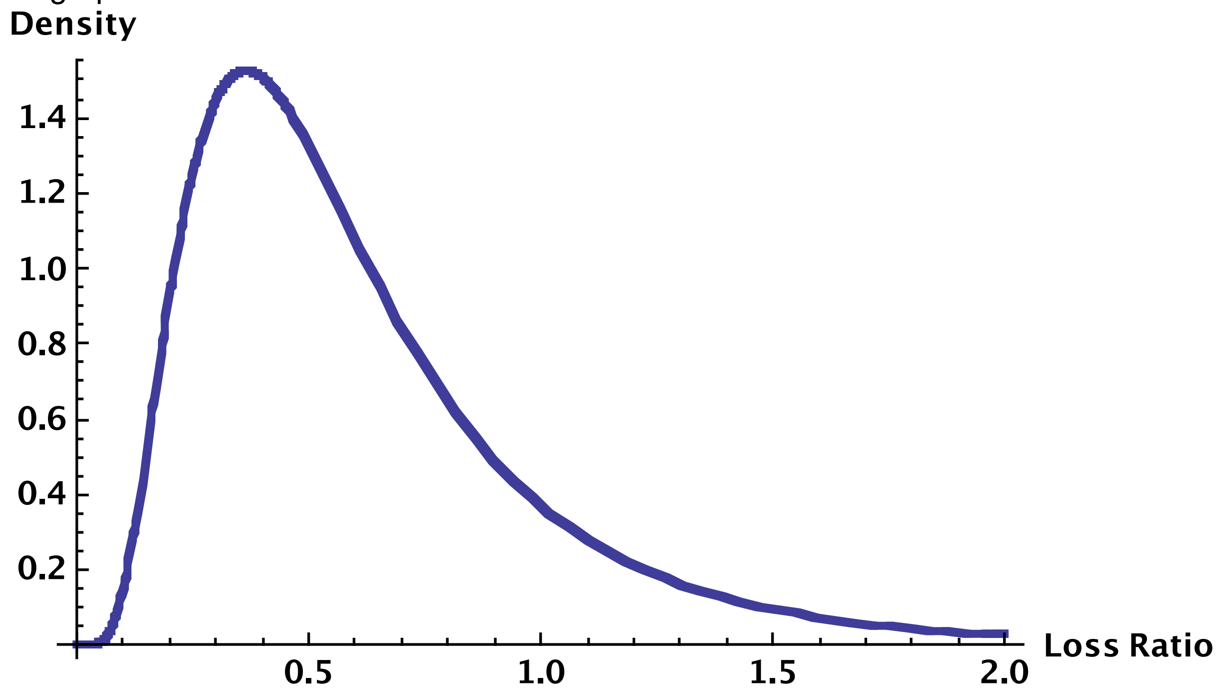
Exercise: Using the above distribution of loss ratios determine the average commission.
The previous sliding scale:
- Minimum commission of 20% at a 70% loss ratio.
- Sliding 1:1 to 40% at a 50% loss ratio.
- Sliding 0.5:1 to a maximum of 50% at a 30% loss ratio.

[Solution: Commission at a 45% loss ratio is: 40% + 5%/2 = 42.5%.
Commission at a 61% loss ratio is: 40% - 11%/1 = 29%.
Average commission is: (10%)(50%) + (30%)(42.5%) + (40%)(29%) + (20%)(20%) = 33.35%.]

A LogNormal Model of Loss Ratios:[31]

For example, let us assume that for a single year the distribution of loss ratios for a reinsured book of business is LogNormal with μ = -0.65 and σ = 0.6.
Then the mean is: $\exp[-0.65 + 0.6^2/2] = 62.5\%$.
The second moment is: $\exp[(2)(-0.65) + (2)(0.6^2))] = 0.560$.
The standard deviation is: $\sqrt{0.560 - 0.625^2} = 41.2\%$.

Here is a graph of this distribution of loss ratios:



The previous sliding scale:
- Minimum commission of 20% at a 70% loss ratio.
- Sliding 1:1 to 40% at a 50% loss ratio.
- Sliding 0.5:1 to a maximum of 50% at a 30% loss ratio.

The commission for a loss ratio x between 30% and 50% is: 50% - (x - 30%)/2 = 0.65 - x/2.
The commission for a loss ratio x between 50% and 70% is: 40% - (x - 50%) = 0.9 - x.
Then the average commission would be:

$$(50\%)\ F(0.3) + \int_{0.3}^{0.5} (0.65 - x/2)\ f(x)\ dx + \int_{0.5}^{0.7} (0.9 - x)\ f(x)\ dx + (20\%)\ S(0.7).$$

[31] Although Clark mentions at page 37 using a LogNormal Distribution to model loss ratios, he does not give a numerical example. Subsequently, I have a summary of the LogNormal Distribution.
See "Loss Distribution for Actuaries" by Bahnemann.

Using a computer, for the LogNormal Distribution with μ = -0.65 and σ = 0.6:

F(0.3) = 0.1779, $\int_{0.3}^{0.5} (0.65 - x/2)\, f(x)\, dx = 0.1323$, $\int_{0.5}^{0.7} (0.9 - x)\, f(x)\, dx = 0.0665$,

and S(0.7) = 0.3125.

Thus the average commission is:

$$(50\%)\ F(0.3) + \int_{0.3}^{0.5} (0.65 - x/2)\, f(x)\, dx + \int_{0.5}^{0.7} (0.9 - x)\, f(x)\, dx + (20\%)\ S(0.7) =$$

(50%)(0.1779) + 0.1323 + 0.0665 + (20%)(0.3125) = 35.0%.

Equivalently, we can compute that for this LogNormal Distribution with μ = -0.65 and σ = 0.6:[32]

Loss Ratio Range	Average in Range	Probability in Range
x ≤ 30%	22.43%	17.79%
30% < x ≤ 50%	39.83%	29.34%
50% < x ≤ 70%	59.25%	21.62%
x > 70%	108.86%	31.25%

Commission at a 39.83% loss ratio is: 40% + 10.17%/2 = 45.085%.
Commission at a 59.25% loss ratio is: 40% - 9.25%/1 = 30.75%.
Thus the average commission is:
(17.79%)(50%) + (29.34%)(45.085%) + (21.62%)(30.75%) + (20%)(31.25%) = 35.0%.

Now let us assume we average the loss ratios over several years rather than just a single year. Then the distribution would have the same mean but a smaller variance.
For example, a LogNormal Distribution with μ = -0.521 and σ = 0.32 would have the same mean but half the standard deviation of the previous LogNormal. We could use this LogNormal Distribution with the smaller variance to try to approximate the effect of a carryforward provision.

Using a computer, for the LogNormal Distribution with μ = -0.521 and σ = 0.32:

F(0.3) = 0.0164, $\int_{0.3}^{0.5} (0.65 - x/2)\, f(x)\, dx = 0.1224$, $\int_{0.5}^{0.7} (0.9 - x)\, f(x)\, dx = 0.1225$,

and S(0.7) = 0.3038.

[32] While this calculation can be done using formulas, you should not be asked to do this on your exam.

Thus the average commission is:

$$(50\%)\ F(0.3) + \int_{0.3}^{0.5} (0.65 - x/2)\ f(x)\ dx + \int_{0.5}^{0.7} (0.9 - x)\ f(x)\ dx + (20\%)\ S(0.7) =$$

(50%)(0.0164) + 0.1224 + 0.1225 + (20%)(0.3038) = 31.4%.

Equivalently, we can compute that for this LogNormal Distribution with $\mu = -0.521$ and $\sigma = 0.32$:

Loss Ratio Range	Average in Range	Probability in Range
x ≤ 30%	26.89%	1.64%
30% < x ≤ 50%	42.23%	27.89%
50% < x ≤ 70%	59.45%	40.09%
x > 70%	87.10%	30.38%

Commission at a 42.23% loss ratio is: 40% + 7.77%/2 = 43.885%.
Commission at a 59.45% loss ratio is: 40% - 9.45%/1 = 30.55%.
Thus the average commission is:
(1.64%)(50%) + (27.89%)(43.885%) + (40.09%)(30.55%) + (20%)(30.38%) = 31.4%.

Profit Commission:[33]

Define the reinsurer's profit if positive as:
100% - (actual loss ratio) - (ceding commission) - (margin for expenses).
A profit commission returns to the insurer a portion of this profit as additional commission.

Exercise: The ceding commission is 28% and the margin for expenses is 12%.
The reinsurer will return to the ceding company as a profit commission 40% of its profit if any.
If the loss ratio is 53% what is the profit commission paid to the insurer?
[Solution: Profit = 100% - 53% - 28% - 12% = 7%. (40%)(7%) = 2.8%.
Comment: If the loss ratio is 60% of more, then there is no profit and no profit commission.]

Assume the following based on a distribution of loss ratios:

Loss Ratio Range	Average in Range	Probability in Range
x ≤ 60%	38%	58%

Then for a loss ratio x ≤ 60%, profit commission is: (40%)(0.6 - x) = 0.24 - 0.4 x.
Thus the average profit commission is: (58%) {0.24 - (0.4)(38%)} = 5.1%.
Equivalently, the profit commission for a loss ratio of 38% is: (40%)(60% - 38%) = 8.8%.
Thus the average profit commission is: (58%)(8.8%) = 5.1%.[34]

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[33] See pages 12 to 13 of Clark. See also 8, 11/15, Q.21.
[34] Like with sliding scale commissions, there is some ambiguity in the handling of any carryforward provision.

Loss Corridors:[35]

A loss corridor provides that the ceding insurer will reassume a portion of the reinsurer's liability if the loss ratio exceeds a certain amount. For example, the corridor may be 60% of the layer from a 75% to a 90% loss ratio. Then if the reinsurer's loss ratio is 95% before the application of the loss corridor, then the insurer will assume: (60%)(90% - 75%) = 9%. After the application of the loss corridor, the reinsurer will have a net ratio of: 95% - 9% = 86%.

For this example, for a loss ratio x before the application of the loss corridor:

Loss Ratio	Assumed by Insurer	Loss Ratio After Loss Corridor
x ≤ 75%	0	x
75% ≤ x ≤ 90%	(0.6)(x - 0.75)	x - (0.6)(x - 0.75) = 0.45 + 0.4x
x ≤ 90%	(0.6)(0.90 - 0.75) = 0.09	x - 0.09

As with sliding scale commissions, in order to estimate the effect of a loss corridor, one needs to assume a distribution of loss ratios.

For example, assume the following based on a distribution of loss ratios:

Loss Ratio Range	Average in Range	Probability in Range
75% ≤ x ≤ 90%	82%	18%
90% ≤ x	120%	8%

Exercise: Calculate the average effect of the loss corridor.
[Solution: For a loss ratio of 82%, the effect of the loss corridor is: (0.6)(82% - 75%) = 4.2%.
For a loss ratio of 90% or more, the effect of the loss corridor is: (0.6)(90% - 75%) = 9%.
Thus the average effect of the loss corridor is: (4.2%)(18%) + (9%)(8%) = 1.476%.
Comment: If for example, the average loss ratio prior to the loss corridor was 61%,
then the average loss ratio after the application of the loss corridor is: 61% - 1.476% = 59.524%.]

[35] See pages 13 to 14 of Clark. See also for example, 8, 11/11, Q.8 and .8, 11/14, Q.23.

Excess of Loss (Non-Proportional) Reinsurance:[36]

Under excess of loss reinsurance arrangements, the reinsurer indemnifies the primary insurer for claims that exceed a specified dollar amount. The common characteristic of all types of excess of loss reinsurance is that the reinsurer responds to a claim only when the claim amount exceeds a specified dollar amount, known as the attachment point (also referred to as retention or limit).

Five major types of excess of loss reinsurance:

● Per risk excess of loss: The reinsurance limit and the retention apply per risk rather than per accident, per event, or in the aggregate. Per risk excess of loss typically covers property insurance and applies separately to each loss occurring to each risk.

● Catastrophe excess of loss: This type of reinsurance protects the primary insurer from an accumulation of retained claims that arise from a single catastrophic event.

● Per policy excess of loss: The attachment point and the reinsurance limit apply separately to each insurance policy issued by the primary insurer regardless of the number of claims occurring under each policy.

● Per occurrence excess of loss: The attachment point and the reinsurance limit apply to the total claims arising from a single event (or occurrence) affecting one or more of the primary insurer's policies.

● Aggregate excess of loss (also referred to as stop-loss and aggregate stop-loss):
The reinsurer participates over a predetermined aggregate limit for a collection of risks over a specified period (usually one year).

[36] See Fundamentals of General Insurance Actuarial Analysis, 2019 Supplement, by Jacqueline Friedland. Taken from Reinsurance Principles and Practices, by Connor M. Harrison, copyright by The Institutes.

Property Per Risk Excess Treaties:[37]

For example, Minime Insurance writes homeowners insurance. Austin Reinsurance might write a treaty to reinsurance the layer from $100,000 to $300,000, in other words $200,000 excess of $100,000.

Then if Minime needs to pay more than $100,000 on an claim, Austin would reimburse Minime for the amount excess of $100,000 up to a maximum of $200,000. If allocated loss adjustment expense is covered by the property excess treaty it will be prorated between the insurer and reinsurer.

Property per-risk excess treaties provide a layer of coverage which applies on a per risk basis, which typically refers to a single property location.[38] This is to be contrasted with a catastrophe treaty which applies on a per occurrence basis to multiple risks.[39]

Exercise: On four insured homes, Minime pays the following net losses, after the application of any deductibles and coverage limits.

Home	Net Loss	ALAE	Loss & ALAE
1	0	$120,000	$120,000
2	$80,000	$30,000	$110,000
3	$170,000	$20,000	$190,000
4	$420,000	$40,000	$460,000

ALAE is prorated. How much does Austin Reinsurance pay?
[Solution:

Net Loss	ALAE	Loss Paid by Austin	ALAE Paid by Austin
0	$120,000	0	0
$80,000	$30,000	0	0
$170,000	$20,000	$70,000	(70/170)($20,000) = $8235
$420,000	$40,000	$200,000	(200/420)($40,000) = $19,048

Austin pays a total of: 70,000 + 200,000 + 8235 + 19,048 = $297,283.]

A treaty may have a co-participation provision. For example, with a 10% co-participation provision, Austin would only pay 90% of what it would have paid without the provision. Such a treaty would be denoted: 90% 200K xs 100K.

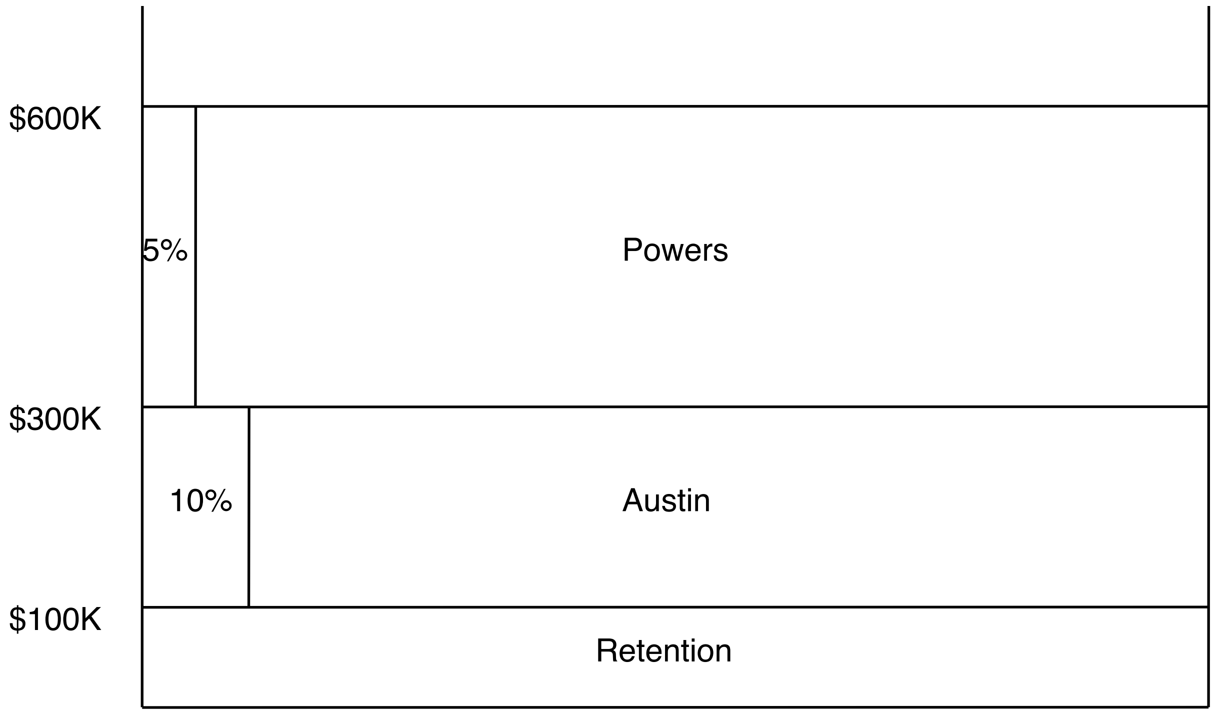
37 See Section 2 of Clark.

38 The definition of a location should be made clear in the treaty. Sometimes disputes arise. For example, it had to be decided in court whether the two towers of the World Trade Center should be considered one or two locations for the purposes of reinsurance after they were destroyed in 2001. The decision depended on the specific language in each reinsurance treaty.

39 To be discussed subsequently. Property catastrophe treaties are discussed in Section 5 of Clark.

A per-risk property treaty would usually have an occurrence limit. For example, this treaty might have an aggregate occurrence limit of $1 million. In that case, Austin would never pay more than $1 million for a single event, no matter how many locations were affected. This would leave Minime unprotected in the case of many large claims due to a catastrophe such as a hurricane or earthquake.[40] It would be expected that Minime would buy a separate treaty to protect against catastrophes.[41]

An insurer such as Minime might buy several different per-risk excess treaties. For example, it might buy a 90% 200K xs 100K treaty from Austin Reinsurance and then also a 95% 300K xs 300K treaty from Powers Reinsurance. We could diagram the coverage provided as follows:[42]



Exercise: For simplicity, assume that ALAE is not covered in each excess treaty.
On three insured homes, Minime suffers losses of: $220,000, $450,000, and $730,000.
How much does each reinsurer pay in each case?
[Solution: For the first loss, Austin pays: (90%)(120,000) = $108,000. Minime retains $112,000.
For the second loss, Austin pays: (90%)(200,000) = $180,000,
Powers pays: (95%)(450,000 - 300,000) = $142,500,
and Minime retains: 450,000 - 180,000 - 142,500 = $127,500.
For the third loss, Austin pays: (90%)(200,000) = $180,000, Powers pays:
(95%)(300,000) = $285,000, and Minime retains: 730,000 - 180,000 - 285,000 = $265,000.]

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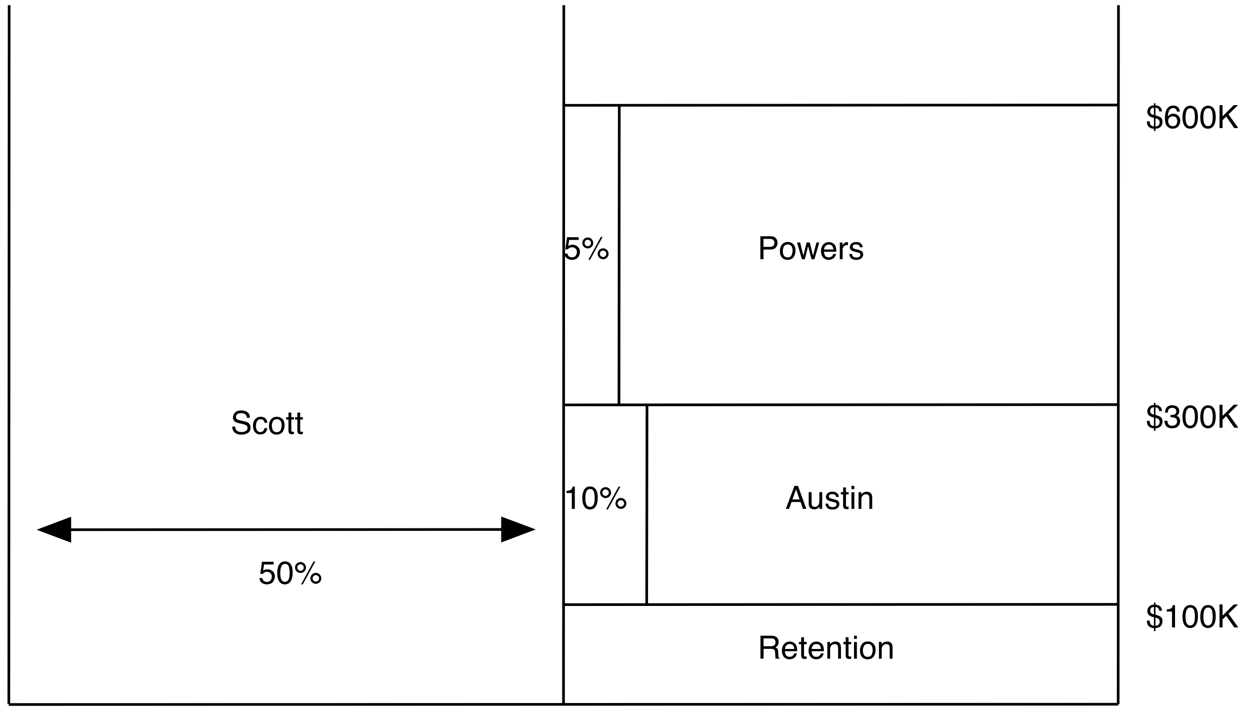
[40] Even without a per occurrence limit, an excess treaty would not protect Minime against getting a very large number of claims due a catastrophe, since Minime is responsible for the first $100,000 of each claim.
[41] To be discussed subsequently. Property catastrophe treaties are discussed in Section 5 of Clark.
[42] See page 125 of Grossi and Kunreuther.

Minime may have bought a 50% quota share treaty from Scott Reinsurance.[43] [44]
If the quota share treaty inures to the benefit of the excess treaties, then the excess treaties apply net of any recoveries from the quota share treaty.[45] Assume that in additional to the quota share, Minime bought a 80% 100K xs 50K treaty from Austin Reinsurance and a 90% 150K xs 150K treaty from Powers Reinsurance. Then the coverage diagram looks as follows:[46]



Exercise: Assume that ALAE is not covered in each excess treaty.
On three insured homes, Minime suffers losses of: $220,000, $450,000, and $730,000.
How much does each excess reinsurer pay in each case?
[Solution: For the first loss, Scott pays: (50%)(220,000) = $110,000.
Austin pays: (80%)(110,000 - 50,000) = $48,000. Minime retains $62,000.
For the second loss, Scott pays: (50%)(450,000) = $225,000.
Austin pays: (80%)(100,000) = $80,000,
Powers pays: (90%)(225,000 - 150,000) = $67,500,
and Minime retains: 450,000 - 225,000 - 80,000 - 67,500 = $77,500.
For the third loss, Scott pays: (50%)(730,000) = $365,000,
Austin pays: (80%)(100,000) = $80,000,
Powers pays: (90%)(150,000) = $135,000,
and Minime retains: 730,000 - 365,000 - 80,000 - 135,000 = $150,000.]

[43] I have assumed that the quota share treaty does not have a limit.
In which case, Scott Reinsurance may have in turn bought excess reinsurance.
[44] If it had bought instead a surplus share treaty, then the proportion of each loss paid by Scott Reinsurance would vary with the value of the property insured, and the example would have been much harder to follow.
[45] This would include any amounts that Minime should have been able to recovery from Scott Reinsurance, whether or not it was able to.
[46] The quota share treaty applies first.
Thus a 100K gross loss acts as a 50K loss net of the quota share reinsurance.
Thus the 100K xs 50K Austin treaty first attaches at a gross loss of 100K and is exhausted at a gross loss of 300K.

Pricing Per Risk Excess Treaties:

There are two techniques for pricing per risk excess treaties:
experience rating and exposure rating.[47]

Experience rating relies on the past losses of the insurer,
while exposure rating does not.

Usually an actuary will perform both types of analyses, and rely on the experience rating to the extent that the data that underlies the experience rating is credible.

The treaty premium is set as a percent of a subject premium base. This subject premium is net of any other reinsurance inuring to the benefit of the per risk treaty, such as a surplus share treaty, but gross of the per risk treaty being priced.[48]

So for example, for a given treaty, experience rating might indicate a rate of 8%, while exposure rating indicates a rate of 5%.[49] A rate of 7% might be selected.

For the reinsured book of business the premium is $400 million. However, there is a quota share treaty which insures to the benefit of the excess treaty. Assume that net of the quota share treaty, the insurer retains $300 million in premium. Then the insurer would pay for the excess treaty: (7%)($300 million) = $21 million.[50] [51]

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[47] Both types of analyses will be discussed subsequently.
[48] If the excess treaty were on a losses occurring basis, this should be earned premium.
If the excess treaty were on a risks attaching basis, this should be written premium.
[49] On the syllabus, you are only responsible for the loss rate not the final rate loaded for reinsurer's expenses and profit, which is discussed in Section 6 of Clark, not on the syllabus.
[50] Note that ceding commissions are used with proportional reinsurance, but not with excess treaties.
[51] For simplicity, I am ignoring the possibility that the 7% rate may be prior to loading for the reinsurer's expenses and profit.

Experience Rating Per Risk Excess Treaties:

Experience rating uses the historical experience, adjusted properly, to predict the future.[52]
Clark lists the following steps:[53]

1. Gather the subject premium and historical losses for the insurer for many years.[54]
   The historical losses should include all losses that would pierce the layer being priced after the application of trend factors.

2. Adjust the subject premium to the future level using rate, price, and exposure inflation factors.

3. Apply severity trend to the historical large losses and determine the amount included in the layer being analyzed. Treat ALAE as it will be in the treaty.[55]

4. Apply excess development factors to the summed losses for each period.[56]
   Apply frequency trend, if needed.

5. Dividing the trended and developed layer losses by the adjusted subject premium produces loss costs by year, which may be averaged to project the expected loss cost.

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[52] Experience rating is sometimes referred to as a "bum cost" model though that phrase more commonly denotes just the raw experience and not the projected cost.
[53] See pages 14 and 15 of Clark.
[54] "Ten years should be sufficient, though the number of years relied upon in the final analysis should be a balance between credibility and responsiveness."
[55] Trend ALAE. If ALAE applies pro-rata in the treaty with losses, it should be added in individually for each loss.
[56] These loss development factors should be appropriate for the layer being reinsured. The loss development factors should be derived from the same ceding company data if possible.

Exercise: You are experience rating a 300K xs 200K property treaty,
that is effective from January 1, 2015 to December 31, 2015, on a losses-occurring basis.
ALAE will apply pro-rata in the treaty with losses.
The primary insurer has the following large claims during 2010:

Claim Number	Loss	ALAE
1	\$190,000	\$20,000
2	\$270,000	\$40,000
3	\$450,000	\$50,000
4	\$610,000	\$70,000

Assume 4% annual severity trend applies to both losses and ALAE.
Determine the 2010 losses and ALAE to use in the experience rating,
prior to loss development and any frequency trend.

[Solution: $(1.04^5)(190{,}000) = 231{,}164$. ⇒ 31,164 in loss.

⇒ $(31{,}164/231{,}164)(1.04^5)(20{,}000) = 3280$ in ALAE.

$(1.04^5)(270{,}000) = 328{,}496$. ⇒ 128,496 in loss.

⇒ $(128{,}496/328{,}496)(1.04^5)(40{,}000) = 19{,}036$ in ALAE.

$(1.04^5)(450{,}000) = 547{,}494$. ⇒ 300,000 in loss.

⇒ $(300{,}000/547{,}494)(1.04^5)(50{,}000) = 33{,}333$ in ALAE.

$(1.04^5)(610{,}000) = 742{,}158$. ⇒ 300,000 in loss.

⇒ $(300{,}000/742{,}158)(1.04^5)(70{,}000) = 34{,}426$ in ALAE.

Total of loss & ALAE is: 31,164 + 3280 + 128,496 + 19,036 + 333,333 + 334,426 = \$849,735.]

Exercise: In a similar manner, 2011, 2012, and 2013 losses and ALAE are trended.
Shown are excess development factors appropriate for this layer.
Also a -1% annual frequency trend is assumed.
Determine the losses for each year to be used in experience rating.

Accident Year	Trended Loss & ALAE in Layer	LDF
2010	\$849,735	1.02
2011	\$688,910	1.07
2012	\$1,255,421	1.15
2013	\$530,542	1.35

[Solution: For 2010: $(849{,}735)(1.02)(0.99^5) = \$824{,}521$.

For 2011: $(688{,}910)(1.07)(0.99^4) = \$708{,}088$.

For 2012: $(1{,}255{,}421)(1.15)(0.99^3) = \$1{,}400{,}853$.

For 2013: $(530{,}542)(1.35)(0.99^2) = \$701{,}979$.]

I have followed Clark at page 17, where the LDFs are applied to the trended losses in a layer for an entire Accident Year rather than to individual losses.[57] If Clark's LDFs for accident years are appropriate for the given layer, then they presumably already include the average impact of the capping effect that can be caused by the width of the layer. Clark's LDFs also include the possibility of yet to be reported losses that may contribute to the layer.

Assume, the following on rate level subject earned premiums for the four years, adjusted for the inflation trend of the insured values:[58] [59] [60]
$11 million, $12 million, $14 million, $17 million.
Then the loss costs as a percent of premium for each year are:

AY	On Level Subject Premium	Trended Ultimate Loss & ALAE in Layer	Loss Cost
2010	$11 million	$824,521	7.5%
2011	$12 million	$708,088	5.9%
2012	$14 million	$1,400,853	10.0%
2013	$17 million	$701,979	4.1%
Total	$54 million	$3,635,441	6.7%

One could use the 6.7% ratio of the totals as the experience rating loss & ALAE cost.[61]

We have implicitly assumed that the portfolio of properties written by the primary insurer in 2015 will be similar to those written during 2010 to 2013. For example, let us assume that the primary insurer is writing homeowners insurance. If during 2014 the primary insurer made a concerted effort to write more very high priced homes, then the assumption of a similar portfolio would not hold. Very high priced homes are more likely to have large claims which can pierce the reinsured layer. Thus the expected loss cost based on past data would probably be too low.

[57] See also 6, 11/07, Q.31 which is along the lines of the calculation shown in Clark.
[58] Clark at pages 7 and 8 has a detailed calculation in his example of pricing proportional reinsurance.
[59] For property insurance, the premium depends on the amount of insurance purchased, which is assumed to increase as the value of the property (usually) increases due to inflation. So even if there were no change in the rates and the primary insurer insured the same properties, we would expect premiums to increase over time.
[60] On rate level should be on the anticipated rate level for 2015 earned premiums for the primary insurer.
[61] Clark takes the ratio of the totals of 8 accident years of data.
One could instead take an average or weighted average of the loss costs for the individual years.

Exposure Curves:[62]

An exposure curve is a graph of the Loss Elimination Ratio as a function size of loss.
Let p be the size loss as a percent of the insured value, denoted by: IV.

$P(p) = \text{LER}(p\ \text{IV}) = E[X \wedge (p\ \text{IV})] / E[X]$.

An important issue is whether the same exposure curve applies regardless of the size of the insured value. "This assumption of scale independence may be appropriate for homeowners business, for which this technique was first developed, but may be a serious problem when applied to large commercial risks."

The probability density function of the severity determines the exposure curve and vice-versa. So the issue is whether the density of severity as a percent of insured value are the same for different sized properties.

For example, let us assume the following very simple discrete distribution of severity is appropriate for a $1 million property:

Loss from One Occurrence as a Percent of Insured Value	Probability
25%	50%
50%	30%
100%	20%

If the same distribution also applied to a $10 million property, then one could use the same exposure curve for both sizes of property. This would not be appropriate, if instead a $10 million property had a different severity distribution such as the following:

Loss from One Occurrence as a Percent of Insured Value	Probability
25%	70%
50%	20%
100%	10%

Scale independence implies that the pure premium per insured value should be the same over the different size ranges.[63] So this is one simple test one can perform. *Clark does not discuss how one could test whether different sizes of properties have the same severity distribution as a percent of insured value. One could use statistical tests such as: the Chi-Square Test, ranksum tests, runs tests, and the two-sample version of the Kolmogorov-Smirnov Test.*[64]

[62] See pages 17 to 18 of Clark. See also the paper by Bernegger on the syllabus of this exam.
[63] The converse is not true.
[64] Such statistical tests are discussed on Exams S and 4.

One would want the exposure curve to be based on loss data as similar as possible to the business being reinsured.[65] [66] *An exposure curve that is appropriate for Homeowners is probably not appropriate for commercial property insurance. A Homeowners exposure curve that is appropriate for masonry construction is probably not appropriate for wood construction. Often it is preferable to have separate exposure curves for different perils. For example, on Homeowners one would expect that the exposure curves for fire and wind would be different. One could do a separate analysis for each major peril as well as all other perils, and weight the results of exposure rating together based on the percentage of expected losses from each peril.*

"In addition to the multiple location problem, several other complicating factors exist in any analysis of commercial property loss experience. First, the coverages provided are not standard across all commercial property policies. Due to the fact that many commercial buildings are leased to tenants, some commercial policies (the owner's) may cover the structure itself, while other policies (the tenant's) may only include contents coverage. Second, even for those policies that provide both building and contents coverages, there isn't the same direct relationship between the building limit and the contents limit as there is with homeowners risks. This lack of a direct percentage relationship with the building limit also extends to the time element (business interruption) coverages, which would typically be included in the definition of loss under a property excess-of-loss reinsurance agreement. A further complicating factor to consider is that while the population of homeowners risks represents a very homogeneous set of exposures (notwithstanding any protection class and/or construction type considerations), under a commercial property policy the class of business (e.g., retail, office, restaurant, etc.) being covered introduces an additional variable into the rating equation, resulting in a less homogeneous set of exposures. Finally, the range of insured values being covered by commercial property policies is much greater than that of homeowners, making it necessary to reexamine the question of whether the relationship between size of loss and insured value is constant across the entire range of insured values."[67]

The exposure curve should usually include the possibility for a loss greater than the insured value.[68]

For example, for Homeowners Insurance the amount of insurance is the Coverage A amount; however, due to the other coverages provided such as contents and additional living expenses, a loss can be significantly more in total than the Coverage A amount. For commercial property insurance, payments for business interruption can lead to total losses greater than the insured value of the property.

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[65] One would also want enough data to be credible.
[66] For example, differences in primary insurance deductible sizes in the data used to develop the exposure curve and the book of business to be reinsured, could cause difficulties.
[67] Quoted from "An Exposure Rating Approach to Pricing Property Excess-of-Loss Reinsurance," by Stephen J. Ludwig, PCAS 1991, <u>not</u> on the syllabus.
[68] See the example at page 18 of Clark.

For commercial property insurance, which can often have more than location per risk, it is important that the exposure curve be consistent with the language of the treaty being priced. Usually the treaty attachment point and limit apply per occurrence per location, in which case the exposure curve should also be based on severity data per location. Then in experience rating the treaty, this exposure curve would be applied to a consistent profile for the subject commercial property business on a per location basis.[69]

[69] See 8, 11/12, Q.10.

Exposure Rating Per Risk Excess Property Treaties:[70]

Unlike experience rating, exposure rating does not use the past loss experience of the primary insurer. Instead, exposure rating uses an exposure curve and a limits profile. A limits profile is a distribution of premium by different ranges of insured values currently being written.
For example, assume the following limits profile for an insurer:[71]

Ceding Company Range of Insured Values ($000)	Subject Premium ($ million)
50 to 100	1.5
100 to 250	2.3
250 to 500	2.0
500 to 1000	1.1

If you are given size ranges as here, assume that properties are at the middle of each interval. Thus, for example, we will assume each property in the second interval is insured for $175,000.

Assume the following exposure curve:[72]

Percent of Insured Value	Exposure Factor
0%	0.00
10%	0.24
20%	0.37
30%	0.47
40%	0.55
50%	0.62
60%	0.68
70%	0.74
80%	0.80
90%	0.86
100%	0.91
110%	0.96
120%	1.00

Assume an excess per risk treaty with an attachment point of $100,000 and a limit of $500,000. Then for an insured value of $175,000, the attachment point is: 100/175 = 57%. Interpolating, the exposure factor is 0.66. The top of the reinsured layer is: 600/175 = 343%. The exposure factor is 1.00. The exposure factor for the reinsured layer is: 1.00 - 0.66 = 0.34. In other words, for an insured value of $175,000, the reinsurer will pay 34% of expected losses.[73]

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[70] See pages 17 to 19 of Clark. See also 6, 11/07, Q.32.
[71] This should be the latest available information, in order to reflect the portfolio to be reinsured.
[72] "The curve does allow for exposure above the insured value; this is due to the fact that often the limits profile provided does not include business interruption coverage for commercial policies or living expenses for homeowners policies."
[73] The portion of losses in a layer is the difference between the loss elimination ratio at the top and the loss elimination ratio at the bottom. Exposure factors are loss elimination ratios.

We would proceed in a similar manner for each interval:

I.V.	100K / I.V.	Exp. Factor	600K / I.V.	Exp. Factor	Exposure Factor for Layer
75K	133%	1.00	800%	1	1 - 1 = 0
175K	57%	0.66	343%	1	1 - 0.66 = 0.34
375K	27%	0.44	160%	1	1 - 0.44 = 0.56
750K	13%	0.28	80%	0.80	0.80 - 0.28 = 0.52

Range of Insured Values	Subject Premium	Exposure Factor	Exposed Premium
50 to 100	1.5	0	0
100 to 250	2.3	0.34	(0.34)(2.3) = 0.782
250 to 500	2.0	0.56	(0.56)(2.0) = 1.12
500 to 1000	1.1	0.52	(0.52)(1.1) = 0.572
Total	6.9		2.474

Assume that the reinsurer estimates the primary insurer's expected loss ratio (excluding ALAE) as 67%.[74] Then the reinsurer's expected losses are: (67%)(2,474,000) = $1,657,580.[75]
Assume the ALAE will be allocated pro-rata, and that ALAE is 8% of losses.[76]
Then, the reinsurer's expected loss and ALAE is: (1.08)($1,657,580) = $1,790,186.
Thus the exposure rate (including ALAE) is: $1,790,186 / $6,900,000 = 25.94%.

Exposure rate is: (1.08) (67%) ($2,474,000) / $6,900,000 = 25.94%.[77]

Usually an excess of loss treaty will have an aggregate limit per occurrence.[78] This exposure rating does not include the impact of any such aggregate limit. This aggregate limit would mainly apply when there are catastrophes, and thus this exposure rate could be based on the insurer's expected non-catastrophe losses rather than its total losses including catastrophes.[79]

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[74] The reinsurer would estimate the insurer's expected loss ratio as is done in pricing proportional reinsurance. The estimation of this expected loss ratio is a very important part of exposure rating.
[75] The reinsurer would make an adjustment if it thought the primary insurer's rates were inadequate. For example, if the primary insurer's rates were inadequate by 10%, then the reinsurer would multiply by 1.1.
[76] The reinsurer should pick an ALAE load that is appropriate for the larger losses that will pierce the reinsured layer. This may differ from the overall ratio of ALAE to loss for the primary insurer.
[77] This exposure rate has not been loaded for the reinsurer's expenses and profit.
[78] This would limit the total amount the excess reinsurer would pay the insurer in the event of a catastrophe such as a hurricane or earthquake. The insurer is expected to buy catastrophe reinsurance to protect against that possibility.
[79] Also the severity distribution per location from catastrophic wind losses and non-cat wind losses differ. See Exhibit 8 in "An Exposure Rating Approach to Pricing Property Excess-of-Loss Reinsurance," by Stephen J. Ludwig, PCAS 1991, not on the syllabus.

Free Cover:[80]

When an insurer provides more coverage, it charges more premium. Similarly, a reinsurer would charge a given insured more for 300K xs 100K than for 200K xs 100K. 300K xs 100K covers the layer from 100K to 400K; this can be thought of as the layer from 100K to 300K plus the layer from 300K to 400K.

When experience rating such treaty, it may happen that no (trended) claims are big enough to penetrate the layer from 300K to 400K.[81] In that case, the experience rate for the layer from 300K to 400K would be zero; if we relied on the experience rate we would charge nothing for this layer.

In general, when experience rating an excess of loss treaty, a "free cover" might appear when there is no trended loss experience in the upper portion of the treaty layer. On an experience only basis this upper layer would be a "free cover" since it has no associated loss cost rate, and we would be giving away additional coverage at no additional cost.

For example, let's assume that experience and exposure rating are applied:

Layer	Experience Rating Loss Cost	Exposure Rating Loss Cost
100K to 300K	10%	8%
300K to 400K	0%	2%

For the 100K to 300K layer, we could select the 10% experience rating loss cost.
For the 300K to 400K layer, we would <u>not</u> select the 0% experience rating loss cost.

Based on exposure rating we estimate that the 300K to 400K layer should cost 2/8 = 1/4 of the 100K to 300K layer. Thus using a mixture of experience and exposure loss rating, we could estimate a rate of: (1/4)(10%) = 2.5% for the 300K to 400K layer.

In general, to price a free cover, first get a experience and exposure rate for the lower layer of the treaty where experience does exist. Next use exposure rating to price the relative cost between the higher layer and the lower layer. Finally, apply this relativity to the experience rated lower layer in order to get a rate for the higher layer.

[80] See page 20 of Clark. See also 8, 11/11, Q.7.
[81] This could happen if the average annual frequency of large claims is small.
The annual frequency of "large" claims depends on the volume of business written by the primary insurer, as well as the value and type of properties written.

Credibility:[82]

The actuary has two estimates of a rate, one from experience rating and one from exposure rating. One could combine them by giving weight Z to the experience rate and weight (1-Z) to the exposure weight. Clark does not go into any detail on how to determine Z; therefore, you can not be asked to determine Z. As mentioned by Clark, assigning credibility here is in part somewhat subjective.

Z would increase as the volume of data on which the experience rating is based increases, all else remaining the same. The volume of data could be measured by either the expected number of excess claims or by the expected dollars of excess losses.[83]

One can examine the random fluctuation in the experience rate from year to year. The less variation in the experience rates based on individual years of data, the more credibility would be given to the experience rate.

The (Buhlmann) credibility is a relative measure of the value of the information contained in the experience rating compared to that in the exposure rate, for the purposes of predicting the future excess losses. Both estimators are imperfect. The exposure rating depends on an exposure curve and limits profile, neither of which is a perfect reflection of the business to be covered by the treaty. The experience rating is affected by the random fluctuation of limited data, as well as trying to adjust for loss development and adjust for changes between the past and the treaty effective period.

Usually, the credibility assigned to the experience rating would decline as the attachment point of the treaty increases, all else being equal.

[82] See pages 20 and 21 of Clark.
[83] One should avoid using the observed number of excess claims or dollars of excess losses, as this is subject to lots of random fluctuation. Using the observed rather than expected values would give more weight when the experience is bad than when the experience is good.

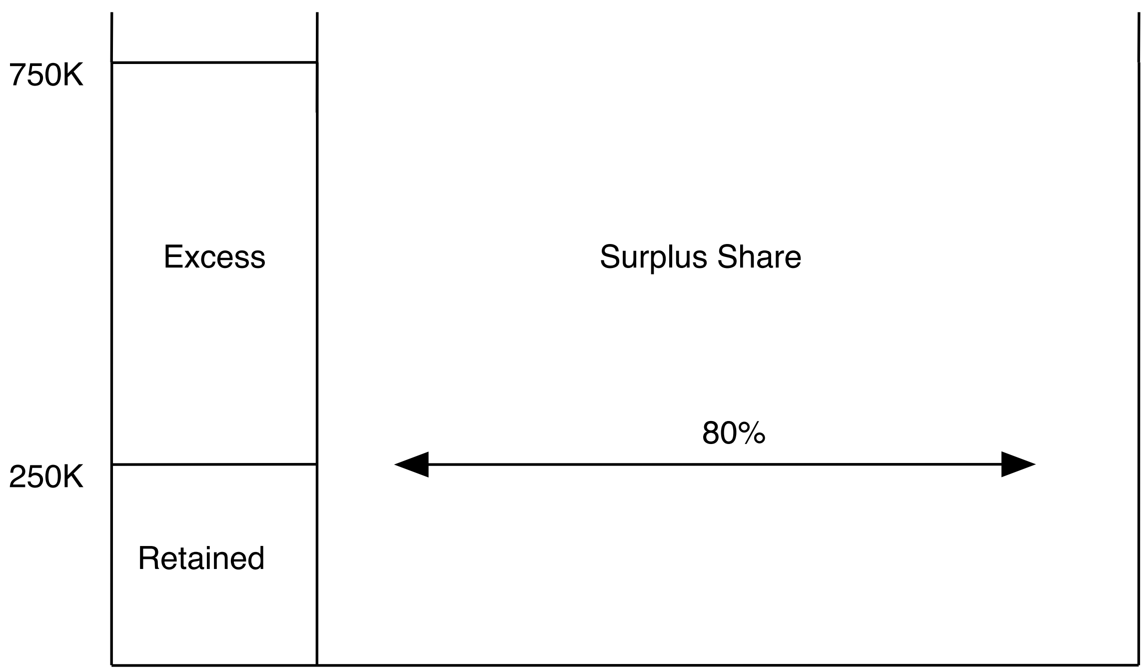
Effect of Inuring Reinsurance:[84]

With respect to a given reinsurance treaty, other reinsurances which are first applied to reduce the loss subject to the given treaty are said to inure to the benefit of the reinsurer of that given treaty. If there is other reinsurance with inures to the benefit of an excess treaty, then we have to carefully take into account the effect when exposure rating.

For example, let us assume that an insurer has purchased a surplus share treaty with a line of $100,000 and 9 lines.[85] This surplus share treaty inures to the benefit of a 100K xs 50K excess treaty. Then for a property insured to a value of $500,000, 80% of the losses are ceded to the writer of the surplus share treaty. The excess treaty only applies to the retained losses.

Exercise: The insurer has the following losses on $500,000 properties: $150,000, $300,000, and $500,000. How much does each reinsurer pay for each loss?
[Solution: For the $150,000 loss the surplus share pays: (80%)(150,000) = $120,000, and the primary insurer retains $30,000. The excess treaty pays nothing.
For the $300,000 loss the surplus share pays: (80%)(300,000) = $240,000, and the primary insurer retains $60,000, prior to the excess treaty.
The excess treaty pays: 60,000 - 50,000 = $10,000. The insurer retains $50,000.
For the $500,000 loss the surplus share pays: (80%)(500,000) = $400,000, and the primary insurer retains $100,000, prior to the excess treaty.
The excess treaty pays: 100,000 - 50,000 = $50,000. The insurer retains $50,000.]

For a $500,000 property, the coverage diagram is:



[84] See pages 21 and 22 of Clark.
[85] If the insurer has purchased a quota share treaty instead of a surplus share treaty, the example would be simpler.

So for a 500,000 property, the 100K xs 50K excess treaty is covering 20% of the layer from 250,000 to $750,000.

Let us assume we had an exposure curve, appropriate for a $500,000 property, then we could exposure rate the excess treaty as follows. $250K / 500K = 50%, so we would look up the exposure factor for 50%; assume that exposure factor is 78%. 750K / 500K = 150%, so we would look up the exposure factor for 150%; assume that exposure factor is 100%. The excess treaty is covering 20% of the layer from $250,000 to $750,000, so we would multiply the losses in the numerator of the exposure rate by 20%. However, the insurer only retains 20% of the premium, after ceding 80% to the quota share reinsurer, thus we would also multiply the premiums in the denominator of the exposure rate by 20%. Thus the exposure rate as a percent of the premium net of the surplus share treaty is the expected loss ratio times: 100% - 78% = 22%.

Alternately, treat the insured property size as the line of $100,000 from the surplus share treaty. 50K/100K = 50%, so we would look up the exposure factor for 50%; however, we look up in the exposure curve appropriate for a $500,000 property.[86] As before, this exposure factor is 78%. 150K / 100K = 150%, with exposure factor of 100%. The exposure rate is the expected loss ratio times: 100% - 78% = 22%.

Exercise: Discuss how to exposure rate a $250,000 property.
[Solution: Treat the insured property size as the line of $100,000 from the surplus share treaty; however, use an exposure curve appropriate for a $250,000 property.
50K/100K = 50%, so we would look up the exposure factor for 50%. 150K / 100K = 150%, so we would look up the exposure factor for 150%. The exposure rate as a percent of the premium net of the surplus share treaty is the difference of these two exposure factors times the expected loss ratio.]

Exercise: Discuss how to exposure rate a $75,000 property.
[Solution: Use an exposure curve appropriate for a $75,000 property.
Since 75,000 ≤ 100,000, the insurer cedes nothing to the surplus share treaty.
50K/75K = 66.7%, so we would look up the exposure factor for 66.7%. 150K / 75K = 200%, so we would look up the exposure factor for 200%. The exposure rate as a percent of the premium net of the surplus share treaty (which for this size property is the same as gross premium) is the difference of these two exposure factors times the expected loss ratio.]

By combining the results for different sized properties in a limits profile, the actuary could determine for this excess treaty an overall exposure rate as a percent of the premium net of the surplus share treaty. Assume that the expected loss and alae ratio of the primary insurer is 68%, and that the excess treaty treats ALAE pro-rata.

[86] Recall, that the appropriate exposure curve can depend on the size of the property.

Range of Insured Value ($000)	MidPoint	Net I.V.	Retention % of I.V.	Ret. + Lim. % of I.V.	Layer Exposure Factor
50 to 100	75	75	66.7%	200%	100% - 85% = 15%
100 to 500	300	100	50%	150%	100% - 78% = 22%
500 to 1000	750	100	50%	150%	100% - 80% = 20%

In each case, the exposure factors are based on an exposure curve appropriate for the given sized property, with somewhat different exposure curves for the different size properties.

The insurer will cede to the surplus share treaty:
0, (300 - 100)/300 = 2/3, and (750 - 100)/750 = 13/15.
Then for example, for the 100 to 500 range, the net premium is: (1 - 2/3)(300) = 100,
and the expected loss and ALAE for the excess treaty is: (68%)(100)(22%) = 14.96.

Range of Insured Value ($000)	Gross Premium	Net Premium	Expected Loss & ALAE Ratio	Exposure Factor	Excess Reinsurer Loss & ALAE
50 to 100	200	200	68%	15%	20.40
100 to 500	300	100	68%	22%	14.96
500 to 1000	150	20	68%	20%	2.72
Total		320			38.08

The loss and ALAE exposure rate as a percent of the premium net of the surplus share treaty is:[87] 38.08 / 320 = 11.9%

[87] This loss and ALAE exposure rate would be loaded for the reinsurer's expenses and profit loading.

<u>Some Principles of Exposure Rating Property Excess of Loss Treaties</u>:[88]

1. Size-of-loss distributions for a homogeneous book of homeowners business can be modeled as a percentage of insured value.

2. The less homogeneous the book, and the wider the range of insured values, the greater will be the disparity in distributions of loss by percentage of insured value across sizes of risk. In general, smaller risks have a greater proportion of severe losses than larger risks.

3. Higher deductibles increase the percentage of net losses in higher layers. As the deductible increases, the primary carrier's premium rate decreases and the reinsurance excess-of-loss treaty rate increases.

4. The relative rates by peril for per-risk excess-of-loss and catastrophe excess-of-loss are different. For instance, fire has a higher per-risk excess-of-loss rate than windstorm, but windstorm has the higher catastrophe excess-of-loss rate.

5. Primary rates depend greatly on claim frequency; reinsurance rates depend on claim severity. Jurisdictions with high claim frequency, and therefore high primary rates, often have low reinsurance excess-of-loss treaty rates.

6. Reinsurers rarely have all the information needed for ideal exposure rating. The reinsurance actuary must find proxies (such as location) for the attributes that influence the excess-of-loss treaty rate (such as construction class, protection class, and peril).

7. The amount of insurance for Homeowners is not the limit for the size of the claim. To the amount of insurance for Coverage A (building) must be added the limit for contents losses, losses on other structures, and loss of use.

8. For small commercial property risks, exposure curves vary by classification and occupancy. In general, "people-oriented" classes, such as restaurants, have a lower frequency of severe losses; properties with flammable contents have a higher frequency of severe losses.

[88] From Sholom Feldblum's discussion of " An Exposure Rating Approach to Pricing Property Excess-of-Loss Reinsurance," Stephen J. Ludwig, PCAS 1993, <u>not</u> on the syllabus.

<u>Casualty Per Occurrence Excess Treaties</u>:[89]

Clark uses casualty lines to refer to: general liability (including products), auto liability, and workers compensation. For example, an insurer who sells general liability insurance could buy an excess treaty for \$200K xs \$100K. Then the reinsurer would pay for the portion of any occurrence in the layer from \$100,000 to \$300,000.[90] In a casualty excess treaty, ALAE can be treated either pro-rata or included in with the losses.

Casualty per occurrence excess treaties are often separated into three categories:

1. **Working Layer**: Low layer attachment which is expected to be penetrated, often multiple times in each annual period.

2. **Exposed Excess**: Excess layer which attaches below some of the policy limits on the underlying business - that is, there are policies for which a full limit loss would cause a loss to the treaty. Typically, these losses will be less frequent and there will be some years in which the treaty layer is not penetrated.

3. **Clash Covers**: A clash treaty is a casualty treaty that attaches above all policy limits; a loss on a single policy will <u>not</u> penetrate the treaty layer. Thus it may be only exposed by:

- extra-contractual obligations (ECO), in other words bad faith claims.[91]
- excess of policy limit damages (XPL).[92]
- catastrophic workers compensation accidents.
- the "clash" of claims arising from one or more loss events involving multiple coverages or policies.[93]
- the method for including allocated loss adjustment expenses in the treaty.

[89] See page 22 of Clark.
[90] Depending on the treaty, this could be the sum of losses from a single occurrence on one policy or could be the sum of losses from a single occurrence on how ever many policies written by the primary insurer that happen to be involved. A clash cover, to be discussed, would use the latter definition.
[91] Extra-contractual obligations (ECO), when used in reinsurance agreements, refers to damages awarded by a court against an insurer or reinsurer that are outside the provisions of the insurance policy, due to the insurer's bad faith, fraud, or gross negligence in the handling of a claim. ECO would include punitive damages. Extra-contractual damage awards most frequently arise from unfair claims handling practices by the insurer (e.g., unjust denial of coverage, failure to settle a claim within policy limits). Damages awarded in excess of policy limits (XPL) are often included in the term ECO.
[92] An obligation on the part of the insurer to cover losses above an insurance contract's stated policy limit.
[93] Subsequently I have some examples.

For example, ALAE can be large for example in professional liability and environmental claims, and ALAE may be included in the limits of the underlying excess of loss treaties.[94] If ALAE is also included with loss for the clash treaty attachment, large ALAE on relatively small indemnity losses may expose the clash layer.[95]

Prices for clash covers are driven by market conditions and are based on a lot of actuarial and underwriting judgement.

There are not definitive boundaries between these three categories.
For example, an insurer might buy the following reinsurance:
Working Cover of $500K excess of $500K, Exposed Layer of $2M excess of $1M,
and a Clash Cover of $7M excess of $3M.

<u>Treatment of ALAE</u>:

ALAE can be treated in one of two ways in a casualty excess treaty:[96]
1. ALAE is shared pro-rata.[97] The attachment point and limit apply to losses.
2. ALAE is included with loss. The attachment point and limit apply to losses plus ALAE.

The insurer pays all losses and ALAE, and is then paid by the reinsurer for any ceded loss and ALAE.

For example, let us assume a 400K xs 100K casualty treaty.
If there is a loss for $250,000 with $50,000 in ALAE, then if ALAE is pro-rata, the reinsurer pays: 150K + (150/250)(50K) = $180,000. If instead ALAE is included with losses, then the reinsurer pays: (250K + 50K) - 100K = $200,000.

Exercise: There is a loss for $80,000 with $40,000 in ALAE.
What does the reinsurer pay under the 400K xs 100K casualty treaty?
[Solution: If ALAE is pro-rata, the reinsurer pays nothing.
If ALAE is included with losses, the reinsurer pays: (80K + 40K) - 100K = $20,000.]

Exercise: There is a loss for $350,000 with $300,000 in ALAE.
What does the reinsurer pay under the 400K xs 100K casualty treaty?
[Solution: If ALAE is pro-rata, the reinsurer pays:
(350K - 100K) + (250/350)(300K) = $464,286.
If ALAE is included with losses, the reinsurer pays: Min[400K, (350K + 300K) - 100K] = $400,000.]

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[94] For the liability policy written by the primary insurer, ALAE is in addition to the liability limits.
[95] This is often called runaway ALAE.
[96] This will be specified in the treaty in advance.
[97] In other words, for each large occurrence, ALAE is prorated between the reinsurer and insurer in proportion to losses.

ALAE can be treated one of two ways in Workers Compensation Large Deductible Policies:
1. The Large Deductible applies to just losses.
2. The Large Deductible applies to losses plus ALAE.

In both cases, the insurer is responsible for handling all of the claims and paying all of the ALAE. In the first case, the insurer expects to be reimbursed by the insured for losses inside the deductible, while in the second case, the insurer expects to be reimbursed by the insured for losses plus ALAE inside the deductible.

A Workers Compensation excess policy is similar to an excess treaty. ALAE can be treated in one of two ways:
1. ALAE is shared pro-rata. The self-insured retention applies to losses.
2. ALAE is included with loss. The self-insured retention applies to losses plus ALAE.

The insured pays all losses and ALAE, and is then paid by the insurer for any loss and ALAE covered by the excess policy.

In retrospective rating, one can use just losses, in which case the average ALAE provision is included within the loss conversion factor. Instead in retrospective rating, one can include ALAE with losses, in which case the loss conversion factor is smaller since it does not include a provision for ALAE.

Experience Rating Casualty Per Occurrence Excess Treaties:[98]

As with property excess, **casualty excess treaties can be priced using experience and exposure rating models**. As the attachment point of the treaty gets higher, the harder it becomes to determine an accurate rate for the treaty, all else being equal.

The steps in the experience rating:

1. Gather the subject premium and historical losses for as many years as possible.
Capture ALAE separately from losses.
Capture the policy limits.[99]
We want Workers Compensation losses on an undiscounted basis.[100]

2. Adjust the subject premium to the future level using rate, price, and exposure inflation factors, appropriate for each line of insurance.

3. Apply severity trend factors to the individual historical losses, appropriate for each line of insurance.[101]
   a) Apply the historical policy limit to each trended loss; this ignores the fact that the insured will generally increase their policy limits over time.
   b) or apply the trend factor to the historical loss without applying a policy limit cap; this assumes that policy limits "drift" upwards to precisely match inflation.[102]

4. Apply excess development factors to the summed losses in the reinsured layer for each year.[103]

5. Divide the trended and developed layer losses by the adjusted subject premium.[104]

[98] See pages 23 to 26 of Clark.
[99] Workers Compensation does not have an explicit limit. For auto losses on a split limits rather than a combined single limit (CSL) basis, care may be needed in order to separately cap losses for bodily injury and property damage.
[100] For Workers Compensation, fatal and permanent total claims are usually reserved on a discounted tabular basis.
[101] ALAE has to be included corresponding to how it will be treated in the treaty. If ALAE is included with losses, then add the trended ALAE to the trended losses. If ALAE is divided pro-rata, then divide the trended ALAE separately for each trended loss.
[102] If this second approach is used, then the subject premium must also be adjusted to the level that would have been charged had the higher limits been in effect.
[103] While Clark does not mention it, any frequency trend would also be applied here.
[104] The resulting loss & ALAE experience rate would be loaded for the reinsurer's expenses and profit load.

Here is an example of Experience Rating, with everything in millions of dollars:

AY	Adjusted Subject Premium	Trended Excess Losses & ALAE	Excess LDF	Loss & ALAE Rate
2011	100	4.2	1.082	(1.082)(4.2) / 100 = 4.54%
2012	115	4.9	1.258	(1.258)(4.9) / 115 = 5.36%
2013	130	3.8	1.303	(1.303)(3.8) / 130 = 3.81%
2014	135	4.3	1.564	(1.564)(4.3) / 135 = 4.98%
Average				4.67%

Alternately, you could take the ratios of the totals:

$$\frac{(1.082)(4.2) + (1.258)(4.9) + (1.303)(3.8) + (1.303)(3.8)}{100 + 115 + 130 + 135} = 4.66\%.$$

Here is example of trending losses, where we use the first method, applying the historical policy limits. We have 3 large losses in the same accident year from policies with \$100,000 limits.[105]
We assume the ground-up unlimited severity trend factor is 1.2.
The treaty being priced is 75K xs 75K, with ALAE included with losses.

Number	Loss	Trended Loss	Limited Trended Loss
1	60,000	72,000	72,000
2	70,000	84,000	84,000
3	100,000	120,000	100,000

Limited Trended Loss	ALAE	Trended ALAE	Loss & ALAE	In Layer
72,000	5000	6000	78,000	3000
84,000	8000	9600	93,600	18,600
100,000	7000	8400	108,400	33,400

If instead ALAE is divided pro-rata, then the ceded losses are: 0, 9000, and 25,000.
The ceded ALAE is: 0, (9/84)(9600) = 1029, and (25/100)(8400) = 2100.

[105] There are other large losses from polices with other limits. We would add together the results for the trended losses & ALAE in the reinsured layer in an accident year.

Let us instead apply the second method of trending to this example. We would instead, not limit any of the trended historical losses:

Number	Loss	Trended Loss
1	60,000	72,000
2	70,000	84,000
3	100,000	120,000

Unlimited Trended Loss	ALAE	Trended ALAE	Loss & ALAE	In Layer
72,000	5000	6000	78,000	3000
84,000	8000	9600	93,600	18,600
120,000	7000	8400	128,400	53,400

While in the historical period, these insureds had a $100,000 policy limit, we are implicitly assuming that during the prospective treaty period these insureds would instead have policy limits of: (1.2)($100,000) = $120,000.[106] The loss that was $100,000 during the historical period, we project would be $120,000 during the prospective treaty period; however, it would not then be limited by a policy limit of $120,000.[107]

If instead we were to deal with inflation using the second method discussed by Clark, we would not have limited the trended losses. Thus the final trended loss of 108,000 would be added to the trended ALAE of 8400 to get 116,400. The resulting contribution to the layer reinsured by the treaty would be 41,400 rather than 33,400.

We see that the second method results in more losses entering the calculation, than the first method. However, using the second method, the subject premium would have to be at current rates and exposure inflation levels for a policy limit of (1.2)(100,000) = 120,000, rather than the 100,000 limit of the policy during the experience period.[108]

If the insureds are on average raising their limits purchased at a rate similar to the severity trend used in the experience rating calculation, then the second method should be more accurate. However, if insureds are on average not raising their policy limits or are doing so only very slowly, then the first method should be more accurate.

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106 Depending on the original policy limit and the severity trend factor, we usually get a trended policy limit that is not actually sold. In this second method, we are making a simplifying assumption which hopefully on average produces approximately the correct result.

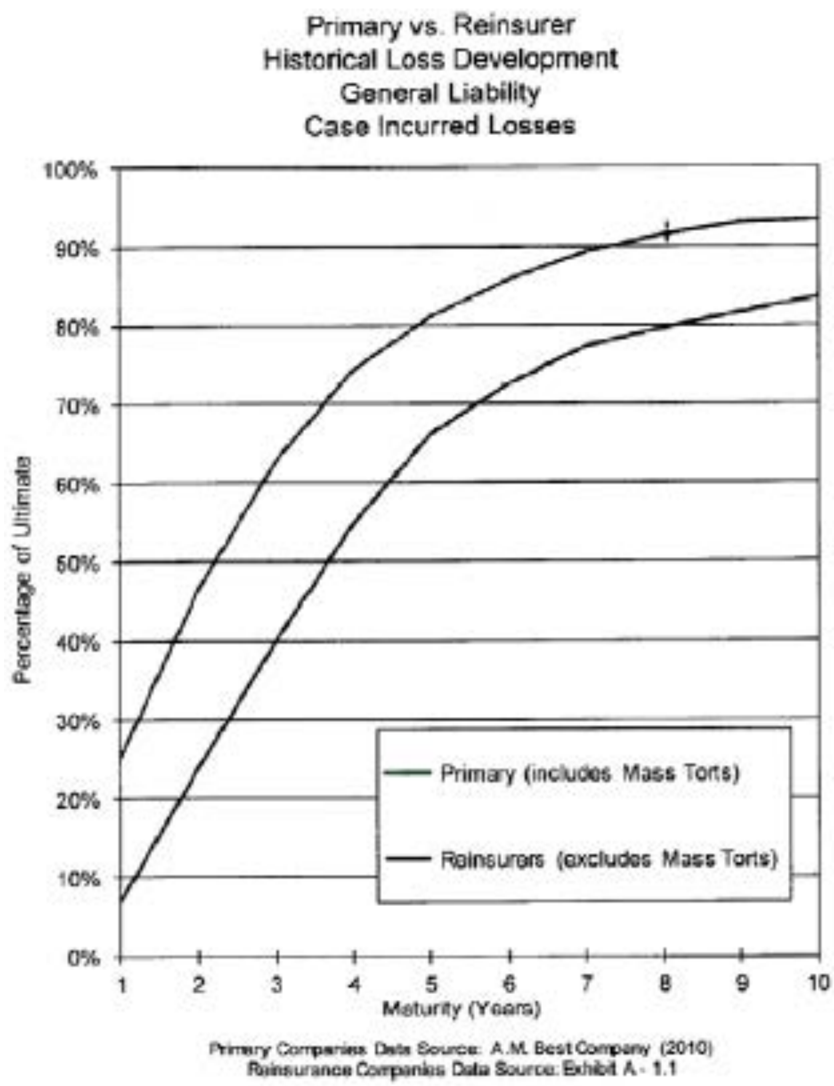
107 If the reported loss of $100,000 had been limited to that value by the historical policy limit, during the prospective treaty period it would be limited by the $120,000 assumed policy limit.

108 Thus when using the second method, both the contributions to the numerator and denominator of the experience rate would be larger than when using the first method of dealing with inflation.

Excess Loss Development:[109]

In experience rating, one has to multiply the summed trended loss and alae by development factors appropriate for the reinsured layer. For casualty lines, this step is critical due to the very large factors needed to reflect future development.

In general, a reinsurer’s losses develop to ultimate much more slowly than those of a primary insurer. For example, for General Liability Insurance, its takes between 4 and 5 years for an average primary insurer’s case incurred losses to get to 80% of ultimate. In contrast, for an average reinsurer it takes between 8 and 9 years for case incurred losses to get to 80% of ultimate.[110]



[109] See pages 25 to 26 of Clark. See also “An Analysis of Excess Loss Development,” by Emanuel Pinto and Daniel F. Gogol, PCAS LXXIV, 1987, not on the syllabus.
“An Enhanced Understanding of Using the RAA Excess Casualty Loss Development Study For Reserve Analysis,” by Chaim Markowitz, CAS E-Forum, Spring 2015.
[110] See the graphs at page 52 of Clark, based on data from the Reinsurance Association of America (RAA).

If possible, historical patterns should be derived for the excess layer using ceding company data. Often this data is not available, or is too sparse to use. Then one can rely on industrywide data, which in any case is more stable.

The Reinsurance Association of America (RAA) publishes a loss development study on a regular basis. While the RAA statistics may be considered a benchmark, there are potential issues.

1. The reporting lag from the occurrence of an event to the establishment of a reinsurer's case reserve may vary by company. Included in the data is retrocessional business which may include several levels of reporting lag.[111]

2. The mix of attachment points and limits is not cleanly broken out. Loss development varies significantly for different attachment points so every effort should be made to adjust the selected factors to the layer of the treaty being priced.

3. The RAA requests data exclusive of Asbestos and Environmental claims which could distort the patterns. It cannot be known if all member companies have done this consistently. Other long term exposure claims, such as products claims for silicone breast implants, are not excluded.

4. For workers compensation, the members may not handle the tabular discount on large claims in a consistent manner. If a ceding company reports a loss on a discounted basis, and the reinsurer establishes a case reserve as the amount of the discounted value that falls into the reinsured layer, a very high development factor may result as the discounting unwinds.[112]

It should be noted that loss development factors can be significantly larger for high layers of loss than for total losses.[113] As claims move from initial reserve to settlement, some of them will increase significantly, piercing an excess layer at ultimate that they did not piece initially.[114] Also for many casualty lines, large claims take longer to report and settle on average.

Clark points out that "having a very slow development pattern will often produce results showing either zero or very high projected ultimate layer losses by year. The [reinsurance] actuary will often need to use smoothing techniques, such as a Bornhuetter-Ferguson approach or Cape Cod (Stanard-Bühlmann), to produce a final experience rate."

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[111] Retrocessional business is reinsurance bought by a reinsurer.
[112] The reserve on an discounted basis is much less than the expected ultimate loss.
[113] Higher attachment points usually have higher loss development.
Wider layers have more room for losses to develop upwards.
[114] The effect of loss dispersion on excess ratios is covered in the
Discussion of "Retrospective Rating: 1997 Excess Loss Factors," by Howard C. Mahler, PCAS 1998.

A Simple Mathematical Example of Excess Loss Development:

Here is an illustration of one reason why loss development factors can be significantly larger for high layers of loss than for total losses. For simplicity, I will ignore unreported claims.[115]

Assume that at first report the distribution of the size of loss is Exponential with mean 50,000.[116] Then at first report the average amount per loss in the layer \$100,000 to \$150,000 is:[117]
$E[X \wedge 150{,}000] - E[X \wedge 100{,}000] = 50{,}000(1 - e^{-150/50}) - 50{,}000(1 - e^{-100/50}) = 4277.$

Assume a dispersion model of development to ultimate, with Gamma loss divisor and thus Inverse Gamma Loss Multipliers.[118] Assume the Inverse Gamma has $\alpha = 5$ and $\theta = 5$. Therefore, the average loss multiplier is $\theta/(\alpha-1) = 5/4 = 1.25$. In other words, the loss development factor from first to ultimate is 1.25.

For loss multiplier of m, the losses at ultimate are Exponential with mean 50,000 m. Thus the losses at ultimate divided by 50,000 are Exponential with mean m. Thus mixing over m, the losses at ultimate divided by 50,000 are Shifted Pareto with $\alpha = 5$ and $\theta = 5$.[119] Therefore, the losses at ultimate are Shifted Pareto with $\alpha = 5$ and $\theta = (5)(50{,}000) = 250{,}000$.

Thus at ultimate the average amount per loss in the layer \$100,000 to \$150,000 is:[120]
$E[X \wedge 150{,}000] - E[X \wedge 100{,}000] =$

$$\frac{250{,}000}{5-1}\left\{1 - \left(\frac{250{,}000}{250{,}000 + 150{,}000}\right)^{5-1}\right\} - \frac{250{,}000}{5-1}\left\{1 - \left(\frac{250{,}000}{250{,}000 + 100{,}000}\right)^{5-1}\right\} = 6733.$$

Thus, the loss development factor from first to ultimate for this excess layer is: 6733 / 4277 = 1.57. This is much bigger than the overall loss development factor from first to ultimate of 1.25.

This is just an illustration. In general, the size of the excess loss development factor would depend on the excess layer as well as the mathematical details of the model.[121]

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[115] Since for many casualty lines large claims take longer to report on average and then longer to settle on average, this is another reason why excess loss development factors are usually large.
[116] Chosen for simplicity rather than realism.
[117] Using the formula for the limited expected value of an Exponential.
[118] The effect of loss dispersion on excess ratios is covered in the
Discussion of “Retrospective Rating: 1997 Excess Loss Factors,” by Howard C. Mahler, PCAS 1998.
At pages 326-27 and Appendix C, Gamma loss divisors are applied to a Shifted Pareto severity, a more complicated model.
[119] For the Inverse Gamma-Exponential, the mixed distribution is Shifted Pareto with the same parameters as the Inverse Gamma.
[120] Using the formula for the limited expected value of the Shifted Pareto Distribution.
[121] Also recall that I have ignored for simplicity any claims that are unreported at first report.

Exposure Rating Casualty Per Occurrence Excess Treaties:[122]

Assume for example a 250,000 xs 250,000 casualty reinsurance treaty.[123]
Assume that ALAE is treated pro-rata in the treaty.[124]
Assume an expected loss ratio (no ALAE) of 65%.
For the primary insurer, for the line of insurance being reinsured, we have the following:

Policy Limit	Increased Limit Factor	Estimated Subject Premium
100,000	1.00	2,000,000
250,000	1.30	3,000,000
500,000	1.40	3,000,000
1,000,000	1.45	2,000,000

Then we calculate the following:

Policy Limit	ILF	% losses Excess of 250K	% losses Excess of 500K
100,000	1.00	0	0
250,000	1.30	0	0
500,000	1.40	(1.40 - 1.30) / 1.40 = 1/14	0
1,000,000	1.45	(1.45 - 1.30) / 1.45 = 3/29	(1.45 - 1.40) / 1.45 = 1/29

Policy Limit	% Losses in Layer	Total Expected Losses	Expected Losses in the Layer
100,000	0	1,300,000	0
250,000	0	1,950,000	0
500,000	1/14	1,950,000	139,286
1,000,000	3/29 - 1/29 = 2/29	1,300,000	89,655
Total			228,941

Assume ALAE is 10% of losses.[125]
Since ALAE is treated pro-rata in the treaty, we could get a loss and ALAE exposure rate of:
(1.1)(228,941) / 10,000,000 = 2.5%.

[122] See pages 27 to 31 of Clark.
[123] We assume that the insurer will buy additional reinsurance to cover higher layers of loss.
[124] As will be discussed, when ALAE is included with losses in the treaty, exposure rating is more complicated.
[125] The reinsurer would pick a loadings for ALAE consistent with the ratio of ALAE to losses for the large losses that will pierce the reinsured layer. The ratio may be different than the ratio for all losses.

Assumptions About Increased Limits Factors:

We have assumed that these Increased Limits Factors are appropriate for the reinsured book of business. If risk loads were included in the ILFs, we would want to remove them for this purpose.

This calculation implicitly assumed as per Clark that the increased limits factors were calculated as:[126]
(Limited Expected Value at Increased Limit) / (Limited Expected Value at Basic Limit).

$$\text{Exposure Factor} = \frac{E[X \wedge \text{Min}(PL, AP+Lim)] - E[X \wedge \text{Min}(PL, AP)]}{E[X \wedge PL]},$$

where PL = ceding company Policy Limit, AP = treaty Attachment Point, Lim = treaty Limit.

If in determining ILFs, expenses were not treated as all variable, and/or if ALAE was not assumed to be the same proportion of losses for all limits purchased, then the manner of using ILFs in the above calculation would not be appropriate.[127]

Assume instead, that all of the ALAE is loaded into the basic limit.
Then the ILF at \$250,000 is: $\dfrac{E[X \wedge 250K] + ALAE}{E[X \wedge 100K] + ALAE}$.

The ILF at \$500,000 is: $\dfrac{E[X \wedge 500K] + ALAE}{E[X \wedge 100K] + ALAE}$.

Thus for the 500K limit policies, the exposure rating factor for the 250K to 500K layer is:

$$\frac{(\text{ILF } 500K) - (\text{ILF } 250K)}{\text{ILF } 500K} = \frac{E[X \wedge 500K] - E[X \wedge 250K]}{E[X \wedge 100K] + ALAE} \ \frac{1}{\text{ILF } 500K}.$$

$$\frac{\text{Total Limit Premium for 500K Limit Policies}}{\text{ILF for 500K}} = \text{Basic Limits Premium for 500K Limit Policies}$$

= (Expected Freq.) ($E[X \wedge 100K]$ + ALAE) / (Basic Limit Expected Loss & ALAE Ratio).

(Expos. Factor) (Total Limits Prem. for 500K Policies) (Basic Limit Expected Loss & ALAE Ratio)
= (Expected Freq.) ($E[X \wedge 500K] - E[X \wedge 250K]$) = Expected Losses in Reinsured Layer.

Thus, when we assume all of the ALAE is loaded into the basic limit, we get a result similar to that in Clark; however, in order to get the expected losses in the reinsured layer we need to multiply by the Loss & ALAE Ratio rather than the pure loss ratio. On your exam, for exposure rating I would expect the example in Clark, where the ILFs are based on the assumption that ALAE is proportional to losses, rather than this alternative based on loading all the ALAE into the basic limit.

[126] See page 27 of Clark.
[127] Different assumptions can be used to determine ILFs, for example how ALAE is affected.

Exposure Rating Treaties Covering Workers Compensation:

If instead the line of insurance is Workers Compensation, then one would use ELFs rather than ILFs.[128] One would want for the primary insurer a list of Workers Compensation standard premium by state and hazard group.[129] For example, assume the following:

State	Hazard Group	Standard Premium ($ million)
X	D	20
X	E	30
Y	E	40
Y	G	10

Exposure factor = ELF_{AP} - $ELF_{AP+Limit}$.
This exposure factor is to be multiplied by Standard Premium.[130]
If we were exposure rating a per-risk per-accident treaty that was 500K xs 250K, we would want the ELFs at 250,000 and 750,000.

State	Hazard Group	Premium	ELF 250K	ELF 750K	Expected Loss in Layer
X	D	20	5.4%	1.5%	780,000
X	E	30	7.5%	2.2%	1,590,000
Y	E	40	10.1%	3.4%	2,680,000
Y	G	10	13.4%	4.3%	910,000
Total		100			5,960,000

For example: (5.4% - 1.5%) ($20 million) = $780,000.

Assume ALAE is 5% of losses.[131] If ALAE is treated pro-rata in the treaty, then we could get a loss and ALAE exposure rate of:[132] (1.05)(5,960,000) / 100,000,000 = 6.3%.

If the ELF included a risk load, then here we would want to use the ELFs excluding the risk load; we are just interested in the excess ratio times the expected loss ratio:
ELF_L = (expected loss ratio) (E[X] - E[X $\wedge$ L]) / E[X].

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128 An Excess Loss Factor (ELF) is intended to be applied to standard premium in order to get expected losses. At page 30, Clark instead takes the ELF as the excess ratio; the ELF as the term is used by the NCCI is the product of the excess ratio and the expected loss ratio.
129 ELFs vary by state and hazard group.
130 At page 31, Clark multiples the ELF times the expected losses, which are standard premium times the expected loss ratio. Instead using the usual definition, ELF times standard premium is the expected excess losses.
131 The reinsurer should select a ratio that is appropriate for the large claims that will pierce the reinsured layer.
132 If ALAE were included with losses in the treaty, then instead we would use Excess Loss and Allocated Expense Factors (ELAEFs). Standard Premium times the ELAEF equals the expected excess loss and ALAE.

ALAE Included with Losses, Casualty Excess Treaties:[133]

As discussed previously, ALAE may be treated in two ways by an excess treaty: either pro-rata or included with losses. If ALAE is handled pro-rata, one can just load the estimated pure loss rate of the treaty for ALAE at the end. If ALAE is included with losses, then when performing an experience rating of the treaty, just include the ALAE with the loss for each claim. This only leaves the case of exposure rating a treaty when ALAE is included with losses.

We are exposure rating an excess treaty 250K excess 150K, with ALAE included with losses. Let us assume we are working on a block of liability policies each with a limit of $500,000 that applies only to loss. On average ALAE is 25% of the losses paid under these policies.

The simplifying assumption that is used here is that for each claim Loss Plus ALAE is 125% of loss; in other words loss and ALAE are perfectly correlated.[134] That is mathematically just like uniform inflation of 25%. Therefore, the average payment per loss in the reinsured layer is:[135]
$1.25\ (E[X \wedge 400K / 1.25] - E[X \wedge 150K / 1.25]) = 1.25\ (E[X \wedge 320K] - E[X \wedge 120K])$.
This would be the numerator of the exposure rating factor.

The denominator of the exposure rating factor is the product of the ALAE loading and the formula for the average payment per loss for the original policy with a limit of 500,000:
$1.25\ E[X \wedge 500K]$.

Thus the exposure factor would be: $\dfrac{E[X \wedge 320K] - E[X \wedge 120K]}{E[X \wedge 500K]}$.[136]

In general, when ALAE is included with losses the exposure factor is:

$$\frac{E[X \wedge \text{Min}[PL, (AP+Lim)/(1+e)]\ ] - E[X \wedge \text{Min}[PL, AP/(1+e)]]}{E[X \wedge PL]},$$

where
PL = underlying Policy Limit applying to loss only
AP = Treaty attachment point applying to ALAE plus loss capped at PL
Lim = Treaty limit applying to ALAE plus loss capped at PL
e = ALAE as a percent of loss capped at PL.

This exposure factor is to be multiplied by the expected losses & ALAE for these policies, in order to get the expected loss and ALAE paid by the excess treaty.

133 See pages 29 to 30 of Clark.
134 In reality, the ratio of capped loss to (unlimited) ALAE varies by claim.
135 This formula for the average payment per loss.
136 The factors of 1.25 in the numerator and denominator cancel.

Exercise: Determine the exposure factor if the excess treaty is instead 400K excess 300K.
[Solution: Min[PL, (AP+Lim)/(1+e)] = Min[500K, 700K/1.25] = 500K.

Min[PL, AP/(1+e)] = Min[500K, 300K/1.25] = 240K.

Thus using the above formula, the exposure factor is: $\frac{E[X \wedge 500K] - E[X \wedge 240K]}{E[X \wedge 500K]}$.

<u>Comment</u>: In this case, (AP+Lim) / (1+e) = (300K + 400K) / 1.25 = 560K > 500K.
We are assuming each loss is limited to 500,000 by the underlying policy. Therefore under our simplifying assumption, the largest possible loss plus ALAE is: (1.25)(500,000) = 625,000.
Thus even though the upper end of the layer reinsured by the treaty is 700,000 (in loss & ALAE), effectively here it is 625,000. "Deflating" this back to a pure loss level, this is the 500,000 which appears in the numerator; Min[500K, 700K/1.25] = 500K.]

Exercise: Determine the exposure factor if the excess treaty is instead 1000K excess 750K.
[Solution: Min[PL, (AP+Lim)/(1+e)] = Min[500K, 1750K/1.25] = 500K.

Min[PL, AP/(1+e)] = Min[500K, 750K/1.25] = 500K.

Thus using the above formula, the exposure factor is: $\frac{E[X \wedge 500K] - E[X \wedge 500K]}{E[X \wedge 500K]} = 0$.

<u>Comment</u>: In this case, AP / (1+e) = 750K / 1.25 = 600K > 500K.
We are assuming each loss is limited to 500,000 by the underlying policy. Therefore under our simplifying assumption, the largest possible loss plus ALAE is: (1.25)(500,000) = 625,000.
Thus even though the lower end of the layer reinsured by the treaty is 750,000 (in loss & ALAE), effectively here it is 625,000. "Deflating" this back to a pure loss level, this is the 500,000 which appears in the second term of the numerator; Min[500K, 750K/1.25] = 500K.
Put another way, since we are making the simplifying assumption that the largest possible loss & ALAE on a claim is: (1.25)(500,000) = 625,000, we do not expect the excess treaty to pay anything for these policies.
However, in reality there could for example be a $500,000 loss with $400,000 in ALAE, for which the excess treaty would pay $150,000.]

ALAE Included with Losses, a Shifted Pareto Example:

For the Shifted Pareto Distribution:

$F(x) = 1 - \left(\frac{\theta}{\theta+x}\right)^{\alpha} > 0.$ $E[X \wedge x] = 1 - \left(\frac{\theta}{\theta+x}\right)^{\alpha}, \alpha \neq 1.$ $R(x) = \left(\frac{\theta}{\theta+x}\right)^{\alpha-1}, \alpha > 1.$

Let us assume that size of loss follows a Shifted Pareto Distribution with $\alpha = 3$ and $\theta = 200{,}000$.

We are exposure rating an excess treaty 250K excess 150K, with ALAE included with losses. Let us assume we are working on a block of liability policies each with a limit of $500,000 that applies only to loss. On average ALAE is 25% of the losses paid under these policies.

Exercise: Calculate the exposure factor.

[Solution: As discussed previously, the exposure factor is:

$$\frac{E[X \wedge \text{Min}[PL, (AP+Lim)/(1+e)]] - E[X \wedge \text{Min}[PL, AP/(1+e)]]}{E[X \wedge PL]}$$

$$= \frac{E[X \wedge \text{Min}[500K, 400K/1.25]] - E[X \wedge \text{Min}[500K, 150K/1.25]]}{E[X \wedge 500K]}$$

$$= \frac{E[X \wedge 320K] - E[X \wedge 120K]}{E[X \wedge 500K]}.$$

$E[X \wedge 120{,}000] = (100{,}000) \{1 - (100/220)^2\} = 79{,}339.$

$E[X \wedge 320{,}000] = (100{,}000) \{1 - (100/420)^2\} = 94{,}331.$

$E[X \wedge 500{,}000] = (100{,}000) \{1 - (100/600)^2\} = 97{,}222.$

Thus, the exposure factor is: (94,331 - 79,339) / 97,222 = 0.1542.

Alternately, in terms of excess ratios, the exposure factor is:

$$\frac{R(120{,}000) - R(320{,}000)}{1 - R(500{,}000)} = \frac{(100/220)^2 - (100/420)^2}{1 - (100/600)^2} = 0.1542.]$$

Thus if for example the expected loss and ALAE for this block of policies is $40 million, then the expected ceded loss and ALAE is: (0.1542)($40 million) = $6.17 million.

Here we have made the simplifying assumption that ALAE is always 25% of the losses for each claim. We could instead make the somewhat more realistic assumption that the ratio of ALAE to loss for a claim is equally likely to be: 10%, 25%, or 40%.[137]

[137] This is similar to the simple dispersion model in the
Discussion of "Retrospective Rating: 1997 Excess Loss Factors," by Howard C. Mahler, PCAS 1998.

Exercise: Calculate the exposure factor if ALAE were always 10% of losses.
[Solution: The exposure factor is:

$$\frac{E[X \wedge 400K /1.1] - E[X \wedge 150K / 1.1]}{E[X \wedge 500K]} = \frac{E[X \wedge 363{,}636] - E[X \wedge 136{,}364]}{E[X \wedge 500{,}000]}.$$

$E[X \wedge 136.364] = (100{,}000) \{1 - (100/236.364)^2\} = 82{,}101.$
$E[X \wedge 363{,}636] = (100{,}000) \{1 - (100/463.636)^2\} = 95{,}348.$
$E[X \wedge 500{,}000] = (100{,}000) \{1 - (100/600)^2\} = 97{,}222.$
Thus, the exposure factor is: (95,348 - 82,101) / 97,222 = 0.1363.]

Exercise: Calculate the exposure factor if ALAE were instead always 40% of losses.
[Solution: The exposure factor is:

$$\frac{E[X \wedge 400K /1.4] - E[X \wedge 150K / 1.4]}{E[X \wedge 500K]} = \frac{E[X \wedge 285{,}714] - E[X \wedge 107{,}143]}{E[X \wedge 500{,}000]}.$$

$E[X \wedge 107{,}143] = (100{,}000) \{1 - (100/207.143)^2\} = 76{,}694.$
$E[X \wedge 285{,}714] = (100{,}000) \{1 - (100/385.714)^2\} = 93{,}278.$
Thus, the exposure factor is: (93,278 - 76,694) / 97,222 = 0.1706.]

Thus, assuming that the ratio of ALAE to loss for a claim is equally likely to be: 10%, 25%, or 40%, the exposure factor is: (0.1363 + 0.1542 + 0.1706) / 3 = 0.1540.
In this example, the exposure factor did not change much from the simplified assumption.

In the case instead of an excess treaty 400K excess 300K, the simplifying assumption for ALAE produced a exposure factor of zero. In contrast, a more realistic assumption for ALAE would produce a small positive exposure factor.

Also note that we have assumed that the distribution of ALAE ratios around its mean as well as the mean itself are the same for all sizes of loss. In situations in which this is not true, this would be another reason why the simplifying assumption that was used is an approximation. "A more refined analysis of the effect of ALAE would require modeling of how ALAE varies with loss size."[138] Modeling a joint distribution of Losses and ALAE would improve exposure rating when ALAE is included with losses, but is beyond what is covered on the syllabus of this exam.[139]

[138] Quoted from page 30 of Clark.
[139] Use of a cupola would help.
See for example Section 9 of "Dependency Modeling," by Sholom Feldblum,
a CAS Study Note formerly on Exam 7.

Examples of Clash Scenarios:[140]

Here are examples where more than one policy written by a ceding insurer for possibly different lines of insurance may be affected, thereby exposing a clash cover.

1. Fire or explosion in a high rise office building. Possible defendants:
- Building operator
- Maintenance service providers (for elevators, electricity or gas supply)
- Security firm
- Construction company
- Architect

2. Auto manufacturer A purchases the steering column from B. The steering column has a design defect which causes a car accident. Defendants:
- Auto manufacturer A
- Steering column supplier B

3. Drug manufacturers A and B independently use the same ingredient in a drug designed for the treatment of the same health condition. The ingredient has serious, unanticipated side effects for numerous patients. Defendants:
- Drug manufacturer A
- Drug manufacturer B
- Ingredient suppliers C, D, E, …

4. A tax advisor is giving training courses to life insurance agents which contain erroneous advise on how to optimize tax savings. The agents then misinform customers to whom they sell life insurance products. Defendants:
- The tax advisor
- The insurance agents

[140] From "Aggregation of Losses and Clash Covers in Liability Treaty Reinsurance," by Werner Bautz, GeneralRe.

Excess of Policy Limits (XPL):[141]

One source of payments under clash covers is Excess of Policy Limits, which covers judgments in excess of the original policy limits against the insured for claims brought by a third party.

Consider an insured who buys a policy with limits of $1 million. The insured loses a lawsuit brought by a third party for a verdict of $10 million. The $9 million above the policy limit may be an XPL judgment. An XPL judgment normally involves alleged questionable claims handling or defense of a lawsuit by the insurer. This often takes the form of the cedant failing to settle a claim within the policy limits when the opportunity was presented. It is due to the handling of the claim that the insurer is considered liable for the excess verdict. The claim for which the damages were awarded must otherwise be covered by the primary policy.

An XPL judgment may include compensatory and punitive damages, but these are not always covered by the primary policy. Compensatory damages can include infliction of emotional distress or loss of business opportunity or business reputation. Punitive damages are usually reserved for situations involving conduct that was exceptionally malicious, egregious, or outrageous.

One example of an XPL claim, affirmed on appeal, can be found in the case of Fortman v. Hemco.[142] This case involves a three-year-old plaintiff who was injured in 1981 when she fell out of a jeep her mother was driving and was run over by a following vehicle. The plaintiff sued Hemco, who manufactured the mold used to form the fiberglass top and doors on the jeep. The doors on the jeep were designed to be rear-hinged and had exposed interior handles. The plaintiff hooked a sleeve on the door handle, opening the latch. The wind caught the door and threw it open, pulling the plaintiff out of the jeep.

Hemco's insurance company had the opportunity to settle the case for $1 million in 1984. They chose to go to court, primarily because Hemco neither designed the jeep top nor manufactured the actual top on the Fortman jeep. In 1986 a jury awarded Ms. Fortman $17.7 million in economic damages for the personal injury claim and $6 million for pain and suffering. The court concluded that Hemco provided expertise and could have corrected the design. Hemco's insurer was required to pay the entire loss, including the amounts in excess of the policy limit.

Under the clash reinsurance program that the insurer purchases today, this would have been a sizable reported loss to the clash layer.

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[141] From "Pricing Extra-Contractual Obligations and Excess of Policy Limits Exposures in Clash Reinsurance Treaties," by Paul Braithwaite and Bryan C. Ware, Spring 1997 CAS Forum, <u>not</u> on the syllabus.
[142] 211 Cal. App. 3d 241 (App. Div., California 1989).

Extra-Contractual Obligations (ECO):[143]

One source of payments under clash covers is Extra-Contractual Obligations (ECO) which covers judgments against the reinsured which are "extra" or outside the policy. The plaintiff in the lawsuit is normally the original insured. The major cost in the judgment is often punitive damages. By nature, these are liabilities not covered under any provision of the policy. ECO claims normally involve wrongful or negligent claims handling by the reinsured. If, in addition, it is found that the cedant dealt with the claim in "bad faith," punitive damages may be awarded.

An example of an ECO loss is the case of Hedrick v. Sentry Insurance Company.[144] This case began with an auto accident on an interstate in May 1986. A truck rear-ended a northbound car and knocked it and a second car over the median into the southbound lanes. One of the cars struck head-on another vehicle containing Virgie Poston and her two adult daughters. All three were killed.

Ms. Poston's grandchildren filed suit against the drivers of both cars and the truck for damages in the deaths of their mothers and grandmother. The insurance companies for the three drivers settled, and the money was split among the grandchildren, the husband of one of the daughters, the driver of the car that struck Ms. Poston's, and Ms. Poston's husband.

Mr. Poston felt that he had been inadequately compensated for the death of his wife. In 1989, he filed suit against his own insurer, State Farm. According to Mr. Poston's attorney, the insurer failed to investigate, lost the claim file, and denied the $20,000 settlement Mr. Poston sought.

Mr. Poston and his son sued State Farm for bad faith arising out of their handling of the claim. On December 10, 1993 the jury awarded them $2.17 million in compensatory damages and $100 million in punitive damages. In March of 1994 while the appeal was pending, this case was settled out of court for an undisclosed amount.

There are a few items of note in this example. The first is the obvious one. It would have been easy for the insurer to settle the claim for a small amount of money early on in the case, and thereby avoid the shock verdict. The settled amount was probably substantially less than the verdict. On the other hand, with the negotiating leverage provided by the shock verdict, the settlement was probably very large. From the reinsurer's perspective, this claim is also likely to have been a substantial loss for any clash reinsurance program the insurer chose to purchase.

The second point is the elapsed time involved here. The accident was in May 1986 and the jury award was late 1993. It is entirely possible that a clash reinsurer would see no reported losses to the layer from a claim such as this for seven years. If appealed, the final value of the claim may not be known for several years after that. This can be a fairly long-tailed coverage.

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[143] From "Pricing Extra-Contractual Obligations and Excess of Policy Limits Exposures in Clash Reinsurance Treaties," by Paul Braithwaite and Bryan C. Ware, Spring 1997 CAS Forum, not on the syllabus.
[144] 96-128100-90 (Dist. Ct., Tarrant Co., Texas).

Truncated Pareto Distribution Increased Limit Factors:[145]

At the time Clark wrote his study note, ISO was using a truncated Pareto Distribution as the basis of its increased limits factors for liability. After that, ISO switched to a mixture of Pareto Distributions. Currently, ISO uses a mixture of many Exponential Distributions.[146] Thus you are unlikely to be asked about the truncated Pareto Distribution on your exam.

Clark gives a formula for the limited expected value:[147] [148]

$$E[X \wedge L] = PS + \frac{1 - P}{Q - 1} \left\{ (B + QT) - (B+L) \left(\frac{B+T}{B+L}\right)^Q \right\}.$$

B is the scale parameter of the Pareto, which would be q in Loss Models.
Q is the scale parameter of the Pareto, which would be a in Loss Models.
T is the left truncation point; "small" losses are of size less than T,
while "large" losses follow a (Shifted) Pareto truncated from below at T.
P is the probability of a "small" loss.
S is the average size of a "small" loss.

Exercise: Let B = 50,000, Q = 2.5, T = 10,000, P = 60%, and S = 6000.
Calculate the limited expected values at 25,000 and 50,000.
[Solution: $E[X \wedge 25{,}000]$ =
$(60\%)(6000) + (40\%/1.5) \{50{,}000 + (2.5)(10{,}000) - (75{,}000)(60/75)^{2.5}\} = 12{,}151$.
$E[X \wedge 50{,}000]$ =
$(60\%)(6000) + (40\%/1.5) \{50{,}000 + (2.5)(10{,}000) - (100{,}000)(60/100)^{2.5}\} = 16{,}164$.
Comment: All values were selected solely for illustrative purposes.]

Exercise: Assuming 25,000 were the basic limit, determine the Increased Limits Factor for 50,000.
[Solution: 16,164 / 12,151 = 1.33.]

Here we are not interested in any detail on the small losses, since we are calculating ILFs and T is below the basic limit.

$$E[X \wedge T] = PS + \frac{1 - P}{Q - 1} \{ (B + QT) - (B+T) \} = PS + (1-P)T.$$

This makes sense, since we are in this case limiting every large loss to size T.

[145] See pages 27 to 28 of Clark.
[146] I believe that ISO uses a mixture of 5 Exponentials, but the number may vary by line of insurance.
[147] I would not memorize this formula.
[148] Under uniform inflation B, T, and S would each be multiplied by the inflation factor.

Letting L go to infinity, $E[X] = PS + \frac{1 - P}{Q - 1}(B + QT)$.

For the untruncated Pareto: $S(x) = \left(\frac{B}{B+x}\right)^Q$, $E[X] = B/(Q-1)$,

and $E[X \wedge x] = \frac{B}{Q - 1}\{1 - \left(\frac{B}{B+x}\right)^Q\}$.

Thus the mean of a Pareto truncated and shifted from below at T is:[149]

$(E[X] - E[X \wedge T]) / S(T) = \frac{B}{Q - 1} \left(\frac{B}{B+T}\right)^{Q-1} / \left(\frac{B}{B+T}\right)^Q = (B+T) / (Q-1)$.

Thus the mean of a Pareto truncated from below at T is: $T + (B+T) / (Q-1) = (B + QT) / (Q-1)$.

Thus the above form for the mean is: $PS + \frac{1 - P}{Q - 1}(B + QT) =$

(probability of small loss)(mean of small loss) + (probability of large loss)(mean of large loss).

Recall that in general: $E[X \wedge L] = \int_0^L S(x)\, dx$.

Thus the survival function is the derivative of the limited expected value.

In this case, for $x \ge T$, we get: $S(x) = (1 - P)\left(\frac{B+T}{B+x}\right)^Q$.

$\Rightarrow S(T) = 1 - P$, which also follows from the definitions of P and T.

For an untruncated Pareto: $S(x) = \left(\frac{B}{B+x}\right)^Q$.

Thus for a Pareto truncated from below at T: $S(x) = \left(\frac{B}{B+x}\right)^Q / \left(\frac{B}{B+T}\right)^Q = \left(\frac{B+T}{B+x}\right)^Q$.[150]

We multiply by (1-P) the probability of a large loss, and get the above form of S(x) used by ISO.

If in the form for S(x), we let B go to zero and Q go to one, then S(x) goes to: $(1-P)\ T/x$, $x \ge T$.
This is (1 - P) times the survival function of a Single Parameter Pareto Distribution as per Loss Models, with $\alpha = 1$ and $\theta = T$.[151]
Then $S(T) = 1 - P$ = probability of a large loss, as it should.
$E[X \wedge T] = P\,S + (1-P)T$. Then integrating the survival function from T to x:
$E[X \wedge x] = P\,S + (1-P)\,T + (1-P)T\{\ln(x) - \ln(T)\} = P\,S + (1-P)\,T\,\{1 - \ln[T/x]\}$, $x \ge T$.

---

[149] This is the average payment per payment is: $(E[X] - E[X \wedge d]) / S(d)$.
By truncated and shifted, I mean that the truncation point is subtracted from each value.
[150] The survival function after left truncating at d, is the original survival function divided by S(d).
[151] At page 28, Clark calls this a LogLogistic, but that does not seem right.

Thus for the limit as B goes to zero and Q goes to one:
$E[X \wedge L] = P S + (1-P) T \{1 - \ln[T/L]\}$, $L \geq T$.

Exercise: Let T = 10,000, P = 60%, and S = 6000.
Calculate the expected losses in the layer from 25,000 to 50,000.
[Solution: $E[X \wedge 25{,}000] = (60\%)(6000) + (40\%)(10{,}000)\{1 - \ln[10{,}000 / 25{,}000]\} = 11{,}265$.
$E[X \wedge 50{,}000] = (60\%)(6000) + (40\%)(10{,}000)\{1 - \ln[10{,}000 /50{,}000]\} = 14{,}038$.
$E[X \wedge 50{,}000] - E[X \wedge 25{,}000] = 14{,}038 - 11{,}265 = 2773$.
Comment: All values were selected solely for illustrative purposes.]

Exercise: Let T = 10,000, P = 60%, and S = 6000.
Calculate the expected losses in the layer from 50,000 to 100,000.
[Solution: $E[X \wedge 100{,}000] = (60\%)(6000) + (40\%)(10{,}000)\{1 - \ln[10{,}000 / 100{,}000]\} = 16{,}810$.
$E[X \wedge 100{,}000] - E[X \wedge 50{,}000] = 16{,}810 - 14{,}038 = 2772$.
Comment: Subject to rounding, the same result as for the previous exercise.]

In both of these exercises, the top of the layer was twice the bottom of the layer, and the expected losses in the layers were the same. In general, this severity distribution has the property that expected losses in layers are equal if the ratio of the top and the bottom of the layers are the same:
$E[X \wedge U] - E[X \wedge L] = E[X \wedge kU] - E[X \wedge kL]$, for any constant k.

Exercise: Prove the above result.
[Solution: $E[X \wedge U] - E[X \wedge L] = (1-P) T \{1 - \ln[T/U]\} - (1-P) T \{1 - \ln[T/L]\} = (1-P) T \ln[U/L]$.
$E[X \wedge kU] - E[X \wedge kL] = (1-P) T \{1 - \ln[T/(kU)]\} - (1-P) T \{1 - \ln[T/(kL)]\} = (1-P) T \ln[(kU)/(kL)]$
$= (1-P) T \ln[U/L] = E[X \wedge U] - E[X \wedge L]$.]

This relationship holds for severities for individual claims but not for treaty loss costs. Treaty loss costs will decrease for higher layers due to fewer policies being exposed.

Mixture of Exponential Distributions, Increased Limits Factors:[152]

A Mixed Exponential, which is a mixture of several exponential distributions, is used by ISO to calculate Increased Limits Factors.

For an exponential with mean θ, $E[X \wedge L] = \theta\ (1 - \exp[-L/\theta])$.
Let a mixture give weight w_i to an Exponential Distribution with mean θ_i.[153]
Then for the mixture: $E[X \wedge L] = \sum w_i\ \theta_i\ (1 - \exp[-L/\theta_i])$.[154]

For example, assume severity is the mixture of four Exponential Distributions:

Mean	Weight
3000	82.5%
20,000	15.9%
300,000	1.4%
2,000,000	0.2%

Exercise: Determine the limited expected value at 100,000.
[Solution: $(82.5\%)(3000)(1 - e^{-100/3}) + (15.9\%)(20,000)(1 - e^{-100/20})$
$+ (1.4\%)(300,000)(1 - e^{-100/300}) + (0.2\%)(2,000,000)(1 - e^{-100/2000}) = 7019$.]

Exercise: Determine the limited expected value at 500,000.
[Solution: $(82.5\%)(3000)(1 - e^{-500/3}) + (15.9\%)(20,000)(1 - e^{-500/20})$
$+ (1.4\%)(300,000)(1 - e^{-500/300}) + (0.2\%)(2,000,000)(1 - e^{-500/2000}) = 9947$.]

Exercise: For a basic limit of $100,000, determine the increased limits factor for $500,000.
[Solution: 9947 / 7019 = 1.417.]

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[152] See pages 28 and 29 of Clark. He does not show an example.
[153] The weights w_i sum to one.
[154] The limited expected value of the mixture is the mixture of the limited expected values.

Fitting Curves to Excess Loss Factors:[155]

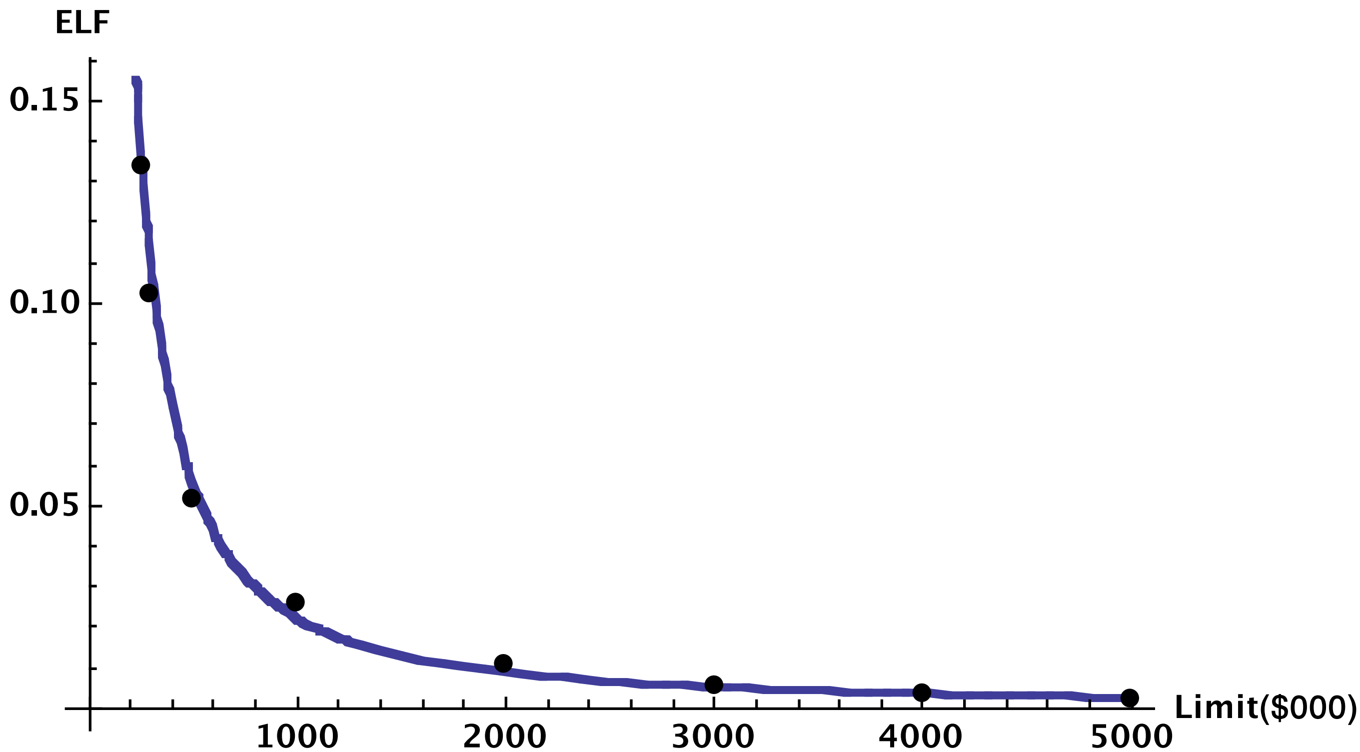
According to Clark, "The NCCI curves, or other excess factors, can easily be approximated by an inverse power curve of the form: $ELF_L = a\ L^{-b}$.
The parameters "a" and "b" are estimable from selected excess factors."[156]

Here is an illustration of this fitting, for higher limits. I took ELFs for Hazard Group 3 from Exhibit 8 of "Workers Compensation Excess Ratios: an Alternative Method of Estimation,": by Howard Mahler.

Limit ($000):	250	300	500	1000	2000	3000	4000	5000
ELF:	0.1337	0.1024	0.0517	0.0256	0.0107	0.0056	0.0033	0.0021

$ELF_L = a\ L^{-b}$. $\Leftrightarrow$ $\ln[ELF_L] = \ln[a] - b \ln[L]$.
Thus I fit a linear regression to ln[L] and $\ln[ELF_L]$.
The resulting fitted curve was for L the limit in thousands of dollars: $214.3 / L^{1.329}$.[157]

Here is a comparison of the ELFs and the fitted curve:



155 See page 30 of Clark. He does not show an example.
156 Clark mistakenly writes the ELF as the excess ratio; an ELF is the expected loss ratio times the excess ratio.
157 The excess ratios for a Shifted Pareto Distribution in the righthand tail are approximately inversely proportional to $x^{\alpha-1}$.

Umbrella Policies:

For example, Matthew buys an automobile policy with a liability limit of $250,000, and a homeowners policy with a liability limit of $250,000. If Matthew has assets worth a lot more than $250,000, then he can obtain additional protection by buying a personal “Umbrella Policy” with a limit of $1 million.

If a claim against Matthew is covered by his auto or homeowners policy, then the insurer will pay up to the limit of $250,000. However, if the damage to the third party or third parties is more than $250,000, then the umbrella policy will pay up to $1 million additional in damages.

Also, there are some claims that would not be covered by Matthew’s automobile policy or homeowners policy. Examples include if Matthew were sued over an incident involving a boat he rented on vacation, a car Matthew rented in Europe, or Matthew’s work on a nonprofit board of directors. In those cases, the umbrella policy provides coverage from such claims from first dollar.

So an umbrella policy both provides additional liability coverage and fills any gaps in the liability coverage in the primary policies. An umbrella policy would provide coverage for claims that may be excluded by other liability policies including: false arrest, libel, slander, and liability coverage on rental units owned by the insured.

Note that in this example, Matthew may have bought all three policies from the same insurer, two of the policies from the same insurer, or all three policies from different insurers.

A business could buy a commercial umbrella policy in addition to a CGL policy. If the CGL policy did not cover pollution claims, then the umbrella policy could cover these claims. The drop down provision in umbrella policies provides that the umbrella will "drop down" over reduced or exhausted underlying policy aggregate limits.[158]

For example, Cunningham Hardware Store buys a CGL policy with basic limits of $100,000 per occurrence and $200,000 aggregate. Cunningham Hardware Store also buys a commercial umbrella policy with a $1 million limit.[159] If Cunningham Hardware Store had a single occurrence for $170,000, then the CGL policy would pay $100,000, while the umbrella policy would pay the remaining $70,000.

If instead in the period covered by this policy there are four occurrences of $75,000, $50,000, $75,000, and $50,000, then the first three occurrences exhaust the aggregate limit of the CGL policy. Thus the CGL policy would pay nothing for the fourth occurrence. The umbrella policy would drop down and pay $50,000 for the fourth occurrence.

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[158] Some umbrellas maintain their own coverage terms when they drop down; others assume those of the primary policy.

[159] It would be more common to buy an umbrella policy over increased limits on a CGL policy.

Umbrella Policies and Casualty Excess Treaties:[160]

If the ceding company sells umbrella polices, this makes it more complicated to price an excess reinsurance treaty. Clark deals with two cases:
1. The umbrella policy and primary policy are written by the same insurer, the ceding company.
2. The umbrella policy is written by the ceding company, but the primary policy is written by another insurer.

In the first case, consider the primary policy and the umbrella policy a single policy with a higher limit.
If this were the case with Matthew, then treat it as a single policy with a $1.25 million limit.[161]

The second case is more complicated to deal with both for experience and exposure rating an excess treaty.

When experience rating, Clark recommends:
Trended Loss = (Loss + Underlying Limit) (Trend Factor) - Underlying Limit.

For example, in the Matthew example, assume his homeowners and umbrella policies are written by separate insurers. A reinsurer is experience rating an excess treaty for the insurer who wrote the umbrella policy.

The ceding company had a loss of $100,000 under the umbrella policy. Assume a trend factor of 1.2. Then the trended loss is:[162] [163]
($100,000 + $250,000) (1.2) - $250,000 = $170,000.

Clark points out that the actuary would not be taking into account large claims that did not exhaust the limit of the homeowners policy. For example, in the experience period there might have been a homeowners liability claim for $225,000, which did not trigger an umbrella claim. The reinsurance actuary would not know about that claim. However, trending that claim it would be for (1.2)(250,000) = $270,000, which would result in a $20,000 loss under the umbrella policy.[164]

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[160] See pages 31 to 33 of Clark.
[161] Matthew buys an automobile policy with a liability limit of $250,000, and a homeowners policy with a liability limit of $250,000. Matthew also buys a personal Umbrella Policy with a limit of $1 million.
[162] The umbrella claim has increased by 70%, since we are assuming that the claim was excess of the $250,000 paid under the homeowners policy. If this were a case where the umbrella policy was filling a gap in the coverage of the homeowners policy, then this would be an incorrect assumption.
[163] We have implicitly assumed that the limits of the primary policy remain the same rather than also increasing over time due to inflation.
[164] This would only have a major impact if there were many such claims and the trend factor was large.
In any case, the limits of the primary policy may also increase over time due to inflation.

Clark also discusses how to exposure rate this situation.

In the Matthew example, the underlying policies have a limit of 250,000 and the umbrella policy has a limit of $1 million.
In other words, the umbrella is covering the layer $1 million excess of $250,000.
$E[X \wedge 1.25 \text{ million}] - E[X \wedge 250{,}000]$,
is the average payment by the umbrella policy per ground up loss.

Let us assume the excess reinsurance treaty is $500,000 excess of $200,000, covering the layer from $200,000 to $700,000.

If the umbrella policy suffers a loss of $320,000, then the excess treaty pays $120,000.
However, this must have corresponded to a ground up loss of:
$250,000 + $320,000 = $570,000.

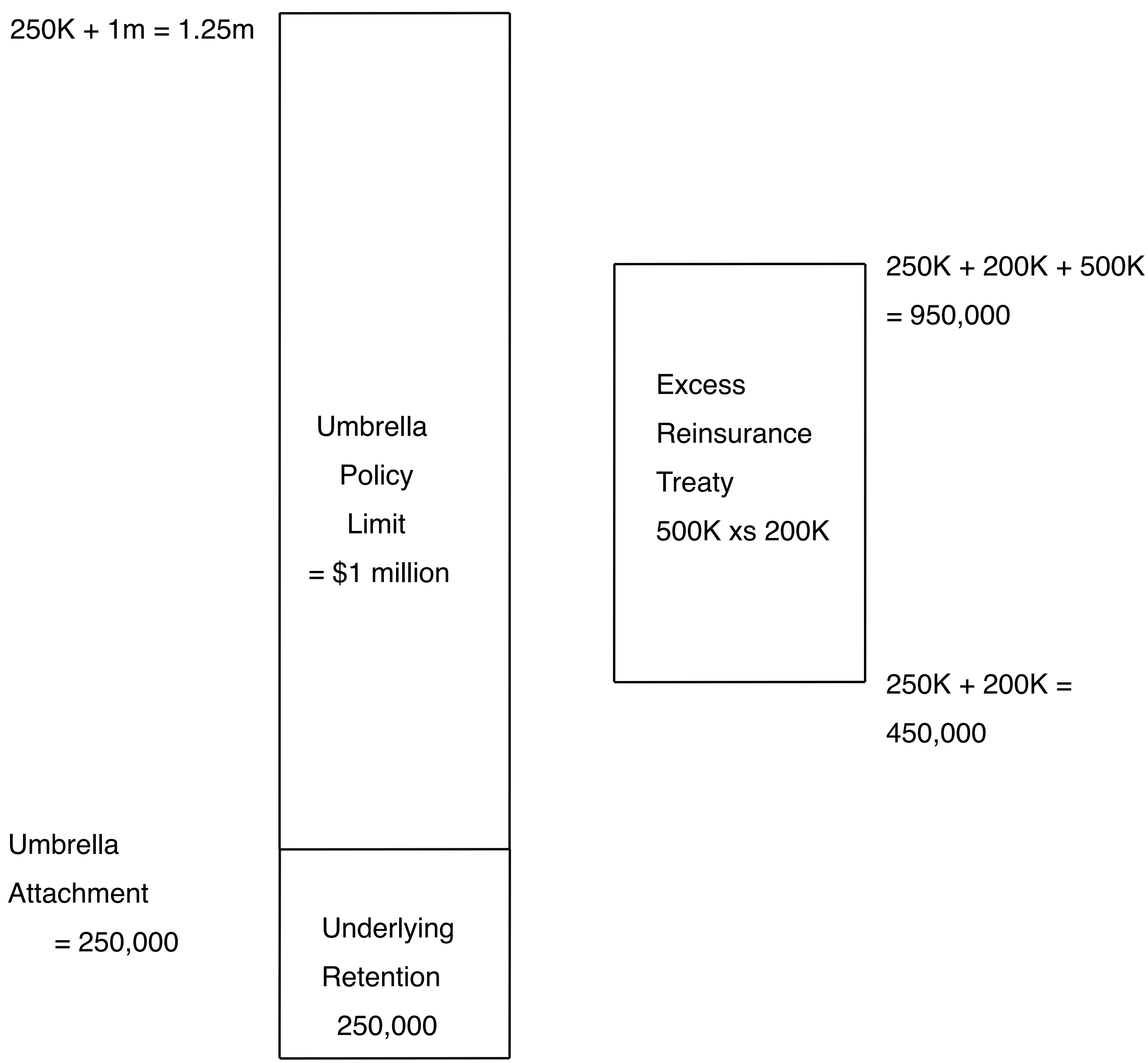
On a ground up basis, the excess treaty is covering the equivalent of the layer
from $250,000 + $200,000 = $450,000,
to $250,000 + $200,000 + $500,000 = $950,000.

$E[X \wedge 950{,}000] - E[X \wedge 450{,}000]$,
is the average payment by the excess treaty per ground up loss.

Thus in this case, the exposure factor is: $\dfrac{E[X \wedge 950{,}000] - E[X \wedge 450{,}000]}{E[X \wedge 1.25 \text{ million}] - E[X \wedge 250{,}000]}$.[165]

[165] This is an exposure factor to be applied to the premium for the umbrella policy written by the ceding insurer.

Here is a diagram for this example:



We note that here the layer covered by the reinsurance treaty is a subset of the layer covered by the umbrella policy.

In general, when exposure rating an excess treaty, when dealing with umbrella treaties written by the ceding company and the primary insurance is written by an insurer other than the ceding company:

$$\text{Exposure Factor} = \frac{E[X \wedge \text{Min}[UL + PL, UL + AP + Lim]] - E[X \wedge \text{Min}[UL + PL, UL + AP]]}{E[X \wedge (UL + PL)] - E[X \wedge UL]},$$

where:
X is the ground up size of loss variable
UL = Limit of Underlying Policies (attachment point of the umbrella)
PL = Policy Limit on Umbrella
AP = Excess Reinsurance Treaty Attachment Point
Lim = Excess Reinsurance Treaty Limit

In the Matthew example, UL = 250,000 and PL = $1 million.
In other words, the umbrella is covering the layer $1 million excess of $250,000.
$E[X \wedge (UL + PL)] - E[X \wedge UL] = E[X \wedge 1.25 \text{ million}] - E[X \wedge 250{,}000]$,
is the average payment by the umbrella policy per ground up loss.

The excess reinsurance treaty is $500,000 excess of $200,000, covering the layer from $200,000 to $700,000. In other words, AP = 200,000 and Lim = 500,000.

If the umbrella policy suffers a loss of $320,000, then the excess treaty pays $120,000.
However, this must have corresponded to a ground up loss of:
$250,000 + $320,000 = $570,000.

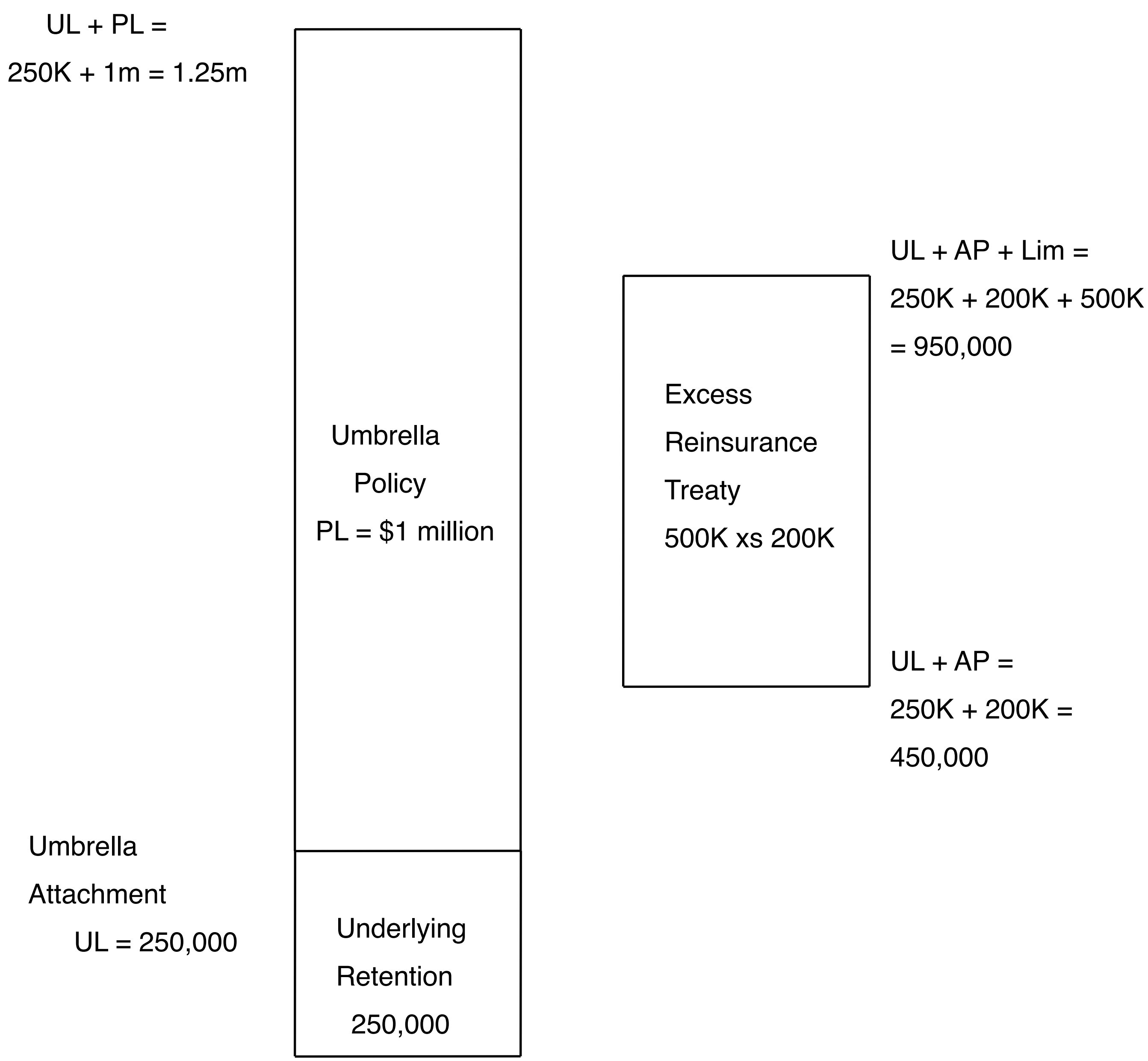
On a ground up basis, the excess treaty is covering the equivalent of the layer
from UL + AP = $250,000 + $200,000 = $450,000,
to IL + AP + Lim = $250,000 + $200,000 + $500,000 = $950,000.

$E[X \wedge \text{Min}[UL + PL, UL + AP + Lim]] - E[X \wedge \text{Min}[UL + PL, UL + AP]] =$
$E[X \wedge \text{Min}[250K + 1000K, 950{,}000] - E[X \wedge \text{Min}[250K + 1000K, 450{,}000]] =$
$E[X \wedge 950{,}000] - E[X \wedge 450{,}000]$,
is the average payment by the excess treaty per ground up loss.

Thus in this case, the exposure factor is: $\dfrac{E[X \wedge 950{,}000] - E[X \wedge 450{,}000]}{E[X \wedge 1.25 \text{ million}] - E[X \wedge 250{,}000]}$,

matching the previous result.

Here is another diagram for this example:[166]



In this example, on a ground up basis, the layer covered by the excess reinsurance is a subset of the layer covered by the umbrella policy. The more complicated formula in Clark also deals with those cases where on a ground up basis the layer covered by the excess reinsurance is not a subset of the layer covered by the umbrella policy.

[166] Similar to the diagram on page 33 of Clark.

Exercise: Assume instead that the excess reinsurance treaty is $900,000 excess of $300,000.
(The reinsurance treaty covers the layer from 300K to 1200K.) Determine the exposure factor.
[Solution: The excess treaty is covering the equivalent on a ground up basis of the layer from $250,000 + $300,000 = $550,000, to $250,000 + $1,200,000 = $1,450,000.
However, the umbrella only covers the equivalent on a ground up basis of the layer from $250,000 to $1,250,000. Therefore, for the excess treaty, $E[X \wedge 1{,}250{,}000] - E[X \wedge 550{,}000]$ is the average payment by the excess treaty per ground up loss.

Thus in this case, the exposure factor is: $\dfrac{E[X \wedge 1.25 \text{ million}] - E[X \wedge 550{,}000]}{E[X \wedge 1.25 \text{ million}] - E[X \wedge 250{,}000]}$.

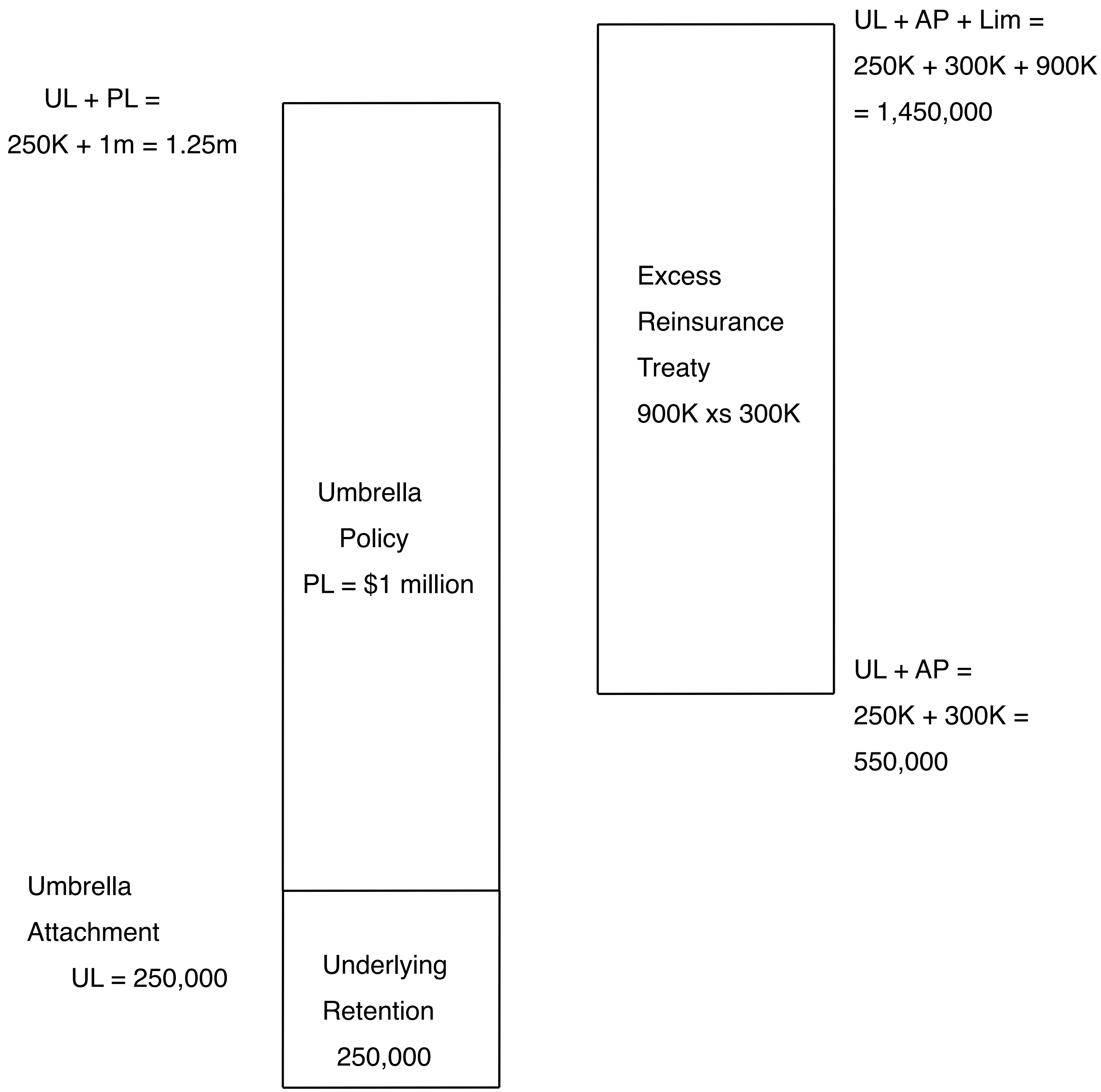
Alternately, $E[X \wedge \text{Min}[UL + PL, UL + AP + Lim]] =$

$E[X \wedge \text{Min}[250K + 1000K, 250K + 300K + 1200K]] = E[X \wedge 1250K]$.
$E[X \wedge \text{Min}[UL + PL, UL + AP]] =$
$E[X \wedge \text{Min}[250K + 1000K, 250K + 300K]] = E[X \wedge 550K]$.
$E[X \wedge (UL + PL)] = E[X \wedge (250K + 1000K)] = E[X \wedge 1250K]$.
$E[X \wedge PL] = E[X \wedge 250K]$.
Thus using the formula in Clark, the exposure factor is:

$$\frac{E[X \wedge \text{Min}[UL + PL, UL + AP + Lim]] - E[X \wedge \text{Min}[UL + PL, UL + AP]]}{E[X \wedge (UL + PL)] - E[X \wedge UL]} =$$

$$\frac{E[X \wedge 1.25 \text{ million}] - E[X \wedge 550{,}000]}{E[X \wedge 1.25 \text{ million}] - E[X \wedge 250{,}000]}.]$$

Here is a diagram for this example:



We note that here the layer covered by the reinsurance treaty extends above the layer covered by the umbrella policy. There are many umbrella policies in the portfolio covered by the treaty, and while some of them may be similar to this situation, others of them may instead be similar to the situations in the previous or following example.

Exercise: Assume instead that the excess reinsurance treaty is $1 million excess of $1.5 million. Determine the exposure factor.
[Solution: The excess treaty is covering the equivalent on a ground up basis of the layer from $250,000 + $1,500,000 = $1,750,000, to $250,000 + $2,500,000 = $2,750,000.
However, the umbrella only covers the equivalent on a ground up basis of the layer from $250,000 to $1,250,000. Therefore in this case, the exposure factor is zero.
Alternately, $E[X \wedge \text{Min}[UL + PL, UL + AP + Lim]] =$

$E[X \wedge \text{Min}[250K + 1000K, 250K + 1500K + 1000K]] = E[X \wedge 1250K]$.
$E[X \wedge \text{Min}[UL + PL, UL + AP]] =$

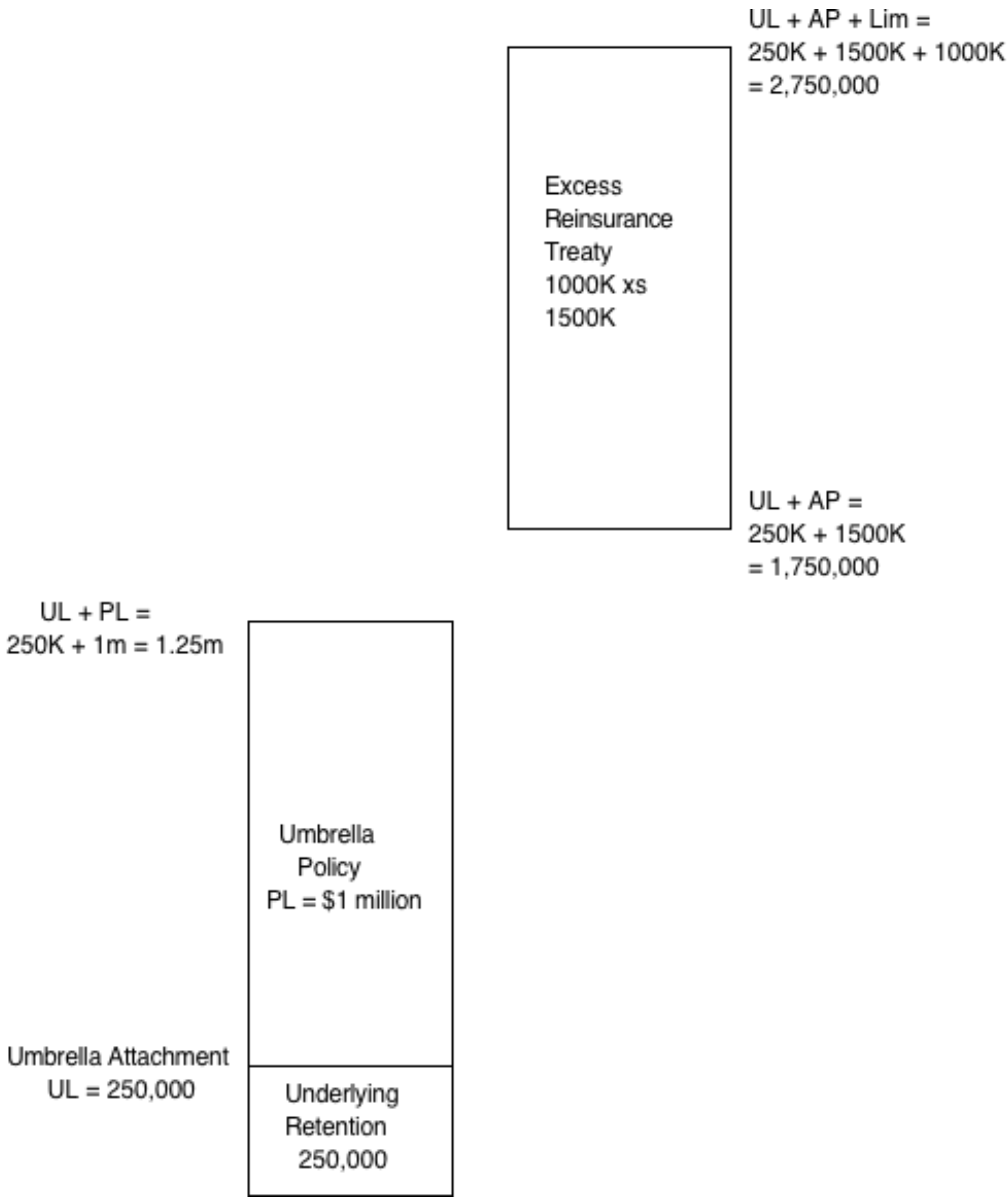
$E[X \wedge \text{Min}[250K + 1000K, 250K + 1500K]] = E[X \wedge 1250K]$.
Using the formula in Clark, the exposure factor is:

$$\frac{E[X \wedge \text{Min}[UL + PL, UL + AP + Lim]] - E[X \wedge \text{Min}[UL + PL, UL + AP]]}{E[X \wedge (UL + PL)] - E[X \wedge UL]} =$$

$$\frac{E[X \wedge 1.25 \text{ million}] - E[X \wedge 1.25 \text{ million}]}{E[X \wedge 1.25 \text{ million}] - E[X \wedge 250{,}000]} = 0.$$

<u>Comment</u>: In exposure rating, some blocks of policies in the limits profile can have an exposure factor of zero. Presumably in this case, other blocks of policies will have a positive exposure factor.]

Here is a diagram for this example:



Here the layer covered by the reinsurance treaty does <u>not</u> intersect with the layer covered by the umbrella policy.

This formula for the exposure factor does not deal with two sources of possible payments by the umbrella policy. As discussed, the umbrella policy may pay for things not covered by the underlying policy; Clark does not deal with this issue. In addition, the umbrella will drop down and pay if the aggregate limits of the underlying policy are exhausted.

Let us assume for example that the underlying policy is a 100/200 basic limit CGL. With the 100,000 per occurrence limit, but no aggregate limit, the expected losses would be larger. These "extra" losses, that are not covered by the CGL policy, would be paid by the umbrella policy.

Let us assume that the ILF for 100/1000 limits is 1.5.[167] Then the expected losses paid by the 100/200 limits policy are 1/1.5 = 2/3 of the expected losses paid by the 100/1000 limits policy. Also let us assume that with a \$100,000 per occurrence limit, a \$1 million aggregate limit is high enough to be virtually the same as infinity. Then of the losses to be paid by a policy with a \$100,000 occurrence limit and no aggregate limit, 1/3 would be paid by the umbrella policy due to the exhausting of the \$200,000 aggregate limit of the basic limits CGL policy.

Clark refers to this 1/3 as ϕ, the aggregate excess factor.
For an underlying CGL policy with limits occ/agg:[168]

$$\phi = \text{aggregate excess factor} = 1 - \frac{\text{expected losses with limits occ/agg}}{\text{expected losses with limits occ/}\infty}.$$

The higher the aggregate limit of the underlying CGL policy, the smaller ϕ.

If ϕ = 1/3, then for 2/3 of the losses limited by the occurrence limit, the umbrella acts as excess above the occurrence limit of the CGL, while for the remaining 1/3 of the losses the umbrella acts as if it were the primary policy. As discussed, in the former case, the average payment per loss is: $E[X \wedge (UL + PL)] - E[X \wedge UL]$. In the latter case, the umbrella drops down and pays up to its policy limit of PL, and thus the average payment per loss is:
$E[X \wedge PL] - E[X \wedge 0] = E[X \wedge PL]$.

Thus the average payment per loss for the umbrella policy is approximately:[169]
$(2/3)(E[X \wedge (UL + PL)] - E[X \wedge UL]) + (1/3)\, E[X \wedge PL] =$

$(1 - \phi)\, (E[X \wedge (UL + PL)] - E[X \wedge UL]) + \phi\, E[X \wedge PL]$.

This will be the denominator of the exposure factor.

As discussed, when the umbrella acts as excess above the occurrence limit of the CGL, the average payment per loss for the excess treaty is:
$E[X \wedge \text{Min}[UL + PL, UL + AP + Lim]] - E[X \wedge \text{Min}[UL + PL, UL + AP]]$.

[167] Assume for simplicity that all expenses are treated as variable and there is no risk load.
[168] Clark does not give an explicit definition. This is mathematically analogous to the Table M insurance charge. The mathematics of determining the aggregate insurance charge are the same as those of pricing aggregate limits for CGL policies, in other words determining increased limits with both an occurrence and an aggregate limit.
[169] ϕ is approximately the probability of the umbrella policy having to drop down.

When due to the drop down provision the umbrella acts as primary,
the average payment per loss for the excess treaty is:
$E[X \wedge \text{Min}[PL, AP + Lim]] - E[X \wedge \text{Min}[PL, AP]]$.

Thus the numerator of the exposure factor is:[170]

$$(1-\phi)\,(E[X \wedge \text{Min}[UL + PL, UL + AP + Lim]] - E[X \wedge \text{Min}[UL + PL, UL + AP]])$$
$$+\ \phi\,(E[X \wedge \text{Min}[PL, AP + Lim]] - E[X \wedge \text{Min}[PL, AP]])$$

Exercise: You are given the following information:

- A ceding company sells umbrella policies with \$2,500,000 limits excess of a \$500,000 underlying limit.
- The expected losses for this portfolio of umbrella polices are \$50 million.
- Based on contract language, the umbrella policy can "drop down" and provide coverage on a first dollar or primary basis. ϕ = aggregate excess factor = 15%.
- The limited expected values for various limits are:

Limit	Limited Expected Value
500,000	130,635
1,000,000	160,564
1,500,000	175,257
2,000,000	184,090
2,500,000	189,993
3,000,000	194,208
3,500,000	197,359

As a reinsurance actuary, you have been asked to price a casualty per occurrence excess treaty with a \$1,000,000 limit excess of a \$1,000,000 retention. The treaty will apply to umbrella losses.
ALAE is pro-rata in the treaty.
Determine the expected ceded losses for the treaty using the exposure rating method.
[Solution: For those situations where it does not drop down, the umbrella policy covers the layer from 0.5M to 3M. $E[X \wedge 3M] - E[X \wedge 0.5M] = 194{,}208 - 130{,}635 = 63{,}573$.
For those situations where it drops down, the umbrella policy covers the layer from 0 to 2.5M. $E[X \wedge 2.5M] = 189{,}993$.
For those situations where the umbrella does not drop down, the reinsurance treaty covers the layer from 1.5M to 2.5M. $E[X \wedge 2.5M] - E[X \wedge 1.5M] = 189{,}993 - 175{,}257 = 14{,}736$.
For those situations where the umbrella does drop down, the reinsurance treaty covers the layer from 1M to 2M. $E[X \wedge 2M] - E[X \wedge 1M] = 184{,}090 - 160{,}564 = 23{,}526$.

The exposure factor is: $\dfrac{(15\%)(23{,}526) + (85\%)(14{,}736)}{(15\%)(189{,}993) + (85\%)(63{,}573)} = 0.1945$.

Expected ceded loss is: (0.1945) (\$50 million) = \$9.725 million.].

[170] Clark shows an example rather than a general formula. See 6, 11/00, Q. 71.

Exercise: What if instead the excess treaty has a $2 million limit excess of a $1 million retention?
[Solution: When the umbrella does not drop down, the reinsurance treaty covers the layer from 1.5M to Min[3M, 3.5M] = 3M. $E[X \wedge 3M] - E[X \wedge 1.5M]$ = 194,208 - 175,257 = 18,951.
For those situations where the umbrella does drop down, the reinsurance treaty covers the layer from 1M to Min[2.5M, 3M] = 2.5M. $E[X \wedge 2.5M] - E[X \wedge 1M]$ = 189,993 - 160,564 = 29,429.

The exposure factor is: $\frac{(15\%)(29{,}429) + (85\%)(18{,}951)}{(15\%)(189{,}993) + (85\%)(63{,}573)} = 0.2487.$

Expected ceded loss is: (0.2487) ($50 million) = $12.435 million.]

<u>Further Discussion of the Aggregate Excess Factor</u>:[171]

A reinsurer writes an excess treaty covering a book of umbrella polices. Each umbrella policy is purchased by a business to provide additional insurance beyond its CGL policy.[172]

There are a number of issues:
- The aggregate limit of the CGL policy has an effect beyond that of just an occurrence limit.
- When the aggregate limit of the CGL policy is exhausted the umbrella policy drops down and becomes like a primary policy.[173]

The aggregate excess factor measures the effect of the CGL aggregate limit.
For example if ϕ = 10%, then on average the aggregate limit eliminates 10% of losses from a CGL policy compared to having no aggregate limit and the same occurrence limit.[174]

If the umbrella policy never dropped down, then the umbrella policy is providing coverage for a layer excess of the CGL occurrence limit. Then using a coverage diagram, we can figure out what the reinsurance treaty is responsible for and the expected gross losses for the umbrella policy.

If the umbrella policy drops down, then the umbrella policy is providing coverage for a layer instead starting at zero. The coverage diagram differs. Therefore, the reinsurance treaty is responsible for a different layer. Then using the appropriate coverage diagram, we can figure out what the reinsurance treaty expects to pay and the expected gross losses for the umbrella policy.

We determine exposures factors for these two separate situations.
Then we use ϕ to weight together the numerators and denominators of these exposures factors.

For example if ϕ = 10%, then we are assuming that approximately 10% of the time the CGL aggregate limit is exhausted and the umbrella policy drops down. We are assuming that the remaining 90% of the time the umbrella policy does <u>not</u> drop down. The numerator of the exposure factor would be a 10%-90% weighted average of the reinsurer's expected losses in these two situations, while the denominator of the exposure factor would be a 10%-90% weighted average of the expected gross losses of the umbrella policy in these two situations.

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[171] This is among the harder concepts.
[172] In a book of business, the umbrella policies would have varying limits. In addition, the underlying CGL policies would have varying occurrence and aggregate limits. In an example, for the sake of simplicity, one will usually ignore these complications.
[173] Clark only deals with the case of the umbrella policy dropping down when the CGL aggregate limit is exhausted.
[174] ϕ would depend on the underlying frequency distribution and the underlying severity distribution, as well as the occurrence and aggregate limits of the CGL policy.
See my section on Bahnemann, where I discuss split increased limits.
See also my Section on Table L, where I use the Panjer Algorithm to illustrate the effect of various accident limits.

Annual Aggregate Deductible, Casualty Excess Treaties:[175]

An excess treaty may have an annual aggregate deductible (AAD). Assume for example that a treaty included an annual aggregate deductible of $250,000. If during the year the total losses in the reinsured layer are less than or equal to $250,000, then the reinsurer pays nothing. If during the year the total losses in the reinsured layer are greater than $250,000, then the reinsurer pays that amount less $250,000.

Exercise: An insurer buys a 200K xs 100K excess treaty with an annual aggregate deductible of $250,000. During a year there are the following large occurrences:
$150,000, $200,000, $300,000, $500,000.
How much does the reinsurer pay?
[Solution: Total losses in the layer are: 50,000 + 100,000 + 200,000 + 200,000 = 550,000.
Thus the reinsurer pays: 550,000 - 250,000 = $300,000.]

With an annual aggregate deductible in the treaty, the insurer gets paid nothing if there are few large occurrences in a year, but does get paid something if there are many large occurrences in the year. With an annual aggregate deductible in the treaty, the insurer gets less coverage and pays less premium.

One can use a distribution of aggregate losses in order to estimate the reduction in the expected amount paid by the reinsurer due to the annual aggregate deductible. Step one is to only include the portion of each occurrence in the reinsured layer. Then one can use the Panjer algorithm to calculate the distribution of the aggregate annual amount in the reinsured layer.[176] Applying the annual aggregate deductible to see how much the reinsurer pays in each case, one can calculate the mean, variance, etc., of the amount the reinsurer pays.

An excess charge factor for a given AAD is defined as:[177]

$$\phi_{AAD} = \frac{\int_{AAD}^{\infty} (y - AAD)\, g(y)\, dy}{E[Y]},$$

where g(y) is the distribution of annual aggregate losses in the reinsured excess layer.

Alternately, $\phi_{AAD} = \dfrac{\text{expected amount ceded with AAD}}{\text{expected amount ceded without AAD}}$.

[175] See pages 33 and 34 of Clark.
[176] Subsequently, I show a simple example using the Panjer Algorithm.
[177] The excess charge factor is mathematically the same as a Table M Insurance charge or an excess ratio with respect to aggregate losses.

ϕ_{AAD} is multiplied by the loss cost on the layer gross of the AAD to estimate the net loss cost.

Exercise: If a given excess reinsurance treaty did not have an AAD, the expected ceded losses would be $22 million. For the AAD contained in the treaty, the excess charge factor is 70%. What are the expected ceded losses with the annual aggregate deductible?
[Solution: (70%)(22 million) = $15.4 million.]

If instead of a layer of loss with a finite top, we took the entire part of each occurrence excess of some attachment point, then this situation would be mathematically equivalent to a large deductible policy with an aggregate limit on deductible reimbursements. The attachment point here is analogous to the amount of the large deductible, while the annual aggregate deductible here is analogous to the aggregate limit on deductible reimbursements.

However, with a finite limit as is usual for an excess treaty, we are dealing with a layer with a finite top, which is mathematically more complicated than the large deductible with an aggregate limit. The large deductible with an aggregate limit is a special case of the situation here, as the limit in the reinsurance treaty approaches infinity.

Swing Plan, Casualty Excess Treaties:[178]

A swing plan is mathematically similar to retrospectively rating the excess treaty.
The final reinsurance premium is partially determined by the treaty's loss experience.

For example, let us assume a swing plan will be applied to a 500K xs 500K liability excess treaty.
Retro Premium is 120% of the actual ceded losses,
subject to a minimum premium of 12% of subject premium,
and subject to a maximum premium of 36% of subject premium.

Then in order to determine the average retro premium after the application of the swing plan, we need the distribution of aggregate ceded losses. Since the retro premium is a linear function of ceded losses in between the minimum and the maximum, we can calculate the average retro premium from the summary information for the aggregate ceded losses under the treaty:

Range of Ceded Losses over Subject Premium	Probability	Average in Range
Less than 10%	15%	7%
10% to 30%	65%	16%
More than 30%	20%	42%

For ceded losses equal to 10% of subject premium, the retro premium is: (1.2)(10%) = 12%.
For ceded losses less than 10% of subject premium, the retro premium is the minimum 12%.
For ceded losses equal to 30% of subject premium, the retro premium is: (1.2)(30%) = 36%.
For ceded losses greater than 30% of subject premium, the retro premium is the maximum 36%.
For ceded losses between 10% and 30% of subject premium, the average retro premium is:
(1.2)(16%) = 19.2%.
Thus the overall average retro premiums as a percent of subject premium is:
(15%)(12%) + (65%)(19.2%) + (20%)(36%) = 21.48%.

In addition, there would be a provisional rate which would be initially paid by the ceding company, in this case it might for example be 20% or 25%.[179] Then once the ceded losses are reported, the swing plan is used to adjust the ceded premiums from the initial level.[180]

This example of a swing plan was <u>not</u> specifically designed to balance to some predetermined level; to do so one would have to carefully select the minimum, maximum, and multiplier, so that the desired average is attained when calculated using the distribution of aggregate ceded losses.[181]

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[178] See pages 34 and 35 of Clark. Swing plans would usually be applied to working cover excess treaties.
[179] This and the other parameters of the swing plan are negotiated in advance between both parties.
The smaller the provisional rate, the more of a cashflow advantage to the primary insurer.
[180] As in a retrospective rating plan, if the experience is good the ceding company gets money back, while if the treaty experience is poor, then the ceding company owes money.(Here the ceding insurer stands in the place of the retrospectively rated insured.) As in a retrospective rating plan, there may be many adjustments until the losses are considered to be at ultimate.
[181] Subsequently, I have an example using the Panjer Algorithm.

Experience Rating Workers Compensation Excess Treaties:[182]

Fatal and permanent total claims for workers compensation are large claims where payments are made for many years.[183] Thus these claims are of great concern to the excess reinsurer.[184] [185]

When reserving fatal and permanent total claims for workers compensation most insurers use tabular reserves for the indemnity losses. Tabular reserves are based on an assumed mortality table and discount factor.[186] For permanent total claims, when the injured worker needs continuing lifetime medical care, the medical reserve can be quite large. Many insurers will also base such medical reserves on an assumed mortality table, discount factor, and inflation rate for medical expenses.

For example, a permanent total claim could be reserved for $2 million. However, the expected cost could be for example $3 million on an undiscounted basis. The reinsurance actuary is interested in the undiscounted value rather than the discounted value.

For example, let us assume the treaty is for $10 million xs $2.5 million. Then on a discounted basis, this permanent total claim does not piece the layer. However, if everything goes according to average, the claim will eventually cost $3 million, and reinsurer will have to pay $0.5 for this claim.[187]

Thus the reinsurer wants to collect sufficient information for individual claimants to project their expected (undiscounted) costs into the treaty layer.
According to Clark, the information needed is:
1. Claimant's current age
2. Claimant's sex (M/F)
3. Estimate of annual indemnity cost including escalation, if any
4. Estimate of annual medical cost
5. Amounts paid to date

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[182] See page 35 of Clark.
[183] For a fatal claim, the dependents of the dead worker would be paid to make up for lost wages, in some cases for their lifetime. For permanent total claims, the insurer would pay for medical care related to the injury and pay the injured worker and his dependents to make up for lost wages, in most cases for the lifetime of the injured worker. The medical care for a permanently disabled worker can be a significant amount of money per year. In some states, the payments for lost wages for fatal and permanent totals are adjusted each year to keep up with inflation.
[184] The reinsurer would also need to include a provision for unreported claims. For workers compensation, claims are reported more quickly than for liability insurance. Thus the pure IBNR to cover unreported claims is relatively small.
[185] Assuming the treaty is on a per accident per policy basis, then a large accident, such as an explosion at a factory, may result in many large workers compensation claims against the primary insurer. This possibility would be of very great concern to the excess reinsurer.
[186] They would also include an assumed escalation rate, where benefits are escalating, in other words adjusted yearly for inflation.
[187] For simplicity I have ignored the severity trend that would be applied in experience rating this treaty.

In addition, using the average value does not take into account the distribution of possible ultimate values.[188] A permanent total claim whose expected value is $3 million has a good chance of costing more than $4 million, which would pierce a layer excess of $4 million. Due to this dispersion effect, using the (undiscounted) average value can seriously underestimate the expected amount the reinsurer would pay for such a permanent total claim.[189]

Exercise: A particular permanent total claim is equally likely to cost the primary insurer either 1, 2, 3, 4, or 5 million dollars.
What are the expected losses ceded to a $2.5 million xs $2.5 million treaty?
[Solution: (1/5)(0) + (1/5)(0) + (1/5)(0.5) + (1/5)(1.5) + (1/5)(2.5) = $0.9 million.
Comment: For the expected claim size of $3 million, the expected ceded loss is only $0.5 million.]

An additional complication is the large variance in severity and relatively low frequency of fatal and permanent total claims. Unless the ceding company has a relatively large book of workers compensation business, there will be a large random element to a rate developed by experience rating the treaty.

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188 As with a retrospective rating plan, at some point prior to all claims being closed, the primary insurer and the reinsurer can close out the excess treaty via a commutation. The reinsurer would make a final payment of an agreed upon figure to the primary insurer in order to settle the treaty. Commutations and the complicated calculation of what might be a fair settlement are not on the syllabus of this exam.
See for example, "Commutation Pricing in the Post Tax Reform Era," by Vincent P. Connor and Richard Alan Olsen, CAS Discussion Paper Program, May 1990.
189 See the Discussion by Howard C. Mahler of "Retrospective Rating: 1997 Excess Loss Factors".

Aggregate Loss Distribution Models:[190]

As has been discussed, estimating adjustable features such as sliding scale commissions, aggregate deductibles, loss corridors, or swing plans, requires a distribution of aggregate losses.

Clark discusses: the Empirical Model, Use of Single Distributions, the Panjer Algorithm, and Other Collective Risk Models.

Empirical Distribution of Aggregate Losses:[191]

The empirical distribution is easy to calculate, and should be examined at least as a check on other methods. However, the experience does not take into account all possible outcomes, and may miss the possibility that the actual result is extremely different than the historical average.

For example, we might have 5 years of data on a certain treaty with loss ratios sorted from smallest to largest of: 55%, 60%, 65%, 70%, 75%.[192] However, a loss ratio well outside this observed range may be possible.

Clark lists three caveats with using the empirical distribution:

1) The experience does not take into account all possible outcomes, and may miss the possibility of events outside of what has been observed.
2) If the volume or mix of business has been changing, then the volatility of the future period may be very different than the historical period.
3) If loss development has been performed using a Bornhuetter-Ferguson or Cape Cod method, then the historical periods may present an artificially smooth sequence of loss ratios that does not reflect future volatility.

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[190] See pages 36 to 43 of Clark.
[191] See page 36 of Clark.
[192] There are a limited number of years on any given treaty that are useful; for years that are too distant one would have to make very large adjustments for trend. Also risk parameters shift over time.

Single Distribution Model of Aggregate Losses:[193]

One can model the aggregate losses by a distribution such as the LogNormal, Inverse Gaussian, or Transformed Gamma Distribution. There are no explicit separate models of frequency and severity, as in the collective risk model. While Clark gives lots of formulas for the LogNormal, I would be surprised if you are asked to use them on this exam.[194] However, you could be asked to use the limited expected values and distribution function values.

Exercise: Premium for an insurer is \$10 million.
Aggregate Losses for this insurer follow a LogNormal Distribution with $\mu = 15.3$ and $\sigma = 0.8$, for which:

Limit (\$ million)	Limited Expected Value (\$ million)	Distribution Function
4	3.277	0.4512
7	4.476	0.7180

A proportional treaty has a sliding scale commission provision:
Minimum Commission of 15% at a 70% loss ratio.
Sliding 1:1 to a Maximum of 45% at a 40% loss ratio.
Determine the average commission.
[Solution: The average size of aggregate losses in the interval 4 to 7 million is:

$$\frac{E[X \wedge 7m] - (7m)\,S(7m) - \{E[X \wedge 4m] - (4m)\,S(4m)\}}{F(7m) - F(4m)} =$$

$$\frac{4.476 - (7)(1 - 0.7180) - \{3.277 - (4)(1 - 0.4512)\}}{0.7180 - 0.4512} = 5.323 \text{ million.}$$

For a loss ratio of 53.23%, the commission would be: 15% + (70% - 53.23%) = 31.77%.
Thus the average commission is:
(0.4512)(45%) + (0.7180 - 0.4512)(31.77%) + (1 - 0.7180)(15%) = 33.0%.]

Advantages of the use of Single Distribution according to Clark:
1. Relatively simple to use, even when the source data is limited.
2. A reasonable fit is provided even when separate frequency and severity distributions are not known.

Disadvantages of the use of Single Distribution according to Clark:
1. There is no allowance for the loss free scenario.[195]
2. There is no easy way to reflect the impact of changing per occurrence limits on the aggregate losses.

[193] See pages 36 to 38 of Clark.
[194] Such calculations would require a Normal Distribution Table.
[195] One would need to add to the model a point mass of probability at zero.

LogNormal Distribution, Review:[196]

$$F(x) = \Phi\left[\frac{\ln(x) - \mu}{\sigma}\right] \qquad f(x) = \frac{\exp\left[-\frac{(\ln(x) - \mu)^2}{2\sigma^2}\right]}{x\,\sigma\sqrt{2\pi}}$$

Mean = $\exp[\mu + \sigma^2/2]$ Second moment = $\exp[2\mu + 2\sigma^2]$ $E[X^n] = \exp[nm + n^2\sigma^2/2]$
Variance = $\exp(2\mu + \sigma^2)\,\{\exp(\sigma^2) - 1\}$ $1 + CV^2 = E[X^2]/E[X]^2 = \exp[\sigma^2]$.

Exercise: A LogNormal Distribution has parameters $\mu = 5$ and $\sigma = 1.5$.
What are the mean and variance?
[Solution: Mean = $\exp[\mu + \sigma^2/2] = \exp[5 + (1.5^2)/2] = 457.145$.
Second Moment = $\exp[2\mu + 2\sigma^2] = \exp[10 + (2)(1.5^2)] = 1{,}982{,}759$.
Variance = $1{,}982{,}759 - 457.145^2 = 1{,}773{,}777$.]

Method of Moments: $\exp[\sigma^2] = 1 + CV^2$. $\Rightarrow \sigma^2 = \ln[1 + CV^2]$. $\Rightarrow \mu = \ln[\text{mean}] - \sigma^2/2$.

$$E[X \wedge x] = \exp(\mu + \sigma^2/2)\,\Phi\left[\frac{\ln(x) - \mu - \sigma^2}{\sigma}\right] + x\left\{1 - \Phi\left[\frac{\ln(x) - \mu}{\sigma}\right]\right\}.$$

Excess Charge Function = $\phi(L)$ = excess ratio at L = $\dfrac{E[X] - E[X \wedge L]}{E[X]}$

$$= 1 - \Phi\left[\frac{\ln(x) - \mu - \sigma^2}{\sigma}\right] - x\,\frac{1 - \Phi\left[\frac{\ln(x) - \mu}{\sigma}\right]}{\exp[\mu + \sigma^2/2]}.$$

Average size of losses in the interval from L to U is:

$$\frac{E[X \wedge U] - U\,S(U) - \{E[X \wedge L] - L\,S(L)\}}{F(U) - F(L)} =$$

$$\exp[\mu + \sigma^2/2]\;\frac{\Phi\left[\frac{\ln[U] - \mu - \sigma^2}{\sigma}\right] - \Phi\left[\frac{\ln[L] - \mu - \sigma^2}{\sigma}\right]}{\Phi\left[\frac{\ln[U] - \mu}{\sigma}\right] - \Phi\left[\frac{\ln[L] - \mu}{\sigma}\right]}.$$ [197]

[196] See page 37 of Clark. It would not hurt to remember the formulas for F(x) and the mean.
[197] For U = ∞, the first normal table lookup is 1. For L = 0, the second normal table lookup is 0.

Panjer Algorithm:[198]

The Panjer Algorithm is a recursive formula for calculating an aggregate distribution. The frequency distribution is assumed to be Poisson, Negative Binomial or Binomial.[199] The severity distribution is defined in discrete steps. The number of points evaluated on the severity distribution can be expanded to closely approximate continuous curves.

For example, if the discrete severity has support at: 0, 1000, 2000, 3000, 4000, ..., then the output of the algorithm is the probability of the aggregate losses being: 0, 1000, 2000, 3000, 4000, ...,

Advantages of the Panjer Algorithm according to Clark:
1. Simple to work with.
2. Provides an accurate handling of low frequency scenarios.

Disadvantages of the Panjer Algorithm according to Clark:
1. For higher expected frequencies, the calculation is inconvenient because all the probabilities up to the desired level must be calculated.[200]
2. Only a single severity distribution can be used in the analysis.

Usually the frequency and severity used as inputs to the algorithm are those for ground-up losses. However, for example if analyzing the effect of an annual aggregate deductible on an excess treaty, an actuary would only be interested in the contributions to the reinsured layer. In that case, the frequency would be that of those occurrences large enough to pierce the reinsured layer, and the severity would be the contribution of such large occurrences to the reinsured layer.

The Panjer Algorithm is a very useful tool which is easily programmed on a computer. However, I think you are very unlikely to be asked a question on your exam that requires you to perform a calculation of the aggregate distribution using the Panjer Algorithm.

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[198] See pages 39 to 41 of Clark.
Called the "Recursive Method" in Loss Models.
[199] As discussed in Loss Models, the algorithm can be generalized to work with the (a, b, 1) class of frequency distributions.
[200] There are ways around this difficulty. One could start the calculation at for example 6 standard deviations below the mean of the aggregate distribution. Then at the end one would have to normalize the distribution of aggregate losses to add to one.

Formulas for the Panjer Algorithm:

Frequency is a member of the (a, b, 0) class.[201] [202]

Distribution	a	b	f(0)
Binomial	q/(1-q)	(m+1)q/(1-q)	$(1-q)^m$
Poisson	0	λ	$e^{-\lambda}$
Negative Binomial	$\beta/(1+\beta)$	$(r-1)\beta/(1+\beta)$	$1/(1+\beta)^r$

Let the discrete severity distribution be S_i, and A_i be the discrete aggregate distribution.

The first step is to calculate the aggregate density at zero.
$A_0 = P(S_0)$, where P is the probability generating function of the frequency distribution.[203]
If there is no probability of the severity being zero, in other words if $S_0 = 0$,
then this simplifies as shown in Clark:
A_0 = probability of the frequency being zero.

Then we compute recursively the remaining densities of the aggregate distribution.

$$A_k = \frac{1}{1 - a\,S_0} \sum_{i=1}^{k} (a + b\,i/k)\, S_i\, A_{k-i}.$$

If $S_0 = 0$, then this simplifies as shown in Clark:[204]

$$A_k = \sum_{i=1}^{k} (a + b\,i/k)\, S_i\, A_{k-i}.$$

[201] For each of these three frequency distributions: f(x+1) / f(x) = a + {b / (x+1)}, x = 0, 1, ...
where a and b depend on the parameters of the distribution.
[202] Clark uses a different parameterization of the Negative Binomial Distribution: $\alpha = r$ and $p = 1/(1+\beta)$.
r = 1 is a Geometric Distribution.
[203] Not discussed by Clark, and thus not on the syllabus of this exam.
[204] The number of non-zero terms in the summation is limited by the maximum possible value of the discrete severity distribution. For numerical work, one would add zeros to the end of the vector of severity probabilities.

An Example of the Calculation of the Aggregate Distribution Using the Panjer Algorithm:[205]

Frequency follows a Negative Binomial Distribution with r = 0.3 and β = 4.
Severity is: one 50% of the time, two 30% of the time, and three 20% of the time.[206]

For the Negative Binomial Distribution, $a = \beta/(1+\beta) = 4/5 = 0.8$, $b = (r - 1)\beta/(1+\beta) = -0.56$.
A_0 = probability of the frequency being zero = $1/(1+\beta)^r = 1/5^{0.3} = 0.6170339$.

Then we compute recursively the remaining densities of the aggregate distribution.

$$A_k = \sum_{i=1}^{k} (a + b\, i/k)\, S_i\, A_{k-i} = \sum_{i=1}^{k} (0.8 - 0.56\, i/k)\, S_i\, A_{k-i}.$$

$A_1 = (0.8 - 0.56)S_1A_0 = (0.24)(0.5)(0.6170339) = 0.0740441$.

$A_2 = (0.8 - 0.56/2)S_1A_1 + \{0.8 - 0.56(2/2)\}S_2A_0$
$= (0.52)(0.5)(0.0740441) + (0.24)(0.3)(0.6170339) = 0.0636779$.

$A_3 = (0.8 - 0.56/3)S_1A_2 + \{0.8 - 0.56(2/3)\}S_2A_1 + \{0.8 - 0.56(3/3)\}S_3A_0$
$= (0.613333)(0.5)(0.0636779) + (0.426667)(0.3)(0.0740441) + (0.24)(0.2)(0.6170339)$
$= 0.0586232$.

$A_4 = (0.8 - 0.56/4)S_1A_3 + \{0.8 - 0.56(2/4)\}S_2A_2 + \{0.8 - 0.56(3/4)\}S_3A_1$
$+ \{0.8 - 0.56(4/4)\}S_4A_0 = (0.66)(0.5)(0.0586232) + (0.52)(0.3)(0.0636779)$
$+ (0.38)(0.2)(0.0740441) + (0.24)(0)(0.6170339) = 0.0349068$.

Continuing in this manner produces the following densities for the aggregate distribution from zero to twenty: 0.617034, 0.0740441, 0.0636779, 0.0586232, 0.0349067, 0.0280473, 0.0224297, 0.0173693, 0.0140905, 0.0114694, 0.00937036, 0.00773602, 0.00641437, 0.00534136, 0.00446732, 0.00374844, 0.00315457, 0.00266188, 0.00225134, 0.00190808, 0.0016202.

The corresponding distribution functions from zero to twenty are:[207]
0.617034, 0.691078, 0.754756, 0.813379, 0.848286, 0.876333, 0.898763, 0.916132, 0.930223, 0.941692, 0.951062, 0.958798, 0.965213, 0.970554, 0.975021, 0.97877, 0.981924, 0.984586, 0.986838, 0.988746, 0.990366.

[205] At pages 39 to 40, Clark has an example for a Poisson frequency.
[206] Chosen for simplicity. For example, severity could be in units of $50,000.
[207] One would continue to use the algorithm to calculate densities beyond twenty in a similar manner.

A Simple Example of an Aggregate Deductible:

Assume that an insurer buys a per policy excess treaty 100K xs 300K.
Then only occurrences of size greater than 300,000 can contribute to the ceded losses.
Let us assume that the number of large occurrences is Poisson with mean 3, and that 1/3 of them are of size $350,000, while 2/3 of them are 400,000 or more.[208] Then the contributions to the reinsured layer are $50,000 and $100,000 with Poisson frequencies of 1 and 2.

Using the Panjer algorithm I have calculated the distribution of the ceded losses; in units of $50,000 starting at zero, the probabilities are: 0.0497871, 0.0497871, 0.124468, 0.107872, 0.151436, 0.116585, 0.120388, 0.0838181, 0.0706712, 0.0451049, 0.032779, 0.0193817, 0.0125415, 0.00692832, 0.00407815, 0.00211943, 0.001152, 0.000566454, 0.00028747, 0.000134383, 0.0000642132, 0.0000286546, 0.0000129776, 0.00000554766, 0.00000239409, ...

Using these probabilities, the mean ceded losses are: $250,000 = $50,000 + (2)($100,000).

If the treaty also had an annual aggregate deductible of $150,000, then if the losses in the layer are less than or equal to $150,000, the reinsurer pays nothing. If for example, the losses in the layer were $350,000, then the insurer would pay only: $350,000 - $150,000 = $200,000.
Using the above probabilities, with a $150,000 aggregate deductible the average amount paid by the reinsurer is $118,670.[209]

The average amounts paid by the reinsurer for a selection of annual aggregate deductibles:[210]

Aggregate Deductible	Average Annual Amount Paid by Reinsurer	Excess Charge Factor
0	250,000	100.0%
50,000	202,489	81.0%
100,000	157,469	63.0%
150,000	118,670	47.5%
200,000	82,266	32.9%
250,000	59,433	23.8%
300,000	39,430	15.8%
350,000	25,446	10.2%
400,000	15,653	6.3%
450,000	9394	3.8%
500,000	5390	2.2%

[208] I have deliberately kept things simple.
[209] The corresponding vector of amounts paid with the $150,000 aggregate deductible is:
($50,000) (0, 0, 0, 0, 1, 2, 3, 4, ...)
[210] The excess charge factor is the ratio of the expected amount paid by the reinsurer with the given AAD to the expected amount the reinsurer would pay with no AAD.

A Simple Example of a Swing Plan:

Assume that an insurer buys a liability excess treaty 250K xs 250K, with subject premium of \$10 million. Then only occurrences of size greater than 250,000 can contribute to the ceded losses.

Assume that the number of large occurrences is Negative Binomial with r = 15 and β = 0.5.[211]
Assume that the severity of large occurrences is: 40% at \$300,000, 30% at \$400,000, and 30% at least \$500,000.[212]

Then the contributions to the reinsured layer are \$50,000, \$150,000, and \$250,000.
The average ceded severity is: (40%)(50,000) + (30%)(150,000) + (30%)(250,000) = \$140,000.
Thus the expected ceded losses are: (7.5)(\$140,000) = \$1.05 million.
This is 10.5% of the subject premium of \$10 million.

In units of \$50,000, the severity distribution is: 40% @ 1, 30% @ 3, and 30% at 5.[213]

Using the Panjer algorithm I have calculated the distribution of the ceded losses; in units of \$50,000 starting at zero, the probabilities are: 0.00228366, 0.00456732, 0.0048718, 0.00710641, 0.00951626, 0.0128266, 0.0171711, 0.0188888, 0.0218712, 0.0255359, 0.0282549, 0.0317442, 0.033569, 0.035214, 0.0374096, 0.0384107, 0.0394303, 0.0397497, 0.0394414, 0.0393164, 0.038519, 0.0374786, 0.0362373, 0.0346065, 0.0330114, 0.0312152, 0.0292923, 0.0273856, 0.0253932, 0.0234612, 0.0215656, 0.0196973, 0.0179299, 0.0162319, 0.0146347, 0.0131461, 0.0117511, 0.0104695, 0.00929109, 0.00821435, 0.00724, 0.00635724, 0.00556491, 0.00485634, 0.00422435, 0.00366458, 0.00316949, 0.00273372, 0.00235177, 0.00201769, 0.00172686, 0.00147428, 0.0012556, 0.00106695, 0.00090456, 0.000765218, 0.000645967, 0.000544148, 0.000457466, 0.000383826, 0.000321421, 0.00026866, 0.000224145, 0.000186674, 0.000155195, 0.000128804, 0.000106724, 0.0000882839, 0.0000729143, 0.000060127, 0.0000495065, 0.0000407015, 0.0000334137, 0.0000273918, 0.0000224238, 0.0000183317, 0.0000149664, 0.0000122028, 0.00000993676, 0.00000808132, 0.00000656421, 0.000005.32545, 0.00000431533, 0.00000349273, 0.00000282371, 0.00000228027, 0.00000183939, 0.00000148215, ...

For a swing plan let us take a minimum premium of 5% and a maximum premium of 20% of subject premium. In between the maximum and minimum, the retro premium will be some constant c times the total ceded losses. One can use the above distribution of total ceded losses to determine the average retro premium for various values of the multiplier c.[214] I then compared these average retro premiums to the expected ceded losses of \$1.05 million.

[211] The mean is: (0.5)(15) = 7.5, and the variance is: (0.5)(1.5)(15) = 11.25.
The mean number of large occurrences is related to the limits sold by the ceding company as well as the size of its book of business. The larger the mean frequency of large occurrences, the smaller the coefficient of variation of the ceded losses, all else being equal. The CV of the ceded losses helps to determine the average impact of the maximum and minimum premiums on average retro premiums.
[212] I have deliberately kept things simple.
[213] Thus the severity distribution used as an input to the Panjer algorithm is the vector: 0, 0.4, 0, 0.3, 0, 0.3, 0, 0, ...
[214] I did all of the calculations on a computer using Mathematica.

Multiplier	Average Retro Premium ($ million)	Average Retro Premium / Expected Ceded Loss
1.00	1.0579	1.0075
1.05	1.1020	1.0495
1.10	1.1444	1.0899
1.15	1.1857	1.1292
1.20	1.2252	1.1669
1.25	1.2631	1.2030
1.30	1.2994	1.2376
1.35	1.3340	1.2704
1.40	1.3667	1.3016

For example, for a multiplier of 1.25:
For ceded losses of $400,000 or less, the retro premium is the minimum premium of $500,000; this has a probability of 9.910%.
For ceded losses of $1,600,000 or more, the retro premium is the maximum premium of $2 million; this has a probability of 15.496%.
There is a 74.594% chance of ceded losses being more than $400,000 and less than $1,600,000; in this case the average ceded losses are $0.96917 million.

Therefore, for a multiplier of 1.25, the average retro premium is:
(9.910%)($500,000) + (15.496%)($2 million) + (74.594%)(1.25)($0.96917 million)
= $1.2631 million.

As calculated previously, the expected ceded losses are: (7.5)($140.000) = $1.05 million.
Then 1.2631/1.05 = 1.2030, as shown above.

We see that all else being equal, the higher the multiplier, the larger the average retro premium. Assuming that the minimum and maximum premiums are acceptable, using the above table, the reinsurance actuary can choose a multiplier such that the ratio of the average retro premium to the expected ceded losses is acceptable.[215]

[215] All the features of the swing plan have to be negotiated in advance, and be acceptable to both the ceding company and the reinsurer.

<u>Other Collective Risk Models</u>:[216]

Collective risk models assume a specific frequency distribution, a specific severity distribution, and that frequency and severity are independent. The Panjer algorithm is one technique of calculating the distribution of aggregate losses for a collective risk model. Other techniques are those of Heckman-Meyers, Robertson, and Wang.[217] One could also use simulation.

If y is aggregate loss, then the expected value given y is in the interval from L to U is:[218]

$$E[y \mid L < y < U] = \frac{E[Y \wedge U] - U\,S(U) - \{E[Y \wedge L] - L\,S(L)\}}{F(U) - F(L)}$$

$$= \frac{E[Y]\,\{R(L) - R(U)\} + L\,S(L) - U\,S(U)}{F(U) - F(L)}.$$

The output of these models can be used to price various reinsurance situations.
Under a Stop Loss Treaty, a reinsurer would pay all of the aggregate losses above an annual aggregate deductible. The distribution of aggregate losses would be useful to price such a treaty.

Exercise: Assume the annual aggregate losses in millions of dollars for an insurer are approximated by the following discrete distribution:
f(5) = 0.6, f(10) = 0.2, f(15) = 0.1, f(20) = 0.05, f(25) = 0.03, f(30) = 0.02.
The insurer purchases a Stop Loss treaty with an annual aggregate deductible of 10 million.
What are the expected losses paid by the reinsurer?
[Solution: For aggregate losses of: 5, 10, 15, 20, 25, and 30, the amounts paid by the reinsurer are respectively: 0, 0, 5, 10, 15, 20.
Thus the expected amount paid by the reinsurer is:
(0)(0.6) + (0)(0.2) + (5)(0.1) + (10)(0.05) + (15)(0.03) + (20)(0.02) = 1.85 million.]

[216] See pages 41 to 43 of Clark.
[217] Clark provides no detail.
[218] The first equation is for the average size conditional on being in an interval, while the second equation is from the top of page 42 of Clark. R(y) is the excess ratio at y for the distribution of aggregate losses, which Clark denotes as $\phi(y)$, the excess charge factor.

Clark lists the following cautions when using one of these algorithms to calculate aggregate losses:

1. The complexity of the calculations can lead to assuming the numbers must be right because of the accuracy of the computer. More than one set of results should be produced, as a check on the sensitivity of the answer to the starting assumptions.[219] Some basic statistics, such as the coefficient of variation and percentiles, should be compared to the empirical data for reasonability.

2. Most models assume that each occurrence is independent of the others and that the frequency and severity distributions are independent of each other. This may be a reasonable assumption in many cases, but could be false in others.

3. Some collective risk models use numerical methods with a large error term for low frequency scenarios. Check the output of the model; the expected error term should be given.

4. The aggregate distribution reflects the process variance of losses but does not reflect the full parameter variance.[220] Some models allow for a prior distribution to apply to the selected parameters. “Model risk,” not being sure if you are even in the right model, is harder to estimate and is best reflected by repeated sensitivity analysis.

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[219] This would be a good idea whenever using a complex model, such as a computer model of catastrophes.
[220] “Process variance” refers to the random fluctuation of actual results about the expected value.
“Parameter variance” in the narrow sense refers to uncertainty about the parameters and may be calculable from outside sources.

Property Catastrophe Treaties:[221]

Catastrophes such as a hurricane or earthquake are infrequent, but their large size can threaten an insurer's solvency or at least impair its continued operation.[222] A property catastrophe reinsurance treaty provides an insurer with protection from a catastrophic event.

A catastrophe treaty applies to the total aggregate loss for the insurer from a single event. Typically, the catastrophe treaty applies to the ceding company's retained losses net of surplus share, per risk excess treaties, etc.; in other words, other reinsurance inures to the benefit of the catastrophe treaty.

For example, a single hurricane may affect multiple locations on multiple policies written by the ceding company. If for a single hurricane the aggregate loss for the ceding company, net of other reinsurance, is $52 million, then a catastrophe treaty $40 million excess of $30 million would pay the insurer $22 million.[223]

Exercise: A primary insurer has a 60% quota share treaty and a $20 million xs $10 million catastrophe treaty. The quota share treaty inures to the benefit of the catastrophe treaty.
Due to an earthquake, the primary insurer pays a total of $40 million in losses.
How much does the catastrophe reinsurance treaty pay?
[Solution: The quota share pays: (60%)(40) = $24 million.
The net losses are: 40 - 24 = 16 million.
The catastrophe reinsurance treaty pays: 16 - 10 = $6 million.]

As with other treaties, a catastrophe treaty may be on either a "losses occurring" or "risks attaching" basis. Assume a large hurricane occurs on July 13, 2014. If the catastrophe treaty is on a losses occurring basis, then the most the reinsurer can pay this primary insurer due to this hurricane is the limit of the treaty. However, if the catastrophe treaties were on a risk attaching basis, then there may be two treaties that apply.[224]

A treaty effective January 1, 2013 to December 31, 2013 would cover policies written by the ceding insurer during 2013; some of these policies would still be in effect on July 13, 2014.
A treaty effective January 1, 2014 to December 31, 2014 would cover policies written by the ceding insurer during 2014; some of these policies would be in effect on July 13, 2014. Thus if the hurricane were large enough the reinsurer could pay its limits on each of these reinsurance treaties for the single hurricane. On the other hand, by splitting the aggregate losses for this hurricane between two treaties, the ceding insurer could end up being paid nothing by the reinsurer when it would have been paid if only a single treaty applied.

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[221] See page 43 of Clark.
[222] In the United States, tornadoes produce about as many total dollars of loss, but the aggregate loss from a single tornado is nowhere near as large as the aggregate loss from a large hurricane.
[223] An insurer may buy more than one catastrophe treaty from different reinsurers, with each treaty covering a different layer. For example, it might buy three catastrophe treaties: $10M excess of $30M, $20M excess of $40M, and $40M excess of $60M.
[224] See pages 45 and 46 of Clark.

<u>Rating Property Catastrophe Treaties</u>:[225]

Since a catastrophe treaty applies to aggregate losses for the ceding company, the size of the primary insurer's book of business is one important consideration in rating the treaty. For two otherwise similar insurers, with the same size and type of properties, same locations of properties, same concentration of properties, etc., the larger insurer will have larger aggregate losses. For example, an attachment point of $50 million in a catastrophe treaty might be rarely reached for a smaller insurer, but much more frequently reached for a similar larger insurer.

As discussed in Grossi and Kunreuther, for perils for which they are available, computer models are now the preferred way to rate catastrophes. Clark mentions two other methods.

One could use the traditional payback approach. The payback approach asks how many years of premium are needed to cover a single loss that exhausts the treaty limit. For example, if the payback period were five years, then the premium would be 20% of the limit.

$$\text{Rate on line} = \frac{\text{Reinsurance Premium}}{\text{Treaty Limit}} = \frac{1}{\text{Payback Period}}.$$

Exercise: A $20 million excess of $40 million catastrophe treaty has a premium of $2 million. Determine the payback period and rate on line.
[Solution: Rate on Line = 2/20 = 10%. Payback period = 1/10% = 10 years.]

The payback method is essentially judgmental pricing, which is no longer used for property reinsurance.[226] The payback approach can be useful for setting relativities between layers. The rate on line for a treaty with a certain limit should decrease as the attachment point increases. For example, if a $20 million excess of $40 million catastrophe treaty has a rate on line of 10%, then an otherwise similar $20 million excess of $60 million catastrophe treaty should have a rate on line of less than 10%.

One could instead use experience rating. Unfortunately, for a single treaty, even many years of data on catastrophes would have little credibility.

Catastrophe models are now the generally accepted approach for pricing of natural and some man-made events.

[225] See pages 43 to 45 of Clark.
[226] Limitations of the payback method according to the previous edition of the study note by Clark:
1. Does not consider the growth in subject premium.
2. Does not consider reinstatement provisions.
3. Does not consider the reinsurer's expenses.
4. The amount of insured value exposed must be included subjectively.
5. The geographic spread of the insured properties must be included subjectively.

Using Catastrophe Models to Rate Property Catastrophe Treaties:[227]

Computer models of catastrophes attempt to estimate the total losses paid by an insurer for hurricanes or earthquakes. The distribution of possible insured loss outcomes is produced by simulating a large number of events.

There are four main components of typical catastrophe models:[228]

1. Event sets that simulate the covered hazards.[229]
2. Calculation of local event intensity for each property within a portfolio.
3. Estimation of damage for each property within a portfolio impacted by a given event.
4. Loss: Insured loss estimates based on policies written by the ceding company.

According to Clark, information needed by a catastrophe model:

1. Measure of exposure.[230]
2. Geographical information on insured properties.
3. Terms of the insurance policies
4. Details of any inuring reinsurance.

The output of a catastrophe model is a distribution of possible losses on the subject business. This can be used to calculate the expected amount in the reinsured layer, the average annual loss (AAL), along with its standard deviation. This can be used as a starting point for estimating a loss cost on a catastrophe treaty.

The Occurrence Exceedance Probability (OEP) curve represents the probability that at least one event during the year will exceed a given loss amount.[231] The OEP is an important output of a catastrophe model. The Annual Exceedance Probability (AEP) represents the probability that the total of all modeled events in a single year exceeds a given loss amount. The AEP is another possible output of a catastrophe model. The output of catastrophe models can also be used to estimate the probable maximum loss (PML).[232]

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[227] See pages 44 to 45 of Clark. Grossi & Kunreuther, on the syllabus of this exam, has more detail.
[228] Grossi & Kunreuther list the four components somewhat differently:
hazard, inventory, vulnerability, and loss.
[229] For example, hurricanes, earthquakes, terrorist events. These events cover the full range of possible sizes of a hazard at all relevant locations and are simulated based on estimates of frequency and intensity at specific locations.
[230] This should be insured values, construction types, occupancies, along with attachment points for excess contracts.
[231] See also Grossi & Kunreuther.
[232] See Grossi & Kunreuther, on the syllabus of this exam, for more detail.

According to Clark, considerations that may need to be included subjectively if they are not considered in the catastrophe model:

1. Workers compensation losses may be included within the treaty. If there is an earthquake during standard working hours, this exposure could be substantial.[233]
2. The inuring reinsurance terms may not be able to be incorporated into the model.[234]
3. Even if earthquake coverage is not sold by the ceding company, there may still be exposure due to a "fire following" the earthquake.[235]
4. Other coverage terms, such as the portion of policyholders purchasing replacement cost coverage instead of actual cash value, may be critical. After a major catastrophe event, there may be increased demand for materials and labor which raise the total cost borne by the insurer. In other words, does the model take into account demand surge.

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[233] In the terrorist attacks on September 11, 2001 on the World Trade Center, there were some insurers who suffered large losses for Workers Compensation due to many workers in the same office dying.
Something similar could happen in an earthquake.
[234] This would not be hard for inuring quota share, but could be hard for more complicated forms of reinsurance.
[235] Many insurance policies that do not cover damage caused directly by earthquakes, still include coverage for damage due a fire started by an earthquake.
For example, the 1906 San Francisco Earthquake started fires which burned down a major part of the city . It has been estimated that up to 90% of the total destruction was the result of the subsequent fires. Within three days, over 30 fires, caused by ruptured gas mains, destroyed approximately 25,000 buildings on 490 city blocks.

Reinstatements, Catastrophe Treaties:[236]

A primary insurance company purchases a property catastrophe reinsurance treaty providing coverage of $40 million excess of $20 million. Then one large event would exhaust the coverage, leaving the ceding insurer without reinsurance protection for the remainder of the year. One solution is to have a reinstatement clause in the treaty that allows the limits to be reset.

Reinstatement clauses deal with the possibility of more than one large event per year. Without additional reinstatements, the catastrophe treaty would provide its limit of $40 million of coverage, but after the full layer is exhausted there is no more protection. Typically a property catastrophe treaty will allow for reinstatements.

Assume for example, that there is an event for $35 million. Then the above treaty would provide that the insured has used up $15 million of the available $40 million in coverage.

The primary insurer could restore the full $40 million in coverage by paying a reinstatement premium.[237] Reinstatement is usually automatic, to provide immediate coverage in case of another catastrophic event. For example, if the reinstatement premium were 110% pro-rata as to amount, then in this case, the reinstatement premium would be the original reinsurance premium times: (110%)(15/40) = 41.25%.

If additional reinstatements are available “pro-rata as to amount” then:

$$\frac{\text{Reinstatement Premium}}{\text{Reinsurance Premium}} = (\text{Specified factor}) \ \frac{\text{Ceded Losses}}{\text{Reinsurance Treaty Limit}}.$$

Exercise: A $40 million excess of $20 million catastrophe treaty has a premium of $3 million.
The reinsurer pays $18 million as the result of a catastrophe.
The reinstatement provision is 110%, pro-rata as to amount.
Determine the reinstatement premium.
[Solution: (110%)(18/40)($3 million) = $1,485,000.]

Less frequently, the reinstatement premium is pro-rata as to time, meaning that the premium would be further reduced to reflect only the amount of time left in the policy period. Given the seasonal nature of some types of catastrophes such as hurricanes, relatively few catastrophes treaties include reinstatements pro-rata as to time.

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236 See page 43 of Clark. See also 6, 11/10, Q.22 and 8, 11/13, Q.21.

237 While it is possible for a catastrophe treaty to have free reinstatements, usually the ceding company has to pay a reinstatement premium. This and other provisions would be spelled out in the reinstatement clause of the catastrophe reinsurance treaty.

Exercise: A $40 million excess of $20 million catastrophe treaty has a premium of $3 million.
The treaty is effective from January 1, 2015 to December 31, 2015 and on a losses occurring basis. The reinsurer pays $23 million as the result of a catastrophe on August 1, 2015.
The reinstatement provision is 110%, pro-rata as to both amount and time.
Determine the reinstatement premium.
[Solution: On August 1, 2015 there are 5 out of 12 months remaining in the year of coverage.
(110%)(23/40)(5/12)($3 million) = $790,625.
Comment: Clark provides no details on pro-rata with respect to time.
If the treaty is instead on a risks attaching basis, then the treaty covers Policy Year 2015.
Ignoring any rate changes, the expected proportion of events that have happened by a given date is the same as the portion of premiums earned by that date. If the primary policies are annual, then the primary insurer has earned $(1/2)(7/12)^2 = 0.170$ of its PY2015 premium by August 1, 2015.
Thus the reinstatement premium would be: (110%)(23/40)(0.830)($3 million) = $1,574,925.]

Most property catastrophes are known immediately after they happen. The primary insurer and the reinsurer would usually know if the attachment point of the catastrophe treaty is likely to be exceeded, triggering the reinstatement clause. However, until the amount of ceded losses are finalized, the final amount of the reinstatement premium would not be known. Usually portions of the restatement premium are due as the reinsurer pays losses under the treaty.[238]

A treaty would usually allow a limited number of reinstatements. The treaty effectively has an aggregate limit equal to one plus the number of reinstatements, times the occurrence limit.

For example, if in this case only two full reinstatements were allowed, then the maximum amount the reinsurer would pay in any single year is: (2 + 1) ($40 million) = $120 million.

Assume for example that there were 4 catastrophe in a year for which the insurer would have to pay (prior to catastrophe reinsurance): 43M, 77M, 46M, and 58M.
Then the 40M xs 20M cat treaty would have paid: 23M + 40M + 26M + 38M = 127M.
However, with only two full reinstatements available, the catastrophe treaty would only pay 120M in total. The catastrophe treaty would only pay 31M rather than 38M for the last catastrophe.

After the third catastrophe of 26M, the remaining coverage without reinstatement would be:
40 - 26 = 14M.
The reinstatement premium would be based on only restoring the coverage to:
120 - (23 + 40 + 26) = 31M.
Thus the reinstatement premium pro rata as to amount would be:
(110%) {(31 - 14)/40} ($3 million) = $1.4025M.

[238] See Reinsurance Principles and Practices by Connor M. Harrison, not on the syllabus of this exam.

The actuary could price the impact of this limited number of reinstatements. While this is not specifically covered by Clark, it is mathematically similar to pricing an aggregate limit in the syllabus reading "Individual Risk Rating". The actuary would also have to estimate the expected annual amount of reinstatement premiums the reinsurer would expect to receive.

"Basing the up-front premium on the average annual loss to a treaty, disregarding reinstatements, is equivalent to assuming that there are unlimited free reinstatements. If, on the other hand, reinstatements are limited and paid, then the up-front premium will be lower because fewer losses will be covered (because the reinstatements are limited) and some of the premium will be paid after an event has occurred (because the reinstatements are paid)."[239]

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[239] See "Pricing Catastrophe Reinsurance with Reinstatement Provisions Using a Catastrophe Model," by Richard R. Anderson and Wemin Dong, Spring 1998 CAS Forum, not on the syllabus.

Finite Risk Reinsurance:[240]

"Finite Reinsurance is a type of reinsurance that transfers only a finite or limited amount of risk to the reinsurer. Risk is reduced through accounting or financial methods, along with the actual transfer of economic risk. By transferring less risk to the reinsurer, the insurer receives coverage on its potential claims at a lower cost than traditional reinsurance. Due to the highly complex structure of these risk instruments, there can be abuses where no risk is transferred and the insurer's income is improved."[241]

Various features in types of traditional reinsurance that have been discussed reduce the risk of the reinsurer: sliding scale commissions, profit commissions, loss corridors, co-participation clauses, reinstatement premiums, etc. Although there is no clear boundary, finite reinsurance is structured so as to transfer less risk to the reinsurer than traditional reinsurance.[242] Since less risk is transferred, there is both less chance for the reinsurer to lose a lot of money (less downside risk) and less chance for the reinsurer to make a lot of money (less upside potential.)

Clark uses the term "finite risk" to refer to property catastrophe covers for which the maximum loss amount is reduced relative to traditional covers.

Finite risk reinsurance usually has:
1. Multiple year features, for example 3 or 5 years.
2. Loss sensitive features such as profit commissions and additional premium formulas.[243]

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[240] See pages 46 to 48 of Clark. He does not go into much detail.
Because the objective of this type of reinsurance is to improve the financial (accounting) results of the primary insurer, it is often called financial reinsurance.
[241] From the website of the National Association of Insurance Commissioners (NAIC).
[242] Analogous to the situation in retrospective rating plans; as one raises the maximum premium and lowers the minimum premium, more risk is retained by the insured and less risk is transferred to the insurer.
[243] The insurer may have to pay additional premiums if there is bad experience under the treaty.
The insurer may have to pay a large upfront premium; in exchange the insurer will get large profit commissions if the experience is good.

"There has been a growing use of reinsurance, especially treaties, whose only or main function is to manage financial results. The word finite means that the reinsurer's assumed risk is significantly reduced by various contractual conditions, sometimes called structure. Of course, the reinsurer's expected margin (expense and profit) is also reduced to reflect this reduced risk transfer. Sometimes these covers are structured to induce a favorable tax treatment for the cedant. Often they are based on the ability of offshore reinsurers to book claims on a discounted basis in anticipation of the future investment income that will be earned from the premium income received before the claims are settled. The reinsurance premium reflects this discounting, thus giving the cedant a statutory and GAAP accounting benefit.
There have been cases where the risk transfer was nonexistent or negligible. In order to stop accounting abuses through reinsurance, the Financial Accounting Standards Board issued FAS 113 in 1992. FAS 113 requires a measurable and significant transfer of risk before a contract can receive the benefit of reinsurance accounting. Although the standard is somewhat ambiguous, it has largely stopped abusive reinsurance practices."[244]

The National Association of Insurance Commissioners (NAIC) and the Financial Accounting Standards Board (FASB) have been concerned about abuses.
Therefore accounting standards require that:[245]
1. The reinsurer assumes significant insurance risk under the reinsured portions of the underlying insurance contracts.
2. It is reasonably possible that the reinsurer may realize a significant loss from the transaction.

Sometimes one can compare a finite risk transfer to a traditional one; if the price would be reasonable for the traditional cover then perhaps it is also reasonable for the finite cover.[246]

"The best approach to [analyzing] these programs is to estimate the different possible outcomes for a one year time horizon. Using a simplifying assumption that any penetrations into the layer will exhaust the full limit, probabilities can be assigned to each scenario using a Poisson or other distribution. Using a frequency distribution is convenient because the mean of the distribution is related to the payback period for traditional risk covers. The payback which produces an acceptable expected result can be compared to the results of catastrophe models or other pricing analysis."[247]

If the reinsurer depends upon contingent additional premium to minimize the downside risk on the contract, then the reinsurer faces credit risk.[248] After experiencing the loss that makes the additional premium necessary, the primary insurer may be unable to pay this additional premium.

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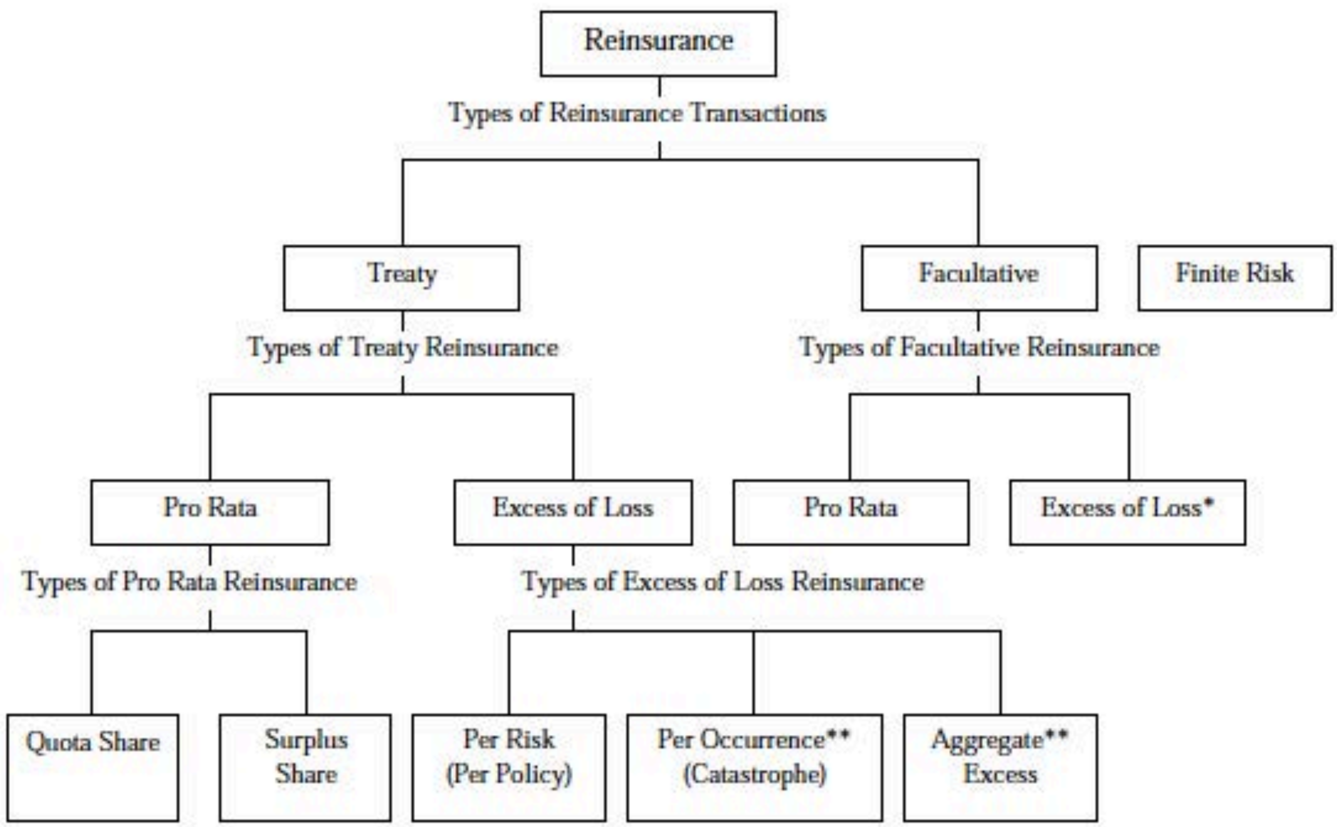
[244] Quoted from "Reinsurance" by Gary S. Patrik, Chapter 7 of Foundations of Casualty Actuarial Science.
[245] This is complicated subject, but Clark does not go into detail.
See for example, Academy of Actuaries' "Risk Transfer Testing Practice Note."
[246] Clark shows an example. See also SOA GIADV Exam, 5/15, Q.8.
[247] Quoted from page 48 of Clark.
[248] Analogous to the situation with large deductible policies.

Different Types of Reinsurance:[249]

Figure H.1 Types of Reinsurance[13]



*Excess of loss reinsurance written on a facultative basis is always on a per risk or per policy basis.

**Per occurrence and aggregate excess of loss reinsurance relate to a type of insurance, a territory, or the primary insurer's entire portfolio of in-force loss exposures rather than to a specific policy or a specific loss exposure.

[249] See Fundamentals of General Insurance Actuarial Analysis, 2019 Supplement, by Jacqueline Friedland. Taken from Reinsurance Principles and Practices, by Connor M. Harrison, copyright by The Institutes.

Calculating the Final Price:[250]

In order to determine the final premium from the estimated loss cost (losses plus alae), one needs to include a provision for the reinsurer’s expenses plus profit and/or risk load.

$$\textbf{Premium} = \frac{\text{(Loss Cost) (1 + ULAE) + Fixed Expenses}}{\text{(1 - Variable Expense \%)}}.$$

While the details differ, this is parallel to ratemaking for primary insurance.[251]

Reinsurer’s expenses can be divided into three categories:
Varying with Premium, Fixed, and Varying with Losses.

Expenses Varying with Premium include (where applicable):[252]
- ceding commission paid to the reinsured
- brokerage fees
- federal excise tax

Fixed expenses include:
- general overhead costs (salaries, real estate)
- underwriting and claims audit expenses

Note that some “fixed” expenses may vary somewhat with the size of the account, but much less than proportionally, as is assumed for variable expenses. Clark mentions that these fixed expenses should not vary based on whether or not an Annual Aggregate Deductible is included.

Expenses varying with losses:
- reinsurer's unallocated loss adjustment expenses

The load for ULAE varies between types of treaties depending on how much work the reinsurer’s claims department expects to have to do. “A working layer excess treaty may require extensive work; a quota share contract may require a review of a loss bordereau, but less claim file review.” For ratemaking, the reinsurance actuary is trying to estimate for a given type of treaty the ratio: $\dfrac{\text{expected ULAE for the reinsurer}}{\text{expected loss + ALAE for the reinsurer}}$.

[250] See Section 6 of Clark, added to the syllabus for Fall 2021.
[251] See Basic Ratemaking by Werner and Modlin.
See also my section on pricing large deductible policies.
[252] Unlike primary insurers, reinsurers do not pay premium tax.

The actuary has to include a loading for profit and/or risk load.[253] [254] Among the items to consider is how much investment income the reinsurer can expect to earn between the time premiums are collected and losses are paid. This can be complicated if treaty includes adjustable features such as swing plans.

For purposes of pricing, the reinsurer may allocate some of its capital to support the treaty.

The riskiness will vary based on the type of treaty. For example, a quota share treaty is usually considered less risky than an excess of loss treaty. Working layers are usually considered less risky than exposed excess, which are in turn less risky than clash covers.
Catastrophe reinsurance can be extremely risky.

“**In general, reinsurance pricing is more uncertain than primary pricing**. Coverage terms can be highly individualized, especially for treaties. These terms determine the coverage period, definition of loss, commission arrangements, premium and loss payment timing, etc. It is often difficult and sometimes impossible to get credible loss experience directly pertaining to the cover being evaluated. Often the claims and exposure data are not as they first seem. So you must continually ask questions in order to discover their true nature. Because of these problems of coverage definition and interpretation of loss and exposure statistics, the degree of risk relative to premium volume is usually much greater for reinsurance than for primary insurance.

Additional risk arises from the low claim frequency and high severity nature of many reinsurance coverages, from the lengthy time delays between the occurrence, reporting, and settlement of many covered loss events, and also from the leveraged effect of inflation upon excess claims. In general, the lower the expected claims frequency, the higher the variance of results relative to expectation, and thus the higher the risk level. In addition, IBNR claims emergence and case reserve development are severe problems for casualty excess business. Claims development beyond ten years can be large, highly variant, and extremely difficult to evaluate. Because of this long tail and extreme variability of loss payments, the matching of assets with liabilities is more difficult. Future predictability is also decreased by the greater uncertainty about claims severity inflation above excess cover attachment points. All these elements create a situation where the variance (and higher moments) of the loss process and its estimation are much more important relative to the expected value than is usually the case for primary coverage. For some reinsurance covers, the higher moments (or at least the underwriter/actuary’s beliefs regarding uncertainty and fluctuation potential) determine the technical rate.”[255]

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[253] This complicated subject of choosing profit provisions and/or risk loads is <u>not</u> covered on the syllabus of this exam. See the syllabus of Exam 9.
[254] Reinsurance markets are usually very competitive. Thus market forces have a large effect on the profit provision and/or risk load a reinsurer will include in its price quote to a primary insurer.
[255] Quoted from “Reinsurance” by Gary S. Patrik, Chapter 7 of <u>Foundations of Casualty Actuarial Science</u>. Emphasis added.

Exercise: You are given the following information for a reinsurance treaty.
● Expected ceded loss and alae = $500,000.
● ULAE is 4% of loss + ALAE.
● Fixed expenses for the reinsurer are $40,000.
● Ceding Commission Rate = 20% (specified in the contract.)
● Brokerage Rate = 5% (specified in the contact.)
● Reinsurer's profit provision and risk load = 15%.
Determine the indicated price for this reinsurance treaty.

[Solution: $\frac{(\$500{,}000)(1.04) + \$40{,}000}{1 - 20\% - 5\% - 15\%}$ = $933,333.

Comment: Unfortunately, Clark does not provide a numerical example.
How to include the reinsurer's profit provision and risk load may differ.]

An exam question might ask you to determine the expected ceded loss and alae, and then given the other items ask you to determine the indicated price.

Problems:

1. (1.5 points) Given the following information:
- A property excess of loss treaty covers losses in the layer \$300,000 excess of \$100,000.
- The reinsurer is to share the cost of ALAE proportionally to loss.
- The reinsurer estimates the ceding company's expected loss ratio excluding ALAE as 66%.
- The reinsurer estimates ALAE will be 10% of the large losses that pierce the reinsured layer.
- The primary insurer writes properties with insured values of either \$200,000 or \$500,000, with the total premium from the former being twice that of the latter.
- The following exposure curve applies to both sizes of property:

Loss as a Percent of Coverage Limit	Exposure Factor
10%	41%
20%	55%
30%	64%
40%	72%
50%	78%
60%	83%
70%	88%
80%	92%
90%	96%
100%	100%

Calculate the indicated exposure rate for this treaty.

2. (1.5 points) A reinsurer writes an excess treaty with an Annual Aggregate Deductible (AAD).
(a) Briefly describe how the AAD operates.
(b) Define the excess charge factor for a given AAD, ϕ_{AAD}.
(c) If the treaty did not have an AAD, the expected ceded losses would be \$13 million.
For the AAD in the treaty, the excess charge factor is 60%.
What are the expected ceded losses with the AAD?

3. (0.5 point) Briefly describe one potential difficulty in experience rating workers compensation excess treaties.

4. (2 points) A reinsurer writes a facultative certificate covering premises and operations for a hotel chain, with 1,000 hotels in 2012 and 1,200 hotels in 2017.
Ten claims exceeding $100,000 are expected for 2012.
Loss cost inflation is a total of 25% for the five years from 2012 to 2017.
The reinsurer has the following size of loss table for the 2012 claims from the hotel chain.

Retention	Percentage of Claims Below Retention	Average Claim Size for Claims Below Retention**
$75,000	56.3%	$49,800
80,000	58.1	51,900
90,000	61.3	56,000
100,000	64.2	59,700
125,000	69.9	67,900
150,000	74.2	74,900
175,000	77.6	80,900
200,000	80.3	86,100
250,000	84.4	94,900
300,000	87.3	101,900
400,000	91.0	112,600
500,000	93.3	120,300
600,000	94.8	126,200
700,000	95.9	130,800
750,000	96.3	132,800
800,000	96.7	134,500
900,000	97.2	137,600
1,000,000	97.7	140,100

** This column shows the accident severity of claims that are below the retention;
it does not include any portion of claims larger than the retention.

a. (0.5 point) How many claims exceeding $100,000 are expected in 2017?
b. (1.5 points) What are the expected losses in the layer $750,000 excess of $250,000 in 2017?

5. (1 point) Quoting from page 1 of "Basics of Reinsurance Pricing," by Clark, "This leads to what might be called the pricing paradox: If you can precisely price a given contract, the ceding company will not want to buy it." Discuss this statement by Clark.

6. (3 points)
A primary insurer writes workers compensation insurance in the state of Midlands.
The expected loss ratio is 62%. The expected loss and ALAE ratio is 72%.
Below are shown standard premiums ($ million) by hazard group as well as
Excess Loss Pure Premium Factors (ELPPFs) and
Excess Loss & Allocated Expense Pure Premium Factors (ELAEPPFs).

$$\text{ELPPF} = \frac{\text{Expected Excess Losses}}{\text{Expected Loss \& ALAE}}. \qquad \text{ELAEPPF} = \frac{\text{Expected Excess Loss \& ALAE}}{\text{Expected Loss \& ALAE}}.$$

Hazard Group	Premium	ELPPF 200K	ELPPF 500K	ELAEPPF 200K	ELAEPPF 500K
B	13	0.207	0.083	0.263	0.111
C	18	0.235	0.101	0.294	0.133
D	9	0.265	0.119	0.329	0.155
F	7	0.361	0.190	0.437	0.241

A reinsurer is exposure rating an excess treaty 300K xs 200K.
(a) If ALAE is pro-rata in the treaty, determine the exposure rate for loss & ALAE.
(b) If ALAE is included with losses in the treaty, determine the exposure rate for loss & ALAE.

7. (1 point) List six steps that should be included in the pricing analysis for proportional treaties.

8. (2.5 points) A property excess-of-loss reinsurance treaty has the following terms:

Effective Date	January 1, 2017
Treaty Limit	$300,000
Attachment Point	$200,000

● ALAE is included with losses
● The treaty is on a losses occurring basis

The following losses have been recorded for the book of business subject to the treaty and have been trended to the effective period of the treaty.

Accident Date	Trended Loss & ALAE ($000)
February 28, 2013	442
August 31, 2013	276
November 30, 2013	617
March 31, 2014	461
June 30, 2014	733
October 31, 2014	340
July 31, 2015	174
September 30, 2015	279
December 31, 2015	321

The excess layer loss development factors applicable to the treaty losses are:

Accident Year	LDFs
2013	1.020
2014	1.080
2015	1.140

Calendar Year	Earned Premium	On Level Factor
2013	20,640,000	0.912
2014	21,709,000	0.925
2015	22,854,000	0.967

Exposure trend is 3% per year.
Determine the experience rating loss cost of this treaty.

9. (1 point) Compare and constant reinsurance treaties written on a "losses occurring" basis and a "risks attaching" basis. What type of data would be used to price each type of treaty?

10. (3 points) Given the following information:
- You are exposure rating a casualty excess treaty 350K xs 200K.
- In the treaty, ALAE is included with loss.
- For every policy limit written by the ceding company, ALAE is 15% of losses.
- The primary insurer writes liability policies with limits of \$100,000, \$250,000, or \$500,000. These limits apply to losses; ALAE is not limited.
- The following set of limited expected values apply to unlimited losses (no ALAE):

Limit	Limited Expected Value		Limit	Limited Expected Value
86,956	34,608		347,826	54,390
100,000	36,797		400,000	55,934
115,000	38,980		434,782	56,802
173,913	45,750		460,000	57,366
200,000	47,267		478,281	57,744
217,391	48,432		500,000	58,166
230,000	49,203		550,000	59,030
250,000	50,317		575,000	59,414
287,500	52,110		632,500	60,200

Determine the exposure factor to be used for each of the three policy limits.

11. (1 point) Briefly describe the following three special features of proportional treaties: sliding scale commissions, profit commissions, and loss corridors.

12. (2.5 points) Given the following to be used in pricing a property excess-of-loss treaty on a losses occurring basis effective January 1, 2015 and expiring December 31, 2015:
- Treaty limit: $400,000
- Attachment point: $200,000
- Annual ground up loss trend: 3%

Historical Losses	
Date of Loss	Ground-up Loss
January 1, 2007	$300,000
April 1, 2008	800,000
July 1, 2009	200,000
October 1, 2010	600,000
January 1, 2011	500,000
July 1, 2012	400,000
April 1, 2013	200,000

Loss Development Factors Applicable to Treaty Layer	
Accident Year	Factor to Ultimate
2007	1.00
2008	1.00
2009	1.05
2010	1.10
2011	1.25
2012	1.40
2013	1.80

Calculate the trended ultimate losses in the treaty layer that would be used to experience rate this treaty.

13. (1 point) An insurer writes property insurance and has purchased a surplus share treaty with a line of $200,000 and 9 lines.
This surplus share treaty inures to the benefit of a 75K xs 75K excess treaty.
Discuss how to exposure rate a $600,000 property.

14. (2 points) You are given the following information:
● Premium for an insurer is \$50 million.
● Aggregate Losses for this insurer follow a LogNormal Distribution with $\mu = 17$ and $\sigma = 0.7$, for which:

Limit (\$ million)	Limited Expected Value (\$ million)	Distribution Function
15	13.862	0.2481
30	22.096	0.6216

● A proportional treaty has a sliding scale commission provision:
Minimum Commission of 25% at a 60% loss ratio.
Sliding 0.5:1 to a Maximum of 40% at a 30% loss ratio.
Determine the average commission.

15. (2.5 points) A primary insurance company purchases a property catastrophe excess-of-loss reinsurance treaty providing coverage of \$50 million over a \$15 million retention. The catastrophe loss event distribution for the primary insurer's direct business is as follows:

Gross Loss (\$ million)	Probability
Less than 15	90%
20	3%
25	2%
30	1%
40	1%
50	1%
60	1%
65 or more	1%

Assume that the annual frequency of a covered event is 17%.
(a) (1 point) Ignoring the possibility of more than one event per year, determine the expected annual amount the reinsurer expects to pay.
(b) (1.5 points) Discuss how reinstatements would work for a treaty such as this one.

16. (3 points) You are given the following information for a finite risk reinsurance treaty:

Annual Premium:	$3,000,000
Occurrence Limit:	$20,000,000
Profit commission:	75% after 12% margin on Annual Premium
Additional Premium:	60% of (Loss + Margin - Annual Premium)

(a) (2 points) Determine the reinsurer's underwriting results excluding its expenses, for a “loss free” scenario and for a “one full loss” scenario.
(b) (1 point) A traditional reinsurance treaty has the same limit as the finite risk treaty. The traditional treaty does not have profit commission or additional premium, but does have a co-participation clause.
Determine the annual premium and the reinsurer’s share of the covered layer, such that for the traditional and finite risk treaties the reinsurer has the same underwriting results excluding its expenses, for a “loss free” scenario and for a “one full loss” scenario.

17. (1 point) Typically, there will be insufficient credibility in the historical loss experience to price a loading for catastrophe potential. However, this amount is critical to the evaluation of property treaties. Briefly discuss how to load the expected non-catastrophe loss ratio for catastrophes.

18. (2.5 points) Given the following information:

Loss as a Percent of Coverage Limit	Exposure Factor
10%	55.4%
20%	68.4%
30%	76.2%
40%	81.8%
50%	86.1%
60%	89.7%
70%	92.8%
80%	95.5%
90%	97.9%
100%	100.0%

Ceding Company Coverage Limit	Direct Premium ($ million)
100,000	19
300,000	25
500,000	31
700,000	27
900,000	15

- The reinsurer estimates the ceding company's expected loss ratio including ALAE as 74%.
- The reinsurer is to share the cost of ALAE proportionally to loss.

Calculate the indicated exposure rate for a property excess of loss treaty covering losses in the layer $250,000 excess of $250,000.

19. (1 point) A carryforward provision allows that if the past loss ratios have been above the loss ratio corresponding to the minimum commission, then the excess loss amount can be included with the current year’s loss in the estimate of the current year’s commission. Discuss two ways of estimating the expected effect of carryforward provisions on sliding scale ceding commissions.
Give a shortcoming of each approach.

20. (1 point) An actuary is pricing the $3,000,000 excess of $1,000,000 layer for an excess of loss treaty. The results of experience and exposure rating are as follows:

Layer	Experience Rating Loss Cost	Exposure Rating Loss Cost
1M xs 1M	9%	11%
1M xs 2M	3%	7%
1M xs 3M	0%	4%

Use a mixture of experience and exposure loss rating to calculate the treaty's loss cost as a percentage of subject premium. Explain what you do.

21. (1.5 points) Clark discusses finite risk reinsurance, sometimes called financial reinsurance.
(a) Define finite risk reinsurance.
(b) List two characteristic common to most finite risk covers.
(c) Due to accounting rules, to be considered insurance two conditions must be met by a finite risk transfer. List them.

22. (1 point) A primary insurer purchases an 80% 150,000 xs 50,000 reinsurance treaty, effective 1/1/2015 through 12/31/2015. You are given the following loss history:

Event Number	Primary Policy Effective Date	Date of Loss	Loss Amount
1	10/01/2014	2/11/2015	110,000
2	2/01/2015	1/20/2016	150,000
3	7/01/2015	9/07/2015	400,000
4	11/01/2015	7/29/2016	70,000

Calculate the total ceded loss.
a. (0.5 point) The treaty is on a risk-attaching basis.
b. (0.5 point) The treaty is on a loss-occurring basis.

23. (1 point) Fully discuss measures of the credibility for experience rating property per risk reinsurance treaties.

24. (0.5 point) When pricing excess reinsurance, list two situations where the use of an exposure rate is likely to be better than the use of an experience rate.

25. (1 point) Casualty per occurrence excess treaties are often separated into three categories. List and describe these three categories.

26. (2 points) You are exposure rating an excess treaty where ALAE is included with losses.
The treaty limit is \$3 million, and the treaty attachment point is \$1 million.
The primary company only writes workers compensation in one state.
The standard premium written by hazard group (\$ million) is: 8 in B, 13 in E, and 5 in G.
You are given the following Excess Loss and ALAE Factors (ELAEFs) by Hazard Group:

Hazard Group	ELAEF @ 1 million	ELAEF @ 3 million	ELAEF @ 4 million
B	0.032	0.009	0.006
E	0.061	0.017	0.012
G	0.123	0.039	0.028

Hint: ELAEF = (expected excess loss & ALAE) / Standard Premium.
Determine the estimated loss & ALAE cost for the treaty as a percent of subject standard premium.

27. (1 point) You are given the following information regarding a $30 million xs $10 million Property Catastrophe Treaty:
- Reinstatement Provision = 120% pro-rata as to amount.
- Payback period = 15 years.

The ceding company experiences a total loss of $19 million due to a major hurricane.
Calculate the reinstatement premium.

28. (3.5 points) An insurer writes property insurance, and has purchased a surplus share treaty with a line of $250,000 and 3 lines.
This surplus share treaty inures to the benefit of a 200K xs 100K per risk excess treaty.
The expected loss and ALAE ratio of the primary insurer is 70%.
The excess treaty treats ALAE pro-rata.
The limits profile for the primary Insurer is:

Range of Insured Value ($000)	Gross Premium ($ million)
100 to 250	10
250 to 500	30
500 to 1000	18

The exposure curve has the following form as per the paper by Bernegger:

$$G(x) = \frac{\ln\left[\frac{(g-1)b + (1 - gb)\, b^x}{1-b}\right]}{\ln[gb]},$$

where the parameters b and g depend on the size of the property.

Insured Value	b	g
$175,000	9.0	7.7
$250,000	8.4	8.7
$375,000	7.8	9.9
$750,000	6.5	13.0

Determine the exposure rate for the excess treaty as a percent of the premium net of the surplus share treaty. Show all work.

29. (3 points) Frequency follows a Poisson Distribution with mean 0.7.
Severity is: S_1 = 40%, S_2 = 20%, and $S_3 = S_4 = S_5 = S_6$ = 10%.
Use the recursive formula of Panjer to calculate the probability that the aggregate losses are 3.

30. (1 point) There are different types of reinsurance treaties.
Describe how surplus share treaties differ from excess loss treaties and quota share treaties.

31. (1 point) In using past data to price casualty per occurrence excess treaties one trends historical losses. Discuss how to deal with policy limits.

32. (2 points) You are exposure rating an excess treaty.
The treaty limit is $3 million, and the treaty attachment point is $2 million.
The primary company only writes workers compensation in one state.
You are given the following Excess Loss Factors (ELFs) by Hazard Group:

Hazard Group	ELF @ 2 million	ELF @ 3 million	ELF @ 5 million
C	0.015	0.009	0.005
D	0.021	0.013	0.007
F	0.026	0.016	0.009

The standard premium written by hazard group ($ million) is: 5 in C, 3 in D, and 2 in F.
Determine the estimated loss cost for the treaty as a percent of subject standard premium.

33. (1 point) Compare and contrast process variance and parameter variance as it relates to the use of collective risk models of aggregate losses.

34. (2 points) An insurer insurers the following properties:

Insured Value Range	Subject Premium ($ million)
$250,000 - $500,000	$20
$501,000 - $1,000,000	$40
$1,000,001- $2,500,000	$60
$2,500,001- $5,000,000	$50

The insurer's expected loss ratio is 68%.
A reinsurer will write a property per-risk excess treaty, $2 million excess of $500,000.
Use the following exposure curve, in order to estimate the losses the reinsurer expects to pay as a percent of subject premium.

Percent of Insured Value	Exposure Factor
0%	0%
5%	43%
10%	54%
20%	67%
40%	84%
60%	91%
80%	95%
100%	98%
120%	100%

35. (2 points) You are given the following information:
- A ceding company sells umbrella policies with \$4,500,000 limits excess of a \$500,000 underlying limit.
- The expected losses for this portfolio of umbrella polices are \$60 million.
- Based on contract language, the umbrella policy can "drop down" and provide coverage on a first dollar or primary basis. The probability of the umbrella policy dropping down and providing first dollar coverage is 10%.

Limit	Limited Expected Value
500,000	125,233
1,000,000	149,566
1,500,000	160,538
2,000,000	166,740
2,500,000	170,685
3,000,000	173,386
3,500,000	175,334
4,000,000	176,793
4,500,000	177,919
5,000,000	178,809
5,500,000	179.526
6,000,000	180,112

As a reinsurance actuary, you have been asked to price a casualty per occurrence excess treaty with a \$2,500,000 limit excess of \$1,000,000 retention. The treaty will apply to umbrella losses. ALAE is pro-rata in the treaty.
Determine the expected ceded losses for the treaty using the exposure rating method.

36. (1 point) In exposure rating excess of loss casualty treaties, one commonly assumes that ALAE varies directly with capped indemnity loss. Briefly discuss this assumption.

37. (1.5 points) You are exposure rating a liability treaty.
- The limited expected value function of size of occurrence capped at L is:
  $E[x; L] = 136,000 - 6000e^{-L/10,000} - 30,000e^{-L/100,000} - 100,000e^{-L/1,000,000}$.
- All of the ceding company's underlying policy limits are \$1,000,000 per occurrence.
- The ceding company's expected ground-up aggregate annual losses are \$70 million.
- The reinsurance treaty attachment point is \$250,000 per policy per occurrence.
- The reinsurance treaty limit is \$750,000 per policy per occurrence.

Calculate the estimated loss cost of this treaty layer.

38. (2 points) A primary insurer buys reinsurance from more than one reinsurer.
In each case, neither reinsurance inures to the benefit of the other.
In each case is there a problem, and briefly discuss why or why not .
a) 60% quota share treaty from Reinsurer X.
200K xs 100K treaty from Reinsurer Y.
b) 200K xs 100K treaty from Reinsurer Y.
700K xs 300K treaty from Reinsurer Z.
c) 50% 200K xs 100K treaty from Reinsurer Y.
50% 250K xs 50K treaty from Reinsurer Z.
d) 50% 200K xs 100K treaty from Reinsurer Y.
70% 400K xs 200K treaty from Reinsurer Z.

39. (1.5 points) Given the following data for an insurer:

Risk	Premium	Insured Value	Loss
1	$20,000	$400,000	$80,000
2	4,000	60,000	40,000
3	50,000	1,500,000	400,000

These policies are covered by a surplus share treaty in which the primary insurer retains a line of $100,000 and the treaty contains nine lines providing a maximum cession of $900,000.
Calculate the total loss ratio for the primary insurer.
Calculate the total loss ratio for the reinsurer.

40. (1 point) You are the actuary for Worldwide Reinsurance. Worldwide is negotiating a proportional treaty with a primary insurer, which includes sliding scale commissions.
The expected loss ratio for the primary insurer is 70%.
You are working with an inexperienced underwriter at Worldwide, Seth Oxnard. Briefly explain to Seth why the expected commission is not equal to the commission at a 70% loss ratio.

41. (4 points) A reinsurer is exposure rating a 2 million xs 0.5 million excess of loss property treaty. The reinsurer will use SwissRe exposure curves as discussed by Bernegger.
The exposure curves have the following form as per the paper by Bernegger:

$$G(x) = \frac{\ln\left[\frac{(g-1)b + (1 - gb)\, b^x}{1-b}\right]}{\ln[gb]}.$$

b = exp[3.1 - 0.15 c (1+c)].   g = exp[c (0.78 + 0.12c)].
Curve Y2 has c = 2, while curve Y3 has c = 3.
The portfolio of insured properties is:

MPL	Gross Premium	Exposure Curve to Use
500,000	40 million	Y2
1,000,000	20 million	Y2
1,500,000	20 million	Y3
2,000,000	10 million	Y3

The primary insurer has an expected loss ratio of 62%.
Determine the exposure rate.

42. (1 point) The steps in the experience rating procedure for casualty excess of loss treaties follow those of property experience rating, but some additional complications arise.
Briefly discuss two of these complications that arise in gathering historical losses.

43. (2 points) A reinsurer is going to write a \$3M xs \$2M excess of loss reinsurance treaty covering a large property. In the treaty, ALAE is prorated.

Limit (\$ million)	Limited Average Severity(\$000)
1	825
2	1201
3	1358
4	1410
5	1442
10	1558
50	1623
100	1632
200	1633
250	1633

The primary insurer charges a premium of \$2 million and has an expected loss ratio of 60%.
ALAE is 6% of losses.
The reinsurer's expenses and profit are 15%.
What is the premium needed for this reinsurance treaty?

44. (1.5 points) There are two casualty excess of loss treaties covering the same book of business. Treaty A insures to the benefit of Treaty B.
Treaty A is 5 million xs 1 million.
Treaty B is 10 million xs 1 million.
The primary insurer pays a claim with $2 million in loss and $10 million in ALAE.
Determine how much each treaty pays.
(a) If for each treaty, ALAE is shared pro-rata.
(b) If for each treaty, ALAE is included with losses.

45. (2 points) A swing plan applies to a casualty excess treaty.
- Retro Premium = (1.25)(Actual Layer Losses)
- Maximum Premium = 40% of Subject Premium
- Minimum Premium = 10% of Subject Premium
- You are given the following distribution of ceded losses as a percent of subject premium:

Range of Loss Cost	Probability	Average in Range
0% < LC < 8%	22%	6.5%
8% < LC < 32%	64%	22.2%
32% < LC	14%	40.9%

Determine the expected retro premium paid by the ceding company under this swing plan, as a percent of the subject premium.
Determine the expected loss ratio for the reinsurer under this swing plan.

46. (2 points)
A reinsurer is exposure rating a 2 million xs 500,000 excess of loss property treaty.
The following exposure curve is appropriate for the insured properties covered by the treaty.

Percent of Property Value	Exposure Factor
0%	0.000
10%	0.333
20%	0.479
30%	0.581
40%	0.661
50%	0.730
60%	0.792
70%	0.849
80%	0.902
90%	0.952
100%	1.000

Assume for simplicity that the primary insurer writes properties each of which are insured for $1 million.
It turns out each of these properties is underinsured and is actually worth $1.25 million.
The reinsurer is not aware of this and falsely assumes that each property is insured to value.
Calculate the effect this will have on the adequacy of the exposure rate the reinsurer calculates.

47. (1 point) For general liability and auto liability, one option is to use the truncated Pareto distribution for loss severity. Briefly discuss two potential difficulties.

48. (2 points) Given the following information:
- You are exposure rating a casualty excess treaty 500K xs 500K.
- In the treaty, ALAE is treated pro-rata.
- For the ceding company, ALAE is 17% of losses.
- The primary insurer writes liability policies with limits of \$2 million.
  These limits apply to losses; ALAE is not limited.
- The expected losses for the primary insurer for the ceded book of business are \$25 million.
- The following limited expected values apply to unlimited losses (no ALAE):

$$E[X; x] = (200{,}000)\ \{1 - \left(\frac{400{,}000}{400{,}000 + x}\right)^2\}.$$

(a) (1.5 points) Determine the expected ceded losses plus ALAE.
(b) (0.5 point) Briefly discuss a problem with your treatment of ALAE in part (a).

49. (1 point) In pricing reinsurance, a single distribution can be used to model aggregate losses, as opposed to the collective risk model.
(a) (0.5 point) List two advantages of this approach.
(b) (0.5 point) List two disadvantages of this approach.

50. (0.5 point) Actuaries use exposure curves for exposure rating property excess of loss reinsurance. Briefly discuss the appropriateness of using the same exposure curve for properties of significantly different sizes.

51. (2 points) A swing plan applies to a casualty excess treaty.
- Retro Premium = (1.4)(Actual Layer Losses)
- Maximum Premium = 30% of Subject Premium
- Minimum Premium = 10% of Subject Premium
- The subject premium is $200 million.
- The ceded losses ($ million) are equally likely to be:
  0, 5, 10, 15, 20, 25, 30, 40, 60, 80.

Determine the expected retro premium paid by the ceding company under this swing plan.
Determine the expected loss ratio for the reinsurer under this swing plan.

52. (2 points)
A reinsurer is exposure rating a 1 million xs 1 million excess of loss property treaty.
The following exposure curve is appropriate for the insured properties covered by the treaty.

Percent of Property Value	Exposure Factor
0%	0%
1%	32.70%
5%	48.68%
10%	58.71%
20%	70.41%
30%	77.78%
40%	83.14%
50%	87.32%
60%	90.72%
70%	93.57%
80%	96.00%
90%	98.13%
100%	100%

The primary insures a property with a $3 million limit and a $100,000 deductible (self-insured retention). (The primary insurer will pay at most $2.9 million for any loss.)
The primary insurer charges $80,000 to insure this property.
The primary insurer has an expected loss ratio of 65%.
Determine the losses the reinsurer expects to pay due to this property.

53. (2 points) You are given the following information:
- A ceding company sells commercial umbrella policies with \$2,500,000 limits excess of a \$500,000 underlying limit.
- The expected losses for this portfolio of umbrella polices are \$40 million.
- Based on contract language, the umbrella policy can "drop down" and provide coverage on a first dollar or primary basis.
- ϕ, the aggregate excess factor on the underlying umbrella policies is 12%.

Limit	Limited Expected Value
500,000	125,233
1,000,000	149,566
1,500,000	160,538
2,000,000	166,740
2,500,000	170,685
3,000,000	173,386
3,500,000	175,334
4,000,000	176,793
4,500,000	177,919
5,000,000	178,809

As a reinsurance actuary, you have been asked to price a casualty per occurrence excess treaty with a \$1,000,000 limit excess of \$1,000,000 retention. The treaty will apply to umbrella losses. ALAE is pro-rata in the treaty.
Determine the expected ceded losses for the treaty using the exposure rating method.

54. (2 points) A proportional reinsurance treaty has a loss corridor provision in which the ceding company will retain 40% of the losses in the 60% to 75% loss ratio layer, and 70% of the losses in the 75% to 90% loss ratio layer.
Given the following loss ratio information:

Range of Direct Loss Ratio	Average Loss Ratio in Range	Probability Loss Ratio is in Range
0%-60%	52%	20%
60%-75%	66%	30%
75%-90%	82%	35%
Over 90%	97%	15%

a. (0.5 point) Calculate the reinsurer's expected loss ratio before the application of the loss corridor.
b. (1.5 points) Calculate the reinsurer's expected loss ratio after the application of the loss corridor.

55. (2 points) Given the following information:

- You are exposure rating a casualty excess treaty 300K xs 300K.
- In the treaty, ALAE is included with loss.
- For the ceding company, ALAE is 20% of losses.
- The primary insurer writes liability policies with limits of \$1 million.
  These limits apply to losses; ALAE is not limited.
- The following limited expected values apply to unlimited losses (no ALAE):
  $E[X; x] = (100{,}000)\ x / (100{,}000 + x)$.

If the expected losses for the primary insurer for the ceded book of business are \$80 million, what are the expected ceded losses plus ALAE?

56. (2.5 points) An insurer writes homeowners insurance, with an expected loss ratio of 64%.
The insurer has purchased a surplus share treaty with a line of \$250,000 and 5 lines.
This surplus share treaty inures to the benefit of a 750K xs 250K per risk excess treaty.
The limits profile for the primary insurer is:

Range of Insured Value (\$000)	Gross Premium (\$ million)
100 to 250	20
250 to 500	40
500 to 1000	25
1000 to 1500	10
1500 to 2000	5

Retention as a Percentage of Insured Value	Exposure Factor (percent)
10	44
20	58
30	67
40	75
50	81
60	85
70	89
80	92
90	94
100	96
110	97
120	98
130	99
140	100

Determine the expected ceded losses for the excess treaty.

57. (2 points) You are pricing excess of loss reinsurance.
You have developed the following exposure rating loss costs:
$250K xs $250K 18%
$500K xs $500K 6%
You also have used 5 years of data in order to experience rate.
The on-level subject premium totals $13 million.
The large trended and developed losses are ($000):
275, 275, 275, 275, 275, 275, 275, 275, 300, 300, 300, 300, 350, 350, 350, 350, 400, 400, 450, 500, 550.
Use a mixture of experience and exposure rating to determine the loss cost for the layer 500K xs. 500K.

58. (1.5 points) Clark discusses using the collective risk model in order to help price reinsurance treaties.
Briefly discuss two reasons to be cautious in relying on the output of such algorithms.

59. (4 points) Given the following information:
- You are experience rating a casualty excess treaty 400K xs 400K.
- In the treaty, ALAE is included with loss.
- During the experience period the primary insurer wrote liability policies with limits of $500,000. These limits apply to losses; ALAE is not limited.
- The subject premium brought to the current rate level and adjusted for exposure trend is $40 million.
- The current increased limit factor for $500,000 is 1.41, while that for $600,000 is 1.49.
- The loss trend factor to apply to each loss in the experience period is 1.20.
- The loss trend factor to apply to ALAE is the same as for losses.
- For simplicity ignore any effects of loss development.
- There were four large losses during the experience period:
  200,000, 300,000, 450,000, and 500,000.
- The corresponding amounts of ALAE amounts are:
  100,000, 80,000, 140,000, and 200,000.

Using each of the two approaches discussed by Clark in order to adjust for inflation of losses, experience rate this treaty.

60. (2.5 points) An actuary is pricing the $20,000,000 excess of $5,000,000 layer for an excess of loss policy. Total insured value of the properties is $25,000,000.
Historical information for this policy is as follows:

Accident Year	Excess Loss Development Factor	On-Level Trended Premium
2011	1.00	$4,800,000
2012	1.00	$4,900,000
2013	1.02	$4,700,000
2014	1.05	$5,000,000
2015	1.10	$5,200,000

Accident Date	Trended Losses ($ million)
March 29, 2013	5.82
October 12, 2015	9.39

The exposure curve below applies to the insured risk:

Retention as a Percentage of Insured Value	Exposure Factor
10	30%
20	50%
40	80%
60	90%
80	95%
100	98%
120%	100%

Calculate the policy's loss cost as a percentage of premium.

61. (3 points) A reinsurance broker has proposed that a reinsurer provide a catastrophe finite risk cover without reinstatements to an insurer with the following terms:
- Annual Premium: 3,000,000
- Occurrence Limit: 20,000,000
- Profit Commission: 80% after 20% margin on Annual Premium
- Additional Premium: 40% of (Loss + Margin - Annual Premium)

A catastrophe model indicates that a loss will fully exhaust the limit once every 10 years and that the probability of a partial loss is negligible.
As usual for simplicity ignore the reinsurer's expenses and profit provision.
(a) (2 points) Determine an equivalent traditional risk cover to which to compare this finite risk cover.
(b) (1 point) Recommend whether or not the reinsurer should accept the proposal.

62. (3.5 points) A reinsurer is offering a ceding company a two-year aggregate stop loss with the following terms:

- The treaty is effective on 1/1/2017 and expires on 12/31/2018.
- The treaty will cover aggregate losses between a 60% and a 75% loss ratio for accident years 2017 and 2018 separately.
- Premium is paid at the beginning of each year and all losses are paid at the end of the year incurred.

Additionally, the reinsurance treaty has the following termination provisions:

- The ceding company has the option to terminate the contract at the end of 2017 and receive a profit commission from the reinsurer calculated as follows:
  Profit commission = (40%) (2017 Ceded Premium) - (2017 Ceded Loss),
  subject to a minimum of 0.
- If the contract is not terminated at the end of 2017, the ceding company will again have the option to terminate the contract at term expiration and receive a profit commission from the reinsurer calculated as follows:
  Profit commission = (40%) (Full Term Ceded Premium) - (Full Term Ceded Loss),
  subject to a minimum of 0.
- The reinsurer cannot terminate the contract at any time.

The reinsurer has simulated five trials of the ceding company's loss ratio subject to the aggregate stop loss as follows:

Simulation	Accident Year 2017 Loss Ratio	Accident Year 2018 Loss Ratio
1	57.5%	62.2%
2	71.8%	64.5%
3	61.0%	75.6%
4	76.2%	66.3%
5	65.1%	59.4%
Subject Premium	300 million	330 million
Ceded Premium	21 million	23 million

For each simulation, assume that the ceding company will only terminate the contract if the profit commission payable is greater than zero.

a. (1.75 points) Calculate the expected profit commission payable at the end of 2017.

b. (1.75 points) Calculate the expected profit commission for the full term of the contract.

63. (1.5 points) You are given the following information:

Policy Year	Incurred Losses in \$1 million excess of \$500,000 Layer	Cumulative Loss Development Factor for Losses in Layer	Subject Premium	Rate Level Adjustment Factor to 2016 Policy Year	Loss Trend to 2016 Policy Year
2011	\$200,000	1.200	\$11 million	0.900	1.160
2012	150,000	1.300	\$10 million	0.980	1.120
2013	300,000	1.500	\$11 million	1.030	1.090
2014	250,000	2.000	\$12 million	1.050	1.060

The given loss trends are appropriate for the given excess layer of loss.
Calculate the 2016 policy year experience rate for the layer \$1 million excess of \$500,000.

64. (2 points) An insurer writes homeowners insurance, with an expected loss ratio of 65%.
The insurer has purchased a surplus share treaty with a line of \$200,000 and 4 lines.
This surplus share treaty inures to the benefit of a 300K xs 200K per risk excess treaty.
The limits profile for the primary insurer is:

Range of Insured Value (\$000)	Gross Premium (\$ million)
50 to 200	50
200 to 1000	90
1000 to 2000	30

Retention as a Percentage of Insured Value	Exposure Factor (percent)
10	44
20	58
30	67
40	75
50	81
60	85
70	89
80	92
90	94
100	96
110	97
120	98
130	99
140	100

Determine the expected ceded losses for the excess treaty.

65. (0.75 points) A property catastrophe treaty covers the layer $150,000,000 excess of $50,000,000 with an annual premium of $5,000,000 and a reinstatement provision that is 110% pro-rata as to amount with no limit on the number of reinstatements.
The treaty is issued for a one-year term effective January 1, 2015 on a losses occurring basis.
a. (0.75 point) Given the three following ground-up catastrophe losses during 2015:
- A loss of $80,000,000 on July 11.
- A loss of $270,000,000 on August 26.
- A loss of $140,000,000 on October 9.

Calculate how much the ceding company pays in reinstatement premiums during 2013.

66. (2 points) A primary insurance company's actuary is evaluating the following three types of reinsurance contracts:
- 70% ceded quota share.
- Four-line surplus share treaty with retained line = $150,000.
- $300,000 xs $200,000 per-risk excess of loss.

In the most recent accident year, the company has experienced the following losses on its policies:

Risk	Insured Value ($000)	Loss ($000)
A	200	100
B	700	330
C	100	80
D	900	650
E	300	40
Total	2,200	1,200

For each reinsurance contract, determine the total retained losses for the insurance company if that reinsurance contract had been in effect.

67. (1.5 points) A proportional reinsurance treaty has sliding scale commission, where the ceding commission depends on the annual loss ratio of the primary insurer.
For loss ratios greater than or equal to 80%, the minimum commission is paid.
For loss ratios less than or equal to 50%, the maximum commission is paid.
The maximum commission is twice the minimum commission.
In between 50% and 80% loss ratios, the commission decreases linearly.
A loss ratio of 70% would produce a ceding commission of $12 million.
A loss ratio of 60% would produce what ceding commission?

68. (1.25 points) An insurance company is writing each year separate excess policies for several different self-insured independent retailers.
Given the following information:
- All policies have a $500,000 per occurrence insured retention.
- All policies have occurrence limits of either $1,000,000 or $2,000,000 in excess of the insured's retention.
- The only claims that are reported to the insurer are those that exceed the insured's retention.

Policy Year	Aggregate Loss in Excess of $500,000 retention	Reported Claim Count
2000	$520,000	1
2001	-	0
2002	$3,750,000	3
2003	$860,000	2
2004	-	0
2005	$1,000,000	1
2006	$330,000	1
2007	$1,290,000	2
2008	-	0
2009	$150,000	1

Using this loss history, two actuaries are tasked with calculating an aggregate loss distribution.
Actuary A wants to use the lognormal distribution to determine the aggregate loss distribution.
Actuary B prefers Panjer's recursive formula, using a Poisson frequency distribution.
Evaluate each actuary's selection and propose the more appropriate method.

69. (1.5 points) The company purchases a catastrophe reinsurance treaty, which provides the following coverage:

Layer 1: 70% of 10 million excess of 10 million
Layer 2: 80% of 10 million excess of 20 million
Layer 3: 90% of 20 million excess of 30 million

The primary insurance company experiences a catastrophe loss of 42 million.
Calculate the total loss retained by the primary insurance company.

70. (3 points) A primary insurer has entered into property catastrophe excess of loss treaties with three reinsurers. The terms of the treaties are as follows:

Reinsurer A: 100% of $100 million excess of $100 million written at a nominal rate on line of 15%.
Reinsurer B: 100% of $100 million excess of $200 million written at a nominal rate on line of 9%.
Reinsurer C: 100% of $100 million excess of $300 million written at a nominal rate on line of 5%.

Each treaty specifies mandatory reinstatement that is 100% pro rata as to amount.
Each treaty has a contract period limit of $200 million.
None of the treaties inures to the benefit of the others.
The primary insurer incurs three covered loss events during the contract period:

Loss Number	Loss Amount
1	$260 million
2	$170 million
3	$440 million

a. (0.75 point) For each loss event, calculate the amount of loss retained by the primary insurer.
b. (2.25 points) For each loss event, calculate the amount of reinstatement premium owed by the primary insurer to each of its reinsurers.

71. (1.5 points) A $400,000 excess of $100,000 per-occurrence excess-of-loss treaty has an annual aggregate deductible (ADD) of $1,000,000.
Based on a collective risk model, the annual aggregate losses in the reinsured layer are:

Aggregate	Probability
$0	5%
$500,000	20%
$1,000,000	30%
$1,500,000	30%
$2,000,000	10%
$2,500,000	5%

(a) Calculate the expected losses ceded to the reinsurer after the effect of the ADD.
(b) Determine ϕ_{AAD}, the excess charge factor for the given AAD.

72. (1.5 points) You are given the following data for a property insurer:

Policy	Premium	Insured Property Value
1	150	50,000
2	300	150,000
3	400	200,000
4	1000	400,000
5	1600	700,000

All policies are covered by the same quota share treaty.
The primary insurer cedes 2070 of premium to the reinsurer.
If the total losses on these policies are 2000, how much of this does the primary insurer retain?

73. (1.5 points) A reinsurer is considering entering into a quota share treaty.
Expenses for the treaty are as follows:

Ceding Commission	25%
Brokerage Fees	4%
Reinsurer's General Expenses and Overhead	3%

The following table represents the expected loss ratio distribution for the primary insurer under the treaty:

Range of Loss Ratios	Average Ratio in Range	Probability of being in Range
0-40%	31.0%	0.20
40-60%	49.9%	0.29
60-80%	69.3%	0.22
80% or above	113.5%	0.29

The treaty includes a loss corridor from 60% to 80% loss ratio.
75% of the losses in the loss corridor are reassumed by the primary insurer.
Determine the expected combined ratio for the reinsurer for this treaty.

74. (2 points) An excess-of-loss reinsurance treaty covers losses and ALAE above the attachment point of $250,000 to a limit of $500,000.
(The treaty covers $500,000 excess of $250,000.)
Historical losses and ALAE for six large claims have been trended for inflation to the future expected level:

Accident Date	Trended Loss & ALAE ($000)
February 11, 2016	383
May 30, 2016	804
August 3, 2016	230
June 22, 2017	908
September 8, 2017	441
November 17, 2017	589

Subject premium adjusted to the current rate level, and loss development factors are:

Accident Year	Subject Premium On-Level	Loss Development Factors
2016	$23 million	1.15
2017	$29 million	1.40

Using experience rating, combining the two years of data, determine the loss ratio of this treaty.

75. (1 point) An insurer writes 200 million in premium.
The insurer enters into a reinsurance treaty:
- 30% Aggregate Quota Share
- Ceding Commission = 20%
- Target ceded profit of 25%, which specifies that if ceded premiums less ceding commission and ceded loss exceeds 25% of ceded premiums, the excess is paid back to the cedent through a profit commission
- Aggregate Ceded Loss Ratio Cap = 160%

Calculate the ceded profit for each of the following gross loss amount scenarios:

Scenario	Gross Loss Amount
1	100,000,000
2	200,000,000
3	300,000,000
4	400,000,000

76. (2 points) An insurer writes $200 million in annual insurance premium.
It is considering buying Stop Loss (Aggregate Excess) Reinsurance with an attachment point of $150 million. The Stop Loss Reinsurance would cost $30 million.
Determine in each case the insurer's loss ratio if it did not buy the reinsurance,
and net of reinsurance if it did buy the Stop Loss treaty.
(a) Annual losses are $100 million.
(b) Annual losses are $300 million.

77. (0.5 point) Discuss determining the loading of ALAE for a clash cover excess treaty.

78. (3 points)
A reinsurer uses the following commission structure to pay a ceding company:

Provisional commission	30%	
Minimum commission	15%	At a 80% loss ratio
Sliding 1:1 to	35%	At a 60% loss ratio
Sliding 0.5:1 to a maximum	50%	At a 30% loss ratio

The aggregate loss distribution model is as follows:

Range of Loss Ratios	Average Loss Ratio in Range	Probability the Loss Ratio is in Range
0-30%	25%	0.10
30-60%	50%	0.30
60-80%	70%	0.35
80%+	100%	0.25

a. (1 point) Calculate the expected loss ratio and expected sliding scale commission using the aggregate loss distribution model above.
b. (0.5 point) Explain what a carryforward provision is and why it is used.
c. (0.5 point) Assume the prior year's loss ratio was above 80%. Explain what effect a carryforward provision would have on the expected commission for the current year, all else being equal.
d. (0.5 point) Briefly explain two approaches used in pricing the impact of a carryforward provision.
e. (0.5 point) Give a shortcoming of each approach in part (d).

79. (1 point) An insurer writes 50 million in premium.
The insurer enters into a reinsurance treaty:
● 20,000,000 xs 60,000,000 Aggregate Excess of Loss
● Rate on Line = 10%
Calculate the ceded profit for each of the following gross loss amount scenarios:

Scenario	Gross Loss Amount
1	25,000,000
2	50,000,000
3	75,000,000
4	100,000,000

80. (2 points) A reinsurance treaty covers $1 million excess of $500,000.
(It covers the layer from $500,000 to $1,500,000.)
You are given historical data on all of the large claims that the primary insurer had.

Claim Number	Ground-up Loss
1	475,000
2	600,000
3	1,100,000
4	1,400,000
5	2,100,000
	5,675,000

If there were 10% inflation, calculate the percentage increase in the amount paid under this reinsurance treaty by the reinsurer.

81. (1 point) You are given the following information for a reinsurance treaty.
● Expected ceded loss and alae = $400,000.
● ULAE is 3% of loss + ALAE.
● Fixed expenses for the reinsurer are $20,000.
● Ceding Commission Rate = 25% (specified in the contract.)
● Brokerage Rate = 6% (specified in the contact.)
● Reinsurer's profit provision and risk load = 20%.
Determine the indicated price for this reinsurance treaty.

82. (2 points) Given the following ground up loss and ALAE for occurrences subject to an umbrella excess-of-loss reinsurance treaty:

Occurrence Number	Ground Up Loss	Ground Up ALAE	Limits of Policies Providing Coverage
1	$2,000,000	$500,000	$500,000 Umbrella over $500,000 Homeowners Liability
2	$1,000,000	$200,000	$1,000,000 Umbrella over $1,000,000 Homeowners Liability
3	$1,200,000	$300,000	$1,000,000 Umbrella over $500,000 Auto Liability

The treaty covers $500,000 excess of $250,000 per occurrence on the umbrella portfolio.
- The treaty covers ALAE pro rata with loss.
- The ceding commission for the treaty is 18% of the reinsurance premium.
- The reinsurance premium for the treaty is $1,300,000.

Calculate the reinsurer's technical ratio for this umbrella treaty.

83. (2 points)
A primary liability insurer has a book of business with an expected loss ratio of 70%.
A reinsurer provides an excess of loss treaty for the layer 500,000 excess of 250,000.
The following table of increased limits factors and premiums by limit sold is available:

Limit	ILF	Manual Direct Premium
100,000	1.00	8 million
250,000	1.40	6 million
500,000	1.70	5 million
750,000	1.85	4 million
1,000,000	1.95	3 million

Calculate the expected losses paid under this treaty.

84. (1 point) A primary insurance company has purchased a $500,000 excess of $250,000 per-occurrence excess-of-loss treaty with an annual aggregate deductible of $1,000,000.
The occurrences subject to the treaty are as follows:

Occurrence	Loss Amount
1	$350,000
2	$100,000
3	$400,000
4	$700,000
5	$1,100,000

Calculate the primary company's net loss.

85. (3 points) An insurer is deciding on a proportional reinsurance strategy for the upcoming two years. A reinsurer proposes the following sliding-scale commission with a carry-forward provision. The contract will be identical for the second year.

Minimum Commission:	25% at a 75% loss ratio
Sliding 1:1 to	35% at a 65% loss ratio
Sliding 0.5:1 to a Maximum	45% at a 45% loss ratio

Assume the insurer places no value on the carry-forward provision after the second year, and ignore the time value of money (i.e., assume a 0% interest rate).
The following table details the expected loss distribution of the underlying business:

Range of Loss Ratios	Average Loss Ratio in Range	Probability of Being in Range
0% - 45%	37%	0.15
45% - 65%	58%	0.35
65% - 75%	71%	0.30
75% or above	82%	0.20

Calculate the reinsurer's expected technical ratio for each year if the insurer buys the sliding-scale commission contract for both years.

86. (5 points)
A property insurer has an existing Quota Share reinsurance treaty in place.
The insurer would like to further reduce its net loss exposure by exploring a proposed Property Per Risk Excess reinsurance treaty. Given the following:

<u>Existing Quota Share Reinsurance Treaty</u>
- 40% Quota Share
- 18% Ceding Commission

<u>Proposed Property Per Risk Excess Reinsurance Treaty</u>
- Subject premium of $50 million net of the Quota Share reinsurance treaty
- Covers net losses in excess of $250,000, up to a limit of $500,000
- The Quota Share reinsurance treaty will inure to the benefit of this treaty
- Reinsurer's Ceded Loss Ratio for this Excess layer is estimated to be 88%

The table below illustrates the insurer's historical experience data net of the Quota Share that is subject to the proposed Property Per Risk Excess layer:

Historical Accident Year	On Level Subject Earned Premium ($ millions)	On Level Trended Ultimate Subject Loss Ratio	On Level Trended Ultimate Layer Loss Cost
2015	35	80%	28%
2016	37	75%	33%
2017	40	85%	38%
2018	42	65%	25%
2019	45	70%	23%

a. (0.5 point) Calculate the loss cost of the Property Per Risk Excess layer using all 5 years of historical experience data provided.
b. (1 point) Assume that the following exposure curve definition applies to the Property Per Risk Excess layer:

$$G(x) = \frac{1 - 0.2^x}{1 - 0.2}$$

where G(x) represents the ratio of pure risk premiums retained by the insurer, and x represents the ground up loss net of the quota share treaty, normalized to the maximum possible net loss of $750,000.
Calculate the loss cost of the Property Per Risk Excess layer retained by the reinsurer based on the exposure curve.
c. (0.5 point)
Calculate the loss cost of the Property Per Risk Excess layer using a blend of the experience loss cost and the exposure loss cost based on a credibility weight of 70%.

QUESTION CONTINUED ON NEXT PAGE

d. (2 points)
The insurer's historical gross expenses are estimated to be 18% of historical gross premiums. Calculate the insurer's expected net underwriting profit after application of both the existing Quota Share reinsurance treaty and the proposed Property Per Risk Excess reinsurance treaty.
e. (1 point) The insurer is also exploring the following proposed modification to the Quota Share reinsurance treaty:
● Profit Commission equal to 30% of reinsurer profit above a 5% reinsurer margin
Calculate the 5 year weighted average ratio of profit commission to ceded premium for the Quota Share reinsurance treaty using the historical experience data provided.

87. (1 point) You are given the following information for a reinsurance treaty.
● Expected ceded loss and alae = $200,000.
● ULAE is 6% of loss + ALAE.
● Fixed expenses for the reinsurer are $10,000.
● Reinsurer’s risk load = $50,000
● Reinsurer’s profit provision = 10%.
Determine the indicated price for this reinsurance treaty.

88. (6, 11/00, Q.12) (0.5 point) True or False. An advantage of quota share reinsurance is that it allows the ceding company to retain the entire premium for smaller risks.

89. (6, 11/00, Q.14) (0.5 point) True or False.
In surplus share reinsurance, the ceding company's percentage retention goes up as the policy limit increases, as long as the policy limit does <u>not</u> exceed the maximum limits ceded to the reinsurer.

90. (6, 11/00, Q.15) (0.5 point) In surplus share reinsurance, is it possible for the ceding company to retain a greater dollar amount than its minimum line?

91. (6, 11/00, Q.27) (0.5 point) True or False.
According to Clark, "Basics of Reinsurance Pricing," one difficulty that arises in exposure rating property per risk excess treaties is the issue of "free cover."

92. (6, 11/00, Q.50) (2 points) A company has purchased five line surplus share reinsurance with a minimum line of $50,000. For each of the following two claims, calculate the ceding company's percentage retention, and the amount of reinsurance recovery. Show all work.

	Limit	Minimum Retention	Loss Amount
Claim 1	$250,000	$50,000	$200,000
Claim 2	$400,000	$50,000	$100,000

93. (6, 11/00, Q.51) (3 points) Respectable Re is a large, U.S.-based reinsurance company licensed in all states that writes many types of reinsurance coverages. Respectable Re's contracts include all of the clauses commonly used in the United States.
Each of the four situations below reflects a reinsurance transaction with a different primary insurance client of Respectable Re. For each situation below, calculate the amount currently payable to each client company. Show all work.
a. (0.75 points) Respectable Re receives a client bordereaux of general liability losses totaling \$500,000, \$200,000 of which has been paid and \$300,000 of which represents case reserves and IBNR. The client has an applicable 50% quota share treaty.
b. (0.75 points) A client writes only commercial automobile insurance with \$1,000,000 limits. The client settles a claim for \$400,000, but has spent an additional \$200,000 in outside legal fees defending its insured. The client company has an applicable excess of loss policy with limits of \$750,000 excess of \$250,000 with Respectable Re, in which allocated loss adjustment expenses are shared pro rata with the reinsurer.
c. (0.75 points) A client settles a workers compensation claim with a single payment of \$525,000. The client has an applicable \$600,000 excess of \$400,000 treaty with a 20% co-participation clause.
d. (0.75 points) The liquidator of an insolvent insurance client settles and closes a claim with its policyholder by paying \$200,000 on a \$250,000 claim. The policyholder was forced to accept less than full value of his claim due to the primary insurer's lack of available funds. The client company has an applicable excess of loss policy with Respectable Re with limits of \$400,000 excess of \$100,000.

94. (6, 11/00, Q.52) *(3 points) Answer the following with respect to reinsurance.*
a. (1 point) Identify and describe the two types of capacity.
b. (2 points) For each of the following types of reinsurance, describe which type of capacity is addressed and how it is addressed.
   i. Quota Share Reinsurance
   ii. Excess of Loss Reinsurance

95. (6, 11/00, Q.53) (1 point) Briefly discuss situations in which a clash layer could be exposed to potential loss.

96. (6, 11/00, Q.55) (3 points)
You are given the following historical information regarding a property excess of loss contract.

Detailed Claims History - Ground Up				
Claim Number	Loss Year	Paid Loss	Case Reserve	Total Incurred
96-001	1996	$472,000	$0	$472,000
97-001	1997	1,175,000	200,000	1,375,000
98-001	1998	180,000	250,000	430,000
98-002	1998	575,000	0	575,000
99-001	1999	775,000	125,000	900,000

	Construction Costs Trend Index Base Year-1996
1996	1.000
1997	1.050
1998	1.092
1999	1.158
2000 and later	1.158

	Rate Change Index Base Year-1996
1996	1.000
1997	1.080
1998	1.102
1999	1.047
2000 and later	1.047

	Subject Earned Premium
1996	$20,000,000
1997	$23,000,000
1998	$24,500,000
1999	$25,500,000

Experience Rating Period: 4 years
Evaluation Date: December 31, 1999
You are asked to develop an experience rate for the reinsurance layer $1,000,000 excess of $500,000. Answer the following:
a. (1.5 points) Calculate the total adjusted losses to be used in the experience rate calculation.
b. (1.5 points) Calculate the total subject premium to be used in the experience rate calculation.

97. (6, 11/00, Q.66) (0.5 point) In his paper "Basics of Reinsurance Pricing," Clark indicates that when property proportional treaties are priced, the expected non-catastrophe loss ratio should be loaded for catastrophes. However, there will typically be insufficient credibility in the historical loss experience to price a loading for catastrophes.
What is the most common procedure to address this issue?
<u>Note</u>: I have rewritten this past exam question in order to match he current syllabus.

98. (6, 11/00, Q.67) (2 points) List two procedures used to price excess-of-loss reinsurance and briefly describe potential problems associated with each procedure.
Note: I have rewritten this past exam question in order to match he current syllabus.

99. (6, 11/00, Q.69) (2 points) You are given the following information:
- Contract Term: 1 year
- Reinsured Loss: All Automobile Liability in the layer $250,000 excess of $250,000 per policy
- Deposit Premium: $1,000,000
- Aggregate Loss Limit: $5,000,000
- Contract Balance: This is defined as cumulative paid premiums, less losses paid to date, plus accumulated investment income on the balance credited at the five-year U.S. Treasury interest rate.
- Profit Sharing: At the end of 3 years, 90% of any positive contract balance will be paid to the ceding company, and losses will be commuted at that time.
- Additional Premium: At the end of each year, the ceding company shall pay to the reinsurer as an additional premium, any amount by which the contract balance is less than zero.
- Expected losses in the layer are $3,000,000.

Would this transaction between a ceding company and a reinsurer qualify for accounting treatment as reinsurance? Explain why or why not.

100. (6, 11/00, Q.71) (3 points) You are given the following information:
- A ceding company sells umbrella policies with $5,000,000 limits excess of a $1,000,000 underlying limit.
- Based on contract language, the umbrella policy can "drop down" and provide coverage on a first dollar or primary basis. The probability of the umbrella policy dropping down and providing first dollar coverage is 20%.
- $E[x; Z] = (Z/1{,}000{,}000)\ (0.9)^{Z/1{,}000{,}000}$
- ULAE = 10% of Loss & ALAE
- Variable expense ratio (including profit load) = 15%
- Fixed expenses = $50,000

Class of Business	Umbrella Subject Premium	Expected Loss and ALAE Ratio
1	$110,000	65%
2	115,000	65%
3	108,000	70%
4	100,000	75%
5	110,000	70%

As a reinsurance actuary, you have been asked to price a casualty per occurrence excess treaty with a $2,000,000 limit excess of $500,000 retention. The treaty will apply to umbrella losses. ALAE is pro-rata in the treaty.
Based on the methods in Clark, "Basics of Reinsurance Pricing," determine the final premium for the treaty using the exposure rating method. Show all work.
Note: I have slightly rewritten this past exam question.

101. (6, 11/01, Q.16) (0.5 point) True or False. According to Clark, "Basics of Reinsurance Pricing," a loss corridor is a common feature of property per risk treaties.

102. (6, 11/01, Q.25) (1 point) Assume an insured incurred a loss of $500,000 and had purchased a policy with a limit of $1,000,000. Rank the amount of the net loss to a primary insurer under the following three types of reinsurance treaties.
1. Quota share of 50%.
2. Surplus share with a retained line of $100,000, a maximum number of lines ceded of four.
3 . Per risk excess of loss with an attachment point of $250,000, a limit of $750,000 excess of $250,000, and a 10% co-participation clause in the layer.

103. (6, 11/01, Q.26) (1 point) A primary insurance company experiences a $40,000,000 catastrophe loss. They have a $90,000,000 excess of $10,000,000 catastrophe reinsurance cover with one reinstatement prorata as to amount and 100% as to time. The original premium the primary insurer paid for the cover was $1,200,000. What is the reinstatement premium the primary insurer would have to pay as a result of the catastrophe?

104. (6, 11/01, Q.27) *(1 point)*
Which of the following best matches the types of reinsurance to the functions of reinsurance?
A. Large Line Capacity: Per occurrence excess
Premium Capacity: Catastrophe excess
Loss Ratio Stability: Quota share
B. Large Line Capacity: Clash Cover
Premium Capacity: Quota share
Loss Ratio Stability: Surplus share
C. Large Line Capacity: Catastrophe excess
Premium Capacity: Quota share
Loss Ratio Stability: Aggregate excess (Stop Loss)
D. Large Line Capacity: Per occurrence excess
Premium Capacity: Quota share
Loss Ratio Stability: Aggregate excess (Stop Loss)
E. Large Line Capacity: Per occurrence excess
Premium Capacity: Aggregate excess (Stop Loss)
Loss Ratio Stability: Surplus share

105. (6, 11/01, Q.28) (1 point) Your insurance company currently purchases catastrophe reinsurance to cover the one-in-100-year catastrophe event. The corporate risk advisor has decided to purchase an additional layer of reinsurance to withstand the one-in-250-year catastrophe event. ABC Re, the company's reinsurer, has offered to provide this additional layer of coverage using a 100-year payback approach.

Percentile	Probable Maximum Loss
99.90	$200,000,000
99.80	150,000,000
99.60	125,000,000
99.50	100,000,000
99.00	80,000,000
98.00	60,000,000

Based upon the table above, what is the <u>additional</u> premium that would be ceded to ABC Re?

106. (6, 11/01, Q.29) (1 point)
According to Clark, "Basics of Reinsurance Pricing," which of the following is true?
A. When including ALAE in a treaty, the pro-rata method always produces a lower amount reinsured than the add-on method.
B. A multiple year feature is a common characteristic of finite risk covers,
C. Loss development statistics produced by the Reinsurance Association of America have stable groupings of data by attachment point and limit.
D. "Free cover" refers to exposure rating a layer that attaches higher than the largest policy limit sold by a ceding company.
E. A loss corridor is a contract provision that provides for the reinsurer's loss experience to be capped so that it remains above a minimum and below a maximum.

107. (6, 11/01, Q.48) (2 points)
You are given the following information from a prospective reinsurance client:

Policy Limit	Manual Direct Premium	Increased Limits Factor
$100,000	$5,000,000	1.000
500,000	3,000,000	1.260
1,000,000	2,000,000	1.400
Total	$10,000,000	

Primary Insurer's Expected Loss Ratio is 60%.
ALAE is to be shared pro-rata.
Reinsurance Loading for ALAE = 1.10.
Reinsurance Loading for Brokerage, Internal Expense, and Underwriting Profit = 1.25.
Calculate the exposure rate for a $500,000 excess of $500,000 reinsurance cover. Show all work.

108. (6, 11/01, Q.49) (1.5 points)
Briefly describe the following bases of attachment for reinsurance treaties.
a. (0.5 point) Policies-attaching basis
b. (0.5 point) Losses-occurring basis
c. (0.5 point) Policies-issued basis

109. (6, 11/01, Q.50) (1 point)
What are two reasons that experience rating is not used in pricing catastrophe treaties?

110. (6, 11/01, Q.51) (2.5 points) You are given the following information:
A primary insurance company buys a quota share treaty from a reinsurer with a sliding scale commission structure with the following terms:

Provisional Commission	27.5%	
Minimum Commission	25.0%	at 66% loss ratio
Sliding 0.750:1 to	28.0%	at 62% loss ratio
Sliding 0.667:1 to a maximum	32.0%	at 56% loss ratio

The reinsurer's operating expenses are 4% of ceded premium.
The primary insurer's expenses are 15% for brokerage, 35% for taxes,
and 9% for operating expenses.
Using the concepts presented in Clark, "Basics of Reinsurance Pricing,” answer the following.
Show all work.
a. (1 point) Calculate the technical ratios for the treaty if its ultimate loss ratios are:
i. 52.0%
ii. 57.0%
b. (0.5 point) What loss ratio produces a 100% combined ratio for the reinsurer?
c. (0.5 point) What is a "balanced" plan?
d. (0.5 point) What is the correct method to interpret the potential profitability of a sliding scale reinsurance agreement from the reinsurer's perspective?

111. (6, 11/01, Q.52) (3 points) You are given the following information:

	Cumulative Distribution of Losses By Cause	
Loss as a % of Insured Value	Cause of Loss	
	Fire	All Other
25%	75%	80%
50%	95%	96%
75%	98%	98%
100%	100%	100%

Policy Limit	Premium by Policy Limit
$50,000	$50,000
100,000	50,000
200.000	50,000

Cause of Loss	Loss Weights
Fire	40%
All Other	60%

● Expected Loss Ratio = 65%
● ALAE as a percent of loss = 10%
● Ceding Company Rate Adequacy = 5% Inadequate
● Reinsurer Expense Load = 1.25
● ALAE is shared pro-rata in the excess treaty.

Calculate the total commercial property exposure rate for the layer $100,000 excess of $100,000.
Show all work.
Note: I have rewritten this past exam question to match the current syllabus.

***112.* (6, 11/02, Q.18)** (1 point)
A primary insurer has two reinsurance treaties covering their property book of business.
Treaty A is a per risk excess of loss for the layer $3,000,000 excess of $1,000,000.
Treaty B is a catastrophe cover with the following structure:

- Primary initial retention: $5,000,000
- 1st layer coverage: 90% of next $4,000,000
- 2nd layer coverage: 50% of next $6,000,000
- Treaty A inures to the benefit of Treaty B

A hurricane results in large claims from three risks, in addition to numerous smaller claims (less than $500,000 per risk), as shown below.

	Loss
Risk #1	$1,500,000
Risk #2	1,000,000
Risk #3	800,000
All other hurricane claims	10,000,000
Total	$13,300,000

What is the total of the primary insurer's reinsurance recoveries from both treaties?

113. (6, 11/02, Q.19) (1 point) An insurance company has purchased a 9 line surplus share reinsurance cover with a retained line of $100,000.
What is the recovery on a $400,000 loss to a property insured for $2,000,000?

114. (6, 11/02, Q.37) (1.5 points) You are given the following information:

- A primary insurance company enters into a 75% pro rata treaty reinsurance arrangement with a reinsurer, effective January 1, 2001.
- The treaty ceding commission is 30%.
- During 2001, the primary insurer has $250,000 of written premium, gross of reinsurance, subject to the treaty.
- As of December 31, 2001, the primary insurer has paid $50,000 of loss and $9,000 of loss adjustment expenses, gross of reinsurance.

a. (1 point) Assuming that no money has been exchanged between the two companies as of December 31,2001, what is the net amount due to the reinsurer under the treaty? Show all work.
b. (0.5 point) Explain the purpose of the ceding commission.

115. (6, 11/02, Q.38) (2 points) You are given the following information:

Policy Limit	Increased Limit Factor	Primary Company Direct Written Premium
$35,000	1.00	$2,000,000
50,000	1.15	3,000,000
100,000	1.32	3,000,000
200,000	1.45	2,000,000
500,000	1.55	5,000,000

Primary company's expected loss ratio is 65%.
Calculate the exposure rate for the layer $50,000 excess of $50,000.
ALAE is shared pro-rata by the treaty.
Note: This question has rewritten to match the current syllabus.

116. (6, 11/02, Q.39) (2 points) You are pricing a casualty excess of loss treaty using the severity distribution implied by the NCCI excess factors for retrospective rating plans.
You are given the following information regarding the exposure and the treaty to be priced.

State	Hazard Group	Standard Premium	Expected Loss Ratio	Excess Loss Factor at $300,000	Excess Loss Factor at $1,000,000
CA	III	$2,360,000	75%	0.080	0.030
FL	IV	1,200,000	80%	0.125	0.055
IL	II	800,000	70%	0.025	0.005
GA	II	640,000	70%	0.040	0.010

Treaty Limit: $700,000
Treaty Attachment Point: $300,000
Based on Clark, calculate the loss cost for the layer $700,000 excess of $300,000.
Show all work.

117. (6, 11/02, Q.40) (2 points)
You are given the following information regarding a proportional property reinsurance treaty:

Range of Loss Ratios	Average Loss Ratio In Range (Before Corridor)	Probability That Loss Ratio is in the Range
0% - 75%	65.0%	60%
75% - 85%	82.0%	25%
85% or above	93.5%	15%

Based on the loss ratio distribution shown above, along with consideration of the ceding commission and expenses, you conclude that this particular treaty will produce an unacceptable combined ratio for the reinsurer. A solution is proposed to incorporate a loss corridor provision in the treaty, whereby the primary company would reassume 70% of the layer from a 75% to 85% loss ratio.
Calculate the reinsurer's loss ratio with and without the proposed loss corridor provision.

118. (6, 11/02, Q.41) (3 points) You are given the following information:

Ceding Company Coverage Limit	Direct Premium
$100,000	$5,000,000
200,000	5,000,000
300,000	5,000,000
400,000	5,000,000
500,000	5,000,000

Loss as % of Insured Value	Exposure Factor*
10%	48.7%
20	61.5
30	71.1
40	78.6
50	84.6
60	89.3
70	93.1
80	96.1
90	98.2
100	100.0

• Assume straight line interpolation is appropriate to calculate additional values.

Ceding Company Expected Loss Ratio (Excluding ALAE):	70%
ALAE to Loss Ratio:	15%
Ceding Company Rates Inadequate by:	10%
Reinsurer Profit and Expenses as Percent of Premiums:	20%

Reinsurer is to share the cost of ALAE pro-rata
Calculate the exposure rate for a property excess-of-loss treaty covering losses in the layer $300,000 excess of $100,000 per occurrence. Show all work.
<u>Note</u>: This question has rewritten to match the current syllabus.

119. (6, 11/02, Q.43) (2 points) From the point of view of the ceding company, describe two advantages and two disadvantages of excess of loss reinsurance over pro rata reinsurance.

120. (6, 11/03, Q.16) (1 point)
Clash contracts provide coverage for which of the following types of loss?
1. Loss from an occurrence involving more than one policy or coverage for an insurer
2. Excessive ULAE
3. Excess of policy limits (XPL)

121. (6, 11/03, Q.17) (1 point) You are given the following information:
● A per occurrence excess of loss treaty covers $1,000,000 excess of $1,000,000.
● There is a $3,000,000 contract year reinsurance limit.
● Occurrences for a contract year are:

Occurrence 1	$900,000
Occurrence 2	1,250,000
Occurrence 3	1,550,000
Occurrence 4	1,750,000
Occurrence 5	1,850,000
Occurrence 6	2,000,000

What is the total amount retained by the primary insurer?

122. (6, 11/03, Q.18) (1 point) You are given the following aggregate loss ratio distribution and sliding scale commission structure:

Commission Structure	
Provisional Commission:	25%
Minimum Commission:	20% at a 60% loss ratio
Sliding 1: 1 to	30% at a 50% loss ratio
Sliding 0.5: 1 to a Maximum	40% at a 30% loss ratio

● Average Loss Ratio: 59.6%

Range of Loss Ratios	Average Loss Ratio in Range	Probability Loss Ratio is in Range
0% - 30%	28.0%	0.050
30% - 50%	41.0%	0.300
50% - 60%	54.0%	0.200
60% or above	78.0%	0.450

● There is no carryforward provision in the contract.
What is the expected sliding scale commission paid by the reinsurer to the ceding company?

123. (6, 11/03, Q.19) (1 point) You are given the following information for a reinsurer:

Range of Loss Ratios	Average Loss Ratio in Range (before corridor)	Probability Loss Ratio is in Range
0% - 75%	60.7%	0.595
75% - 90%	81.2%	0.228
90% or above	106.2%	0.177

• The ceding company will reassume 50% of the loss ratio layer from 75% to 90%.
What is the reinsurer's expected loss ratio after the application of the loss corridor?

124. (6, 11/03, Q.37) (3 points) A catastrophic event triggers the following losses under an insurer's commercial multiple peril (CMP) policies:

CMP Liability				CMP Property	
Loss #	Policy Limit	Loss Amount		Loss #	Loss Amount
1	$100,000	$100,000		1	$5,000,000
2	1,000,000	500,000		2	2,500,000
3	2,000,000	1,000,000		3	1,500,000
				All Others*	30,000,000

● All Others are CMP Property losses less than $1,000,000 per loss.

Three reinsurance programs are in place:
- A two line surplus share covering CMP Liability with a minimum per occurrence line of $200,000.
- A property per risk excess of loss treaty covering CMP Property for $3,000,000 excess of $1,000,000 per loss.
  While the $3,000,000 limit applies per risk for any one occurrence, the liability of the reinsurer under this treaty shall not exceed $10,000,000 in total for any one occurrence.
- A catastrophe treaty covering CMP for $30,000,000 excess of $1,000,000 per catastrophe with a 10% co-participation clause.

Assuming the surplus share and property per occurrence excess of loss treaties inure to the benefit of the catastrophe treaty, calculate the loss retained by the primary insurer.
Show all work.
<u>Comment</u>: I have rewritten this past exam question to match the current syllabus.

125. (6, 11/03, Q.39) (1 point) A primary insurer purchases a casualty excess of loss treaty covering $400,000 excess of $100,000 per occurrence.
The treaty is effective from January 1, 2002 through December 31, 2002.
The loss history is:

	Effective Date of Underlying Policy	Date of Loss	Amount of Loss
Occurrence 1	December 1, 2001	June 15, 2002	$300,000
Occurrence 2	January 15, 2002	July 15, 2002	500,000
Occurrence 3	April 1, 2002	April 20, 2002	90,000
Occurrence 4	May 31, 2002	May 15, 2003	120,000

Calculate the total amount ceded to the reinsurer if the treaty is on a:
a. (0.5 point) Policies-attaching basis. Show all work.
b. (0.5 point) Losses-occurring basis. Show all work.

126. (6, 11/03, Q.40) (2 points)
You are given the following information for a casualty excess of loss treaty:

Policy Limit	Increased Limit Factor	Direct Written Premium
$500,000	1.000	$500,000
1,000,000	1.180	3,500,000
2,000,000	1.350	2,222,000
5,000,000	1.710	9,000,000
10,000,000	1.800	4,650,000
15,000,000	1.880	1,075,000
25,000,000	1.990	1,428,000

- ALAE is shared pro-rata in the treaty.
- Primary insurer's Expected Loss Ratio is 55%.

Calculate the indicated exposure rate for the $9,000,000 excess of $1,000,000 layer.
Show all work.
Comment: This past exam question has been rewritten to match the current syllabus.

127. (6, 11/03, Q.41) (1.5 points) You are given the following 3 claims:

Claim #	Loss	ALAE
1	$600,000	$360,000
2	50,000	150,000
3	200,000	400,000

There are two common methods to handle ALAE in reinsurance contracts.
List these two methods and calculate the total loss and ALAE in the $400,000 excess of $100,000 layer for each method. Show all work.
Comment: This past exam question has been reworded to match the current syllabus.

128. (6, 11/03, Q.42) (1.5 point) You are given the following information for a per occurrence catastrophe reinsurance treaty effective January 1, 2002 to December 31, 2002.

- The annual premium is $2,000,000.
- The treaty covers $15,000,000 excess of $10,000,000 per occurrence, with one paid reinstatement.
- The treaty covers risks attaching.
- The reinstatement premium is 75% pro rata as to amount.
- The reinstatement premium is pro rata as to time.
- The primary insurer had $20,000,000 of ground up loss from a catastrophe on July 1, 2002.

Calculate the reinstatement premium charge. Show all work.

129. (6, 11/03, Q.43) (2 points) You are given the following information:

Ceding Company Coverage Limit	Direct Premium
$100,000	$5,000,000
200,000	5,000,000
250,000	5,000,000

Loss as % of Insured Value	Exposure Factor
10%	32.5%
20	42.2
30	49.7
40	56.4
50	62.3
60	67.6
80	76.8
100	84.5
200%	100.0%

- The ceding company's expected loss ratio (excluding ALAE) is 60%.
- The ratio of ALAE to loss is 8%.
- The reinsurer is to share the cost of ALAE proportionally.
- The ceding company's rates are adequate.

Calculate the exposure rate for a property excess-of-loss treaty covering losses in the layer $100,000 excess of $100,000 per occurrence. Show all work.
Comment: This past exam question has been rewritten to match the current syllabus.

130. (6, 11/04, Q.11) (1 point)
You are given the following information regarding a casualty excess of loss treaty:

- Treaty Basis: Losses Occurring
- Treaty Effective Date: January 1, 2003
- Treaty Expiration Date: December 31, 2003
- ALAE Treatment: Part of Loss
- Subject Premium: $20,000,000
- Reinsurance Rate: 6%
- Layer: $400,000 excess of $100,000
- Primary company loss experience:

Claim #	Policy Effective Date	Loss Date	Loss	ALAE
1	December 15, 2002	May 15, 2003	$900,000	$125,000
2	February 15, 2003	November 13, 2003	345,000	50,000
3	March 10, 2003	October 31, 2003	75,000	200,000
4	June 30, 2003	July 15, 2003	50,000	45,000
5	October 15, 2003	January 15, 2004	250,000	100,000

What is the reinsurer's loss ratio for this treaty?

131. (6, 11/04, Q.13) (1 point) You are given the following exposure curve in order to price property per-risk excess of loss reinsurance:

Retention as a Percentage of Insured Value	Exposure Factor (percent)
10	48
20	62
30	71
40	79
50	85
60	89
70	93
80	96
90	98
100	100

For a particular property with a total insured value of $500,000, the primary insurer is charging a premium that assumes expected losses of $5,000.
Based on the table above, what is the expected loss for a reinsurance treaty with a limit of $200,000 excess of $100,000?
Note: I have rewritten this past exam question to match the current syllabus.

132. (6, 11/04, Q.14) (1 point)
A casualty excess of loss treaty provides a limit of $900,000 excess of $100,000.
For a particular claim, the primary insurer made direct payments of $500,000 for indemnity and $120,000 for ALAE. How much more would the reinsurer pay on this claim if ALAE were included as part of the reinsurance limit, rather than being shared pro rata?

133. (6, 11/04, Q.15) (1 point) You are given the following information:

State	Standard Premium	Expected Loss Ratio	Excess Loss Factor at $500,000	Excess Loss Factor at $1,000,000	Excess Loss Factor at $1,500,000
X	$10,000	80%	0.040	0.030	0.022
Y	$10,000	70%	0.037	0.027	0.019

What is the loss cost rate for a casualty excess of loss treaty with a limit of $1,000,000 and attachment point of $500,000?

134. (6, 11/04, Q.16) (1 point)
You are given the following information for a per occurrence excess of loss treaty:
● Ceded per occurrence layer of $1,000,000 excess of $1,000,000
● 10% co-participation provision
● ALAE included in the limit
● Claim values for occurrences subject to the treaty:

	Loss	ALAE
Occurrence 1	$1,200,000	$75,000
Occurrence 2	$900,000	$120,000
Occurrence 3	$2,000,000	$200,000

What is the total loss and ALAE ceded to the reinsurer?

135. (6, 11/04, Q.17) (1 point) A surplus share treaty has a retention of $150,000, a minimum line of $150,000, and a cession of up to 3 lines. It covers two risks whose policy limits are $525,000 and $900,000, whose premiums are $100,000 and $120,000, respectively.
What percentage of the premium will be ceded to the reinsurer?

136. (6, 11/04, Q.18) *(1 point) Which one of the following statements regarding reinsurance ceding commissions is true?*
1. They compensate the primary insurer for policy acquisition costs.
2. They provide surplus relief to the primary insurer under U.S. Statutory Accounting Principles.
3. They are generally applicable to pro rata reinsurance.
4. They are usually higher for more profitable underlying books of business.
Note: I have rewritten this past exam question.

137. (6, 11/04, Q.43a) (1 point) You are pricing a property per risk excess of loss treaty.
You are provided with the following limits profile and exposure curve.

Limits	Subject Premium
$50,001 to 150,000	$180,000
150,001 to 350,000	275,000
350,001 to 450,000	250,000
450,001 to 550,000	120,000

Retention as a Percentage of Insured Value	Exposure Factor
10	46%
20	57%
30	66%
40	75%
50	82%
60	87%
70	92%
80	95%
90	98%
100	100%

The loss ratio of the primary insurer 70%.
Calculate the reinsurer's expected losses for a $300,000 excess of $200,000 layer.
Show all work.
<u>Note</u>: I have rewritten this past exam question to match the current syllabus.

138. (6, 11/04, Q.45) (3 points)
A primary insurer has the following catastrophe reinsurance program:

Primary Insurer:	\$1,000,000 retention; 20% co-participation on all reinsurance layers
Reinsurer A:	\$4,000,000 excess of \$1,000,000
Reinsurer B:	\$5,000,000 excess of \$5,000,000
Reinsurer C:	\$10,000,000 excess of \$10,000,000

An earthquake results in a gross loss of \$15,000,000.

a. (1 point) How much of this \$15,000,000 loss is paid by each party?
Assume there is no other reinsurance that inures to the benefit of these treaties.

b. (1 point) Assume now that an excess of loss treaty inures to the benefit of these treaties.
The excess of loss reinsurer is responsible for \$3,000,000 excess of \$12,000,000 of the gross loss. Which of the party or parties listed above will benefit from this inuring reinsurance? How much less, if any, will each party pay when compared to their obligations without such inuring reinsurance?

c. (0.5 point) When a reinsurer prices a catastrophe treaty. it must take into account any inuring reinsurance, and any changes therein. Give an example of a change to inuring reinsurance that would make the catastrophe reinsurer insist on higher premiums from the primary insurer.

d. (0.5 point) The premium paid to Reinsurer B is \$500,000.
Calculate the rate on line associated with this treaty.

139. (6, 11/04, Q.46) (2.5 points)

a. (1 point) Define finite reinsurance and list two characteristics.

b. (0.5 point) How does loss portfolio reinsurance differ from other types of finite reinsurance?

c. (1 point) What are the two FASB guidelines that must be met for a finite transaction to be considered reinsurance?

<u>Note</u>: I have rewritten this past exam question to match the current syllabus.

140. (6, 11/04, Q.47) *(2 points) A primary insurer writes \$10 million in direct premium one year. It spends 20% of that in acquisition costs. At year-end, half the premium remains unearned, the insurer has paid \$2 million in direct loss and LAE, and has direct loss and LAE reserves of \$1.5 million. If the insurer had a 75% quota share cession to a reinsurer with a 30% ceding commission, what was its statutory surplus relief? Show all work.*

141. (6, 11/04, Q.48) (3.5 points)
You are given the following information regarding a homeowners book of business:

	Exposure Factors		
Loss as a Percent of Insured Value	Fire	Wind	All Other
20%	41.1%	95.6%	99.0%
40%	55.6%	97.3%	99.4%
60%	67.1%	98.2%	99.7%
80%	76.5%	98.8%	99.9%
100%	84.2%	99.2%	100.0%
120%	90.3%	99.5%	100.0%
140%	94.8%	99.7%	100.0%
160%	97.7%	99.9%	100.0%
180%	99.2%	100.0%	100.0%
200%	100.0%	100.0%	100.0%

Coverage A Limit	Direct Premium		Distribution of Historical Losses by Cause of Loss	
$50,000	$500,000		Fire	60.0%
100,000	750,000		Wind	30.0%
250,000	1,200,000		All Other	10.0%
500,000	800,000			
Total	$3,250,000			

- The pure loss component of the direct premium is 60%.
- The reinsurer will share the cost of ALAE pro-rata.
- LAE as a percent of loss is 5%.
- Reinsurer's loading for expense and profit is 15%.
- The primary rates are adequate.

Calculate the indicated exposure rate for an excess of loss reinsurance cover for the layer $400,000 excess of $100,000. Show all work.
Note: I have rewritten this past exam question to match the current syllabus.

142. (6, 11/05, Q.28) (2.5 points) You are given the following information:

Policy Year	Incurred Losses in \$500,000 excess of \$500,000 Layer	Cumulative Loss Development Factor for Losses in Layer	Subject Premium	Rate Level Adjustment Factor to 2005 Policy Year	Loss Trend to 2005 Policy Year
2000	\$25,000	1.100	\$100,000	1.200	1.300
2001	35,000	1.200	120,000	1.100	1.150
2002	30,000	1.300	130,000	1.050	1.070
2003	15,000	1.500	140,000	1.000	1.000

The given loss trends are appropriate for the given excess layer of loss.
a. (1.5 points) Calculate the 2005 policy year experience rate for the layer \$500,000 excess of \$500,000.
b. (1 point) The primary insurer is also interested in purchasing an excess of loss cover for the layer \$10,000,000 excess of \$10,000,000. Some members of the management team believe that because they have experienced no losses in this layer, no premium should be charged for this layer. How would you respond to this assertion, and how would you price this layer?
Note: I have revised this past exam question.

143. (6, 11/05, Q.29) (2 points)
You are given the following information about a catastrophe reinsurance program:

Layer	Co-participation
Retention: \$2,000,000	NA
1st Layer: \$5,000,000 excess of \$2,000,000	10%
2nd Layer: \$10,000,000 excess of \$7,000,000	10%
3rd Layer: \$10,000,000 excess of \$17,000,000	15%

● Total premium for the catastrophe reinsurance program is \$8 million.
● No other reinsurance applies to the catastrophe program.
a. (1 point) Given a gross catastrophe loss of \$19 million,
what is the net loss ceded to the reinsurers?
b. (1 point) The program allows one reinstatement across all layers.
Assuming the premium for the reinstatement is pro-rata as to amount and 100% as to time, calculate the total reinstatement premium following the gross catastrophe loss of \$19 million.
Note: I have rewritten this past exam question to match the current syllabus.

144. (6, 11/05, Q.31) *(4 points) You have been asked by the chief financial officer (CFO) of a primary insurance company to recommend a reinsurance program.*
The CFO has the following concerns:
- *Underwriting results are less predictable than the CFO would like.*
- *Rapid growth has caused the premium to surplus ratio to become too high.*
- *The company is writing higher policy limits than it has in the past.*
- *There has been an increase in the number of claims with very high amounts of ALAE.*

Discuss the reinsurance program you would recommend to address each concern.
Identify the type of reinsurance used, its advantages and disadvantages, and how or why it would address the CFO's concerns.

145. (6, 11/05, Q.32) (2 points)
You are given the following information regarding a reinsurance treaty:

Effective Date	January 1, 2005
Termination Date	December 31, 2005
Basis of Attachment	Policies Attaching
Retention	$100,000
Limit	$250,000
Co-participation	10%
ALAE treatment	pro-rata

- All subject policies have a term of twelve months.

Policy Number	Policy Effective Date	Claim Number	Accident Date	Gross Loss	Gross ALAE
1	September 1, 2004	1	November 15, 2004	$175,000	$35,000
		2	February 15, 2005	150,000	45,000
2	March 1, 2005	1	July 1, 2005	375,000	30,000
		2	September 1, 2005	5,000	45,000
3	July 1, 2005	1	November 1, 2005	225,000	15,000
		2	February 15, 2006	175,000	70,000

Calculate the ceded amounts for <u>each</u> of these six claims.
<u>Note</u>: I have rewritten this past exam question to match the current syllabus.

146. (6, 11/05, Q.33) (2.5 points) According to Clark, a problem arises when trended losses are capped at policy limits when using experience rating to price a casualty excess treaty.
a. (0.5 point) Briefly state the problem.
b, (1 point) List the two possible approaches for addressing the problem.
c. (1 point) Briefly describe a disadvantage or concern for each approach listed in part b. above.

147. (6, 11/05, Q.34) (4 points) You are given the following information about a company's property book of business and reinsurance treaties.

Insured Value of Each Risk	Number of Risks	Total Gross Premium
$20,000,000	10	$500,000
60,000,000	8	800,000
120,000,000	5	900,000

● The exposure curve applicable to these risks is shown in the table below.

Percent of Insured Value	Exposure Factor
0%	0%
10%	28%
20%	42%
30%	52%
40%	60%
50%	67%
60%	73%
70%	79%
80%	83%
90%	88%
100%	92%
110%	97%
120%	100%

● The gross expected loss ratio for this book is 60%.
● The company has a per-risk treaty with attachment point $250,000 and limit $750,000.
● The company also has a surplus share treaty under which it retains a maximum of $1,000,000 on anyone risk.
● The surplus share treaty inures to the benefit of the per-risk treaty.
a. (2.5 points) Calculate the exposure-rated loss cost for the per-risk excess treaty net of the surplus share treaty.
b. (0.5 point) How should the surplus share treaty be reflected when experience rating the per-risk excess treaty?
c. (1 point) List and briefly describe two reasons why exposure rating can be distorted when deductible levels change.

148. (6, 11/05, Q.35) (1.5 points) A primary insurance company has entered into a 75% pro-rata treaty reinsurance agreement with a ceding commission of 30%, effective January 1, 2004.
The primary insurer has $1,000,000 of written premium, $300,000 of loss and $45,000 of loss adjustment expenses, gross of reinsurance, subject to the treaty.
a. (1 point)
What is the net amount owed by the primary company to the reinsurer under the treaty?
b. (0.5 point) Briefly explain the purpose of the ceding commission.

149. (6, 11/05, Q.36) (2 points)
a. (1 point) Give two reasons why report lags are longer for reinsurance companies than for primary carriers.
b. (1 point) An actuary is doing an independent reserve analysis for a reinsurance company and is using loss development data from the Reinsurance Association of America (RAA) as benchmarks. What are two items the actuary should consider when using these patterns?

150. (6, 11/05, Q.37) (1.5 points) A primary insurer has two reinsurance treaties:

Catastrophe	
Retention:	$40,000,000
Limit:	$60,000,000
Co-participation:	40%

Per-Risk Excess of Loss	
Retention:	$5,000,000
Limit:	$35,000,000
Co-participation:	40%

● The per-risk excess of loss contract inures to the benefit of the catastrophe contract.
● The gross amount of a single occurrence covered by both contracts is $75,000,000.
Calculate the losses retained by the primary insurer.
Note: I have rewritten this past exam question to match the current syllabus.

151. (6, 11/06, Q.10) (1 point) Given the following information regarding a loss covered by an excess of loss reinsurance treaty:

Ground-Up Loss	$775,000
Ground-Up ALAE	400,000
Treaty Attachment	500,000
Treaty Limit	600,000
ALAE Treatment	Pro Rata with Loss

If the treaty had been written to treat ALAE included with loss, what difference would it have made?

152. (6, 11/06, Q.11) (1 point) Given the following information regarding the coverage provided by a catastrophe reinsurance treaty (in $000,000):

Retention	100
Layer 1	85% of 100 excess of 100
Layer 2	90% of 100 excess of 200
Layer 3	95% of 300 excess of 300

Given a catastrophic loss of $450,000,000, what is the total loss retained by the primary insurer?

153. (6, 11/06, Q.36) *(2.5 points) A primary insurer sells only homeowners policies, specializing in non-coastal homes valued under $1,000,000. Its business has been profitable for the past 10 years. It has a long-standing treaty reinsurance agreement to cover the excess portion of losses above $750,000. The treaty allows for generous profit sharing, assuming favorable loss experience.*
The primary insurer has recently been subject to market pressure to liberalize its underwriting guidelines, with respect to both the maximum home value and coastal restrictions.
a. (1.5 points) Describe two problems, one from the perspective of the primary insurer and one from the perspective of the reinsurer, that might arise from guideline liberalization.
b. (1 point) How might the insurer and reinsurer address the problems identified in part a. above?

154. (6, 11/06, Q.37) *(3.5 points)*
A primary insurer enters into a quota share agreement with a reinsurer to cover its commercial property business, which is concentrated in an area with hurricane exposure.

Primary Insurer Loss Retention	*40%*
Maximum Per Risk Limit of Treaty	*$5,000,000*
Per-Occurrence Limit of Treaty	*$20,000,000*
Ceding Commission	*20%*
Range of Insured Building Value	*$100,000 to $25,000,000*
Primary Insurer's Surplus	*$500,000,000*

a. (1 point) Describe two potential concerns for the primary insurer under this reinsurance agreement.
b. (1 point) How might the primary insurer address each of these concerns?
c. (1.5 points) The primary insurer believes it has the capacity to retain exposure on smaller risks. However, it worries that the larger insured values may present too much risk to wholly retain. It also desires to maintain a ceding commission.
Identify two types of reinsurance treaties that the primary insurer might use to address these concerns. Explain how these treaties would meet each concern.

155. (6, 11/06, Q.38) (2 points) Within the context of an excess of loss reinsurance agreement, define each of the following terms and contrast it with the corresponding concept in a pro rata reinsurance agreement.
a. (0.5 point) Attachment point
b. (0.5 point) Rate
c. (0.5 point) Subject premium
d. (0.5 point) Co-participation provision

156. (6, 11/06, Q.39) *(2.25 points) The Chief Executive Officer of a small regional primary homeowner insurer has directed the company to generate new business growth at a rate twice what the industry is expected to achieve. The Chief Marketing Officer has decided that the best way to generate the new business is to write homes with values above current underwriting guidelines. The Chief Financial Officer is concerned that such an approach would raise the premium to surplus ratio above acceptable levels. The Chief Underwriting Officer has expressed concern that the company does not have the underwriting expertise to write the higher valued homes at an acceptable loss ratio.*
a. (0.75 points) What type of reinsurance could address the Chief Financial Officer's concern? Explain your answer.
b. (1.5 points) What two types of reinsurance could address the Chief Underwriting Officer's concern? Explain your answer.

157. (6, 11/06, Q.40) (2 points) A primary property insurer has a book of business with the following limit/premium distribution and increased limit factors (ILFs).
(Limits are for combined loss and ALAE.)

Limit	Expected Losses	ILF
$1 Million	$6 Million	1.00
2 Million	8 Million	1.25
3 Million	12 Million	1.45
4 Million	10 Million	1.60
5 Million	4 Million	1.70

A prospective reinsurer wants to write an excess of loss treaty for the layer $3 million excess of $1 million with an expected loss ratio of 60%, net of the brokerage fee of 10% of reinsurance premium. How much would the primary insurer have to pay for this coverage?
Note: I have rewritten this past exam question.

158. (6, 11/06, Q.42) (1.5 points) A homeowners insurer wishes to buy excess of loss reinsurance for its book, whose premium distribution by Coverage A limits is:

Coverage A Limit	Direct Written Premium
$100,000	$10,000,000
200,000	15,000,000
500,000	20,000,000
1,000,000	30,000,000

The following Salzmann table (exposure curve) applies, for total losses as a function of losses to Coverage A:

Percent of Coverage A	Exposure Factor
20%	10%
40%	25%
60%	45%
80%	65%
100%	85%
120%	90%
140%	94%
160%	97%
180%	99%
200%	100%

An excess of loss treaty covers the primary insurer for $500,000 excess of $600,000.
What fraction of the primary insurer's total expected losses would this treaty cover?
<u>Note</u>: I have rewritten this past exam question.

159. (6, 11/06, Q.43) (2 points)
For a particular building, you are given the following information for property insurance:

Amount of Insurance	Average Expected Annual Loss
$100,000	$22,084
150,000	22,364
200,000	22,791
250,000	23,321
300,000	23,875
350,000	24,430
400,000	25,000

Reinsurer Operating Expenses	15.0%
Reinsurer Profit and Contingency	7.5%
Primary Carrier Operating Expenses	24.0%
Primary Carrier Profit and Contingency	4.0%
Primary Rates Adequacy/(Deficiency)	(5.0%)

A primary carrier has written a property policy with a $350,000 limit on this property.
It is asking the reinsurer to provide reinsurance for the $200,000 excess of $100,000 layer.
a. (1 point) What premium should the reinsurer receive for this cover?
b. (0.5 point) Based on all the information, what premium did the primary carrier charge for this risk?
c. (0.5 point) If the excess premium charge were obtained by applying the percentage of expected losses ceded to the primary carrier's subject premium, would the resulting premium be adequate? Explain.
Note: I have rewritten this past exam question.

160. (6, 11/06, Q.44) (1.5 points)
A primary insurer has experienced the following losses on four of its issued policies:

Policy	Amount of Insurance	Loss
A	$200,000	$10,000
B	$50,000	$5,500
C	$20,000	$7,000
D	$10,000	$600

a. (0.5 point) What is the primary insurer's retained loss if it has a 65% quota share treaty?
b. (1 point) What is the primary insurer's retained loss if the line is $20,000 for a four-line surplus share treaty?

161. (6, 11/07, Q.2) (1 point) Given the following information for an insured property covered by an excess-of-loss reinsurance treaty:

Loss as a Percentage of Insured Value	Exposure Factor
10%	36%
20%	48%
30%	57%
40%	66%
50%	73%
60%	80%
70%	86%
80%	91%
90%	96%
100%	100%

Insured value of the property:	$400,000
The total expected loss for the property:	$88,000
Loss layer covered by the reinsurance treaty:	$200,000 excess of $120,000

Calculate the reinsurer's expected loss for this property.
Note: I have rewritten this past exam question to match the current syllabus.

162. (6, 11/07, Q.23) (1.5 points)
Given the following information about a quota share reinsurance treaty:

Primary insurer's pro rata share:	25%
Reinsurer's pro rata share:	75%
Flat ceding commission:	20%
Premium subject to reinsurance treaty:	$1,000,000
Loss subject to reinsurance treaty:	$250,000
Reinsurer's expense ratio:	10%
Reinsurer's expected loss ratio:	60%

The reinsurance treaty includes a profit-sharing commission to be paid to the primary insurer at the end of the treaty year if the reinsurer earns greater than expected profits. The profit sharing commission is 30% of the amount by which the reinsurer’s profit exceeds expected.
a. (0.75 point) Calculate the reinsurer's profit before the profit-sharing commission is considered.
b. (0.75 point) Calculate the amount of the profit-sharing commission.
Note: I have rewritten this past exam question

163. (6, 11/07, Q.24) (2.5 points) Given the following information:
- A reinsurance treaty covers 85% of the layer $5 million excess of $1 million.
- The treaty experiences a claim with $7 million of loss and
  $1.5 million of allocated loss adjustment expenses (ALAE).

a. (1 point) Calculate the combined loss and ALAE retained by the primary insurer, using the following ALAE approaches:
   i. The "pro rata in addition" method.
   ii. The "included in the limit" method.

b. (0.5 point) Calculate the combined loss and ALAE ceded to the reinsurer, using the following ALAE approaches:
   i. The "pro rata in addition" method.
   ii. The "included in the limit" method.

c. (1 point) Does either method generally favor one party over the other? Explain.

164. (6, 11/07, Q.25) (1.5 points) Explain three reasons why reinsurers must be cautious when using Reinsurance Association of America casualty excess loss development data.

165. (6, 11/07, Q.27) (2.5 points) Given the following data for an insurer:

Policy	Premium	Policy Limit	Loss
1	$12,000	$200,000	$150,000
2	3,000	50,000	50,000
3	30,000	500,000	300,000

a. (0.5 point) Assume these policies are covered by a 60% quota share treaty.
   i. Calculate the retained premium for the primary insurer.
   ii. Calculate the retained losses for the primary insurer.

b. (1 point) Assume these policies are covered by a surplus share treaty in which the primary insurer retains a line of $100,000 and the treaty contains nine lines providing a maximum cession of $900,000.
   i. Calculate the retained premium for the primary insurer.
   ii. Calculate the retained losses for the primary insurer.

c. (0.5 point) Provide one reason for a primary insurer to purchase surplus share reinsurance instead of quota share reinsurance.

d. (0.5 point) Provide one reason for a primary insurer to purchase quota share reinsurance instead of surplus share reinsurance.

166. (6, 11/07, Q.28) (2 points) A primary insurer has a 75% quota share treaty.
The insurer also has a $10 million excess of $10 million catastrophe treaty.
The insurer experiences a $15 million catastrophe loss covered by both treaties.
a. (0.5 point) Define the term inuring reinsurance.
b. (0.75 point) If the quota share treaty inures to the benefit of the catastrophe treaty, calculate how much of the $15 million loss is paid by the:

i. primary insurer
ii. quota share reinsurer
iii. catastrophe reinsurer

c. (0.75 point) If the catastrophe treaty inures to the benefit of the quota share treaty, calculate how much of the $15 million loss is paid by the:

i. primary insurer
ii. quota share reinsurer
iii. catastrophe reinsurer

***167. (6, 11/07, Q.29)** (2 points) An insurance company has been writing property business with policy limits of $300,000 in a specific geographic area. The insurance company has been retaining 100% of these risks.*
Due to increases in construction costs the insurance company expects some of its insureds to request an increase in policy limits to $500,000. The insurer is uncomfortable with the increase in loss exposure and would like to develop a reinsurance plan that allows it to retain all exposure on the $300,000 limit policies but only $400,000 on each of the $500,000 limit policies.
Describe in detail two different reinsurance treaty types that the insurer can use to achieve its goals. Be specific as to the appropriate treaty terms, including the desired retention for each policy.

168. (6, 11/07, Q.30) (1.5 points) Given the following information for a reinsurance treaty:
Sliding-scale commission:

Minimum Commission:	10% at a 70% loss ratio
Sliding 1.5: 1 to:	25% at a 60% loss ratio
Sliding 0.5: 1 to a maximum of:	30% at a 50% loss ratio

Aggregate loss distribution:

Range of Loss Ratio	Average in Range	Probability Loss Ratio is in Range
0%- 50%	40%	0.20
50% - 60%	56%	0.40
60% - 70%	64%	0.30
70% or above	73%	0.10

Calculate the expected technical ratio.

169. (6, 11/07, Q.31) (2.5 points)
A property excess-of-loss reinsurance treaty has the following terms:

Effective Date	January 1, 2007
Treaty Limit	$500,000
Attachment Point	$500,000

The following losses have been recorded for the book of business subject to the treaty and have been trended to the effective period of the treaty.

Accident Date	Trended Losses
April 30, 2003	$662,719
August 31, 2003	267,526
October 31, 2004	564,610
March 31, 2004	1,417,777
September 30, 2005	974,679
December 31, 2005	678,321

The on-level, trended subject premiums and excess layer loss development factors applicable to the treaty losses are:

Accident Year	On Level Trended Premium	LDFs
2003	$1,600,000	1.000
2004	1,632,000	1.050
2005	1,664,640	1.100

Determine the experience rating loss cost of this treaty.

170. (6, 11/07, Q.32) (2 points) Given the following information:

Loss as a Percent of Coverage Limit	Exposure Factor
10%	36%
20%	48%
30%	57%
40%	66%
50%	73%
60%	80%
70%	86%
80%	91%
90%	96%
100%	100%

Ceding Company Coverage Limit	Direct Premium
$50,000	$500,000
100,000	800,000
200,000	1,500,000
500,000	1,000,000

- Ceding company's expected loss ratio (excluding ALAE): 70%
- Ratio of ALAE to loss: 12%
- Ceding company rate inadequacy: 5%
- The reinsurer is to share the cost of ALAE proportionally to loss.

Calculate the indicated exposure rate for a property excess of loss treaty covering losses in the layer $400,000 excess of $100,000.
Note: I have rewritten this past exam question in order to match the current syllabus.

171. (6, 11/08, Q.28) (1.5 points) A primary insurance company has purchased an $800,000 excess of $200,000 per-occurrence excess-of-loss treaty with an annual aggregate deductible of $1,000,000. The occurrences subject to the treaty are as follows:

Occurrence	Loss Amount
1	$150,000
2	250,000
3	700,000
4	500,000
5	1,250,000
6	250,000

Calculate the primary company's net loss.

172. (6, 11/08, Q.29) (2 points) Given the following occurrences subject to a $900,000 excess of $100,000 per-occurrence excess-of loss treaty:

Occurrence	Incurred Loss	Incurred ALAE
1	$200,000	$100,000
2	50,000	150,000
3	1,000,000	200,000
4	500,000	200,000
5	1,500,000	300,000

a. (1 point) Calculate the total amount ceded to the treaty if allocated loss adjustment expense is to be included with loss.
b. (1 point) Calculate the total amount ceded to the treaty if allocated loss adjustment expense is to be shared pro rata with loss.

***173.* (6, 11/08, Q.31)** (3.25 points) Given the following as of December 31, 2007 for pricing a proposed quota share agreement on a book of property business:

Accident Year	Historical Earned Premium ($000,000)	Reported Losses ($000,000)	On-Level Factor	Loss Development Factor	Loss Trend Factor	Property Value Inflation Factor
2003	1,000	555	1.250	1.010	1.338	1.159
2004	1,100	1,638	1.200	1.025	1.262	1.126
2005	1,210	614	1.120	1.065	1.191	1.093
2006	1,331	552	1.075	1.250	1.124	1.061
2007	1,464	481	1.015	1.750	1.060	1.030
Total	6,105	3,840				

- Accident year 2004 has a $1,000,000,000 reported catastrophe loss included in the above data; no other years have experienced catastrophe losses.

Reinsurer Expense Item	Loading as a Percentage of Reinsurance Premium
Administrative Expenses	1.5%
Brokerage Fees	6.0%
Unallocated Expenses	1.5%

a. (2 points)
Calculate the five-year total ultimate non-catastrophe loss ratio projected to the treaty period.
b. (0.25 point) Based on the output of a catastrophe model, the indicated catastrophe load is 15% as a percentage of non-catastrophe losses.
Calculate the expected ultimate loss ratio for the treaty period.
c. (1 point) The ceding company is requesting a 27.5% ceding commission for this treaty.
The reinsurer's target combined ratio is 95%.
Explain whether the reinsurer should accept or reject the 27.5% ceding commission.

174. (6, 11/08, Q.32) (2.25 points) A primary insurance company has two treaties with a single reinsurance company. The first is a 10% quota share subject to a $100,000 maximum coverage amount for each policy. The second is a $900,000 excess of $100,000 per-occurrence excess-of-loss treaty where the primary company has a 5% co-participation. There have been four occurrences, each on a separate underlying policy, subject to the treaty:

Occurrence	Ground-up loss
1	$500,000
2	50,000
3	850,000
4	1,150,000

a. (1 point) Assuming that neither treaty inures to the benefit of the other, calculate the primary insurance company's total retained loss.
b. (1 point) Assuming that the quota share treaty inures to the benefit of the excess-of-loss treaty, calculate the primary insurance company's total retained loss.
c. (0.25 point) Briefly describe the purpose of a co-participation provision in a reinsurance treaty.

175. (6, 11/08, Q.33) (1.5 points) Given the following primary company loss data for policies subject to a four-line surplus share reinsurance treaty with a net line of $200,000:

Policy Number	Policy Limit	Incurred Loss
1	$200,000	$150,000
2	$1,000,000	$500,000
3	$500,000	$50,000
4	$450,000	$225,000

Calculate the primary company's net loss.

176. (6, 11/08, Q.38) (2 points) Given the following for a reinsurance treaty:
- Provisional commission of 32.5%
- Minimum commission of 27.5% at a 62.5% loss ratio
- Sliding 1:1 to 40% at a 50% loss ratio
- Sliding 1:3 to a maximum of 50% at a 20% loss ratio

Loss Ratio Range	Average in Range	Probability in Range
$x \le 20\%$	18.0%	0.15
$20\% < x \le 50\%$	44.0%	0.20
$50\% < x \le 62.5\%$	57.5%	0.25
$x > 62.5\%$	80.0%	0.40

Calculate the expected ceding commission for this book of business.

177. (6, 11/08, Q.39) (2.5 points) Given the following to be used in pricing a property excess-of-loss treaty effective January 1, 2008 and expiring December 31, 2008:
● Treaty limit: $500,000
● Attachment point: $100,000
● Annual ground up loss trend: 4%

Historical Losses

Date of Loss	Ground-up Loss
January 1, 2000	$200,000
January 1, 2001	200,000
July 1, 2001	225,000
July 1, 2002	600,000
January 1, 2004	475,000
July 1, 2005	90,000

Loss Development Factors Applicable to Treaty Layer

Accident Year	Factor to Ultimate
2000	1.00
2001	1.00
2002	1.05
2003	1.10
2004	1.30
2005	1.50
2006	2.00

Calculate the trended ultimate losses in the treaty layer that would be used to experience rate this treaty.

178. (6, 11/09, Q.21a) (2 points) Given the following contract language for a property per-risk excess-of-loss reinsurance contract:
The company shall retain and be liable for the first $250,000 of ultimate net loss as respects to anyone risk, each loss occurrence. The reinsurer shall then be liable for the amount by which such ultimate net loss exceeds the company's retention, but the liability of the reinsurer shall not exceed $600,000 as respects anyone risk, each loss occurrence.
The liability of the reinsurer under this contract in any one loss occurrence shall in no event exceed $1,000,000.

The following losses covered by the reinsurance contract have occurred:

Risk	Dwelling	Contents	Date of Loss
1	$600,000	$30,000	Feb. 3, 2008
2	575,000	25,000	Feb. 3, 2008
3	700,000	35,000	Feb. 3, 2008
4	250,000	25,000	May 5, 2008
5	150,000	10,000	Oct. 5, 2008
6	800,000	200,000	Oct. 5, 2008

Losses on the same date are considered one occurrence.
Calculate the amount of loss retained by the primary insurer.
Note: Part b of this past exam question is no longer on the syllabus.

179. (6, 11/09, Q.22) (1.5 points)
Given the following information regarding a casualty excess-of-loss treaty:
- Treaty Effective Date: January 1, 2008
- Treaty Expiration Date: December 31, 2008

Primary company loss experience:

Claim Number	Policy Effective Date	Policy Type	Date of Loss	Loss to Treaty
1	Nov. 1, 2007	New	Jan. 23, 2008	\$50,000
2	Dec. 1, 2007	Renewal	Dec. 17, 2007	200,000
3	Dec. 15, 2007	Renewal	Feb. 8, 2008	220,000
4	Jan. 1, 2008	New	Apr. 10, 2008	65,000
5	Jan. 1, 2008	Renewal	Mar. 4, 2008	120,000
6	Feb. 15, 2008	New	Jun. 16, 2008	400,000
7	Apr. 1, 2008	New	Aug. 12, 2008	185,000
8	Jun. 1, 2008	New	Sep. 25, 2008	90,000

a. (0.75 point) Calculate the losses covered by the treaty, assuming the treaty is written on a risks attaching basis. Include the definition for risks attaching basis in your answer.
b. (0.75 point) Calculate the losses covered by the treaty, assuming the treaty is written on a losses occurring basis. Include the definition for losses occurring basis in your answer.
Note: Parts c and d of this past exam question are no longer on the syllabus.

180. (6, 11/09, Q.25) (2 points) A primary insurer has a 50% quota share treaty and a \$10 million excess of \$20 million catastrophe treaty.
a. (1 point) A policy covered by these treaties experiences a \$30 million loss. The catastrophe treaty inures to the benefit of the quota share treaty. Calculate how much of the loss is covered by each of the two treaties and how much is retained by the primary insurer.
b. (1 point) A policy covered by these treaties experiences a \$45 million loss. The quota share treaty inures to the benefit of the catastrophe treaty. Calculate how much of the loss is covered by each of the two treaties and how much is retained by the primary insurer.

181. (6, 11/09, Q.26) (1.5 points) A primary insurer purchases a $300,000 excess of $200,000 per occurrence excess-of-loss reinsurance treaty.
The following ground-up losses are subject to the treaty:

Claim Number	Loss	Loss Adjustment Expense
1	$200,000	$300,000
2	300,000	200,000
3	300,000	100,000
4	600,000	200,000

Calculate the primary insurer's total recovery for each of the following scenarios:
a. (0.75 point) Loss adjustment expense is included in the limit.
b. (0.75 point) Loss adjustment expense is shared pro-rata in addition to loss.

182. (6, 11/09, Q.27a) (2.5 points) A homeowners insurer wishes to buy excess-of-loss reinsurance. Given the following information for this insurer:

Coverage A Limit	Direct Written Premium
$100,000	$10,000,000
250,000	25,000,000
500,000	20,000,000
1,000,000	10,000,000

The following exposure curve (Salzmann table) applies for total losses as a function of losses to Coverage A:

Percent of Coverage A	Exposure Factor
20%	10%
40%	25%
60%	45%
80%	65%
100%	85%
120%	90%
140%	94%
160%	97%
180%	99%
200%	100%

An excess-of-loss treaty covers the primary insurer for $750,000 excess of $250,000.
Using linear interpolation where necessary, calculate the fraction of the primary insurer's total expected losses covered by this treaty.
Note: Part b of this past exam question is no longer on the syllabus.

183. (6, 11/09, Q.28) (1 point) When pricing an excess-of-loss treaty using experience rating, an actuary may come across the issue of free cover.
a. (0.5 point) Describe what is meant by free cover.
b. (0.5 point) Describe a pricing approach that addresses this issue.

184. (6, 11/09, Q.29) (3 points) Given the following information as of September 30, 2009 for a property quota share treaty that will renew January 1, 2010:

Accident Year	Earned Premium ($000)	Incurred Loss & ALAE ($000)	Premium On-Level Factors	Loss & ALAE Development Factor
2005	7,400	4,200	1.188	1.000
2006	8,000	3,800	1.142	1.015
2007	8,500	4,600	1.100	1.045
2008	9,100	4,400	1.048	1.150
2009	6,800	2,600	1.000	1.800

Annual loss trend	5%
Annual premium trend	3%
Ceding commission	30% of premium
Brokerage fees	6% of premium
Administrative expenses	1% of premium
ALAE	8% of loss
Catastrophe load	11% of non-catastrophe loss and ALAE
ULAE	3% of total loss and ALAE

- The treaty term is 12 months.
- The treaty is written on a losses occurring basis.
- There are no reported catastrophe losses.
- The full year earned premium for AY 2009 will be $9,400,000.

Calculate the projected combined ratio for the 2010 treaty renewal period.

185. (6, 11/09, Q.30) (2 points) A proportional reinsurance treaty has a loss corridor provision in which the ceding company will retain 80% of the losses in the 75% to 100% loss ratio layer.
Given the following loss ratio information:

Range of Direct Loss Ratio	Average Loss Ratio in Range	Probability Loss Ratio is in Range
0%-50%	44%	6%
50%-75%	67%	64%
75%-100%	86%	22%
Over 100%	109%	8%

a. (0.5 point)
Calculate the reinsurer's expected loss ratio before the application of the loss corridor.
b. (1.5 points)
Calculate the reinsurer's expected loss ratio after the application of the loss corridor.

186. (6, 11/10, Q.20) (3.5 points) Given the following information regarding a primary insurance company's property book of business:

Range of Insured Values ($000s)	Subject Premium
100 - 200	$300,000
200 - 300	250,000
300 - 700	423,000
700 - 1,300	871,000

● Exposure curve applicable to property risks:

Percent of Insured Value	Exposure Factor
0%	0%
20%	45%
40%	60%
60%	73%
80%	85%
100%	95%
120%	100%

● The primary insurance company's expected loss ratio: 60%
● A per risk treaty covers the layer of loss $300,000 excess of $200,000.
● The exposure rating method and linear interpolation is used to determine exposure factors.
● Reinsurer's expenses:
ULAE as a percent of loss: 10%
Fixed expense as a percent of subject premium: 5%
Variable expense ratio: 20%
a. (2.5 points) Calculate the reinsurer's expected loss cost as a percentage of subject premium.
b. (1 point) Calculate the final premium for the reinsurance treaty.

187. (6, 11/10, Q.21) (1.5 points)
Given the following information for a catastrophe reinsurance program:

	Layer of Ceded Loss	Layer	Reinsurance Premium
Reinsurer	$0	Participation	Paid ($000,000)
A	125 excess of 75	100%	10
B	100 excess of 200	60%	4

● Gross written premium for the primary insurer's property program is $35 million.
● One $82.5 million catastrophe loss occurs during the treaty year.
a. (0.5 point) Calculate the rate-on-line for each reinsurer.
b. (0.5 point) Calculate the payback period for each reinsurer.
c. (0.5 point) Calculate the ratio of excess losses to total subject premium,
in other words the burning cost, for the total reinsurance program.

188. (6, 11/10, Q.22) (1.25 points)
Given the following information for a property catastrophe reinsurance treaty:

Effective date of treaty	January 1, 2009
Expiration date of treaty	December 31, 2009
Attachment point	$30,000,000
Treaty limit	$20,000,000
Annual premium	$3,600,000
Reinstatement provision	1.15

One $45,000,000 gross loss subject to the treaty occurs on July 31, 2009.
a. (0.5 point)
If the reinstatement provision is pro rata as to amount, calculate the reinstatement premium.
b. (0.5 point) If the reinstatement provision is pro rata as to both amount and time, calculate the reinstatement premium.
c. (0.25 point) Briefly explain why property catastrophe reinstatement provisions are typically not pro rata as respects to time.

189. (6, 11/10, Q.24a) (1 points)
Given the following information for a surplus share treaty:
● The primary carrier retains a line of $250,000 and cedes three lines.
● Insured values and losses by risk:

Risk	Insured Value	Incurred Loss
1	$200,000	$75,000
2	500,000	400,000
3	1,250,000	200,000

Calculate the total amount of loss retained by the primary insurer.
Note: Parts b and c of this past exam question are not covered on the current syllabus of this exam.

190. (6, 11/10, Q.25) (1.5 points)
Given the following information for a primary insurer's book of liability policies:

Policy Limit	Earned Premium
$5,000,000	$20,000,000
10,000,000	50,000,000
20,000,000	100,000,000

● The expected loss ratio (gross of reinsurance) is 60%.
● Selected Increased Limit Factors (ILF):

Size of Loss	ILF
$1,000,000	1.0
2,000,000	1.6
3,000,000	1.9
4,000,000	2.1
5,000,000	2.2
10,000,000	2.5
20,000,000	3.0

Calculate the expected loss for the reinsurance layer $6,000,000 excess of $4,000,000.

191. (6, 11/10, Q.26) (2 points) Given the following ground up loss and ALAE for occurrences subject to an umbrella excess-of-loss reinsurance treaty:

Occurrence Number	Ground Up Loss	Ground Up ALAE	Limits of Policies Providing Coverage
1	$1,100,000	$100,000	$500,000 Umbrella over $500,000 Homeowners Liability
2	900,000	100,000	$1,000,000 Umbrella over $1,000,000 Homeowners Liability
3	1,000,000	100,000	$1,000,000 Umbrella over $750,000 Auto Liability

The treaty covers $250,000 excess of $250,000 per occurrence on the umbrella portfolio.
● The treaty covers ALAE pro rata with loss.
● The ceding commission for the treaty is 15% of the reinsurance premium.
● The reinsurance premium for the treaty is $900,000.
Calculate the reinsurer's technical ratio for this umbrella treaty.

192. (6, 11/10, Q.27) (2.25 points)
Given the following information for a primary company's reinsurance program:
● Ground up losses:

Loss Number	Number	Occurrence Loss Amount
1	1	$100,000
2	1	200,000
3	1	300,000
4	2	200,000
5	2	100,000
6	3	450,000

● Subject earned premium equals $1,500,000.
● Treaty 1 covers the layer $150,000 excess of $200,000 per occurrence.
● Treaty 2 covers the layer $500,000 excess of $250,000 per occurrence.
● Treaty 1 inures to the benefit of Treaty 2.
● Reinsurance premium for Treaty 1 equals $200,000.
● Reinsurance premium for Treaty 2 equals 7.5% of net subject premium.
a. (0.75 point) Calculate the expected recovery by the primary company from Treaty 1.
b. (0.75 point) Calculate the expected recovery by the primary company from Treaty 2.
c. (0.75 point) Calculate the ceded loss ratio for Treaty 2.

193. (6, 11/10, Q.28) (1 point) A primary insurer has the following reinsurance program:
- Excess-of-loss treaty: $3,000,000 excess of $500,000 per occurrence
- Clash cover: $9,000,000 excess of $1,000,000

Given the following losses from a single occurrence subject to the treaties:

Policy	Ground Up Loss
1	$200,000
2	1,500,000
3	2,000,000
4	1,000,000

a. (0.5 point) Calculate the total ceded losses under the excess-of-loss treaty.
b. (0.5 point) Calculate the total ceded losses under the clash treaty.

194. (8, 11/11, Q.7) (4 points) An actuary is pricing the $4,000,000 excess of $1,000,000 layer for an excess of loss policy on a property with a total insured value of $10,000,000.

Historical policy information is as follows:

Accident Year	LDF	On-Level Trended Subject Premium
2008	1.00	$6,400,000
2009	1.04	$6,900,000
2010	1.08	$7,400,000

Historical loss information is as follows:

Accident Date	Actual Loss	Trended Loss
March 4, 2008	$2,544,684	$2,976,920
October 12, 2008	$831,360	$972,574
April 2, 2009	$1,969,985	$2,215,965
June 6, 2009	$1,394,744	$1,564,398
August 14, 2010	$2,525,492	$2,731,572

The exposure curve below applies to the insured's risk profile:

% of Insured Value	Exposure Factor
10%	35%
20%	46%
30%	55%
40%	63%
50%	70%
60%	77%
70%	83%
80%	88%
90%	92%
100%	95%
110%	98%
120%	100%

Use a mixture of experience and exposure loss rating to calculate the policy's loss cost as a percentage of subject premium. Include a discussion on the appropriate layer for experience rating and the appropriate layer for exposure rating.

195. (8, 11/11, Q.8) (2 points) The table below represents a reinsurer's aggregate loss distribution for a single ceding company's treaty:

Range of Loss Ratios	Average in Range	Probability Loss Ratio is in Range
0% to 60%	47.9%	0.45
60% to 75%	67.8%	0.29
75% to 90%	81.5%	0.17
90% or above	99.6%	0.09

The ceding company will reassume 50% of the losses from a 60% to 75% loss ratio and 80% of the losses from a 75% to 90% loss ratio.
Calculate the reinsurer's expected loss ratio after the application of the loss corridor.

196. (8, 11/12, Q.7) (2.5 points)
A reinsurer uses the following commission structure to pay a ceding company:

Provisional commission	30%	
Minimum commission	20%	At a 75% loss ratio
Sliding 1:1 to	35%	At a 60% loss ratio
Sliding 0.5:1 to a maximum	45%	At a 40% loss ratio

The aggregate loss distribution model is as follows:

Range of Loss Ratios	Average Loss Ratio in Range	Probability the Loss Ratio is in Range
0-40%	35%	0.04
40-60%	55%	0.32
60-75%	68%	0.24
75%+	80%	0.40

a. (1 point) Calculate the expected loss ratio and expected sliding scale commission using the aggregate loss distribution model above.
b. (0.5 point) Explain what a carryforward provision is and why it is used.
c. (0.5 point) Assume the prior year's loss ratio was above 75%. Explain what effect a carryforward provision would have on the expected commission for the current year, all else being equal.
d. (0.5 point) Briefly explain two approaches used in pricing the impact of a carryforward provision.

197. (8, 11/12, Q.10) (2 points) An actuary has calculated an exposure curve using 10 years of the company's historical commercial property data, comprised of the following portfolio.
All insured values used to calculate the exposure curve are on a per location basis.

Insured Value Range	Number of Risks	Total Premium	Pure Premium per Location
$500,000 - $1,000,000	415	$23,810,000	$34,539
$1,000,001- $2,500,000	650	$90,400,000	$80,665
$2,500,001- $5,000,000	350	$92,970,000	$172,659
$5,000,001- $10,000,000	180	$94,199,600	$345,399

a. (1 point)
Evaluate the actuary's decision to use this book of business to produce an exposure curve.
b. (1 point) The actuary is considering using the exposure curve to price a commercial property excess of loss treaty. The actuary is given the following information:
- A per risk limits profile for the subject commercial property business.
- Ten years of historical ultimate loss ratios for the commercial property business:

Year	Ultimate Loss Ratio
2011	120%
2010	40%
2009	90%
2008	15%
2007	150%
2006	30%
2005	75%
2004	10%
2003	105%
2002	60%

Evaluate the actuary's decision to use the exposure curve to price the treaty.

198. (8, 11/13, Q.21) (1.5 points) A property catastrophe treaty covers the layer $50,000,000 excess of $50,000,000 with an annual premium of $3,000,000 and a reinstatement provision that is 120% pro-rata as to amount with no limit on the number of reinstatements.
The treaty is issued for a one-year term effective January 1, 2013.
a. (0.75 point) Given the three following ground-up catastrophe losses during 2013:
- A loss of $65,000,000 on June 1.
- A loss of $85,000,000 on September 1.
- A loss of $115,000,000 on November 1.

Calculate how much the ceding company pays in reinstatement premiums during 2013.
b. (0.5 point) Calculate the annual total reinstatement premium with the same three losses as above if the reinstatement provision was pro-rata as to amount and pro-rata to time.
c. (0.25 point) Briefly explain why relatively few contracts include reinstatements pro-rata as to time.

199. (8, 11/13, Q.23) (2 points) A primary insurance company's actuary is evaluating the following three types of reinsurance contracts:

- 60% ceded quota share.
- Five-line surplus share treaty with retained line = $100,000.
- $400,000 xs $100,000 per-risk excess of loss.

In the most recent accident year, the company has experienced the following losses on its policies:

Risk	Insured Value	Loss
A	$250,000	$120,000
B	$1,000,000	$245,000
C	$85,000	$85,000
D	$1,250,000	$490,000
E	$400,000	$180,000
Total	$2,985,000	$1,120,000

Determine which reinsurance contract would result in the lowest retained losses for the insurance company.

200. (8, 11/13, Q.25) (1.25 points) An insurance company is considering a sliding-scale commission structure for its niche casualty excess of loss program, comprised of independent retailers. Using the program's loss history, two actuaries are tasked with calculating an aggregate loss distribution.
Given the following information:

● All policies have a $2,000,000 per occurrence insured retention.
● All policies have occurrence limits of either $1,000,000 or $2,000,000 in excess of the insured's retention.
● All policies have a $2,000,000 aggregate limit.
● The only claims that are reported to the insurer are those that exceed the insured's retention.

Policy Year	Aggregate Loss in Excess of $2,000,000	Reported Claim Count
2000		0
2001	$1,506,002	2
2002	$1,070,358	1
2003		0
2004	$977,602	1
2005	$2,490,714	2
2006		0
2007	$512,933	1
2008		0
2009		0

Actuary A wants to use the lognormal distribution to determine the aggregate loss distribution.
Actuary B prefers Panjer's recursive formula, using a Poisson frequency distribution.
Evaluate each actuary's selection and propose the more appropriate method.

201. (SOA GIADV Exam, 5/14, Q.1) (4 points)
Property R Us Insurance Company is a small insurer that insures commercial buildings of all sizes. It has purchased surplus share reinsurance with a retained line of 100,000.
(a) (0.5 point) Explain why Property R Us may have purchased surplus share reinsurance instead of quota share reinsurance.
(b) (1.5 points) The surplus share treaty has a sliding scale commission with terms as follows:

Minimum Commission	20% at a 70% loss ratio
Sliding 1:1 to	30% at a 60% loss ratio
Sliding 0.5:1	to a Maximum 40% at a 40% loss ratio

There is a 20% probability of a loss ratio below 40%, a 20% probability of a loss ratio above 70%, and the loss ratio is uniformly distributed in the range from 40% to 70%.
Calculate the expected commission.
(c) (1 point) State whether the expected commission will increase or decrease as Property R Us grows its business and writes more risks. Support your conclusion.
(d) (1 point) Property R Us wishes to purchase a 50,000 excess of 50,000 property per risk excess treaty. The surplus share reinsurance will inure to the benefit of the property per risk treaty. Explain how an exposure curve can be used to price risks with an insured value of 1,000,000 for the property per risk treaty.

202. (SOA GIADV Exam, 5/14, Q.8) (5 points)
Casualty R Us Reinsurance Company has been presented with an opportunity to write a casualty per occurrence excess treaty covering the layer 500,000 excess of 500,000 on a swing plan. The following limits profile has been provided:

Subject Premium	Underlying Limit	Policy Limit	
5,000,000	0	500,000	
5,000,000	0	750,000	
15,000,000	0	1,000,000	
10,000,000	0	1,500,000	
2,000,000	500,000	1,000,000	Umbrella Policies

ALAE is shared pro-rata by the treaty.
(a) (3 points) Calculate the expected losses in the layer using an exposure rating approach with an expected loss ratio of 60% and the following increased limits factors:

Policy Limit	Increased Limits Factor
500,000	1.00
750,000	1.20
1,000,000	1.35
1,500,000	1.56

(b) (0.5 point) ALAE on the underlying business is expected to be 20% of losses, and ALAE is allocated to each layer in proportion to losses. Explain why applying a loading of 20% of layer losses to account for ALAE in the layer is problematical.
(c) (1 point) Explain one method for calculating probabilities when using a collective risk model approximation to the aggregate distribution to set the terms of the swing plan.
(d) (0.5 point) Recommend whether or not this ceding company should purchase any other casualty per occurrence excess coverage. Justify your answer.
Note: I have revised this past exam question.

203. (SOA GIADV Exam, 11/14, Q.1) (4 points) Property R Us Reinsurance Company uses the following exposure factors to price its commercial property per risk excess treaties:

Percent of Insured Value	Exposure Factor
0%	0%
10%	37%
20%	49%
30%	57%
40%	64%
50%	70%
60%	76%
70%	81%
80%	85%
90%	89%
100%	93%
110%	97%
120%	100%

(a) (0.5 point)
Explain why the exposure curve allows for exposure above 100% of the insured value.
(b) (2 points) Property R Us has received a submission requesting 400,000 excess of 100,000 property per risk coverage for insured values and subject premium as follows:

Insured Value	Subject Premium
200,000	1,000,000
500,000	1,000,000
1,000,000	1,000,000

Calculate the expected loss in the requested layer assuming an expected loss ratio of 60%.
(c) (0.5 point)
State the key assumption underlying the use of a single exposure curve to price this treaty.
(d) (1 point) The submission is revised to reflect the purchase of surplus share reinsurance with a retained line of 500,000, which will inure to the benefit of the property per risk treaty.
Calculate the revised expected loss in the layer.

204. (SOA GIADV Exam, 11/14, Q.8) (5 points) Casualty R Us Reinsurance Company has been presented with an opportunity to write a casualty per occurrence excess treaty covering the layer 500,000 excess of 500,000. The following limits profile has been provided:

Subject Premium	Underlying Limit	Policy Limit	
5,000,000	0	500,000	
5,000,000	0	1,000,000	
4,000,000	0	1,500,000	
2,000,000	500,000	500,000	Umbrella Policies
3,000,000	500,000	1,000,000	Umbrella Policies

(a) (2 points) Calculate the expected losses in the layer using an exposure rating approach with an expected loss ratio of 60% and the following increased limits factors:

Policy Limit	Increased Limit Factor
500,000	1.00
1,000,000	1.35
1,500,000	1.56

(b) (1.5 points) Identify the two methods for handling trend and policy limits when an experience rating approach is used to calculate the expected losses.
State the assumption that underlies each method.

(c) (1 point) Individual losses of 600,000 and 750,000 are incurred.
Identify, for each loss, the range of ALAE amounts for which allocating ALAE to the layer in proportion to losses would result in Casualty R Us paying more than it would if ALAE were included with losses.

(d) (0.5 point) Identify any additional casualty per occurrence excess coverage that the ceding company should consider purchasing. Justify your answer.

205. (8, 11/14, Q.20) (1.5 points) An insurer is evaluating the experience of an annual umbrella policy that is renewing on April 1, 2015.

Attachment Point:	\$3,000,000
Policy Limit:	10,000,000

- The insurer paid a \$4,000,000 claim for a loss that occurred on July 1, 2014.
- Trend is applied to the midpoint of a prospective policy year

a. (0.75 point) The insurer has trended the July 1, 2014 claim to \$4,395,940.
Determine the insurer's annual trend factor.

b. (0.75 point) Explain how the upward drift of policy limits and attachment points on the underlying and umbrella policies can distort the trending of historical losses and if the trended claim would likely be overstated or understated had the loss occurred in 2004.

206. (8, 11/14, Q.21) (3.5 points) Company A is pricing an umbrella policy with an effective date of July 1, 2014 for a large commercial risk that is written above Company B's underlying layer. Company A is considering various options for the treatment of allocated loss adjustment expenses (ALAE).

a. (2 points) Fully explain how the following options rank with respect to the relative cost of the umbrella policy:

1. ALAE is included within both the underlying and umbrella limits.
2. ALAE is in addition to both the underlying and umbrella limits.
3. ALAE is included within the underlying layer's limit, but is in addition to the umbrella's limit.

b. (1 point) The following loss history of the risk is provided by Company B below:

Date of Loss	Paid Loss	Paid ALAE	Reserved Loss	Reserved ALAE
September 16, 2009	$240,030	$324,235	$0	$0
December 14, 2009	43,658	8,750	0	0
March 1, 2010	2,000,000	140,000	0	0
March 14, 2010	50,000	891,320	0	0
August 11, 2010	0	75,500	25,000	174,500
January 2, 2011	1,257,902	124,870	0	0
March 14, 2012	200,000	45,040	1,800,000	55,960
July 1, 2012	32,320	175,340	0	0
November 30, 2012	1,000,000	22,430	1,000,000	250,000

Explain two reasons why the pricing actuary for Company A should be wary of using the historical data to determine the ALAE load.

c. (0.5 point) The pricing actuary has been asked to opine on the treatment of ALAE for a clash cover excess treaty. The actuary states that because penetration of high excess layers is infrequent, the calculation of ALAE is difficult and the loading is insignificant. Evaluate this statement.

207. (8, 11/14, Q.22) (2.5 points)

The following information is given for a one-year reinsurance treaty effective January 1, 2012.

Risk	Effective Date	Insured Value	Loss	Loss Date
1	June 1, 2011	$200,000	$100,000	March 1, 2012
2	January 1, 2012	500,000	400,000	June 1, 2012
3	June 1, 2012	1,500,000	1,500,000	September 1, 2012

a. (0.5 point) Calculate the reinsurer's loss liability for a 30% quota share agreement written on a risks attaching basis.

b. (1 point) Calculate the reinsurer's loss liability for a 5-line surplus share agreement written on a losses occurring basis with a retained line of $100,000.

c. (1 point) The primary insurance company purchased a treaty on a risks attaching basis for the time period from January 1, 2013 to December 31, 2013 time.

They then decided to purchase the same treaty but on a losses occurring basis for the time period from January 1, 2014 to December 31, 2014.

Describe a coverage issue that could arise with these two treaties.

Explain how the ceding company and reinsurer can structure the treaties to avoid this issue.

208. (8, 11/14, Q.23) (2 points) While negotiating the terms of a quota share treaty, a reinsurer is considering an agreement with a sliding scale commission and an agreement with a loss corridor. The terms of the sliding scale commission are as follows:

Provisional Commission:	20% at a 65% loss ratio
Sliding 1: 1 to a minimum:	10% at a 75% loss ratio
Sliding 0.5:1 to a maximum:	30% at a 45% loss ratio

The terms of the loss corridor are as follows:

Commission:	15%
Loss Corridor:	60% of 75% to 85% loss ratio

The expected loss ratio is 73%
a. (0.5 point) Assume that the ceding company is interested in optimizing the amount and the timing of cash flows. Identify and briefly describe one advantage of the sliding scale commission option when compared to the loss corridor option.
b. (1 point) The insurer and reinsurer agree that the performance of the underlying business is highly volatile. Describe two ways the insurer can stabilize its results for its sliding scale commission structure over time.
c. (0.5 point) Explain whether the smoothing mechanisms in part b. above should be used in the determination of an aggregate loss distribution model.

209. (8, 11/14, Q.25) (1.5 points) An actuary is using exposure rating to calculate increased limit factors for an auto liability treaty. The actuary has selected a severity distribution for the exposures being considered. The expected value function of losses capped at L is:

$$E[x; L] = 30 + 900\{1 - \ln(1000/L)\}$$

Additionally, the actuary has the following information:
- All of the ceding company's underlying policy limits are $1,000,000
- The reinsurance treaty attachment point is $250,000
- The reinsurance treaty limit is $750,000

a. (1 point) Calculate the exposure factor.
b. (0.5 point)
Calculate the ground up expected loss if the estimated loss cost of the treaty layer is $243,500.

210. (SOA GIADV Exam, 5/15, Q.1) (6 points) Casualty R Us Reinsurance Company has been presented with an opportunity to write a casualty per occurrence excess treaty for accident year 2016 covering the layer 500,000 excess of 500,000.
You are given the following information:
● The following loss experience has been provided, evaluated as of 12/31/2014:

Accident Date	Untrended Loss	Untrended ALAE
7/1/2012	400,000	500,000
7/1/2012	750,000	100,000
7/1/2013	450,000	0
7/1/2013	900,000	1,000,000
7/1/2014	500,000	100,000
7/1/2014	1,000,000	0

● All losses of at least 400,000 are shown.
● All policy limits throughout the experience period are 1,000,000 and are expected to remain at this level through 2016.
● On level subject premium is 10,000,000 for each year from 2012-2014.
● ALAE is allocated to layer in proportion to losses.
● Loss and ALAE trend are each 5% per year.
● The following accident year development factors are applicable to both loss and ALAE in the layer 500,000 excess of 500,000:

12-Ultimate	3.00
24-Ultimate	1.50
36-Ultimate	1.10

(a) (4 points) Estimate the experience rating loss and ALAE cost as a percentage of the subject premium.
(b) (1.5 points) State an alternative model to experience rating, and identify any additional information you would need to apply this alternative.
(c) (0.5 points) Recommend the model to which you would assign greater credibility in this case.
Justify your recommendation.

211. (SOA GIADV Exam, 5/15, Q.8) (4 points) A reinsurance broker has proposed that Property R Us Reinsurance Company provide a finite risk cover without reinstatements to All Cat Insurance Company with the following terms:
● Annual Premium: 10,000,000
● Occurrence Limit: 100,000,000
● Profit Commission: 90% after 10% margin on Annual Premium
● Additional Premium: 50% of (Loss + Margin - Annual Premium)
(a) (0.5 points) Calculate the nominal rate on line.
(b) (0.5 points) Calculate the underwriting loss (excluding expenses) to Property R Us if a loss fully exhausts the limit.
(c) (0.5 points) Calculate the premium for an equivalent traditional risk cover.
(d) (0.5 points) Calculate the rate on line for an equivalent traditional risk cover.
(e) (2 points) A catastrophe model indicates that a loss will fully exhaust the limit once every 15 years and that the probability of a partial loss is negligible.
Recommend whether or not Property R Us should accept the proposal.
If your answer is no, offer a counterproposal. Justify your answer.

***212.* (SOA GIADV Exam, 11/15, Q.1)** (5 points) Casualty R Us Reinsurance Company has been presented with an opportunity to write a casualty per occurrence excess treaty covering the layer 1,000,000 excess of 1,000,000 on a portfolio of umbrella policies.
The following limits profile has been provided:

Subject Premium	Underlying Limit	Umbrella Policy Limit
5,000,000	1,000,000	1,000,000
4,000,000	1,000,000	2,000,000
2,250,000	1,000,000	3,000,000
2,000,000	2,000,000	1,000,000
1,500,000	2,000,000	2,000,000

An analysis of historical data indicates that losses excess of 1,000,000 in this portfolio will follow a distribution with cumulative distribution function

$$F(x) = 1 - \left(\frac{x}{1{,}000{,}000}\right)^{-2}, \ x > 1{,}000{,}000.$$

(a) (3 points) Calculate the expected losses in the layer using an exposure rating approach with an expected loss ratio of 60%, ignoring “drop down” exposure.
(b) (0.5 points) Define “drop down” exposure.
(c) (0.5 points) Explain how your analysis would have to be modified to take into account “drop down” exposure.
(d) (1 point) An experience rating analysis may require the use of excess loss development factors from industry sources such as the Reinsurance Association of America.
Describe two cautions that should be considered when using such factors.

213. (8, 11/15, Q.21) (3.75 points) A reinsurer is offering a ceding company a two-year aggregate stop loss with the following terms:
● The treaty is effective on 1/1/2016 and expires on 12/31/2017.
● The treaty will cover aggregate losses between a 65% and a 70% loss ratio for accident years 2016 and 2017 separately.
● Premium is paid at the beginning of each year and all losses are paid at the end of the year incurred.
Additionally, the reinsurance treaty has the following termination provisions:
● The ceding company has the option to terminate the contract at the end of 2016 and receive a profit commission from the reinsurer calculated as follows:
Profit commission = [35% * 2016 Ceded Premium] - [2016 Ceded Loss], subject to a minimum of 0.
● If the contract is not terminated at the end of 2016, the ceding company will again have the option to terminate the contract at term expiration and receive a profit commission from the reinsurer calculated as follows:
Profit commission = [35% * Full Term Ceded Premium] - [Full Term Ceded Loss], subject to a minimum of 0.
● The reinsurer cannot terminate the contract at any time.

The reinsurer has simulated five trials of the ceding company's loss ratio subject to the aggregate stop loss as follows (values in millions):

	Accident Year 2016				Accident Year 2017		
Simulation	Loss Ratio		Loss Amount		Loss Ratio		Loss Amount
1	73.9%		517		68.0%		510
2	64.8%		454		67.9%		509
3	66.8%		468		65.5%		491
4	65.6%		459		65.3%		490
5	65.1%		456		58.7%		440
Mean	67.2%		471		65.1%		488
Subject Premium		700				750	
Ceded Premium		9.8				10.5	

For each simulation, assume that the ceding company will only terminate the contract if the profit commission payable is greater than zero.
a. (1 point) Calculate the expected profit commission payable at the end of 2016.
b. (2.75 points) Calculate the expected profit commission for the full term of the contract.

214. (8, 11/15, Q.23) (2.5 points) An actuary is pricing the $9,000,000 excess of $1,000,000 layer for an excess of loss policy. Total insured value of the properties is $10,000,000.
Historical information for this policy is as follows:

Accident Year	Excess Loss Development Factor	On-Level Trended Premium
2012	1.01	$5,200,000
2013	1.05	$5,700,000
2014	1.10	$5,900,000

Accident Date	Actual Ground- Up Loss	Trend Factor	Trended Losses
July 18, 2012	$3,500,000	1.10	$3,850,000
February 12, 2013	$2,000,000	1.08	$2,160,000
August 15, 2013	$1,000,000	1.07	$1,070,000
March 3, 2014	$3,000,000	1.04	$3,120,000
November 2, 2014	$920,000	1.01	$929,200

The exposure curve below applies to the insured risk:

% of Insured Value	Exposure Factor
10%	30%
20%	41%
30%	52%
40%	63%
50%	71%
60%	75%
70%	80%
80%	84%
90%	89%
100%	90%
120%	100%

Calculate the policy's loss cost as a percentage of premium.

215. (SOA GIADV Exam, 5/16, Q.1) (5 points)
Beta Reinsurance Company writes property per risk excess reinsurance and uses an exposure curve to price its treaties that is defined by, where x is the proportion of insured value,

$$\text{Exposure Factor} = 1 - \left(1 - \frac{x}{1.2}\right)^2, \ 0 < x < 1.2.$$

Beta has been presented with an opportunity to write a property per risk excess treaty covering the layer 400,000 excess of 100,000 on a portfolio with insured values
and subject premium as follows:

Insured Value	Subject Premium
100,000	1,000,000
200,000	1,000,000
500,000	1,000,000
1,000,000	1,000,000

A surplus share treaty with a retained line of 500,000 and a maximum of 2 lines ceded will inure to the benefit of the excess treaty.
(a) (3 points) Calculate the expected loss in the layer assuming an expected loss ratio of 60%.
(b) (1 point) Explain why you might elect not to use an experience rating approach to price this treaty because of the issue of "free cover."
(c) (0.5 points) The ceding company has also purchased a property catastrophe cover. State whether the property catastrophe cover will typically inure to the benefit of the surplus share reinsurance.
(d) (0.5 points) State whether the property catastrophe cover will typically inure to the benefit of the property per risk excess treaty.

216. (SOA GIADV Exam, 5/16, Q.8) (4 points)
With a proportional reinsurance treaty, there will often be disagreement between the ceding company and the reinsurer about the expected loss ratio and the appropriate ceding commission. To resolve these differences, an adjustable feature is often built into the treaty.
(a) (1.5 points) Provide an example of each of the following adjustable features:
(i) Sliding scale commission
(ii) Profit commission
(iii) Loss corridor
(b) (1.5 points) The impact of an adjustable feature is best evaluated using an aggregate loss distribution model. Identify one disadvantage specific to each of the following approaches:
(i) Empirical distribution
(ii) Single distribution model
(iii) Recursive formula
(c) (1 point) Carryforward provisions complicate evaluation of the impact of adjustable features. Describe one approach to pricing the impact of a carryforward provision and identify one problem with that approach.

217. (SOA GIADV Exam, 11/16, Q.1) (4 points) You are exposure rating a workers compensation treaty that covers the layer 750,000 excess of 250,000 and are given:
- 60% of Standard Premium is in State X and 40% is in State Y.
- The expected loss ratio is 60% in State X and 70% in State Y.
- 50% of Standard Premium in each state is in Hazard Group A and 50% is in Hazard Group B.
- Both states have the following excess loss factors:

	Excess Loss Factors	
Loss Size	Hazard Group A	Hazard Group B
250,000	0.0500	0.1000
500,000	0.0250	0.0500
750,000	0.0167	0.0333
1,000,000	0.0125	0.0250

(a) (2 points) Calculate the loss cost rate for the treaty.
(b) (1 point) You are experience rating the same treaty. Explain how you would handle:
(i) Policy limits
(ii) Discounting
(c) (1 point) Identify an adjustment (unrelated to policy limits or discounting) you would need to make to:
(i) The historical premium
(ii) The historical losses

218. (SOA GIADV Exam, 11/16, Q.8) (5 points)
Property R Us Reinsurance Company has been presented with an opportunity to write a quota share treaty with two different adjustable feature options:
Option 1: Minimum commission of 20% at a 70% loss ratio sliding 0.5:1 to a maximum commission of 30% at a 50% loss ratio
Option 2: Flat commission of 27.5% with the ceding company reassuming 50% of the loss between a 60% and 70% loss ratio
The loss ratio is uniformly distributed in the range from 30% to 80%.
(a) (1.5 points) Calculate the expected technical ratio (loss ratio plus commission ratio) for Option 1.
(b) (1.5 points) Calculate the expected technical ratio for Option 2.
(*c) (2 points) Which option would result in the larger variance of technical ratios?*
Hint: The second moment of a uniform distribution on [a, b] is: $(b^3 - a^3) / \{3(b-a)\}$.
Note: I have rewritten part (c) and eliminated part (d) of this past exam question.

219. (8, 11/16, Q.20) (3 points) A primary insurer has entered into property catastrophe excess of loss treaties with three reinsurers. The terms of the treaties are as follows:

- Reinsurer A: 100% of $100 excess of $200 written at a nominal rate on line of 10%
- Reinsurer B: 100% of $100 excess of $300 written at a nominal rate on line of 7%
- Reinsurer C: 100% of $100 excess of $400 written at a nominal rate on line of 4%

Each treaty specifies a single mandatory reinstatement that is 100% pro rata as to amount.
The primary insurer incurs three covered loss events during the contract period:

Loss Number	Loss Amount
1	$380
2	$260
3	$600

a. (0.75 point) For each loss event, calculate the amount of loss retained by the primary insurer.
b. (2.25 points) For each loss event, calculate the amount of reinstatement premium owed by the primary insurer to each of its reinsurers.

220. (SOA GIADV Exam, 5/17, Q.1) (6 points) Casualty R Us Reinsurance Company has been presented with an opportunity to write a casualty per occurrence excess treaty for accident year 2018 covering the layer 200,000 excess of 400,000.
You are given the following information:
● The following loss experience has been provided, evaluated as of 12/31/2016:

Accident Date	Untrended Loss	Untrended ALAE
7/1/2014	200,000	100,000
7/1/2014	400,000	200,000
7/1/2015	550,000	0
7/1/2015	1,000,000	500,000
7/1/2016	600,000	300,000
7/1/2016	450,000	0

● All losses of at least 200,000 are shown.
● All policy limits throughout the experience period are 1,000,000 and are expected to remain at this level through 2018.
● On level subject premium is 10,000,000 for each year from 2014-2016.
● ALAE is allocated to layer in proportion to losses.
● Loss and ALAE trend are each 6% per year.
● The following accident year development factors are applicable to both loss and ALAE in the layer 200,000 excess of 400,000:

12-Ultimate	2.00
24-Ultimate	1.50
36-Ultimate	1.10

(a) (4 points) Estimate the experience rating loss and ALAE cost as a percentage of the subject premium.
(b) (2 points) The ceding company requests alternative quotes on the following two layers:
(i) 200,000 excess of 300,000
(ii) 200,000 excess of 200,000
Explain what additional information you would need, if any, to experience rate each layer.

***221.* (SOA GIADV Exam, 5/17, Q.8)** (6 points) You project that the number of catastrophe losses next year for your company will follow a Poisson distribution with mean 2 and that each loss size will have the following probability function:

Loss Size	Probability
1 billion	0.4
2 billion	0.3
3 billion	0.2
4 billion	0.1

Loss sizes are assumed to be independent of one another and independent of the number of losses.
The aggregate distribution of catastrophe losses has the following probability function for aggregate losses below 10 billion:

Aggregate Losses	Probability
0 billion	0.1353
1 billion	0.1083
2 billion	0.1245
3 billion	0.1306
4 billion	0.1230
5 billion	0.0982
6 billion	0.0804
7 billion	0.0621
8 billion	0.0453
9 billion	0.0318

(a) (3 points) Calculate the probability that aggregate catastrophe losses will be 10 billion.
(b) (1.5 points) Calculate the mean and coefficient of variation of aggregate catastrophe losses.
(c) (1.5 points) You decide to approximate aggregate catastrophe losses with a lognormal distribution. Calculate the parameters of the lognormal distribution using your answer to part (b).

222. (SOA GIADV Exam, 11/17, Q.1) (5 points) Casualty R Us Reinsurance Company has been presented with an opportunity to write a casualty per occurrence excess treaty covering the layer 500,000 excess of 500,000 on a portfolio of umbrella policies.
The following limits profile has been provided:

Subject Premium	Underlying Limit	Policy Limit
1,200,000	1,000,000	1,000,000
785,000	1,000,000	2,000,000

An analysis of historical data indicates that the following increased limits factors apply to this portfolio:

Policy Limit	Increased Limits Factor
500,000	1.00
1,000,000	1.50
1,500,000	1.80
2,000,000	2.00
2,500,000	2.10
3,000,000	2.15

The aggregate excess factor ϕ on the underlying policies is 0.10.
(a) (0.5 points) Explain how the aggregate excess factor is analogous to a Table M charge factor.
(b) (3 points) Calculate the approximate expected losses in the layer using an exposure rating approach with an expected loss ratio of 60%.
(c) (0.5 points) Casualty R Us has also been presented with an opportunity to write a casualty per occurrence excess treaty covering the layer 3,000,000 excess of 2,000,000 on the same portfolio of umbrella policies.
Identify the treaty category under which this treaty would fall.
(d) (1 point) Describe two ways that a loss on this treaty could occur.

223. (SOA GIADV Exam, 11/17, Q.8) (5 points) You project that the number of catastrophe losses next year for your company will follow a distribution with mean 1, variance 2 and probability function $p(n) = (0.5)^{n+1}$, n = 0, 1, 2, ...
You also project that each loss size will have the following probability function:

Loss Size	Probability
1 billion	0.4
2 billion	0.3
3 billion	0.2
4 billion	0.1

Loss sizes are assumed to be independent of one another and independent of the number of losses.
The aggregate distribution of catastrophe losses has the following probability function for aggregate losses below 10 billion:

Aggregate Losses	Probability
0 billion	0.5000
1 billion	0.1000
2 billion	0.0950
3 billion	0.0840
4 billion	0.0661
5 billion	0.0403
6 billion	0.0311
7 billion	0.0231
8 billion	0.0166
9 billion	0.0119

(a) (3 points) Calculate the probability that aggregate catastrophe losses will be 10 billion.
(b) (1.5 points) Calculate the mean and coefficient of variation of aggregate catastrophe losses.
(c) (0.5 points) Identify one disadvantage of using the recursive formula to calculate aggregate distribution probabilities.

224. (8, 11/17, Q.19) (3.0 points)
A primary insurer wants to institute a swing plan with the following terms:
● Layer: $500,000 xs $500,000 per claim
● Premium = Layer Losses * 1.3
● Minimum Premium = $100,000
● Maximum Premium = $325,000
The reinsurer has analyzed the insurer's experience.
All claims are closed and trend has been applied.

Accident Year	On-Level Premium	Claim #1	Claim #2	Claim #3	Claim #4
2012	400,000	465,899	230,567	512,017	
2013	420,000	240,785	125,766	80,154	470,666
2014	1,200,000	704,895	106,528		
2015	2,000,000	45,988	349,145	376,011	1,246,852
2016	2,200,000	437,100	654,158	250,000	156,750

a. (1.5 points) Calculate the expected loss ratio for the reinsurer based on the empirical distribution model using equal weights for each year.
b. (0.5 point) Determine if an empirical distribution model is appropriate for this account and give one reason in support of your answer.
c. (1 point) The use of a collective risk model is an alternative approach.
Explain two concerns when using this type of model.

225. (SOA GIADV Exam, 5/18, Q.2) (6 points) SafePort Insurance Company is a small company that provides property coverage to hotels in a specific region.
SafePort’s reinsurance arrangements for windstorm losses are applied in the following order:

Reinsurance Treaty	Description
Surplus Share	5 lines with 2,000 retained line
Per Risk Excess of Loss	4,000 excess of 1,000
Catastrophe	8,000 excess of 6,000

During 2017, a severe windstorm caused the following covered losses:

Property	Insured Value	Loss
A	2,000	400
B	20,000	16,000
C	8,000	3,200
D	12,500	12,500
E	4,000	1,200

(a) (2 points) Calculate the total losses recoverable under each treaty.
(b) (0.5 points) You are provided with the following information on the catastrophe treaty:
● The annual premium is 600.
● There is a reinstatement provision that is 125% pro-rata as to amount.
Calculate the reinstatement premium for the catastrophe treaty.
(c) (0.5 points) Discuss whether a reinstatement pro-rata as to time would be appropriate for this type of cover.
(d) (1 point) Explain with an example why a catastrophe cover is usually written on a losses occurring basis rather than on a risks attaching basis.

Question Continued on the Next Page

(e) (2 points) The per risk excess of loss treaty was underwritten by Windy Reinsurance, Inc. Windy priced the treaty using the following exposure factors:

Percent of Insured Value	Exposure Factor
0%	0%
10%	37%
20%	49%
30%	57%
40%	64%
50%	70%
60%	76%
70%	81%
80%	85%
90%	89%
100%	93%
110%	97%
120%	100%

Expected gross losses for each property were assumed to be 5% of insured value.
Calculate the expected losses in the excess layer underwritten by Windy for each of the following properties:
(i) Property A
(ii) Property B

226. (SOA GIADV Exam, 5/18, Q.8) (4 points) You project that the number of flood losses in the upcoming year will have a binomial distribution with M = 2 and p = 0.5. Hence, there will be 0 losses with 25% probability, 1 with 50% probability, and 2 with 25% probability.
You also project that each loss size will have the following probability function:

Loss size (billions)	Probability
1	0.25
2	0.25
3	0.25
4	0.25

Loss sizes are assumed to be independent of one another and independent of the number of losses.
The aggregate distribution of flood losses has the following probability function:

Aggregate Losses (billions)	Probability
0	0.250000
1	0.125000
2	0.140625
3	0.156250
4	
5	0.062500
6	0.046875
7	0.031250
8	

(a) (1.5 points) Calculate the probability that aggregate flood losses will be:
   (i) 4 billion
   (ii) 8 billion

(b) (1 point) Calculation of the full set of aggregate probabilities can be done by either using a recursive formula or employing basic probability rules.
Explain the advantages of using a recursive formula.

(c) (1.5 points) Calculate the mean and coefficient of variation of aggregate flood losses.

227. (SOA GIADV Exam, 11/18, Q.1) (6 points) Tolerant Reinsurance Company has written a proportional treaty with the following characteristics:
Sliding scale commission:

Loss Ratio	Commission
40% or below	20%
40%-60%	Sliding 0.5:1
60% or above	10%

Profit commission:

Margin for Expenses	15%
Percent Returned to Ceding Insurer	40%

The loss ratio for 2017 was 65%.
(a) (1 point) Calculate the technical ratio (loss ratio plus commission ratio) for 2017.
(b) (4 points) Any losses in excess of a 60% loss ratio are carried forward to the following year's commission calculations. The 2018 loss ratio is uniformly distributed from 30% to 70%.
Calculate the expected technical ratio for 2018.
(c) (1 point) Describe two complications with pricing the effect of carryforward provisions.

228. (SOA GIADV Exam, 11/18, Q.8) (5 points)
Very Patient Reinsurance Company has been asked to price a property per risk excess treaty for accident year 2019 covering the layer 4,000,000 excess of 1,000,000.
You are given the following information:
● The following loss experience has been provided, evaluated as of 12/31/2017:

Accident Year	Untrended Loss
2015	1,100,000
2015	2,200,000
2016	900,000
2016	1,400,000
2017	800,000
2017	1,800,000

● All losses of at least 500,000 are shown.
● On level subject premium is 6,000,000 for each year from 2015-2017.
● Loss trend is 6% per year.
● The insured value of each property is 5,000,000.
● The following accident year development factors are applicable to losses in the layer 4,000,000 excess of 1,000,000:

12-Ultimate	1.50
24-Ultimate	1.20
36-Ultimate	1.00

(a) (2 points) Estimate the experience rating loss cost as a percentage of the subject premium.
(b) (0.5 points) Define free cover.

Question is continued on the next page

(c) (2 points) The following exposure factors are considered appropriate for pricing this treaty:

Percent of Insured Value	Exposure Factor
10%	30%
20%	50%
30%	61%
40%	69%
50%	75%
60%	80%
70%	85%
80%	89%
90%	92%
100%	95%
110%	98%
120%	100%

Calculate a suitable adjustment to the loss cost using these exposure factors to estimate the cost of free cover.

(d) (0.5 points) Assess whether these exposure factors would be appropriate for pricing coverage on properties valued at 20 million.

229. (5, 11/18, Q.15) (2.75 points) Given the following:

Claim ID	Accident Date	Transaction Date	Gross Amount Paid on Transaction Date ($)	Gross Ending Case Outstanding ($)
A	May 30, 2015	December 24, 2015	1,000	550
		August 1, 2016	500	225
		June 1, 2017	725	0
B	August 28, 2015	August 29, 2015	300	1,050
		February 6, 2016	600	375
		June 14, 2016	450	150
C	April 21,2016	April 25, 2016	1.200	575
		March 3, 2017	700	250
		December 1, 2017	200	0
D	October 11, 2016	October 12, 2016	400	900
		May 17, 2017	800	625

60%	Quota share ceded percentage for reinsurance that applies to claims occurring in 2015.
$1,500	Per claim excess of loss retention for reinsurance that applies to claims occurring in 2016.

a. (0.25 point) Calculate calendar year 2015 reported claims, gross of reinsurance.
b. (0.75 point) Calculate calendar year 2016 paid claims, net of reinsurance.
c. (0.75 point) Calculate calendar year 2016 reported claims, gross of reinsurance.
d. (1 point) Calculate calendar year 2017 reported claims, net of reinsurance.

230. (8, 11/18, Q.15) (1.75 points) A reinsurer has priced a quota share treaty to achieve an expected combined ratio of 90%. Expenses for the treaty are as follows:

Ceding Commission	20%
Brokerage Fees	5%
Administrative Expenses	1%
Unallocated Expenses	1%

The following table represents the expected loss ratio distribution for the primary insurer under the treaty:

Range of Loss Ratios	Average Ratio in Range	Probability of being in Range
0-40%	37.3%	0.03
40-60%	53.2%	0.21
60-80%	66.1%	0.55
80% or above	91.1%	0.21

The treaty includes a loss corridor from 60% to 80% loss ratio.
Calculate the percent of loss reassumed by the primary insurer in the loss corridor.

231. (SOA GIADV Exam, 5/19, Q.2) (6 points)
Casualty R Us Reinsurance Company is evaluating a proposed casualty per occurrence excess treaty covering the layer 2,000,000 excess of 1,000,000.
The following information has been provided:

Subject Premium	Underlying Limit	Policy Limit
3,000,000	0	1,000,000
4,000,000	0	2,000,000
5,000,000	0	3,000,000
8,000,000	1,000,000	2,000,000
9,000,000	1,000,000	3,000,000

Policy Limit	Increased Limits Factor
1,000,000	1.00
2,000,000	1.16
3,000,000	1.28
4,000,000	1.38

The expected loss ratio is 60%.
(a) (3 points) Calculate the expected losses in the layer using an exposure rating approach.
(b) (2 points) A swing plan has been proposed with the following features:
● Retro Premium = (Actual Layer Losses) x 100 / 80
● Provisional Rate = 20%
● Maximum Premium = 37.5% x Subject Premium
● Minimum Premium = 12.5% x Subject Premium
The following information has been provided:

Range of Loss Cost	Probability	Average Loss Cost in Range
0-10%	0.10	4%
10-30%	0.75	19%
30%+	0.15	44%

Calculate the expected loss ratio on this swing plan.
(c) (0.5 points) Explain the concept of "balance" in the context of swing plans.
(d) (0.5 points) Explain why the provisional rate may be too low.

232. (SOA GIADV Exam, 5/19, Q.8) (5 points) Specialist Reinsurance Company has proposed offering a finite risk cover without reinstatements with the following terms:
● Annual Premium: 50,000,000
● Occurrence Limit: 200,000,000
● Profit Commission: 95% after 10% margin on Annual Premium
● Additional Premium: 60% of (Loss + Margin - Annual Premium)
Any single loss will fully exhaust the limit.
(a) (2 points) Calculate the net profit or loss for Specialist assuming:
(i) No losses
(ii) One or more losses
(b) (1 point) Calculate the rate on line for an equivalent traditional risk cover.
(c) (1.5 points) Specialist wishes to modify the terms of the proposed finite risk cover so that the rate on line for an equivalent traditional risk cover is 15%.
Calculate the Additional Premium percentage required to match the rate on line of 15%.
(d) (0.5 points) State the two conditions that a finite reinsurance arrangement must fulfill for a ceding company to consider it insurance.

233. (SOA GIRR Exam, 11/19, Q.13) (5 points)
There are many functions of reinsurance coverage.
For example, reinsurance can allow insurers to access technical expertise from a reinsurer.
(a) (0.5 points)
Identify a function of reinsurance that is best addressed by proportional reinsurance.
(b) (0.5 points)
Identify a function of reinsurance that is best addressed by excess of loss reinsurance.
(c) (1 point) Insurer A purchases a per occurrence excess of loss contract from Reinsurer B for its general liability portfolio. The policy attaches at 1,000,000. Insurer A and Reinsurer B equally share the 1,000,000 to 2,000,000 layer. Reinsurer B assumes the full 2,000,000 to 5,000,000 layer. A policyholder submits a claim of 4,000,000 to Insurer A.
Determine the net amount paid by each of the insurer and the reinsurer.
(d) (1 point) There is an additional 500,000 of ALAE associated with the 4,000,000 of indemnity. Calculate the total net amount paid by by each of the insurer and the reinsurer with pro-rata treatment of ALAE.
(e) (1 point) Calculate the total net amount paid by each of the insurer and the reinsurer when ALAE is considered within the retention.
(f) (1 point) Insurer A is considering revising the reinsurance agreement for next year. The new treaty would attach at 1,500,000 with Reinsurer B assuming the 1,500,000 to 5,000,000 layer. This alternative would provide the same maximum net retention for Insurer A.
Explain how net claims paid by Insurer A under the alternative coverage could differ from the current reinsurance coverage despite having the same maximum net retention.

234. (SOA GIADV Exam, 11/19, Q.1) (5 points) Property R Us Reinsurance Company uses the following exposure factors to price its commercial property per risk excess treaties:

Percent of Insured Value	Exposure Factor
0%	0%
10%	37%
20%	49%
30%	57%
40%	64%
50%	70%
60%	76%
70%	81%
80%	85%
90%	89%
100%	93%
110%	97%
120%	100%

(a) (0.5 points)
Explain why the exposure curve allows for exposure above 100% of the insured value.
(b) (2.5 points) Property R Us has received a submission requesting 300,000 excess of 100,000 property per risk coverage for insured values and subject premium as follows:

Insured Value	Subject Premium
200,000	800,000
500,000	1,000,000
1,000,000	1,200,000

Calculate the expected loss in the requested layer assuming an expected loss ratio of 50%.
(c) (0.5 points) State the key assumption underlying the use of a single exposure curve to price this treaty.
(d) (1.5 points) The submission is revised to reflect the purchase of surplus share reinsurance with a retained line of 500,000, which will inure to the benefit of the property per risk treaty. Calculate the revised expected loss in the layer.

235. (SOA GIADV Exam, 11/19, Q.8) (6 points) You are assessing your company's reinsurance program for the management of catastrophe risk.
The annual number of catastrophe losses has a Poisson distribution with mean 1.3 and the loss size distribution is:

Loss Size (billions)	Probability
1	6
2	3
3	1

Loss sizes are independent of one another and independent of the number of losses.
(a) (1.5 points) Demonstrate that the mean and coefficient of variation of aggregate losses are 1.95 billion and 0.961, respectively.
(b) (2 points) The following is an extract from the probability function for aggregate losses:

Aggregate Losses (billions)	Probability
0	0.2725
1	
2	0.1892
3	0.1399
4	0.0849
5	0.0498
6	
7	0.0132
8	0.0063

Calculate the probability that aggregate losses will be:
(i) 1 billion
(ii) 6 billion
(c) (1.5 points) You decide to approximate aggregate losses with a lognormal distribution.
Demonstrate that the method of moments estimates are $\mu = 0.341$ and $\sigma^2 = 0.654$.
(d) (1 point) The 95% cumulative probability level of the standard normal distribution is 1.645.
Calculate the retention for a treaty covering aggregate catastrophe losses that is expected to be triggered once in 20 years, using the lognormal model.

236. (8, 11/19, Q.17) (3 points) An insurer is deciding on a proportional reinsurance strategy for the upcoming two years. A reinsurer proposes the following sliding-scale commission with a carry-forward provision. The contract will be identical for the second year.

Minimum Commission:	12.5% at an 80% loss ratio
Sliding 1:1 to	27.5% at a 65% loss ratio
Sliding 0.5:1 to a Maximum	35% at a 50% loss ratio

Assume the insurer places no value on the carry-forward provision after the second year, and ignore the time value of money (i.e., assume a 0% interest rate).
The following table details the expected loss distribution of the underlying business:

Range of Loss Ratios	Average Loss Ratio in Range	Probability of Being in Range
0% - 50%	43%	0.15
50% - 60%	57%	0.25
60% - 80%	68%	0.45
80% or above	85%	0.15

Calculate the reinsurer's expected technical ratio for each year if the insurer buys the sliding-scale commission contract for both years.

237. (8, 11/19, Q.18) (5.5 points)
A property insurer has an existing Quota Share reinsurance treaty in place.
The insurer would like to further reduce its net loss exposure by exploring a proposed Property Per Risk Excess reinsurance treaty. Given the following:

<u>Existing Quota Share Reinsurance Treaty</u>
- 25% Quota Share
- 15% Ceding Commission

<u>Proposed Property Per Risk Excess Reinsurance Treaty</u>
- Subject premium of $100 million net of the Quota Share reinsurance treaty
- Covers net losses in excess of $500,000, up to a limit of $500,000
- The Quota Share reinsurance treaty will inure to the benefit of this treaty
- Ceded Loss Ratio for this Excess layer is estimated to be 90%

The table below illustrates the insurer's historical experience data net of the Quota Share that is subject to the proposed Property Per Risk Excess layer:

Historical Accident Year	On Level Subject Earned Premium ($ millions)	On Level Trended Ultimate Subject Loss Ratio	On Level Trended Ultimate Layer Loss Cost
2014	100	75%	25%
2015	120	75%	30%
2016	150	90%	45%
2017	80	75%	24%
2018	100	80%	36%

a. (0.5 point) Calculate the loss cost of the Property Per Risk Excess layer using all 5 years of historical experience data provided.
b. (1 point) Assume that the following exposure curve definition applies to the Property Per Risk Excess layer:

$$G(x) = \frac{1 - 0.32428^x}{1 - 0.32428}$$

where G(x) represents the ratio of pure risk premiums retained by the insurer, and x represents the ground up loss normalized to the maximum possible loss of $1 million.
Calculate the loss cost of the Property Per Risk Excess layer retained by the reinsurer based on the exposure curve.
c. (0.5 point)
Calculate the loss cost of the Property Per Risk Excess layer using a blend of the experience loss cost and the exposure loss cost based on a credibility weight of 80%.

CONTINUED ON NEXT PAGE

d. (2 points)
The insurer's historical gross expenses are estimated to be 15% of historical gross premiums. Calculate the insurer's expected net underwriting profit after application of both the existing Quota Share reinsurance treaty and the proposed Property Per Risk Excess reinsurance treaty.

The insurer is also exploring the following proposed modifications to the Quota Share reinsurance treaty:
- Option 1: Profit Commission equal to 100% of reinsurer profit above a 5% reinsurer margin
- Option 2: 20% Ceding Commission

e. (1 point) Calculate the 5 year weighted average ratio of profit commission to ceded premium for the Quota Share reinsurance treaty using the historical experience data provided.
f. (0.5 point) Briefly describe one advantage and one disadvantage of implementing the proposed profit commission versus the higher ceding commission from the insurer's perspective.

238. (SOA GIRR Exam, 5/20, Q.6) (4 points)
(a) (1.5 points) State three reasons why a reinsurer often experiences greater variability in claims than a primary insurer.
(b) (1 point) An annual aggregate deductible (AAD) is a reinsurance contract provision that can result in ceded claims that vary based on experience.
Describe one other possible reinsurance contract provision.
(c) (1.5 points) Primary insurer PPP purchases a per risk, excess of loss reinsurance contract from Reinsurer RRR with a 5 million excess 1 million limit and a 10 million AAD.
PPP experiences the following claims covered by the policy:

Claim #	Ultimate Claims
1	1,000,000
2	5,000,000
3	6,000,000
4	12,000,000
5	5,000,000
6	1,000,000

Calculate the amount paid by PPP and RRR for each claim.

239. (SOA GIADV Exam, 5/20, Q.1) (5 points) You are exposure rating a workers compensation treaty that covers the layer 600,000 excess of 200,000 and are given:

State	Expected Loss Ratio
X	60%
Y	80%

State	Hazard Group	Standard Premium
X	J	80,000
X	K	120,000
Y	J	110,000
Y	K	90,000

Both states have the following NCCI excess loss factors (ELF):

	NCCI Excess Loss Factors	
Loss Size	Hazard Group J	Hazard Group K
250,000	0.032	0.060
1,000,000	0.008	0.020

You approximate the NCCI ELF curves with an inverse power curve of the form
$ELF_L = a\ L^{-b}$.

(a) (1 point) Calculate the values of a and b for each hazard group.
(b) (3 points) Calculate the loss cost rate for the treaty.
(c) (1 point) You are experience rating the same treaty. Explain how you would handle:
    (i) Policy limits
    (ii) Discounting

240. (SOA GIADV Exam, 5/20, Q.8) (4 points)
(a) (0.5 points) Explain why adjustable features are common in proportional reinsurance treaties.
(b) (1.5 points) Illustrate with examples how the following adjustable features work:
(i) Sliding scale commission
(ii) Profit commission
(iii) Loss corridor
(c) (1 point) The impact of an adjustable feature is best evaluated using an aggregate loss distribution model. Identify one disadvantage of each of the following approaches:
(i) Empirical distribution
(ii) Single distribution model
(d) (0.5 points) Carryforward provisions complicate evaluation of the impact of adjustable features. Describe one approach to pricing the impact of a carryforward provision.
(e) (0.5 points) Identify one problem with the approach described in part (d).

241. (SOA GIADV Exam, 11/20, Q.1) (5 points)
Casualty Reinsurance Company is writing a casualty per occurrence excess treaty for accident year 2021 covering the layer 750,000 excess of 250,000.
You are given the following information:

Loss Experience Evaluated as of December 31, 2019		
Accident Date	Untrended Loss	Untrended ALAE
July 1, 2017	200,000	150,000
July 1, 2017	350,000	400,000
July 1, 2018	225,000	0
July 1, 2018	900,000	450,000
July 1, 2019	250,000	50,000
July 1, 2019	800,000	275,000

- All losses of at least 200,000 are shown.
- All policy limits throughout the experience period are 1,000,000 and are expected to remain at this level through 2021.
- On level subject premium is 10,000,000 for each year from 2017-2019.
- ALAE is allocated to layer in proportion to losses.
- Loss and ALAE trend are each 5% per year.
- The following accident year development factors are applicable to both loss and ALAE in the layer 750,000 excess of 250,000:

12-Ultimate	2.40
24-Ultimate	1.40
36-Ultimate	1.10

Estimate the experience rating loss cost, including ALAE, as a percentage of the subject premium.

242. (SOA GIADV Exam, 11/20, Q.8) (5 points) Specialist Reinsurance Company is offering finite reinsurance to Ceding Insurance Company to cover its aggregate annual losses.
The annual number of losses has a Poisson distribution with mean 1.5.
The loss size distribution is:

Loss Size (millions)	Probability
1	0.5
2	0.4
3	0.1

Loss sizes are independent of one another and independent of the number of losses.
(a) (2 points) Complete the following aggregate loss probability table:

Aggregate Losses (millions)	Probability
0	
1	0.1673
2	0.1966
3	0.1496
4	
5	
6	0.0411
7	
8	
9	
10	
11	
12	
13	
14	0.0001
15	0.0000

Question is continued on the next page

(b) (2 points) The finite risk cover has the following terms:
- Annual Premium: 2.5 million
- Limit: 10 million
- Profit Commission: 80% after 10% margin on Annual Premium
- Additional Premium: 50% of (Loss + Margin - Annual Premium)

The underwriting results for different aggregate losses are:

Aggregate Losses (millions)	Underwriting Result (millions)
0	0.700
1	0.500
2	0.300
3	-0.125
4	-0.625
5	-1.125
6	-1.625
7	-2.125
8	-2.625
9	-3.125
10+	-3.625

Verify the following underwriting results for Specialist:
(i) A profit of 0.3 million if aggregate losses are 2 million.
(ii) A loss of 1.125 million if aggregate losses are 5 million.
(c) (0.5 points) State the two conditions that a finite reinsurance arrangement must fulfill for a ceding company to consider it insurance.
(d) (0.5 points) Explain whether the finite reinsurance can be considered insurance by Ceding Insurance Company.

Solutions:

1.

I.V.	100K / I.V.	Exp. Factor	400K / I.V.	Exp. Factor	Exposure Factor for Layer
200K	50%	0.78	200%	1	1 - 0.78 = 0.22
500K	20%	0.55	80%	0.92	0.92 - 0.55 = 0.37

Assume for example that the total subject premium is 3.

Insured Value	Subject Premium	Exposure Factor	Exposed Premium
200K	2	0.22	0.44
500K	1	0.37	0.37
Total	3		0.81

Exposure rate is: (1.1)(66%) (0.81) / 3 = **19.6%**.
Comment: Similar to 6, 11/07, Q.32.

2. (a) With for example an annual aggregate deductible of \$1 million, the ceding company would retain the first \$1 million in losses in the reinsured excess layer in a year. If the annual losses in the reinsured excess layer exceeded \$1 million, then the reinsurer would pay the amount by which they exceeded \$1 million.

(b) $\phi_{AAD} = \dfrac{\int_{AAD}^{\infty} (y - AAD)\, g(y)\, dy}{E[Y]}$,

where g(y) is the distribution of annual aggregate losses in the reinsured excess layer.

Alternately, $\phi_{AAD} = \dfrac{\text{expected amount ceded with AAD}}{\text{expected amount ceded without AAD}}$.

(c) (60%)(\$13 million) = \$7.8 million.
Comment: See pages 33 and 34 of Clark.

3. Experience rating for workers compensation may be distorted depending on how tabular discounts are taken into effect. A way to avoid this distortion is to collect sufficient information for individual claimants to project their (undiscounted) expected costs into the treaty layer.
<u>Comment</u>: See page 35 of Clark.
"The information needed is:
1. Claimant's current age
2. Claimant's sex (M/F)
3. Estimate of annual indemnity cost including escalation, if any
4. Estimate of annual medical cost
5. Amounts paid to date
For claims with the potential for penetrating the layer, all future payments (both indemnity and medical costs adjusted for escalation) should be determined. For those potential payments which fall within the excess layer, an appropriate mortality factor should be applied to determine the expected amount in the treaty layer. It is important to note that some claims, for which the incurred amount reported by the ceding company falls below the treaty retention, will show an expected amount in the layer. A smaller development factor would then be needed to include only true IBNR claims."

4. (a) The expected number of claims exceeding $100,000 is 10 in 2012.
These are 1 - 64.2% = 35.8% of all claims, so the expected number of claims in 2012 is:
10 / 0.358 = 27.933.
A $100,000 claim in 2017 is a $100,000/1.25 = $80,000 claim in 2012.
In 2012, 1 - 58.1% = 41.9% of claims exceed $80,000;
these are: (41.9%)(27.933) = 11.704 claims.
The exposure increases 20% from 2012 to 2017, so the expected number of claims in 2017 exceeding $100,000 are: (11.704)(1.2) = **14.045 claims**.

(b) The losses in the layer $750,000 excess of $250,000 in 2017 are in the layer $600,000 excess of $200,000 in 2012.
In 2012, $E[X \wedge 200{,}000]$ = (80.3%)($86,100) + (1 - 80.3%)($200,000) = $108,538.30.
In 2012, $E[X \wedge 800{,}000]$ = (96.7%)($134,500) + (1 - 96.7%)($800,000) = $156,461.50.
In 2012, the loss per claim in the layer from $200,000 to $800,000 is:
$156,461.50 - $108,538.30 = $47,923.2.
Thus in 2017, the loss per claim in the layer from $250,000 to $1,000,000 is:
(1.25)($47,923.2) = $59,904.
The exposure increases by 20%, so the number of claims expected in 2017 is:
(1.2)(27.933) = 33.520.
Thus the expected cost in 2017 in the layer from $250,000 to $1,000,000 is:
(33.520)($59,904) = **$2.008 million**.

5. "If the historical experience is stable enough to provide data to make a precise expected loss estimate, then the reinsured would be willing to retain that risk. As such, the basic pricing tools are usually only a starting point in determining an adequate premium. The Actuary earns his or her money by knowing when the assumptions in these tools are not met and how to supplement the results with additional adjustments and judgement."
"A major difference between reinsurance and primary insurance is that a reinsurance program is generally tailored more closely to the buyer; there is no such thing as the average reinsured or the average reinsurance price. Each contract must be individually priced to meet the particular needs and risk level of the reinsured."

6. (a) If the ALAE is pro-rata, then we get the expected losses using ELPPFs.
From the definition of ELPPF's in the question:
Expected excess loss without ALAE = (ELPPF)(Expected Losses & ALAE).

H.G.	Premium	200K	500K	Expected Loss in Layer
B	13	0.207	0.083	(0.207 - 0.083)(72%)(13) = 1.16064
C	18	0.235	0.101	(0.235 - 0.101)(72%)(18) = 1.73664
D	9	0.265	0.119	(0.265 - 0.119)(72%)(9) = 0.94608
F	7	0.361	0.190	(0.361 - 0.190)(72%)(7) = 0.81184
Total	47			4.70520

This is the expected losses in the layer without ALAE.
We need to load for ALAE by multiplying by 72%/62%.
Loss and ALAE exposure rate of: (72%/62%)(4.70520) / 47 = **11.6%**.

(b) If the ALAE is included with the treaty, then we first get the expected loss & ALAE using ELAEPPFs. Expected excess loss & ALAE = (ELAEPPF)(Expected Losses & ALAE).

H.G.	Premium	200K	500K	Expected Loss & ALAE in Layer
B	13	0.263	0.111	(0.263 - 0.111)(72%)(13) = 1.42272
C	18	0.294	0.133	(0.294 - 0.133)(72%)(18) = 2.08656
D	9	0.329	0.155	(0.329 - 0.155)(72%)(9) = 1.12752
F	7	0.437	0.241	(0.437 - 0.241)(72%)(7) = 0.98784
Total	47			5.62464

Loss and ALAE exposure rate of: 5.62464 / 47 = **12.0%**.
Comment: The ELAEPPFs are used with the ALAE option; ALAE is included with losses.
In the question I have used the NCCI definitions, which are not specified in the syllabus reading:
ELPPF = (excess losses) / (expected loss plus ALAE), and
ELAEPPF = (excess loss plus ALAE) / (expected loss plus ALAE).
If not defined in a exam question, just be sure to make clear what you are doing. On the exam, one could instead assume that ELPPF times expected losses = expected excess losses.
Page 31 of Clark has an example of exposure rating using ELFs.

7.
1. Compile the historical experience on the treaty.
2. Exclude catastrophe and shock losses.
3. Adjust experience to ultimate level and project to future period.
4. Select the expected non-catastrophe loss ratio for the treaty.
5. Load the expected non-catastrophe loss ratio for catastrophes.
6. Estimate the combined ratio given ceding commission and other expenses.

8. Combine the losses in the same accident year.

AY	Trended Losses in Layer	LDFs	Ultimate Trended Losses in Layer
2013	242 + 76 + 300 = 618	1.020	(1.02)(618,000) = 630,360
2014	261 + 300 + 140 = 701	1.080	(1.08)(701,000) = 757,080
2015	0 + 79 + 121 = 200	1.140	(1.14)(200,000) = 228,000

CY	Earned Premium	On Level Factor	Trend	On Level Trended Premium
2013	20,640,000	0.912	1.03^4	$21,186,218
2014	21,709,000	0.925	1.03^3	$21,942,860
2015	22,854,000	0.967	1.03^2	$23,445,697

AY	On Level Trended Premium	Ultimate Trended Losses in Layer	Loss Cost
2013	$21,186,218	630,360	2.98%
2014	$21,942,860	757,080	3.45%
2015	$23,445,697	228,000	0.97%
Total	$66,574,775	1,615,440	**2.43%**

1,615,440 / 66,574,775 = 2.43%.
Alternately, one could average the loss costs for the three years:
(2.98% + 3.45% + 0.97%)/3 = 2.47%.
Comment: Similar to 6, 11/07, Q.31. See pages 16 and 17 of Clark.
As here, In Clark the LDFs are applied to the trended losses in a layer for an entire Accident Year rather than to individual losses. If Clark's LDFs for accident years are appropriate for the given layer, then they presumably already include the average impact of the capping effect that can be caused by the width of the layer. Clark's LDFs also include the possibility of yet to be reported losses that may contribute to the layer.

9. For a reinsurance treaty written on a loss occurring basis, the coverage trigger is whether or not the loss event for the primary insurer occurs during the treaty effective period.
In contrast, for a reinsurance treaty written on a risks attaching basis, the coverage trigger is whether or not the loss event for the primary insurer is covered by a primary policy written during the treaty effective period.
To price a losses occurring basis treaty one uses calendar year earned premium and accident year losses, in other words Calendar/Accident Year data.
To price a risks attaching basis treaty one uses calendar written premium and the losses covered by those policies are used, or one uses Policy Year data if available.
Comment: See page 4 of Clark.

10. The excess treaty will pay for loss & ALAE in the layer 350K xs 200K.
We assume this corresponds to losses in the layer:
350K / 1.15 xs 200K / 1.15 = 304,348 xs 173,913.
This layer of loss is from 173,913 to 478,261.
However, primary policy has a limit of 250K.
$\Rightarrow$ Numerator of exposure factor is: 1.15 (E[X $\wedge$ 250,000] - E[X $\wedge$ 173,913]).
Denominator of exposure factor is: 1.15 E[X $\wedge$ 250,000].

$$\Rightarrow \text{Exposure Factor: } \frac{E[X \wedge 250K] - E[X \wedge 173{,}913]}{E[X \wedge 250K]} = \frac{50{,}317 - 45{,}750}{50{,}317} = \mathbf{9.08\%}.$$

Alternately, in each case, use the formula from page 29 of Clark. The exposure factor is:

$$\frac{E[X \wedge \text{Min}[PL, (AP+Lim)/(1+e)]\,] - E[X \wedge \text{Min}[PL, AP/(1+e)]]}{E[X \wedge PL]},$$

PL = underlying Policy Limit applying to loss only
AP = Treaty attachment point applying to ALAE plus loss capped at PL = 200,000.
Lim = Treaty limit applying to ALAE plus loss capped at PL = 350,000.
e = ALAE as a percent of loss capped at PL = 15%.
(AP+Lim) / (1+e) = (200K + 350K) / 1.15 = 478,261. AP / (1+e) = 200K / 1.15 = 173,913.
For the policies with a limit of 100K, the exposure factor is:

$$\frac{E[X \wedge \text{Min}[100K, 478261]\,] - E[X \wedge \text{Min}[100K, 173913]]}{E[X \wedge 100K]} = \frac{E[X \wedge 100K] - E[X \wedge 100K]}{E[X \wedge 100K]} = \mathbf{0}.$$

For the policies with a limit of 250K, the exposure factor is:

$$\frac{E[X \wedge \text{Min}[250K, 478261]\,] - E[X \wedge \text{Min}[250K, 173913]]}{E[X \wedge 250K]} = \frac{E[X \wedge 250K] - E[X \wedge 173{,}913]}{E[X \wedge 250K]} =$$

$$\frac{50{,}317 - 45{,}750}{50{,}317} = \mathbf{9.08\%}.$$

For the policies with a limit of 500K, the exposure factor is:

$$\frac{E[X \wedge \text{Min}[500K, 478261]\,] - E[X \wedge \text{Min}[500K, 173913]]}{E[X \wedge 500K]} = \frac{E[X \wedge 478{,}261] - E[X \wedge 173{,}913]}{E[X \wedge 500K]} =$$

$$\frac{57{,}744 - 45{,}750}{58{,}166} = \mathbf{20.62\%}.$$

<u>Comment</u>: Each exposure factor would be multiplied by the losses & ALAE expected for that block of policies, in order to get the expected loss and ALAE paid by the excess treaty.

11. Sliding scale commission – ceding commission paid by the reinsurer to the ceding company varies with the actual loss experience, subject to minimum and maximum amounts.
Profit commission – subtracts actual loss ratio, ceding commission, and a margin for expenses from the treaty premium to calculate the “profit”. If the reinsurer has a profit, then a portion is paid to the ceding company.
Loss corridor – ceding company will reassume a portion of the reinsurer’s liability if the loss ratio exceeds a certain amount. For example the ceding company might assume 60% of the liability in the corridor from an 80% to 95% loss ratio.

12. The treaty is on a losses occurring basis, so that the trend to date is July 1, 2015.

Date of Loss	Trended Loss	Losses in Layer, 400K xs 200K
January 1, 2007	$(300,000)(1.03^{8.5}) = 385,689$	185,689
April 1, 2008	$(800,000)(1.03^{7.25}) = 991,197$	400,000
July 1, 2009	$(200,000)(1.03^{6}) = 238,810$	38,810
October 1, 2010	$(600,000)(1.03^{4.75}) = 690,443$	400,000
January 1, 2011	$(500,000)(1.03^{4.5}) = 571,133$	371,133
July 1, 2012	$(400,000)(1.03^{3}) = 437,090$	237,090
April 1, 2013	$(200,000)(1.03^{2.25}) = 213,754$	13,754

Combine the losses in the same accident year. (In this case there is one loss per accident year.)
Applying loss development factors applicable to the treaty layer.

AY	Losses in Layer	LDF	Product
2007	185,689	1	185,689
2008	400,000	1	400,000
2009	38,810	1.05	40,751
2010	400,000	1.10	440,000
2011	371,133	1.25	463,916
2012	237,090	1.40	331,926
2013	13,754	1.80	24,757
	Total		**1,887,039**

Comment: Similar to 6, 11/08, Q.39.
My solution follows Clark as per pages 16 and 17.

13. Treat the insured property size as the line of $200,000 from the surplus share treaty; however, use an exposure curve appropriate for a $600,000 property.
75K / 200K = 37.5%, so we would look up the exposure factor for 37.5%.
150K / 200K = 75%, so we would look up the exposure factor for 75%.
The exposure rate as a percent of the premium net of the surplus share treaty is the difference of these two exposure factors times the expected loss ratio.
Alternately, the ceding percent for the 600,000 property is: (600 - 200) / 600 = 2/3.
Thus if the insurer has a 225K loss, the loss net of the surplus share reinsurance is:
225K / 3 = 75K.
Thus the lower end of the excess layer corresponds to: 225K / 600K = 37.5% of insured value.
If the insurer has a 450K loss, the loss net of the surplus share reinsurance is: 450K / 3 = 150K.
Thus the upper end of the excess layer corresponds to: 450K / 600K = 75% of insured value.
Proceed as before.
<u>Comment</u>: See pages 21 and 22 of Clark.
If we had a property of value more than $2 million, then it would exceed the 9 lines of the surplus share treaty; one needs to be careful.
For example for a property of size $3 million, the ceding percent is: (9) (200K) / 3000K = 60%.
Thus for a loss of size 187.5K, the net loss is: (40%)(187.5K) = 75K.
Thus the lower end of the excess layer corresponds to: 187.5K / 3000K = 6.25% of insured value.
For a loss of size 375K, the net loss is: (40%)(375K) = 150K.
Thus the upper end of the excess layer corresponds to: 375K / 3000K = 12.5% of insured value.

14. A 30% loss ratio corresponds to $15 million in losses.
A 60% loss ratio corresponds to $30 million in losses.
The average size of aggregate losses in the interval 15 to 30 million is:

$$\frac{E[X \wedge 30m] - (30m)\, S(30m) - \{E[X \wedge 15m] - (15m)\, S(15m)\}}{F(30m) - F(15m)} =$$

$$\frac{22.096 - (30)(1 - 0.6216) - \{13.862 - (15)(1 - 0.2481)\}}{0.6216 - 0.2481} = 21.849 \text{ million.}$$

For a loss ratio of 21.849 / 50 = 43.70%, the commission would be:
25% + (60% - 43.70%)/2 = 33.15%.
Thus the average commission is:
(0.2481)(40%) + (0.6216 - 0.2481)(33.15%) + (1 - 0.6216)(25%) = **31.8%**.

15. (a) If there is an event, the average amount the reinsurer pays is:
(3%)(5) + (2%)(10) + (1%)(15) + (1%)(25) + (1%)(35) + (1%)(45) + (1%)(50) = 2.05.
Thus the expected annual amount paid by the reinsurer is: (17%)(2.05 million) = **$348,500**.
(b) Reinstatement clauses deal with the possibility of more than one (large) event per year.
Typically a property catastrophe treaty will allow for reinstatements.
Assume for example, that there is an event for $25 million.
Then the treaty would provide that the insured has used up $10 million of the available $50 million in coverage. The primary insurer could restore the full $50 million in coverage by paying a reinstatement premium. Reinstatement is usually automatic, to provide immediate coverage in case of another event. For example, if the reinstatement premium were 110% pro-rata as to amount, then in this case, the reinstatement premium would be the original reinsurance premium times: (110%)(10/50) = 22%.
A treaty would allow a limited number of reinstatements. For example, if in this case only two full reinstatements were allowed, then the maximum amount the reinsurer would pay in any single year is: (2 + 1)($50 million) = $150 million.
Comment: For example, a $25 million event occurs. The reinsurer pays the insurer $10 million.
The insured pays its reinstatement premium.
Another event of size $40 million occurs in the same year. The reinsurer pays the insurer $25 million. The insurer pays another reinstatement premium.
As a practical matter, when the second large event occurred, the insurer might still be settling claims on the first event, and therefore the exact amount of the recovery from the reinsurer for the first event nor the exact amount of the reinstatement premium would be known at the time of the second event.
This is why reinstatements are usually automatic.
It is possible for a catastrophe treaty to have free reinstatements.
If there are only two full reinstatements allowed, then there is the equivalent of a $150 million aggregate limit. The actuary could price the impact of this aggregate limit. (In this example, the effect would be very small, but would reduce the expected annual amount paid by the reinsurer.)
While this is not specifically covered by Clark, it is mathematically similar to pricing an aggregate limit in other readings such as Fisher. Then the actuary would also have to estimate the expected annual amount of reinstatement premiums the reinsurer would expect to receive.
See “Pricing Catastrophe Reinsurance with Reinstatement Provisions Using a Catastrophe Model,” by Richard R. Anderson and Wemin Dong, Spring 1998 CAS Forum,
not on the syllabus.
“Basing the up-front premium on the average annual loss to a treaty, disregarding reinstatements, is equivalent to assuming that there are unlimited free reinstatements. If, on the other hand, reinstatements are limited and paid, then the up-front premium will be lower because fewer losses will be covered (because the reinstatements are limited) and some of the premium will be paid after an event has occurred (because the reinstatements are paid).”

16. (a) For the loss free scenario, the profit is: 3 million - 0 - (12%)(3 million) = 2.64 million.
The reinsurer pays a profit commission of: (75%)(2.64 million) = $1.98 million.
Thus the underwriting result excluding expense is: 3 million - 1.98 million = **$1.02 million**.
For the one full loss, there is no profit and thus no profit commission.
The additional premium is: (60%){20 million + (12%)(3 million) - 3 million} = $10.416 million.
Thus the underwriting result excluding expenses is:
3 million + 10.416 million - 20 million = -**$6.584 million**.
(b) Let P be the premium, and x be the reinsurer's share of the covered layer.
For the loss free scenario: 1.02 million = xP.
For the one full loss scenario: -$6.584 million = xP - x 20 million.
Subtracting the two equations: 7.604 million - x 20 million. ⇒ x = **38.02%.**
⇒ P = **$2.683 million**.
Comment: See page 47 of Clark.

	Loss Free Scenario	One Full Loss Scenario
Premium	$3,000,000	$3,000,000
Loss	$0	$20,000,000
Profit Commission	$1,980,000	$0
Additional Premium	$0	$10,416,000
Underwriting Result	$1,020,000	-$6,584,000

17. "In the past, reinsurers had priced catastrophe loads based on spreading large losses over expected payback periods. A 1-in-20-year event would be included as a loading of 5% of the loss amount. The payback approach may still be used for casualty events but is only referenced as a reasonability check for property. The most common procedure is now for a company to select a property catastrophe load based on an engineering-based model that incorporates the risk profile of the ceding company."
Comment: See page 5 of Clark.

18.

I.V.	250K / I.V.	Exp. Factor	500K / I.V.	Exp. Factor	Exposure Factor for Layer
100K	250%	1.000	500%	1	1 - 1 = 0
300K	83.3%	0.963	166.6%	1	1 - 0.963 = 0.037
500K	50%	0.861	100%	1	1 - 0.861 = 0.139
700K	35.7%	0.794	71.4%	0.932	0.932 - 0.794 = 0.138
900K	27.8%	0.745	55.6%	0.881	0.881 - 0.745 = 0.136

Insured Value	Subject Premium	Exposure Factor	Exposed Premium
100K	19	0	0
300K	25	0.037	(0.037)(25) = 0.925
500K	31	0.139	(0.139)(31) = 4.309
700K	27	0.138	(0.138)(27) = 3.726
900K	15	0.136	(0.136)(15) = 2.040
Total	117		11.000

Exposure rate is: (74%) (11.000) / 117 = **6.96%**.
Comment: Similar to 6, 11/07, Q.32.

19. 1. Include any carryforward from past years and estimate the impact on the current year only. This amounts to shifting the slide by the amount of the carryforward.
The problem with this approach is that it ignores the potential for carryforward beyond the current year. For example, in the first year of the program we would calculate the expected commission for the current year as though the program would be cancelled at the end of the year.
2. Look at the "long run" of the contract. The sliding scale is modeled as applying to a longer block of years rather than just the single current year. The variance of the aggregate distribution would be reduced on the assumption that individual bad years would be smoothed by good experience on other years. The variance of the average loss ratio for a block of years should be significantly less that the variance of the loss ratio for a single year.
The first problem with this approach is that the method for reducing the variance is not obvious. A second problem is that it ignores the fact that the contract may not renew the following year, potentially leaving the reinsurer with no carryforward benefit.
Comment: See pages 11 to 12 of Clark.

20. Due to the experience rate of zero for 1M xs 3M, we would have a free cover.
Based on exposure rating, we expect the layer 1M xs 3M to be 4/(11+ 7) = 2/9 of the cost of the layer 2M xs 1M.
Therefore, we can can estimate the cost for 1M xs 3M as: (2/9)(9% + 3%) = 2.67%.
For the lower layers I will select the experience rating rate.

Layer	Experience Rating	Exposure Rating Loss Cost	Selected
1M xs 1M	9%	11%	9%
1M xs 2M	3%	7%	3%
1M xs 3M	0%	4%	2.67%
Total	12%	22%	**14.67%**

Alternately, one could base the rate for the layer 1M xs 3M on the experience rate for the layer 1M xs 1M: (4/11)(9%) = 3.27%.
For the 1M xs 1M layer, I will select the experience rating rate.
For the 1M xs 2M layer, I suspect that the experience rate is based on very few claims, so I will select the average of the experience rating rate and the exposure rating rate.

Layer	Experience Rating	Exposure Rating Loss Cost	Selected
1M xs 1M	9%	11%	9%
1M xs 2M	3%	7%	5%
1M xs 3M	0%	4%	3.27%
Total	12%	22%	**17.27%**

<ins>Comment</ins>: See page 20 of Clark.
Other reasonable selections could be made.

21. (a) Finite risk refers to property catastrophe covers for which the maximum loss amount is reduced relative to traditional covers. This distinction is very soft.
(b) 1. Multiple year features.
2. Loss sensitive features such as profit commissions and additional premium formulas.
(c) 1. The reinsurer assumes significant insurance risk under the reinsured portions of the underlying insurance contracts.
2. It is reasonably possible that the reinsurer may realize a significant loss from the transaction.
<ins>Comment</ins>: See page 46 of Clark.

22. (a) The first loss is not covered by the treaty since it is from a policy with effective date before 1/1/2015.
Ceded losses: (80%)(100,000 + 150,000 + 20,000) = **$216,000**.
(b) The second and fourth losses are not covered by the treaty since they occur after 12/31/2015.
Ceded losses: (80%)(60,000 + 150,000) = **$168,000**.

23. A first measure of credibility is the number of claims expected during the historical period. Note that this is not the same as the actual number observed during the period. If credibility is set based solely on the historical number, then more credibility will be assigned to experience rating projections that are fortuitously worse than average.
(Excess of loss reinsurance has relatively few expected claims and thus the number of observed claims is subject to much more random fluctuation than is primary insurance.)
As a second measure of credibility, it is appropriate to look at the year-to-year variation in the projected loss cost from each of the historical periods. Stability in this rate should add credibility even if the number of claims is relatively small.
In general, the issue of assigning credibility is a subjective exercise. Often significant credibility is given to experience rating simply because there are too many limitations to the exposure rating alternative.
Comment: See pages 20 to 21 of Clark.

24. 1. When there is very little data on which to base an experience rate.
2. When the mix of policies written by the primary insurer has changed significantly over the experience period, and thus does not reflect the mix of business to be reinsured.

25.
1. Working Layer: Low layer attachment which is expected to be penetrated, often multiple times in each annual period.
2. Exposed Excess: Excess layer which attaches below some of the policy limits on the underlying business - that is, there are policies for which a full limit loss would cause a loss to the treaty. Typically, these losses will be less frequent and there will be some years in which the treaty layer is not penetrated.
3. Clash Covers: High layer attachment excess; typically a loss on a single policy will not penetrate the treaty layer. A clash cover will be penetrated due to multiple policies involved in a single occurrence, or when extra-contractual obligations (ECO) or rulings awarding damages in excess of policy limits (XPL) are determined in a settlement. The method for including allocated loss adjustment expenses in the treaty may also expose the clash layer.
Comment: See pages 22 of Clark. There are not definitive boundaries between these categories.

26. The exposure factor in each case is ELAEF at $1 million - ELAEF at $4 million.
For example: (0.032 - 0.006)($8 million) = $208,000.

Hazard Group	ELAEF for 1m	ELAEF for 4m	Standard Prem.	Treaty Loss & ALAE
B	0.032	0.006	$8,000,000	$208,000
E	0.061	0.012	$13,000,000	$637,000
G	0.123	0.028	$5,000,000	$475,000
Total			$26,000,000	$1,320,000

The loss & ALAE cost for the treaty is: $1,320,000 / 26,000,000 = **5.08%**.
Comment: The definition of ELAEF given in the question differs from that at pp. 30-31 of Clark.

27. Reinsurance Premium = $30 million / 15 = $2 million.
The treaty pays: 19 - 10 = $9 million.
The reinstatement premium is: (120%)(9/30)($2 million) = **$720,000**.
Comment: Payback = (Treaty Limit) / (Reinsurance Premium).

28.

Range of Insured Value ($000)	MidPoint	b	g	I.V. net of surplus share
100 to 250	175	9.0	7.7	175
250 to 500	375	7.8	9.9	250
500 to 1000	750	6.5	13.0	250

We need to use the exposure curve appropriate for each midpoint, but then take the ratios of the endpoints of the reinsured layer to the insured value net of the surplus share treaty, which inures to the benefit of the excess treaty.

b	g		Net I.V.		100 / Net I.V.	Exp. Factor		300 / Net I.V.	Exp. Factor
9.0	7.7		175		57.1%	0.734		171%	1.000
7.8	9.9		250		40%	0.627		120%	1.000
6.5	13.0		250		40%	0.650		120%	1.000

For the first interval, $G(0.571) = \dfrac{\ln\left[\dfrac{(7.7-1)(9.0) + \{1-(7.7)(9.0)\}\, 9.0^{0.571}}{1 - 9.0}\right]}{\ln[(7.7)(9.0)]} = 0.734.$

For the second interval, $G(0.4) = \dfrac{\ln\left[\dfrac{(9.9-1)(7.8) + \{1-(9.9)(7.8)\}\, 7.8^{0.4}}{1 - 7.8}\right]}{\ln[(9.9)(7.8)]} = 0.627.$

For the third interval, $G(0.4) = \dfrac{\ln\left[\dfrac{(13-1)(6.5) + \{1-(13)(6.5)\}\, 6.5^{0.4}}{1 - 6.5}\right]}{\ln[(13)(6.5)]} = 0.650.$

The insurer will cede to the surplus share treaty:
0, (375 - 250)/375 = 1/3, and (750 - 250)/750 = 2/3.
Then for example, for the 250 to 500 range, the net premium is: (1 - 1/3)(30) = 20,
and the expected loss and ALAE for the excess treaty is: (70%)(20)(100% - 62.7%) = 5.222.

Range of Insured Value ($000)	Gross Premium	Net Premium	Expected Loss & ALAE Ratio	Exposure Factor	Exc. Reinsurer Loss & ALAE
100 to 250	10	10	70%	0.266	1.862
250 to 500	30	20	70%	0.373	5.222
500 to 1000	18	6	70%	0.350	1.470
Total		36			8.554

The exposure rate for the excess treaty as a percent of the premium net of the surplus share treaty: 8.554 / 36 = **23.8%**.
<u>Comment</u>: See pages 21-22 of Clark. See equation 3.3 in Bernegger.
We are interested in the losses net of the surplus share treaty. For example, for a property with value 375K, 125K/375K = 1/3 of the each loss is ceded to the surplus share. Therefore, if there is a loss of for example 330K, then only (2/3)(330K) = 220K is retained by the primary.
Thus the excess reinsurer pays: 220K - 200K = 20K.

29. For the Poisson Distribution, a = 0, b = λ = 0.7.
A_0 = probability of the frequency being zero = $e^{-\lambda} = e^{-0.7}$ = 0.4965853.

Then we compute recursively the remaining densities of the aggregate distribution.

$$A_k = \sum_{i=1}^{k} (a + b\ i/k)\ S_i\ A_{k-i} = (0.7/k) \sum_{i=1}^{k}\ i\ S_i\ A_{k-i}\ .$$

$A_1 = (0.7/1)(1)S_1A_0$ = (0.7)(0.4)(0.4965853) = 0.1390439.

$A_2 = (0.7/2)\ \{S_1A_1 + 2S_2A_0\}$
= (0.35) {(0.4)(0.1390439) + (2)(0.2)(0.4965853)} = 0.0889881.

$A_3 = (0.7/3)\ \{S_1A_2 + 2S_2A_1 + 3S_3A_0\}$
= (0.7/3) {(0.4)(0.0889881) + (2)(0.2)(0.1390439) + (3)(0.1)(0.4965853)}
= **0.0560440**.

Comment: Similar to the example at pages 39 to 40 of Clark.
I would be somewhat surprised if you were asked a question like this on your exam,
but this formula is also discussed in NCCI Circular CIF-2018-28, on the syllabus of this exam.
We could continue and calculate A_4, A_5, ...
The average severity is: (0.4)(1) + (0.2)(2) + (0.1)(3) + (0.1)(4) + (0.1)(5) + (0.1)(6) = 2.6.
Thus the average aggregate loss is: (0.7)(2.6) = 1.82.
The second moment of severity is:
$(0.4)(1^2) + (0.2)(2^2) + (0.1)(3^2) + (0.1)(4^2) + (0.1)(5^2) + (0.1)(6^2) = 9.8$.
Thus since frequency is Poisson, the variance of aggregate losses is: (0.7)(9.8) = 6.86.

30. Surplus share take a fixed dollar amount for a given treaty, and covers the amount above that point as a percent of insured value; the retained line is fixed per a treaty, but the percent ceded, which starts at dollar one, is different for each risk. So for example, for a small risk the primary insurer would retain everything, while for a large risk the primary insurer might cede 75% of losses and premiums.
Quota share contracts cover a fixed percent for all risks in the treaty, and the ceding percentage does not vary from risk to risk.
Excess loss only covers the portion of losses that pierce a given threshold and thus are in a given layer.

31. The trended losses must then be capped at applicable policy limits. However, there is no generally accepted solution. Theoretically, we want to cap losses at the limit applicable if the same policy were written in the future treaty period.
One possible approach is to apply the historical policy limit to each trended loss; this leaves out the fact that the insured will generally increase their policy limits over time.
A second approach is to apply the trend factor to the historical loss without applying a policy limit cap; this assumes that policy limits “drift” upwards to precisely match inflation. If this second approach is used, then the subject premium must also be adjusted to the level that would have been charged had the higher limits been in effect; otherwise an overstatement of the expected loss cost will result.
Comment: See pages 23 and 24 of Clark.

32. The exposure factor in each case is ELF at $2 million - ELF at $5 million.
For example: (0.015 - 0.005)($5 million) = $50,000.

Hazard Group	ELF for 2m	ELF for 5m	Standard Prem.	Treaty Losses
C	0.015	0.005	$5,000,000	$50,000
D	0.021	0.007	$3,000,000	$42,000
F	0.026	0.009	$2,000,000	$34,000
Total			$10,000,000	$126,000

The loss cost for the treaty is: 126,000 / 10,000,000 = **1.26%**.
Comment: At page 31, Clark multiples the ELF times the expected losses, which are standard premium times the expected loss ratio. However, instead here I have used the NCCI definition, ELF times standard premium is the expected excess losses. The ELF (without risk load) is the excess ratio times the expected loss ratio to standard premium.

$$\text{ELF} = \frac{\text{Expected Excess Losses}}{\text{Standard Premium}}.$$

33. The aggregate distribution that is the output of the collective risk model, reflects the process variance of losses but does not reflect the full parameter variance.
“Process variance” refers to the random fluctuation of actual results about the expected value.
“Parameter variance” in the narrow sense refers to uncertainty about the parameters and may be calculable from outside sources. Some models allow for a prior distribution to apply to the selected parameters.
“Parameter variance” in the broader sense of not being sure if you are even in the right model is harder to estimate and is best reflected by repeated sensitivity analysis. This broader sense could perhaps be called “model risk”.
Comment: See pages 42 and 43 of Clark.

34. Linearly interpolate exposure factors.

Midpoint of Range	\$500K Over I.V.	Expos. Factor	\$2.5M Over I.V.	Expos. Factor
0.375	133.3%	100%	666.7%	100%
0.750	66.67%	92.33%	333.3%	100%
1.750	28.57%	74.28%	142.9%	100%
3.750	13.33%	58.33%	66.67%	92.33%

Insured Value Range	Subject Premium	Exposure Factor
\$250,000 - \$500,000	\$20	0%
\$500,001 - \$1,000,000	\$40	100% - 92.33% = 7.67%
\$1,000,001- \$2,500,000	\$60	100% - 74.28% = 25.72%
\$2,500,001- \$5,000,000	\$50	92.33% - 58.33% = 34.00%

The losses the reinsurer expects to pay as a percent of subject premium:

$$68\% \ \frac{(0)(20) + (7.67\%)(40) + (25.72\%)(60) + (34.00\%)(50)}{20 + 40 + 60 + 50} = \mathbf{14.20\%}.$$

35. For those situations where it does not drop down, the umbrella policy covers the layer from 0.5M to 5M. $E[X \wedge 5M] - E[X \wedge 0.5M] = 178{,}809 - 125{,}233 = 53{,}576$.
For those situations where it drops down, the umbrella policy covers the layer from 0 to 4.5M. $E[X \wedge 4.5M] = 177{,}919$.
For those situations where the umbrella does not drop down, the reinsurance treaty covers the layer from 1.5M to 4M. $E[X \wedge 4M] - E[X \wedge 1.5M] = 176{,}793 - 160{,}538 = 16{,}255$.
For those situations where the umbrella does drop down, the reinsurance treaty covers the layer from 1M to 3.5M. $E[X \wedge 3.5M] - E[X \wedge 1M] = 175{,}334 - 149{,}566 = 25{,}768$.

The exposure factor is: $$\frac{(10\%)(25{,}768) + (90\%)(16{,}255)}{(10\%)(177{,}919) + (90\%)(53{,}576)} = 0.2607.$$

Expected ceded loss is: (0.2607)(\$60 million) = **\$15.64 million**.
Comment: Similar to 6, 11/00, Q. 71.
Clark uses the "aggregate excess factor" to approximate the percent of time the umbrella policy will drop down (due to the exhausting of the underlying policies aggregate limit. The umbrella may also drop down in order to fill coverage gaps.)

36. This is simplifying assumption. "This is not an accurate model in that ALAE is not a constant percent of any given loss. For example, losses which close without an indemnity payment may still incur a large expense. In general, as the size of a loss increases, the ALAE as a percent of the loss will tend to decrease. The assumption that loss and ALAE are perfectly correlated will tend to result in an overstatement of expected amounts in the higher layers.
When ALAE is included with losses, a limitation of the resulting formula is that an exposure factor of zero will be applied to high layers which are indeed exposed. For example, if the underlying policy limit is \$1,000,000 and the ALAE loading is 1.500, then a treaty attaching at \$1,500,000 will not be considered exposed by this formula.
A more refined analysis of the effect of ALAE would require modeling of how ALAE varies with loss size."
One could model the joint distribution of loss and ALAE, as ISO has done.
Comment: See page 29 and 30 of Clark.

37. E[X ; 1 million] = 136,000 - 6000e^{-100} - 30,000e^{-10} - 100,000e^{-1} = 99,211.
E[X ; 250,000] = 136,000 - 6000e^{-25} - 30,000$e^{-2.5}$ - 100,000$e^{-0.25}$ = 55,657.

Exposure factor is: $\dfrac{E[X\ ;\ 250K + 750K] - E[X\ ;\ 250K]}{E[X\ ;\ 1000K]} = \dfrac{99{,}211 - 55{,}657}{99{,}211} = 43.90\%$.

The estimated loss cost of this treaty layer is: (43.90%)($70 million) = **$30.7 million**.
<u>Comment</u>: Similar to 8, 11/14, Q.25.
The limited expected value function is for a mixture of three Exponential Distributions.
The treaty would have an aggregate limit per occurrence, for example $2 million, which for simplicity has been ignored in this question.

38. a) There is a problem.
If for example the primary insurer had a loss of size $250,000 then reinsurer X would pay (60%)($250,000) = $150,000 and reinsurer Y would pay: 250K - 100K = $150,000.
The primary insurer would recover $300,000 from reinsurance for a $250,000 loss.
Not only is the primary insurer being reimbursed for the same dollars twice, there is a moral hazard; the insurer has an incentive to pay <u>more</u> for some large losses.
b) No problem.
It is common for different reinsurers to each reinsure different (non-overlapping) excess layers.
Y covers the layer from 100K to 300K, while Z covers the layer from 300K to 1000K.
c) No problem. Y covers 50% of the layer from 100K to 300K, while Z covers the other 50% of that layer. Z also covers 50% of the layer from 50K to 100K.
Two or more reinsurers can divide the coverage of layers between them; there is no problem as long as the total of the percentages for any layer is at most 100%
d) There is a problem.
In the layer from 200K to 300K, the insurer will recover from reinsurance 50% + 70% = 120% of losses. Not only is the primary insurer being reimbursed for the same dollars twice, there is a moral hazard; the insurer has an incentive to pay <u>more</u> for some large losses.

39. Percentages ceded are: 300/400 = 75%, 0, and 900/1500 = 60%.
Primary insurer premiums: (25%)(20K) + (100%)(4K) + (40%)(50K) = 29,000.
Primary insurer losses: (25%)(80K) + (100%)(40K) + (40%)(400K) = 220,000.
Primary insurer loss ratio: 220,000/29,000 = **758.6%**.
Reinsurer premiums: (75%)(20K) + (0%)(4K) + (60%)(50K) = 45,000.
Reinsurer losses: (75%)(80K) + (0%)(40K) + (60%)(400K) = 300,000.
Reinsurer loss ratio: 300,000/45,000 = **666.7%**.
<u>Comment</u>: Presumably, there were many other properties the insurer insured which had no losses.

40. While the average loss ratio is 70%, in some years the loss ratio will be higher and some years it will be lower. We need to apply the commission schedule to each of these possible loss ratios and weight them together by their probabilities. Since the commission schedule is somewhat complex (for an inexperienced underwriter like Seth) the result is not equal to the commission at a 70% loss ratio.
<u>Comment</u>: If the average commission were equal to that at the 70% expected loss ratio, then the plan would be called balanced.

41. The excess treaty is not exposed by the first group of policies.

MPL	Gross Premium	500K / MPL	Exposure Curve to Use
500,000	40 million	1	Y2
1,000,000	20 million	1/2	Y2
1,500,000	20 million	1/3	Y3
2,000,000	10 million	1/4	Y3

For c = 2, b = exp[3.1 - (0.15)(2)(3)] = 9.025. g = exp[(2){0.78 + (0.12)(2)}] = 7.691.

$$G(1/2) = \frac{\ln\left[\frac{(7.691 - 1)(9.025) + \{1 - (7.691)(9.025)\}\, 9.025^{0.5}}{1 - 9.025}\right]}{\ln[(7.691)(9.025)]}$$

= ln[18.085] / ln[69.411] = 0.683.

For c = 3, b = exp[3.1 - (0.15)(3)(4)] = 3.669. g = exp[(3){0.78 + (0.12)(3)}] = 30.569.

$$G(1/3) = \frac{\ln\left[\frac{(30.569-1)(3.669) + \{1 - (30.569)(3.669)\}\, 3.669^{1/3}}{1 - 3.669}\right]}{\ln[(30.569)(3.669)]}$$

= ln[23.588] / ln[112.16] = 0.670.

$$G(1/4) = \frac{\ln\left[\frac{(30.569-1)(3.669) + \{1 - (30.569)(3.669)\}\, 3.669^{1/4}}{1 - 3.669}\right]}{\ln[(30.569)(3.669)]}$$

= ln[16.993] / ln[112.16] = 0.600.

Exposed premium: (1M) {(1 - 0.683)(20) + (1 - 0.670)(20) + (1 - 0.600)(10)} = $16.94 million.
The expected ceded losses are: (62%)(16.94) = $10.5028 million.
The exposure rate is: $10.5028 million / $90 million = **11.67%**.

42. "Along with the historical losses, it is very important that allocated loss adjustment expenses (ALAE) be captured separately from losses. For general liability and auto liability losses, the underlying policy limit should also be listed. For auto losses on a split limits rather than a combined single limit (CSL) basis, other modifications may be needed in order to separately cap losses for bodily injury and property damage.
Workers compensation (WC) losses will not have an explicit limit associated with them. However, because large workers compensation losses are often shown on a discounted case reserve basis, a request should be made for these losses on a full undiscounted basis."
Comment: See page 23 of Clark.

43. Expected Loss = ($2M)(0.60) = $1.2M.
Portion of loss in layer = (1442 - 1201) / 1633 = 0.14758.
(1.06) ($1.2M) (0.14758) / (1 - 0.15) = **$220,849**.

44. (a) Treaty A pays $1 million of the loss, and shares 50% of the ALAE or $5 million.
Thus, Treaty A pays $6 million. Treaty B pays nothing. The primary insurer pays $6 million.
(b) Treaty A pays $5 million of loss and ALAE, its limit.
The insurer retains $1 million, and Treaty B pays the remaining $6 million of loss and ALAE
Comment: Part (b) is an example of how if a claim has very large ALAE, a higher layer of coverage can be pierced.

45. If the ceded losses are 8% of subject premium, then the ceding company pays a retro premium equal to the minimum. If the ceded losses are 32% of the subject premium then the ceding company pays the maximum retro premium.
In the middle range the retro premium is: (1.25)(Ceded Losses).
The average retro premium for the middle range is: (1.25)(22.2%).
The average retro premium as a percent of subject premium is:
(22%)(10%) + (64%)(1.25)(22.2%) + (14%)(40%) = **25.56%**.
The average ceded losses as a percent of subject premium are:
(22%)(6.5%) + (64%)(22.2%) + (14%)(40.9%) = 21.364%.
The expected loss ratio for the reinsurer is: 21.364% / 25.56% = **83.58%**.
Comment: See pages 34 and 35 of Clark.
Assume the subject premium is for example $100 million.

Range of Ceded Losses	Probability	Average in Range
Less than 8 million	22%	6.5 million
8 to 32 million	64%	22.2 million
More than 32 million	14%	40.9 million

Average ceded losses are: (22%)(6.5) + (64%)(22.2) + (14%)(40.9) = 21.364 million.
In the first range the retro premium is $10 million. In the last range the retro premium is $40 million.
In the middle range the retro premium is: (1.25)(Ceded Losses).
The average retro premium for the middle range is: (1.25)(22.2 million).
Thus the average retro premium is:
(22%)(10) + (64%)(1.25)(22.2) + (14%)(40) = 25.56 million.
The expected loss ratio for the reinsurer is: 21.364 / 25.56 = 83.58%.

46. Let G(x) be the exposure curve.
Assuming the property is worth $1 million, the reinsurer is responsible for portion of losses:
G(1/1) - G(0.5/1) = 1 - 0.730 = 0.270.
The property is worth 1.25 million, but since the policy limit is $1 million, I assume that there are no losses of size greater than $1 million. Thus the reinsurer still expects to pay for the layer from $500,000 to $1 million. The reinsurer is responsible for portion of losses:
G(1/1.25) - G(0.5/1.25) = G(0.8) - G(0.4) = 0.902 - 0.661 = 0.241.
Another question is whether or not the premium charged by the primary insurer is adequate.
Assuming that the rate per $100 of value is correct, then the premium should be 25% more.
Then the reinsurer's expected losses are: (1.25)(0.241) = 0.301 versus 0.270.
0.301 / 0.270 = 1.115. The reinsurer's estimate of its **expected loss should be 11.5% highe**r.
Comment: Reinsurer's need to worry about whether the properties in the reinsured portfolio are underinsured and whether the rates charged by the primary insurer are adequate.
The exposure curve is as per Section 4.3 of Bernegger with c = 2.5.

47. 1. The formula for the limited expected value only applies for losses above the truncation point T. As a practical matter, this is not a problem as that parameter is set at an amount well below any treaty attachment point.
2. The excess factors for higher layers become very dependent on the shape parameter Q.
This parameter must be watched very carefully when the curves are updated.
Comment: See pages 27 and 28 of Clark.

48. (a) $E[X \wedge 500{,}000] = (200{,}000)\{1 - (400/900)^2\} = 160{,}494$.
$E[X \wedge 1{,}000{,}000] = (200{,}000)\{1 - (400/1400)^2\} = 183{,}673$.
$E[X \wedge 2{,}000{,}000] = (200{,}000)\{1 - (400/2400)^2\} = 194{,}444$.
Exposure Factor: $\dfrac{E[X \wedge 1000K] - E[X \wedge 500K]}{E[X \wedge 2000K]} = \dfrac{183{,}673 - 160{,}494}{194{,}444} = 11.92\%$.

Thus the expected ceded loss and ALAE is: (11.92%)(25 million)(1.17) = **$3.49 million**.
(b) The ALAE is shared pro-rata, so for a claim that pierces the reinsured layer, the ratio of ALAE to loss is the same for the ceded and retained portions. However, in general, as the size of a loss increases, the ALAE as a percent of the loss will tend to decrease. The ceded losses result from larger claims, for which ALAE is on average a smaller ratio to losses than the overall 17%. Thus using the 17% from the primary company overstates the expected ceded ALAE.
Comment: The formula given for the Limited Expected Values is for a Shifted Pareto Distribution with $\alpha = 3$ and $\theta = 400{,}000$.
For part (b), see page 29 of Clark.

49. (a) Advantages of the use of single distribution according to Clark:
1. Relatively simple to use, even when the source data is limited.
2. A reasonable fit is provided even when separate frequency and severity distributions are not known.

(b) Disadvantages of the use of single distribution according to Clark:
1. There is no allowance for the loss free scenario.
(One would need to add to the model a point mass of probability at zero.)
2. There is no easy way to reflect the impact of changing per occurrence limits on the aggregate losses.

Comment: See page 38 of Clark.

50. "This assumption of scale independence may be appropriate for homeowners business, for which this technique was first developed, but may be a serious problem when applied to large commercial risks."
Comment: See page 19 of Clark.

51. The minimum retro premium is: (10%)(200 million) = $20 million.
The maximum retro premium is: (30%)(200 million) = $60 million.

Ceded Loss	1.4 times Ceded Loss	Retro Premium
0	0	20
5	7	20
10	14	20
15	21	21
20	28	28
25	35	35
30	42	42
40	56	56
60	84	60
80	112	60

Average retro premium is:
(20 + 20 + 20 + 21 + 28 + 35 + 42 + 56 + 60 + 60) / 10 = **$36.2 million**.
Average ceded losses are:
(0 + 5 + 10 + 15 + 20 + 25 + 30 + 40 + 60 + 80) / 10 = $28.5 million.
The expected loss ratio for the reinsurer is: 28.5 / 36.2 = **78.7%**.

52. $100,000 is 3.333% of insured value with an exposure factor of:
32.70% + (48.68% - 32.70%) (3.333% - 1%) / (5% - 1%) = 42.02%.
The primary pays $1 million for a loss of $1.1 million.
$1.1 million is 36.667% of insured value with an exposure factor of:
77.78% + (83.14% - 77.78%) (36.667% - 30%) / (40% - 30%) = 81.35%.
The primary pays $2 million for a loss of $2.1 million.
$2.1 million is 70% of the insured value of $3 million with an exposure factor of 93.57%.
The primary insurer covers the layer from $100,000 to $3 million.
Thus the reinsurer's expected losses are:

$$(65\%)(\$80{,}000)\ \frac{93.57\% - 81.35\%}{100\% - 42.02\%} = \mathbf{\$10{,}960}.$$

Comment: (65%)(80,000) = 52,000 are the expected losses with the deductible.
52,000 / (100% - 42.02%) = 89,686 are the expected losses without a deductible.
The reinsurer would assume 93.57% - 81.35% = 12.22% of these total expected losses.
(12.22%) (89,686) = 10,960.

Another way to think of it: $\frac{\text{Reinsurer}}{\text{Insurer}} = \frac{93.57\% - 81.35\%}{100\% - 42.02\%} = 0.21076.$

Note that the numerator and denominator are each in the same units, as a portion of total losses without a deductible. Then: (0.21076)(52,000) = 10,960.

53. For those situations where it does not drop down, the umbrella policy covers the layer from 0.5M to 3M. $E[X \wedge 3M] - E[X \wedge 0.5M] = 173,386 - 125,233 = 48,153$.
For those situations where it drops down, the umbrella policy covers the layer from 0 to 2.5M.
$E[X \wedge 2.5M] = 170,685$.
For those situations where the umbrella does not drop down, the reinsurance treaty covers the layer from 1.5M to 2.5M. $E[X \wedge 2.5M] - E[X \wedge 1.5M] = 170,685 - 160,538 = 10,147$.
For those situations where the umbrella does drop down, the reinsurance treaty covers the layer from 1M to 2M. $E[X \wedge 2M] - E[X \wedge 1M] = 166,740 - 149,566 = 17,174$.

The exposure factor is: $\dfrac{(12\%)(17,174) + (88\%)(10,147)}{(12\%)(170,685) + (88\%)(48,153)} = 0.1748$.

Expected ceded loss is: (0.1748)($40 million) = **$6.99 million**.
Comment: See page 33 of Clark.

ϕ = aggregate excess factor = $1 - \dfrac{\text{expected losses with limits occ/agg}}{\text{expected losses with limits occ/}\infty}$.

The higher the aggregate limit of the underlying CGL policy, the smaller ϕ.

54. (a) (52%)(20%) + (66%)(30%) + (82%)(35%) + (97%)(15%) = **73.45%**.
(b) The effect of the loss corridor is:
(40%) {(30%)(66% - 60%) + (35% + 15%)(75% - 60%)}
+ (70%) {(35%)(82% - 75%) + (15%)(90% - 75%)} = 3.72% + 3.29% = 7.01%.
Reinsurer's expected loss ratio after the application of the loss corridor is:
73.45% - 7.01% = **66.44%**.

55. The excess treaty will pay for loss & ALAE in the layer 300K xs 300K.
We assume this corresponds to losses in the layer:
300K / 1.2 xs 300K / 1.2 = 250K xs 250K.
This layer of loss is from 250K to 500K.
$\Rightarrow$ Numerator of exposure factor is: 1.2 (E[X $\wedge$ 500,000] - E[X $\wedge$ 250,000]).
Denominator of exposure factor is: 1.2 E[X $\wedge$ 1,000,000].
E[X $\wedge$ 250,000] = (100,000) (250/350) = 71,429.
E[X $\wedge$ 500,000] = (100,000) (500/600) = 83,333.
E[X $\wedge$ 1,000,000] = (100,000) (1000/1100) = 90,909.

$$\Rightarrow \text{Exposure Factor: } \frac{E[X \wedge 500K] - E[X \wedge 250K]}{E[X \wedge 1000K]} = \frac{83{,}333 - 71{,}429}{90{,}909} = 13.10\%.$$

The primary insurer has expected loss and ALAE of: (1.2)(80 million) = \$96 million.
Thus the expected ceded loss and ALAE is: (13.10%)(96 million) = **\$12.6 million**.
Alternately, in each case, use the formula from page 29 of Clark. The exposure factor is:

$$\frac{E[X \wedge \text{Min}[PL, (AP+Lim)/(1+e)]\,] - E[X \wedge \text{Min}[PL, AP/(1+e)]]}{E[X \wedge PL]},$$

PL = underlying Policy Limit applying to loss only = 1,000,000
AP = Treaty attachment point applying to ALAE plus loss capped at PL = 300,000.
Lim = Treaty limit applying to ALAE plus loss capped at PL = 300,000.
e = ALAE as a percent of loss capped at PL = 20%.
(AP+Lim) / (1+e) = (300K + 300K) / 1.2 = 500,000. AP / (1+e) = 300K / 1.2 = 250,000.
For the policies with a limit of 1000K, the exposure factor is:

$$\frac{E[X \wedge \text{Min}[1000K, 500K]\,] - E[X \wedge \text{Min}[1000K, 250K]]}{E[X \wedge 1000K]} = \frac{E[X \wedge 500K] - E[X \wedge 250K]}{E[X \wedge 1000K]} =$$

$$\frac{83{,}333 - 71{,}429}{90{,}909} = 13.10\%.$$

The primary insurer has expected loss and ALAE of: (1.2)(80 million) = \$96 million.
Thus the expected ceded loss and ALAE is: (13.10%)(96 million) = **\$12.6 million**.
<u>Comment</u>: The formula given for the Limited Expected Values is for a Shifted Pareto Distribution with $\alpha = 2$ and $\theta = 100{,}000$.

56. First figure out how much is retained by the primary insurer under the surplus share treaty.

Range of Insured Value	Midpoint	% Retained under Surplus Share
100 to 250	175K	100%
250 to 500	375K	250/375 = 2/3
500 to 1000	750K	250/750 = 1/3
1000 to 1500	1250K	250/1250 = 0.2
1500 to 2000	1750K	1 - 1250/1750 = 2/7

Midpoint	Portion Retained	250K / (midpoint times portion retained)	Exposure Factor
175K	100%	1.429	100%
375K	2/3	1	96%
750K	1/3	1	96%
1250K	0.2	1	96%
1750K	2/7	0.5	81%

Midpoint	Portion Retained	1000K / (midpoint times portion retained	Exposure Factor
175K	100%	5.714	100%
375K	2/3	4	100%
750K	1/3	4	100%
1250K	0.2	4	100%
1750K	2/7	1.5	100%

Range of Insured Value	Net Premium ($ million)	Expected Ceded Losses
100 to 250	(20)(100%) = 20	(64%)(100% - 100%)(20) = 0
250 to 500	(40)(2/3) = 26.667	(64%)(100% - 96%)(26.667) = 0.683
500 to 1000	(25)(1/3) = 8.333	(64%)(100% - 96%)(8.333) = 0.213
1000 to 1500	(10)(0.2) = 2	(64%)(100% - 96%)(2) = 0.051
1500 to 2000	(5)(2/7) = 1.429	(64%)(100% - 81%)(1.429) = 0.174

Expected ceded losses = 0 + 0.683 + 0.213 + 0.051 + 0.174 = **$1.121 million**.
Comment: See pages 21 and 22 in Clark.
In each case we should use an exposure curve appropriate to the size of property (prior to the effect of the surplus share treaty). For homeowners it often makes sense to use a single exposure curve for all sizes of property, as was done here.
Expected Losses =
(Expected Loss Ratio) (Difference in Exposure Factors) (Prem. Retained Net of Surplus Share).

57. The trended and developed losses in the 250K xs 250K layer are:
(8)(25) + (4)(50) + (4)(100) + (2)(150) + 200 + (2)(250) = 1.8 million.
The experience loss cost for this layer is: 1.8/13 = 13.85%.
The trended and developed losses in the 500K xs 500K layer are: 50,000.
The experience loss cost for this layer is: 0.05/13 = 0.38%.
The experience rate for the loss cost for the 500K xs. 500K layer is based on only one claim, so I will instead rely on the ratio of the exposure rating loss costs and the experience loss cost for the lower layer: (6%/18%)(13.85%) = **4.6%**.
<u>Comment</u>: See page 20 of Clark.
While relying on the experience rating in this case would not result in a free cover for the 500K xs. 500K layer, it would be close to free.

58. 1. The complexity of the calculations can lead to a "black box" mentality - assuming the numbers must be right because of the accuracy of the computer. Whenever possible, more than one set of results should be produced, as a check on the sensitivity of the answer to the starting assumptions. Some basic statistics, such as the coefficient of variation (standard deviation over mean) and percentiles, should be compared to the empirical data for reasonability.
2. Most models assume that each occurrence is independent of the others and that the frequency and severity distributions are independent of each other. This may be a reasonable assumption in many cases, but could be false in others.
3. Some collective risk models use numerical methods with a large error term for low frequency scenarios. Check the output of the model; the expected error term should be given.
4. The aggregate distribution reflects the process variance of losses but does not reflect the full parameter variance. "Process variance" refers to the random fluctuation of actual results about the expected value. "Parameter variance" in the narrow sense refers to uncertainty about the parameters and may be calculable from outside sources. Some models allow for a prior distribution to apply to the selected parameters. "Parameter variance" in the broader sense of not being sure if you are even using the right model is harder to estimate and is best reflected by repeated sensitivity analysis. This broader sense could perhaps be called "model risk".
<u>Comment</u>: I have quoted from pages 42 and 43 of Clark.
Discuss only two reasons.

59. In the first approach, the trended losses are capped at the historical policy limit.

Number	Loss	Trended Loss	Limited Trended Loss
1	200,000	240,000	240,000
2	300,000	360,000	360,000
3	450,000	540,000	500,000
4	500,000	600,000	500,000

Limited Trended Loss	ALAE	Trended ALAE	Loss & ALAE	In Layer
240,000	100,000	120,000	360,000	0
360,000	80,000	96,000	456,000	56,000
500,000	140,000	168,000	668,000	268,000
500,000	200,000	240,000	740,000	340,000

Experience Rate is: (56,000 + 268,000 + 340,000) / (40 million) = **1.66%**.

A second approach is to apply the trend factor to the historical loss without applying a policy limit cap; this assumes that policy limits "drift" upwards to precisely match inflation. If this second approach is used, then the subject premium must also be adjusted to the level that would have been charged had the higher limits been in effect.
The subject premiums for a limit of (1.2)(500,000) = 600,000 are:
(1.49/1.41) (40 million) = 42.270 million.

Trended Loss	ALAE	Trended ALAE	Loss & ALAE	In Layer
240,000	100,000	120,000	360,000	0
360,000	80,000	96,000	456,000	56,000
540,000	140,000	168,000	708,000	308,000
600,000	200,000	240,000	840,000	400,000

Experience Rate is: (56,000 + 308,000 + 400,000) / (42.270 million) = **1.81%**.
Comment: See pages 23 to 24 of Clark.
One would normally include a factor for excess loss development.
Usually, the trend from date and thus the trend period and trend factor would differ for individual losses in the experience period.

60. We determine the trended losses in the reinsured layer:

Accident Date	Trended Losses ($ million)	Trended Loss in Layer
March 29, 2013	5.82	0.82
October 12, 2015	9.39	4.39

Apply the Excess Loss Development Factor to each accident year:
(0.82)(1.02) = 0.8364.
(4.39)(1.10) = 4.829.

$$\text{Experience Rate} = \frac{0.8364 + 4.829}{4.8 + 4.9 + 4.7 + 5.0 + 5.2} = 23.0\%.$$

However, there are no trended losses in the layer from 10M to 25M.
Therefore, we would be providing a free cover; to avoid that we use the exposure curve.
The exposure rate for the layer from 5M to 10M is:
(80% - 50%) (expected loss ratio) = (30%) (expected loss ratio).
The exposure rate for the layer from 10M to 25M is:
(98% - 80%) (expected loss ratio) = (18%) (expected loss ratio).
Estimate rate for 10M to 25M layer: (23.0%) (18%/30%) = 13.8%.
Thus the policy's loss cost as a percentage of premium is: 23.0% + 13.8% = **36.8%**.
Comment: Similar to 8, 11/15, Q.23.

61. (a) 20% margin on annual premium is 0.6 million.
Profit Commission = (80%) (2.4 million - ceded losses), if positive.
Additional Premium: (40%) (Loss - 2.4 million), if positive.
If there is a loss of 20 million, then the additional premium is: (40%) (20M - 2.4M) = 7.04 million.
The underwriting loss is: 3M - 20M + 7.04M = -9,960,000.
In the case of no loss, Profit Commission = (80%) (2.4 million - 0) = 1.92 million.
The loss free underwriting result is: 3M - 1.92M = 1,080,000.
We want an equivalent traditional risk cover with the same underwriting results.
We can charge a premium of 1.08 million with no profit commission; this will produce the same result if there is no loss.
If we change the occurrence limit to 11.04 million, then without a provision for additional premium, if there is a large loss the underwriting result is: 1.08M - 11.04M = -9.96M, the same result as before.
(b) The rate on line for the equivalent traditional risk cover is: 1.08M / 11.04M = 9.78%.
A loss will fully exhaust the limit once every 10 years and the probability of a partial loss is negligible. We are ignoring the reinsurer’s expenses and profit provision. Therefore, we want the rate on line for the equivalent traditional risk cover to be more than 1/10 = 10%.
Since 9.78% < 1/10, the reinsurer should **reject the proposal**.
Comment: See page 47 of Clark. Similar to SOA GIADV Exam, 5/15, Q.8.
So that the rate on line for the equivalent traditional risk cover is greater than 1/10,
a counterproposal would include some or all of the following: an increase in premium,
a decrease in profit commission , an increase in margin, an increase in the additional premium.

62.

	Accident Year 2017		Accident Year 2018	
Simulation	Loss Ratio	Ceded Loss	Loss Ratio	Ceded Loss
1	57.5%	0	62.2%	(2.2%)(330) = 7.26
2	71.8%	(11.8%)(300) = 35.4	64.5%	(4.5%)(330) = 14.85
3	61.0%	(1.0%)(300) = 3	75.6%	(15%)(330) = 49.5
4	76.2%	(15%)(300) = 45	66.3%	(6.3%)(330) = 20.79
5	65.1%	(5.1%)(300) = 15.3	59.4%	0

(a) For 2017, 40% of the ceded premium is: (40%)(21) = 8.4.
Thus there is a profit commission if there is a ceded loss of less than 8.4.
Thus only in simulations #1 and #3 does the insurer terminate the contract.
The profit commission for simulation #1 is: 8.4M - 0 = 8.4M.
The profit commission for simulation #3 is: 8.4M - 3 = 5.4M.
Expected profit commission payable at the end of 2017 is: (8.4 + 5.4)/5 = **2,760,000**.
(b) In simulation #2 the profit commission is: (40%)(21 + 23) - (35.4 + 14.85) = -32.65. ⇒ 0.
In simulation #4 the profit commission is: (40%)(21 + 23) - (45 + 20.79) = -48.19. ⇒ 0.
In simulation #5 the profit commission is: (40%)(21 + 23) - (15.3 + 0) = 2.3.
Expected profit commission for the full term of the contract is: (8.4 + 5.4 + 2.3)/5 = **3,220,000**.
<u>Comment</u>: Similar to 8, 11/15, Q.23.

63. Exposure Rate = (Ultimate Trended Losses) / (On-Level Premium) =

$$\frac{(0.20)(1.2)(1.16) + (0.15)(1.3)(1.12) + (0.30)(1.5)(1.09) + (0.25)(2.0)(1.06)}{(11)(0.90) + (10)(0.98) + (11)(1.03) + (12)(1.05)} = \frac{1.5173}{43.63} = \mathbf{3.48\%}.$$

<u>Comment</u>: Similar to 6, 11/05, Q.28a.

64. First figure out how much is retained by the primary insurer under the surplus share treaty.

Range of Insured Value	Midpoint	% Retained under Surplus Share
50 to 200	125K	100%
200 to 1000	600K	200/600 = 1/3
1000 to 2000	1500K	1 - 800/1500 = 7/15

Midpoint	Portion Retained	200K / (midpoint times portion retained)	Exposure Factor
125K	100%	1.6	100%
600K	1/3	1	96%
1500K	7/15	0.286	66%

For example, 200K / {(1500K) (7/15)} = 2/7 = 0.286.

Midpoint	Portion Retained	500K / (midpoint times portion retained)	Exposure Factor
125K	100%	4	100%
600K	1/3	2.5	100%
1500K	7/15	0.714	89%

Range of Insured Value	Premium Net of Surplus Share($ million)	Expected Ceded Losses
50 to 200	(50)(100%) = 50	(65%)(100% - 100%)(50) = 0
200 to 1000	(90)(1/3) = 30	(65%)(100% - 96%)(30) = 0.78
1000 to 2000	(30)(7/15) = 14	(65%)(89% - 66%)(14) = 2.093

Expected ceded losses = 0 + 0.78 + 2.093 = **$2.873 million**.
<u>Comment</u>: See pages 21 and 22 of Clark.
In each case we should use an exposure curve appropriate to the size of property (prior to the effect of the surplus share treaty). For homeowners it often makes sense to use a single exposure curve for all sizes of property, as was done here.
Expected Ceded Losses =
(Expected Loss Ratio) (Difference in Exposure Factors) (Prem. Retained Net of Surplus Share).

65. The catastrophe reinsurance covers the layer from 50M to 200M.
In each case we multiply by the percentage of the width of the layer that has been exhausted.

Date	Losses in Layer	Reinstatement Premium
July 11	30,000,000	($5m) (1.1) (30/150) = $1.1 million
August 26	150,000,000	($5m) (1.1) (150/150) = $5.5 million
October 9	90,000,000	($5m) (1.1) (90/150) = $3.3 million
Total		**$9.9 million**

<u>Comment</u>: Similar to 8, 11/13, Q.21a.
For the second catastrophe, the layer of protection provided by this catastrophe treaty was exhausted. Thus assuming the insurer has no additional reinsurance protection, it would retain: 270 - 150 = $120 million of the losses from the second catastrophe.

66. Under the 70% ceded quota share, the insurer retains 30% of each loss and thus 30% of the total losses: (30%)($1,200,000) = **$360,000**.

Under the four-line surplus share treaty with retained line = $150,000:

Risk	Insured Value	Percent retained	Loss	Amount Retained
A	200	150/200	100	$75,000
B	700	150/700	330	$70,714
C	100	100%	80	$80,000
D	900	1 - 600/900	650	$216,667
E	300	150/300	40	$20,000
Total				**$462,381**

Under the $300,000 xs $200,000 per-risk excess of loss:

Risk	Loss	Amount Ceded	Amount Retained
A	$100,000	0	100,000
B	$330,000	130,000	200,000
C	$80,000	0	80,000
D	$650,000	300,000	350,000
E	$40,000	0	40,000
Total			**$770,000**

<u>Comment</u>: Similar to 8, 11/13, Q.23.
For the surplus share treaty, if X is the insured value, then the percent ceded is:

$$\begin{cases} 0 & \text{for } X < 150{,}000 \\ 1 - 150{,}000/X & \text{for } 150{,}000 \le X \le 750{,}000 \\ 600{,}000/X & \text{for } 750{,}000 < X \end{cases}$$

67. Let X be commission for an 80% loss ratio,
and 2X be the commission for a 50% loss ratio.
Then $12 million = (2/3)(X) + (1/3)(2X) = 4X/3. ⇒ X = $9 million.

Thus the commission for a 60% loss ratio is: (1/3)(9) + (2/3)(18) = **$15 million**.

68. Disadvantages to the use of the lognormal distribution to model aggregate losses:
1) There is no allowance for the loss free scenario.
Since in this example we have 3 out 10 loss free years, and even in years with claims some retailers had no claims, there is a large probability of no reinsured loss from a retailer in a year.
This argues very strongly against using the lognormal distribution here.
2) There is no easy way to reflect the impact of changing per occurrence limits on the aggregate losses.
Since in this example we have occurrence limits of either $1,000,000 or $2,000,000 in excess of the insured's retention this argues against using the lognormal distribution here.

The Panjer algorithm works well in low frequency situations, such as we have here.
In order to use the Panjer algorithm:
1) Since we have chosen to use a Poisson frequency, we would need to estimate its mean λ.
2) We would need to model severity (excess of the retention and prior to the impact of the occurrence limits) as a discrete distribution
with support at evenly spaced points, such as for example $100,000.
(We would then apply the appropriate occurrence limit.)

While **the Panjer algorithm is clearly the superior approach of the two in this example**, there will still be problems.
One could estimate the mean frequency per policy for an average independent retailer from the number of claims and the number of policies covered in each of the years. Unfortunately, with such a low frequency, there is considerable potential estimation error due to random fluctuation.
(A complication is that due to inflation, claims that were within the retention in the past will exceed the retention in the future.)
More importantly, with only 11 claims it will be difficult to model severity.
(One would want to take into account the effects of inflation.)
Perhaps some relevant data other than from this book of business can be obtained to help model severity.
Comment: Similar to 8, 11/13, Q.25.
See Section 4 of Clark on Reinsurance Pricing.

69. As computed below, the reinsurer pays 25.8 million.

Layer	Loss in Layer	Percent Paid by Reinsurer	Amount Paid by Reinsurer
Below 10 million	10,000,000	0%	0
10 to 20 million	10,000,000	70%	7,000,000
20 to 30 million	10,000,000	80%	8,000,000
30 to 50 million	12,000,000	90%	10,800,000
Above 50 million	0	0%	0
Total	42,000,000		25,800,000

Thus the primary insurer retains: 42 million - 25.8 million = **16.2 million**.
Alternately, one can compute the amount retained in each layer:

Layer	Loss in Layer	Percent Retained by Insurer	Amount Retained by Insurer
Below 10 million	10,000,000	100%	10,000,000
10 to 20 million	10,000,000	30%	3,000,000
20 to 30 million	10,000,000	20%	2,000,000
30 to 50 million	12,000,000	10%	1,200,000
Above 50 million	0	100%	0
Total	42,000,000		**16,200,000**

70. (a) For the first loss, A pays 100 and B pays 60. The insurer retains 100.
For the second loss, A pays 70, and the insurer retains 100.
For the third loss, since there is \$200 million contract period limit of \$200 million,
A pays only 200 - 170 = 30;
B pays 100 and C pays 100. Thus the insurer retains: 440 - 30 - 100 - 100 = 210.
(b) The premium for A is: (15%)(100M) = 15M. The premium for B is: (9%)(100M) = 9M.
The premium for C is: (5%)(100M) = 5M.
After the first loss, the insurer pays reinstatement premiums of (100/100)(15M) = 15M to A,
and (60/100)(9) = 5.4M to B.
After the second loss, the insurer pays no reinstatement premium to A, since due to the contract period limit there is no more coverage available to be reinstated.
After the third loss, the insurer pays no reinstatement premium to A; the contract period limit is exhausted. Reinsurer B receives a second reinstatement premium on loss event 3 as they had not fully exhausted their limit after loss event 1 and therefore only a partial reinstatement premium was paid after that loss event; the insurer pays B a reinstatement premium of:
9M - 5.4M = 3.6M = (40/100)(9M).
The insurer pays a reinstatement premium to C of: (100/100)(5M) = 5M.
Comment: Similar to 8, 11/16, Q.20.
Beyond the level of detail covered in Clark on reinstatement clauses.

71. (a) The reinsurer pays nothing if the aggregate loss in the reinsured layer is less than or equal to $1 million.
If the aggregate loss in the reinsured layer is $1.5 million then the reinsurer pays $500,000.
(30%)(500K) + (10%)(1000K) + (5%)(1500K) = **$325,000**.
(b) Without the ADD, the expected ceded losses would be:
(5%)(0) + (20%)(500K) + (30%)(1000K) + (30%)(1500K) + (10%)(2000K) + (5%)(2500K) = 1,175,000.

$$\phi_{AAD} = \frac{\text{expected amount ceded with AAD}}{\text{expected amount ceded without AAD}} = 325 / 1175 = \mathbf{0.277}.$$

Comment: Assume for example that one wants to use the Panjer algorithm. One would modify the ground up size of occurrence distribution to get the contributions to the reinsured layer. An occurrence of size 100,000 or less contributes nothing. A medium sized occurrence contributes its value minus 100,000. A large occurrence contributes 400,000 to the layer. Then one would use (a discrete version of) this distribution in the Panjer Algorithm, in order to estimate the distribution of aggregate losses in the layer.

72. The insurer cedes some fixed percent (specified in the treaty) of all premiums and losses. The total premiums are 3450. Thus the percent ceded is: 2070/3450 = 60%.
Thus the insurer retains 40% of the total losses or (40%)(2000) = **800**.

73. The primary insurer's direct loss ratio is:
(31.0%)(0.20) + (49.9%)(0.29) + (69.3%)(0.22) + (113.5%)(0.29) = 68.832%.
The losses reassumed in the loss corridor are:
(0.75) {(69.3% - 60%)(0.22) + (20%)(0.29)} = 5.8845%.
Thus the reinsurer's expected loss ratio is: 68.832% - 5.8845% = 62.9475%.
Thus the expected combined ratio for the reinsurer is: 62.9475% + 25% + 4% + 3% = **94.95%**.
Comment: Similar to 8,11/18, Q. 15.

74.

Accident Date	Trended Loss & ALAE ($000)	Amount in Layer ($000)
February 11, 2016	383	133
May 30, 2016	804	500
August 3, 2016	230	0
June 22, 2017	908	500
September 8, 2017	441	191
November 17, 2017	589	339

Combine the losses in the same accident year.

AY	Trended Losses in Layer ($000)	LDFs	Ultimate Trended Losses in Layer ($000)
2016	633	1.15	727.95
2017	1030	1.40	1442
AY	On Level Premium ($000)	Ultimate Trended Losses in Layer ($000)	Loss Ratio
2016	23,000	727.95	3.17%
2017	29,000	1442	4.97%
Total	52,000	2169.95	**4.17%**

2,169,950 / 52,000,000 = **4.17%**.

75. The ceded premium is: (30%)(200,000,000) = 60 million.
The ceding commission is: (20%)(60 million) = 12 million.
Thus net of the ceding commission, the reinsurer receives 48 million.
If the ground up losses are 110 million, then the ceded losses are (0.3)(110) = 33 million, and the reinsurer's profit is: (48 million - 33 million) / 60 million = 25%.
Thus the profit commission is paid when the ground-up losses are less than 110 million.
This is the case for scenario 1.
For scenario 1, (30%)(100M) = 30M in loss is ceded; the profit commission is:
60M - 12M - 30M - (25%)(60M) = 3M.
Scenario 4 exceeds the 160% aggregate ceded loss ratio cap.
Thus in this case, only (160%)(60 million) = 96 million of loss is ceded to the reinsurer.

Scenario	Ceded Loss	Ceded Premium - Ceding Commission	Profit Commission	Profit
1	30,000,000	48,000,000	3,000,000	**15,000,000**
2	60,000,000	48,000,000	0	**-12,000,000**
3	90,000,000	48,000,000	0	**-42,000,000**
4	96,000,000	48,000,000	0	**-48,000,000**

Comment: Similar to 8, 11/18, Q. 1c.

76. (a) Without reinsurance: 100/200 = 50%.
With reinsurance: 100/(200 - 30) = 58.9%.
(b) Without reinsurance: 300/200 = 150%.
With reinsurance: 150/(200 - 30) = 88.2%
Comment: Stop Loss Reinsurance stabilizes the insurer's annual loss ratios.
Hopefully, a very bad year such as $300 million in losses while possible is unlikely.

77.
Because penetration of high excess layers is infrequent, the calculation of ALAE is difficult.
If ALAE is included with loss for the attachment point of the clash treaty, then large ALAE may expose the clash layer. (This is sometimes called runaway ALAE.)
Clash covers are also penetrated when extra-contractual obligations (ECO) or rulings awarding damages in excess of policy limits (XPL) are determined in a settlement, both of which can have significant associated ALAE. Thus the loading for ALAE may be a significant part of the cost of the clash cover; it is an empirical question.
Comment: Similar to 8. 11/14, Q.21c.

78. (a) The expected loss ratios is:
(0.10)(25%) + (0.30)(50%) + (0.35)(70%) + (0.25)(100%) = **67.0%**.

The commission for a 50% loss ratio is: 35% + (50% - 35%) (60 - 50) / (60 - 30) = 40%.
The commission for a 70% loss ratio is: 15% + (35% - 15%) (80 - 70) / (80 - 60) = 25%.
Thus the average commission is:
(0.10)(50%) + (0.30)(40%) + (0.35)(25%) + (0.25)(15%) = **29.5%**.
(Note that the commission schedule is linear in the intervals from 30% to 60% and from 60% to 80%, and therefore, it is appropriate to use the average loss ratio in those intervals.)

(b) “A carryforward provision allows that if the past loss ratios have been above the loss ratio corresponding to the minimum commission, that the excess loss amount can be included with the current year’s loss in the estimate of the current year’s commission. In the long run, this should help smooth the results.”

(c) 80% is the loss ratio corresponding to the minimum commission. Therefore, the prior year’s loss excess of a 80% loss ratio would be included with the current year’s losses in order to compute the current year’s commission. Thus the losses entering the calculation will be bigger than average and the expected commission will be lower than the average of 29.5%.

(d) 1. Include any carryforward from past years and estimate the impact on the current year only. This amounts to shifting the slide by the amount of the carryforward.
2. Look at the “long run” of the contract. The sliding scale is modeled as applying to a longer block of years rather than just the single current year. The variance of the aggregate distribution would be reduced on the assumption that individual bad years would be smoothed by good experience on other years. The variance of the average loss ratio for a block of years should be significantly less that the variance of the loss ratio for a single year (roughly equal to dividing by the number of years in the block).

(e) 1. The problem with this approach is that it ignores the potential for carryforward beyond the current year. For example, in the first year of the program we would calculate the expected commission for the current year as though the program would be cancelled at the end of the year.
2. The first problem with this approach is that the method for reducing the variance is not obvious.
A second problem is that it ignores the fact that the contract may not renew the following year, potentially leaving the reinsurer with no carryforward benefit.
<u>Comment</u>: Similar to 8, 11/12, Q.7.

79. The reinsurance premium is: (10%)(20 million) = 2 million.
For scenarios 1 and 2 the ceded loss is zero, and the reinsurer’s profit is **2 million**.
For scenario 3 the ceded loss is 15 million, and the reinsurer’s profit is: 2 - 15 = **-13 million**.
For scenario 4 the ceded loss is 20 million, and the reinsurer’s profit is: 2 - 20 = **-18 million**.
<u>Comment</u>: Similar to 8, 11/18, Q. 1c.

80. The amount paid by the excess of loss treaty is:
Minimum(1 million, Maximum(0, X - 500,000)).

Original Loss	Paid by Reinsurer	Inflated Loss	Paid by Reinsurer
475,000	0	522,500	22,500
600,000	100,000	660,000	160,000
1,100,000	600,000	1,210,000	710,000
1,400,000	900,000	1,540,000	1,000,000
2,100,000	1,000,000	2,310,000	1,000,000
5,675,000	2,600,000	6,242,500	2,892,500

The percentage increase in the amount paid by the reinsurer:
2,892,500/2,600,000 - 1 = **11.25%**.
Comment: Most excess of loss treaties would cover a layer as here.
For a layer, the rate of inflation can either be higher or lower than the overall rate of inflation.

81. $\frac{(\$400{,}000)(1.03) + \$20{,}000}{1 - 25\% - 6\% - 20\%}$ = **$881,633**.

82. For Occurrence #1, the umbrella policy pays $500,000 in loss, so the reinsurer pays for $250,000 in loss. The reinsurance treaty covers ALAE pro-rata with respect to loss.
The reinsurer pays 250K of the total 2000K of the first loss.
Thus the reinsurer also pays ALAE of: (250/2000)($500,000) = $62,500.
For Occurrence #2, the umbrella policy pays nothing, and so does the reinsurer.
For Occurrence #3, the umbrella policy pays $700,000 in loss, so the reinsurer pays $450,000.
The reinsurer pays 450K of the total 1200K of the first loss.
Thus the reinsurer also pays ALAE of: (450/1200)($300,000) = $112,500.
Thus the reinsurer's loss and alae ratio is:
(250 + 62.5 + 450 + 112.5) / 1300 = 67.31%.
To get the technical ratio, add in the ceding commission of 18%: 67.31% + 18% = **85.31%**.
Comment: Similar to 6, 11/10, Q.26.

83. For policies with limits of 100K and 250K, no losses should pierce the reinsured layer.
For the primary policies with a 500,000 limit, the reinsurer expects to pay for the reinsured layer from 250,000 to 750,000 as a percent:

$$\frac{\text{ILF(500K) - ILF(250K)}}{\text{ILF(500K)}} = \frac{1.70 - 1.40}{1.70} = 0.1765. \quad (70\%)(5{,}000{,}000)(0.1765) = 0.618 \text{ million.}$$

For the primary policies with a 750,000 limit, the reinsurer expects to pay for the reinsured layer from 250,000 to 750,000 as a percent:

$$\frac{\text{ILF(750K) - ILF(250K)}}{\text{ILF(750K)}} = \frac{1.85 - 1.40}{1.85} = 0.2432. \quad (70\%)(4{,}000{,}000)(0.2432) = 0.681 \text{ million.}$$

For the primary policies with a 1,000,000 limit, the reinsurer expects to pay for the reinsured layer from 250,000 to 750,000 as a percent:

$$\frac{\text{ILF(750K) - ILF(250K)}}{\text{ILF(1000K)}} = \frac{1.85 - 1.40}{1.95} = 0.2308. \quad (70\%)(3{,}000{,}000)(0.2308) = 0.485 \text{ million.}$$

Limit	ILF	Manual Direct Premium	Exposure Factor	Expected Ceded Losses
100,000	1.00	8,000,000	0	0
250,000	1.40	6,000,000	0	0
500,000	1.70	5,000,000	0.1765	617,647
750,000	1.85	4,000,000	0.2432	681,081
1,000,000	1.95	3,000,000	0.2308	484,615
Total				**1,783,344**

84. Ignoring the aggregate annual deductible, the primary insurance company would retain:
250 + 100 + 250 + 250 + (1100 - 500) = $1.45M,
and the reinsurer would pay: 100 + 0 + 150 + 450 + 500 = $1.2M.
However, the annual aggregate deductible of $1,000,000 reduces the amount paid by the reinsurer by $1M. Thus the insurer retains: $1.45M + $1M = **$2.45 million**.
<u>Comment</u>: Similar to 6, 11/08, Q.28.

85. The average loss ratio is:
(0.15)(37%) + (0.35)(58%) + (0.30)(71%) + (0.20)(82%) = 63.55%.
For a loss ratio of 58%, the ceding commission is: 35% + 7%/2 = 38.5%.
For a loss ratio of 71%, the ceding commission is: 25% + 4% = 29%

Range of Loss Ratios	Average Loss Ratio in Range	Probability of Being in Range	Ceding Commission	Carry Forward
0% - 45%	37%	0.15	45%	
45% - 65%	58%	0.35	38.5%	
65% - 75%	71%	0.30	29%	
75% or above	82%	0.20	25%	7%

The average ceding commission for the first year is:
(0.15)(45%) + (0.35)(38.5%) + (0.30)(29%) + (0.20)(25%) = 33.925%
The reinsurer's expected technical ratio for the first year:
expected loss ratio + expected commission = 63.55% + 33.925% = **97.475%**.
If there is no carry-forward, then the average commission in year two is the same as that in year one: 33.925%.
On average there will be a 7% carry-forward when the loss ratio in the first year is above a loss ratio of 75%; this has a 20% chance. For the second year with a 7% carry-forward:

Range of Loss Ratios	Average Loss Ratio in Range	Probability of Being in Range	Average Loss Ratio plus 7%	Ceding Commission
0% - 45%	37%	0.15	44%	45%
45% - 65%	58%	0.35	65%	35%
65% - 75%	71%	0.30	78%	25%
75% or above	82%	0.20	89%	25%

The average ceding commission for second year with 7% carry-forward is:
(0.15)(45%) + (0.35)(35%) + (0.30)(25%) + (0.20)(25%) = 31.5%.
Combining the two cases, the average commission for the second year is:
(20%)(31.5%) + (1 - 20%)(33.925) = 33.44%.
The reinsurer's expected technical ratio for the second year: 63.55% + 33.44% = **96.99%**.
Comment: Similar to 8, 11/19, Q.17.

86. (a) I will use the totals for the five years combined:

Historical Accident Year	On Level Subject Earned Premium ($ millions)	On Level Trended Ultimate Layer Loss Cost	On Level Trended Ultimate Layer Losses
2015	35	28%	9.8
2016	37	33%	12.21
2017	40	38%	15.2
2018	42	25%	10.5
2019	45	23%	10.35
Total	199		58.06

The experience rate is: 58.06/199 = **29.18%**.
This corresponds to: ($50 million)(29.18%) = **$14.590,000**.
(b).The bottom of the layer, $250,000, corresponds to a normalized value of 250/750 = 1/3.

$$G(1/3) = \frac{1 - 0.2^{1/3}}{1 - 0.2} = 0.5190.$$

The top of the layer, $750,000, corresponds to a normalized value of 1.
G(1) = 1.
G(1) - G(1/3) = 1 - 0.5190 = 0.4810.

Historical Accident Year	On Level Subject Earned Premium ($ millions)	On Level Trended Ultimate Subject Loss Ratio	On Level Trended Ultimate Losses
2015	35	80%	28
2016	37	75%	27.75
2017	40	85%	34
2018	42	65%	27.3
2019	45	70%	31.5
Total	199		148.55

$$\text{Primary Loss Ratio} = \frac{(80\%)(35) + (75\%)(37) + (85\%)(40) + (65\%)(42) + (70\%)(45)}{35 + 37 + 40 + 42 + 45}$$

$$= 148.55 / 199 = 74.65\%.$$

The exposure rate is: (0.4810)(74.65%) = **35.91%**.
This corresponds to: ($50 million)(35.91%) = **$17,955,000**.

(c) (70%)(29.18%) + (30%)(35.91%) = **31.20%**.
(70%)($14.590,000) + (30%)($17,955,000) = **$15,600,000**.
(d) Since the ceding commission is equal to the insurer's expenses, the Quota Share treaty will result in no underwriting profit or loss for the insurer. On this ceded business, the insurer keeps no premiums, pays no losses, and its expenses are covered by the ceding commission.
The subject premium net of the Quota share is $50 million.
The corresponding insurer expenses are: (18%)(50 million) = $9 million.
The ceded loss ratio for this Excess layer is 88%.
Thus using the result of part (c), the reinsurer will charge: $15,600,000/0.88 = $17,727,000.
From previously, the insurer's expected gross loss ratio is: 148.55 / 199 = 74.65%.
The corresponding expected losses net of the Quota Share are:
(74,65%)($50 million) = $37.325 million.
Then using the result of part (c), the expected losses net of the Excess Treaty are:
$37.325 million - $15.600 million = $21.725 million.
Thus the insurer's expected net underwriting profit after application of both the existing Quota Share reinsurance treaty and the proposed Property Per Risk Excess reinsurance treaty:
$50 million - $9 million - $21.725 million - $17.727 million = **$1.548 million**.
(e) The quota share reinsurer's profit =
100% - (actual loss ratio) - (18% ceding commission) - (5% margin) = 77% - (actual loss ratio).
The profit commission is equal to this times 30% of the ceded premium, when it is positive.
Since the Quota Share is 40%, the ceded premium is in each case (0.4)/(1-0.4) = 2/3 of the net premium. However, using the net premiums instead of the ceded premiums will not affect the weighted average.

Historical Accident Year	On Level Subject Earned Premium ($ millions)	On Level Trended Ultimate Subject Loss Ratio	Profit Commission
2015	35	80%	0
2016	37	75%	0.222
2017	40	85%	
2018	42	65%	1.512
2019	45	70%	0.945
Total	199		

For example, for AY16: (77% - 75%)(37 million)(30%) = 0.222 million.
5 year weighted average ratio of profit commission to ceded premium: 2.679/199 = **1.346%**.
<u>Comment</u>: Similar to 8, 11/19, Q.18.
In part (a), one could instead use a straight average of the separate annual loss costs.

87. $\frac{(\$200{,}000)(1.06) + \$10{,}000 + \$50{,}000}{1 - 10\%}$ = **$302,222**.

88. **False**. It would be true for surplus share.

89. False. The percentage retention goes down.

90. As long as the policy limit does not exceed the maximum limits ceded to the reinsurer, the ceding company will retain at most its minimum line. However, if the number of lines in the surplus share treaty are exhausted, then the ceding company will retain more than its minimum line.
Comment: For example, assume a line of 100,000, with a maximum of 4 lines. Then for property of value 300,000, the insurer retains 100,000. However, for a property of value 600,000, the insurer cedes only 400,000 and retains 200,000.

91. False. This is a potential problem with experience rating.

92. Reinsurer's maximum limit = (5)(50,000) = 250,000.
1. percentage retained: (5/25) = 1/5. Amount recovered: (1 - 1/5)(200,000) = $160,000.
2. percentage retained: (1 - 25/40) = 37.5%. Amount recovered: (25/40)(100,000) = $62,500.

93. a. Now reimburse only on those claims paid. (50%)(200,000) = **$100,000**.
b. Ceded loss: 400K - 250K = 150K. Ceded ALAE: (150/400)(200K) = 75K.
Total amount paid by the reinsurer: 150K + 75K = **$225,000**.
c. (80%)(525K - 400K) = **$100,000**.
d. With the applicable clause in the contract, Respectable Re must reimburse as if the full $250,000 was paid to the insured. 250K - 100K = **$150,000**.
Comment: *The insolvency clause in most reinsurance treaties states that the reinsurer agrees to pay its reinsurance obligations if the primary insurer becomes insolvent whether or not the primary insurer has paid its obligations to the underlying policyholders. This clause ensures that the reinsurer pays the state insurance department liquidator the same amount as they would have paid had the primary insurer been solvent.*

94. a. Large line capacity - ability of the insurer to write large risks.
Premium capacity - amount of premium that can be written based on the company's surplus.
b. i - Quota Share - addresses premium capacity because it provides a ceding commission for primary insurers acquisition expenses. This increases capacity because acquisition expenses are reduced. It does not address large line capacity because insurer is still responsible for proportional share of losses.
ii - Excess of Loss - It addresses large line capacity because insurer can cede risk (losses in higher layers) to reinsurer allowing the insurer to write larger risks than it would without reinsurance.

95. "A clash cover will be penetrated due to multiple policies involved in a single occurrence, or when extra-contractual obligations (ECO) or rulings awarding damages in excess of policy limits (XPL) are determined in a settlement. The method for including allocated loss adjustment expenses in the treaty may also expose the clash layer." Also Workers Compensation has no specific limit, so losses above the high attachment point are possible.
Comment: See pages 22-23 of Clark.

96. a.

Claim	Incurred Loss	Trend for Cost Index	Trended Loss	Loss in Layer
96-1	472,000	1.158/1.00	546,576	46,576
97-1	1,375,000	1.158/1.05	1,516,429	1,000,000
98-1	430,000	1.158/1.092	455,989	0
98-2	575,000	1.158/1.092	609,752	109,752
99-1	900,000	1.158/1.158	900,000	400,000
				1,556,328

b.

Year	Premium	Trend for Cost Index	On Level Factor	Adjusted Premium
96	20,000,000	1.158/1.00	1.047/1.000	24,248,520
97	23,000,000	1.158/1.05	1.047/1.080	24,590,651
98	24,500,000	1.158/1.092	1.047/1.102	24,684,088
99	25,500,000	1.158/1.158	1.047/1.047	25,500,000
				99,023,259

Comment: Clark would include loss development. There is no mention of catastrophes; we would normally exclude catastrophe losses and put in a loading for catastrophes.

97. Remove any catastrophe losses from the historical experience and then use a computer model to determine a catastrophe load to use.
Comment: Clark at page 5:
"The most common procedure is now for a company to select a property catastrophe load based on an engineering-based model that incorporates the risk profile of the ceding company."

98. Experience Rating:
Because of lack of experience in higher layers there may be credibility concerns.
(This can lead to the problem of a free cover.)
You may not get all the loss information necessary to price the layer because of trending of losses. For example, if you are pricing a contract excess of 250K, then you will need losses below 250K for prior years; this information may not be available.
The mix of business of the primary insurer changes over time. Experience rating presumes that the hazards have not changed significantly between the past experience period and future policy period.
Exposure Rating:
It may be difficult to obtain appropriate exposure curve(s) to use. Different sizes and types of properties follow different exposure curves. Different perils, such as fire and wind, follow different exposure curves.
The actuary needs to get very detailed information on the book of business to be reinsured.
The actuary needs to estimate the adequacy of the primary insurer's rates.
If the properties are not insured to value, the use of exposure curves based on insurance to value may be problematic.
If deductibles are significant and differ widely between insured properties, as they can for commercial properties, this complicates exposure rating.
Comment: Clark does not discuss these issues in detail.

99. This does <u>not</u> appear to qualify for accounting treatment as reinsurance due to the fact that the reinsurer is not exposed to the risk of significant loss. The additional premium provision permits the reinsurer to collect additional premium equal to the amount by which the balances of this contract are less than zero. This provision eliminates the risk of "significant loss" to the reinsurer.

100. For those situations where it does not drop down, the umbrella policy covers the layer from 1M to 6M. $E[X \wedge 6M] - E[X \wedge 1M] = 6\ (0.9^6) - (1)(0.9^1) = 2.289$ M.
For those situations where it drops down, the umbrella policy covers the layer from 0 to 5M.
$E[X \wedge 5M] = 5\ (0.9^5) = 2.952$ M.
For those situations where the umbrella does not drop down, the reinsurance treaty covers the layer from 1.5M to 3.5M. $E[X \wedge 3.5M] - E[X \wedge 1.5M] = 3.5\ (0.9^{3.5}) - 1.5\ (0.9^{1.5}) = 1.140$ M.
For those situations where the umbrella does drop down, the reinsurance treaty covers the layer from 0.5M to 2.5M. $E[X \wedge 2.5M] - E[X \wedge 0.5M] = 2.5\ (0.9^{2.5}) - 0.5\ (0.9^{0.5}) = 1.447$ M.
The exposure factor is: $\dfrac{(20\%)(1.447\text{ M}) + (80\%)(1.140\text{ M})}{(20\%)(2.952\text{ M}) + (80\%)(2.289\text{ M})} = 0.496.$
The expected loss and ALAE for the umbrella policies are:
(1000) {(110)(65%) + (115)(65%) + (108)(70%) + (100)(75%) + (110)(70%)} = $373,850.
Expected ceded loss and ALAE is: (0.496)($373,850) = $185,430.
Reinsurance Premium: {($185,430)(1.1) + ($50,000)} / (1 - 15%) = **$298,792**.
<u>Comment</u>: Clark uses the "aggregate excess factor" to approximate the percent of time the umbrella policy will drop down (due to the exhausting of the underlying policies aggregate limit. The umbrella may also drop down in order to fill coverage gaps.)
The given formula for the limited expected value is not based on any well known size of loss distribution.

101. False. Loss Corridors are used with proportional treaties.

102. (1) The primary insurer retains (50%)(500,000) = $250,000.
(2) The lines are exhausted, so the insurer cedes: (4/10)($500K) = $200K, and retains $300,000.
(3) The insurer cedes (90%)(500K - 250K) = 225K, and retains $275,000.
The rank of the net loss to the primary insurer is: 1 < 3 < 2.

103. The reinsurer pays 30M for the catastrophe.
The reinstatement premium is: (30M / 90M) ($1,200,000) = **$400,000**.

104. D. Per Occurrence excess provides large line capacity; the insurer can write larger limits of insurance than it otherwise would be prudent to retain.
Quota share provides premium capacity; the insurer gets surplus relief.
Aggregate Excess (Stop Loss) stabilizes the loss ratio of the primary insurer; the attachment point and limit are net of inuring reinsurance in terms of either the annual aggregate losses or loss ratio.

105. The one-in-100-year event is the 99th percentile: \$80 million.
1/250 = 0.4%. The one-in-250-year event is the 99.6th percentile: \$125 million.
The layer of extra coverage has width of: 125 - 80 = \$45 million.
Using a 100-year payback approach, the additional premium is: \$45 million / 100 = **\$450,000**.

106. B. Statement A is false. Which is bigger depends on the details.
B Is true; see page 46 of Clark.
C is false; see page 26 of Clark.
"The mix of attachment points and limits is not cleanly broken out. In recent studies, the RAA has begun publishing statistics by attachment point ranges, but this data is considerably less stable than the total triangle."
D is false; see page 20 of Clark.
Free cover refers to an <u>experience</u> rating in which no losses trend into the highest portion of the layer being priced.
E is false; see page 13 of Clark.
"A loss corridor provides that the ceding company will reassume a portion of the reinsurer's liability if the loss ratio exceeds a certain amount."

107. Only the final set of policies pierce the reinsured layer.
The exposed premium is: (2 million) (1.400 - 1.260) / 1.400 = \$200,000.
Expected ceded losses: (60%)(\$200,000) = \$120,000.
Loading for ALAE and Reinsurer's Expenses and Profit, the exposure rate is:
(1.10)(1.25)(\$120,000) / \$10,000,000 = **1.65%**.
<u>Comment</u>: The final step is beyond what is shown in the part of Clark on the syllabus.

108. a. Policies attaching basis covers policies that are new or renewed during the term of the reinsurance treaty.
b. Losses occurring basis covers policies that are already in-force, new or renewed during the term of the reinsurance treaty.
c. Policies issued basis covers only policies that are newly issued during the term of the reinsurance treaty.

109. 1. An insurer's catastrophe losses are too sparse to accurately estimate future losses and premiums
2. The insurer's book of business can change so that actual losses might not be useful in accurately estimating future losses.
<u>Comment</u>: While in theory one could use several decades of experience, due to changes in the primary insures book of business, the data from a decade or more in the past is unlikely to be reflective of the primary insurer's book of business that will be covered by the treaty.
For example, expected hurricane losses depend on how many insured properties are near vulnerable coasts. For an individual insurer this can change very significantly over a decade.

110. a) Technical ratio is the sum of the loss ratio and the commission ratio.
For a 52% loss ratio, the maximum commission, 32%, is received, so technical ratio is **84%**.
For a 57% loss ratio, commission is: 32% - (1/6)(4%) = 31.33%. The technical ratio is **88.33%**.
b) For combined ratio = 100%, technical ratio must be 1 - 4% = 96%.
For example, for a loss ratio of 66%, the technical ratio is: 66% + 25% = 91%.
The technical ratio is 100% at a loss ratio of **71%**.
c) A balanced plan is one in which the expected sliding scale commission equals the sliding scale commission at the expected loss ratio.
d) A probability distribution should be associated within intervals of possible loss ratios. For each interval, the sliding scale commission is calculated. These commissions should be weighted by the probabilities to determine the expected commission ratio. This expected commission ratio can be added to the expected loss ratio and the reinsurer's operating expenses to get the reinsurer's expected combined ratio. This expected combined ratio can be used to decide whether the reinsurer finds this treaty together with the sliding scale of commissions sufficiently profitable.

111.

Policy Limit	100K / Policy Limit	200K / Policy Limit
50K	2	4
100K	1	2
200K	50%	1

Thus only the final group of policies contributes to the reinsured layer.
The expected ceded losses are:
(65%)($50,000) {(40%)(100% - 95%) + (60%)(100% - 96%)} = $1430.
Loading for ALAE, and taking into account the rate inadequacy, the expected loss and ALAE is:
(1.1)(1.05)($1430) = $1652.
Including the reinsurer’s expense load, the exposure rate is:
(1.25)($1652) / $150,0000 = **1.38%**.

112. From the excess treaty the insurer recovers from the loss to Risk #1: 0.5 million.
Thus there is $12.8 million for the cat treaty to deal with.
From the cat treaty the insurer recovers:
(90%)(4 million) + (50%)(12.8M - 5M - 4M) = 5.5 million.
The total recovery from both treaties is: 5.5 + 0.5 = **$6 million**.

113. The 9 lines are exhausted, so only 900K/2000K is ceded: (9/20)(400K) = **$180,000**.

114. (a) (250,000)(0.75) = 187,500 = written premium ceded.
(187,500)(0.30) = 56,250 = ceding commission.
187,500 - 56,250 = 131,250 premium due reinsurer.
50,000 losses. 9,000 ALAE.
(50,000 + 9000) (0.75) = 44,250 ceded and due from reinsurer.
Premium - loss = 131,250 - 44,250 = **$87,000** is due to the reinsurer.
(b) The purpose of the ceding commission is for the reinsurer to pay for the primary insurer’s expenses that were incurred on the business ceded to the reinsurer.
(The ceding commission is negotiated as a way to price the proportional reinsurance treaty.)

115. The exposed premium is:
(3M)(1.32 - 1.15)/1.32 + (2M)(1.32 - 1.15)/1.45 + (5M)(1.32 - 1.15)/1.55 = $1,169,234.
Expected ceded loss: (0.65)($1,169,234) = $760,002.
Exposure rate is: $760,002 / $15 million = **5.07%**.
Comment: The reinsurer would load this rate for ALAE and its expenses and profit.

116. Expected ceded losses: (0.080 - 0.030)(75%)($2,360,000) +
(0.125 - 0.055)(80%)($1,200,000) + (0.025 - 0.005)(70%)($800,000) +
(0.040 - 0.010)(70%)($640,000) = $180,340.
Total Standard Premium is $5 million.
Loss cost is: $180,340 / $5 million = **3.61%**.
Alternately, NCCI ELF's already include the expected loss ratio;
ELF times Standard Premium = Expected Excess Losses.
Expected ceded losses: (0.080 - 0.030)($2,360,000) + (0.125 - 0.055)($1,200,000) +
(0.025 - 0.005)($800,000) + (0.040 - 0.010)($640,000) = $237,200.
Total Standard Premium is $5 million.
Loss cost is: $237,200 / $5 million = **4.74%**.
Comment: The first solution follows Clark at page 31.
We have implicitly assumed that ALAE is shared pro-rata under the treaty.

117. Expected loss ratio without corridor:
(60%)(65%) + (25%)(82%) + (15%)(93.5%) = **73.525%**.
The effect of the loss corridor is: (70%) {(25%)(82% - 75%) + (15%)(85% - 75%)} = 2.275%.
Expected loss ratio with corridor: 73.525% - 2.275% = **71.25%**.
Alternately, for the range 70% to 85%, with the loss corridor, the average loss ratio becomes:
82% - (70%)(82% - 75%) = 77.1%.
For the range above 85%, with the loss corridor, the average loss ratio becomes:
93.5% - (70%)(85% - 75%) = 86.5%.
Reinsurer Loss Ratio After Loss Corridor is:
(65%)(0.6) + (77.1%)(0.25) + (86.5%)(0.15) = **71.25%**.

118.

Limit	100K/Limit	Factor	400K/Limit	Factor
100K	1	100.0%	4	100.0%
200K	0.5	84.6%	2	100.0%
300K	1/3	73.6%	4/3	100.0%
400K	1/4	66.3%	1	100.0%
500K	0.2	61.5%	0.8	96.1%

Exposed Premium is:
($5M) {(100% - 84.6%) + (100% - 73.6%) + (100% - 66.3%) + (96.1% - 61.5%)} = $5.505 M.
Expected ceded loss and ALAE taking into account that the rates are 10% inadequate is:
(1.1)(1.15)(70%)($5.505 M) = $4,874,678.
Loading for the reinsurer's profit and expenses, the exposure rate is:
($4,874,678 / $25 million) / (1 - 20%) = **24.37%**.
Comment: The final part of the solution is somewhat beyond what is in the portion of Clark on the syllabus; his Section 6, not on the syllabus, covers calculating the final price.

119. Advantages
1) Can retain larger portion of premium on small profitable risks.
2) Helps stabilize underwriting results.
Disadvantages
1) No benefit to insurer's surplus.
2) An increase in frequency of small losses can deteriorate the loss ratio.

120. Only 1 and 3 are true.
Comment: See pages 22 and 23 of Clark. "Clash Covers: High layer attachment excess - typically a loss on a single policy will not penetrate the treaty layer. A clash cover will be penetrated due to multiple policies involved in a single occurrence, or when extra-contractual obligations (ECO) or rulings awarding damages in excess of policy limits (XPL) are determined in a settlement.
The method for including allocated loss adjustment expenses in the treaty may also expose the clash layer."

121. 0 + 250K + 550K + 750K + 850K + 1000K = 3.4 million.
However, the aggregate limit, means that the reinsurer pays only 3 million in total.
Primary retains: 9.3 million - 3 million = **$6.3 million**.

122. In the interval from 50% to 60% loss ratio, the commission is: 80% - LR.
Thus the average commission in this interval is: 80% - 54% = 26%.
In the interval from 30% to 50% loss ratio, the commission is: 55% - LR/2.
Thus the average commission in this interval is: 55% - 41%/2 = 34.5%.
Expected Commission is: (0.45)(20%) + (0.2)(26%) + (0.3)(34.5%) + (0.05)(40%) = **26.55%.**
Comment: No use is made of the average loss ratio.
(28%)(0.05) + (41%)(0.3) + (54%)(0.2) + (78%)(0.45) = 59.6%.
Note that the commission at the average loss ratio is: 80% - 59.6% = 21.4% ≠ 26.55%.

123. Average loss ratio (before corridor):
(60.7%)(0.595) + (81.2%)(0.278) + (106.2%)(0.177) = 73.4275%.
In the 75% to 90% interval, the average effect of the corridor is: (81.2%- 75%)/2 = 3.1%.
Above 90%, the effect of the corridor is: (90%- 75%)/2 = 7.5%.
The average effect of the loss corridor is: (3.1%)(0.228) + (7.5%)(0.177) = 2.0343%.
Thus the reinsurer's expected loss ratio after the application of the loss corridor is:
73.42755 - 2.0343% = **71.3932%**.

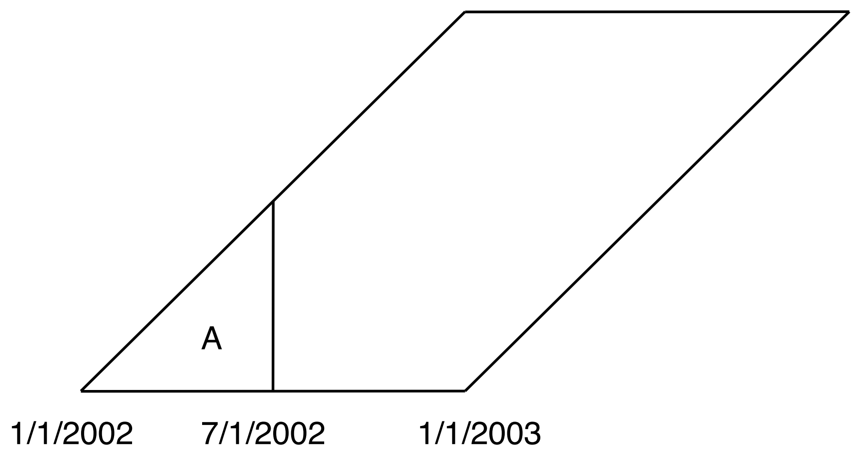
124. The Liability losses ceded: 0 + (500K)(4/10) + (1000K)(4/20) = 600K.
Property losses ceded: 3M + 1.5M + 0.5M = 5 million.
Losses retained (prior to cat treaty): 40.6M - 0.4M - 5M = 35.2M.
Thus the cat treaty pays: (90%)(30M) = 27M.
After the cat treaty, the insurer retains: 35.2M - 27M = **$8.2 million**.

125. (a) For a policies-attaching basis, only the last three occurrences are covered by this treaty.
Ceded: 400K + 0 + 20K = **$420,000**.
(b) For a losses-occurring basis, only the first three occurrences are covered by this treaty.
Ceded: 200K + 400K + 0 = **$600,000**.

126. Exposed premium is: (2222)(1.35 - 1.18)/1.35 + (9000)(1.71 - 1.18)/1.71
+ (4650)(1.80 - 1.18)/1.80 + (1075)(1.80 - 1.18)/1.88 + (1428)(1.80 - 1.18)/1.99 =
$5,470,370.
The expected ceded loss is: (55%)($5,470,370) = $3,008,703.
The subject premium totals $22,375,000.
The exposure rate is: $3,008,703 / $22,375,000 = **13.45%**.
<u>Comment</u>: The reinsurer would load this rate for ALAE.
In his final section, <u>not</u> on the syllabus, Clark shows how to calculate the final price taking into account the expenses of the reinsurer.
The reinsurer might also want to load the rate for extra-contractual obligations (ECO) and rulings awarding damages in excess of policy limits (XPL).

127. One could handle ALAE pro-rata.
Ceded Losses: 400K, 0, 100K.
Ceded ALAE: (400/600)(360K) = 240K, 0, (100/200)(400K) = 200K.
Total ceded Loss and ALAE: 400K + 100K + 240K + 200K = **940,000**.
Include ALAE with losses.
Ceded Loss plus ALAE: 400K + 100K + 400K = **900,000**.

128. Assuming that all of the $20 million is from policies covered by this treaty,
then the treaty pays 10M, 2/3 of its limit.
The treaty is on a risks attaching basis, and covers PY 2002.
As of July 1, 2002, the expected portion of the events that have occurred is the same as the portion of premium that has been earned by the primary insurer on the polices covered by the treaty. Assuming the underlying policies are annual:



Area A is a triangle with area: (1/2)(1/2)(1/2) = 1/8.
Thus 1/8 of the premium has been earned and 7/8 has yet to be earned.
Reinstatement premium is: (75%)(2/3)(7/8)($2 million) = **$0.875 million**.
Comment: At page 43 of Clark, “Less frequently, the reinstatement premium is pro-rata as to time, meaning that the premium would be further reduced to reflect only the amount of time left in the policy period. Given the seasonal nature of some types of catastrophes (e.g., hurricanes), relatively few contracts include reinstatements pro-rata as to time.”
Clark does not go into any detail with respect to pro-rata with respect to time.
If instead the treaty had been on a loss occurring basis, then the factor for pro-rata with respect to time would have been 1/2, and the reinstatement premium would have been:
(75%)(2/3)(1/2)($2 million) = $0.5 million.

129. We compare the bottom and top of the reinsured layer to each coverage limit.

Limit		100K / Limit	Factor		200K / Limit	Factor
100K		100%	84.5%		200%	100%
200K		50%	62.3%		100%	84.5%
250K		40%	56.4%		80%	76.8%

The exposed premium is:
(5M)(100% - 84.5%) + (5M)(84.5% - 62.3%) + (5M)(76.8% - 56.4%) = $2.905 million.
Thus the expected ceded loss and alae is: (1.08)(60%)($2.905 million) = $1.88244 million.
Exposure rate is: $1.88244 million / $15 million = **12.55%**.
Comment: If for example the reinsurer's profit and expense ratio were 25%, then the full exposure rate would be: 12.55% / (1 - 25%) = 16.73%.

130. Claim 5 is for a loss occurring in 2004 and thus is <u>not</u> covered by the treaty.
The treaty covers the layer of loss plus alae from 100K to 500K.
Ceded amounts for the first 4 claims are: 400,000, 295,000, 175,000, and 0.
The reinsurer's premium for the treaty is: (6%)(20 million) = $1.2 million.

Reinsurer's loss ratio for this treaty: $\frac{400{,}000 + 295{,}000 + 175{,}000 + 0}{1{,}200{,}000}$ = **72.5%**.

131. The reinsured layer is from 20% to 60% of insured value;
100/500 = 20%, 300/500 = 60%.
The corresponding exposure factors are: 62% and 89%.
The reinsurer's expected loss is: (89% - 62%)($5000) = **$1350**.

132. For ALAE included in the limit, the reinsurer pays $520,000.
For ALAE treated pro rata, the ceded loss is $400,000, and the ceded ALAE is:
(4/5)(120,000) = $96,000. The reinsurer pays $496,000.
The difference is: $520,000 - $496,000 = **$24,000**.

133. For State X, expected ceded loss: (0.040 - 0.022)(10,000)(80%) = 144.
For State Y, expected ceded loss: (0.037 - 0.019)(10,000)(70%) = 126.

Loss cost rate for the treaty is: $\frac{144 + 126}{10{,}000 + 10{,}000}$ = **1.35%**.

Alternately, the NCCI ELFs already include the expected loss ratio.

Loss cost rate for the treaty is: $\frac{(10{,}000)(0.040 - 0.022) + (10{,}000)(0.037 - 0.019)}{10{,}000 + 10{,}000}$ = **1.80%**.

<u>Comment</u>: The first solution follows what is done in Clark.

134. (90%) (275,000 + 20,000 + 1,000,000) = **$1,165,500**.

135. For the first property, ceded premium is: ($100,000) $\frac{525 - 150}{525}$ = $71,429.

For the second property, 900,000 > (4)(150,000), so the three lines are exhausted;
ceded premium is: ($120,000) $\frac{450}{900}$ = $60,000.

Percentage of the premium ceded to the reinsurer:
(71,429 + 60,000) / (100,000 + 120,000) = **59.7%**.

136. These statements are all true.

137. For each interval use the average of the limits.

Limits	Premium	200K / I.V.	Factor	500K / I.V.	Factor
$100,000	$180,000	2	1	5	1
$250,000	275,000	0.8	0.95	2	1
$400,000	250,000	0.5	0.82	1.25	1
$500,000	120,000	0.4	0.75	1	1

Reinsurer's expected losses are:
(70%)(1000) {(275)(1 - 0.95) + (250)(1 - 0.82) + (120)(1 - 0.75)} = **$62,125**.

138. (a) Reinsurer A: (0.8)(4 million) = 3.2 million.
Reinsurer B: (0.8)(5 million) = 4 million.
Reinsurer C: (0.8)(5 million) = 4 million.
The primary insurer pays the remaining:
15 - 3.2 - 4 - 4 = **3.8 million** = 1 million + (0.2)(14 million).
(b) I am assuming the excess treaty has no co-participation.
Then as far as the catastrophe treaties are concerned, it is as if the insured had a loss of $12 million.
Thus Reinsurer C pays only: (0.8)(2 million) = 1.6 million rather than 4 million.
Reinsurer C has benefited by: 4 - 1.6 = **2.4 million**.
Also since the excess treaty pays 3 million, the primary insurer pays only:
15 - 3.2 - 4 - 1.6 - 3 = 3.2 million.
The primary insurer has benefited by: 3.8 - 3.2 = **0.6 million** = (0.2)(3).
(c) Change in inuring reinsurance: limit is lowered and/or the attachment point is raised.
(d) Taking into account the 20% co-participation: $\frac{500{,}000}{(0.8)(5{,}000{,}000)}$ = **12.5%**.

<u>Comment</u>: In would be more common for an excess of loss treaty to be per occurrence rather than based on the aggregate loss as in part (b).

139. (a) Finite reinsurance refers to property catastrophe covers for which the maximum loss amount is reduced relative to traditional covers.
1. Multiple year features.
2. Loss sensitive features such as profit commissions and additional premium formulas.
(b) Loss portfolio reinsurance is for losses that have occurred but have not yet been paid.
Most finite reinsurance is for future losses on events that have not yet happened.
(c) 1. The reinsurer assumes significant insurance risk under the reinsured portions of the underlying insurance agreements.
2. It is reasonably possible that the reinsurer may realize a significant loss from the transaction.
<u>Comment</u>: Finite reinsurance performs a specific financial objective for the primary insurer.

140.

Effect on Surplus	Without Quota Share	With Quota Share
Earned Premium	$5,000,000	$1,250,000
Acquisition Expense	-2,000,000	-2,000,000
Paid Loss & ALAE	-2,000,000	-500,000
Loss & ALAE Reserve	-1,500,000	-375,000
Ceding Commission	N/A	2,250,000
Change in Surplus	-500,000	625,000

Surplus relief = $625,000 - (-$500,000) = **$1,125,000**.

141.

Limit	100K / Limit	500K / Limit
50K	200%	1000%
100K	100%	500%
250K	40%	200%
500K	20%	100%

Exposed Premium for Fire:
(1000) {(1 - 0.842)(750) + (1 - 0.556)(1200) + (0.842 - 0.411)(800)} = $996,100.
Exposed Premium for Wind:
(1000) {(1 - 0.992)(750) + (1 - 0.973)(1200) + (0.992 - 0.956)(800)} = $67,200.
Exposed Premium for All Other: (1000) {(1 - 0.994)(1200) + (1 - 0.990)(800)} = $15,200.
Part of Reinsurer Premium for Fire: (0.6)(60%)(996,100)(1.05) / (1 - 0.15) = $442,972.
Part of Reinsurer Premium for Wind: (0.3)(60%)(67,200)(1.05) / (1 - 0.15) = $14,942.
Part of Reinsurer Premium for All Other: (0.1)(60%)(15,200)(1.05) / (1 - 0.15) = $1127.
Indicated reinsurer's premium: $442,972 + $14,942 + $1127 = $459,041.
The indicated exposure rate is: $459,041 / $3,250,000 = **0.141**.

142. (a) Exposure Rate = (Ultimate Trended Losses) / (On-Level Premium) =

$$\frac{(25)(1.1)(1.3) + (35)(1.2)(1.15) + (30)(1.3)(1.07) + (15)(1.5)(1)}{(100)(1.2) + (120)(1.1) + (130)(1.05) + (140)(1)} = \frac{148.280}{528.500} = \mathbf{0.2806}.$$

(b) i. A premium still needs to be charged because there is a chance of loss to the layer. Just because a rare event has not happened in the last few years does not mean it will not happen in the future. In any case, reinsurers will not provide a free cover.
ii. I would exposure rate the 500k xs 500k layer and the 10M xs 10M layer, calculate the relativity between the two layers, and multiply the experience rate for the 500K xs 500K layer by this relativity.
Alternately, I would exposure rate the 10M xs 10M layer.

143. a. (5m)(90%) + (10m)(90%) + (19m - 2m - 5m - 10m)(85%) = **$15.2 million**

Retention	Layer 2M	Part of 19M 2M	Co-participation	Ceded
1st	2M – 7M	5M	0.1	4.5M = (0.9)(5m)
2nd	7M – 17M	10M	0.1	9.0M
3rd	17M - 27M	2M	0.15	1.7M
Total				15.2M

b. Assume the original premium already accounts for co-participation.

Layer	Amount Exhausted	Total Available
1st	5M	5M
2nd	10M	10M
3rd	2M	10M
Total	17M	25M

% Exhausted = 17M / 25M = 68%.
Total Reinstatement Premium = (68%) ($8M) = **$5.44 million**.
Comment: In part (b) one would have to read the reinstatement clause of the treaty to know exactly how it works. The maximum possible payment is:
(0.9)(5m) + (0.9)(10M) + (0.85)(10M) = 22M.
The percent of possible paid is: 15.2M / 22M = 69.1%, somewhat different than the 68%.

144. 1. U/W results – treaty excess of loss reinsurance: will stabilize loss ratios because higher layers of loss will be ceded to the reinsurer;
advantage: only retain losses up to a certain retention;
disadvantage: it will not help if an increase in the frequency of smaller retained losses occur.
Note also catastrophe excess of loss treaties would limit catastrophe losses.
2. Rapid growth – pro rata reinsurance: quota share provides surplus relief which will lower the premium to surplus ratio and allow the company to continue growing;
advantage: ceding commission reimburses primary company for their policy acquisition costs and provides surplus relief;
disadvantage: the primary insurer will cede premium on profitable business, not just unprofitable business.
Note that surplus share would instead allow the primary insurer to keep smaller risks, and cede away a portion of larger risks.
3. Higher policy limits – excess of loss reinsurance: provides large line capacity so the primary company can write higher policy limits than it could on its own;
advantage: can write higher policy limits without retaining higher losses;
disadvantage: can get expensive, and does not provide relief for an increase in frequency.
4. Increase in ALAE – a reinsurance program that covers ALAE pro-rata in addition to limits rather than including ALAE within the limits;
advantage: lowers ceding company's ALAE costs;
disadvantage: will pay some ALAE on all claims above the retention, instead of just those where ALAE exceeds reinsured limits.
Alternately, the insurer could buy a clash cover.
advantage: clash cover should provide reinsurance for claims with excessive ALAE.
disadvantage: The clash cover would have a very high attachment point.

145. Claims 1 and 2 are on a policy written in 2004 and thus not covered by the treaty; zero is ceded on this treaty due to claims 1 and 2.

3. Ceded Loss: (90%)(250K) = 225K.	Ceded ALAE: (225/375)(30K) = 18K.
4. Ceded Loss: 0.	Ceded ALAE: 0.
5. Ceded Loss: (90%)(125K) = 112.5K.	Ceded ALAE: (112.5/225)(15K) = 7.5K.
6. Ceded Loss: (90%)(75K) = 67.5K.	Ceded ALAE: (67.5/175)(70K) = 27K.

146. a. The trended losses must be capped at applicable policy limits. Theoretically, we want to cap losses at the limit applicable if the same policy were written in the future treaty period, when the policy limits are likely to be higher than in the historical period.
b. 1. Apply the historical policy limit to each trended loss.
2. Apply the trend factor to the historical loss without applying a policy limit cap.
c. 1. This leaves out the fact that the insured will generally increase its policy limits over time.
2. This assumes that policy limits "drift" upwards to precisely match inflation.
If this second approach is used, then the subject premium must also be adjusted to the level that would have been charged had the higher limits been in effect; otherwise an overstatement of the expected loss cost will result.
<u>Comment</u>: See page 24 of Clark.
"This represents another problem for which there is no generally accepted solution."

147. (a) Although it is not stated, assume the surplus share treaty has enough lines.
Then for the first type of policies the insurer retains 1/20, for the second type 1/60, and for the third type 1/120.
Using the subject premium for each type, the retained premium is:
($100,000)(5/20 + 8/60 + 9/120) = $45,833
Since the insurer retains $1 million in each case, $250,000 the bottom of the excess layer corresponds to 25% of insured value, while $1 million the top of the excess layer corresponds to 100% of insured value. The exposure factors are 47% (interpolated) and 92%.
Thus using the subject premium for each type, the expected losses under the excess treaty are:
(92% - 47%)(60%)($100,000)(5/20 + 8/60 + 9/120) = $12,375.
The exposure-rated loss cost for the per-risk excess treat is: $12,375/$45,833 = **27.00%**.
Alternately, (60%)(92% - 47%) = **27%**.
(b) You apply the surplus share treaty to each loss and experience rate the remaining retained loss.
retained loss amount = (original loss amount) (% retained after the surplus share treaty).
(c) 1. When deductibles increase, primary rates decrease and since the expected excess losses don't change, the exposure rate has to increase to offset the primary price decrease.
2. As deductible levels increase, it is difficult to determine the appropriate exposure factor because ground-up losses below the deducible that have not pierced the deducible are unknown.

148. (a) The primary insurer cedes to the reinsurer premiums: (75%)($1 million) = $750,000, less a ceding commission of: (30%)($750,000) = $225,000.
The reinsurer pays loss & ALAE of: (75%)($345,000) = $258,750.
The net amount owed by the primary company to the reinsurer under the treaty is:
$750,000 - $225,000 - $258,750 = **$266,250**.
(b) The ceding commission reflects the fact that the primary insurer has higher underwriting expenses than the reinsurer.
Alternately, the ceding commission is used to compensate the primary insurer for their policy acquisition costs as well as determine the final price of reinsurance.
<u>Comment</u>: The loss experience was good. If instead it had been bad, then in part (a) the reinsurer would have owed a net amount to the primary insurer.

149. (a) 1. The primary insurer must first have notice of the loss and then tell the reinsurer about it. So it has to go through the primary insurer first, which takes time.
2. A primary insurer may not have realized that a given loss would pierce the reinsured layer, but later it does, resulting in longer report lags.
3. Retrocessional agreements between reinsurers result in longer reporting lags.
(b) 1. The reporting lag from the occurrence of an event to the establishment of a reinsurer's case reserve may vary by company. Included in the data is retrocessional business which may include several levels of reporting lag.
2. The mix of attachment points and limits is not cleanly broken out. In recent studies, the RAA has begun publishing statistics by attachment point ranges, but this data is considerably less stable than the total triangle. Loss development varies significantly for different attachment points so every effort should be made to adjust the selected factors to the layer of the treaty being priced.
3. The RAA requests data exclusive of Asbestos and Environmental claims which could distort the patterns. It cannot be known if all member companies have done this consistently. Other long term exposure claims, such as medical products, mold, or tobacco, are not excluded.
4. For workers compensation, the members may not handle the tabular discount on large claims in a consistent manner. If a ceding company reports a loss on a discounted basis, and the reinsurer establishes a case reserve as the amount of the discounted value that falls into the reinsured layer, a very high development factor may result due to the unwinding of the discount.
<u>Comment</u>: The difference in report lags for primary insurers and reinsurers is very significant for casualty lines, as shown in the graphs at page 52 of Clark.
For part (b), see page 26 of Clark.

150. Applying the excess of loss treaty, the reinsurer pays:
(1 - 40%)($35 million) = $21 million.
Thus the primary insurer retains: 75 - 21 = $54 million.
Then the catastrophe treaty pays: (1 - 40%)(54 - 40) = $8.4 million.
Thus the primary insurer retains: 54 - 8.4 = **$45.6 million**.

151. With ALAE pro-rata as to loss, the reinsurer pays:
775,000 - 500,000 = 275,000 for losses.
The insurer pays: (275/775) (400,000) = 141,935 for ALAE.
The insurer pays in total: 275,000 + 141,935 = $416,935.
With ALAE included with loss, the reinsurer would pay: 775,000 + 400,000 - 500,000 = 675,000; however, this is capped at the treaty limit of $600,000.
Thus if the treaty had been written to treat ALAE included with loss, it would have made a difference of $600,000 - $416,935 = **$183,065 in favor of the primary insurer**.

152. The insurer retains: 100 + (15%)(100) + (10%)(100) + (5%)(450 - 300) = **$132,500,000**.

153. a) The primary insurer is not accustomed to writing large home values or business on the coast, so it has the risk of underpricing its business.
The reinsurer priced its excess of loss treaty according to the current policy limit profile and geography of exposures. Changes to these could result in unanticipated loss and loss expenses.
Alternately, the primary insurer should be concerned about catastrophes with liberalization of the former coastal restrictions on underwriting. There could be increased total losses for a large number of properties at once. This will hurt with frequency of claims but also have less profit sharing from reinsurer.
Reinsurer should be concerned about increase in mix and value of homes. There now could be more losses in the excess portion and the insurer premiums to reinsurer may not be adequate.
b) The insurer may look to reinsurer for guidance on pricing this unfamiliar type of business. Moreover, the insurer could seek out facultative reinsurance for the extremely different risks.
Reinsurer may wish to renegotiate terms of the treaty, at the very least price according to liberalized guidelines.
Alternately, the reinsurer may want to get retrocessional reinsurance and cede layers above theirs to other reinsurance companies. The primary insurer could get catastrophe coverage to protect itself where there is a coverage gap.

154. a) A hurricane could result in many of the policies under the treaty experiencing losses thereby causing big losses to the insurer because the per-occurrence limit could be penetrated making the insurer retain more than 40%, and the maximum per-risk limit can easily be penetrated since the insurer offers exposure up to $25 million.
Alternately, the per-risk limit of the treaty is $5M, but the range of insured values is up to $25M. This means that individual losses between $5M and $25M will not be reinsured.
The per-occurrence limit is $20M, which could be reached by accumulation of small or large losses that result from a hurricane.
b) The insurer needs to purchase catastrophe reinsurance attaching at $20 million, and excess of loss per-risk reinsurance attaching at $5 million, both with appropriate coinsurance provisions.
Alternately, the insurer can buy excess of loss reinsurance for the layer between $5M and $25M, plus catastrophe reinsurance to indemnify the insurer against losses from a hurricane.
(The catastrophe reinsurance would be cheaper if the quota share agreement inures to its benefit.)
c) Surplus Share reinsurance: still provides a ceding commission and the insurer sets a retained line where it would wholly retain policies with limits below that line.
Variable Quota Share: this functions like surplus share. The insurer would seek reinsurance that enables it to retain low-limit policies. Percent of retention can vary depending on the insured value and a lower percent is used for higher valued policies. It is likely that this would still provide a ceding commission.

155. a) Excess of loss: Dollar threshold above which the reinsurer has liability to the primary insurer (per claim or per occurrence.)
Pro-rata: Reinsurer pays on each claim from the first dollar.
Alternately, the primary insurer retains all of the losses below the attachment point point. There is no such concept in pro-rata as all losses, regardless of ground-up size, are ceded/retained using the pro-rata percent.
b) Excess of loss: Percent of subject premium, charged for excess of loss cover.
Pro-rata: Reinsurer pays a given percentage of each claim and receives the same percentage of premium. Ceding commission for a pro-rata treaty can play the pricing role assumed by the rate in a excess of loss treaty.
c) Excess of loss: Premium charged by the primary insurer for the policies underlying the treaty.
Pro-rata: subject premium is used in the context as well, but gross losses are directly shared in the same proportion as premium
d) Excess of loss: Percent of loss in reinsured layer that the primary insurer must retain.
Pro-rata: the pro-rata percentage is the complement of the percent retained. Pro-rata is in essence a co-participation provision on the whole book from first dollar.

156. a. Pro-rata reinsurance such as quota share reinsurance or surplus share reinsurance would address the premium to surplus ratio issue because they provide surplus relief in the form of a ceding commission.
b.. Per risk excess of loss reinsurance could address the issue of writing high valued homes since it provides large line capacity; the primary insurer would cede off the higher layers to the reinsurer.
Aggregate excess of loss (Stop Loss) reinsurance provides loss ratio stability because once retained losses reach a certain level, the reinsurer begins offering protection.
Comment: A reinsurer's expertise could to some extent supplement the underwriting expertise of the primary insurer.

157.

Limit	ILF	Expected Portion of Losses Ceded
$1 Million	1.00	0
2 Million	1.25	(1.25 - 1.00) / 1.25
3 Million	1.45	(1.45 - 1.00) / 1.45
4 Million	1.60	(1.60 - 1.00) / 1.60
5 Million	1.70	(1.60 - 1.00) / 1.70

Expected ceded losses are:
(1 million) {(0.25/1.25)(8) + (0.45/1.45)(12) + (0.60/1.60)(10) + (0.60/1.70)(4)} = $10,485,903.
Loading to get a 60% loss ratio net of brokerage commission: $10,485,903 / 0.6 = $17,476,505.
Loading for the brokerage commission, the reinsurance premium is:
$17,476,505 / (1 - 0.1) = **$19,418,339**.

158. The reinsurer is recovering the layer from 600K to 1100K.
In each case we get an entry ratio by dividing an endpoint of the layer by the coverage A amount.

Cov. A	Bottom	Expos. Factor	Top	Expos. Factor	Net Expos. Factor
100,000	6	100%	11	100%	0
200,000	3	100%	5.5	100%	0
500,000	1.2	90%	2.2	100%	100% - 90% = 10%
1,000,000	0.6	45%	1.1	87.5%	42.5%

Fraction of the primary insurer's total expected losses covered by this treaty:

$$\frac{(0)(10) + (0)(15) + (10\%)(20) + (42.5\%)(30)}{10 + 15 + 20 + 30} = 14.75/75 = \mathbf{19.67\%}.$$

Comment: See pages 18 to 19 of Reinsurance Pricing by Clark.

159. (a) (23,875 - 22,084) / (1 - 15% - 7.5%) = **$2311**.
(b) The expected loss for the primary insurer is $24,430.
The appropriate premium would have been: $24,430 / (1 - 24% - 4%) = $33,931.
However, the primary insurer's rates are 5% deficient, so it charged: ($33,931)(0.95) = **$32,234**.
(c) Ceding percentage is: (23,875 - 22,084) / 24,430 = 7.331%.
(7.331%)(32,234) = $2363.
This premium would be adequate; it is more than the 2311 from part (a).
On the one hand, the primary insurer's rates are inadequate.
On the other hand, the insurer's expense loading is much more than that of the reinsurer.
On balance, the $2363 is more than enough for the reinsurer to get for this cover.
Comment: The method used in part (a) is called the first loss scale approach.

160. (a) The insurer retains 35% of each loss:
(35%)(10,000 + 5500 + 7000 + 600) = **$8085**.
(b)

Amount of Insurance	Loss	Percent Ceded	Retained Loss
$200,000	80/200	$10,000	(0.6)(10,000) = 6000
50,000	30/50	5,500	(0.4)(5500) = 2200
20,000	0	7,000	7000
10,000	0	600	600

Retained losses: 6000 + 2200 + 7000 + 600 = **$15,800.**

161. 120,000 / 400,000 = 30%; exposure factor is 57%.
320,000 / 400,000 = 80%; exposure factor is 91%. (91% - 57%)($88,000) = **$29,920**.
Comment: See pages 18 to 19 of Clark.

162. (a) Profit as percent of ceded premium: 100% - 10% - 20% - 250,000/1,000,000 = 45%.
Profit: (45%)(75%)($1 million) = **$337,500**.
(b) Reinsurer's expected profit as a percent of ceded premium: 100% - 10% - 20% - 60% = 10%.
Reinsurer's expected profit is: (10%)(75%)($1 million) = $75,000.
Profit commission is: (30%)($337,500 - $75,000) = **$78,750**.

7.163. (a) i. Losses ceded are: (85%)($5 million) = $4.25 million.
Losses retained are: 7 - 4.25 = $2.75 million.
ALAE retained is: (2.75/7)($1.5 million) = $589,286.
Total loss & ALAE retained is: $2.75 million + $589,286 = **$3,339,286**.
ii. Losses and ALAE ceded are: (85%)($5 million) = $4.25 million.
Losses & ALAE retained are: 8.5 - 4.25 = **$4.25 million**.
(b) i. Loss & ALAE ceded is: 8.5 million - 3,339,286 = **$5,160,714**.
ii. Loss & ALAE ceded is: 8.5 million - 4.25 million = **$4.25 million**.
(c) It depends on the type of business, type of loss, ALAE patterns, etc., to determine which way of handling ALAE (pro-rata versus included) is best.
Assuming the reinsurance is priced properly, neither party should be particularly favored.

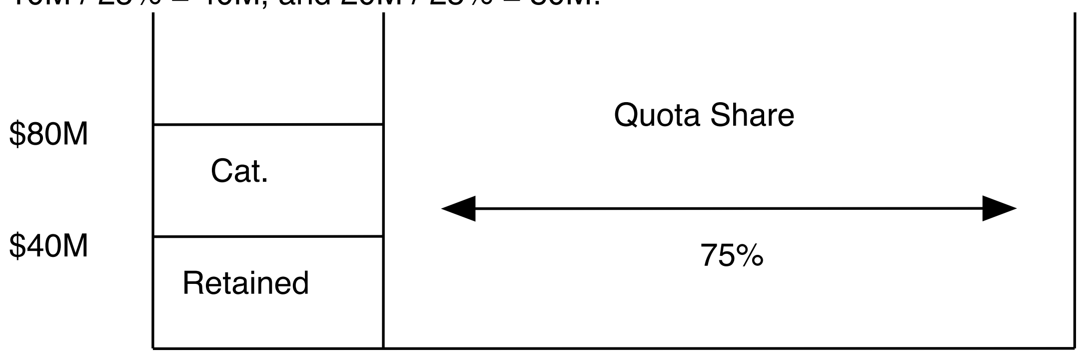
164. 1. The reporting lags are a mix of many ceding companies, and also retrocessional lags are included which are usually longer.
2. The data combines various policy limit and attachment points.
The mix of attachment points and limits are not cleanly broken out.
3. Inconsistencies with the reporting of asbestos and environmental claims among companies; they are supposed to be excluded but RAA can not confirm that this is done consistently.
4. Workers compensation tabular discount may be applied or not applied; it is not consistent across companies.
Comment: See page 26 of Clark.
"1. The reporting lag from the occurrence of an event to the establishment of a reinsurer's case reserve may vary by company.
Included in the data is retrocessional business which may include several levels of reporting lag.
2. The mix of attachment points and limits is not cleanly broken out.
In recent studies, the RAA has begun publishing statistics by attachment point ranges, but this data is considerably less stable that the total triangle.
Loss development varies significantly for different attachment points so every effort should be made to adjust the selected factors to the layer of the treaty being priced.
3. The RAA requests data exclusive of Asbestos and Environmental claims which could distort the patterns. It cannot be known if all member companies have done this consistently. Other long term
exposure claims, such as products claims for silicone breast implants, are not excluded.
4. For workers compensation, the members may not handle the tabular discount on large claims in a consistent manner. If a ceding company reports a loss on a discounted basis, and the reinsurer establishes a case reserve as the amount of the discounted value that falls into the reinsured layer, a very high development factor may result."

165. (a) Retained premium: (1 - 60%) (12,000 + 3000 + 30,000) = **$18,000.**
Retained losses: (1 - 60%) (150,000 + 50,000 + 300,000) = **$200,000**.
(b)

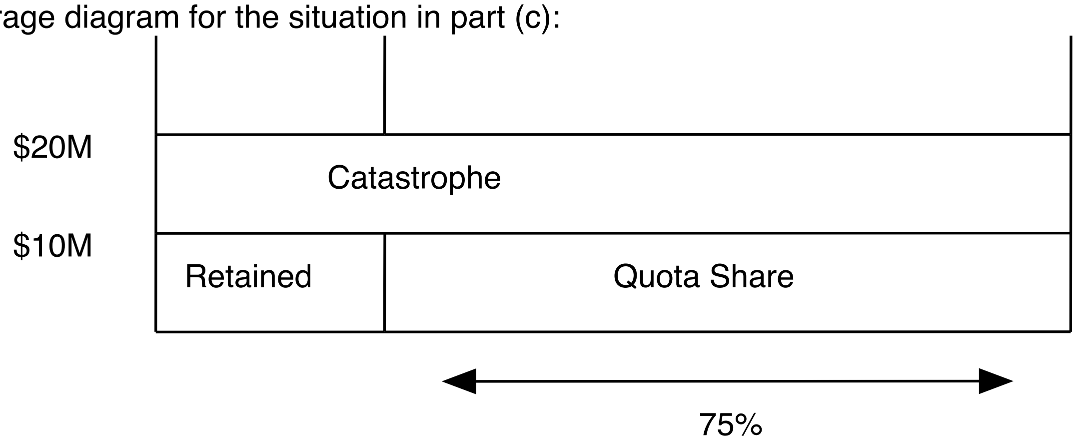
Policy	Policy Limit	Portion Ceded	Portion Retained
1	$200,000	100/200	1/2
2	50,000	0	1
3	500,000	400/500	1/5

Retained premium: (1/2)(12,000) + 3000 + (1/5)(30,000) = **$15,000**.
Retained losses: (1/2)(150,000) + 50,000 + (1/5)(300,000) = **$185,000**.
(c) Surplus share allows the insurer to retain more of their business for the smaller limit policies where they have enough capacity, while ceding a greater portion for the higher limit policies where they need the help with capacity provided by reinsurance.
(d) Due to the different ceding percentages for each policy, surplus share requires more complicated and expensive accounting procedures than the simpler quota share treaty.

166. (a) With respect to a given reinsurance treaty, other reinsurances which are first applied to reduce the loss subject to the given treaty are said to inure to the benefit of the reinsurer of that given treaty.
(b) The quota share treaty pays: (75%)($15 million) = $11.25 million.
Remaining is: (25%)($15 million) = $3.75 million.
Applying the catastrophe treaty to this $3.75 million, the catastrophe treaty pays nothing.
Thus the primary insurer retains $3.75 million.
i) Primary Insurer = $3.75M
ii) Quota Share = $11.25M
iii) Catastrophe = 0
(c) The catastrophe treaty pays: $15 million - $10 million = $5 million.
$10 million remains.
Applying the quota share treaty to this remaining $10 million, the quota share treaty pays:
(75%)($10 million) = $7.5 million.
Thus the primary insurer retains: $10 million - $7.5 million = $2.5 million.
i) Primary Insurer = $2.5M
ii) Quota Share = $7.5M
iii) Catastrophe = $5M
<u>Comment</u>: Coverage diagram for the situation in part (b),
where 10M / 25% = 40M, and 20M / 25% = 80M:



Coverage diagram for the situation in part (c):



167. 1. Excess of loss treaty of 200K excess of 300K with a co-participation provision of 50%.
2. Excess of loss treaty of 100K excess of 400K, no co-participation provision.
3. Variable quota share, with the insurer ceding 0% for the 300K risks
and ceding 20% on the 500K policies.
Comment: A surplus share treaty with a line of $300,000, would allow the insurer to retain all of a $300,000 limit policy and 3/5 of a $500,000 limit policy.

168. For a loss ratio of 56%, the commission is: 25% + (0.5)(4%) = 27%.
For a loss ratio of 64%, the commission is: 25% - (1.5)(4%) = 19%.
Expected Commission is: (0.2)(30%) + (0.4)(27%) + (0.3)(19%) + (0.1)(10%) = 23.5%.
Expected Loss Ratio is: (0.2)(40%) + (0.4)(56%) + (0.3)(64%) + (0.1)(73%) = 56.9%.
Expected technical ratio is: 23.5% + 56.9% = **80.4%**.
Comment: Technical Ratio = Loss Ratio + Ceding Commission.

169. Combine the losses in the same accident year.

AY	Trended Losses in Layer	LDFs	Ultimate Trended Losses in Layer
2003	162,719 + 0 = 162,719	1.000	162,719
2004	64,610 + 500,000 = 564,610	1.050	(1.05)(564,610) = 592,841
2005	474,679 + 178,321 = 653,000	1.100	(1.1)(653,000) = 718,300

AY	On Level Trended Premium	Ultimate Trended Losses in Layer	Loss Cost
2003	$1,600,000	162,719	10.2%
2004	1,632,000	592,841	36.3%
2005	1,664,640	718,300	43.2%
Total	4,896,640	1,473,860	30.1%

1,473,860 / 4,896,640 = 30.1%.
Alternately, one could average the loss costs for the three years:
(10.2% + 36.3% + 43.2%)/3 = 29.9%.
Comment: See pages 16 and 17 of Clark.

170.

Limit	Prem.	100k/Limit	Table Val.	500k/Limit	Table Val.	Exp. Factor	Exp. Prem.
50k	500k	200%	1	1000%	1	1-1 = 0	0
100k	800k	100%	1	500%	1	1-1 = 0	0
200k	1.5m	50%	0.73	250%	1	1-0.73 = 0.27	405,000
500k	1m	20%	0.48	100%	1	1-0.48 = 0.52	520,000
	3.8m						925,000

Exposure rate is: $(1.12)\ \dfrac{(0.70)(925{,}000)}{3{,}800{,}000}\ /\ (1 - 0.05) =$ **20.1%**.

Comment: The insurer’s rates are 95% of what they should be; in other words, the expected losses are 70% of 925,000 / 95%. Therefore, I divided by 95% to increase the exposure rate to reflect expected losses.

171. Ignoring the aggregate annual deductible, the primary insurance company would retain:
150 + 200 + 200 + 200 + (1250-800) + 200 = \$1.4M,
and the reinsurer would pay: 0 + 50 + 500 + 300 + 800 + 50 = \$1.7M.
However, the annual aggregate deductible of \$1,000,000 reduces the amount paid by the reinsurer by \$1M. Thus the insurer retains: \$1.4M + \$1M = **\$2.4 million**.
Comment: The reinsurer pays: \$1.7m - \$1m = \$700,000.
An annual aggregate deductible (AAD) is discussed at pages 33 and 34 of Clark.
The AAD allows the ceding company to retain the first losses in the reinsured layer, but maintain protection in case there are more losses than anticipated.
So for example, if in this case only the first four occurrences had taken place, then due to the AAD the insurer would have retained all of the loss and the reinsurer would have paid nothing.

172. (a)

Occurrence	Incurred Loss + ALAE	Ceded
1	\$300,000	\$200,000
2	200,000	100,000
3	1,200,000	900,000
4	700,000	600,000
5	1,800,000	900,000
Total		**\$2,700,000**

(b) For example, for occurrence number four: 400,000 in losses is ceded,
and (4/5)(200) = \$160,000 in ALAE is also ceded.

Occurrence	Incurred Loss	Ceded Loss	Incurred ALAE	Ceded ALAE
1	\$200,000	\$100,000	\$100,000	\$50,000
2	50,000	0	150,000	0
3	1,000,000	900,000	200,000	180,000
4	500,000	400,000	200,000	160,000
5	1,500,000	900,000	300,000	180,000
Total		**\$2,300,000**		**\$570,000**

Total amount ceded: \$2,300,000 + \$570,000 = **\$2,870,000**.

173. (a) Premiums: (1000)(1.250)(1.159) + (1100)(1.200)(1.126) + (1210)(1.120)(1.093) + (1331)(1.075)(1.061) + (1464)(1.015)(1.030) = 7465.
Losses: (555)(1.010)(1.338) + (1638 - 1000)(1.025)(1.262) + (614)(1.065)(1.191) + (552)(1.250)(1.124) + (481)(1.750)(1.060) = 4022.
Loss Ratio is: 4022 / 7465 = **53.88%**.
(b) (1.15)(53.88%) = **62.0%**.
(c) 62.0% + 27.5% + 1.5% + 6.0% + 1.5% = 98.5%.
Since 98.5% is more than 95%, the reinsurer should reject the 27.5% ceding commission.
Comment: See pages 6 to 9 of Clark.
Part (a) is just like doing part of an overall rate indication for the primary insurer.

174. (a) The co-participation provision in excess of loss treaty requires the insurer to retain 5% of the reinsured layer of loss.

Occurrence	Ground-up loss	Ceded Quota Share	Ceded Excess
1	$500,000	$50,000	(95%)($400,000)
2	50,000	$5000	0
3	850,000	$85,000	(95%)($750,000)
4	1,150,000	$100,000	(95%)($900,000)
Total		$240,000	$1,947,500

Total Retained loss are: $2,550,000 - $240,000 - $1,947,500 = **$362,500**.
(b) For example, for occurrence three, the excess-of-loss treaty pays: (95%)(765,000 - 100,000).

Occurrence	Ground-up loss	Ceded Quota Share	Ceded Excess
1	$500,000	$50,000	(95%)($350,000)
2	50,000	$5000	0
3	850,000	$85,000	(95%)($665,000)
4	1,150,000	$100,000	(95%)($900,000)
Total		$240,000	$1,819.250

Total Retained loss are: $2,550,000 - $240,000 - 1,819,250 = **$490,750**.
(c) A co-participation provision is used to provide an incentive for the primary insurer to efficiency manage losses that exceed the attachment point. It requires the primary insurer to retain a percentage above the attachment point. For example, an excess of loss treaty 75% 400K xs 100K would require the primary insured to retain 25% of the losses in the reinsured layer.

175.

Policy Number	Policy Limit	Incurred Loss	Ceded
1	$200,000	$150,000	0
2	$1,000,000	$500,000	(4/5)($500,000) = $400,000
3	$500,000	$50,000	(3/5)($50,000) = $30,000
4	$450,000	$225,000	(25/45)($225,000) = $125,000
Total		$925,000	$555,000

Retained losses: $925,000 - $555,000 = **$370,000**.

176. Commission at a 44% loss ratio is: 50% - 24%/3 = 42%.
Commission at a 57.5% loss ratio is: 40% - 7.5%/1 = 32.5%.
Average commission is: (50%)(0.15) + (42%)(0.20) + (32.5%)(0.25) + (27.5%)(0.4) = **35.025%.**
<u>Comment</u>: See page 10 and 11 of Clark.
We make no use of the given provisional commission. 32.5% ceding commission would be paid by the reinsurer as a percent of the ceded premiums; however this would be adjusted once the experience on the proportional treaty is known, as per the schedule of sliding scale commissions.
If the insurer's experience was good, then the reinsurer would owe additional ceding commissions, while if the experience was bad, then the insurer would have to return some of the ceding commissions. This is conceptually similar to a retrospective rating plan.

177. Assume the treaty is on a losses occurring basis, so that the trend to date is July 1, 2008.

Date of Loss	Trended Loss	Losses in Layer, 500K xs 100K
January 1, 2000	$(200,000)(1.04^{8.5}) = 279,134$	179,134
January 1, 2001	$(200,000)(1.04^{7.5}) = 268.398$	168,398
July 1, 2001	$(225,000)(1.04^{7}) = 296,085$	196,085
July 1, 2002	$(600,000)(1.04^{6}) = 759,191$	500,000
January 1, 2004	$(475,000)(1.04^{4.5}) = 566,688$	466,668
July 1, 2005	$(90,000)(1.04^{3}) = 101,238$	1238

Applying loss development factors applicable to the treaty layer, but limiting the losses in the layer to a maximum of $500,000, the width of the reinsured layer:

Losses in Layer	LDF	Product	Limited to 500,000
179,134	1	179,134	179,134
168,398	1	168,398	168,398
196,085	1	196,085	196,085
500,000	1.05	525,000	500,000
466,668	1.30	606,668	500,000
1238	1.50	1857	1857
			$1,545,474

Comment: See pages 16 and 17 of Clark. However, in Clark the LDFs are applied to the trended losses in a layer for an entire Accident Year rather than to individual losses.
If the treaty were on a risk attaching basis, then it would cover insurance policies written during 2008; if these were annual policies, the trend to date would be January 1, 2009.

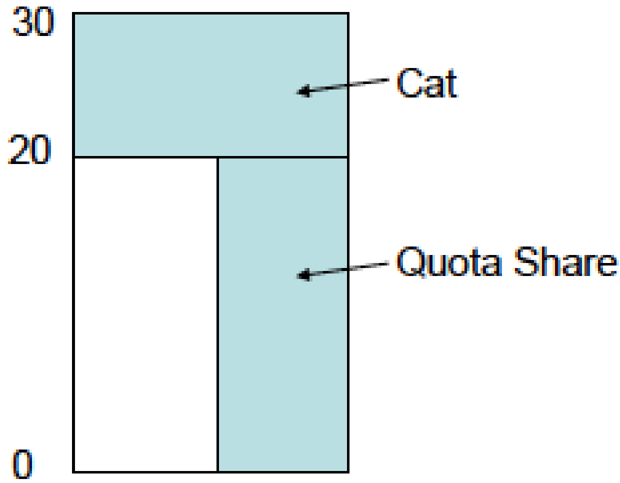
178. For each risk, we combine dwelling and contents, and the reinsurer pays for the layer from $250,000 to $850,000

Risk	Dwelling + Contents	Ceded
1	$630,000	$380,000
2	600,000	$350,000
3	735,000	$485,000
4	275,000	$25,000
5	160,000	0
6	1,000,000	$600,000

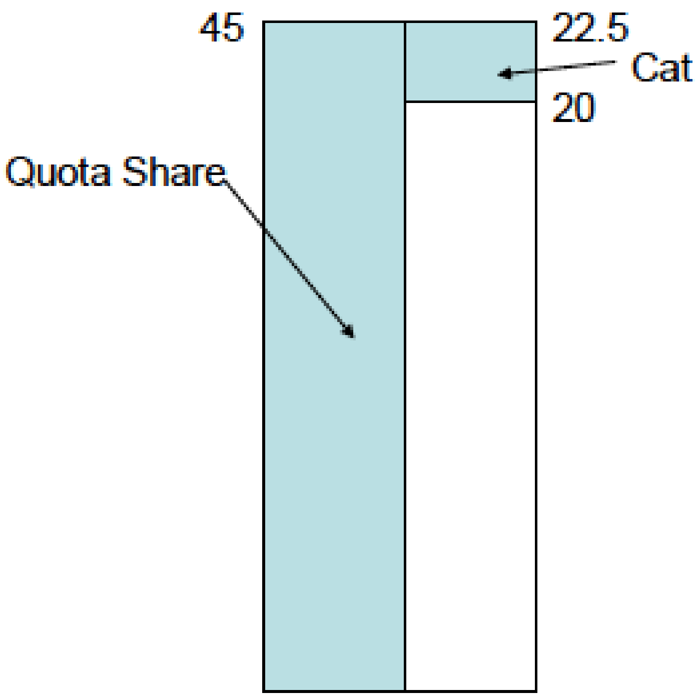
However, the first three losses are due to the same occurrence.
Therefore, the reinsurer pays for these three losses only $1 million rather than $1,215,000.
(The final two losses are also from a single occurrence, but this turns out to have no effect.)
Thus the total ceded is: $1 million + $25,000 + $600,000 = $1,625,000.
Thus the total retained is: $3.4 million - $1.625 million = **$1.775 million**.

179. (a) Risk attaching basis is similar to a policy year of data; we include those losses on policies written during the policy period of the reinsurance treaty, in this case with effective dates during 2008. This is claims 4 to 8: (1000)(65 + 120 + 400 + 185 + 90) = **$860,000**.
(b) Losses occurring basis is similar to an accident year of data; we include those losses that occur during the policy period of the reinsurance treaty, in this case with accident dates during 2008. This is claims 1 and 3 to 8: (1000)(50 + 220 + 65 + 120 + 400 + 185 + 90) = **$1,130,000**.

180. (a) Catastrophe treaty pays $10M. At that point insurer has retained $20M.
Thus quota share pays: (50%)($20M) = $10M. Primary insurer ends up retaining $10M.



(b) Quota share pays: (50%)($45M) = $22.5M. At that point insurer has retained $22.50M.
Catastrophe treaty pays $2.5M. Primary insurer ends up retaining $20M.



181. Assume each claim is from a different occurrence.
(a)

Claim Number	Loss + LAE	Recovery
1	$500,000	$300,000
2	500,000	300,000
3	400,000	200,000
4	800,000	300,000
Total		**$1,100,000**

(b) The total recovery is: $500,000 + $200,000 = **$700,000**.

Claim Number	Loss	Recovery	LAE	Recovery %	Recovery
1	$200,000	0		$300,000	0
2	300,000	100,000	200,000	1/3	66,667
3	300,000	100,000	100,000	1/3	33,333
4	600,000	300,000	200,000	300/600 = 1/2	100,000
Total		$500,000			$200,000

Comment: See page 25 of Clark on Reinsurance Pricing.

182. For example, for a coverage A limit of $1 million, $250,000 is 25% and $1,000,000 is 100% of the coverage A amount. The exposure factor for 25% of Coverage A is 13,75% by linear interpolation. Thus the expected percent of losses ceded is: 85% - 13.75% = 71.25%.

Coverage A Limit	Direct Written Premium	Expected % of Losses Ceded
$100,000	$10,000,000	0
250,000	25,000,000	100% - 85% = 15%
500,000	20,000,000	100% - 35% = 65%
1,000,000	10,000,000	85% - 13.75% = 71.25%

The fraction of the primary insurer's total expected losses covered by this treaty is:

$$\frac{(0)(10) + (15\%)(25) + (65\%)(20) + (71.25\%)(10)}{10 + 25 + 20 + 10} = 23.875 / 65 = \mathbf{36.73\%}.$$

183. (a) When experience rating an excess of loss treaty, a "free cover" might appear when there is no trended loss experience in the upper portion of the treaty layer.
On an experience only basis this upper layer would be a "free cover" since it has no associated loss cost rate, and we would be giving away additional coverage at no additional cost.
For example, if there were no trended losses in the layer from $2M to $3M, then based solely on experience rating we would charge the same for a treaty covering $2M excess of $1M as for a treaty covering $1M excess of $1M.
(b) To price a free cover, first get a experience and exposure rate for the lower layer of the treaty where experience does exist. Next use exposure rating to price the relative cost between the upper "free" layer and the lower layer. Finally, apply this relativity to the experience rated lower layer in order to get a rate for the "free cover".
Comment: See page 20 of Clark on Reinsurance.

184. Since the treaty is written on a losses occurring basis, we want to trend to the equivalent of AY2010; thus we apply for example 5 years of trend to AY2005 data.
The AY2009 premium is incomplete as of September 30, 2009, so we start instead with the full year earned premium given as \$9.4M. (Presumably this was based on applying a premium development factor to the given \$6.8M.)
Trended and on level premium for the five years:
$(1.188)(1.03^5)(7.4) + (1.142)(1.03^4)(8) + (1.1)(1.03^3)(8.5) + (1.048)(1.03^2)(9.1) + (1.03)(9.4) =$ \$50.491M.
Trended and developed Loss & ALAE for the five years:
$(1.05^5)(4.2) + (1.015)(1.05^4)(3.8) + (1.045)(1.05^3)(4.6) + (1.15)(1.05^2)(4.4) + (1.8)(1.05)(2.6) =$ \$26.106M.
We need to load this loss and ALAE for catastrophes and ULAE:
(1.11)(1.03)(\$26.106M) = \$29.847M.
Thus the loss ratio is: \$29.847M / \$50.491M = 59.1%.
Thus, the projected combined ratio for the 2010 treaty renewal period is:
59.1% + 30% + 6% + 1% = **96.1%**.
Comment: See pages 6 to 9 of Clark on Reinsurance Pricing.
To get the loss ratio, we perform the same steps as we would in an overall indication for the primary insurer.
No use is made of the given ALAE as a percent of losses.
If there had been a catastrophe in the historical period, we would need to subtract out its losses.

185. (a) (44%)(6%) + (67%)(64%) + (86%)(22%) + (109%)(8%) = **73.16%**.
(b) The average loss ratio in the 75%-100% range will be: 75% + (20%)(86% - 75%) = 77.2%.
The average loss ratio in the over 100% range will be:
75% + (20%)(100% - 75%) + (109% - 100%) = 89%.
Thus, the reinsurer's expected loss ratio after the application of the loss corridor is:
(44%)(6%) + (67%)(64%) + (77.2%)(22%) + (89%)(8%) = **69.624%**.

186. (a) For the midpoint of each interval translate the layer of coverage into percents.
200/150 = 1.33. Exposure factor is: 100%.
200/250 = 0.8. Exposure factor is: 85%. 500 / 250 = 2. Exposure factor is: 100%.
200/500 = 0.4. Exposure factor is: 60%. 500 / 500 = 1. Exposure factor is: 95%.
200/1000 = 0.2. Exposure factor is: 45%. 500 / 1000 = 0.5. Exposure factor is: 66.5%.
For example, for the last interval, the total expected losses are: (60%)(871,000),
and the expected losses paid by the reinsurer are: (66.5% - 45%)(60%)(871,000).
Thus expected loss cost is:
(60%)(1000) {(0%)(300) + (100% - 85%)(250) + (95% - 60%)(423) + (66.5% - 45%)(871))} = \$223,689.
Reinsurer's expected loss cost as a percentage of subject premium is:
\$223,689 / \$1,844,000 = **12.13%**.
(b) $\dfrac{(\$233{,}689)(1.1) + (\$1{,}844{,}000)(0.05)}{1 - 0.2}$ = **\$422,822**.
Comment: There is no mention of ALAE in this exam question. If ALAE was treated pro-rata in the treaty, then one would load the expected excess loss for ALAE.

187. (a) Rate-on-line = reinsurance premium / {(width of covered layer)(participation in layer)}.
For A: 10 / 125 = 8%.
For B: 4 / {(60%)(100)} = 6.67%.
(b) Payback period is the inverse of the rate-on-line.
For A: 1/8% = 12.5 years = 125/10.
For B: 1/6.67% = 15 years = (100)(60%) / 4.
(c) For A, the losses ceded are: (100%) (82.5 - 75) = 7.5 million.
For B, the losses ceded are zero.
Burning cost is: $7.5 / $35 = 0.214.
Comment: Not covered in detail in Clark.
At the time, there were other readings on reinsurance on Exam 6.
The denominator of the rate-on-line is the amount the reinsurer would have to pay in the event of one very large catastrophe.

188. (a) The reinsurer pays $15 million for the loss, which is 75% of the treaty limit of $20 million. The reinstatement premium is: (75%)($3.6 million)(1.15) = **$3.105 million**.
(b) July 31 is 7/12 of the way through the year.
The reinstatement premium is: (1 - 7/12)(75%)($3.6 million)(1.15) = **$1,293,750.**
Alternately, after the July 1 date of loss, there are 153 days remaining in the treaty period.
Reinstatement premium is: ($3,600,000)(1.15)(15/20)(153/365) = **$1,301,548**.
(c) "Given the seasonal nature of some types of catastrophes (e.g. hurricanes), relatively few contracts include reinstatements pro-rata as to time."
Pro-rata as to time assumes instead of seasonality that the exposure to property catastrophes is constant throughout the year.

189.

Risk	Insured Value	Percent Ceded	Incurred Loss	Amount Retained
1	$200,000	0%	$75,000	$75,000
2	500,000	250/500 = 50%	400,000	200,000
3	1,250,000	750/1250 = 60%	200,000	80,000
Total				**$355,000**

Comment: See page 3 of Clark on Reinsurance Pricing.

190. For example, for the $5 million policy limit, the total losses are proportional to 2.2, while the reinsured layer from $4M to $5M is proportional to: 2.2 - 2.1.

Policy Limit	% Losses in Reinsured Layer	Earned Premium
$5,000,000	(2.2 - 2.1)/2.2	$20,000,000
10,000,000	(2.5 - 2.1)/2.5	50,000,000
20,000,000	(2.5 - 2.1)/3.0	100,000,000

The expected loss for the reinsurance layer $6,000,000 excess of $4,000,000 is:
(60%)($1 million) {(20)(2.2 - 2.1)/2.2 + (50)(2.5 - 2.1)/2.5 + (100)(2.5 - 2.1)/3.0}
= **$13.345 million**.
Comment: This past exam question did not say how ALAE is treated in the treaty.
I have used the technique applicable when ALAE is treated pro-rata.
If ALAE were instead included with losses, then exposure rating is more complicated.

191. For Occurrence #1, the umbrella policy pays \$500,000 in loss, so the reinsurer pays for \$250,000 in loss. The reinsurance treaty covers ALAE pro-rata with respect to loss.
The reinsurer pays 250K of the total 1100K of the first loss.
Thus the reinsurer also pays ALAE of: (250/1100)(\$100,000) = \$22,727.
For Occurrence #2, the umbrella policy pays nothing, and so does the reinsurer.
For Occurrence #3, the umbrella policy pays \$250,000 in loss, so the reinsurer pays nothing.
Thus the reinsurer's loss and alae ratio is: \$272,727 / \$900,000 = 30.30%.
To get the technical ratio, add in the ceding commission of 15%: 30.30% + 15% = **45.30%**.

192. (a) From occurrence #1 of size \$600,000: 150,000.
From occurrence #2 of size \$300,000: 100,000.
From occurrence #3 of size \$450,000: 150,000.
Recovery by the primary company from Treaty 1: 150,000 + 100,000 + 150,000 = **\$400,000**.
(b) Net of recoveries from Treaty #1, the occurrences are: 450,000, 200,000, and 300,000.
Thus Treaty #2 pays: \$200,000, 0, and \$50,000. For a total of **\$250,000**.
(c) Since Treaty 1 inures to the benefit of Treaty 2, the net subject premium is:
\$1,500,000 - \$200,000 = \$1,300,000.
Thus the reinsurance premium for Treaty #2 is: (7.5%)(\$1,300,000) = \$97,500.
Thus the ceded loss ratio for Treaty 2 is: \$250,000 / \$97,500 = **256.4%**.
<u>Comment</u>: I do not know why the question says “expected recovery” rather than “recovery”.

193. (a) These losses are all from one occurrence and total \$4,700,000.
Thus the excess-of-loss treaty pays **\$3,000,000**.
(b) The insurer retains \$1,700,000 after the excess of loss treaty.
Then the primary insurer retains \$1 million, and the clash cover pays: **\$700,000**.

194. Assume that these are excess loss development factors.
For the trended losses, take the amounts excess of \$1 million, multiplied by the appropriate LDF: (1)(1,976,920) + 0 + (1.04)(1,215,965) + (1.04)(564,398) + (1.08)(1,731,572) = 5,698,595.
Divide by the on-level trended subject premium to get the experience rate:
5,698,595 / (6.4 million + 6.9 million + 7.4 million) = 27.53%.
However, none of the losses pierce the layer excess of \$3 million.
Thus to avoid the free cover, we charge the layer from \$1 million to \$3 million based on the experience rate and the layer from \$3 million to \$5 million based on a combination of exposure rating and experience rating.
1M is 10% of the insured value of 10M.
The layer from 1M to 3M has exposure of: 55% - 35% = 20%.
The layer from 3M to 5M has exposure of: 70% - 55% = 15%.
Thus we charge for the layer from 3M to 5M: (15% / 20%) (27.53%) = 20.65%.

Layer	Experience Rating Loss Cost	Exposure Rating Loss Cost	Selected
2M xs 1M	27.53%	20%	27.53%
2M xs 3M	0	15%	20.65%*
4M xs 1M			**48.18%**

* 20.65% = (27.53%) (15%/20%)
<u>Comment</u>: See page 20 of Clark on Reinsurance Pricing, as well as the background in Section 2A.

195. The first range is unaffected by the loss corridor.
In the second range, net of the loss corridor the average loss ratio is:
60% + (50%)(67.8% - 60%) = 63.9%.
In the third range, net of the loss corridor the average loss ratio is:
60% + (50%)(15%) + (20%)(81.5% - 75%) = 68.8%.
In the fourth range, net of the loss corridor the average loss ratio is:
60% + (50%)(15%) + (20%)(15%) + (99.6% - 90%) = 80.1%.
Thus, the reinsurer's expected loss ratio after the application of the loss corridor is:
(0.45)(47.9%) + (0.29)(63.9%) + (0.17)(68.8%) + (0.09)(80.1%) = **58.991%**.

Alternately, the average reduction due to the loss corridor is:
(0.29)(50%)(67.8% - 60%) + (0.17) {(50%)(15%) + (80%)(81.5% - 75%)}
+ (0.09) {(50%)(15%) + (80%)(15%)} = 5.045%.
The average loss ratio prior to the effect of the loss corridor is:
(0.45)(47.9%) + (0.29)(67.8%) + (0.17)(81.5%) + (0.09)(99.6%) = 64.036%.
Thus, the reinsurer's expected loss ratio after the application of the loss corridor is:
64.036% - 5.045% = **58.991%**.

Comment: See page 13 of Clark on Reinsurance Pricing.

196. (a) The expected loss ratios is:
(0.04)(35%) + (0.32)(55%) + (0.24)(68%) + (0.40)(80%) = **67.32%**.

The commission for a 68% loss ratio is: 20% + (35% - 20%) (75 - 68) / (75 - 60) = 27%.
The commission for a 55% loss ratio is: 35% + (45% - 35%) (60 - 55) / (60 - 40) = 37.5%.
Thus the average commission is:
(0.04)(45%) + (0.32)(37.5%) + (0.24)(27%) + (0.40)(20%) = **28.28%**.
(Note that the commission schedule is linear in the intervals from 60% to 75% and from 40% to 60%, and therefore, it is appropriate to use the average loss ratio in those intervals.)

(b) “A carryforward provision allows that if the past loss ratios have been above the loss ratio corresponding to the minimum commission, that the excess loss amount can be included with the current year’s loss in the estimate of the current year’s commission. In the long run, this should help smooth the results.”

(c) 75% is the loss ratio corresponding to the minimum commission. Therefore, the loss excess of a 75% loss ratio would be included with the current year’s losses in order to compute the current year’s commission. Thus the losses entering the calculation will be bigger than average and the expected commission will be lower than the average of 28.28%.

(d) 1. Include any carryforward from past years and estimate the impact on the current year only. This amounts to shifting the slide by the amount of the carryforward.
2. Look at the “long run” of the contract. The sliding scale is modeled as applying to a longer block of years rather than just the single current year. The variance of the aggregate distribution would be reduced on the assumption that individual bad years would be smoothed by good experience on other years. The variance of the average loss ratio for a block of years should be significantly less that the variance of the loss ratio for a single year (roughly equal to dividing by the number of years in the block).

<u>Comment</u>: See pages 9-12 of Clark on Reinsurance Pricing.

197. (a) “An implicit assumption in the exposure rating approach outlined above is that the same exposure curve applies regardless of the size of the insured value. For example, the likelihood of a $10,000 loss on a $100,000 risk is equal to the likelihood of a $100,000 loss on a $1,000,000 risk. This assumption of scale independence may be appropriate for homeowners business, for which this technique was first developed, but may be a serious problem when applied to large commercial risks.”

In this case, we can check for scale independence by comparing the ratio of pure premiums to insurance to value:

Insured Value Range	MidPoint	Pure Premium	P.P. Over Insured Value
$500,000 - $1,000,000	0.75	$34,539	46,052
$1,000,001- $2,500,000	1.75	$80,665	46,094
$2,500,001- $5,000,000	3.75	$172,659	46,042
$5,000,001- $10,000,000	7.5	$345,399	46,045

The ratio of pure premiums to insurance to value is the same for the different size categories. Based on the above information, it appears that the assumption of scale independence is reasonable for this portfolio.
Therefore, the actuary's decision is sound.

Alternately, Clark states this is not as ideal for a commercial book as for homeowners. Commercial risks are very heterogeneous; therefore, over the large spectrum of size of risks, it may not be the case that for example a 10% loss is as likely for a $1 million property as for a $10 million property. In other words, scale independence is unlikely to hold, which is the key assumption of constructing an exposure curve.
Therefore, the actuary's decision is <u>not</u> sound.

(b) We are given a per <u>risk</u> limits profile, but the exposure curve was calculated based on a per <u>location</u> basis. This creates a problem, since different risks have different numbers of locations. Also, the historical loss ratios are so volatile that it will be difficult to determine an expected loss ratio for the rating period.
Therefore, the actuary's decision is <u>not</u> sound.

<u>Comment</u>: See pages 17 to 19 of Clark on Reinsurance.

Note that while scale independence implies that the pure premium per insured value should be the same over the different size ranges, the converse is not true. With more detailed information, we could do further investigations as to whether scale independence applies to this book of business.

198. (a) The catastrophe reinsurance covers the layer from 50M to 100M.
In each case we multiply by the percentage of the width of the layer that has been exhausted.

Date	Losses in Layer	Reinstatement Premium
June 1	15,000,000	(\$3m) (1.2) (15/50) = \$1,080,000
Sept. 1	35,000,000	(\$3m) (1.2) (35/50) = \$2,520,000
Nov. 1	50,000,000	(\$3m) (1.2) (50/50) = \$3,600,000
Total		\$7,200,000

(b) In each case we also multiply by the percentage of the year of coverage remaining.

Date	Losses in Layer	Reinstatement Premium
June 1	15,000,000	(\$3m) (1.2) (15/50) (7/12) = \$630,000
Sept. 1	35,000,000	(\$3m) (1.2) (35/50) (4/12) = \$840,000
Nov. 1	50,000,000	(\$3m) (1.2) (50/50) (2/12) = \$600,000
Total		\$2,070,000

(c) “Given the seasonal nature of some types of catastrophes (e.g. hurricanes), relatively few contracts include reinstatements pro-rata as to time.”
Pro-rata as to time assumes instead of seasonality that the exposure to property catastrophes is constant throughout the year.

Comment: Similar to 6, 11/10, Q. 22.
See page 43 of Clark.
We are implicitly assuming that there is no other reinsurance, which would inure to the benefit of the catastrophe reinsurance.
In the Northern Atlantic Ocean, a distinct hurricane season occurs from June 1 to November 30, sharply peaking from late August through September.
If the major peril we were concerned about was hurricanes in the United States, then after the June 1 catastrophe, almost all of the hurricane season would still be to left, so that a pro-rata as to time factor of 7/12 would be too low.
If instead the major peril we were concerned about was earthquakes, then reinstatements pro-rata as to time would make sense.

199. Under the 60% ceded quota share, the insurer retains 40% of each loss and thus 40% of the total losses: (40%)($1,120,000) = **$448,000**.

Under the five-line surplus share treaty with retained line = $100,000:

Risk	Insured Value	Percent retained	Loss	Amount Retained
A	$250,000	100/250	$120,000	$48,000
B	$1,000,000	1 - 500/1000	$245,000	$122,500
C	$85,000	100%	$85,000	$85,000
D	$1,250,000	1 - 500/1250	$490,000	$294,000
E	$400,000	100/400	$180,000	$45,000
Total				**$594,500**

Under the $400,000 xs $100,000 per-risk excess of loss:

Risk	Loss	Amount Ceded	Amount Retained
A	$120,000	20,000	100,000
B	$245,000	145,000	100,000
C	$85,000	0	85,000
D	$490,000	390,000	100,000
E	$180,000	80,000	100,000
Total			**$485,000**

The least is retained by the insurer in total for the **60% ceded quota share**.

<u>Comment</u>: One could instead compute the amounts ceded, and say what you are doing. Then the most ceded corresponds to the least retained.

For the surplus share treaty, if X is the insured value, then the percent ceded is:

$$\begin{cases} 0 & \text{for } X < 100{,}000 \\ 1 - 100{,}000/X & \text{for } 100{,}000 \le X \le 600{,}000 \\ 500{,}000/X & \text{for } 600{,}000 < X \end{cases}$$

For the surplus share treaty, if X is the insured value, then the percent retained is:

$$\begin{cases} 1 & \text{for } X < 100{,}000 \\ 100{,}000/X & \text{for } 100{,}000 \le X \le 600{,}000 \\ 1 - 500{,}000/X & \text{for } 600{,}000 < X \end{cases}$$

As the amount of insurance increases from $100,000 to $600,000, the percent ceded increases linearly from 0 to 5/6. As the amount of insurance increases beyond $600,000, the percent ceded decreases; for X > $600,000, the percent ceded is $500,000/X.

200. I am assuming that the reinsurer is writing each year separate policies for several different self-insured independent retailers.

Disadvantages to the use of the lognormal distribution to model aggregate losses:
1) There is no allowance for the loss free scenario.
    Since in this example we have 5 out 10 loss free years, and even in years with claims some retailers had no claims, there is a large probability of no reinsured loss from a retailer in a year.
    This argues very strongly against using the lognormal distribution here.
2) There is no easy way to reflect the impact of changing per occurrence limits on the aggregate losses.
    Since in this example we have occurrence limits of either $1,000,000 or $2,000,000 in excess of the insured's retention this argues against using the lognormal distribution here.

The Panjer algorithm works well in low frequency situations, such as we have here.
In order to use the Panjer algorithm:
1) Since we have chosen to use a Poisson frequency, we would need to estimate its mean l.
2) We would need to model severity (excess of the retention and prior to the impact of the occurrence and aggregate limits) as a discrete distribution with support at evenly spaced points, such as for example $250,000.
    (We would then apply the appropriate occurrence limit and aggregate limit.)

While **the Panjer algorithm is clearly the superior approach of the two in this example**, there will still be problems.
One could estimate the mean frequency per policy for an average independent retailer from the number of claims and the number of policies covered in each of the years. Unfortunately, with such a low frequency, there is considerable potential estimation error due to random fluctuation.
(A complication is that due to inflation, claims that were within the retention in the past will exceed the retention in the future.)
More importantly, with only 7 claims it will be difficult to model severity.
(One would want to take into account the effects of inflation.)
Perhaps some relevant data other than from this book of business can be obtained to help model severity.

Comment: See Section 4 of Clark on Reinsurance Pricing.
Panjer algorithm is discussed at pages 39-41 of Clark on Reinsurance Pricing.

201. (a) A key difference between the two types of reinsurance is that surplus share reinsurance limits the retention per risk while quota share, being proportional, has no limit. Property R Us is a small company and so will likely prefer surplus share as this limits its exposure and allows larger amounts of coverage to be sold. With a retained line of $100,000, and a sufficient number of lines, Property R Us can write properties of size greater than 100,000 without using up any more of its surplus's limited capacity to support business than if the property were of size 100,000. At the same time it retains all of the premiums and losses for properties of size 100,000 or less.
(b) There is a 60% probability of a loss ratio between 40% and 70%.
There is a 20% chance of a loss ratio between 60% and 70%, and an average commission of 25%.
There is a 40% chance of a loss ratio between 40% and 60%, and an average commission of 35%.
Average commission is: (20%)(40%) + (20%)(20%) + (20%)(25%) + (40%)(35%) = **31%**.
(c) As Property R Us grows its business and writes more risks the variance of its distribution of loss ratios should get less. There is not enough information to adequately model the effect.
Let us just as example assume that the loss ratios will be uniformly distributed between 40% and 70%. Then the average commission would be: (1/3)(25%) + (2/3)(35%) = 31.67%. This is slightly more than currently.
(d) Assuming the surplus share treaty has at least 9 lines, then we can treat the 1,000,000 property as if it were of size 100,000, the retained line. We would look up the exposures factors at 50,000/100,000 = 1/2 and 100,000/100,000 = 1, using an exposure curve appropriate for a $1 million property. In addition the expected losses by which we multiply would be only 100,000 / 1 million = 1/10 of those for the primary policy.

202.
(a) For the first three policy types, the reinsurer is covering the layer from 500K to 1000K.
For the last set of policies, the reinsurer is covering the layer from 500K + 500K to 1500K.

Expected Losses	Underlying Limit	Policy Limit	Fraction of Insured Losses Ceded
3,000,000	0	500,000	0
3,000,000	0	750,000	(1.20 - 1) / 1.20 = 1/6
9,000,000	0	1,000,000	(1.35 - 1) / 1.35 = 7/27
6,000,000	0	1,500,000	(1.35 - 1) / 1.56 = 35/156
1,200,000	500,000	1,000,000	(1.56 - 1.35) / (1.56 - 1.00) = 3/8

Expected ceded losses are:
(1 million) {(1/6)(3) + (6/27)(9) + (35/156)(6) + (3/8)(1.2)} = **4,296,154**.
(b) Applying 20% assumes that ALAE is a constant percentage of each loss. However, ALAE usually varies as a percentage of loss with different sizes of loss. The large losses that will pierce the reinsured layer may have a different ratio of ALAE to loss than overall. Thus 20% is likely to be incorrect for the reinsured layer.
(c) 1. The Panjer algorithm is a recursive method to calculate the aggregate distribution. It requires an assumed frequency distribution such as a Poisson or Negative Binomial, and a discrete severity distribution.
2. Numerical methods, such as the Heckman-Meyers algorithm.
3. Simulation, generating the number of losses followed by the amount of an individual loss
(d) For the 4th and 5th sets of policies, any portion of a ground up loss excess of 1 million is not reinsured. Yet these policies would pay something more up to a ground up loss of 1.5 million. Thus the primary insurer should consider buying a 500K xs 1 million per occurrence treaty. (The primary should consider buying in addition a clash cover, for example 10 million xs 1.5 million.)
Comment: How the ALAE ratio varies with size of loss is an empirical issue.
The fact that the treaty was written on a swing plan was not used to answer the questions.
As discussed at page 34 of Clark, a "swing plan" which is a type of retrospective rating program. Actual losses to the layer are loaded for expenses and the result is charged back to the ceding company, subject to maximum and minimum constraints.

203. (a) Often insured values in the limits profile provided does not include business interruption coverage for commercial policies (or living expenses for homeowners policies.)
(b)

I.V.	100/I.V.	Exposure Factor	500/I.V.	Exposure Factor
200	0.5	70%	2.5	100%
500	0.2	49%	1	93%
1000	0.1	37%	0.5	70%

Expected losses in the reinsured layer from 100K to 500K:
(60%) (1,000,000) {(1)(100% - 70%) + (1)(93% - 49%) + (1)(70% - 37%)} =
180,000 + 264,000 + 198,000 = **642,000**.
(c) That the exposure factors, which are loss elimination ratios, are independent of the size of the property. In other words, that the severity distribution as percent of the value of the property is the same for different size properties. In other words, the key assumption is that the same exposure curve applies regardless of the insured value.
(d) For the 1000K properties, the insurer will only retain half of the premium and losses.
Thus the excess treaty will only pay when the ground up loss is 200K or more.
The top of the reinsured layer correspond to a 1000K ground up loss.
Thus the exposure factors are for: 200/1000 = 0.2 and 1000/1000 = 1.
Expected losses in the layer for these properties are:
(1/2)(60%)(1 million)(93% - 49%) = 132,000.
Total expected losses in the layer are: 180,000 + 264,000 + 132,000 = **576,000**.
Alternately, for the $1 million properties, treat the insured property size as the line of $500,000 from the surplus share treaty, and look up the exposure factors for:
100/500 = 0.2 and 500/500 = 1.
The expected losses net of the surplus share treaty are: (1/2)(60%)(1 million) = 300,000.
(93% - 49%) (300,000) = 132,000.
Total expected losses in the layer are: 180,000 + 264,000 + 132,000 = **576,000**.
<u>Comment</u>: See pages 18 and 19 of Clark.
"This assumption of scale independence may be appropriate for homeowners business, for which this technique was first developed, but may be a serious problem when applied to large commercial risks."
The appropriate exposure curves would differ by construction and occupancy.

204. (a) For the first three policy types, the reinsurer is covering the layer from 500K to 1000K.
For the last two sets of policies, the reinsurer is covering the layer from 500K + 500K to 1500K.

Expected Losses	Underlying Limit	Policy Limit	Fraction of Insured Losses Ceded
3,000,000	0	500,000	0
3,000,000	0	1,000,000	(1.35 - 1.00) / 1.35 = 7/27
2,400,000	0	1,500,000	(1.35 - 1.00) / 1.56 = 35/156
1,200,000	500,000	500,000	0
1,800,000	500,000	1,000,000	(1.56 - 1.35) / (1.56 - 1.00) = 21/56

Expected ceded losses are: (1 million) {(7/27)(3) + (35/156)(2.4) + (21/56)(1.8)} = **1,991,239**.
(b) One method is to trend losses but neither policy limits nor subject premiums. This method assumes that policy limits do not increase over time.
A second method is to trend losses and policy limits and adjust subject premiums to match the higher limits. This method assumes that policy limits increase over time.
"One possible approach is to apply the historical policy limit to each trended loss; this leaves out the fact that the insured will generally increase its policy limits over time. A second approach is to apply the trend factor to the historical loss without applying a policy limit cap; this assumes that policy limits drift upwards to precisely match inflation. If this second approach is used, then the subject premium must also be adjusted to the level that would have been charged had the higher limits been in effect; otherwise an overstatement of the expected loss cost will result."
(c) I assume the primary policy has no deductible (nor is it an umbrella policy with an underlying limit) and has a limit large enough to pay for the loss.
Let x be the amount of ALAE. Then for the $600,000 loss, the reinsurer pays:
ALAE within limit: Min[500,000, 100,000 + x],
ALAE pro-rata: 100,000 + x/6.
These are equal when: x = (6)(400K) = 2.4 million.
The reinsurer pays more under pro rata when **ALAE > 2.4 million**.
For the $750,000 loss, the reinsurer pays:
ALAE within limit: Min[500,000, 250,000 + x],
ALAE pro-rata: 250,000 + x2.5/7.5.
These are equal when: x = (3)(250K) = 750,000.
The reinsurer pays more under pro rata when **ALAE > 750,000**.
(d) For the 3rd and 5th sets of policies, any portion of a ground up loss excess of 1 million is not reinsured. Yet these policies would pay something more up to a ground up loss of 1.5 million.
Thus the primary insurer should consider buying a 500K xs 1 million per occurrence treaty.
(The primary should consider buying in addition a clash cover, for example 10 million xs 1.5 million.)
<u>Comment</u>: See Section 3 of Clark.
In part (a), I have used the technique applicable when ALAE is treated pro-rata.
If ALAE were instead included with losses, then exposure rating is more complicated.

205. (a) Assume that this loss is not one for which the umbrella policy had to drop down in order to fill a coverage gap in the primary policy. Also assume that the attachment point of the umbrella policy that was in effect during 2014 was also \$3 million (and the policy limit was at least 4 million.)
Then the ground up size of loss was: 3 + 4 = 7 million.
The trend to date is 6 months beyond April 1, 2015; thus the trend period is 1.25 years.
The trended ground up loss was: 3 million + 4,395,940 = 7,395,940.
$\Rightarrow (1+r)^{1.25}$ (7) = 7.395940. $\Rightarrow$ r = **4.5%**.
(b) Assume for simplicity that there is one underlying policy and that each year the limit of the underlying policy matches the attachment point of the umbrella policy.
Policy limits of the underlying policy and of the umbrella policy both tend to drift up over time with inflation; however, they also change due to other changes in the circumstances of the insured.
In any case, changes in policy limits usually occur at irregular intervals and do not match the continuous effects of inflation.
Let us for example assume 50% inflation between 2004 and 2014. Then if the loss had been in 2004 it would have been of ground up size: 7/1.5 = 4.666. Assuming an umbrella policy in 2004 with attachment point of 3/1.5 and limit 10/1.5, then the umbrella policy would have paid 7/1.5 - 3/1.5, and we could determine the ground up loss to be: 7/1.5 = 4.666. Then we could appropriately trend it to 2014 when it would have been 7.
Thus the changes in policy limits and attachment points should create no inherent problem if they keep up with inflation and provided we have complete information about the past.
Unfortunately, the policy limit in 2004 could have been anything, the umbrella policy limit could have been anything, this insurer may not have written an umbrella policy for the insured who had the loss, etc. The trended claim could be either overstated or understated had the loss occurred in 2004.
Alternately, if we mistakenly assume that the attachment point in 2004 was 3 million, the same as currently, when in fact it was 2 million, we would overestimate the trended loss. A \$4 million payment in 2004 would be mistakenly assumed to come from a 3 + 4 = 7 million loss, while it in fact came from 6 million ground up loss; this would result in a trended loss that is bigger than it should be.
The upward drift of policy limits and attachment points can distort trending of historical losses if historical losses are mistakenly made subject to the current limits and attachment points before being trended to the prospective year.
Comment: While umbrella policies are mentioned at pages 31-33 of Clark in the context of pricing reinsurance, the details of umbrella polices are not covered in the syllabus readings.
“This procedure will still leave out losses from the underlying policy which historically did not exhaust the underlying limit, but which would have after the application of a trend factor.”
In part (b), the alternate solution presented is that intended by the CAS, which personally would have required me to read the mind of the question writer in order to know what he or she had intended.

206. (a) Assuming the underlying policy is CGL, ALAE is <u>not</u> included in the limit; to do so would be impractical and I do not know how it would work.
It is unclear what would be meant by including ALAE in the limit of the umbrella policy. The insurer who wrote the umbrella policy is not responsible for paying any portion of the ALAE incurred by the insurer who wrote the primary policy. The writer of the umbrella policy may choose to get involved in the settlement or trial relating to large claims and thus incur some its own ALAE. Any ALAE incurred by the writer of the umbrella policy either due to such situations or due to the umbrella policy dropping down would <u>not</u> be included in the limit; to do so would be impractical and I do not know how it would work. Option #2 is how things work in the real world; nevertheless I will try my best to answer this exam question.
The greatest cost would be option #3, because including ALAE in the limit of the underlying policy will mean more dollars will reach the umbrella layer, and for the umbrella policy ALAE is in addition to the umbrella limit.
For option #1, including ALAE in the limit of the underlying policy will mean that more dollars are reaching the umbrella layer than option #2; however, including ALAE in the limit of the umbrella policy caps the total loss & ALAE paid by the umbrella policy, resulting in less paid than option #3.
For option #2, not as many dollars will reach the umbrella layer than for the other two options. However, the umbrella policy could in theory pay more for loss plus ALAE than its limit. There should be no ALAE for the umbrella to pay from the underlying policy since the underlying policy has paid it all. Thus, option #2 is likely to be less expensive for the umbrella policy than option #1. However, if the umbrella carrier incurs a lot of its own ALAE, option #2 could be more expensive than option #1.
On average I would expect option #1 to be more expensive than option #2, since the effect of more losses reaching the umbrella layer for option #1 should be greater than the effect of the possibility of more ALAE being paid under option #2.
For example, let us assume that Company B writes a policy with a limit of \$100K, and Company A writes an umbrella policy with a limit of \$400K.
Let assume we have a large claim with \$400K in loss.
Company B incurs \$100K in ALAE and Company A incurs \$50K in ALAE.
1. ALAE is included within both the underlying and umbrella limits.
B pays nothing to the third party since it has incurred 100K of ALAE, exhausting the limit.
A pays 350K to the third party, since it has incurred 50K of ALAE.
(Presumably the insured is responsible for the remaining 50K owed to the third party.)
2. ALAE is in addition to both the underlying and umbrella limits.
B pays 100K to the third party claimant as well as 100K for its ALAE.
A pays the remaining 300K to the third party claimant as well as 50K for its own ALAE.
3. ALAE is included within the underlying layer's limit, but is in addition to the umbrella's limit.
B pays nothing to the third party since it has incurred 100K of ALAE, exhausting the limit.
A pays 400K to the third party, as well as 50K for its own ALAE.
For this example, Option 3 costs the most for the umbrella policy, followed by Option 1, with Option 2 costing the umbrella policy the least.

(b) The umbrella policy will not be responsible for any of the ALAE incurred by Company B, and Company B will pay its own ALAE in addition to any policy limits. Thus there is in fact no need for Company A to pay any attention at all to the ALAE data supplied by Company B.
In any case, the data supplied by Company B is too sparse to be credible; the ratio of ALAE to loss is volatile.
Company A should be particularly concerned with its ALAE costs when the umbrella policy drops down due to exhausted aggregate limits or provides coverage for perils not covered by the policy written by Company B. Thus the ALAE may have a significantly different relationship to loss.
“ALAE is not a constant percent of any given loss. For example, losses which close without an indemnity payment may still incur a large expense. In general, as the size of a loss increases, the ALAE as a percent of the loss will tend to decrease. The assumption that loss and ALAE are perfectly correlated will tend to result in an overstatement of expected amounts in the higher layers.”
Thus since the umbrella policy is covering high layers of loss, Company A is only concerned with ALAE on large claims, which will be a smaller percent of losses on average than for all claims.
Also open claims will be affected by loss dispersion; in many cases the paid plus reserves will develop upwards. Thus as explained in Mahler’s Discussion of “Retrospective Rating: 1997 Excess Loss Factors” the percent of losses and ALAE that will expose the umbrella treaty at ultimate are more on average more than at immature reports such as in the data from Company B.
From the Examiner’s Report: “The underlying layer appears to have a limit of 2 million.
It looks like losses may be capped, which distorts the ALAE ratio.”
From the Examiner’s Report: “There are multiple losses where paid ALAE exceeds paid loss. This could be evidence of extensive litigation and aggressive defense of claims that could inflate the ALAE ratio.”
(c) Because penetration of high excess layers is infrequent, the calculation of ALAE is difficult.
If ALAE is included with loss for the attachment point of the clash treaty, then large ALAE may expose the clash layer. (This is sometimes called runaway ALAE.)
Clash covers are also penetrated when extra-contractual obligations (ECO) or rulings awarding damages in excess of policy limits (XPL) are determined in a settlement, both of which can have significant associated ALAE. **Thus there is no reason to assume the loading for ALAE is an insignificant part of the cost of the clash cover**; it is an empirical question.
Comment: The writer of this question and I disagree about how commercial liability policies and commercial umbrella policies work in the real world.
Part (b) of this question is not covered in the syllabus readings on this exam.
Part (c) is briefly referred to by Clark at pages 22 and 23 “Clash Covers: High layer attachment excess - typically a loss on a single policy will not penetrate the treaty layer. A clash cover will be penetrated due to multiple policies involved in a single occurrence, or when extra-contractual obligations (ECO) or rulings awarding damages in excess of policy limits (XPL) are determined in a settlement. The method for including allocated loss adjustment expenses in the treaty may also expose the clash layer.” Prices for clash covers are driven by market conditions and are based on a lot of actuarial and underwriting judgement.

207. (a) On risks attaching basis, the reinsurer is essentially reinsuring policy year 2012, so only risks 2 and 3 are covered by the treaty.
The reinsurer pays: (30%)($400,000 + 1,500,000) = **$570,000**.
(b) On losses occurring basis, the reinsurer is essentially reinsuring accident year 2012, so all three losses are covered by the treaty.
The percent ceded depends on the insured value. The reinsurer pays:
($100,000)(200 - 100)/200 + ($400,000)(500 - 100)/500 + ($1,500,000)(500/1500) = **$870,000**.
(c) The first treaty in essence covers PY2013 while the second treaty in essence covers AY2014.
However, some losses on policies written in 2013 will occur during 2014. Therefore, some losses would be paid for under each treaty. Not only would the insurer be reimbursed twice for the same loss, but there would be a morale hazard. The insurer would have an incentive to pay more for a claim so that after being reimbursed twice it would end up in better shape.
One could include in these treaties an "interlocking clause", designed to equitably apportion losses that may be covered under more than one contract.
Alternately, the reinsurer may add a clause in the 2014 treaty to exclude claims that would be covered by the 2013 treaty.
Alternately, commute the risks attaching policy at 1/1/14 and replace it with the loss occurring treaty.
Comment: A commutation is a settlement agreement reached between a reinsured and a reinsurer by which the reinsurance obligation is terminated. The reinsurer agrees to pay funds at present value that are not yet due under the reinsurance agreement. A commutation allows the reinsured to receive cash now to invest for the payment of claims that will come due in the future. The reinsurer's obligations for future payments are terminated and the reinsurance contract is terminated.

208. (a) Under the sliding scale commission, the ceding company gets a provisional 20% which is later adjusted up or down when the experience under the treaty becomes available.
Instead under the loss corridor option the ceding company gets a 15% commission when the treaty is written, which is later in essence reduced if the loss ratio under the treaty exceeds 75%.
Thus the ceding insurer is better off initially with the sliding scale commission option.
The ceding insurer will receive more cash up front with the sliding scale commission option, which they can invest; with the sliding scale commission the ceding insurer has a timing cashflow advantage compared to the loss corridor.
Under the sliding scale commission:

Loss Ratio	Commission
45%	30%
65%	20%
70%	15%
75%	10%
80%	10%
85%	10%
95%	10%

Under the Loss Corridor:

Loss Ratio	Commission	Loss Corridor Effect	Commission - Loss Corridor
45%	15%	0	15%
65%	15%	0	15%
70%	15%	0	15%
75%	15%	0	15%
80%	15%	(60%)(80% - 75%) = 3%	12%
85%	15%	(60%)(85% - 75%) = 6%	9%
95%	15%	6%	9%

Thus once the experience is known, for most loss ratios the ceding company is better off with the sliding scale option. However, at loss ratios between 70% and 83.3% the ceding company would be somewhat better off with the loss corridor option. There is no information on the distribution of loss ratios needed in order to calculate an average, but it is likely that on average the ceding company is also better off eventually with the sliding scale commission option.
(b) A carryforward provision allows that if the past loss ratios have been above the loss ratio corresponding to the minimum commission, then the excess loss amount can be included with the current year's loss in the estimate of the current year's commission. In the long run, this should help smooth the results for both the insurer and the reinsurer. One could also allow a case where the past loss ratios have been below the loss ratio corresponding to the maximum commission, then the credited loss amount can be included with the current year's loss in the estimate of the current year's commission.
Another way to stabilize results would be to reduce the maximum commission and increase the minimum commission in the schedule. Alternately, one could change the slopes; for example, sliding 0.5: 1 to a minimum: 10% at an <u>85%</u> loss ratio.

(c) Two approaches may be taken to pricing the impact of carryforward provisions. The first is to include any carryforward from past years and estimate the impact on the current year only. This amounts to shifting the slide by the amount of the carryforward. (It does not really affect the determination of an aggregate loss distribution model.) The problem with this approach is that it ignores the potential for carryforward beyond the current year. For example, in the first year of the program we would calculate the expected commission for the current year as though the program would be cancelled at the end of the year. The same price would result with or without the carryforward provision, which does not seem right because the benefit of the carryforward is ignored.
A second approach is to look at the "long run" of the contract. The sliding scale is modeled as applying to a longer block of years rather than just the single current year. The variance of the aggregate distribution would be reduced on the assumption that individual bad years would be smoothed by good experience on other years. The variance of the average loss ratio for a block of years should be significantly less than the variance of the loss ratio for a single year.
The first problem with this approach is that the method for reducing the variance is not obvious. A second problem is that it ignores the fact that the contract may not renew the following year, potentially leaving the reinsurer with no carryforward benefit.
If the maximum commission, minimum commission, or slopes are changed this should be taken into account in pricing, although it does not affect the determination of an aggregate loss distribution model.
<u>Comment</u>: In my opinion, this question is poorly worded.
Part (c) does not flow smoothly from part (b).
“c. Explain whether the smoothing mechanisms in part b. above should be used in the determination of an aggregate loss distribution model.” However, part (b) does not specify that the mechanisms be “smoothing.” Also “ways the insurer can stabilize its results for its sliding scale commission structure over time” from part (b) do not directly translate into models of aggregate loss distributions used to quantify the effect of the commission structure. Rather the actuary has to make some adjustment to the model or his use of the model of aggregate loss distributions in order to try to quantify the impact of whatever change is made to the sliding scale commission structure in part (b).
Based on the Examiner’s Report, the examiners do not seem to realize that if one stabilizes the insurer’s results for its sliding scale commission structure over time, then one has also automatically stabilized the reinsurer’s results for its sliding scale commission structure over time, and vice-versa.
Part (a) says in part “the ceding company is interested in optimizing the <u>amount</u> and the timing of cash flows.” Yet based on the Examiner’s Report, the examiners seemed to ignore any comparison of the amount of the eventual cashflows under the two options, which of course would be a very important consideration for the ceding company.
In part (b), apparently the examiners intended the two ways the insurer can stabilize its results for its sliding scale commission structure over time to be “the carryforward provision and its extension in considering a block of years to model the carryforward.” I do not consider this two different ways; I believe that the examiners confused the provision in the treaty and different ways of pricing it.

209. (a) E[X ; 1 million] = 30 + (900){1 - ln(1/1000)} = 7147.
E[X ; 250,000] = 30 + (900){1 - ln(1/250)} = 5899.
Exposure factor is: $\frac{\text{E[X ; 250K + 750K] - E[X ; 250K]}}{\text{E[X ; 1000K]}} = \frac{7147 - 5899}{7147}$ = 17.46%.

(b) (17.46%) (ground-up expected loss) = \$243,500.
⇒ ground-up expected loss = \$243,500 / 0.1746 = **\$1.395 million**.
Comment: I do not know what the purpose was of making part (b) backwards.
The given formula for the limited expected value is not from a well-known size of loss distribution, although it is increasing and concave downwards. However, as L gets small, the limited expected value becomes negative; for example E[X ; 300] = -154. Also as L approaches infinity, the limited expected value approaches infinity; therefore, there is no finite mean.
If the severity distribution is per claim, then we have to assume so is the treaty.
If the severity distribution is per occurrence, then we have to assume so is the treaty.
One does not "use exposure rating to calculate increased limit factors for an auto liability treaty."

210. (a) Trended loss and ALAE in layer for each claim:
• (400,000) (1.05^4) = 486,203 < 500,000. ⇒ There is 0 loss in layer. ⇒ Ceded ALAE is 0.
• (750,000) (1.05^4) = 911,630. Loss in layer is: 911,630 - 500,000 = 411,630.
ALAE is (100,000) (1.05^4) = 121,551. Ceded ALAE is: (121,551) (411,630/911,630) = 54,884.
• (450,000) (1.05^3) = 520,931. Loss in layer is 520,931 - 500,000 = 20,931.
ALAE is 0, so ceded ALAE is 0.
• (900,000) (1.05^3) = 1,041,863. Loss in layer is: 1,000,000 - 500,000 = 500,000.
ALAE is (1,000,000) (1.05^3) = 1,157,625.
Ceded ALAE is: (1,157,625) (500,000/1,000,000) = 578,813.
• (500,000) (1.05^2) = 551,250. Loss in layer is 551,250 - 500,000 = 51,250.
ALAE is (100,000) (1.05^2) = 110,250. Ceded ALAE is: (110,250) (51,250/551,250) = 10,250.
• (1,000,000) (1.05^2) = 1,102,500. Loss in layer is: 1,000,000 - 500,000 = 500,000.
ALAE is 0, so ceded ALAE is 0.
Developed losses and ALAE for each year:
2012: (411,630 + 54,884) (1.10) = 513,165
2013: (20,931 + 500,000 + 578,813) (1.50) = 1,649,616
2014: (51,250 + 10,250 + 500,000) (3.00) = 1,684,500
Total is 3,847,281.
Rate is: 3,847,281 / 30,000,000 = **12.8%**.
(b) Exposure rating is an alternative approach. Additional information needed is:
Expected loss ratio, relevant increased limits factors,
and an assumption regarding how ALAE relates to the distribution of losses.
(c) The exposure rating approach should receive greater credibility. There are only one or two losses per year in the layer. This is not enough data to provide a credible experience rate.

211. 10% margin on annual premium is 1 million.
Profit Commission = (90%) (9 million - ceded losses), if positive.
Additional Premium: (50%) (Loss - 9 million), if positive.
(a) The nominal rate on line is 10,000,000/100,000,000 = **10%**.
(b) If there is a loss of 100 million, then the additional premium is:
(50%) (100M - 9M) = 45.5 million.
The underwriting loss is:
10M - 100M + 45.5M = **–44,500,000**.
(c) In the case of no loss, Profit Commission = (90%) (9 million - 0) = 8.1 million.
The loss free underwriting result is: 10M - 8.1M = 1,900,000.
We want an equivalent traditional risk cover with the same underwriting results.
We can charge a premium of **1.9 million** with no profit commission; this will produce the same result if there is no loss.
(d) If we change the occurrence limit to 46.4 million, then without a provision for additional premium, if there is a large loss the underwriting result is: 1.9M - 46.4M = -44.5M,
the same result as before.
The rate on line for the equivalent traditional risk cover is: 1.9M / 46.4M = **4.1%**.
(e) A loss will fully exhaust the limit once every 15 years and the probability of a partial loss is negligible. We are ignoring the reinsurer’s expenses and profit provision. (Section 6 of Clark “Calculating the Final Price” is not on the syllabus.) Therefore, we want the rate on line for the equivalent traditional risk cover to be more than: 1/15 = 6.67%.
Since 4.1% < 1/15, Property R Us should **reject the proposal**.
A counterproposal should include:
- An increase in premium;
- A decrease in profit commission;
- An increase in margin; and/or
- An increase in the additional premium

so that the rate on line for the equivalent traditional risk cover is greater than 1/15.
<u>Comment</u>: See page 47 of Clark.

212. (a) For L > 1 million, the expected payment in the layer from 1,000,000 to L is:

$$\int_{1 \text{ million}}^{L} S(x)\ dx = \int_{1 \text{ million}}^{L} \left(\frac{x}{1{,}000{,}000}\right)^{-2} dx = 1{,}000{,}000\ \{1 - 1{,}000{,}000/L\}.$$

The expected loss for the treaty is 0 if the umbrella policy limit is less than or equal to 1,000,000.
The second umbrella policy (ignoring drop down) covers the layer from 1M to 3M.
Therefore, the reinsurance treaty covers the layer from 2M to 3M.

Expected losses are: $(60\%)(4\text{M}) \dfrac{(1 - 1/3) - (1 - 1/2)}{(1 - 1/3) - (1 - 1/1)} = 2.4\text{M}\ (2/3 - 1/2) / (2/3) = 0.6\text{ M}.$

The third umbrella policy (ignoring drop down) covers the layer from 1M to 4M.
Therefore, the reinsurance treaty covers the layer from 2M to 3M.

Expected losses are: $(60\%)(2.25\text{M}) \dfrac{(1 - 1/3) - (1 - 1/2)}{(1 - 1/4) - (1 - 1/1)} = 1.35\text{M}\ (2/3 - 1/2) / (3/4) = 0.3\text{ M}.$

The fifth umbrella policy (ignoring drop down) covers the layer from 2M to 4M.
Therefore, the reinsurance treaty covers the layer from 3M to 4M.

Expected losses are: $(60\%)(1.5\text{M}) \dfrac{(1 - 1/4) - (1 - 1/3)}{(1 - 1/4) - (1 - 1/2)} = 0.9\text{M}\ (3/4 - 2/3) / (3/4 - 1/2) = 0.3\text{ M}.$

The expected losses for the treaty is: 0.6 M + 0.3 M + 0.3 M = **1.2 million**.
(b) Drop down exposure arises because umbrella policies provide coverage that would otherwise be provided by the underlying policy when the aggregate limit in the underlying policy is exhausted.
(c) A portion of the exposure, ϕ, would be rated assuming an underlying limit of zero.
(Clark calls ϕ the aggregate excess factor. The higher the aggregate limit, the smaller ϕ.)
(d) 1. The report lag may vary by company.
2. The mix of attachment points and limits may not be clearly broken out.
3. Data may or may not be exclusive of asbestos and environmental claims.
4, There may be inconsistent handling of tabular discounts of workers compensation claims.

213.

	Accident Year 2016		Accident Year 2017	
Simulation	Loss Ratio	Ceded Loss	Loss Ratio	Ceded Loss
1	73.9%	(5%)(700) = 35	68.0%	(3%)(750) = 22.5
2	64.8%	0	67.9%	(2.9%)(750) = 21.75
3	66.8%	(1.8%)(700) = 12.6	65.5%	(0.5%)(750) = 3.75
4	65.6%	(0.6%)(700) = 4.2	65.3%	(0.3%)(750) = 2.25
5	65.1%	(0.1%)(700) = 0.7	58.7%	0

(a) For 2016, 35% of the ceded premium is: (35%)(9.8) = 3.43.
Thus there is a profit commission if there is a ceded loss of less than 3.43.
Thus only in simulations #2 and #5 does the insurer terminate the contract.
The profit commission for simulation #2 is: 3.43M - 0 = 3.43M.
The profit commission for simulation #5 is: 3.43M - 0.7 = 2.73M.
Expected profit commission payable at the end of 2016 is: (3.43 + 2.73)/5 = **1,232,000**.
(b) In simulation #1 the profit commission is: (35%)(9.8 + 10.5) - (35 + 22.5) = -50.395. ⇒ 0.
In simulation #3 the profit commission is: (35%)(9.8 + 10.5) - (12.6 + 3.75) = -9.245. ⇒ 0.
In simulation #4 the profit commission is: (35%)(9.8 + 10.5) - (4.2 + 2.25) = 0.655.
Expected profit commission for the full term of the contract is:
(3.43 + 2.73 + 0.655)/5 = **1,363,000**.
Comment: I do not understand their division of points between parts (a) and (b).

214. We determine the trended losses in the reinsured layer:

Accident Date	Trended Losses	Loss in Layer
July 18, 2012	$3,850,000	2,850,000
February 12, 2013	$2,160,000	1,160,000
August 15, 2013	$1,070,000	70,000
March 3, 2014	$3,120,000	2,120,000
November 2, 2014	$929,200	0

Apply the Excess Loss Development Factor to each accident year:
(2,850,000)(1.01) = 2,878,500.
(1,160,000 + 70,000)(1.05) = 1,291,500.
(2,120,000 + 0)(1.10) = 2,332,000.

$$\text{Experience Rate} = \frac{2{,}878{,}500 + 1{,}291{,}500 + 2{,}332{,}000}{5{,}200{,}000 + 5{,}700{,}000 + 5{,}900{,}000} = 38.7\%.$$

However, there are no trended losses in the layer from 4M to 10M.
Therefore, we would be providing a free cover; to avoid that we use the exposure curve.
The exposure rate for the layer from 1M to 4M is:
(63% - 30%) (expected loss ratio) = (33%) (expected loss ratio).
The exposure rate for the layer from 4M to 10M is:
(90% - 63%) (expected loss ratio) = (27%) (expected loss ratio).
Estimate rate for 4M to 10M layer: (38.7%) (27%/33%) = 31.7%.
Thus the policy's loss cost as a percentage of premium is: 38.7% + 31.7% = **70.4%**.
Comment: "Candidates who selected prorated layers other than 3M xs 1M and 6M xs 4M were also given credit as long as selection was reasonable and/or justified."

215. (a) For the smaller properties, the insurer cedes nothing under the surplus share treaty.
In the case of the 1 million property, under the surplus share treaty the primary insurer will cede half of the premiums and losses.
Treat the insured property size of the 1 million property as the retained line of 500,000 from the surplus share treaty; use the exposure factors for 100K / 200K = 20%, and 500K / 500K = 100%.
Alternately, the ceding percent for the 1 million property is: (1000 - 500) / 1000 = 1/2.
Thus if the insurer has a 200K loss, the loss net of the surplus share reinsurance is:
200K / 2 = 100K.
Thus the lower end of the excess layer corresponds to: 200K / 1000K = 20% of insured value.
If the insurer has a 1000K loss, the loss net of the surplus share reinsurance is:
1000K / 2 = 500K.
Thus the upper end of the excess layer corresponds to: 500K / 1000K = 100% of insured value.
In each case, the exposure factor for the layer is the difference of the exposure factors at the top and the bottom of the layer.

Insured Value	Exposure Factor for the Excess Layer
100,000	$1 - \{1 - \left(1 - \frac{1}{1.2}\right)^2\} = 0.027778.$
200,000	$1 - \{1 - \left(1 - \frac{0.5}{1.2}\right)^2\} = 0.340278.$
500,000	$1 - \left(1 - \frac{1}{1.2}\right)^2 - \{1 - \left(1 - \frac{0.2}{1.2}\right)^2\} = 2/3.$
1,000,000	$1 - \left(1 - \frac{1}{1.2}\right)^2 - \{1 - \left(1 - \frac{0.2}{1.2}\right)^2\} = 2/3.$

The gross expected loss for each size category is: (60%)(1 million) = 600,000.
However, net of the surplus share treaty, the expected losses for the 1 million properties are only: (50%)(600,000) = 300,000.
Thus the expected loss in the layer of the excess treaty is:
(600,000) (0.027778 + 0.340278 + 2/3) + (300,000) (2/3) = **820,834**.
(b) There may be no trended losses in for example the layer from 400K to 500K. In that case the experience rating approach would indicate zero premium for this part of the excess layer. Thus, in that case the cover would turn out to be free. A reinsurer should not give away additional coverage for free!
(c) The answer is no. The reverse is typically true. The surplus share reinsurance will inure to the benefit of the property catastrophe cover.
(d) The answer is no. The reverse is typically true. The property per risk excess treaty will inure to the benefit of the property catastrophe cover.
<u>Comment</u>: We are assuming that the same exposure curve is appropriate for all of the different sizes of properties we have here.
While there is a typical pattern, which reinsurance treaties inure to the benefit of other treaties would be specified in the language of the reinsurance treaties. The property catastrophe treaty provides protection against many insured properties being damaged by the same catastrophe, and is intended to step in after the other elements of the primary insurer's reinsurance program have provided their protection to the primary insurer.

216. (a)
(i) A sliding scale commission is a percent of premium paid by the reinsurer to the ceding company which "slides" with the actual loss experience, subject to set minimum and maximum amounts.
For example, 25% above a 65% loss ratio, slides 1:1 to be 35% at a 55% loss ratio, then slides 0.5:1 to be 45% at a 35% loss ratio, at which point it remains constant.
For example, at a 62% loss ratio, the commission is: 25% + (1)(65% - 62%) = 28%.
For example, at a 40% loss ratio, the commission is 45% - (1/2)(40% - 35%) = 42.5%.
(ii) A profit commission subtracts the actual loss ratio, ceding commission and a "margin" for expenses from the treaty premium and returns a percent of this as additional commission.
For example, assuming an actual loss ratio of 62%, a ceding commission of 20%, a margin of 8%, and a percent returned of 40%.
The profit commission is: (40%) (100% - 62% - 20% - 8%) = 4%.
(iii) A loss corridor provides that the ceding company will reassume a portion of the reinsurer's liability if the loss ratio exceeds a certain amount. For example, the corridor may be 70% of the layer from an 80% to a 90% loss ratio. If the reinsurer's loss ratio is 105% before the application of the loss corridor, then after the application of the loss corridor, the reinsurer will have a net ratio of: 80% + (30%)(10%) + (105% - 90%) = 98%.
If instead the reinsurer's loss ratio is 86% before the application of the loss corridor, then after the application of the loss corridor, the reinsurer will have a net ratio of:
80% + (30%)(6%) = 81.8%.
(b) (i) Empirical distribution:
• The experience does not take into account all possible outcomes, and may miss the possibility of events outside of what has been observed.
• If the volume or mix of business has been changing, then the volatility of the future period may be very different than the historical period.
• If loss development has been performed using a Bornhuetter-Ferguson or Cape Cod method, then the historical periods may present an artificially smooth sequence of loss ratios that does not reflect future volatility.
(ii) Single distribution model:
• There is no provision for the loss free scenario.
• There is no easy way to reflect the impact of changing per occurrence limits on the aggregate losses.
(iii) Recursive formula:
• Calculation is difficult when the expected frequency is high.
• Only one severity distribution can be used.
(c) • Include the carryforward from past years and estimate its effect on the current year only. This approach ignores the potential effect on later years.
• Look at the long run of the contract; apply to a longer block of years, incorporating a reduction in the variance. For this method there is no obvious way to reduce the variance. A second problem is that it ignores the fact that the treaty may not renew the following year, potentially leaving the reinsured with no carryforward benefit.
<u>Comment</u>: In part (b) one need only list one disadvantage for each approach.
In part (c), either of the two possible approaches is acceptable as an answer.

217. (a) Of the standard premium: 30% is State X Hazard Group A, 30% is State X Hazard Group B, 20% is State Y Hazard Group A, and 20% is State Y Hazard Group B,

Combination	Weight	ELF 250K	ELF 1000K	Difference of ELFs	Loss Ratio
X and A	30%	0.0500	0.0125	0.0375	60%
X and B	30%	0.1000	0.0250	0.0750	60%
Y and A	20%	0.0500	0.0125	0.0375	70%
Y and B	20%	0.1000	0.0125	0.0750	70%

The loss cost rate for the treaty is:
(30%)(0.0375)(60%) + (30%)(0.0750)(60%) + (20%)(0.0375)(70%) + (20%)(0.0750)(70%) = **0.036**.
(b) Because workers compensation insurance does not have policy limits, no adjustment is needed. For discounting, loss data should be requested on a full undiscounted basis. Alternately, one can request from the primary insurer sufficient information for individual claimants in order to project their expected undiscounted costs into the treaty layer.
(c) Historical premiums should be adjusted for rate changes, development, and for exposure (payroll) inflation.
Historical losses should be adjusted for trend, development, and any benefit changes.
(Historical losses are not used in exposure rating rather than experience rating.)
Comment: The ELFs would usually differ between states.
At page 31, Clark multiples the ELF times the expected losses; that is what I have done here. However, in the NCCI definition, ELF times standard premium is the expected excess losses.

218. (a) From 30 to 50%, the commission is 30% and the average loss ratio is 40%.
From 50 to 70%, the average commission is 25% and the average loss ratio is 60%.
From 70 to 80%, the commission is 20% and the average loss ratio is 75%.
The weighted average technical ratio is:
{(20)(30% + 40%) + (20)(25% + 60%) + (10)(20% + 75%)} / 50 = **81%**.
Alternately, the average commission is: {(20)(30%) + (20)(25%) + (10)(20%)} / 50 = 26%.
The average loss ratio is: (30% + 80%)/2 = 55%.
Thus the expected technical ratio is: 26% + 55% = **81%**.
(b) From 30 to 60%, the average loss ratio is 45%, and nothing is reassumed.
From 60 to 70%, the average loss ratio is 65%, and the average amount reassumed is:
(50%)(5%) = 2.5%.
From 70 to 80%, the average loss ratio is 75%, and the amount reassumed is:
(50%)(10%) = 5%.
The weighted average of the loss ratio minus reassumed is:
{(30)(45%) + (10)(65% - 2.5%) + (10)(75% - 5%)} / 50 = 53.5%.
Thus the expected technical ratio is: 27.5% + 53.5% = **81%**.
(c) Option 1: From 30 to 50%, the technical ratio is uniform from 60% to 80%.
From 50 to 70%, the technical ratio is uniform from 80% to 90%.
From 70 to 80%, the technical ratio is uniform from 90% to 100%.
Second moment is:

$$\frac{(20)(0.8^3-0.6^3)/\{(3)(0.8-0.6)\} + (20)(0.9^3-0.8^3)/\{(3)(0.9-0.8)\} + (10)(1.0^3-0.9^3)/\{(3)(1.0-0.9)\}}{50}$$

= 0.66733. Thus for Option 1, the variance is: $0.66733 - 0.81^2 = 0.01123$.
For Option 2, the commission is constant, so we just need the variance of the loss ratios after reassumption.
From 30 to 60%, the loss ratio is uniform from 30% to 60%.
From 60 to 70%, the loss ratio after reassumption is uniform from 60% to 65%.
From 70 to 80%, the loss ratio after reassumption is uniform from 65% to 75%.
Second moment is:

$$\frac{(30)(0.6^3-0.3^3)/\{(3)(0.6-0.3)\}+(10)(0.65^3-0.6^3)/\{(3)(0.65-0.6)\}+(10)(0.75^3-0.65^3)/\{(3)(0.75-0.65)\}}{50}$$

= 0.30233.
The average loss ratio after reassumptions is:
{(30)(45%) + (10)(62.5%) + (10)(70%)} / 50 = 53.5%
Thus for Option 2, the variance is: $0.30233 - 0.535^2 = 0.01611$.
Option 2 has the larger variance.
<u>Comment</u>: Option 2 is an example of a loss corridor.
Given that the two options have the same mean technical ratio, since Option 1 has the smaller variance the reinsurer may prefer it.

219. (a) For the first loss, A pays 100 and B pays 80. The insurer retains 200.
For the second loss, A pays 60, and the insurer retains 200.
For the third loss, since there was only one reinstatement, A pays only 100 - 60 = 40;
B pays 100 and C pays 100. Thus the insurer retains: 600 - 40 - 100 - 100 = 360.
(b) The premium for A is: (10%)(100) = 10. The premium for B is: (7%)(100) = 7.
The premium for C is: (4%)(100) = 4.
After the first loss, the insurer pays reinstatement premiums of (100/100)(10) = 10 to A,
and (80/100)(7) = 5.6 to B.
After the second loss, the insurer pays no reinstatement premium to A, since there is only one reinstatement.
After the third loss, the insurer pays no reinstatement premium to A, since there is only one reinstatement and the coverage is exhausted. Reinsurer B receives a second reinstatement premium on loss event 3 as they had not fully exhausted their limit after loss event 1 and therefore only a partial reinstatement premium was paid after that loss event; the insurer pays B a reinstatement premium of: 7 - 5.6 = 1.4 = (20/100)(7).
The insurer pays a reinstatement premium to C of: (100/100)(4) = 4.
<u>Comment</u>: Presumably all of the figures are actually in millions of dollars.
Beyond the level of detail covered in Clark on reinstatement clauses.
I have assumed that A does not inure to the benefit of B, nor do A and B inure to the benefit of C;
this is the way an insurer would set up a cat coverage, with the layer from 200 to 500 covered.
Cat treaties often have annual aggregate limits; here with only one reinstatement each reinsurer's payment during the year is capped at twice its per event limit.
Reinsurer B would pay up to (2)(100) - 80 - 100 = 20 for a fourth catastrophe.

220. (a) Trended losses are limited to the policy limit of 1,000,000; this affects the fourth accident.
For example for the second accident, $(400,000)(1.06^4) = 504,991$.
504,991 - 400,000 = 104,991 is in the layer 200K xs 400K.
(252,495)(104,991/504,991) = 52,495, the ALAE proportionally allocated to the treaty.

Accident Date	Untrended Loss	Trended Loss	Trended Loss in Layer	Untrended ALAE	Trended ALAE	Trended ALAE Allocated
7/1/2014	200,000	252,495	0	100,000	126,248	0
7/1/2014	400,000	504,991	104,991	200,000	252,495	52,495
7/1/2015	550,000	655,059	200,000	0	0	0
7/1/2015	1,000,000	1,000,000	200,000	500,000	595,508	119,102
7/1/2016	600,000	674,160	200,000	300,000	337,080	100,000
7/1/2016	450,000	505,620	105,620	0	0	0

Developed loss and ALAE in the layer are:
AY2014: (104,991 + 52,495) (1.10) = 173,235
AY2015: (200,000 + 200,000 + 119,102) (1.50) = 778,653
AY2016: (200,000 + 100,000 + 105,620) (2.00) = 811,240
Total is 1,763,128
The percentage of subject premium is 1,763,128 / 30,000,000 = **5.9%**.
(b) For both cases, revised and likely smaller development factors would be needed.
For the layer 200,000 excess of 200,000, information about untrended losses below 200,000 would be needed. Assuming the 6% trend factor applies to these losses, untrended losses of $200,000/1.06^4 = 158,419$ (likely rounded to 150,000) or larger would be required.

221. (a) One needs to use the Panjer Algorithm / Recursive Method.
With the probability of severity of zero, $S_0 = 0$, then as shown at page 41 of Clark:

$$A_k = \sum_{i=1}^{k} (a + b\ i/k)\ S_i\ A_{k-i}\ .$$

For the Poisson Distribution, a = 0 and b = λ.

Thus, $A_k = (\lambda/k) \sum_{i=1}^{k} i\ S_i\ A_{k-i}$.

$A_{10} = (2/10)\ \{1\ S_1A_9 + 2\ S_2A_8 + 3\ S_3A_7 + 4\ S_4A_6\}$
= (0.2) {(1)(0.4)(0.0318) + (2)(0.3)(0.0453) + (3)(0.2)(0.0621) + (4)(0.1)(0.0804)} = **0.021864**.
(b) Mean severity is: (0.4)(1) + (0.3)(2) + (0.2)(3) + (0.1)(4) = 2.
Second Moment of severity is: $(0.4)(1^2) + (0.3)(2^2) + (0.2)(3^2) + (0.1)(4^2) = 5$.
Mean Aggregate = λ (Mean Severity) = (2)(2) = **4 billion**.
Variance of Aggregate = λ (Second Moment of Severity) = (2)(5) = 10 billion^2.
CV = Standard Deviation / Mean = $\sqrt{10}$ /4 = **0.7906**.

(c) mean = $\exp[\mu + \sigma^2/2]$ = 4 billion.
$\exp[\sigma^2] = 1 + CV^2 = 1 + 0.7906^2$. ⇒ σ = **0.6968**. ⇒ μ = ln(4 billion) - $0.6968^2/2$. = **21.87**.

Alternately, set the second moment of aggregate equal to the second moment of the LogNormal:
$\exp[2\mu + 2\sigma^2]$ = 10 billion^2 + (4 billion)^2 = 26 billion^2 = 26 x 10^{18}. ⇒ $2\mu + 2\sigma^2 = 44.7046$.

Also, $\exp[\mu + \sigma^2/2]$ = 4 billion. ⇒ $\mu + \sigma^2/2 = 22.1096$.

⇒ $\sigma^2 = 0.4854$. ⇒ σ = **0.6967**. ⇒ μ = ln(4 billion) - $0.6967^2/2$ = **21.87**.

222. (a) The Table M charge accounts for the effect of aggregate losses excess of the equivalent of the maximum premium, while the aggregate excess factor accounts for the effect of aggregate losses excess of the aggregate limit of the underlying policy.
The aggregate excess factor is the average amount of loss in excess of the aggregate limit, divided by the expected loss. The Table M charge factor is the average amount of loss in excess of r_G times the expected loss, divided by the expected loss. Hence, they measure the same thing, with a slightly different definition of the point above which the excess is calculated.
(b) First handle those cases where the umbrella policy does not drop down due to the exhausting of the aggregate limit.
For the first set of policies the umbrella covers the layer from 1M to 2M.
Thus the excess treaty covers the layer from 1.5M to 2M.
For the second set of policies the umbrella covers the layer from 1M to 3M.
Thus the excess treaty covers the layer from 1.5M to 2M.
Now handle those cases where the umbrella policy drops down.
In both cases, the excess treaty covers the layer from 0.5M to 1M.
For the first set of polices, the umbrella covers the layer from 0 to 1M.
For the second set of polices, the umbrella covers the layer from 0 to 2M.
Thus for the first set of policies, the exposure factor is:

$$\frac{(0.9)(2 - 1.8) + (0.1)(1.5 - 1)}{(0.9)(2 - 1.5) + (0.1)(1.5 - 0)} = 0.383.$$

Expected ceded losses are: (60%)(1,200,000)(0.383) = 276,000.
For the second set of policies, the exposure factor is:

$$\frac{(0.9)(2 - 1.8) + (0.1)(1.5 - 1)}{(0.9)(2.15 - 1.5) + (0.1)(2 - 0)} = 0.293.$$

Expected ceded losses are: (60%)(785,000)(0.293) = 138,000.
Total expected ceded losses are: 276,000 + 138,000 = **414,000**
(c) A clash cover.
(d) i. Extra-contractual obligations (ECO), in other words bad faith claims.
ii. Excess of policy limit damages (XPL).
iii. The clash of claims arising from one or more loss events involving multiple policies.
iv. ALAE being included with losses in the treaty and the total exceeding the policy limit of the umbrella policy.

223. (a) Using the notation in Clark, the frequency distribution is a Negative Binomial with $1 - p = 0.5$ and $\alpha = 1$. Therefore, in the Panjer algorithm / recursive method:
$a = 1 - p = 1/2$, and $b = (\alpha-1)(1-p) = 0$.
With the probability of severity of zero, $S_0 = 0$, then as shown at page 41 of Clark:

$$A_k = \sum_{i=1}^{k} (a + b\, i/k)\, S_i\, A_{k-i} = 0.5 \sum_{i=1}^{k} S_i\, A_{k-i}.$$

$A_{10} = (0.5)\{(0.4)(0.0119) + (0.3)(0.0166) + (0.2)(0.0231) + (0.1)(0.0311)\} = \mathbf{0.008735}$.
(b) The mean severity is: (1)(0.4) + (2)(0.3) + (3)(0.2) + (4)(0.1) = 2 billion.
Mean aggregate = (1)(2 billion) = **2 billion**.
The second moment of severity is: $(1^2)(0.4) + (2^2)(0.3) + (3^2)(0.2) + (4^2)(0.1) = 5$ billion^2.
Variance of severity is: $5 - 2^2 = 1$ billion^2.
Variance of the aggregate = (1)(1 billion^2) + (2)(2 billion)^2 = 9 billion^2.
Coefficient of variation of aggregate is: $\sqrt{9}/2 = \mathbf{1.5}$.
(c) ● Only a single severity distribution can be used in the analysis.
● For higher expected frequencies, the calculation is inconvenient because all the probabilities up to the desired level must be calculated.
Comment: The given frequency distribution is called a Geometric, a special case of a Negative Binomial Distribution. Using the notation in Loss Models, r = 1 and $0.5 = \beta/(1+\beta)$.
Using the notation in Bahnemann r = 1 and q = p = 1 - 0.5.

222. (a) 2012: (12,017)(1.3) = 15,622. So premium would be the minimum \$100,000.
Loss Ratio = 12,017/100,000 = 12.017%.
2013: (0)(1.3) = 0. So premium would be the minimum \$100,000. Loss ratio = 0%.
2014: (204,895)(1.3) = 266,364. This is the premium.
Loss ratio is: 204,895/266,364 = 76.923%.
2015: (500,000)(1.3) = 650,000. So the premium would be the maximum \$325,000.
Loss ratio is: 500,000/325,000 = 153.846%.
2016: (154,158)(1.3) = 200,405. This is the premium.
Loss ratio is: 154,158/200,405 = 76.923%.
I assume that “using equal weights for each year” means averaging the five loss ratios:
(12.017% + 0 + 76.923% + 153.846% + 76.923%)/5 = **63.94%**.
(b) I would not rely on the empirical distribution to price this account. There are only 4 claims in five years that pierce the layer; thus there is a lot of uncertainly in the estimate of the frequency of such large claims. Also only one of the claims exhausts the layer, so again there is much uncertainty in the estimate of how often this would be expected to happen in the future. The data is not credible.
Alternately, I would not rely on the empirical distribution to price this account.
The on-level premiums of the primary insurer have increased tremendously over the historical period. I assume this is due to the expected annual frequency increasing tremendously (presumably due to increased exposures.) Therefore, one can not combine these years in order to usefully predict the future.

(c) 1. One would have determine the frequency distribution to use, both the form and the parameters. One would have to determine the severity distribution to use, both the form and the parameters. This is often difficult to do with limited data, particularly when we are interested in the righthand tail. (In this case we only have 17 claims in 5 years.)
2. Most models assume that each occurrence is independent of the others and that the frequency and severity distributions are independent of each other. This may be a reasonable assumption in many cases, but could be false in others.
3. The aggregate distribution reflects the process variance of losses but does <u>not</u> reflect the full parameter variance, let alone the risk of whether one has picked the correct model.
4. The complexity of the calculations can lead to a "black box" mentality - assuming the numbers must be right because of the accuracy of the computer. Be sure to check results for reasonableness.
5. Some collective risk models use numerical methods with a large error term for low frequency scenarios. Check the output of the model; the expected error term should be given.
6. Panjer's Recursive Method is an example of a collective risk model. For higher expected frequencies, the calculation is inconvenient because all the probabilities up to the desired level must be calculated.
7. Panjer's Recursive Method is an example of a collective risk model. Only a single severity distribution can be used in the analysis.
<u>Comment</u>: For part (c), see pages 41-42 of Clark.
If the on-level premiums had not grown so rapidly during the historical data period, I think one could make a case that it is okay to use the empirical distribution to price this account. The reinsured layer is a working layer that is relatively narrow. Also the swing in the plan is relatively small. Therefore, the reinsurer would get a reasonable estimate of the different types of years and the premiums it can expect in the future for this account. What other information/technique would the actuary use instead that would be superior? Of course the actuary would not have to rely on the estimate from a single technique.

225. (a) First apply the surplus share treaty and then apply the excess treaty:

	A	B	C	D	E	Total
Property Value	2000	20,000	8000	12,500	4000	
Loss	400	16,000	3200	12,500	1200	33,300
Surplus Insured Value	0	10,000	6000	10,000	2000	
Surplus Ceded %	0	50%	75%	80%	50%	
Surplus Ceded Loss	0	8000	2400	10,000	600	21,000
Surplus Retained Loss	400	8000	800	2,500	600	12,300
XS Ceded	0	4000	0	1500	0	5,500
XS Retained	400	4000	800	1000	600	6,800

The total losses that are subject to the catastrophe treaty are:
400 + 4000 + 800 + 1000 + 600 = 6800.
Thus the catastrophe treaty pays: 6800 - 6000 = 800.
The insurer retains the remaining 6000.
(b) Reinstatement premium = (premium) (loss in layer / layer) (factor) =
(600) (800/8000) (1.25) = **75**.
(c) Reinstatement pro-rata as to time is uncommon and usually inappropriate for windstorm coverage, which is seasonal; thus the exposure to risk is not uniform over the coverage period.
(d) A losses occurring basis provides coverage for events that occur in the treaty coverage period, regardless of when policies are written. Risks attaching basis provides cover for events covered by policies written during the treaty coverage period. This can lead to the reinsurer paying twice the treaty layer for a single event.
“Consider a treaty renewing on 1/1/95 for a layer of $10,000,000. A loss event takes place on 3/15/95. The ceding company has policies that are affected, some effective 7/1/94 and some effective 1/1/95. The catastrophe reinsurance treaty effective 1/1/94 covers the losses on the 7/1/94 policies and the treaty effective 1/1/95 covers the losses on the 1/1/95 policies. The reinsurer may end up paying $20,000,000 for the single event.”
In contrast, if written on a losses occurring basis, only the 1995 Treaty will cover a 1995 hurricane.

(e) Excess treaty insures the layer from 1000 to 5000.
(i) Property A has value 2000, and expected gross losses of (5%)(2000) = 100.
The surplus share treaty has no effect.
Bottom of layer: 1000/2000 = 50% ⇔ 70% exposure factor.

Top of layer: 5000/2000 = 250% ⇔ 100% exposure factor.

Expected ceded losses for the excess treaty: (100)(100% - 70%) = **30**.
(ii) Property B has value of 20,000.
The surplus share treaty results in 50% of each loss being ceded.
The expected losses retained from the surplus share treaty: (50%)(5%)(20,000) = 500.
Bottom of layer: 1000/10,000 = 10% ⇔ 37% exposure factor.

Top of layer: 5000/10,000 = 50% ⇔ 70% exposure factor.

Expected ceded losses for the excess treaty: (500)(70% - 37%) = **165**.

226. (a) Aggregate losses of 8 can occur only if there are two claims and both are for 4.
The probability that this happens is: $(0.25)(0.25^2)$ = **0.015625**.
The only possible aggregate losses are 0, 1, 2, …, 8.
Thus the probability that aggregate losses are 4 is 1 minus the sum of the other probabilities, which is **0.171875**.
Alternatively, an aggregate loss of 4 can arise from a single claim of 4, two claims with values 1 and 3 in either order, or two claims with values 2 and 2.
The total probability is: $(0.5)(0.25) + (0.25)(2)(0.25^2) + (0.25)(0.25^2)$ = **0.171875**.
Alternately, using the notation in Clark, the frequency distribution is binomial distribution with M = 2 and p = 0.5. Therefore, in the Panjer algorithm / recursive method:
a = p/(p-1) = -1, and b = (M+1)p/(1- p) = 3.
With the probability of severity of zero, $S_0 = 0$, then as shown at page 41 of Clark:

$$A_k = \sum_{i=1}^{k} (a + b\ i/k)\ S_i\ A_{k-i} = \sum_{i=1}^{k} (-1 + 3\ i/k)\ S_i\ A_{k-i}.$$

A_4 = (-1 + (3)(1/4))(0.25)(0.156250) + (-1 + (3)(2/4))(0.25)(0.140625)
+ (-1 + (3)(3/4))(0.25)(0.125000) + (-1 + (3)(4/4))(0.25)(0.250000) = **0.171875**.
A_8 = (-1 + (3)(1/8))(0.25)(0.031250) + (-1 + (3)(2/8))(0.25)(0.046875)
+ (-1 + (3)(3/8))(0.25)(0.062500) + (-1 + (3)(4/8))(0.25)(0.171875) = **0.015625**.
(b) "The recursive formula has the major advantage of being simple to work with and providing an accurate handling of low frequency scenarios. The number of points evaluated on the severity distribution can be expanded to closely approximate continuous curves."
(c) Mean frequency is: (2)(0.5) = 1. Variance of Binomial frequency is: (2)(0.5)(1 - 0.5) = 0.5.
Mean severity is 2.5.
Variance of severity is: $(0.25)(1.5^2) + (0.25)(0.5^2) + (0.25)(0.5^2) + (0.25)(1.5^2) = 1.25$.
Mean aggregate is: (1)(2.5) = **2.5 billion**.
Variance of aggregate: $(1)(1.25) + (2.5^2)(0.5) = 4.375$ billion^2.
Coefficient of variation of aggregate is: $\sqrt{4.375}$ / 2.5 = **0.837**.

<u>Comment</u>: The recursive method is easily programmed on a computer.

227. (a) The sliding scale commission is 10%, because the loss ratio is 65% ≥ 60%.
Premium - loss ratio - sliding scale commission - expenses = 100% - 65% - 10% - 15% = 10%.
Thus the profit commission is: (40%)(10%) = 4%.
The technical ratio is: 65% + 10% + 4% = **79%.**
(b) The 2017 loss ratio of 65% was 5% in excess of 60%. We shift the ranges in the schedule for the sliding scale commission by adding this 5% carryforward to the 2018 loss ratio.
Loss ratio bands are 35-40 (probability 1/8), 40-60 (probability 4/8), and 60-75 (probability 3/8).

Loss Ratio	Probability	Average L.R.	Commission	Profit Commission
35% to 40%	0.125	37.5%	20.00%	11.00%
40% to 60%	0.5	50%	15.00%	8.00%
60% to 75%	0.375	67.5%	10.00%	3.00%
Average			13.75%	6.50%

For example, (0.4)(100% - 37.5% - 20% - 15%) = 11%.
The average loss ratio (without the carryforward) is: (30% + 70%)/2 = 50%.
The average technical ratio is: 50% + 13.75% + 6.5% = **70.25%.**
(c) ● One can calculate the effect of the carryforward over a single year, but this ignores the potential for carryforward beyond the current year.
● One can instead try to calculate the long run effect. However, it is difficult to take into account the uncertainty about whether the treaty will be renewed.
● In order to apply the long run approach, one would have to reduce the variance of the aggregate distribution, but there is no obvious method for doing so.

228. (a) Trend each loss, apply the layer, and then apply development.

Accident Year	Untrended Loss	Trended Loss	Loss in Layer	Developed Loss
2015	1,100,000	1,388,725	388,725	388,725
2015	2,200,000	2,777,449	1,777,449	1,777,449
2016	900,000	1,071,914	71,914	86,297
2016	1,400,000	1,667,422	667,422	800,907
2017	800,000	898,880	0	0
2017	1,800,000	2,022,480	1,022,480	1,533,720
Total				4,587,098

The on-level subject premium for the three years is: (3)(6,000,000) = 18,000,000.
The experience rate is: 4,587,098 / 18,000,000 = **0.2548**.
(b) Free cover occurs when no losses trend into the highest portion of the layer covered.
In that case, the primary insurer would be getting additional coverage for no additional premium.
(c) There are no trended losses in the layer 3M to 5M.
The layer from 1M to 3M is from 20% to 60% of the insured value of 5M.
The exposure rate for the layer 1M to 3M is: 80% - 50% = 30%.
The layer from 3M to 5M is from 60% to 100% of the insured value of 5M.
The exposure rate for the layer 3M to 5M is: 95% - 80% = 15%.
Thus based on the experience rate for the layer 1M to 3M of 0.2548, I estimate a rate for the layer 3M to 5M of: (15%/30%) (0.2548) = **0.1274**.
The total rate for the layer 1M to 5M is: 0.2548 + 0.1274 = **0.3822**.
(d) Using these exposure factors would imply that the factors are scale invariant. While this would be reasonable for homeowners insurance, it is **not reasonable** for commercial property.
Comment: "This assumption of scale independence may be appropriate for homeowners business, for which this technique was first developed, but may be a serious problem when applied to large commercial risks."

229. (a) Claim A: 1000 + 550 = 1550. Claim B: 300 + 1050 = 1350.
Total = 1550 + 1350 = **2900**.
(b)

Claim	Gross Paid	Reinsurance	Paid in CY16 Net
A	500	(500) (0.6) = 300	500 - 300 = 200
B	600 + 450 = 1050	(1050) (0.6) = 630	1050 - 630 = 420
C	1200	0	1200
D	400	0	400
Total			**2220**

(c) Claim A: 500 + (225 - 550) = 175. Claim B: 600 + 450 + (150 - 1050) = 150.
Claim C: 1200 + 575 = 1775. Claim D: 400 + 900 = 1300.
Total = 175 + 150 + 1775 + 1330 = **3400**.
(d) For Claim C: Total Ceded - CY 2016 Ceded = CY 2017 Ceded

Claim	CY 2017 Gross Reported	CY 2017 Reinsurance Ceded	CY 2017 Net Reported
A	725 + (0 - 225) = 500	(0.6)(725 - 225) = 300	500 - 300 = 200
B	0	0	0
C	700 + 200 + (0 - 575) = 325	(1200 + 700 + 200 - 1500) - (1200 + 575 - 1500) = 325	325 - 325 = 0
D	800 + (625 - 900) = 525	400 + 800 + 625 - 1500 = 325	525 - 325 = 200
Total			**400**

Alternately, Claim A: (0.4)(725 - 225) = 200.
Claim B, since no incurred claims in CY 2017: 0.
Claim C, since retention reached in CY 2016: 0.
Claim D: 1300 incurred at start of CY 2017.
Gross incurred at end of CY 2017 = 1300 + 800 + (625 - 900) = 1,825.
Since the Excess of Loss limit is reached, net incurred = 1500 - 1300 = 200.
Total = 200 + 0 + 0 + 200 = **400**.

230. The reinsurer's expected loss ratio is: 90% - 20% - 5% - 1% - 1% = 63%.
The primary insurer's direct loss ratio is:
(37.3%)(0.03) + (53.2%)(0.21) + (66.1%)(0.55) + (91.1%)(0.21) = 67.777%.
Thus the loss corridor must have the primary insurer reassume: 67.777% - 63 = 4.777%.
If all the losses in the loss corridor were reassumed:
(66.1% - 60%)(0.55) + (20%)(0.21) = 7.555%.
Thus the percent of loss reassumed by the primary insurer in the loss corridor must be:
4.777%/7.555% = **63.2%**.
Comment: I see no educational value to making this a backwards question.

231. (a) $(0.6)\{(0)(3) + \frac{1.16 - 1.00}{1.16 - 0}(4) + \frac{1.28 - 1.00}{1.28 - 0}(5) + \frac{1.28 - 1.16}{1.28 - 1.00}(8) + \frac{1.38 - 1.16}{1.38 - 1.00}(9)\}$

= **6.17 million**.

(b) The average loss cost is: (0.10)(4%) + (0.75)(19%) + (0.15)(44%) = 21.25%.
The loaded loss costs are 100/80 times the average loss cost in each range:
5%, 23.75%, and 55%.
Applying the minimum and maximum changes them to 12.5%, 23.75%, and 37.5%.
The average is: (0.10)(12.5%) + (0.75)(23.75%) + (0.15)(37.5%) = 24.69%.

Range of Loss Cost	Probability	Average Loss Cost in Range	Loaded Loss Cost	Capped Premium
0-10%	0.10	4%	5%	12.5%
10-30%	0.75	19%	23.75%	23.75%
30%+	0.15	44%	55%	37.5%
Average		21.25%		24.69%

The expected loss ratio is: 21.25%/24.69% = **86.1%**.
(c) In a balanced swing plan, the expected loss ratio is the same as the reciprocal of the loading. In other words, on average putting in place the swing plan neither raises nor lowers the loss ratio.
(d) The provisional rate of 20% is well below the expected ultimate swing plan premium rate of 24.69%. This provides a cashflow advantage to the ceding company.
Comment: For part (b), see the top of page 35 of Clark.
For part (d), the cashflow advantage to the ceding company due to the low provisional rate, may have been part of the negotiation process that led to the writing of this treaty.

232. (a) In both cases the premium is 50 and the margin is 10%, or 5.
(i) If there are no losses, the loss is 0 and the profit is: 45 - 0 = 45.
The profit commission is: (0.95)(45) = 42.75.
⇒ Loss + margin - premium = 0 + 5 - 50 = -45. ⇒ Additional premium is 0.

⇒ Net profit = premium - loss - profit commission + additional premium

= 50 – 0 - 42.75 + 0 = **7.25 million**.
(ii) If there are one or more losses, since any single loss will fully exhaust the limit, the loss is 200 and the profit is: 45 - 200 = -155. The profit commission is 0.
⇒ loss + margin - premium = 200 + 5 - 50 = 155.

⇒ Additional premium is: (0.6)(155) = 93.

⇒ Net profit = premium - loss - profit commission + additional premium

= 50 - 200 - 0 + 93 = **-57 million**.
(b) With no loss the net profit is 7.25 while with a loss it is -57.
The equivalent traditional risk cover would have a premium of 7.25, and a layer of coverage of: 7.25 - (-57) = 64.25,
The rate on line is: 7.25/64.25 = **11.3%**.
(c) The no loss net profit is 7.25.
For a rate on line of 15%, the layer of coverage of the equivalent traditional risk cover would have to be: 7.25/0.15 = 48.333.
⇒ With a loss the net profit is 7.25 - 48.333 = -41.083.

⇒ Net profit = premium - loss - profit commission + additional premium

⇒ -41.083 = 50 - 200 - 0 + (additional premium). ⇒ additional premium = 108.917.

With a loss, Loss + Margin - Annual Premium = 200 + 5 - 50 = 155.
⇒ The additional premium percentage is: 108.917/155 = **70.3%**.

(d) ● The reinsurer must assume significant insurance risk.
● It must be reasonably possible that the reinsurer will realize a significant loss.
Comment: For part (b), see page 47 of Clark.
For part (d), see page 46 of Clark.

233. (a) Either one of the following:
● Increased capacity
● Enhance financial strength
(b) Any one of the following:
● Increased capacity
● Catastrophe protection
● Stabilize loss experience
(c) Insurer: 1,000,000 + (0.5)(1,000,000) = **1,500,000**.
Reinsurer: 4,000,000 - 1,500,000 = **2,500,000**.
(d) The insurer pays ALAE of: (1.5/4)(500,000) = 187,500.
The insurer pays a total of: 1,500,000 + 187,500 = **1,687,500**.
The reinsurer pays ALAE of: (2.5/4)(500,000) = 312,500.
The reinsurer pays a total of: 2,500,000 + 312,500 = **2,812,500**.
(e) Insurer: 1,000,000 + (0.5)(1,000,000) = **1,500,000**.
Reinsurer: 4,500,000 - 1,500,000 = **3,000,000**.
(f) If the claim amount is greater than 1,000,000 and less than 2,000,000, then the net amount retained by the insurer will be more under the alternative treaty than the current treaty.
For example, for a loss of 1,600,000:
● Current Treaty: insurer retention = 1,000,000 + (50%)(600,000) = 1,300,000.
● Alternative Treaty: insurer retention = 1,500,000.
<u>Comment</u>: In part (f), the amount retained by the insurer would be the same for a claim of size at most 1 million or for a claim of size at least 2 million.

234. (a) There can be additional coverages, for example business interruption coverage, that lead to total payments above the insured value.
(b) For an insured value of 200K, the layer 100K to 400K is 50% to 200%.
The exposure factors are 70% and 100%.
The expected loss is: (50%)(800K)(100% - 70%) = 120K.
For an insured value of 500K, the layer is 20% to 80%.
The exposure factors are 49% and 85%.
The expected loss is: (50%)(1000K)(85% - 49%) = 180K.
For an insured value of 1000K, the layer is 10% to 40%.
The exposure factors are 37% and 64%.
The expected loss is: (50%)(1200K)(64% - 37%) = 162K.
The total expected loss is **462,000**.
(c) The same exposure curve applies, regardless of the insured value.
(d) Only the third case is affected; the treaty shares: (1000K - 500K)/1000K = 50%.
Now for the excess treaty, the retained insured value is 500K and the retained premium is 600K.
The layer is 20% to 80% of the retained insured value.
The exposure factors are 49% and 85%.
The expected loss is: (50%)(600K)(85% - 49%) = 108K.
The revised total expected loss is **408,000**.
<u>Comment</u>: In part (d), it does not matter how many lines the surplus share treaty has, since the insurer only makes use of the first line for the properties with insured value of 1 million.
In part (c), "This assumption of scale independence may be appropriate for homeowners business, for which this technique was first developed, but may be a serious problem when applied to large commercial risks."

235. (a) E[N] = 1.3. E[X] = (0.6)(1) + (0.3)(2) + (0.1)(2) = 1.5. E[Agg] = (1.3)(1.5) = **1.95**.
$E[X^2] = (0.6)(1^2) + (0.3)(2^2) + (0.1)(3^2) = 2.7$. Var[Agg] = (1.3)(2.7) = **3.51**.
Coefficient of Variation of Aggregate = $\sqrt{3.51}/1.95$ = **0.961**.

(b) Using the Panjer algorithm:

$p(1) = \frac{1.3}{1}$ (1)(0.6)(0.2725) = **0.2126**.

$p(6) = \frac{1.3}{6}$ {(1)(0.6)(0.0498) + (2)(0.3)(0.0849) + (3)(0.1)(0.1399)} = **0.0266**.

(c) Set equal the empirical and theoretical first and second moments:
$1.95 = \exp[\mu + \sigma^2/2]$. $3.51 + 1.95^2 = 7.3125 = \exp[2\mu + 2\sigma^2]$.
Divide the second equation and by the square of the first equation.
$7.3125/1.95^2 = \exp[\sigma^2]$. $\Rightarrow \sigma^2 = 0.651$. $\Rightarrow \mu = \ln(1.95) - 0.654/2 = 0.341$.

(d) The normal distribution 95th percentile is: $0.341 + 1.645\sqrt{0.654} = 1.6713$.

The lognormal value is: exp(1.6713) = **5.319 billion**.
Comment: In part (b), one could instead enumerate all the ways in which the indicated aggregate loss can occur.

236. The average loss ratio is:
(0.15)(43%) + (0.25)(57%) + (0.45)(68%) + (0.15)(85%) = 64.05%.
For a loss ratio of 57%, the ceding commission is: 35% - 7%/2 = 31.5%.
For a loss ratio of 68%, the ceding commission is: 27.5% - 3% = 24.5%

Range of Loss Ratios	Average Loss Ratio in Range	Probability of Being in Range	Ceding Commission	Carry Forward
0% - 50%	43%	0.15	35%	
50% - 60%	57%	0.25	31.5%	
60% - 80%	68%	0.45	24.5%	
80% or above	85%	0.15	12.5%	5%

The average ceding commission for the first year is:
(0.15)(35%) + (0.25)(31.5%) + (0.45)(24.5%) + (0.15)(12.5%) = 26.025%
The reinsurer's expected technical ratio for the first year:
expected loss ratio + expected commission = 64.05% + 26.025% = **90.075%**.
If there is no carry-forward, then the average commission in year two is the same as that in year one: 26.025%.
On average there will be a 5% carry-forward when the loss ratio in the first year is above a loss ratio of 80%; this has a 15% chance. For the second year with a 5% carry-forward:

Range of Loss Ratios	Average Loss Ratio in Range	Probability of Being in Range	Average Loss Ratio plus 5%	Ceding Commission
0% - 50%	43%	0.15	48%	35%
50% - 60%	57%	0.25	62%	29%
60% - 80%	68%	0.45	73%	19.5%
80% or above	85%	0.15	90%	12.5%

For a loss ratio of 62%, the ceding commission is: 35% - 12%/2 = 29.0%.
For a loss ratio of 73%, the ceding commission is: 27.5% - 8% = 19.5%.
The average ceding commission for second year with 5% carry-forward is:
(0.15)(35%) + (0.25)(29%) + (0.45)(19.5%) + (0.15)(12.5%) = 23.15%.
Combining the two cases, the average commission for the second year is:
(15%)(23.15%) + (1 - 15%)(26.025%) = 25.594%.
The reinsurer's expected technical ratio for the second year: 64.05% + 25.594% = **89.644%**.

Comment: Unusually, here the ranges in the data do not match up with the breakpoints in the schedule of sliding scale commissions. This introduces a small inaccuracy in the calculation of the expected ceding commissions using the average loss ratio for a range.
In the calculation of the effect of the carry-forward, for the first interval, I have ignored cases where the loss ratio is for example 49%, and thus with the 5% carry-forward the ceding commission is less than 35%. Thus for the first interval, the average ceding commission would be slightly less than 35%.
In the calculation of the effect of the carry-forward, for the third interval, I have ignored cases where the loss ratio is for example 79%, and thus with the 5% carry-forward the ceding commission is 12.5%. Thus for the third interval, the average ceding commission would be slightly more than 19.5%.

237. (a) I will use the totals for the five years combined:

Historical Accident Year	On Level Subject Earned Premium ($ millions)	On Level Trended Ultimate Layer Loss Cost	On Level Trended Ultimate Layer Losses
2014	100	25%	25
2015	120	30%	36
2016	150	45%	67.5
2017	80	24%	19.2
2018	100	36%	36
Total	550		183.7

The experience rate is: 183.7/550 = **33.4%**.
This corresponds to: ($100 million)(33.4%) = **$33,400,000**.
(b) I will assume that the values x in the exposure curve are net of the quota share.
The bottom of the layer, $500,000, corresponds to a normalized value of 0.5.

$$G(0.5) = \frac{1 - 0.32428^{0.5}}{1 - 0.32428} = 0.63716.$$

The top of the layer, $1,000,000, corresponds to a normalized value of 1.

G(1) = 1.
G(1) - G(0.5) = 1 - 0.63716 = 0.36284.

Historical Accident Year	On Level Subject Earned Premium ($ millions)	On Level Trended Ultimate Subject Loss Ratio	On Level Trended Ultimate Losses
2014	100	75%	75
2015	120	75%	90
2016	150	90%	135
2017	80	75%	60
2018	100	80%	80
Total	550		440

$$\text{Primary Loss Ratio} = \frac{(75\%)(100) + (75\%)(120) + (90\%)(150) + (75\%)(80) + (80\%)(100)}{100 + 120 + 150 + 80 + 100}$$

$$= 440 / 550 = 80\%.$$

The exposure rate is: (0.36284)(80%) = **29.027%**.
This corresponds to: ($100 million)(29.027%) = **$29,027,000**.

(c) (80%)(0.334) + (20%)(0.29027) = **32.525%**.
(80%)($33,400,000) + (20%)($29,027,000) = **$32,525,000**.
(d) Since the ceding commission is equal to the insurer’s expenses, the Quota Share treaty will result in no underwriting profit or loss for the insurer. On this ceded business, the insurer keeps no premiums, pays no losses, and its expenses are covered by the ceding commission.
The subject premium net of the Quota share is $100 million.
The corresponding insurer expenses are $15 million.
The ceded loss ratio for this Excess layer is 90%.
Thus using the result of part (c), the reinsurer will charge: $32,525,000/0.9 = $36,138,000.
From previously, the insurer’s expected gross loss ratio is: 440/550 = 80%.
The corresponding expected losses net of the Quota Share are:
(80%)($100 million) = $80 million.
Then using the result of part (c), the expected losses net of the Excess Treaty are:
$80 million - $32.525 million = $47.475 million.
Thus the insurer’s expected net underwriting profit after application of both the existing Quota Share reinsurance treaty and the proposed Property Per Risk Excess reinsurance treaty:
$100 million - $15 million - $47.475 million - $36.138 million = **$1.387 million**.
(e) The quota share reinsurer’s profit =
100% - (actual loss ratio) - (15% ceding commission) - (5% margin) = 80% - (actual loss ratio).
The profit commission is equal to this times the ceded premium, when it is positive.
Since the Quota Share is 25%, the ceded premium is in each case (0.25)/(1-0.25) = 1/3 of the net premium. However, using the net premiums instead of the ceded premiums will not affect the weighted average.

Historical Accident Year	On Level Net Earned Premium ($ millions)	On Level Trended Ultimate Loss Ratio	Profit Commission
2014	100	75%	5
2015	120	75%	6
2016	150	90%	0
2017	80	75%	4
2018	100	80%	0
Total	550		15

5 year weighted average ratio of profit commission to ceded premium: 15/550 = **2.73%**.
(f) Advantage: If the insurer keep its loss ratios lower than 75%, the profit commission will be more favorable for the insurer.
Also, the profit commission can provide an incentive for risk control and the insurer can get money back if that risk control is successful.
Disadvantage: The increased ceding commission of 20% is 5% more than currently, while the profit commission would on average be only 2.73% more than currently. Thus on average the profit commission is less favorable for the insurer.
Also, the profit commission is not as stable as Option 2.

Comment: In part (a), one could instead use a straight average of the separate annual loss costs: (0.25 + 0.3 + 0.45 + 0.24 + 0.36) / 5 = 0.32.
“Ceded Loss Ratio for this Excess layer is estimated to be 90%.” This 90% is the expected loss ratio for the reinsurer, which will be used to get the price of the treaty from the expected ceded losses. This could have been made clearer in the question.
In part (b), we are using exposure rating to get the expected ceded losses; this only depends on the exposure factor and the primary insurer’s expected losses. The primary insurer’s expected losses are gotten from our estimate of the loss ratio for the primary insurer.
Under a proportional treaty, a profit commission usually returns to the insurer a portion of the profit of the reinsurer as additional commission. The margin is to take into account the expenses of the reinsurer.
I did not see a real advantage to the insurer of the profit commission compared to the increased ceding commission. There is no reason given as to why the reinsurer would be willing to consider either of the two proposed modifications.
The given exposure curve has the form discussed in Bernegger with bg = 1 and b = 0.32428.

238. (a)
● Longer lags common with the reporting and settlement of reinsured claims
● Severity of large claims above high attachment points
● Lower frequency at higher reinsured layers
● Greater variability inherent in the types of claims typically covered by reinsurance
● The fact that case estimates are set by numerous claims adjusters working at many different primary insurers, each with its own processes and philosophies about claim management
● Limitations in the extent of detailed information that is available for reinsured claims
● Development beyond ten years can be large and highly variant.
(b)
● A **swing plan** is a type of retrospective rating program. Actual losses to the layer are loaded for expenses and the result is charged back to the ceding company, subject to maximum and minimum constraints.
● A common adjustable feature is the **sliding scale commission**. A sliding scale commission is a percent of premium paid by the reinsurer to the ceding company which "slides" with the actual loss experience, subject to set minimum and maximum amounts.
● A **profit commission** subtracts the actual loss ratio, ceding commission and a "margin" for expenses from the treaty premium and returns a percent of this as additional commission.
● A **loss corridor** provides that the ceding company will reassume a portion of the reinsurer's liability if the loss ratio exceeds a certain amount.
(c)

Claim #	Ultimate Claims	Paid by PPP	Paid by RRR
1	1,000,000	1,000,000	0
2	5,000,000	1,000,000	4,000,000
3	6,000,000	1,000,000	5,000,000
4	12,000,000	7,000,000	5,000,000
5	5,000,000	0	5,000,000
6	1,000,000	0	1,000,000
Total	30,000,000	**10,000,000**	**20,000,000**

Once the primary insurer PPP reaches the AAD limit of 10,000,000, no further amounts are paid by PPP.
Comment: Part (a) is not discussed by Clark, but is a reason why it is usually harder to price reinsurance than primary insurance.
In part (b), describe only one provision.
In part (c), the payments made on claims by the primary insurer may not be in the order in which the claims were reported. Thus the situation can be more complicated than shown in my solution; nevertheless, the total amount paid by the primary insurer is capped at 10,000,000 and the reinsurer pays the rest.

239. (a) For Hazard Group J: a $250{,}000^{-b} = 0.032$. a $1{,}000{,}000^{-b} = 0.032$.
Dividing the two equations. ⇒ $4^b = 4$. ⇒ b = 1. ⇒ a = 8000.

For Hazard Group K: a $250{,}000^{-b} = 0.060$. a $1{,}000{,}000^{-b} = 0.020$.
Dividing the two equations. ⇒ $4^b = 3$. ⇒ b = ln[3] / ln[4] = 0.7925. ⇒ a = 1138.

(b) Use the results of part (a) to calculate ELFs for 200,000 and 800,000.
For Hazard Group J: 800/200,000 = 0.040. 800/800,000 = 0.010. 0.040 - 0.010 = 0.030.
For Hazard Group K: $1138/200{,}000^{0.7925} = 0.0716$. $1138/1{,}000{,}000^{0.7925} = 0.0239$.
0.0716 - 0.0239 = 0.0477.

State	Hazard Group	Standard Premium	Loss Ratio	Exposure Factor	Expected Treaty Losses
X	J	80,000	60%	0.0300	1,440
X	K	120,000	60%	0.0477	3,434
Y	J	110,000	80%	0.0300	2,640
Y	K	90,000	80%	0.0477	3,434
					10,949

Total premium is 400,000.
Loss cost rate = 10,949/400,000 = **2.74%**.
(c) Because workers compensation insurance does not have policy limits, no adjustment is needed.
For discounting, loss data should be requested on a full undiscounted basis.
Alternately, one can request from the primary insurer sufficient information for individual claimants in order to project their expected undiscounted costs into the treaty layer.

240. (a) There will often be disagreement between the ceding company and reinsurer about the expected loss ratio and the appropriate ceding commission. An adjustable feature can resolve these differences.
(b) Sliding scale commission: For example, commission of 25% at a loss ratio (LR) above 65% that slides 1:1 to be 35% at a 55% LR, then 0.5:1 to be 45% at a 35% LR, then constant.
So, at a 40% LR, it is 15% below 55% and so at 0.5:1 adds 7.5% to the commission, for a commission of: 35% + 7.5% = 42.5%.
Profit commission: Begin with 100%, then subtract the actual Loss Ratio, for example 55% for 45% left. Then subtract the ceding commission, for example 23%, for 22% left. Then subtract a margin, for example 10%, for a profit of 12%. The profit commission is a fixed percentage for example 50% of the profit, so in that case the profit commission is 6%.
Loss corridor: The ceding company reassumes a portion of the liability if the Loss Ratio exceeds a certain amount. For example, the corridor may be 75% of the layer from 80% to 90%. If the actual LR is 84%, then the ceding company will reassume 75% of 84% - 80%, or 3%.
(c) Empirical distribution disadvantages (only need to give one):
● Does not account for all possible outcomes.
● If the volume or mix of business has been changing, then the volatility of the future period may be very different than the historical period.
● If using Bornhuetter-Ferguson or Cape Cod method, it may provide an artificially smooth sequence of loss ratios that does not reflect future volatility.
Single (aggregate) distribution model disadvantages (only need to give one):
● No provision for a positive probability at zero.
● Difficult to reflect changes in per-occurrence limits.
● Cannot split into frequency and severity components.
(d) Approaches to pricing the impact of a carryforward provision (only need to give one):
● Include the carryforward from past years and estimate its effect on the current year only.
● Look at the long run of the contract. The sliding scale is modeled as applying to a longer block of years rather than just the single current year. The variance of the aggregate distribution would be reduced on the assumption that individual bad years would be smoothed by good experience on other years.
(e) ● The first approach ignores the potential effect on later years.
● The second approach has no obvious way to reduce the variance.
● The second approach does not reflect the possibility that the contract may not renew.

241. The trend period from 2017 to 2021 is 4 years.

Accident Date	Untrended Loss	Trend Factor	Trended Losses	Trended Loss in Layer	% of Losses in the Layer
July 1, 2017	200,000	1.2155	243,101	0	0.00%
July 1, 2017	350,000	1.2155	425,427	175,427	41.24%
July 1, 2018	225,000	1.1576	260,466	10,466	4.02%
July 1, 2018	900,000	1.1576	1,041,863	750,000	71.99%
July 1, 2019	250,000	1.1025	275,625	25,625	9.30%
July 1, 2019	800,000	1.1025	882,000	632,000	71.66%
Accident Date	Untrended ALAE	Trend Factor	Trended ALAE	Covered ALAE	
July 1, 2017	150,000	1.2155	182,326	0	
July 1, 2017	400,000	1.2155	486,203	200,488	
July 1, 2018	0	1.1576	0	0	
July 1, 2018	450,000	1.1576	520,931	375,000	
July 1, 2019	50,000	1.1025	55,125	5,125	
July 1, 2019	275,000	1.1025	303,188	217,250	
Accident Date	Covered Loss and ALAE	Develop. Factor	Develop. Trended Loss & ALAE		
July 1, 2017	0	1.10	0		
July 1, 2017	375,915	1.10	413,507		
July 1, 2018	10,466	1.40	14,652		
July 1, 2018	1,125,000	1.40	1,575,000		
July 1, 2019	30,750	2.40	73,800		
July 1, 2019	849,250	2.40	2,038,200		
			4,115,159		

Calculate the rate as the total developed trended layer loss and ALAE divided by the total on level subject premium for the period 2017 to 2019: $\frac{4,115,159}{(3)\ (10,000,000)}$ = **13.7%.**

242. (a)

For aggregate losses of 0, the probability is the density at zero of the Poisson: $e^{-1.5} = 0.2231$.

The Panjer Algorithm, when there is no chance of a loss of size zero:

$$A_k = \sum_{i=1}^{k} (a + b\ i/k)\ S_i\ A_{k-i}.$$ For the Poisson, a = 0 and b = λ = 1.5.

Thus, $A_k = (1.5/k)\ \{(1)(0.5)\ A_{k-1} + (2)(0.4)\ A_{k-2} + (3)(0.1)\ A_{k-3}\}$.

Thus for example, $A_4 = (1.5/4)\ \{(0.5)(0.1496) + (0.8)(0.1966) + (0.3)(0.1673)\} = 0.1059$.

Filling in the whole spreadsheet:

Aggregate Losses (millions)	Probability
0	0.2231
1	0.1673
2	0.1966
3	0.1496
4	0.1059
5	0.0695
6	0.0411
7	0.0231
8	0.0122
9	0.0062
10	0.0030
11	0.0014
12	0.0006
13	0.0003
14	0.0001
15	0.0000

(b) Profit is premium less the losses and margin.
(i) Profit is: 2.5 - 2 - (0.1)(2.5) = 0.25.
Since the profit is positive, there is a profit commission of 80% of the profit.
The underwriting result is premium plus additional premium less losses less profit commission.
2.5 - 2 - (0.8)(0.25) = **0.3**
(ii) Profit is: 2.5 - 5 - (0.1)(2.5) = -2.75.
Since the profit is negative, there is additional premium of 50% of (losses plus the margin minus the annual premium): (50%){5 + (0.1)(2.5) - 2.5} = 1.375.
The underwriting result is premium plus additional premium less losses less profit commission.
2.5 + 1.375 - 5 = **-1.125**.
(c)
1. The reinsurer assumes significant insurance risk under the reinsured portions of the underlying insurance agreements.
2. It is reasonably possible that the reinsurer may realize a significant loss from the transaction.

(d) The reinsurer has a loss if the aggregate loss is 3 or more.
For example, the reinsurer will lose at least 1.125 million if the aggregate loss is 5 or more; note that 1.125 is 45% of the premium of 2.5.
The probability of an aggregate loss of at least 5 is 15.75%.
Thus in my opinion, it is reasonably possible that the reinsurer may realize a significant loss from the transaction. Thus this reinsurance can be considered as insurance for purposes of financial reporting to insurance regulators.
Comment: In part (d), there is no single correct answer. An explanation should consider loss sizes relative to the premium as well as their probabilities.