Mahler's Guide to

Grossi and Kunreuther, Catastrophe Modeling

CAS Exam 9

prepared by
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Mahler's Guide to Grossi and Kunreuther, Catastrophe Modeling[1 2 3 4]

Information in bold or sections whose title is in bold are more important for passing the exam. Larger bold type indicates it is extremely important. Information presented in italics (including subsections whose titles are in italics) should rarely be needed to directly answer exam questions and should be skipped on first reading. It is provided to aid the reader's overall understanding of the subject, and to be useful in practical applications.

I have doubled underlined highly recommended questions to do on your first pass through the material, underlined recommended questions to do on your second pass, and starred additional questions to do on a third pass through the material.[5] No questions were labeled from the 2011 exam or later, in order to allow you to use them as practice exams.

Solutions to problems are at the end.[6]

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He has published dozens of major research papers and won the 1987 CAS Dorweiler prize.
He served 12 years on the CAS Examination Committee including three years as head of the whole committee (1990-1993).

Please send me any suspected errors by Email.

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[1] Catastrophe Modeling: A New Approach to Managing Risk, Edited by Patricia Grossi and Howard Kunreuther, 2005, Springer, Chapters 2-6 (excluding references at the end of each chapter), plus Errata for Section 2.4. Note the helpful Glossary at the end of the book.
[2] Added to the syllabus for 2011.
[3] Prior to 2024, Grossi & Kunreuther was on Exam 8.
The current material was part of my study guides for Exam 8.
[4] Related material not on the syllabus:
ASOP No. 39: Treatment of Catastrophe Losses in Property/Casualty Insurance Ratemaking,
"Uses of Catastrophe Model Output", American Academy of Actuaries, July 2018.
[5] Obviously feel free to do whatever questions you want. This is just a guide for those who find it helpful.
[6] Note that problems include both some written by me and some from past exams. The latter are copyright by the Casualty Actuarial Society and are reproduced here solely to aid students in studying for exams. The solutions and comments are solely the responsibility of the author; the CAS bears no responsibility for their accuracy. While some of the comments may seem critical of certain questions, this is intended solely to aid you in studying and in no way is intended as a criticism of the many volunteers who work extremely long and hard to produce quality exams. There are also some past exam questions copyright by the Society of Actuaries.

It's difficult to make predictions, especially about the future.[7] This is particularly true for low frequency high severity events such as catastrophes. Thus catastrophes have to be treated specially in ratemaking. One would remove any catastrophes from the recent data used for ratemaking, and then load back in an average provision for catastrophes. Sometimes, the actuary will rely on actual insured experience to estimate a provision for catastrophes. For example, for hailstorms one might use decades of data for the entire insurance industry.

In the 1980s, computer models of the insurance losses due to hurricanes started to be developed. Since then computer models of insurance losses due to catastrophes have been developed and refined. They are now widely used for hurricanes and earthquakes, as well as other types of catastrophes.[8] Computer simulation of events can estimate expected annual losses, as well as the distribution of annual losses.

This reading discusses various aspects and uses of catastrophe modeling.
It was written for insurance professionals rather than specifically for actuaries.[9]

"The output from catastrophe models provides important information for insurers to manage their risk. By modeling the risk, insurers can more accurately estimate the premiums to charge for insurance coverage for natural disasters. In addition, insurers and reinsurers are able to tailor their coverage to reduce the chance of insolvency. They can develop new strategies for managing their portfolios so as to avoid losses that might otherwise cause an unacceptable reduction in surplus."[10]

The output of a catastrophe model can be used for: **ratemaking by an insurer or reinsurer**, determining risk loads, underwriting / risk selection, loss mitigation strategies, portfolio management, **determination of the Probable Maximum Loss**, enterprise risk management (ERM), helping to determine insurance territories and classes, designing a reinsurance program, determining whether an insurer has adequate surplus for the insurance being written, determining the number of people who will be killed, injured, and displaced by a disaster in order for governments to be better prepared to respond, helping to design and revise building codes, etc.

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[7] Danish proverb.
[8] A vendor will run an insurer's portfolio though their model and give the insurer the results. An insurer might use two different vendors and compare the results of their models. A large insurer could build its own computer model.
[9] However, one of the contributors, David Lalonde, is a Fellow of the CAS.
Also the assistant editor Chandrakant "Chandu" C. Patel is a Fellow of the CAS.
[10] Quoted from page 41 of Grossi and Kunreuther.

The chapters on the syllabus are:

2. An Introduction to Catastrophe Models and Insurance
    - *2.1 History of Catastrophe Models*
    - 2.2 **Structure of Catastrophe Models**
    - 2.3 Uses of Catastrophe Models for Risk Management
    - 2.4 **Derivation and Use of an Exceedance Probability Curve**[11]
        - 2.4.1 Generating an Exceedance Probability Curve
        - 2.4.2 Stakeholders and the Exceedance Probability Curve
    - 2.5 Insurability of Catastrophe Risks
        - 2.5.1 Conditions for Insurability of A Risk
        - 2.5.2 Uncertainty of Losses
        - 2.5.3 Highly Correlated Losses
        - 2.5.4 Determining Whether to Provide Coverage
    - 2.6 Framework to Integrate Risk Assessment with Risk Management

3. Risk Assessment Process: The Role of Catastrophe Modeling in Dealing with Natural Hazards
    - 3.1 Introduction
    - 3.2 **Hazard Module**
        - 3.2.1 Location of Potential Future Events
        - 3.2.2 Frequency of Occurrence
        - 3.2.3 Parametrizing Severity at the Hazard's Source
        - 3.2.4 Parameters for Local Intensity and Site Effects
    - 3.3 Inventory Module
    - 3.4 Vulnerability Module
        - 3.4.1. Identification of Typical Buildings
        - 3.4.2 Evaluation of Building Performance
    - 3.5 Loss Module

4. Sources, Nature, and Impact of Uncertainties on Catastrophe Modeling
    - 4.1 Introduction
    - 4.2 Classification of Uncertainty
    - 4.3 Sources of Uncertainty
    - 4.4 Representing and Quantifying Uncertainty
        - 4.4.1 Logic Trees
        - 4.4.2 Simulation Techniques
        - 4.4.3 Uncertainty and the Exceedance Probability Curve
    - 4.5 Case Studies in Uncertainty
        - 4.5.1 Hurricane Hazard: Florida
        - 4.5.2 Earthquake Hazard: Charleston, South Carolina

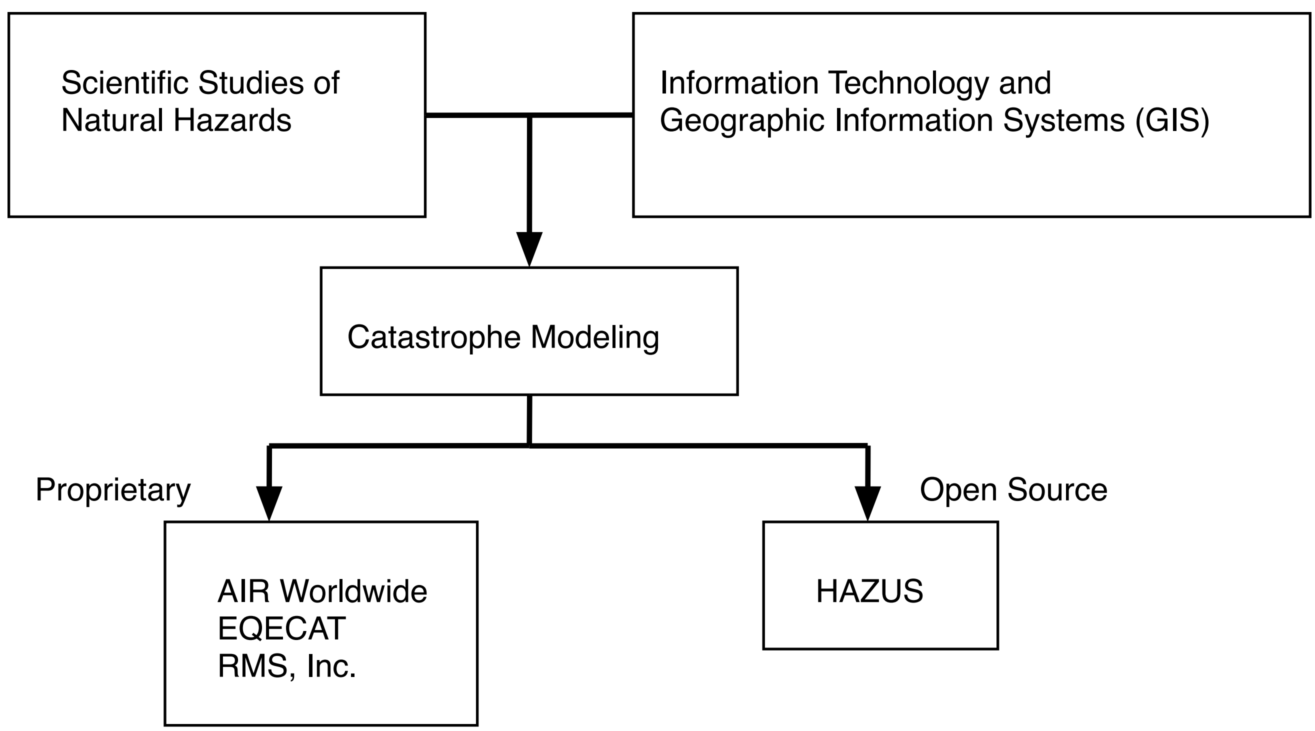
[11] Note the CAS Errata for Section 2.4.

Some Types of Events that may lead to Catastrophic Insurance Losses:

A catastrophe is "An unexpected or unanticipated natural or man-made event that has a wide ranging negative socioeconomic impacts; also known as a disaster."[12]

Hurricanes (Intense Tropical Storms)[13]
Earthquakes
Wild Fires or Firestorms
Tornadoes
Severe Winter Storms
Hailstorms
Floods
Terrorist Attacks
Freezing (due to cold weather in southern states)
Large Explosion (for example of a rocket fuel factory)

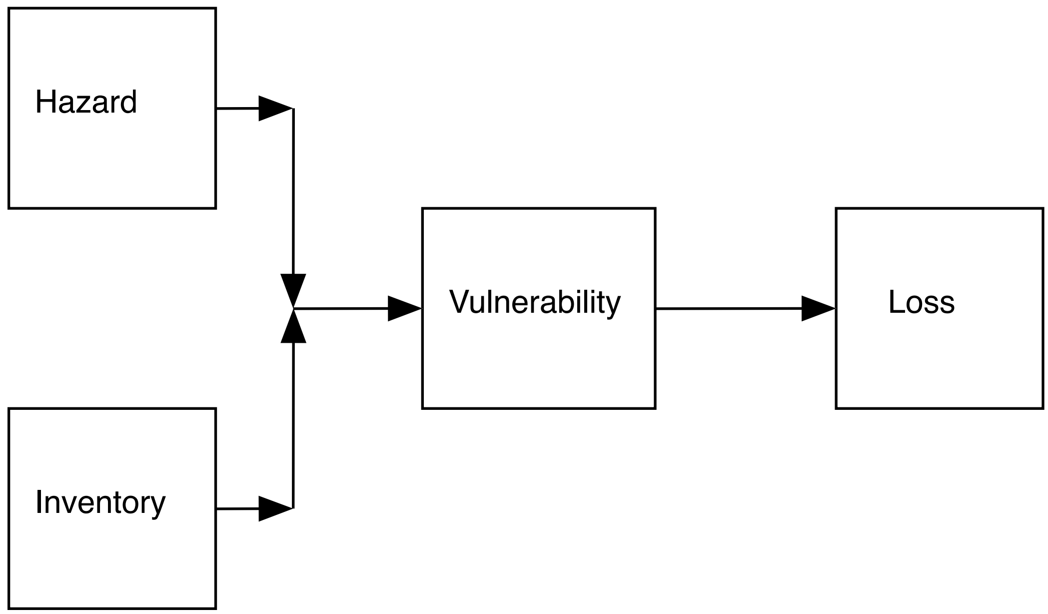
Development of Catastrophe Models:[14]



[12] See the Glossary at the back of Grossi and Kunreuther.
[13] Hurricanes are the largest source of catastrophe losses in the United States.
ISO has done a study "A Half Century of Hurricane Experience." www.iso.com/studies_analyses/hurricane_experience/historical.html
Hurricanes are called cyclones or typhoons in other parts of the world.
Only about 3% of the worlds hurricanes/cyclones/typhoons make landfall in the United States.
[14] See Figure 2.1 in Grossi and Kunreuther.

Structure of Catastrophe Models:[15]

The four components of a catastrophe model are: **hazard, inventory, vulnerability, and loss**.



The hazard component specifies the risk from whatever catastrophe is being insured against. So for example, in an earthquake model we would specify the epicenter and magnitude of a quake. For a hurricane one would specify its path and its strength along each part of that path. In addition, frequencies would be assigned to each of possible modeled earthquakes or hurricanes. This component would rely heavily on the input of scientist who study earthquakes or hurricanes.

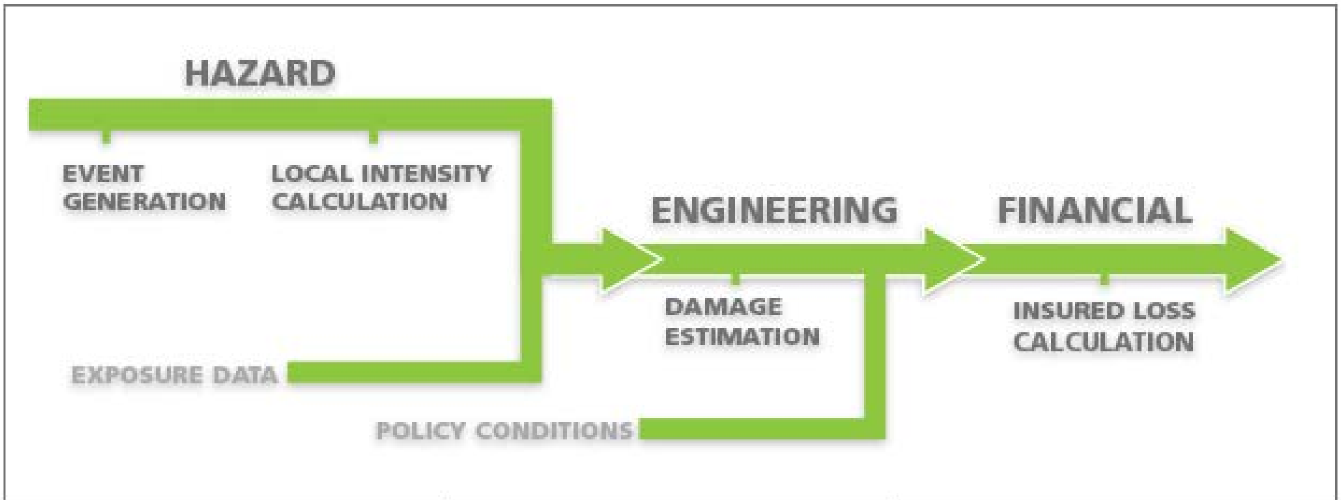
The inventory component specifies the properties that are being insured, with detailed information on each property in the portfolio, such as its location (street address), replacement value, construction, age, occupancy, etc. The more exposure detail the better.

The vulnerability component brings together the hazard and inventory, to determine the damage that results to the insured properties.
For example, for a given earthquake and a given property at a certain location, there could be a distribution of the percentage of damage to the building, contents, etc. For a hurricane we would consider peak wind speed and the construction and location of the property.
This component would rely heavily on the input of structural engineers.

The loss component translates a given level of damage into an amount of insurance loss, based on the coverage features of the relevant insurance policy. For example, an earthquake policy might have a 15% deductible that has to be taken into account.
One might have separate damage curves for buildings, contents, business interruption, etc.

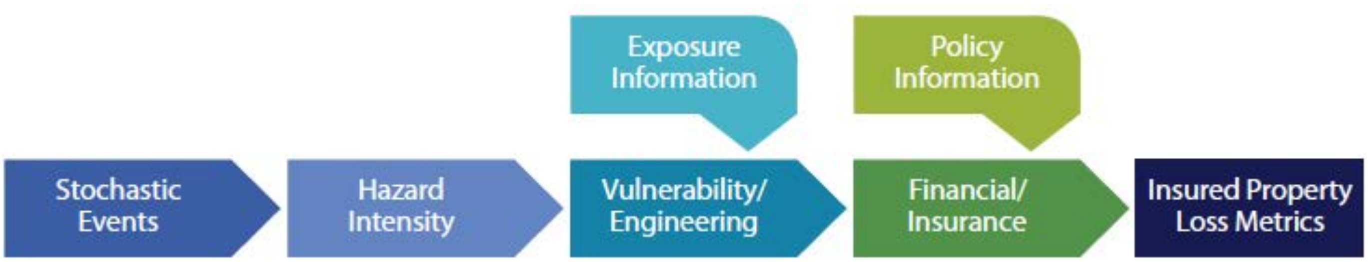
[15] See Section 2.2 of Grossi and Kunreuther.

Here is somewhat different diagram of catastrophe models, from the modeling firm AIR:[16]



Hazard:	Engineering:	Financial:
Large catalogs of simulated catastrophes are generated, representing the entire spectrum of plausible events. These catalogs answer the questions: Where are future events likely to occur? How large or severe are they likely to be? And how frequently are they likely to occur? For each simulated event, the model calculates the intensity at each location.	Measures of intensity—wind speed, ground shaking, flood depth—are then applied to highly detailed information about the properties that are exposed to them. Estimates of physical damage are translated into estimates of monetary damage.	For each simulated event, insured losses are calculated by applying policy conditions (deductibles and limits, for example) to the total damage estimates.

Another somewhat different diagram of catastrophe models, from the AAA:[17]



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[16] Air Worldwide. "About Catastrophe Modeling." http://www.airworldwide.com/Models/About-Catastrophe-Modeling/.
[17] Uses of Catastrophe Model Output", American Academy of Actuaries, July 2018.

Advantages of Catastrophe Models:[18]

Catastrophe models provide comprehensive information on current and future loss potential.

Computer Model:
- Large number of simulated years creates a comprehensive distribution of potential events.
- Use of current exposures represents the latest population, building codes and replacement values.

Use of Historical Data:
- Historical experience is not complete or reflective of potential due to limited historical records, infrequent events, and potentially changing conditions.
- Historical data reflects population, building codes, and replacement values at time of historical loss.
- Coastal population concentrations and replacement costs have been rapidly increasing.

Typical actuarial methods, looking at the latest few years of data, don't work with Catastrophes because they are low frequency, high severity events - recent history doesn't cover all the possible outcomes.

Uses of Catastrophe Models for Risk Management:[19]

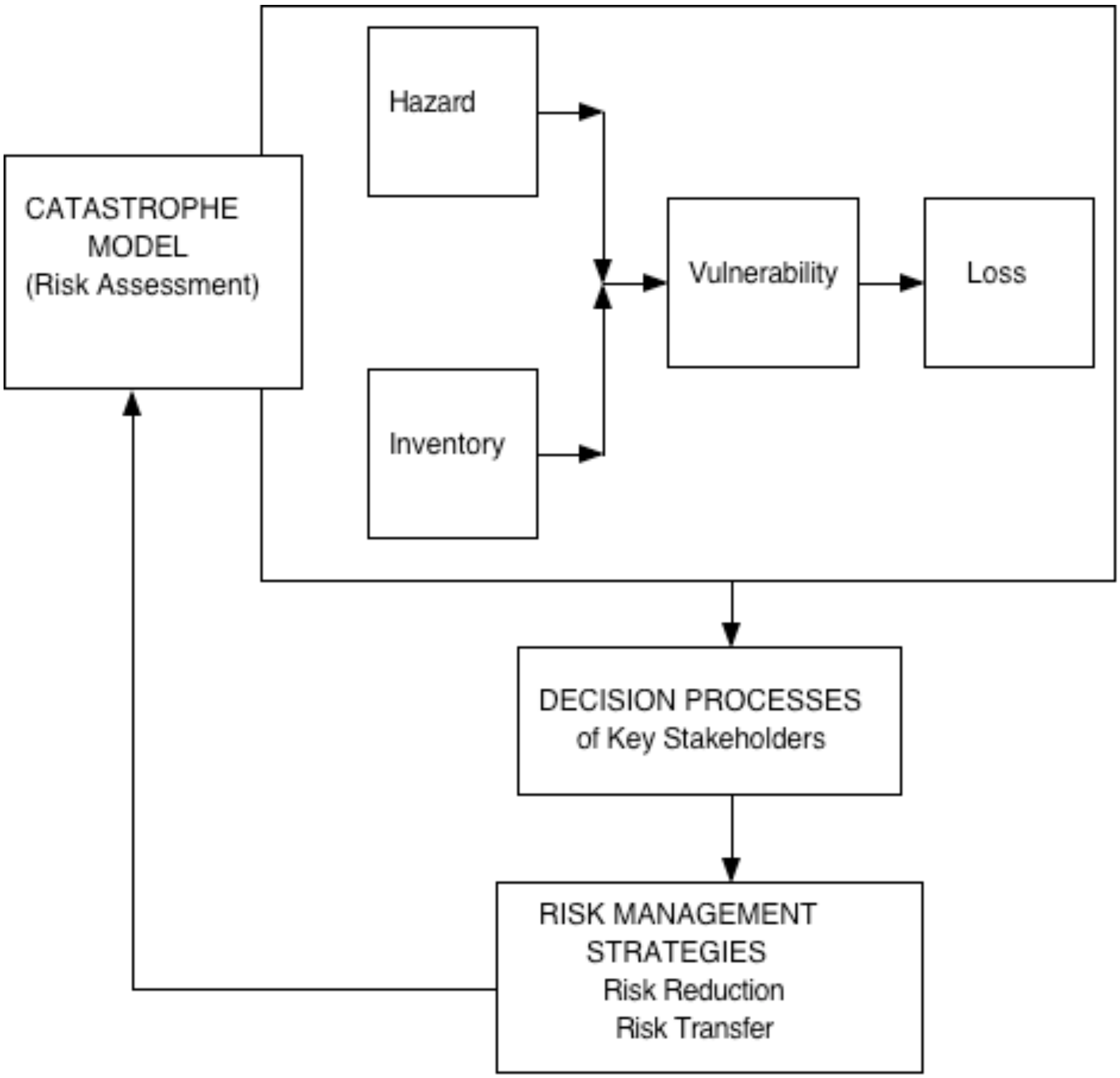
Insurers can use the output of catastrophe models to determine the distribution of their aggregate annual loss. This can be used to decide what to charge, which insureds to write, what coverages to offer, what reinsurance to buy, etc. Models can be used in a similar manner by reinsurers.

Catastrophe models have been a key to being able to accurately price and sell catastrophe bonds.[20]

A government can use catastrophe models to plan for a disaster, for example determining how many people will be left homeless in various neighborhoods in the case of a particular modeled earthquake or hurricane.[21]

[18] Taken from the CAS Fundamentals of Catastrophe Modeling Case Study.
[19] See Section 2.3 of Grossi and Kunreuther.
[20] Catastrophe bonds are risk-linked securities that transfer a specified set of risks from a sponsor to investors. If no catastrophe occurs, then the insurance company will pay a coupon to the investors, who made a healthy return. On the contrary, if a catastrophe does occur, then the principal would be forgiven and the insurance company would use this money to pay the resulting insurance claims.
[21] See Figure 2.3 in Grossi and Kunreuther.

Framework for linking risk assessment with risk management:[22]



[22] Figure 2.9 at page 40 of Grossi and Kunreuther.

Figure 5.1 illustrates how catastrophe modeling can be used in conjunction with data on capital allocation to undertake financial modeling of an insurance company.[23]

Figure 5.1, role of catastrophe modeling in an insurance company's financial management:

[23] See pages 97 and 98 of Grossi and Kunreuther.

Exceedance Probabilities:[24]

Occurrence Exceedance Probability (OEP):
The OEP is the probability that at least one event in a year exceeds the specified loss amount.[25]

For example, there might be a 15% chance that during a year at least one occurrence will exceed $100 million. If there is a 2% chance that during a year at least one occurrence will exceed $800 million, then a one-in-50 year Probable Maximum Loss (PML) would be $800 million.

Aggregate Exceedance Probability (AEP):[26]
The AEP is the probability that the sum of all losses during a given period exceeds some amount.

For example, there might be a 1% chance that during a year the aggregate losses will exceed $1 billion.

Conditional Exceedance Probability (CEP):
The CEP is the probability that the amount on a single event exceeds a specified loss amount; this is the survival function of the size of event.

For example, given that there is an event, the probability that it exceeds size $100 million might be 10%; in other words S($100 million) = 10%.

Exercise: An insurance company is exposed to three independent catastrophic events:

Event	Size ($ billion)	Probability
1	2	7%
2	3	4%
3	4	2%

While each event can only happen once, the total number of events per year is not limited to one. Determine the occurrence exceedance probabilities.
[Solution: Prob[at least one event exceeds $1 billion] = 1 - (93%)(96%)(98%) = 0.1251.
The probability that at least one event exceeds $2 billion is: 1 - (96%)(98%) = 0.0592.
The probability that at least one event exceeds $3 billion is: 2%.
The probability that at least one event exceeds $4 billion is 0.
Comment: Each event is a catastrophe, which results in thousands of claims.]

Exercise: In the previous exercise, determine the conditional exceedance probabilities.
[Solution: S(1 billion) = 1. S(2 billion) = (4% + 2%) / (7% + 4% + 2%) = 46.15%.
S(3 billion) = 2% / (7% + 4% + 2%) = 15.38%. S(4 billion) = 0.
Comment: Mean occurrence is:
{(7%)(2) + (4%)(3) + (2%)(4)} / (7% + 4% + 2%) = $2.615 billion.]

[24] See the CAS errata to Section 2.4 of Grossi and Kunreuther.
[25] The distribution of the largest event in the year is not the same as the distribution of the event size.
[26] AEP equals OEP if and only if there is at most one event per year.

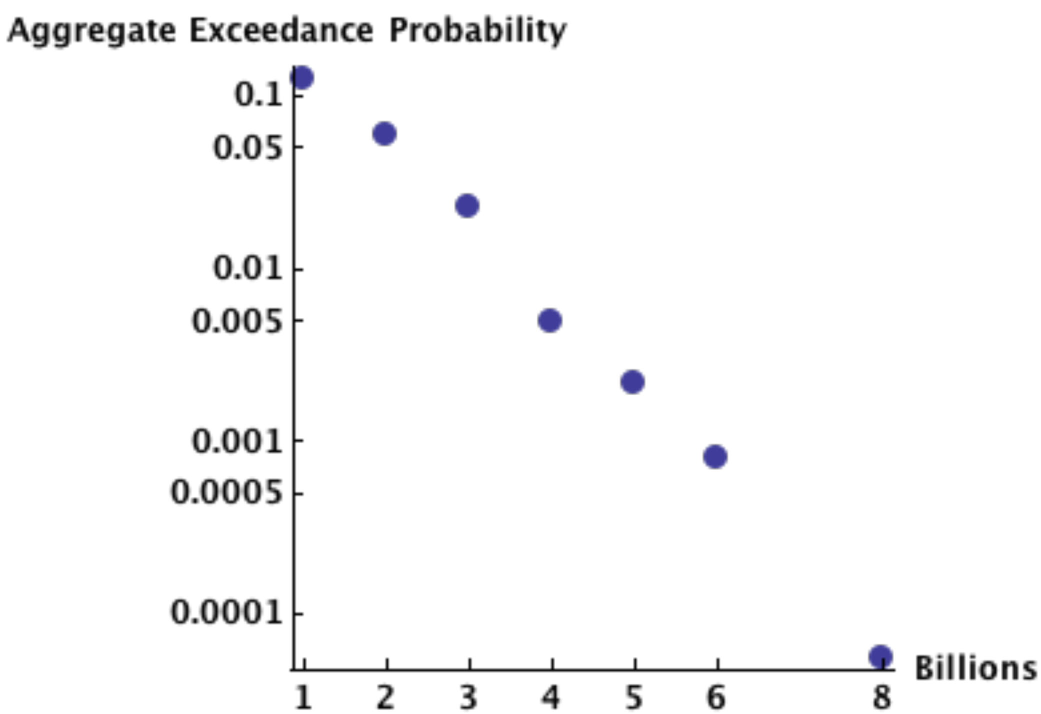
Exercise: An insurance company is exposed to three independent catastrophic events:

Event	Size ($ billion)	Probability
1	2	7%
2	3	4%
3	4	2%

While each event can only happen once, the total number of events per year is not limited to one. Determine the aggregate exceedance probabilities.
[Solution:

Events #s Occurring	Aggregate ($ billion)	Probability
None	0	(0.93)(0.96)(0.98) = 0.874944
1	2	(0.07)(0.96)(0.98) = 0.065856
2	3	(0.93)(0.04)(0.98) = 0.036456
3	4	(0.93)(0.96)(0.02) = 0.017856
1, 2	5	(0.07)(0.04)(0.98) = 0.002744
1, 3	6	(0.07)(0.96)(0.02) = 0.001344
2, 3	7	(0.93)(0.04)(0.02) = 0.000744
1, 2, 3	9	(0.07)(0.04)(0.02) = 0.000056

Now we need to compute the probability of the aggregate loss being greater than a certain values. S(9) = 0. S(8) = S(7) = 0.000056. S(6) = S(7) + 0.000744 = 0.0008.
S(5) = S(6) + 0.001344 = 0.002144. S(4) = S(5) + 0.002744 = 0.04888.
S(3) = S(4) + 0.017856 = 0.022744. S(2) = S(3) + 0.036456 = 0.0592.
S(0) = S(1) = S(2) + 0.065856 = 0.125056. S(0) + 0.874944 = 1. Check.



Comment: See 8, 11/11, Q.5.
The mean aggregate annual loss is: (7%)(2) + (4%)(3) + (2%)(4) = $0.34 billion.]

It can be useful to determine exceedance probability curves for: total ground up losses, the insured, the insurer after policy provisions but gross of reinsurance,
the insurer after policy provisions and net of reinsurance, or reinsurer(s).[27]
One needs to apply the policy and reinsurance treaty provisions to each simulated loss or group of losses from a simulated catastrophe in order to determine who would pay what, prior to combining the results into an exceedance probability curve relevant to a given stakeholder.

The return period or return time for a given size of event is the inverse of the exceedance probability. Return periods can refer either to the aggregate or occurrence exceedance curves. For example, if there is a 1% probability of the aggregate annual loss from hurricanes exceeding \$5 billion, then \$5 billion in aggregate has a 100 year return time. If there is is 0.4% probability per year that the largest earthquake will exceed \$10 billion, then for occurrences \$10 billion has a 250 year return time.

Often one will define the Probable Maximum Loss (PML) in terms of a return time for an occurrence. For example, if we use a 100 year return time, the PML would be the size of loss where the occurrence exceedance curve is 1%.

<u>Simulating the Aggregate Annual Losses</u>:[28]

One can use a random number and aggregate exceedance probabilities in order to simulate the annual aggregate loss. For example, we can use the discrete distribution from the previous exercise.

F(0) = 0.830208.
F(2) = F(0) + 0.072192 = 0.902400.
F(3) = F(2) + 0.052992 = 0.955392.
F(4) = F(3) + 0.034592 = 0.989984.
F(5) = F(4) + 0.004608 = 0.994592.
F(6) = F(5) + 0.003008 = 0.999760.
F(7) = F(6) + 0.002208 = 0.999808.
F(9) = F(7) + 0.000192 = 1.000000.

Assume a random number between zero and one: 0.983627.[29]
We see where the random number is first exceeded by the distribution function.[30]
This is at 4; F(4) = 0.989984 > 0.983627.
Thus we simulate aggregate losses of 4 billion.

Exercise: Given a random number of 0.670173, what are the simulated aggregate losses?
[Solution: F(0) = 0.830208 > 0.670173. The simulated losses are zero.]

Exercise: Given a random number of 0.996142, what are the simulated aggregate losses?
[Solution: F(6) = 0.999760 > 0.996142. The simulated losses are 6 billion.]

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[27] See Figure 2.7 in Grossi and Kunreuther.
[28] See Section 4.4.2 in Grossi and Kunreuther, and 8, 11/15, Q.22b.
[29] A computer will give one random numbers to many more decimal places than this.
[30] This is the usual technique for simulating from discrete distributions.

A Practical Example of Exceedance Curves:

I will illustrate these definitions of exceedance curves by applying them to data on actual hurricanes rather than the output of a computer model. The data is for the entire insurance industry and not broken down by state or smaller subdivision.[31]

Collins and Lowe presented data on the 164 hurricanes hit the continental United States from 1900 to 1999.[32] [33] They report insured losses adjusted to a year 2000 level for inflation, changes in per capita wealth (to represent the changes in property value above the rate of inflation), changes in insurance utilization, and changes in number of housing units (by county).

In each case, the year of the hurricane is followed by the insured loss ($000) normalized to the 2000 level:

{1900, 16485683}, {1901, 76846}, {1901, 366142}, {1903, 2124106}, {1903, 61970},
{1904, 646193}, {1906, 894836}, {1906, 525681}, {1906, 662658}, {1906, 687544},
{1908, 37659}, {1909, 1119560}, {1909, 87098}, {1909, 189900}, {1909, 7976601},

{1910, 75760}, {1910, 2735157}, {1911, 438296}, {1911, 58145}, {1912, 27091},
{1912, 65024}, {1913, 66228}, {1913, 534237}, {1915, 16146375}, {1915, 43577},
{1915, 1709809}, {1916, 3096434}, {1916, 15474}, {1916, 17866}, {1916, 147702},
{1916, 208433}, {1916, 65139}, {1917, 28690}, {1918, 775971}, {1919, 10009409},

{1920, 348405}, {1920, 18497}, {1921, 31069}, {1921, 1624995}, {1923, 9557}, {1924, 12256},
{1924, 86278}, {1925, 155351}, {1926, 1755434}, {1926, 305313}, {1926, 49728840},
{1928, 132787}, {1928, 9816472}, {1929, 18946}, {1929, 356558},

{1932, 836911}, {1932, 32860}, {1933, 67732}, {1933, 1356989}, {1933, 368245},
{1933, 1163819}, {1933, 75739}, {1934, 133959}, {1934, 17976}, {1935, 1191386},
{1935, 1371030}, {1936, 17658}, {1936, 20289}, {1936, 18891}, {1938, 9005},
{1938, 9965606}, {1939, 41746},

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[31] Exceedance curves would most commonly be applied to the output of computer model run versus one insurer's book of business. Since the output of the computer model is broken down by geography, one could construct exceedance curves for any geographical subset if one wanted to.
[32] See "A Macro Validation Dataset for U.S. Hurricane Models", by Douglas J. Collins and Stephen P. Lowe, CAS Forum, Winter 2001, not on the syllabus. The data I am using comes from their Exhibit 3.
[33] Even 164 hurricanes is less data than one would like in order to estimate the righthand tails of the exceedance curves.

{1940, 8223}, {1940, 293910}, {1941, 64533}, {1941, 942310}, {1942, 13296},
{1942, 1028039}, {1943, 970828}, {1944, 8796}, {1944, 2087738}, {1944, 5855343},
{1945, 20416}, {1945, 825054}, {1945, 3762550}, {1946, 465074}, {1947, 10278},
{1947, 5432151}, {1947, 1460391}, {1948, 17116}, {1948, 668635}, {1948, 224907},
{1949, 11446}, {1949, 2728296}, {1949, 217219},

{1950, 13449}, {1950, 194890}, {1950, 2853627}, {1952, 55046}, {1953, 19612},
{1953, 63152}, {1953, 10799}, {1954, 6265912}, {1954, 643598}, {1954, 8196810},
{1955, 1378549}, {1955, 696402}, {1955, 362090}, {1956, 275001}, {1957, 1176396},
{1959, 5717}, {1959, 393073}, {1959, 605316},

{1960, 4709959}, {1960, 11837}, {1961, 3476218}, {1963, 3954}, {1964, 3746855},
{1964, 403169}, {1964, 596026}, {1964, 122518}, {1965, 11518111}, {1966, 194630},
{1966, 16208}, {1967, 888088}, {1968, 96877}, {1969, 2949789}, {1969, 2439},

{1970, 4568366}, {1971, 18825}, {1971, 71158}, {1971, 31447}, {1972, 956927},
{1974, 118642}, {1975, 783072}, {1976, 127951}, {1977, 11414}, {1979, 34636},
{1979, 547711}, {1979, 3686521},

{1980, 283869}, {1983, 3912101}, {1984, 133682}, {1985, 29419},
{1985, 58548}, {1985, 1650468}, {1985, 1435127}, {1985, 192283}, {1985, 189781},
{1986, 42825}, {1986, 19357}, {1987, 502}, {1988, 19065}, {1989, 69972}, {1989, 5529261},
{1989, 63918},

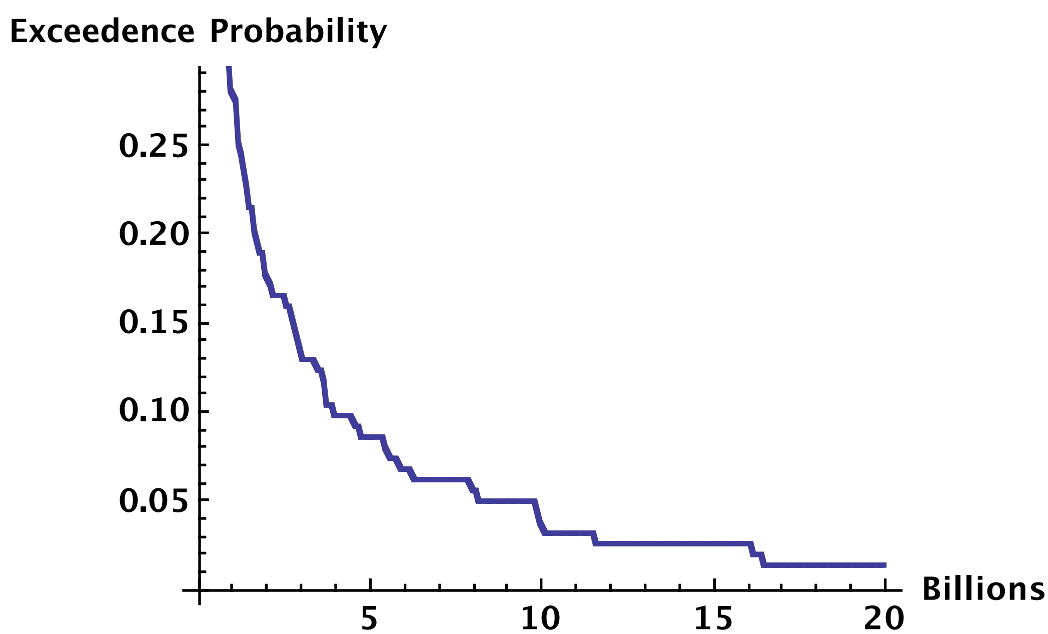
{1991, 923918}, {1992, 24486691}, {1993, 47299}, {1995, 484223}, {1995, 2584891},
{1996, 169071}, {1996, 1910703}, {1997, 41277}, {1998, 400501}, {1998, 19929},
{1998, 1270333}, {1999, 31388}, {1999, 1979274}.

These 164 hurricanes over 100 years total $287.3 billion, adjusted to the 2000 year level.
Their average size is $1.752 billion.

The simplest one of three Exceedance Probability Curves to calculate is the Conditional Exceedance Probability (CEP). The CEP is the probability that the amount on a single event exceeds a specified loss amount; this is the survival function of the size of event.

The first step is to arrange the sizes of the insured loses due to the 164 individual hurricanes from smallest to largest ($000):
502, 2439, 3954, 5717, 8223, 8796, 9005, 9557, 10278, 10799, 11414, 11446, 11837, 12256, 13296, 13449, 15474, 16208, 17116, 17658, 17866, 17976, 18497, 18825, 18891, 18946, 19065, 19357, 19612, 19929, 20289, 20416, 27091, 28690, 29419, 31069, 31388, 31447, 32860, 34636, 37659, 41277, 41746, 42825, 43577, 47299, 55046, 58145, 58548, 61970, 63152, 63918, 64533, 65024, 65139, 66228, 67732, 69972, 71158, 75739, 75760, 76846, 86278, 87098, 96877, 118642, 122518, 127951, 132787, 133682, 133959, 147702, 155351, 169071, 189781, 189900, 192283, 194630, 194890, 208433, 217219, 224907, 275001, 283869, 293910, 305313, 348405, 356558, 362090, 366142, 368245, 393073, 400501, 403169, 438296, 465074, 484223, 525681, 534237, 547711, 596026, 605316, 643598, 646193, 662658, 668635, 687544, 696402, 775971, 783072, 825054, 836911, 888088, 894836, 923918, 942310, 956927, 970828, 1028039, 1119560, 1163819, 1176396, 1191386, 1270333, 1356989, 1371030, 1378549, 1435127, 1460391, 1624995, 1650468, 1709809, 1755434, 1910703, 1979274, 2087738, 2124106, 2584891, 2728296, 2735157, 2853627, 2949789, 3096434, 3476218, 3686521, 3746855, 3762550, 3912101, 4568366, 4709959, 5432151, 5529261, 5855343, 6265912, 7976601, 8196810, 9816472, 9965606, 10009409, 11518111,16146375, 16485683, 24486691, 49728840.

Then for example, the observed probability of a hurricane being greater than $10 billion is 6/164.
The observed probability of a hurricane being greater than $5 billion is 14/164.
Here is the Conditional Exceedance Probability (CEP) curve:

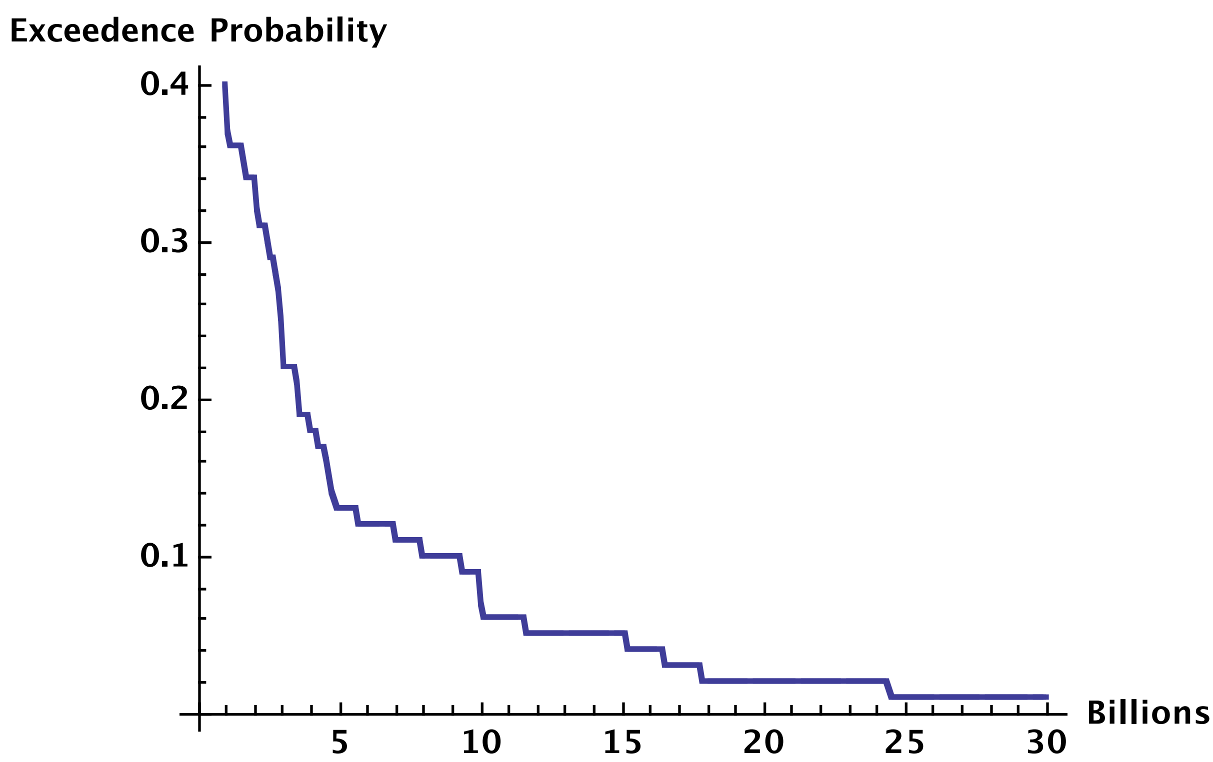


The Aggregate Exceedance Probability (AEP) is the probability that the sum of all losses during a given period of time, which is usually a year, exceeds some amount.

We need to add up all of the hurricanes in a given year.
For example there are two hurricanes in 1901, totaling in thousands of dollars:
76,846 + 366,142 = 442,988.

Here are the aggregate losses($000) for each of the 100 years, sorted from smallest to largest:[34]
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 502, 3954, 9557, 11414, 19065, 28690, 37659, 41277, 41746, 47299, 55046, 56838, 62182, 92115, 93563, 96877, 98534, 118642, 121430, 127951, 133682, 151935, 155351, 210838, 275001, 283869, 302133, 366902, 375504, 442988, 465074, 496441, 600465, 646193, 775971, 783072, 869771, 888088, 910658, 923918, 956927, 970828, 1004106, 1006843, 1041335, 1176396, 1656064, 1690763, 2010662, 2079774, 2186076, 2437041, 2562416, 2770719, 2810917, 2952228, 2956961, 3032524, 3061966, 3069114, 3476218, 3551048, 3555626, 3912101, 4268868, 4568366, 4608020, 4721796, 4868568, 5663151, 6902820, 7951877, 9373159, 9949259, 9974611, 10009409, 11518111, 15106320, 16485683, 17899761, 24486691, 51789587.

Here is the (annual) Aggregate Exceedance Probability (AEP) curve:



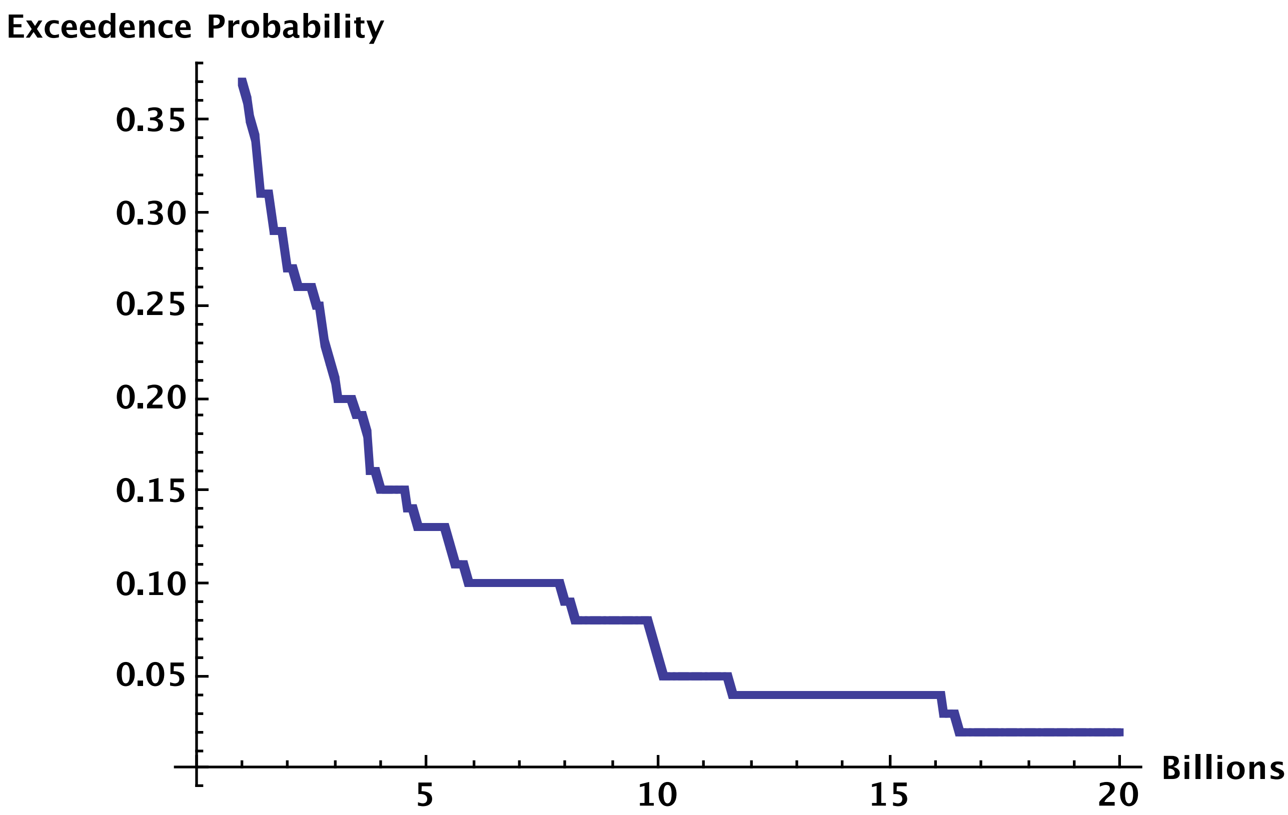
[34] 18 of the 100 years have no hurricane, while two of the hundred years have 6 hurricanes each.

The Occurrence Exceedance Probability (OEP) is the probability that at least one event in a year exceeds the specified loss amount.

We need to find the maximum size of all of the hurricanes in a given year.[35]
For example there are two hurricanes in 1901: 76,846 and 366,142.
The maximum of the two is in thousands of dollars: 366,142.

The maximum occurrences ($000) for each of the 100 years, sorted from smallest to largest:[36]
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 502, 3954, 9557, 11414, 19065, 20289, 28690, 37659, 41277, 41746, 42825, 47299, 55046, 63152, 65024, 71158, 86278, 96877, 118642, 127951, 133682, 133959, 155351, 194630, 275001, 283869, 293910, 348405, 356558, 366142, 438296, 465074, 534237, 605316, 646193, 668635, 775971, 783072, 836911, 888088, 894836, 923918, 942310, 956927, 970828, 1028039, 1176396, 1270333, 1356989, 1371030, 1378549, 1624995, 1650468, 1910703, 1979274, 2124106, 2584891, 2728296, 2735157, 2853627, 2949789, 3096434, 3476218, 3686521, 3746855, 3762550, 3912101, 4568366, 4709959, 5432151, 5529261, 5855343, 7976601, 8196810, 9816472, 9965606, 10009409, 11518111, 16146375, 16485683, 24486691, 49728840.

Here is the Occurrence Exceedance Probability (OEP) curve:



[35] For years in which there is no hurricane, the result is zero.
[36] 18 of the 100 years have no hurricane, while two of the hundred years have 6 hurricanes each.

Insurability of Catastrophe Risks:[37]

Catastrophe models maximize the use of available information on the risk, hazard and inventory, to estimate the potential losses from natural hazards.

Conditions for the insurability against an uncertain event:
1. Ability to identify and quantify, or at least estimate partially, the frequency of the event and the severity of the losses.
2. Ability to set premiums by risk or by class of similar risks.

Potential difficulties in setting adequate rates: regulation by the states, market competition, uncertainty of loss, highly correlated losses, adverse selection, and moral hazards. For natural hazards, neither adverse selection nor moral hazards are major problems in setting rates.[38] [39]

The uncertainty of catastrophe losses can be illustrated by comparing for a given type of catastrophe the median total loss from a type of event and the maximum total loss from such an event. The maximum is much bigger than the median.[40] In other words, the distribution of total losses from a catastrophe of a given type (earthquake, hurricane, flood, etc.) has a very heavy righthand tail. These extremely large and uncertain losses is one reason to use catastrophe models.

Natural disasters involve spatially correlated losses or simultaneous occurrences of many losses from a single event. Insurers seek portfolios of risks which are spatially and otherwise independent, which enables the Law of Large Numbers to operate to reduce risk and yield more reliable estimates of expected losses. However, natural disasters do not follow the Law of Large Numbers due to a lack of independence. Thus an individual insurer may need to limit the number of homes it insures in a given locality, even if these would otherwise be good risks to write.

According to James Stone, insurers are interested in maximizing expected profits subject to a constraint on the insurer's probability of survival.[41] As discussed previously, catastrophe models can help insurers to estimate the exceedance probability curves and thus estimate their probability of running out of surplus due to a catastrophe.

The maximum number of policies n that satisfy the survival constraint is such that the probability of insolvency is less than p_1:

Probability[Total Loss > (n z + A)] ≤ p_1,

z = average policy premium
A = surplus

---

[37] See Section 2.5 of Grossi and Kunreuther.
[38] Adverse selection would occur when the insured using information unknown to the insurer selects a coverage option more favorable to the insured. For example, if the insurer offers different wind deductibles, within a group of insureds rated the same by the insurer, those with more expected wind losses may chose the lower deductible.
[39] An example of moral hazard would be an insured moving unwanted furniture into the basement so that it will be destroyed by an impending flood.
[40] See Figure 2.8 in Grossi and Kunreuther.
[41] "A Theory of Capacity and the Insurance of Catastrophe Risks", by James Stone, Journal of Risk and Insurance, 1973, formerly on the syllabus of the CAS.

Framework to Integrate Risk Assessment with Risk Management:[42]

For example, based on a catastrophe model run with its current book of business, an insurer might have a 1% chance each year of the total loss due to a catastrophe exceeding its premium and surplus.[43] [44] If the insurer (or insurance regulator) considers that too high of a probability, then the insurer can take actions: reduce its book of business in areas prone to catastrophes, revise its book of business to be more geographically diverse, raise its premiums, purchase more reinsurance protection, revise policy provisions, stop insuring certain types of structures particularly vulnerable to catastrophes, etc.

Property owners can mitigate the potential damage from catastrophes. For example, straps that attach the roof to the rest of the house, reduce the probability that a hurricane will lift off the roof and thereby expose the interior of the house to wind and water damage.[45] Building codes, if properly enforced, will reduce the expected losses due to catastrophes. Land use regulations can reduce the potential for catastrophe losses, for example by preventing new construction on land very vulnerable to flooding.

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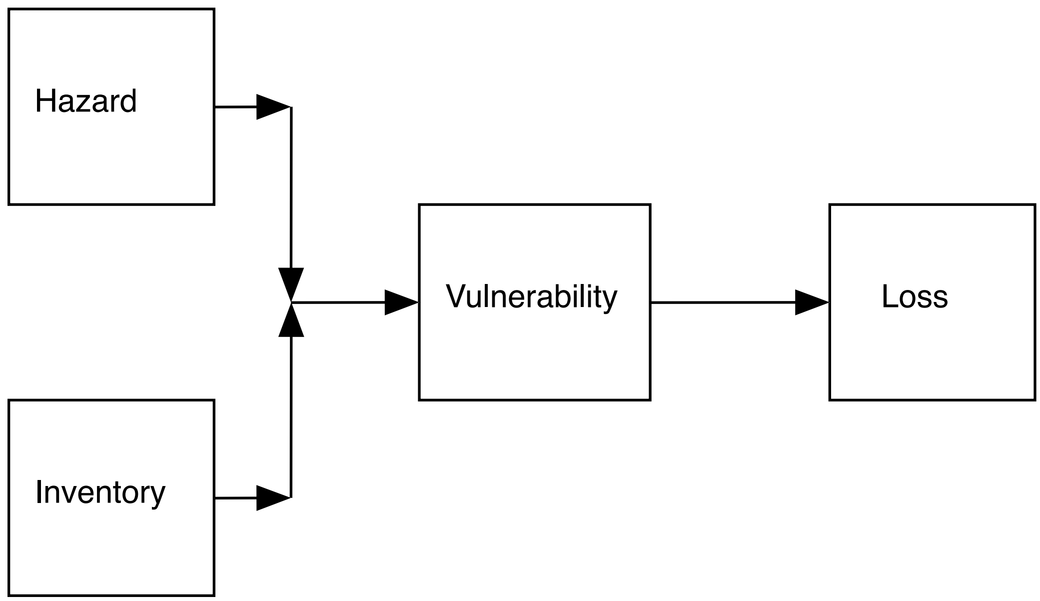
[42] See Section 2.6 in Grossi and Kunreuther, including Figure 2.9 shown previously.
[43] Insurers can go insolvent for other reasons than catastrophes.
Even when the law of large numbers applies, bad luck / random fluctuation can lead to insolvency or impaired surplus; a small chance of unusually bad results is not the same as no chance.
Some reasons why an insurer can be ruined besides bad luck / random fluctuation or catastrophes:
decline in value of investments, changes in interest rates, uncollectible reinsurance,
underestimating expected losses and/or the timing of payments, underreserving for liabilities,
poor selection of risks, bad policy language, too rapid growth,
fraud by employees, agents, or insureds, mismanagement, actions by regulators, actions by legislatures,
actions by courts, loss of key employees, post-retirement obligations, high expenses,
changes in market conditions, and technological changes.
[44] An insurer and/or insurance regulator would also be concerned if for example a single catastrophe reduced an insurer's surplus by half.
[45] Insurers may provide premium discounts to encourage such mitigation efforts.

The Risk Assessment Process:[46]

"A probabilistic risk approach to catastrophe loss analysis is the most appropriate way to handle the abundant sources of uncertainty in all natural hazard related phenomena.
It is highly complex and multifaceted."

The four modules of a catastrophe model are: **hazard, inventory, vulnerability, and loss**.



Hazard Module:[47]

The hazard module estimates the probability that the physical components of the hazard, wind speed or ground acceleration, will have various intensities.

One has to specify the module domain, the likely locations of potential future events, for example, earthquake faults.[48] Unfortunately, not all earthquakes occur on known faults.[49] Thus models statistically smooth the historical data, to create "seismic source zones." Also, there can be earthquakes in regions with no recent seismic activity. This is included in the model as "background seismology." Earthquake hazard is modeled as a weighted average of (known) faults, seismic source zones, and background seismology.

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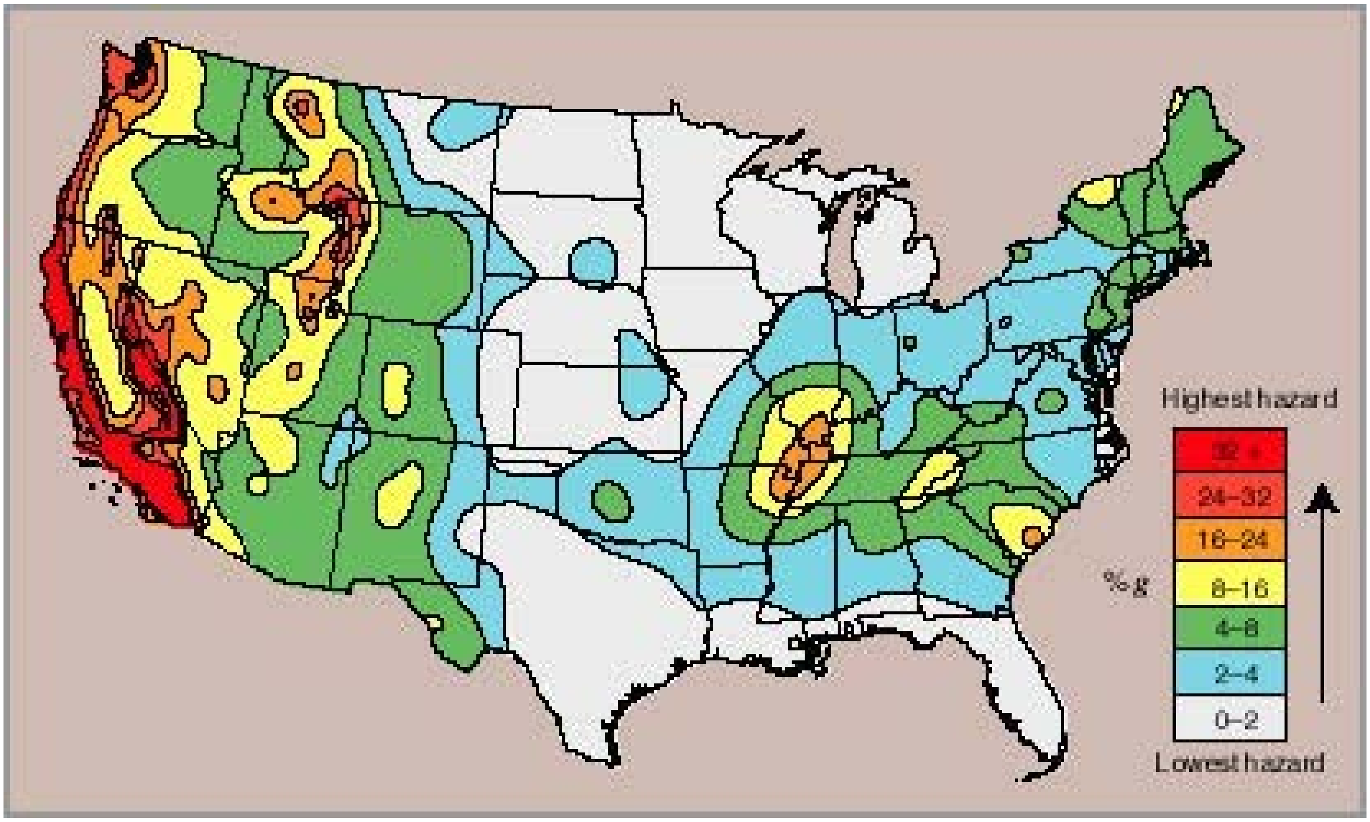
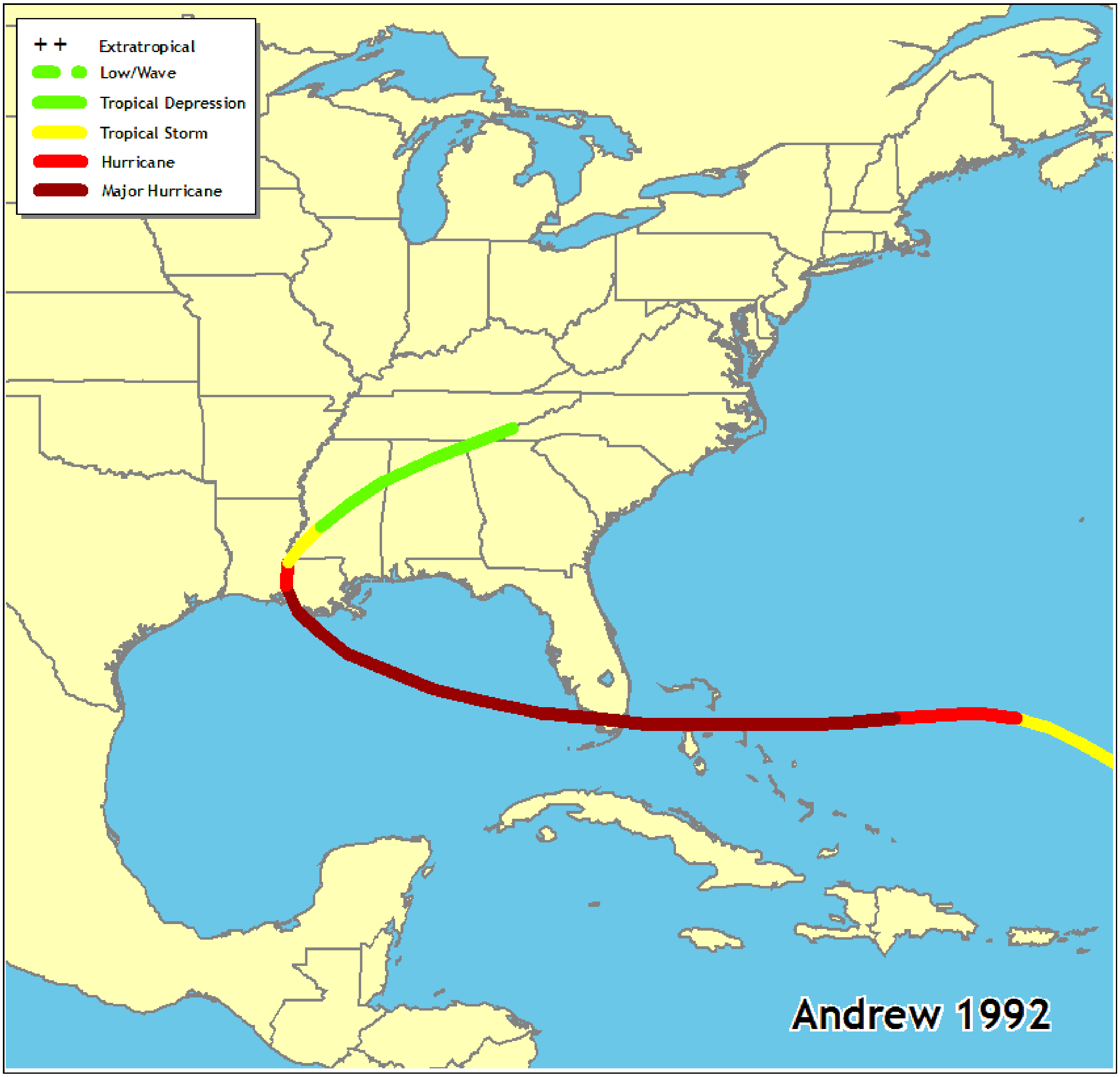
[46] See Section 3.1 of Grossi and Kunreuther
[47] See Section 3.2 of Grossi and Kunreuther
[48] Figure 3.2 in Grossi and Kunreuther maps the location of earthquakes in the United Sates since 1700.
[49] The return time between large earthquakes on a fault can be centuries, so the recent data is supplemented with evidence of prehistoric earthquakes, paleoseismic data.

Here is a map of estimated earthquake hazard across the United States from the U.S. Geological Survey:[50]



[50] For catastrophe modeling, one would need much more detail in states like California. Subsequently, I show a similar map for California. Even within a city like San Francisco, differences in soil conditions a few blacks apart can lead to significant differences in expected losses from earthquakes.

For hurricanes, a starting point is the historical storm tracks over the last century or century and a half.
Here is the storm track for Hurricane Andrew which caused $26.5 billion in property damage:

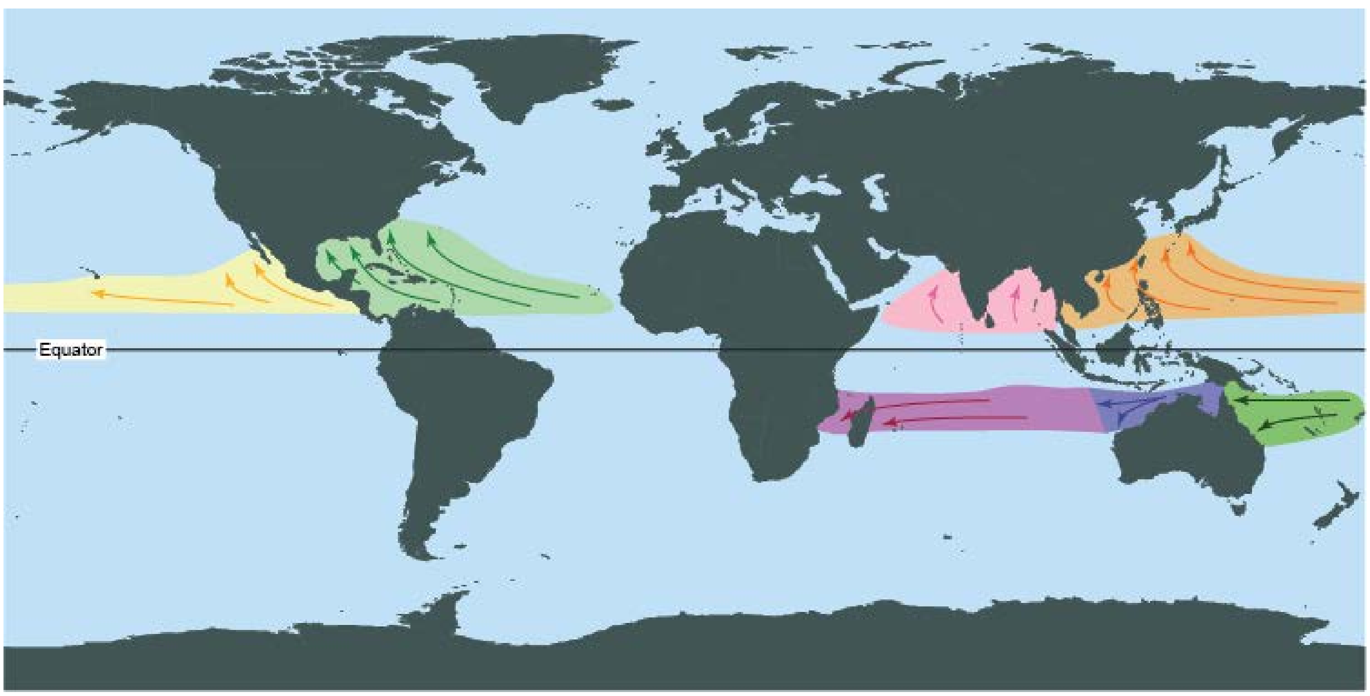


While historical storm tracks are a useful starting point, they may have missed a particular location or hit a particular location more frequently than expected. Models have been developed of the future direction of movement of a storm based on its current location and direction of movement. Such probabilistic models allow the simulation of complete storm tracks which are similar in kind to what is observed.

One wishes to simulate a lot of different storm tracks that correspond to the possible future storm tracks with the appropriate probabilities. For example, probabilities of different locations of landfall for hurricanes is based on historical data, which is then smoothed.[51] A modeler would construct a catalog of for example 5000 storm tracks which could then be run against different portfolios of insured structures.[52]

The most critical and uncertain part of the hazard module is the probability of occurrence of catastrophes. **After specifying the locations to be simulated, then one has to specify the frequency of occurrence of these events**. For example, the model needs to specify the return time for earthquakes of a given magnitude for a given fault.[53]

The most common months for hurricanes in the Northern hemisphere are August and September. Figure 3.5 in Grossi and Kunreuther shows the average number of hurricanes formed per year for each ocean basin.[54] "Given that sea surface temperatures need to be at least 80°F (27°C) for tropical cyclones to form, it is natural that they form near the equator. However, with only the rarest of occasions, these storms do not form within 5° latitude of the equator. This is due to the lack of sufficient Coriolis Force, the force that causes the cyclone to spin. Tropical cyclones form in seven regions around the world."[55] See the map below:



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[51] See Figure 3.3 in Grossi and Kunreuther.
[52] I do not know how many storm tracks a firm like AIR has in its catalog.
The catalog would be updated on a regular or semi-regular basis in order to incorporate new information.
[53] Subsequently I have a subsection on the Gutenberg-Richter magnitude distribution.
[54] Many of these hurricanes never make landfall. North Atlantic 5.4, SW Indian 4.4, N Indian 2.5,
Aus/SE Indian 3.4, NW Pacific 16.0, SW Pacific 4.3, NE Pacific 8.9.
[55] Quoted from the National Weather Service.

Next one has to parametrize the severity at the hazard source. For hurricanes, parameters are: central barometric pressure, forward speed, radius of maximum winds, and track angle at landfall. For earthquakes, one needs to specify: magnitude, focal depth, and fault-rupture characteristics.[56]

Finally, one must estimate the parameters of the hazard at the sites of insured buildings.

For earthquakes one can use attenuation equations. The strength of the ground motion at a given location depends on: the magnitude of the earthquake, the frequency of the seismic waves, the distance form the source to the site, the rupture mechanism, and local soil effects.[57]

For hurricanes one simulates the storm's movement along a track, and estimates wind speeds for building sites near that track. Adjustments are made for: storm asymmetry, the increase in central pressure as the storm moves inland, and local surface terrain.[58] Also of importance is how fast the storm is moving; the longer a storm remains over a given area the greater the expected damage.

<u>Inventory Module</u>:[59]

One has to determine the portfolio of buildings (and contents) that can be damaged by the catastrophe being modeled. This may be either for the entire insurance industry or for an individual insurer.

One would want detailed information about each building: location, value, occupancy, construction, age, height, etc. Details about roof pitch, floor wall connections, mitigation efforts, etc. are useful. One would also want to know the line of business (residential, commercial, industrial) and details of the insurance coverage provided.

A large and valuable building may be subjected to an onsite inspection in order to get details about its construction.

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[56] An earthquake that measures 4.0 corresponds to a $10^{1.5}$ = 31.6 times larger release of energy than one that measures 3.0. (The shaking amplitude is 10 times as big, but the energy released goes up at a higher rate.)
In a subsequent subsection, I briefly discuss the different types of earthquakes.

[57] See Figure 3.6 in Grossi and Kunreuther.

[58] Hurricanes weaken as they pass move inland.

[59] See Section 3.3 in Grossi and Kunreuther.

Vulnerability Module:[60]

The vulnerability module estimates the level of building damage expected from different levels of the external forces imposed, for example high winds in the case of a hurricane. There are two main approaches, based on judgement and based on building response analysis.

Building response analysis is preferred and has two major steps:

1. Identification and definition of typical buildings.
Define as many building classes as is practical so as to divide the whole portfolio into subsets that will act reasonably similarly under the external force such as earthquake ground motion.[61] Useful subsets will differ somewhat depending on what catastrophe is being modeled.

2. Analysis of the vulnerability of these typical buildings to the external forces of different strengths. This is based on engineering studies and/or the observation of damage sustained in actual catastrophes.

A damage function gives a probability distribution of the damage as a percent of the replacement cost.[62] Its coefficient of variation is one way to quantify the uncertainty.

---

[60] See Section 3.4 in Grossi and Kunreuther.
[61] Classes can be based on building materials, structural system, height, etc.
Secondary modifiers can include roof type and foundation type.
[62] See Figures 3.7 and 6.5 in Grossi and Kunreuther.

Figure 3.8 in Grossi and Kunreuther shows a detailed schematic flow of the damage calculation process for an earthquake.[63] This calculation is performed for each site for each simulated event. Separate calculations are done for buildings, contents, and time element.[64]

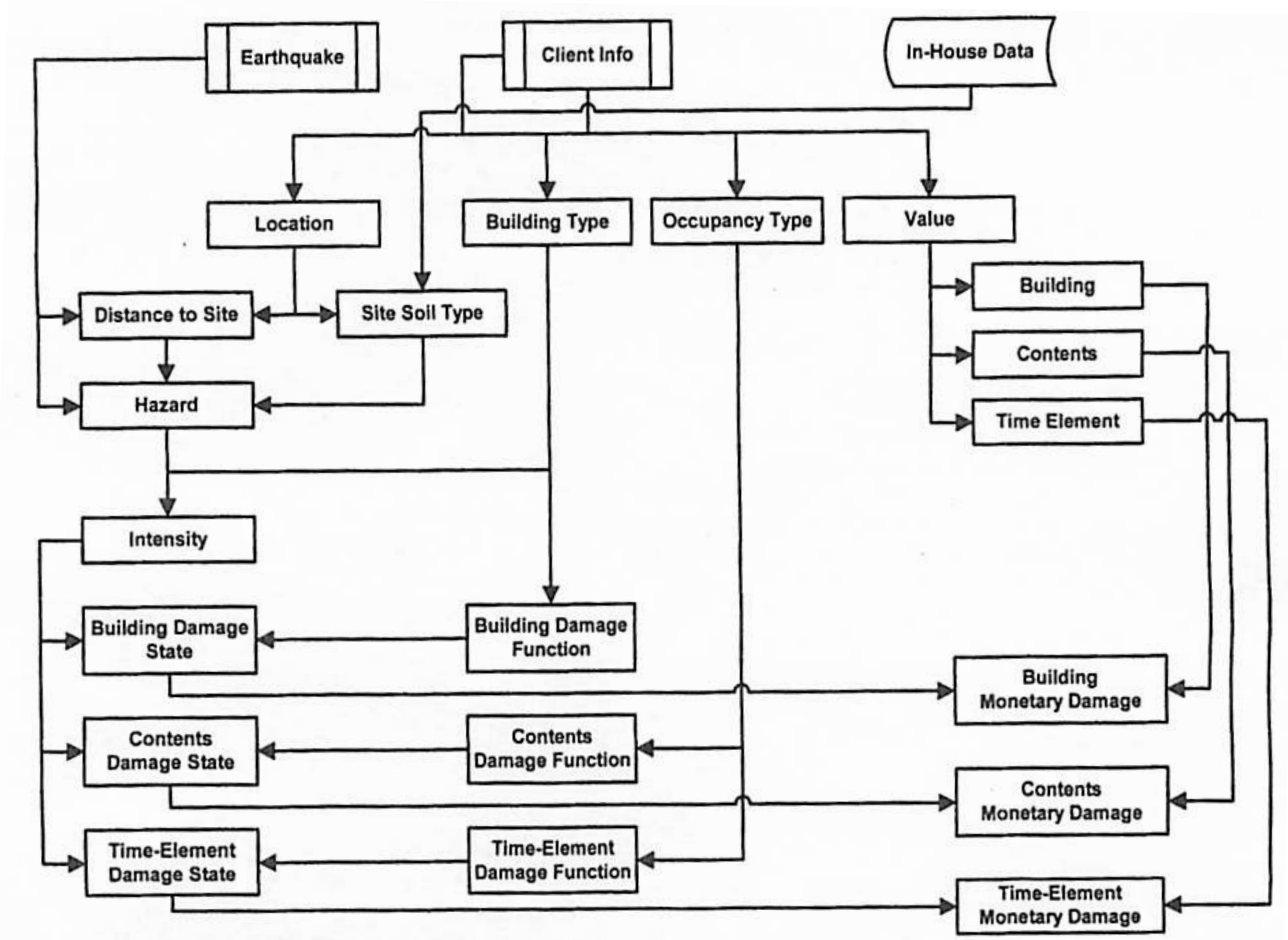


Figure 3.8. Schematic for process of damage calculation.

<u>Loss Module</u>:[65]

The loss module translates estimates of physical damage to a building, from the vulnerability module, into monetary costs to repair and replace damaged parts of the building. Consideration is also included of inspection costs, setup costs, and debris removal. Then the monetary costs are converted to insurance losses based on the coverage provisions of the insurance policy.

---

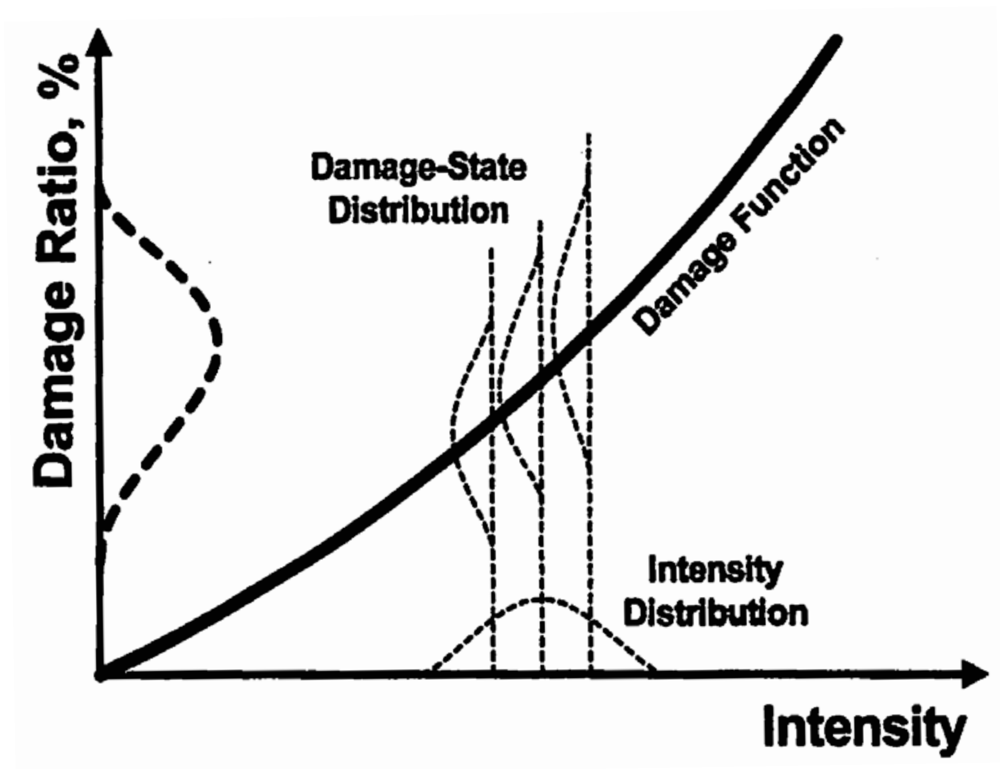
[63] I would not worry about any details of Figure 3.8.
[64] Time element would include business interruption losses.
[65] See Section 3.5 in Grossi and Kunreuther.

Figure 3.7 in Grossi and Kunreuther:[66]

The damage ratio is: $\dfrac{\text{repair cost}}{\text{replacement cost of the building}}$.

The expected damage ratio for an earthquake is a function of the intensity of the external excitation at the location of the building due to the earthquake.

The following is an illustration of a typical damage function for earthquakes:[67]



The solid line labeled damage function represents the expected damage ratio.
For any intensity of external excitation, the damage ratio is uncertain; this distribution for each intensity around the expected value is shown by the "damage-state distribution".

In addition, the intensity of excitation is uncertain. The distributions of possible intensities is shown as the intensity distribution.[68]

Combining the distribution of intensities with the damage-state distributions, produces the curve of damage ratios shown at the left of the above figure.

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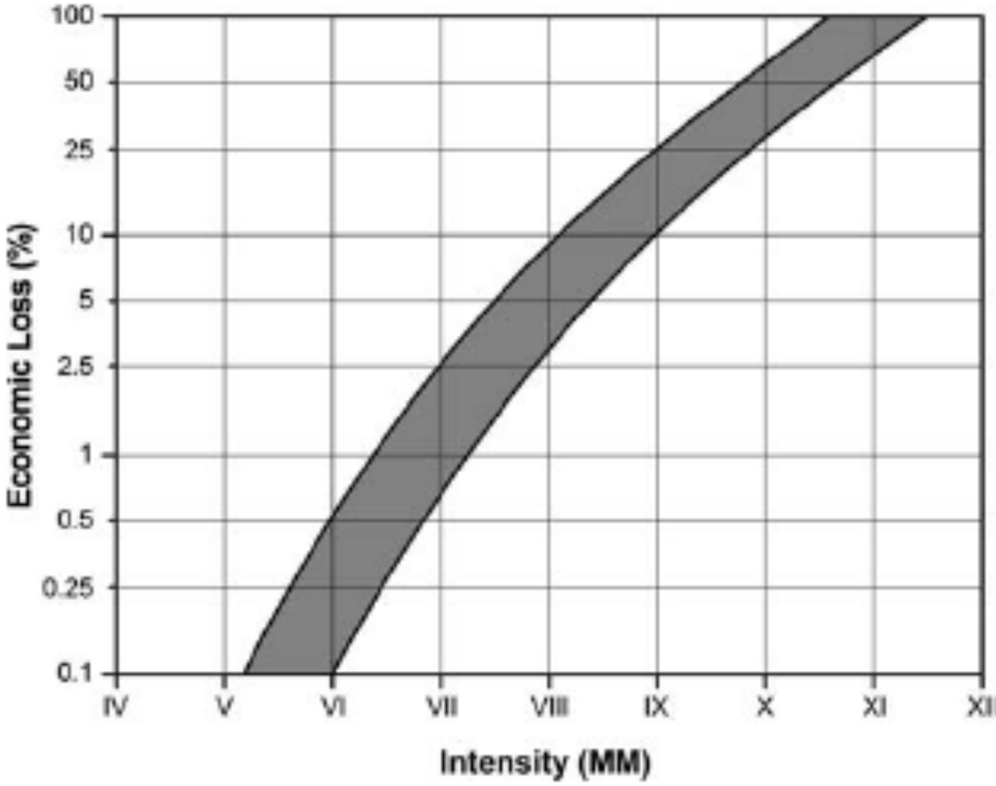
[66] See pages 62 to 64 of Grossi and Kunreuther.
See 8, 11/17, Q.20.
[67] See Figure 3.7 in Grossi and Kunreuther.
[68] It is not clear to me whether these are the possible intensities for a given (simulated) earthquake, or the combined results for all (simulated) earthquakes that could affect this location.

Here is another damage function for earthquakes, where the band shows the upper and lower limits of the economic loss (damage ratio) as a function of earthquake intensity:[69]



Exercise: For a certain building at a given location, in the coming year there will be an earthquake of magnitude 7.0 with 1% probability, of magnitude 7.2 with 2% probability, or of magnitude 7.4 with 1% probability.
(There is a large chance of no earthquake affecting this building in a given year.)
For intensity 7.0, the damage ratio is either 20% or 40% with equal probability.
For intensity 7.2, the damage ratio is either 40% or 60% with equal probability.
For intensity 7.4, the damage ratio is either 60% or 80% with equal probability.
Determine the resulting distribution of damage ratios for this building from this earthquake.
[Solution: For example, there is a 2% chance of an earthquake of magnitude 7.2.
This leads to damage ratios of 40% or 60%.
The contribution to 40% damage ratio is: (50%)(2%) = 1%.
The contribution to 60% damage ratio is: (50%)(2%) = 1%.
Combined damage ratios: 20% at (1%)(50%) = 0.5%, 40% at (1%)(50%) + (2%)(50%) = 1.5%, 60% at (2%)(50%)+ (1%)(50%) = 1.5%, 80% at (1%)(50%) = 0.5%.
Comment: Not intended to be realistic.
The probability distributions would be continuous rather than discrete.]

---

[69] Figure 1 from “Preliminary Quantitative Assessment of Earthquake Casualties and Damages”, by Badal, Vasquez, and Gonzalez.
Earthquake intensity is measured on the Modified Mercalli scale (MM):
VI = strong, VII = very strong, VIII = severe, IX = violent, X to XII = extreme.
For example, earthquakes of magnitude 6.0 to 6.9 would have intensities (MM) of VII to IX.
An earthquake of magnitude 7.0 or more would have intensity (MM) of VIII or higher.

Exercise: The building in the previous exercise has a replacement value of $100 million.
It is being insured for earthquakes. Assume no deductible.
Determine the premium for this building given the following information:
- The insurer's expense and profit load is 20% of premium.
- The risk load is set to 5% of the standard deviation of the loss.

[Solution: Mean damage ratio is:
(20%)(0.5%) + (40%)(1.5%) + (60%)(1.5%) + (80%)(0.5%) = 2.0%.
Mean damage is: (2.0%)($100 million) = $2 million.
Second moment of the damage is:
$(20 \text{ million})^2\ (0.5\%) + (40 \text{ million})^2\ (1.5\%) + (60 \text{ million})^2\ (1.5\%) + (80 \text{ million})^2\ (0.5\%) = 112 \times 10^{12}$.
The standard deviation of the damage is: $\sqrt{112 \times 10^{12} - 2 \text{ million}^2} = 10{,}392{,}305$.
Premium is: $\dfrac{2 \text{ million} + (5\%)(10.392305 \text{ million})}{1 - 20\%} = \$3.15$ million.
<u>Comment</u>: See 8, 11/17, Q20b.]

Types of Uncertainty in Catastrophe Models:[70]

Grossi and Kunreuther define two types of uncertainty, aleatory and epistemic.
“Aleatory uncertainty is the inherent randomness associated with natural hazard events.”
“Epistemic uncertainty is the uncertainty due to lack of information or knowledge of the hazard.”[71]

Aleatory Uncertainty ⇔ Process Risk.

Epistemic Uncertainty ⇔ Parameter Risk and Modeling Risk.

For example, in some ratemaking situation you might model frequency as Poisson with mean 6%. The aleatory uncertainty is due to the inherent randomness of the process.

Epistemic uncertainty can be due to uncertainty in what is the correct mean frequency for the situation being modeled; this is what an actuary would call parameter uncertainty or risk.[72]
Epistemic uncertainty can also be due to uncertainty in what is the correct model for frequency, perhaps it is in fact Negative Binomial; this what an actuary would call modeling risk.[73]

For example, in hurricane models, there is an inherent randomness as to what hurricanes there will be in a given year. What will be their paths and what will be their categories (magnitudes)? This produces aleatory uncertainty due to the inherent randomness of the process, similar to the roll of a die.

Hurricane models consist of many parts; there can be epistemic uncertainty in any of them.
For example, the assumed probabilities of various hurricane paths and magnitudes, and their relationships is likely to be somewhat different than the true distribution of probabilities.

Epistemic uncertainty can be reduced by collecting more data.
For example, there is an assumed relation between the construction and type of building and the damage that will result from a given wind speed. The more detailed data that is collected on the damage from actual hurricanes, the less uncertain will be a model of the relationship between damage and construction and type of building.

[70] See Section 4.2 of Grossi and Kunreuther. See also SOA GIIRR Exam, 11/13, Q.7.
[71] However, “developers of catastrophe models do not necessarily distinguish between these two types of uncertainty; instead model developers concentrate on not ignoring or double counting uncertainties and clearly documenting the process in which they represent and quantify uncertainties.”
[72] Earlier exams discuss how to estimate the variance of parameters estimated via maximum likelihood.
[73] Earlier exams discuss statistical tests to try to determine which is the better model.

Sources of Epistemic Uncertainty:[74]

Some examples of sources of epistemic uncertainty mentioned in Grossi and Kunreuther:[75]
- How to model the recurrence of earthquakes along a fault line.
- Lack of information on repair costs.[76]
- Lack of information on business interruption costs.
- Attempts to quantify the effects of demand surge.[77]
- Lack of information to create the Geographical Systems database (GIS).
- Lack of sufficiently detailed information on the underlying soil in a region.
- Lack of information on previous damage to a structure.
- Lack of information on the location of a structure relative to neighboring structures.
- Lack of accurate information on the values of properties.
- Use of laboratory testing and/or expert opinion to develop the vulnerability component.[78]

Each time you run a given simulation model, you will get a somewhat different result due to aleatory uncertainty, in other words due to the inherent randomness of the process being modeled. However, by running the model a sufficient number of times, you can reduce the effect of this randomness on the estimate of some quantity of interest such as the mean or the 95th percentile.

Simply running a single catastrophe model, will not quantify the epistemic uncertainty inherent in that model. One way to try to quantify this epistemic uncertainty is to compare the results of catastrophe models developed by different vendors applied to the same situation.[79] The more similar the outputs are from independently developed models, the less uncertainty an actuary would have in the results of any single model.[80]

For example, let us assume we run three different catastrophe models for the same portfolio of properties. If the 99th percentiles, in other words the one-in-100 year events, were $2.9 billion, $3.0 billion and $3.1 billion, that would give an actuary some confidence in the results of the three different models.[81] [82] If instead the estimates from the three models were $2 billion, $3 billion, and $4 billion, that would give an actuary less confidence in the results of the three different models.

---

[74] See Section 4.3 in Grossi and Kunreuther.
[75] It would not hurt to have a couple of examples ready to use if needed.
[76] For example what would be the effect of demand surge after a major catastrophe.
[77] I discuss demand surge in a subsequent subsection.
[78] Expert opinions can be combined using the Delphi Method. The experts answer questionnaires in two or more rounds. After each round, a facilitator provides an anonymous summary of the experts' answers from the previous round as well as the reasons they provided for their judgments. Thus, experts are encouraged to revise their earlier answers in light of the replies of other members of their panel.
[79] See for example, Figure 4.4 in Grossi and Kunreuther.
[80] Often an actuary will average the results of two or more catastrophe models.
See Table 4.2 in Grossi and Kunreuther.
[81] Of course, an important question is how independent the development of the different models were from each other. Some of the same data and models from engineering, meteorology, and/or geology may have contributed to the different models.
[82] One should make other comparisons between the models, both at different percentiles and for different portfolios of insured properties.

Logic Trees:[83]

Another way to estimate the effect of epistemic uncertainty is to use Logic Trees.

One of the standard techniques an actuary will use to test a model, is to see the sensitivity of results to the values of key parameters or assumptions. The actuary will change one or more of the parameters or assumptions within the range of reasonable possibility, and see how the results are affected. The more sensitive the output is, the more epistemic uncertainty.

In any catastrophe model, there are parameters, probability distributions, and scientific models. In a Logic Tree one would vary each of these and weight the possibilities. For example, one might give a key parameter, such as the average insurance to value, one of three values with equal probability. At the same time, for example, you could equally weight two competing engineering, meteorological, or geological models.[84]

You then end up with a large number of sets of combinations of different aspects, each with a probability.[85] In essence you have created a spectrum of models similar in concept but different in detail.

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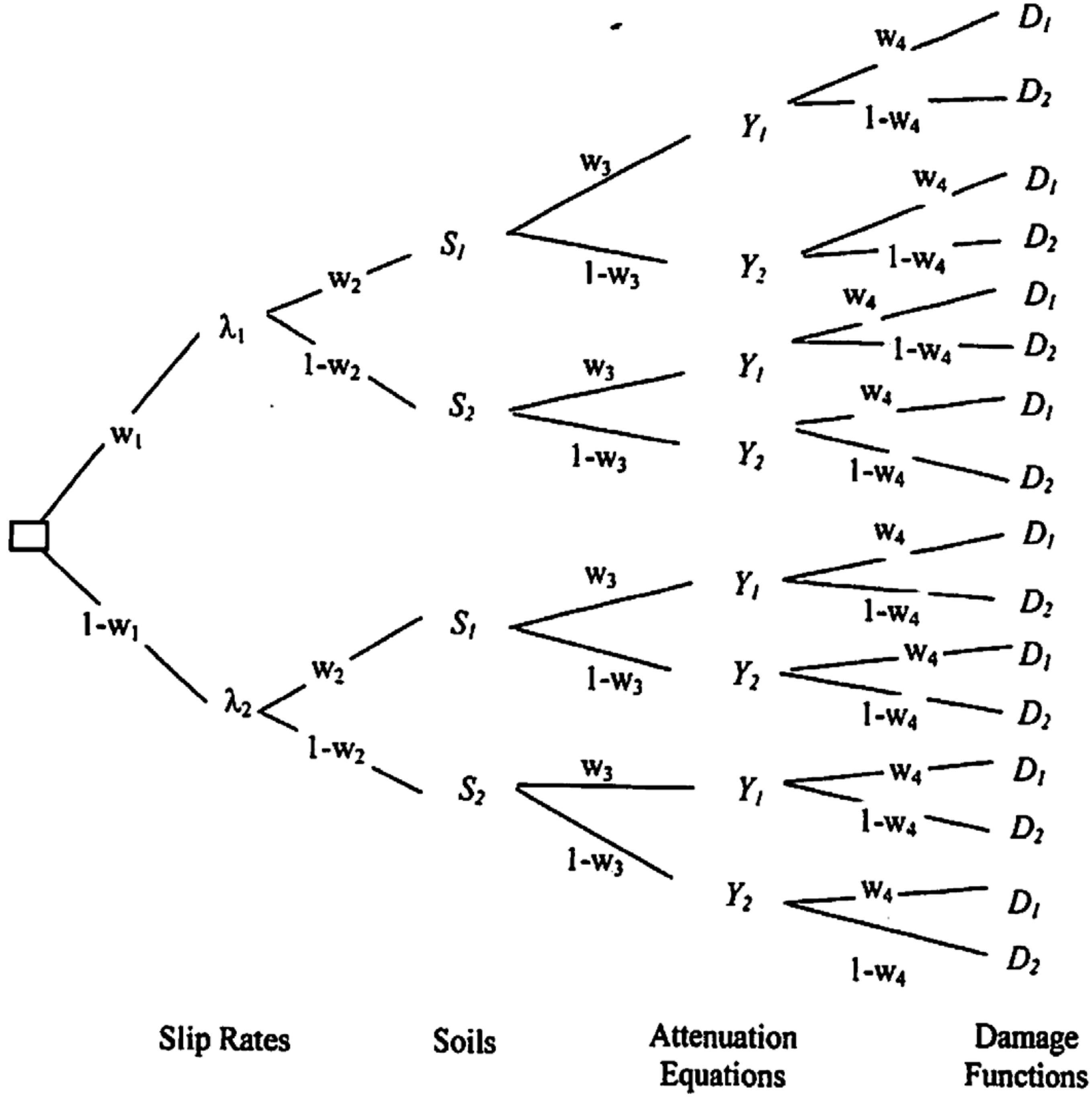
[83] See Section 4.4.1 and Figure 4.1 of Grossi and Kunreuther. See also SOA GIIRR Exam, 11/13, Q.7.
[84] Grossi and Kunreuther give the earthquake example of two competing models of strong ground motion in the central and eastern United States.
[85] For example, if you had two possibilities for each of ten aspects of a catastrophe model, you would have 2^{10} = 10,324 different sets of assumptions.

Figure 4.1 in Grossi and Kunreuther shows an example of a logic tree for an earthquake model:



There are two possibilities for each of four aspects: slip fault rate, soil type, attenuation equations, and damage functions.[86] This results in $2^4 = 16$ different models in total. One could give each possibility either equal or different weights. For example, one could give one of the two possible attenuation equations 60% weight and the other equation 40% weight, indicating that the modeler believes that the first equation is probably a better model of reality than the second.

Then for a given portfolio of properties, one could compute the exceedance curve for each of these different models. These results plus the associated probabilities from the logic tree can be used to get confidence intervals.[87]

For example, using the logic tree, we would have a large set of estimates of the size of the one-in-100 year event. Let us assume that 5% of the probability is associated with one-in-100 year events less than or equal to $1.1 billion, while another 5% of the probability is associated with one-in-100 year events greater than or equal to $1.7 billion. Then a 90% confidence interval for the size of the one-in-100 year event is $1.1 billion to $1.7 billion.[88]

Let us assume that 5% of the probability is associated with a return time for $2 billion of less than or equal to 230 years, while another 5% of the probability is associated with a return time for $2 billion of greater than or equal to 380 years. Then a 90% confidence interval for the return time for $2 billion is 230 to 380 years.

[86] Due to imperfect information, there are two possibilities for the soil type at a particular location.
[87] See for example, Figure 4.5 in Grossi and Kunreuther.
[88] Of course, this is based to some extent on the judgment of the person who constructed the logic tree as to what is a range of reasonable possibilities for parameters, distributions, models, etc. If there are possibilities that person has not considered or if that person was too cautious, then the confidence interval will be too narrow.
For example, think about the poor performance of models of mortgage backed securities. Housing prices started to decline significantly in 2006 and 2007 in much of the United States. Most of the modelers had not considered this type of significant decline in most of the country as a possibility, so it was not included in their measures of uncertainty of their models.

Simulation:[89]

Another way to estimate the effect of epistemic uncertainty is to assume a distribution of parameters, etc., and then pick assumptions at random via simulation.

In an actuarial model we might assume a Poisson frequency. Then as part of estimating the effect of epistemic uncertainty, the mean λ may be assumed to have a distribution of values rather than a single value. If λ is assumed to have a simple discrete distribution, this is basically the same as what was done with Logic Trees. However, if instead lambda is assumed to follow a continuous distribution, then each time we ran the model we would simulate a random value of lambda.[90]

For a catastrophe model, we would end up with sets of assumptions as per Logic Trees, however, there may be too many to list or due to the use of continuous distributions it may be impossible to list all of them. Thus we simulate a large number of the possible sets of assumptions, and run the catastrophe model for each such set.[91] Then the output can be used similarly to that from Logic Trees, in order to get a confidence interval for the exceedance probability curve.

Uncertainty and the Exceedance Probability Curve:[92]

Using the logic tree or simulation approach we have created a set of different versions of a catastrophe model, with corresponding probabilities. We can then run each version of the catastrophe model to produce an exceedance probability curve.[93]

Then we could weight these individual curves using the corresponding probabilities; this is mathematically just like a mixed survival function. More interestingly, we can use the range of these individual curves to obtain an estimate of the epistemic uncertainty of the exceedance probabilities estimated by this catastrophe model.

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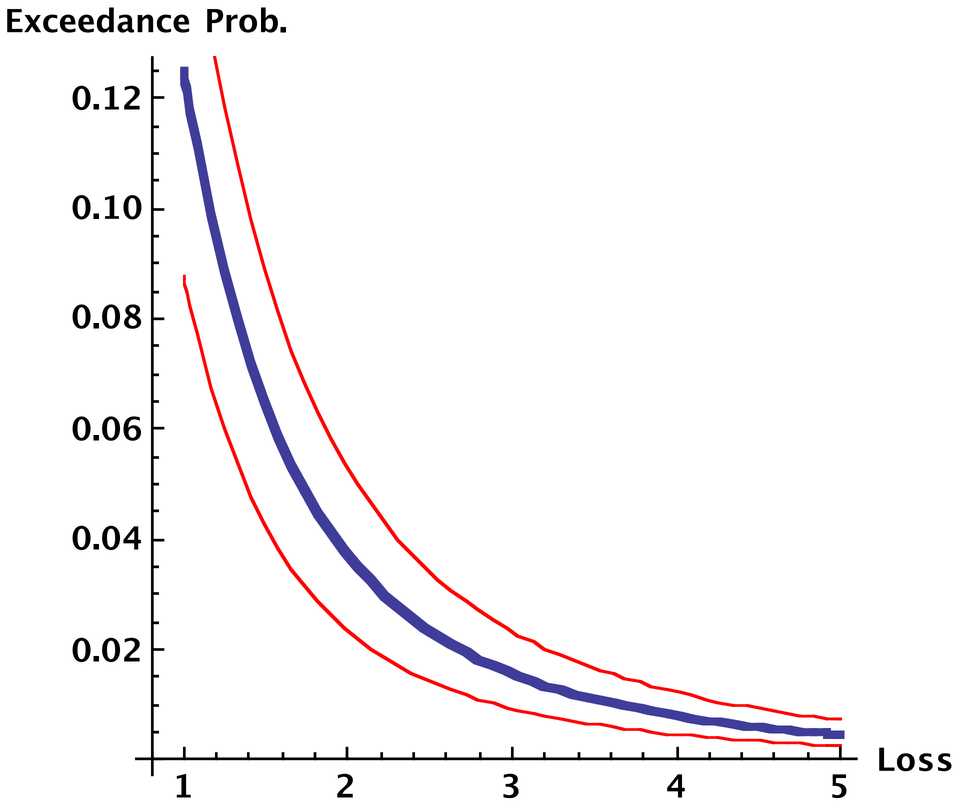
[89] See Section 4.4.2 of Grossi and Kunreuther. The explanation in the text concentrates on the general concept of simulation rather than explaining how to use simulation to estimate epistemic uncertainty. Their example of Monte Carlo simulation seems to be quantifying aleatory uncertainty or process risk.
They mention Latin Cube Sampling, which is one way to increase the efficiency of simulation.
[90] For example, we might assume lambda was uniform on an interval. Instead, we might assume lambda followed a Gamma Distribution, in which case the mixed frequency distribution is Negative Binomial.
[91] In bootstrapping via simulation, we simulate a large number of subsets out of the larger number of possible subsets. Here we simulate a large number of sets of assumptions out of the larger number of possible sets of assumptions.
[92] See Section 4.4.3 of Grossi and Kunreuther.
[93] This would apply separately to any of the three types of exceedance probability curves:
aggregate, occurrence, or conditional.

Doing so would produce something like the following, where losses are in billions of dollars, and the two thin lines represent a 95% confidence interval for the exceedance probabilities:[94]



In a different example, let us concentrate on the probability of exceeding $1 billion. Running the different versions of the model, we rank the values of S($1 billion) from smallest to largest:
0.0238 with probability 1%
0.0254 with probability 2%
0.0260 with probability 2%
⋮
0.0281 with probability 3%
0.0295 with probability 2%

Then a 90% confidence interval for the exceedance probability at $1 billion is
from 0.0260 to 0.0281. This leaves 10% probability in the tails, <u>including</u> 0.0260 and 0.0281.

Alternately, a 95% confidence interval for the exceedance probability at $1 billion is
from 0.0260 to 0.0281. This leaves 5% probability in the tails, <u>excluding</u> 0.0260 and 0.0281.[95]

[94] Similar to Figure 4.3 in Grossi and Kunreuther.
[95] Either way of looking at things has some validity.

Earthquake Hazard: Charleston, South Carolina:[96]

Four earthquake models were compared: RMS, AIR, EQECAT, and FEMA's HAZUS Model. A common set of assumptions was specified in order to estimate the damage due to earthquakes in the region of Charleston, South Carolina.[97] [98]

For each of the three commercial models separately, three exceedance probability curves were produced: the mean exceedance probability curve and two additional curves representing the 5^{th} and 95^{th} percentiles of loss, producing a symmetric 90% confidence intervals around the mean loss.[99] While no details are given, I assume that for each model separately a logic tree or simulation was used, as discussed in my previous subsection, in order to get a confidence interval for the exceedance probability curve.

Then these results for the mean for the three commercial models were combined by averaging them, with 1/3 weight to each. For example, if the three mean estimates of the probability of exceeding \$5 billion, S(\$5 billion), were: 0.17%, 0.18%, and 0.25%,
then the combined estimate is 0.20%.

The 5^{th} percentile estimates were combined by taking the minimum, while the 95^{th} percentile estimates were combined by taking the maximum.[100] For example, let us assume that the three estimates of the 5^{th} percentile of S(\$5 billion) were: 0.13%, 0.12%, and 0.16%. Then the 5^{th} percentile in the combined curve is taken as 0.12%. If the three estimates of the 95^{th} percentile of S(\$5 billion) were: 0.27%, 0.33%, and 0.41%. Then the 95^{th} percentile in the combined curve is taken as 0.41%.

[96] See Section 4.5.2 of Grossi and Kunreuther. I cover Section 4.5.1 on Florida hurricanes in a later subsection.
[97] Table 4.3 in Grossi and Kunreuther specifies the assumptions. I would not worry about the details.
[98] Ground up unlimited losses to buildings were estimated without worrying about provisions in insurance policies.
[99] It is not specified whether these are aggregate or occurrence exceedance curves, but I suspect they are occurrence exceedance probability curves (OEP).
[100] In the extreme righthand tail there were some cases where one model did not have had an estimate of the 5th or 95th percentiles, due to modeling uncertainty, in which case the minimum or maximum of the results of the other two models was taken.

The result of combining the output of the three commercial models is shown in Figure 4.5 in Grossi and Kunreuther, with the equally weighted mean exceedance probability and the upper bound (95th percentile) and the lower bound (5th percentile), and the losses in billions of dollars:

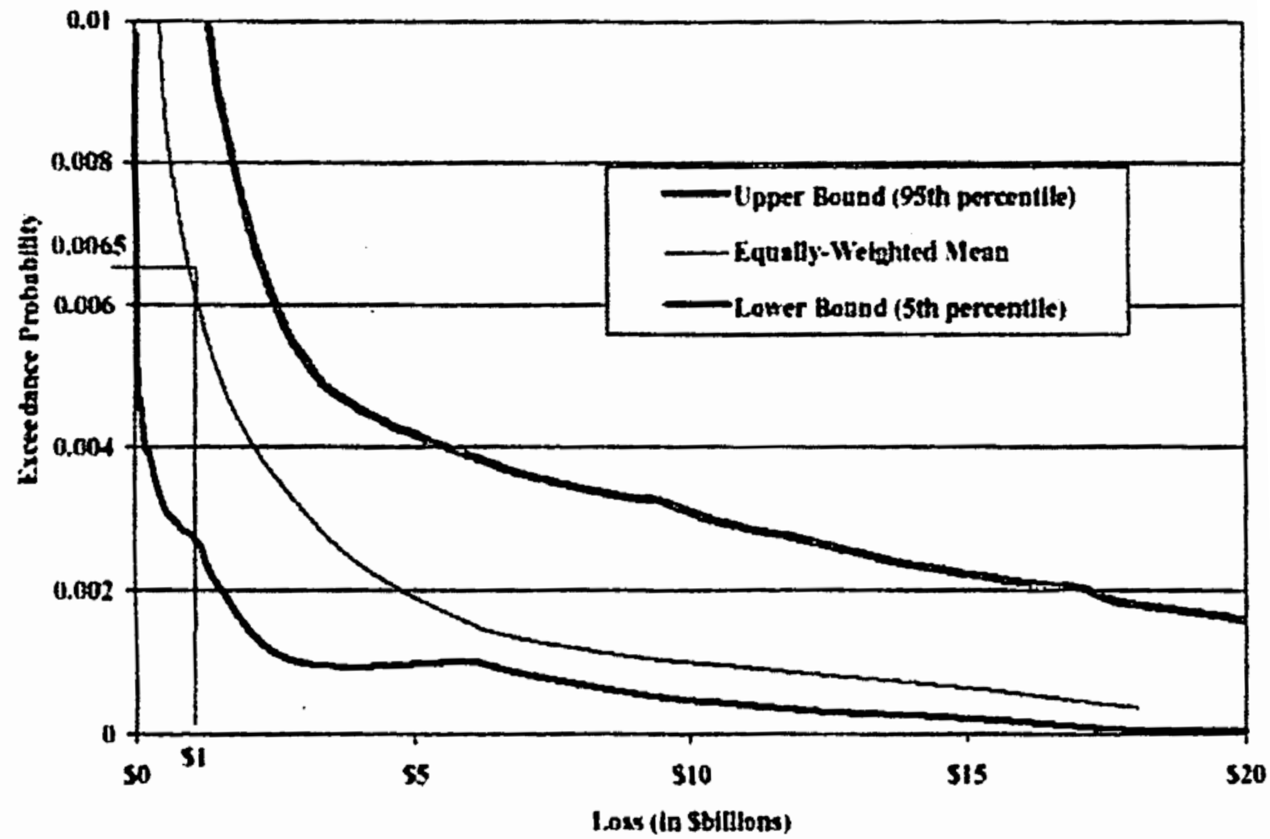


Figure 4.5. Composite exceedance probability curves for Charleston region.

The HAZUS model was developed by the U.S. Federal Emergency Management Agency (FEMA). Currently, Hazus can model four types of hazards: flooding, hurricanes, coastal surge, and earthquakes, however, for this study only the earthquake model was used.[101]

The model was run for a set of 156 (fictional) earthquakes in the Charleston region, versus a given portfolio of buildings, in order to produce an exceedance probability curve. Then the assumptions of the model were varied, in order to test its sensitivity. Assumptions that were varied included: occupancy of structures, attenuation relationships, duration of earthquakes, and types of soils. As an example, Figure 4.7 in Grossi and Kunreuther compares two exceedance curves, one assuming all stiff soil and the other assuming a range of soil types.[102]

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[101] In order to produce exceedance probability curves for this study, additional programing was added to the HAZUS model.
[102] For example, for all stiff soils, the 1-in-500 year event is $8.4 billion, while for varied soil types it is $6.7 billion.

CAS Principles of Ratemaking:[103] [104]

The CAS Principles of Ratemaking stated:
1. A rate is an estimate of the expected value of future costs.
2. A rate provides for all costs associated with the transfer of risk.
3. A rate provides for the costs associated with an individual risk transfer.
4. A rate is reasonable and not excessive, inadequate, or unfairly discriminatory if it is an actuarially sound estimate of the expected value of all future costs associated with an individual risk transfer.

Catastrophe models support this principle by providing increased accuracy in projecting expected future cost of a risk transfer compared to the prior methods used. Catastrophe models provide better estimates of the effect on losses of location, structural attributes, occupancy, mitigation measures, line of business, etc.

This enables better classification of risks and thus increased equity in rates between insureds. Catastrophe models provide more accurate estimates of the expected value of all future costs associated with an individual risk transfer.

One must determine an appropriate exposure unit or premium basis that varies with hazard, and is practical, and verifiable.

[103] See Section 5.2 of Grossi and Kunreuther, where they refer to the attachment of the CAS Principles of Ratemaking (May 1988) to the former Actuarial Standard of Practice number 9.
[104] In December of 2020, the CAS rescinded several Statements of Principles as no longer necessary due to the adoption of many Actuarial Standards of Practice over the previous decades. However in May 2021, the CAS reinstated the CAS Principles of Ratemaking for reference for U.S.-regulated ratemaking.

Use of Catastrophe Models in Rate Making:[105]

The actuary can use the Catastrophe Model to estimate the Average Annual Loss (AAL).

Exercise: For a county, given the following output from a catastrophe model, what is the AAL?

Probability	Annual Loss per $1000 of value
97%	$0
1%	$100
1%	$300
1%	$500

[Solution: (1%)(100) + (1%)(300) + (1%)(500) = $9 per $1000 of value.
Comment: See the output in Table 5.1 in Grossi and Kunreuther.]

Natural hazard perils primarily differ from other perils due to highly correlated losses. This puts a strain on the insurer's surplus. Thus the insurer will want to include a risk load in order to compensate for the opportunity cost of holding capital, in other words for the Surplus Cost.

Table 5.2 in Grossi and Kunreuther shows insurance rates based on the values in Table 5.1. Although it is not explained, in each case, rate = mean + (standard deviation)/2.[106]
In other words, the risk load is taken as half of the standard deviation.[107]

Exercise: In the previous exercise, compute the standard deviation.
[Solution: Second moment is: (1%)(100^2) + (1%)(300^2) + (1%)(500^2) = 3500.
Standard Deviation = $\sqrt{3500 - 9^2}$ = 58.47.]

Thus the risk load would be 58.47/2 = 29.23.
Ignoring expenses, the rate would be: 9 + 29.23 = 38.23 per $1000 of value.

Premium = AAL + Risk Load + Expenses.

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[105] See Section 5.3 in Grossi and Kunreuther.
[106] For example, 9.02 + 45.98/2 = 32.01.
[107] Multiplying the standard deviation by some constant is one simple way to calculate a risk load.
In "On the Theory of Increased Limits and Excess of Loss Pricing," Robert S. Miccolis, formerly on the syllabus, determines risk loads to include in the calculation of ILFs by multiplying the variance times some constant.
There are more sophisticated ways to calculate a risk load, which are not covered on the syllabus of this exam.

Structural Attributes:[108]

Two ways of differentiating risk: structural attributes and location, are discussed in detail. These should be used to help determine equitable insurance rates for catastrophes.

Examples of structural attributes of a building include: building materials, building codes, year of construction, and occupancy. Structural attributes help determine the performance of the building when hit by a catastrophe.

Wood frame is better for earthquakes than masonry, while the reverse is true for hurricanes.

Over time building codes have improved. The codes apply to new and retrofitted buildings. Thus newer buildings are likely to be less damaged by a given catastrophe.[109] Also over time construction techniques have changed. In addition, over time the size and style of new houses has changed. Thus year of construction is a useful (categorical) rating variable.

For commercial properties, occupancy, in other words what the building is used for affects the layout of the building and how vulnerable it may be to a catastrophe.[110] The layouts of walls, windows, doors, equipment affect building performance in a catastrophe. Occupancy has a great effect on the potential for business interruption losses due to a catastrophe. Occupancy also affects the potential for losses due to contents.

Location Attributes:[111]

Where a building is located determines the forces it will be subjected to by a given catastrophe.

In the case of earthquake: distance from an earthquake fault, and local soil conditions.
For example, if a building is sitting on landfill it may be much more damaged by an earthquake than a similar building much closer to the earthquake.[112]

In the case of hurricanes: distance from the coast, surface roughness and topography.[113]
Surface roughness and topography can locally slow down the winds from a hurricane.

In the case of floods: whether the building is in the 100-year flood plain.[114]

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[108] See Section 5.3 in Grossi and Kunreuther.
[109] Building codes and how well they are enforced may vary by locality.
[110] A restaurant differs from a department store, which differs from an office building, which differs from an automobile factory, which differs from a chemical plant, etc.
[111] See Section 5.3 in Grossi and Kunreuther.
[112] The effects of soil amplification is shown by comparing Figures 5.3 and 5.4 in Grossi and Kunreuther.
[113] See Figure 5.2 in Grossi and Kunreuther.
[114] As sea level rises it is important to update coastal flood plain maps.
As rainfall patterns change it is important to update river flood plain maps.

Regulation and Catastrophe Models:[115]

In the United Sates, insurance is largely regulated by each of the fifty states. Each state, particularly those significantly affected by catastrophes such as hurricanes and earthquakes, has had to deal with the use of catastrophe models in ratemaking.[116]

When catastrophe models were first introduced, insurers had an uphill battle in trying to convince most state insurance regulators to allow the output of such models in rate filings. Regulators were particularly concerned with the resulting large indicated increases in Homeowners rates in certain parts of their states.[117] Over time, many states have developed ways for insurers to use such models in their rate filings.[118] [119]

Catastrophe models can be helpful to insurance departments in their role of regulating solvency.[120] These models can help regulators assess whether an insurer has accumulated too many risks which can be affected by a single catastrophe and whether the insurer has sufficient reinsurance protection.
For example, insurance regulators in Canada required insurers to report Probable Maximum Losses (PMLs) based on earthquake models, in order to see if the insurers had sufficient surplus in light of the business being written and their reinsurance protection.[121]

There are a number of issues unique to catastrophe models that challenge insurance regulators.
The models are complex and involve input from multiple disciplines.
The models include many inputs and assumptions; often there is less data than would be desirable. The models are usually from vendors, who consider their models proprietary, and thus do not want the details made public. Different models from different vendors produce somewhat different results.

Grossi and Kunreuther go into detail on how Florida handled the use of hurricane models in ratemaking and how California handled the use of earthquake models in ratemaking. I will discuss these next.

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[115] See Section 5.4 of Grossi and Kunreuther.
[116] While catastrophe models are also used to help price reinsurance, that is largely unregulated by the states. However, when catastrophe models first became widely used, an increase in the price of reinsurance affected the willingness of insurers to write Homeowners in certain parts of certain states at the then current rates.
[117] Due to availability problems for Homeowners Insurance, some states have set up or expanded residual markets.
[118] Different states have come to somewhat different solutions. Some states have adapted what some other state has done. (This is a standard technique of insurance departments when something new comes along.) States also cooperate through the NAIC, the National Association of Insurance Commissioners.
[119] Grossi and Kunreuther was written in 2005, and things have evolved somewhat since then.
[120] One job of insurance regulators is to monitor the solvency of insurers; does an insurer have sufficient surplus for the risks being written.
[121] See Sidebar 1 at page 109 of Grossi and Kunreuther.

Ratemaking for Hurricanes in Florida:[122]

Prior to the widespread use of catastrophe models, the older methods used to estimate hurricane losses in Florida proved to very significantly underestimate the potential losses due to hurricanes such as Andrew in 1992. After Hurricane Andrew, insurance companies tried to substantially raise their homeowners rates in much of Florida and/or substantially decrease their amount of business written. Florida had to somehow deal with this situation, including the desire of many insurers to rely on catastrophe models for hurricanes.

Florida developed a review process by which modeling firms must show that their products meet technical criteria. The Florida Commission on Hurricane Loss Projection Methodology (FCHLPM) consists of independent experts.

To be certified for use in establishing Homeowners rates, a catastrophe model undergoes a rigorous yearly review by FCHLPM. A professional team conducts an onsite audit; the team consists of a statistician, actuary, computer scientist, civil engineer, and meteorologist.

As part of the certification process of a model, the modeling firm must submit an exceedance probability curve for a specified portfolio of residential structures in Florida. Table 4.2 in Grossi and Kunreuther shows a comparison of exceedance probabilities for different models.

Exercise: For three models, the 1-in-500 year events are in millions of dollars:
63.7, 52.0 and 56.5.
Take a weighted average, with 50% weight to the middle value and 25% weight to the others.
[Solution: (50%)(56.5) + (25%)(52.0) + (25%)(63.7) = 57.2.
Comment: Matches the result in the last column of Table 4.2.
The 1-in-500 year event corresponds to a exceedance probability of 0.20%.]

If an insurer submits a rate filing that relies on one or more of the approved catastrophe models, then the findings of the FCHLPM are admissible as evidence at any rate hearing.[123] [124]

Due to availability problems, in 1993 a residual market pool for wind insurance was established, the Florida Hurricane Catastrophe Fund (FHCF) was established. FHCF bases it rates on an average of the output of models approved by the Florida Commission on Hurricane Loss Projection Methodology.

[122] See Section 4.5.1 of Grossi and Kunreuther.
[123] Many large insurers will rely on models from two different vendors.
[124] Insurers are not required to use any of the approved models.

Ratemaking for Earthquakes in California:[125]

The large Northridge earthquake in 1994, led to availability problems for homeowners insurance.
In 1996, this prompted the formation of California Earthquake Authority (CEA) in order to sell homeowners earthquake insurance.

Rate-Setting Considerations for CEA, as per the establishing legislation:[126]

1. Rates established by the Authority shall be actuarially sound so as to not be excessive inadequate, or unfairly discriminatory. Rates shall be based on the best available scientific data. Consider at least: proximity to faults, soil type, construction, and age.

2. If scientific information is used, such information must be consistent with available geophysical data and state of the art scientific knowledge.

3. Scientific information that is used to establish different rates between the most populous rating territories in northern and southern California cannot be used unless that information is analyzed and approved by outside experts.

4. The legislature does not intend to mandate a uniform statewide flat rate for residential policies.

5. Rates established shall not be adjusted to provide rates lower than are justified for classifications of high risk of loss or higher than are justified for classifications of low risk of loss.

6. Policyholders who have retrofitted homes to (better) withstand earthquake shake damage shall receive a 5% premium discount, as long as it is determined to be actuarially sound.

The CEA came up with rates based on a catastrophe model as well as the provisions of the policies it was offering. These rates were the subject of a long and hotly contested hearing before the California Insurance Department. Grossi and Kunreuther discuss in detail the important issues raised at this rate hearing:[127]

1. Earthquake Recurrence Rates: Since historical data in California is limited to 150 years, determining the long-term rate of earthquakes is critical. One of the most significant finds was that the models produced frequencies that were more than twice the historical record. This finding challenged the acceptability of the CEA model's frequency. It was ultimately determined to be valid by comparing that the model compared favorably to models produced by the California Division of Mines and Geology.

[125] See Section 5.5 of Grossi and Kunreuther.
[126] See Sidebar 2 in Grossi and Kunreuther.
[127] It would not hurt to remember one or two of these issues. The Commissioner of Insurance ruled in favor of CEA loss estimates based on catastrophe modeling on almost all major issues.

2. Damage Estimates: Model-based damage estimates are derived by associating a given level of ground shaking severity at a site with the vulnerability to shaking damage for a specific class of structure defined by age, type of construction, number of stories, etc. Prior to the Northridge Earthquake, earthquake damage curves were based on engineering opinions and judgments published by the Applied Technology Council (ATC). However, the model based its curves on over 50,000 claims from the Northridge quake. It was argued that the ATC-13 curves, which were in the public domain since 1985, should be relied upon as opposed to model-based proprietary curves, which were derived from principally one event. Testimony from representatives of the ATC itself supported the use of claims-based curves as the best available source of information for the link between shaking intensity and damage.

3. Underinsurance Factor: Model-based damage estimates are expressed as a percent of the building's value. Accordingly, if the value used is less than the replacement cost, damage and loss estimates are understated. In addition, the policy deductible is likely to be understated since it is typically defined as a percent of the policy limit. Because of inflation and lack of accurate valuation, the insurance to value ratio for most buildings is usually less than 1.0. In other words, most buildings are underinsured. Since the residential insurers in California did not readily have an estimate of the degree of underinsurance, consumer groups challenged the initial model assumptions of 13% derived from surveys of insurance actuaries in the state. They claimed that there was 0% underinsurance and that the properties were fully insured. Ultimately, a 6% underinsurance figure was agreed to and rates were lowered from the initial projections to reflect this compromise.

4. Demand Surge: The CEA testified that insurers estimated a 20% impact for demand surge following the Northridge earthquake.[128] Since the vulnerability curves were based on Northridge data, the curves used were adjusted and initially reduced by 20% to eliminate the demand surge effect. Then the curves were increased by an adjustable factor, relating demand surge to the size of loss from each stochastic event in the model's probabilistic database. Although interveners argued that demand surge does not exist, the CEA actuarial group's testimony was accepted as reasonable even though little empirical data exists to support this assertion.

5. Policy Sublimits: Although a catastrophe model was used to establish the loss costs through such risk factors as location, soil conditions, age and type of structure, the model could not determine the contribution to losses from certain CEA policy features such as sublimits on masonry chimney damage, walkways, awnings, etc., because insurance claims data do not identify sources of loss from these categories. Hence, actuaries had to reduce the modeled loss costs to account for the specific CEA policy sublimits which were not reflected in the claims data used in the damage estimates produced by the model.

[128] I discuss demand surge in a subsequent subsection.

6. Rating Plan-Deviation: The CEA capped rates in two territories because of affordability issues and spread the capped costs to other territories. The commissioner ruled that a rating plan does not have to base premiums on risk in view of affordability issues, and that the plan was actuarially sound and not unfairly discriminatory.

7. Retrofit discount: The CEA offered discounts for three mitigation measures: bolting walls to the house foundation, cripple wall bracing, and water heater tie-down. Based on conversations with structural engineers, the CEA concluded that these measures would reduce losses; the statutory minimum of 5% was used as the discount. The Commissioner ruled the discount was appropriate, even though there was no empirical scientific guidance on the loss reduction.

8. Changing Deductibles and Coverage Limits: The CEA proposed a 15% deductible, a $5000 limit on contents and $1500 limit on additional living expenses. Testimony supported that the insurance costs would be halved from what they were with the previous 10% deductible with much higher limits for contents and additional living expenses.

The hearings associated with the California Earthquake Authority (CEA) raised a number of questions that require future research:[129]

1. The hearing highlighted the significant disagreement among earth scientists on frequency estimates, maximum magnitude and time dependent calculations. Disagreements are likely to persist. The challenge is to select credible and representative research, and in some cases to include more than one methodology in the catastrophe models.

2. Insurance claims data from catastrophic loss are by nature very limited. Yet, it is the single best source from which to estimate future losses. Insurers need to capture and preserve loss data and portfolio exposures for each loss event.

3. Mitigating future catastrophic losses via structural retrofits, with commensurate insurance premium reductions, is strongly desired by politicians and the public. Models have the ability to quantify the various wind or earthquake mitigation applications, but are hampered by the lack of detailed loss data, since insurers typically do not distinguish losses by structural component, such as roof, chimney, foundations, or non-load bearing walls. The states of Florida, California, and Hawaii are encouraging research and studies to assist in estimating such benefits.

4. Demand surge increases repair and building costs after a major catastrophe; one wishes to better understand and quantify this effect.

[129] See Section 5.5.3 of Grossi and Kunreuther.

Open Issues, Using Catastrophe Models in Ratemaking:[130]

While the use of catastrophe models for insurance ratemaking has become common practice, there are still some open issues:

1. Regulatory Acceptance: Proprietary sophisticated models create a problem for regulators who are unlikely to have the technical expertise to judge the reasonableness of the inputs, assumptions and outputs.

2. Public Acceptance: As expected public acceptance of models has been low, principally because their use resulted in substantial increases in wind or earthquake rates.
No insured likes to get a rate increase.

3. Actuarial Acceptance: Since the models are outside the actuary's usual professional expertise, it is necessary for them to become familiar with the model components.
The Actuarial Standards Board has published Standard of Practice No. 38 that requires actuaries to
(a) determine appropriate reliance on experts,
(b) have a basic understanding of the model,
(c) evaluate whether the model is appropriate for the intended application,
(d) determine that appropriate model validation has occurred, and
(e) determine the appropriate use of the model.

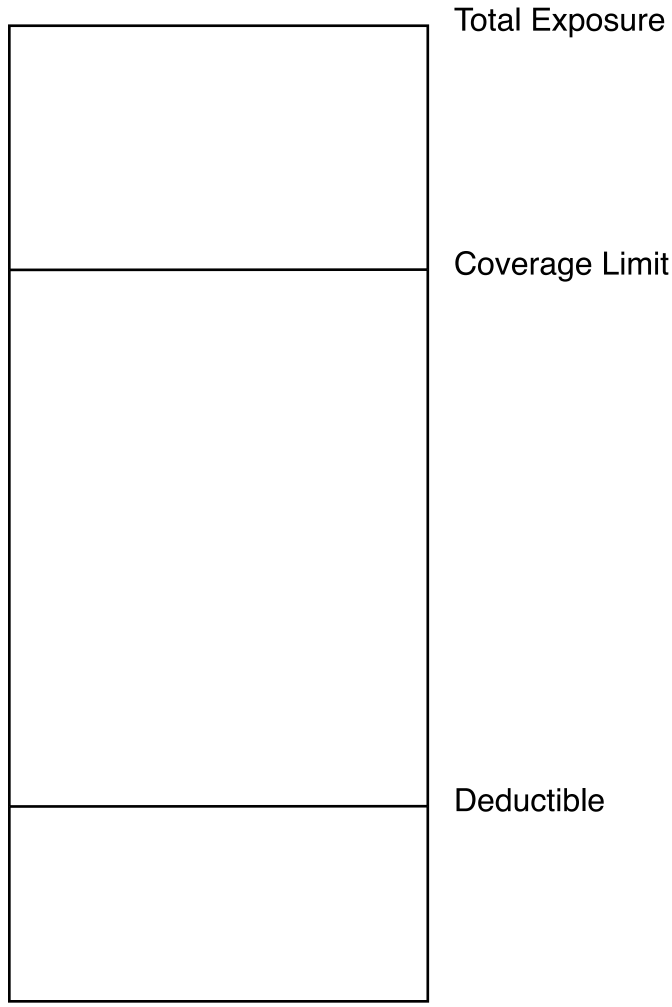
4. Model-to-Model Variance: Given the inherent uncertainty in catastrophe loss estimates, significant differences in loss estimates from one model to another do occur. Risk, and uncertainty in estimating the risk of loss, derives not only from the randomness of the event occurrence but also from the limits in knowledge and different interpretations by experts. It is unlikely that science will provide us all the answers, thus leading to continued differences in model results.

[130] See Section 5.6 in Grossi and Kunreuther.

Insurance Portfolio Management:[131]

An insurance portfolio consists of the insurance policies written. Each policy may cover a single property or multiple properties at different locations. As discussed previously, one would want details on the location and structural attributes of each covered property.

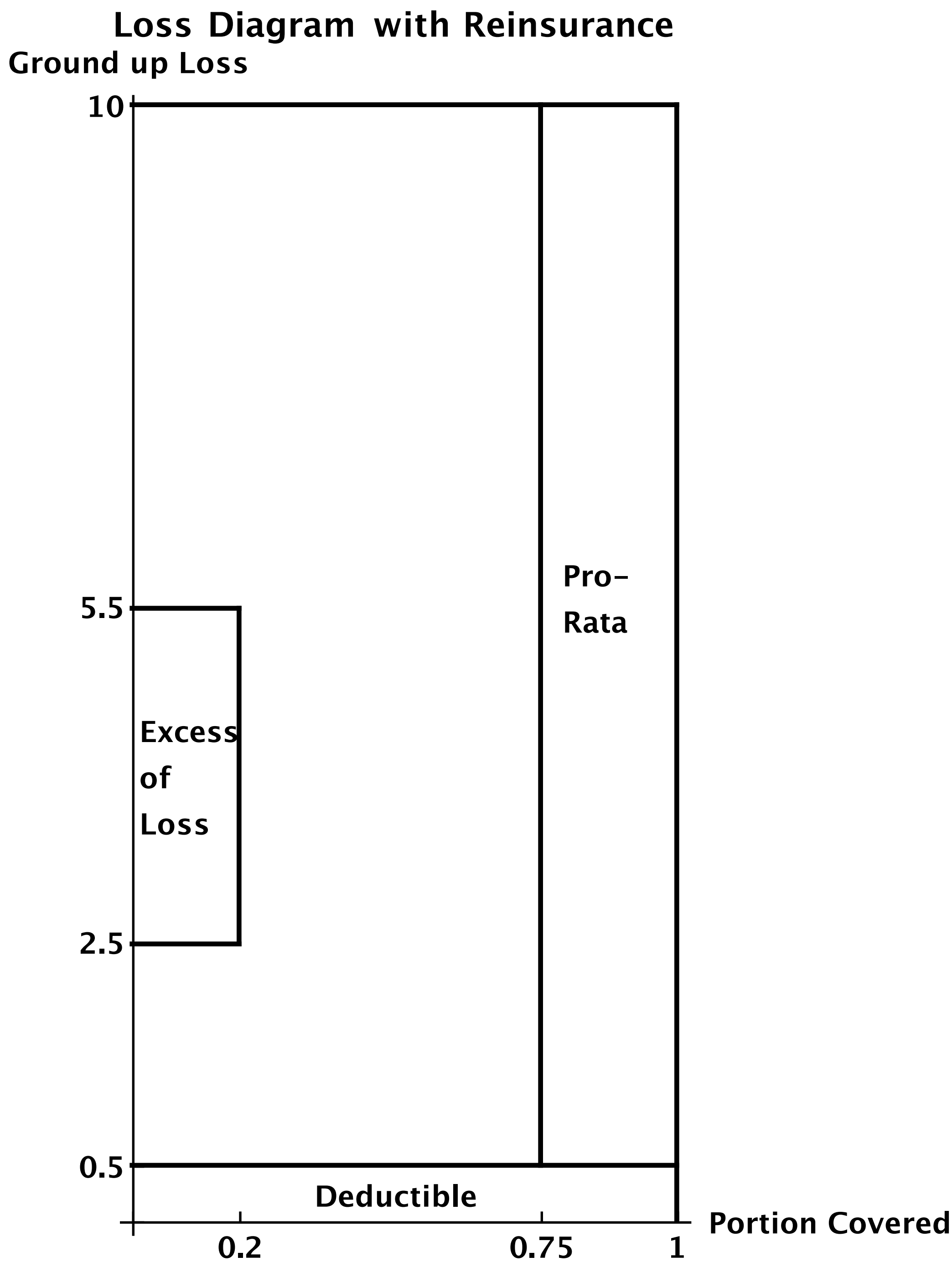
In running a computer model, one would include each of the polices written, including policy provisions such as deductibles and limits. Figure 6.3 shows a loss diagram for a property:



One could run the catastrophe model either without taking into account reinsurance or including possible reinsurance arrangements, in order to compare results.
For any given loss, one can divide it between the amounts paid by the insured, the primary insurer, and the reinsurer(s).

[131] See Chapter 6 in Grossi and Kunreuther.

Sidebar 1 shows a loss diagram with reinsurance for an insured property:[132]



The insured has a \$0.5 million deductible and a coverage limit of \$10 million (maximum payment by the insurer is \$9.5 million.) The insurer buys 25% pro-rata reinsurance.
The insurer also buys 20% 3M xs 2M excess reinsurance.
(Neither treaty inures to the benefit of the other.)

[132] Commercial policies often cover multiple properties at different locations, which could make things more complicated.

Exercise: If there is a ground up loss of $2 million, how much is paid by each party?
[Solution: The insured pays $0.5 million, the deductible.
The insurer would pay $1.5 million prior to reinsurance. However, the pro-rata treaty pays (25%)($1.5 million) = $375,000, while the insurer retains $1,125,000.]

Exercise: If there is a ground up loss of $4.5 million, how much is paid by each party?
[Solution: The insured pays $0.5 million, the deductible.
The insurer would pay $4 million prior to reinsurance.
However, the pro-rata treaty pays
(25%)($4 million) = $1 million.
Also the excess treaty pays: (20%)(4 - 2) = $400,000.
The insurer retains: 4M - 1M - 0.4M = $2.6 million.]

Exercise: If there is a ground up loss of $7.5 million, how much is paid by each party?
[Solution: The insured pays $0.5 million, the deductible.
The insurer would pay $7 million prior to reinsurance.
However, the pro-rata treaty pays
(25%)($7 million) = $1.75 million.
Also the excess treaty pays: (20%)(3) = $600,000.
The insurer retains: 7M - 1.75M - 0.6M = $4.65 million.]

Outputs from the catastrophe model for the insurer would include the Average Annual Loss (AAL) and the exceedance probability curves discussed previously. As discussed, aggregate exceedance probability curves can be used to see if for example the insurer has enough surplus to pay the one-in-200 year loss, the 99.5th percentile of the distribution of aggregate annual losses.

One can use the output of a catastrophe model to help make underwriting decisions. For example, one could test what the effect would be of writing either more or less business in certain zipcodes.

The catastrophe model can also help to make decisions about what reinsurance program to buy.

An insurer has three portfolios that are exposed to the same five catastrophic events.
The insurer is considering which of the three portfolios to eliminate in order to reduce risk.[133]
The details for each event are as follows, with losses in millions of dollars:[134]

Event (E_j)	Probability (p_j)	Loss for Portfolio 1	Loss for Portfolio 2	Loss for Portfolio 3
1	1.0%	100	0	100
2	1.0%	100	100	100
3	1.0%	200	0	100
4	1.0%	0	200	100
5	1.0%	0	100	0

The expected value of losses for each of the portfolios are each equal to 4 million.[135]

Portfolio 2 is negatively correlated with portfolio 1; the correlation is -0.79.
Portfolio 3 is positively correlated with portfolio 1; the correlation is 0.53.
Portfolio 3 is close to uncorrelated with portfolio 2; the correlation is -0.13.

Thus the variance of the sum of portfolios 1 plus 2 will be less than the sum of any other two portfolios. Thus to reduce its risk, the insurer should eliminate portfolio #3.

With the sum of portfolios 1 and 2, the losses are: 100, 200, 200, 200, 100.
With the sum of portfolios 1 and 3, the losses are: 200, 200, 300, 100, 0.
With the sum of portfolios 2 and 3, the losses are: 100, 200, 100, 300, 100.

Thus for the sum of portfolios 1 and 2 the maximum possible loss from a single event is $200 million, while it is $300 million for the other two combinations, which is another reason to eliminate portfolio #3.

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[133] Assume that the distribution of non-catastrophe losses are similar for the three portfolios.
[134] See 8, 11/11, Q.6, and Table 6.2 in Grossi and Kunreuther.
[135] Here the means of the portfolios are equal as in Table 6.2 of Grossi and Kunreuther.
The actuary is interested in the heaviness of the righthand tail of the aggregate loss distribution.
Here the coefficient of variation (standard deviation divided by the mean) would be one measure of how heavy is the righthand tail of the aggregate loss distribution; the larger the CV the heavier the righthand tail and the more likely an usually large annual aggregate loss.

A portfolio manager faces two critical questions with regard to dealing with catastrophic risks:

1. What is the Average Annual Loss?
This is used to help set the premium rate.

2. What is the likelihood that the insurer may become insolvent?
The insurer wants the probability of insolvency to be small.
It is important to adequately model the righthand tail of the aggregate exceedance curve.

Micromanagement addresses individual policies, while macromanagement considers the aggregate portfolio.

For example, an insurer may want to limit its one-in-200 loss to be less than $1 billion. However, using the output of a catastrophe model, it may find that its probability of exceeding $1 billion is 1% rather than the desired 0.5%.

By carefully examining which types of catastrophes were included among those producing aggregate losses greater than $1 billion, the insurer can identify potential areas to modify in its portfolio. For example, the insurer may decide to target earthquake losses and reduce its portfolio in vulnerable locations so that the aggregate losses will be less than $1 billion if those events occur. For example, the insurer may non-renew and decline to cover unreinforced masonry buildings in areas prone to earthquakes, or may avoid areas with soil that is prone to landslides or liquefaction.[136]

When I got my first actuarial job with the Continental Insurance Company in New York City in the late 1970s, that was over a decade before the use of catastrophe models. Continental Insurance wrote homeowners insurance for many high priced homes in Hilton Head and other locations along the Carolina coast. A catastrophe model would have identified the potential of a very large aggregate loss due to a hurricane.[137] This happened in 1989 when Hurricane Hugo caused billions of dollars of loss in South and North Carolina. Unfortunately Continental Insurance had to pay a significant share of those losses.[138]

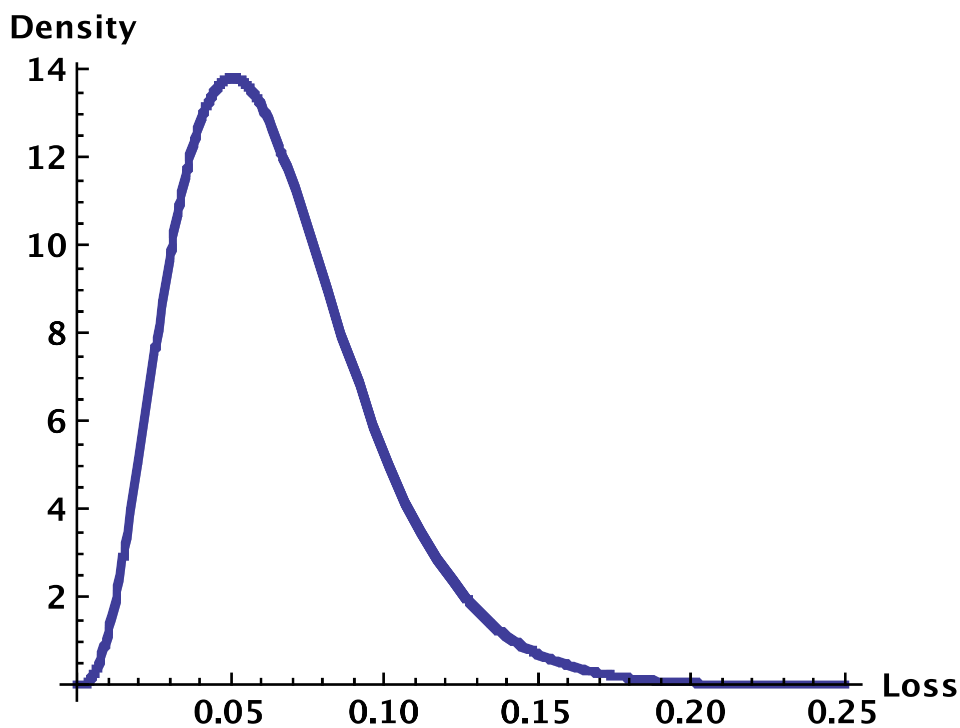
With the help of the information from a catastrophe model, the management of Continental Insurance could have identified and taken measures in advance to alleviate this potential problem. It could have reduced its concentration of risks along the coast, bought more reinsurance protection, introduced a significant deductible for wind losses, etc.

[136] Liquefaction is a phenomenon in which the strength and stiffness of a soil is reduced by earthquake shaking or other rapid loading. It has been responsible for tremendous amounts of damage in historical earthquakes.
[137] Prior to Hurricanes Hugo and Andrew, many insurers significantly underestimated the potential for a truly massive aggregate loss due to a single hurricane, or at least upper management did not act as if they took the possibility sufficiently seriously.
[138] I had left the company many year before.

The insurer could instead use catastrophe modeling to help determine whether to add a policy or a group of policies to its insured book of business.[139] For example, if the effect would be to increase the size of the one-200-year event by too much, the insurer may decline to add this set of policies. This type of analysis could be particularly useful when a reinsurer is considering whether or not to write a large catastrophe property treaty.

As with all actuarial work, the quality of the data used in a catastrophe model is a key. One needs to have accurate and detailed information on the insured structures, their locations, policy provisions, insurance to value, etc. In addition, one wants to use up to date scientific, geological, meteorological, and engineering information. Specifically damage curves should be based in part on data from recent catastrophes.

For example, for a certain modeled catastrophe, the estimated damage to a certain type of building has the following distribution of damage as a ratio to the replacement cost:[140]



As discussed previously such damage curves are an important part of a catastrophe model. They would be based in part on severity data from past catastrophes.[141] The damage curve would depend on the characteristics of the catastrophe as well as the structural characteristics and location of the insured property. Such a damage curve could then be used to calculate the expected loss to this property from this catastrophe.

[139] It would likely be impractical to run a catastrophe model to help underwrite a single policy unless it were an extremely large commercial lines policy.
[140] Similar to Figure 6.5 in Grossi and Kunreuther.
[141] See Figure 6.4 in Grossi and Kunreuther.

Exercise: For a certain modeled catastrophe, the estimated damage to a certain insured building has the following distribution of damage as a ratio to the replacement cost:

Damage	Probability		Damage	Probability
5%	40%		50%	10%
10%	20%		75%	10%
20%	10%		100%	10%

If the insurance policy includes a deductible of 10% of the replacement value of the building, determine the amount expected to be paid by each of the insured and insurer for this catastrophe.

Solution: Expected amount paid by the insured: (5%)(40%) + (10%)(60%) = 8%.

Expected amount paid by the insurer:

(20% - 10%)(10%) + (50% - 10%)(10%) + (75% - 10%)(10%) + (100% - 10%)(10%) = 20.5%.

Comment: See Table 6.5 in Grossi and Kunreuther.]

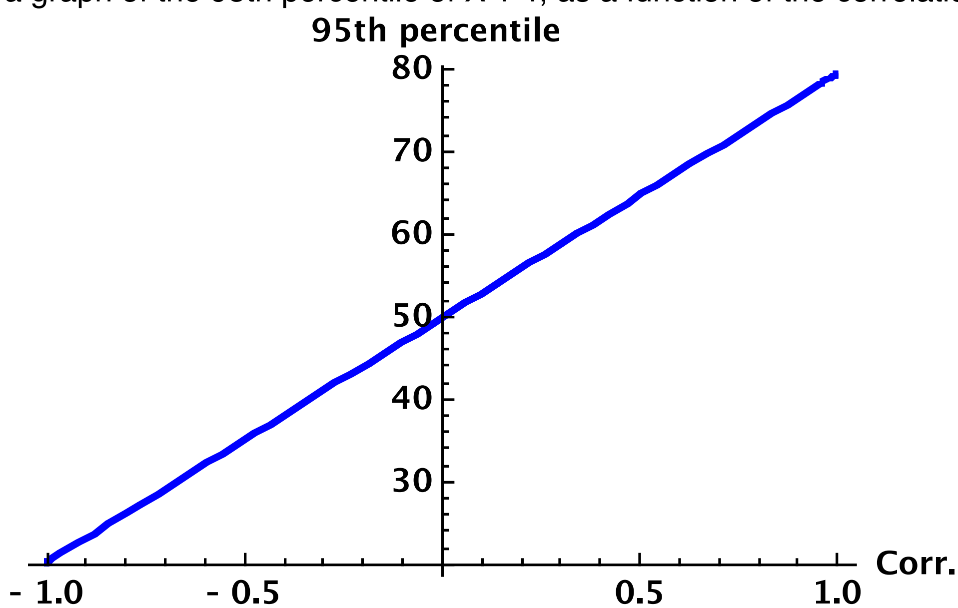
Impact of Correlation:[142]

For example, let us assume that we have three possible situations:
1. An insurer writes 1000 homes in Charleston South Carolina.
2. An insurer writes 1000 homes in Pensacola Florida.
3. An insurer writes 500 homes in Charleston South Carolina
and 500 homes in Pensacola Florida.

Both Charleston South Carolina and Pensacola Florida are along the coast and subject to possible large losses from hurricanes. However, a single hurricane can not severely damage both cities.[143] Therefore, the third portfolio is less hazardous for the insurer; the maximum possible loss from any single hurricane is lower.

In general, when one adds variables: $\text{Var}[X + Y] = s_X^2 + s_Y^2 + 2\ \text{Corr}[X, Y]\ s_X\ s_Y$.
Therefore, if X and Y are positively correlated, the variance of their sum is greater than if they are uncorrelated or negatively correlated. Thus combining correlated portfolios of insurance policies increases the variance and thus PML. The more positive the correlation, the bigger the effect.

For example, lets us assume X and Y are identically distributed Normals with $\mu = 10$ and $\sigma = 2$.
Then here is a graph of the 95th percentile of X + Y, as a function of the correlation:



[142] See Section 6.4.3 of Grossi and Kunreuther.
[143] On October 10, 2018, Hurricane Michael hit Pensacola with tropical storm force winds. Then the next day, it did the same thing to Charleston. However, in neither case were there hurricane force winds. Hurricane Michael passed to the east of Pensacola and to the west of Charleston.

The losses due to hurricane for the different homeowners policies in one of the cities are positively correlated. The losses due to hurricane for the different homeowners policies in the different cities are not positively correlated. Therefore, writing 500 houses in both cities has a smaller PML than writing 1000 houses in either one of the cities.

Insurers have been worried about concentration of risks for more than a century.[144] Simple ideas like my example with two cities do not require computer modeling. Computer modeling allows one to get a more detailed analysis of the effect of an insurer altering its portfolio of insured properties.

Grossi and Kunreuther have an example in their Table 6.2.
Here is a very simplified example, with losses in millions of dollars:[145] [146]

Event (E_j)	Probability (p_j)	Loss for Portfolio 1	Loss for Portfolio 2	Loss for Portfolio 3
1	1.0%	20	0	100
2	1.0%	0	20	100
3	2.0%	100	100	10

All three portfolios have the same expected loss of 2.2. However, portfolios 1 and 2 are positively correlated.[147] In contrast, portfolio 3 is negatively correlated with both portfolios 1 and 2.[148]

Therefore, an insurer who wrote the first two portfolios would have more variance than an insurer who wrote the first and third portfolios. The former would face a possible loss of 200 from a single event, while the latter would face a possible loss of 120 from a single event.

Event (E_j)	Probability (p_j)	Loss for Portfolio 1 plus 2	Loss for Portfolio 1 plus 3
1	1.0%	20	120
2	1.0%	20	100
3	2.0%	200	110

The loss due to earthquakes would be correlated for properties near the same fault line.

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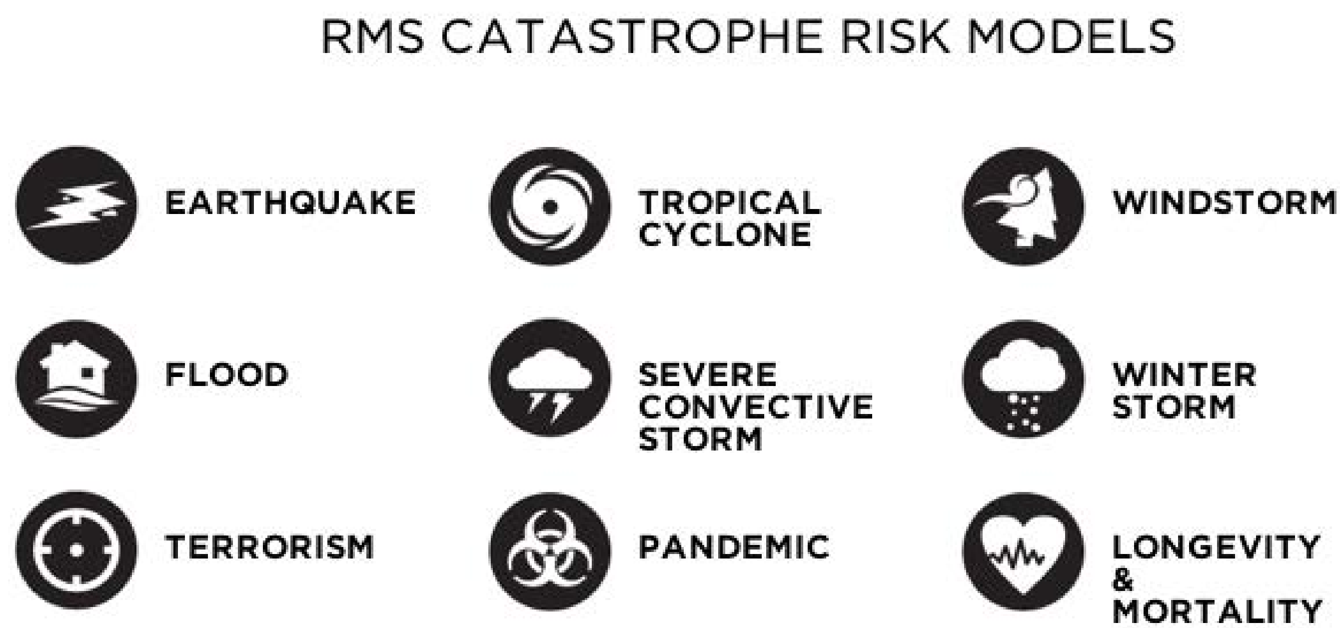
[144] Fire insurers used to maintain maps of cities with pins marking their insured properties. Underwriters were concerned if too many pins were close to one another, since one large fire could wipe out all of these buildings.
[145] See 8, 11/11, Q.6.
[146] Presumably this from the output of a computer simulation model.
[147] The correlation of the first and second portfolios is 0.952.
[148] The correlation of portfolio 3 with either portfolio 1 or 2 is -0.988.

<u>Types of Catastrophes Modeled</u>:

The syllabus concentrates on models of hurricanes (tropical cyclones) and earthquakes.[149]
Here is a list of catastrophes modeled by the firm Risk Management Solutions (RMS):[150]

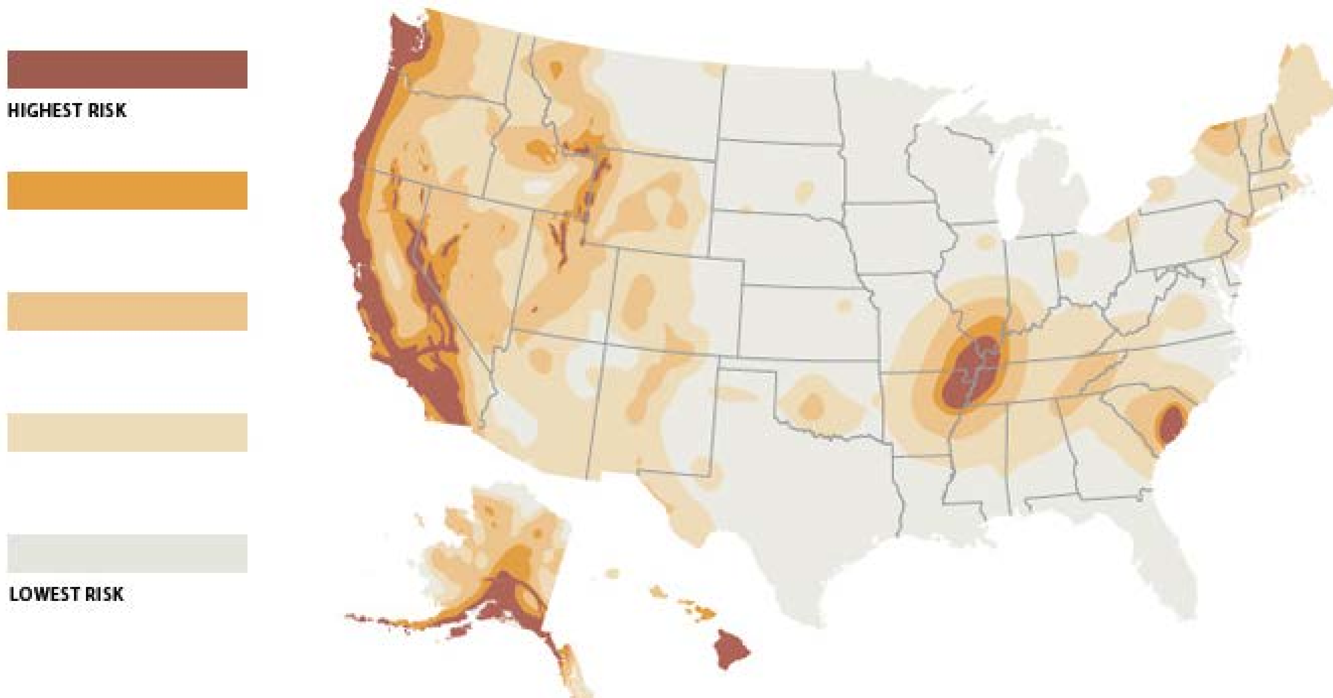


[149] Chapter 10 of Grossi and Kunreuther, <u>not</u> on the syllabus, discusses modeling terrorism.
[150] http://www.rms.com/models/models-cat.
"RMS models risk in nearly 100 countries, allowing insurers, reinsurers, and other stakeholders to analyze the probability of loss from catastrophic events. Our models are built using detailed data reflecting localized variations in hazard, and a databases capturing property and human exposures. They are continually updated with the latest scientific research and data."

Maps of Catastrophe Risks in the United States:[151]

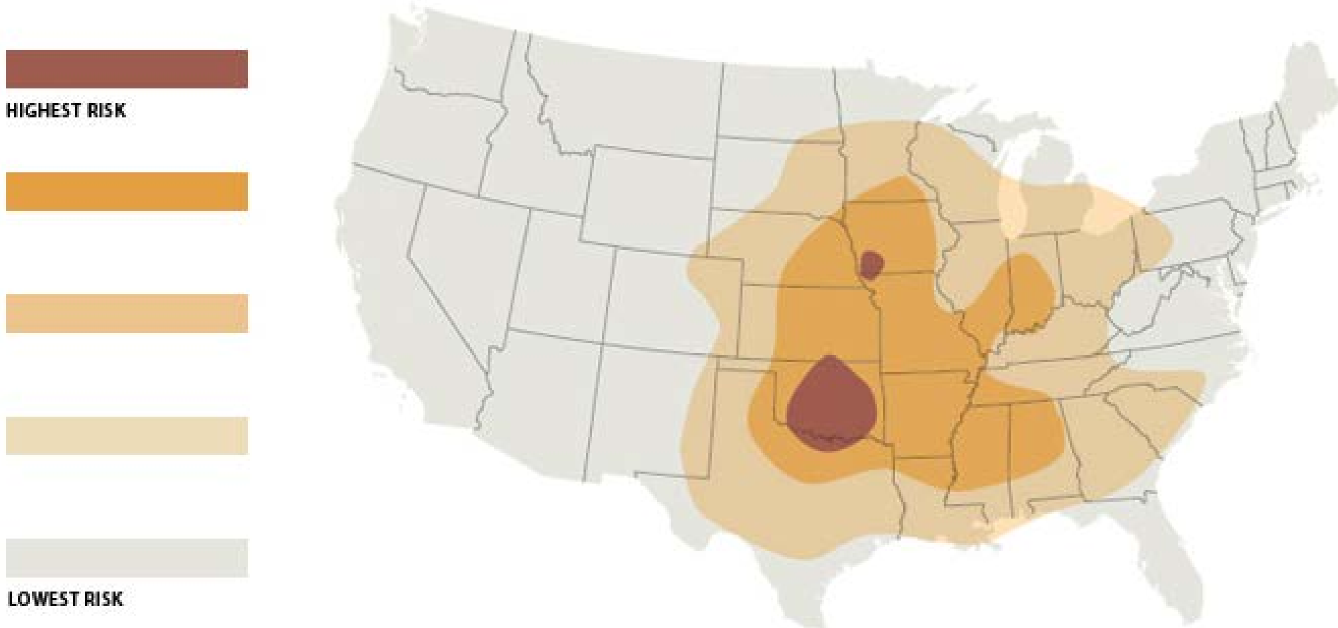
Hurricanes

Earthquakes



[151] http://www.crisishq.com/why-prepare/us-natural-disaster-map/

Tornadoes



Gutenberg-Richter Law:[152]

The **Gutenberg-Richter law** expresses the relationship between the magnitude and total number of earthquakes in any given region and time period of at least that magnitude.

$\text{Log}_{10}(N) = a - b\ M$,

where:

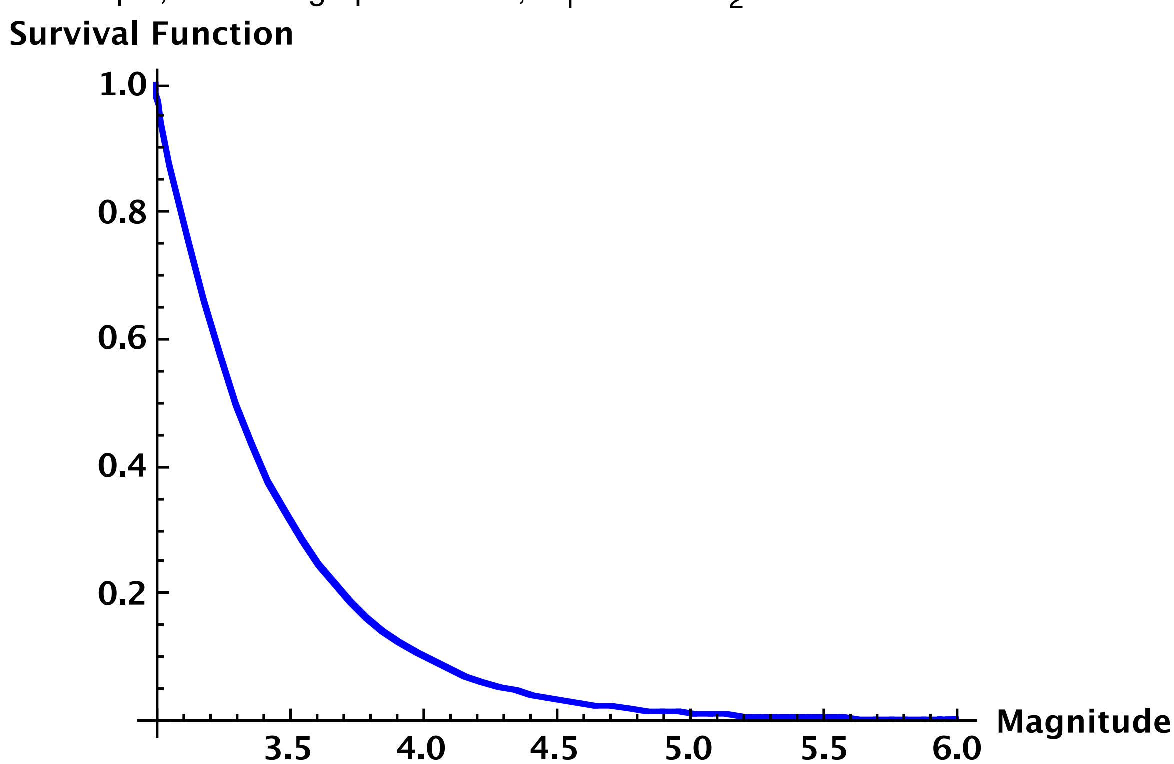
N is the number of events having a magnitude at least M
and a and b are positive constants.

$N = 10^a / 10^{bM}$.

Thus the parameter "a" determines the total number of earthquakes above some reference magnitude, while the parameter "b" determines the rate at which frequency declines with severity.

The constant b is typically equal to 1.0 in seismically active regions. This means that for every magnitude 4.0 earthquake there will be 10 magnitude 3.0 quakes and 100 magnitude 2.0 quakes.[153]

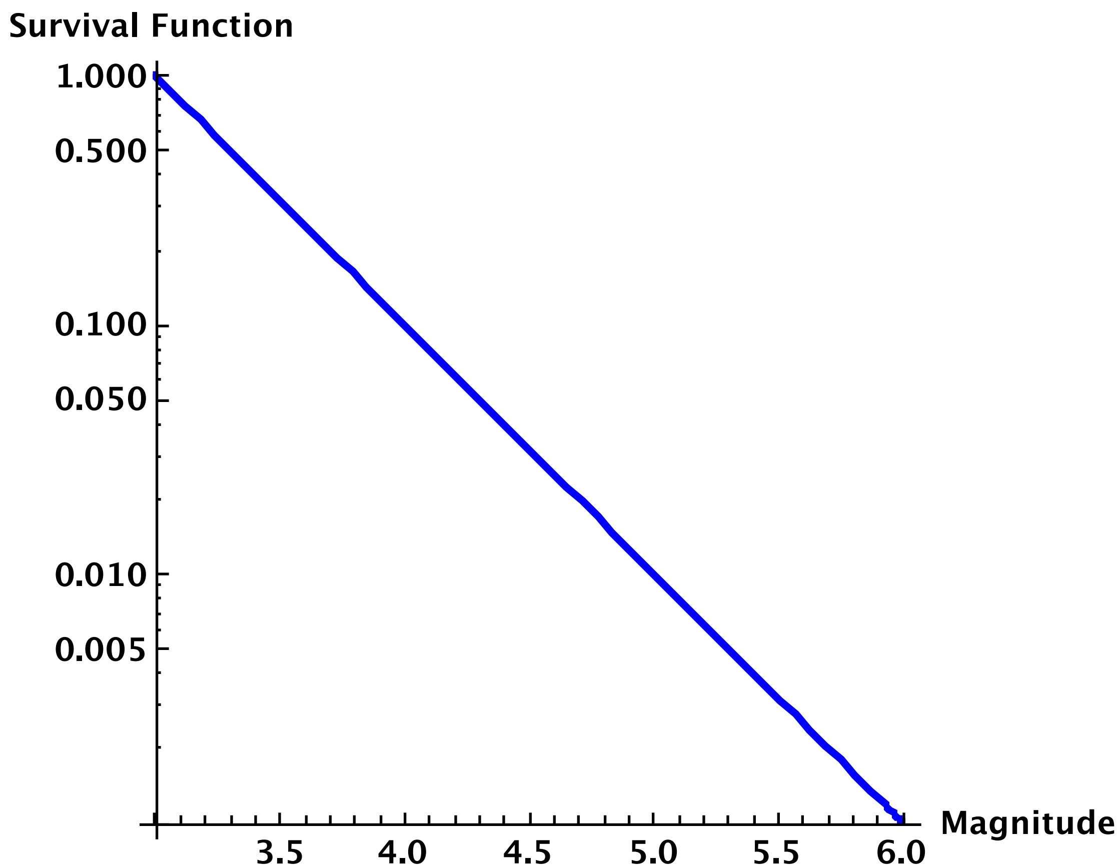
The Gutenberg-Richter law is assumed to apply for some range of magnitudes, between M_1 and M_2. For example, here is a graph for b = 1, M_1 = 3 and M_2 = 6:



[152] See pages 52-53 of Grossi and Kunreuther. Named for Charles Francis Richter and Beno Gutenberg.

[153] An earthquake that measures 4.0 corresponds to a $10^{1.5}$ = 31.6 times larger release of energy than one that measures 3.0.

If instead we put the vertical axis of the graph on a log scale, then the Gutenberg-Richter law is on a straight line:[154]



There are two consequences of the Gutenberg-Richter law for catastrophe models for earthquakes:
1. Severe earthquakes that cause large amounts of damage are rare.
2. The much more common smaller earthquakes that cause little or no damage can be used to help to estimate the frequency of larger earthquakes.

For modeling, it is very important to choose a realistic value of M_2, the maximum magnitude of earthquake for the given fault.[155] An unrealistic choice of M_2 would result in modeling too many or too few large earthquakes; the frequency of large earthquakes is what drives the estimated aggregate losses.

[154] See Figure 3.4 in Grossi and Kunreuther.
[155] For example, the very large San Francisco earthquake of 1906 was 7.9 on the moment magnitude scale. The Northridge California earthquake of 1994 was magnitude 6.7.

<u>Problems with Using a Longterm Average of Hurricane Losses</u>:

Prior to the introduction of computer model for hurricanes, actuaries would take an average of wind losses, including hurricanes, from many years, for example 35, for a state or region. However, in states that have significant hurricane loss potential, traditional loss smoothing approaches have some major limitations.

There is not enough data.
Only 164 hurricanes hit the continental United States from 1900 to 1999.
Homeowners data is only available from 1960.
Expected hurricane frequency varies over time.[156]

The use of long-term data does not fully reflect changes in population density in coastal areas. *The population density in coastal areas exposed to hurricanes has increased tremendously. Therefore, the same hurricane would produce much more insured damage than a few decades ago, even adjusted for inflation.*[157]

Grouping states into regions can distort the true hurricane potential of each state.

Methods that use long-term data are still too responsive to the occurrence of individual large hurricanes.

The traditional approach does not reflect differences between territories within a state adequately. The loss potential from hurricanes can vary tremendously within a state based on factors such as the distance from the coast. The historical data is way too sparse to be used to determine how the expected hurricane losses vary within a state.

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[156] See for example, "Climate and Hurricanes, What happened in 2006?", by David A. Lalonde, March/April 2007 Contingencies.
[157] See "A Macro Validation Dataset for U.S. Hurricane Models", by Douglas J. Collins and Stephen P. Lowe, CAS Forum, Winter 2001, <u>not</u> on the syllabus.

<u>*Various Types of Windstorms*</u>:

Hailstorms: the falling of hailstones (balls of ice), which are generated by the updraft of a thunderstorm.
Hurricanes: Tropical cyclones with sustained winds of 74 miles per hour or more.
Nor'easters: cyclonic (winter) storms of the east coast of North America.
Tornadoes: strong, violently rotating columns of air extending from the base of a cumulonimbus cloud to the ground.
Tropical Cyclones: low pressure weather systems in which the central core is warmer than the surrounding atmosphere; for example tropical storms and hurricanes.

<u>*Hurricanes*</u>:[158]

Hurricanes are tropical storms with sustained winds of 74 miles per hour or more.

The tropics supply the essential ingredients for a hurricane.
Hurricanes consist of high speed winds blowing circularly around a low-pressure center.
The lower the central pressure in the eye of a hurricane, in other words the larger the difference from normal, the stronger the storm.
Hurricanes are placed in Saffir/Simpson categories based on their sustained wind speed.[159]
The strongest hurricanes are placed in Saffir/Simpson category 5.

Four ways in which hurricanes inflict property damage:[160]
1. High wind speeds[161]
2. Intense rain
3. Projected missiles
4. High Water (flooding and storm surge)[162]

Three key characteristics of a hurricane:[163]
1. Central pressure differential
2. Forward velocity
3. Track Angles

[158] For an brief discussion of hurricanes, see for example http://www.ihc.fiu.edu/about_us/meteorology.htm
[159] See http://www.nhc.noaa.gov/aboutsshs.shtml
The Saffir/Simpson categories are used in the North Atlantic basin.
Other similar scales are used elsewhere in the world.
[160] See for example http://www.ihc.fiu.edu/about_us/hurricane_hazards.htm
[161] Besides their overall high wind speeds, hurricanes often spawn tornadoes upon reaching landfall.
[162] Damage from flooding and storm surge is not covered by standard homeowners policies.
[163] For an interesting discussion of the effect of tracking angles on storm surge see http://www.escambia-emergency.com/Hurricane_Preparedness/general_info.htm

<u>Demand Surge</u>:

Recent very large earthquakes and hurricanes have led to observations that construction costs increase in the period immediately after a large catastrophic event. Because this is the time in which insured losses are adjusted, these increased costs have led to larger insured losses than expected and are of potential concern to the insurer. After a large disaster, construction material and labor can temporarily be in short supply, so construction costs are inflated.

The larger the impact of the event on the local economy, the larger the effect of demand surge (or "price gouging"). For example, an event that causes a $10 billion insurance industry loss might cause demand surge to increase construction costs by 5%, while an event that causes a $40 billion insurance industry loss might cause demand surge to increase construction costs by 25%.[164]

Also there are only a limited number of trained claims adjusters. Even if some claims adjusters are brought in from other states, they may be overwhelmed by the tremendous number of claims that can be generated by a large hurricane. This can lead to excessive claim settlements.

[164] Values chosen solely for illustrative purposes.

Storm Surge:

Storm surge is water that is pushed toward the shore by the force of the winds swirling around the hurricane. This advancing surge combines with the normal tides to create the hurricane storm tide, which can increase the mean water level 15 feet or more. In addition, wind driven waves are superimposed on the storm tide. This rise in water level can cause severe flooding in coastal areas, particularly when the storm tide coincides with the normal high tides.

If the loss was totally due to storm surge, then the insurance policy excluded coverage. However, any portion of the loss that was due to wind, was covered. The hurricane model should include a realistic division of the losses that will actually be paid by homeowners insurance and those that will not. For example, after Hurricane Katrina, there was much controversy about how much if anything homeowner's insurers should pay for homes where all that was left was just a slab.[165]

Storm surge is a hybrid, neither wind nor flood. Even relatively weak wind events can push huge volumes of water to surge over the land, causing billions of dollars of damage.
"It has to do with bathymetry, the study of the underwater landscape," Kay Cleary, FCAS says. "Water pushing against the ocean floor starts to build up and inundates the land, creating a surge flood. Initially, it made a lot of sense to have surge factor off the wind, but we found that surge has its own characteristics not necessarily related to the wind. We upgraded our model to account for not only the severity of what can happen, but also the types of things to worry about."[166]

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[165] In the cases where homes were so badly damaged by flooding that only their foundation slabs remained, it was impossible to determine whether wind had contributed to the loss. In many cases, the insurers refused to pay anything to those homeowners. In Mississippi, under pressure in the courts and by state government, one very large insurer agreed to offer at least 50 percent of the value of a homeowner's policy in such "slab cases."
The difficulty of determining what is covered by insurance when there is a hurricane is discussed for example in "Vase or Two Faces, Policyholders and Insurers View Their Coverage", by Rhonda D. Orin, in Nov./Dec. 2007 Contingencies.

[166] Quoted from "Whatever Happened to the 100-Year Event?", by Steven Sullivan, in the Nov.-Dec. 2013 Actuarial Review.

Hurricanes and Homeowners Ratemaking:[167]

Many actuaries split out the rate for hurricanes from the rest of the homeowners rate. The hurricane rate would be determined by pure premium ratemaking applied to the output of the hurricane model. The non-catastrophe portion of the homeowners rate would be determined by applying loss ratio ratemaking to recent experience.[168]

The hurricane portion of the rate would have its own territories and classes.

Hurricane expected pure premiums vary across a state in a different pattern than do fire, liability and other homeowners losses. For example, hurricane expected pure premiums are very sensitive to distance from the coast. One would take the output of the computer model by zipcode and group together similar zipcodes into territories.[169] [170]

Hurricanes and Homeowners Classification:[171]

The factors effecting hurricane losses are different than those affecting losses due to fire: construction and fire protection. For large hurricanes, the key is to protect the envelope of the building from penetration, in other words to protect the windows and the roof. Hence, the relative fire resistance of the construction is essentially irrelevant for the hurricane peril. Therefore, it makes sense to have a separate class plan for hurricanes. Some factors considered are: hurricane shutters, roof type, location, and town building code.

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[167] Not discussed in any detail in the syllabus readings.

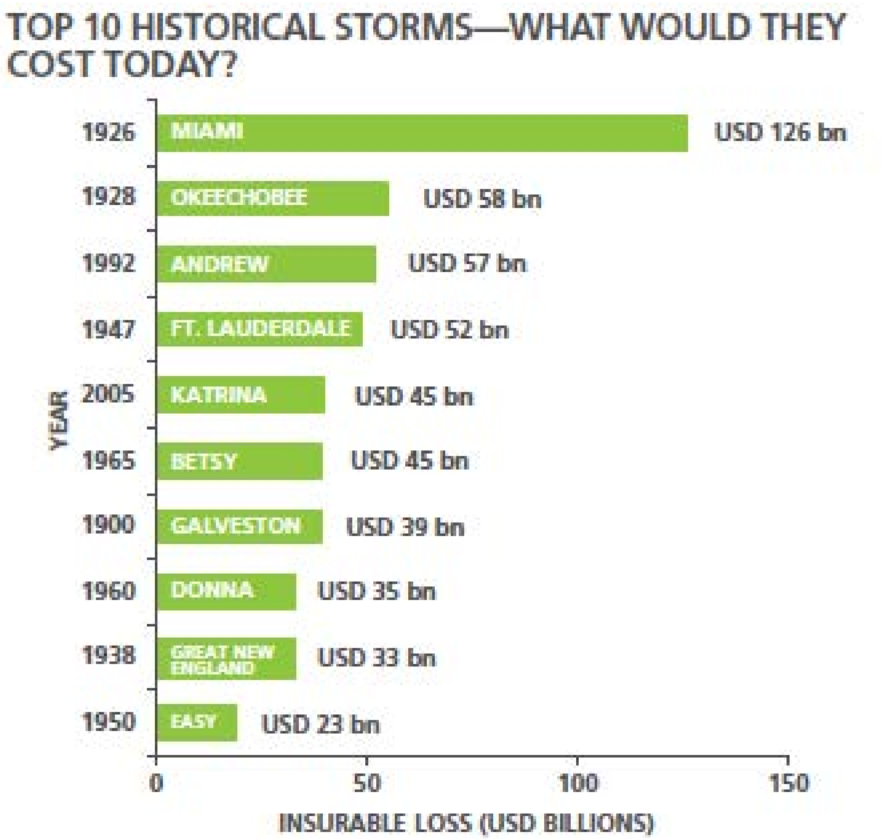
[168] Non-catastrophe wind losses are usually split out from non-catastrophe non-wind losses.
Usually the most recent five years of non-catastrophe non-wind losses are used.
A longer period of non-catastrophe wind losses would be analyzed.

[169] See for example, "How to Best Use Engineering Risk Analysis Models and Geographic Information Systems to Assess Financial Risk from Hurricanes," by Auguste Boissonnade and Peter Ulrich,
1995 CAS Discussion Paper Program.

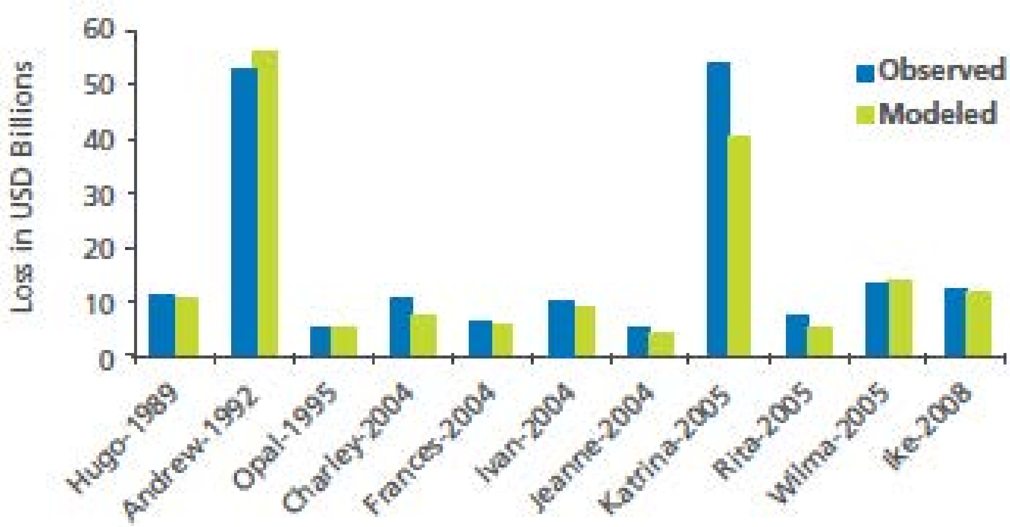
[170] For the purposes of constructing hurricane territories, many insurers would divide coastal zipcodes into smaller pieces. For example, a barrier island may be only part of a zipcode. Homes on the barrier island would be much more vulnerable to hurricanes than those slightly inland.

[171] Not discussed in any detail in the syllabus readings.

Historical Large Hurricanes:[172]



Modeled loss to property, contents, and business interruption and additional living expenses, including demand surge. Based on 2012 exposures.



Modeled industry losses compare well to actual losses (as reported by PCS).

172 From "AIR Hurricane Model for the United States."

Summary of AIR Hurricane Model:[173]

MODEL AT A GLANCE

MODELED PERIL	Hurricane-induced wind and storm surge.
MODEL DOMAIN	Alabama, Arkansas, Connecticut, Delaware, Washington DC, Florida, Georgia, Illinois, Indiana, Kentucky, Louisiana, Maine, Maryland, Massachusetts, Mississippi, Missouri, New Hampshire, New Jersey, New York, North Carolina, Ohio, Oklahoma, Pennsylvania, Rhode Island, South Carolina, Tennessee, Texas, Vermont, Virginia, and West Virginia.
SUPPORTED CONSTRUCTION CLASSES AND OCCUPANCIES	65 construction classes and 110 occupancy classes, including general residential, temporary lodging, apartment/condo, and retail.
SUPPORTED POLICY CONDITIONS	Supports a wide variety of location, policy, and reinsurance conditions, including limits and deductibles by site or by coverage, blanket and excess layers, minimum and maximum deductibles, and sublimits. Reinsurance terms include facultative certificates and various types of risk-specific and aggregate treaties with occurrence and aggregate limits.

MODEL HIGHLIGHTS

- Standard and warm sea surface temperature catalogs provide two robust and scientifically defensible views of the risk
- Realistic wind modeling in four dimensions captures the variability in storm structure
- Uses the latest United States Geological Survey (USGS) land use/land cover data to capture the directional effects of surface friction on wind speeds
- Explicitly accounts for the effect of wind duration, which has been shown to be a significant driver of loss
- Explicitly models temporal and spatial variations in vulnerability, allowing for the most precise risk differentiation
- Supports more than 20 individual building characteristics that were developed based on structural engineering analyses and building damage observations made in the aftermath of historical hurricanes
- Damage to industrial facilities modeled using an objective, engineering-based, component-level approach, which is superior to traditional approaches that treat a facility as a single entity
- Supports the builder's risk line of business with damage functions that vary according to the phase of construction, as well as time-variable replacement cost curves
- Estimates damage to pleasure boats and yachts based on boat type, age, length, and depreciated value, among other inputs
- Indirect business interruption (BI)—often a significant source of loss—estimated using an event tree approach, based on published research and detailed loss data

[173] From "AIR Hurricane Model for the United States." This is a marketing document, but nevertheless provides some useful details about current state of the art models of hurricanes.

Global Warming and Hurricane Models:[174]

Two frequently asked questions on global warming and hurricanes are the following:
• Have humans already caused a detectable increase in Atlantic hurricane activity?
• What changes in hurricane activity are expected for the late 21st century, given the pronounced global warming scenarios from current IPCC models?

The main conclusions are:

• It is premature to conclude that human activities--and particularly greenhouse gas emissions that cause global warming--have already had a detectable impact on Atlantic hurricane activity. That said, human activities may have already caused changes that are not yet detectable due to the small magnitude of the changes or observational limitations, or are not yet properly modeled (e.g., aerosol effects).

• Anthropogenic warming by the end of the 21st century will likely cause hurricanes globally to be more intense on average (by 2 to 11% according to model projections for an IPCC A1B scenario). This change would imply an even larger percentage increase in the destructive potential per storm, assuming no reduction in storm size.

• There are better than even odds that anthropogenic warming over the next century will lead to an increase in the numbers of very intense hurricanes in some basins, an increase that would be substantially larger in percentage terms than the 2-11% increase in the average storm intensity. This increase in intense storm numbers is projected despite a likely decrease (or little change) in the global numbers of all tropical storms.

• Anthropogenic warming by the end of the 21st century will likely cause hurricanes to have substantially higher rainfall rates than present-day hurricanes, with a model-projected increase of about 20% for rainfall rates averaged within about 100 km of the storm center.

[174] Geophysical Fluid Dynamics Laboratory/NOAA, last revised: December 30, 2011.

American Academy of Actuaries on Climate Change:[175]

The National Oceanic and Atmospheric Administration (NOAA) recorded 80 U.S. weather/ climate events that each had losses exceeding $1 billion between 2004 and 2013, compared with only 46 events in the previous decade. Here is NOAA's breakdown of weather related events:

● Western U.S. has experienced hotter and drier temperatures over the past decade, which has led to more wild fires and crop failures. There were 14 drought and wild fire events where each loss exceeded $1 billion in 2004-2013, according to NOAA data, compared with 10 similar events between 1994 and 2003.

● Damage from winter storms and freezes, which generally hit the eastern half of the United States, fell over the past decade. NOAA reported three winter storm and freeze events where losses exceeded $1 billion between 2004 and 2013, compared with seven similar events in 1994-2003.

● Water damage has surged over the past 10 years, in large part caused by increased hurricane activity. NOAA reported 23 flood and hurricane events with losses exceeding $1 billion between 2004 and 2013, compared with 16 from 1994-2003.

● The biggest increase in damage from weather events over the past decade came from severe storms, which NOAA classifies as tornadoes, hail storms, severe thunderstorms, derechos[176], and flash floods. There were 40 such events with losses exceeding $1 billion from 2004-2013, compared with 13 between 1994 and 2003.

In order to monitor climate changes, the American Academy of Actuaries is part of a group of other North American actuarial organizations jointly developing the Actuaries Climate Index (ACI), which will focus on measuring the frequency and intensity of extremes in key climate indicators based on controlled observational data of temperature, precipitation, drought, wind, sea level, and soil moisture. The ACI initially will cover the United States and Canada, but later could be expanded to other parts of the world where reliable data is available.

As a follow-on to the release of the ACI, the Actuaries Climate Risk Index (ACRI) will assess who and what is at risk because of climate change, and quantify that risk. The ACRI will review where people live and the surrounding infrastructure, and look for relationships between climatic and socioeconomic factors. Both indexes will function as a useful tool for actuaries, policymakers, and the general public.

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[175] Essential Elements, January 2015.

[176] http://www.spc.noaa.gov/misc/AbtDerechos/derechofacts.htm A derecho is a widespread, long-lived wind storm associated with bands of rapidly moving showers or thunderstorms. Although a derecho can produce destruction similar to that of a tornado, the damage typically occurs in one direction along a relatively straight path. As a result, the term "straight-line wind damage" sometimes is used to describe derecho damage. If the swath of wind damage extends for more than 240 miles, includes wind gusts of at least 58 mph along most of its length, and several, well-separated 75 mph or greater gusts, then the event may be classified as a derecho.

Updated Report of AAA on the Effects of Climate Change:[177]

The American Academy of Actuaries in January 2020 introduced the first model and preliminary findings of the Actuaries Climate Risk Index (ACRI), which provides results associating dollar estimates of property losses in the United States with changes in extreme weather.

Preliminary ACRI results show **an estimated additional $24 billion of property damage in the United States from 1991 to 2016** associated with changes in extreme weather for that period. The dollar amount is over and above what would have been expected relative to climate and climate risk data from a 1961–1990 reference period, and after adjustment for changes in property damage estimates that could be expected purely due to increases in population and development.

[177] As reported in the AAA, Casualty Quarterly, Spring 2020.

How to Construct a Catastrophe Model:[178]

1. Science Module
Incorporate the physics of the natural phenomena in a module that simulates as closely as possible the actual event, for example a hurricane.
Event generator module must be tested before it is used to reproduce historical events and simulate hypothetical or probabilistic results.
Should be tested for reasonableness by predicting wind speeds for hypothetical events.
Predictive accuracy is limited by the fact that data are not captured for some factors that may affect an individual property. Thus we should not expect a model to exactly reproduce a historical event, but we should verify that it can adequately simulate hypothetical events with a given set of parameters.
Actual future events don't require major modifications of the model, but rather provide additional information to further refine the model.

2. Engineering Module
Damageability functions are needed to estimate the damage to a property subject to an event of a given intensity.
Damageability functions should vary by line of business, region, construction, and coverage.
Accuracy of the damage functions is improved by analyzing actual past events.
Onsite visits to the locations of catastrophes can provide additional information.
Refinement of the damage functions is an ongoing process that is dependent on input generally provided by the engineering community

3. Insurance Module
The science and engineering modules must be integrated with the insurance module to determine the resulting insured loss from a given event.
One must develop and maintain a database of in-force exposures that captures the relevant factors that can be used in assessing the damage to a given risk.

4. Validation
Modeler must verify how the modules interact by completing an overall analysis of the results of using the entire model.

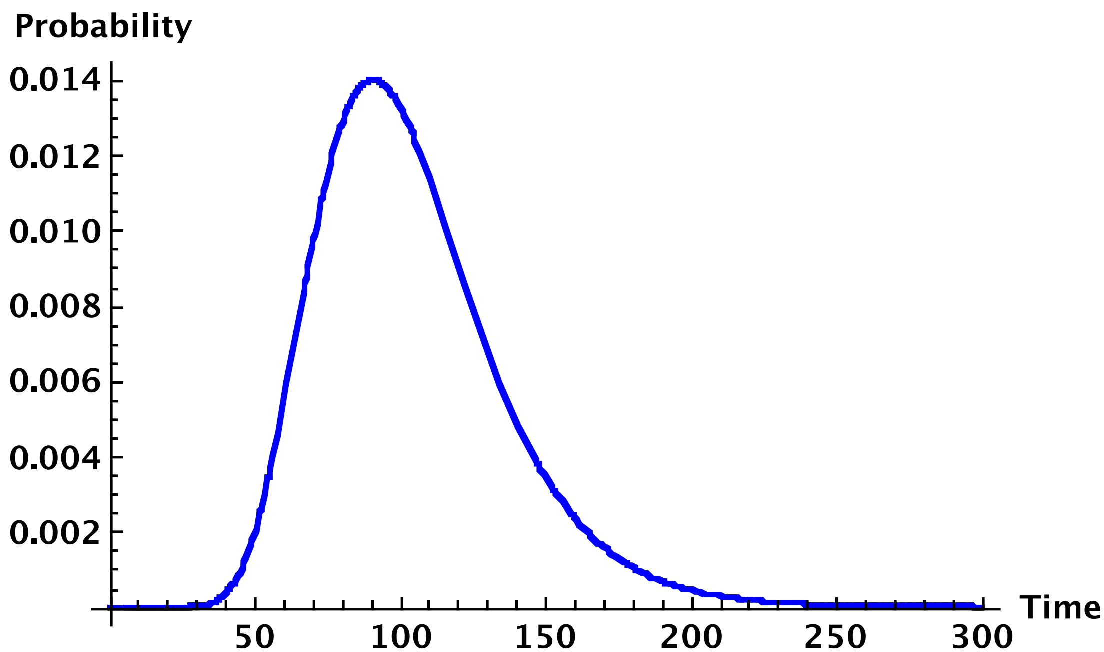
[178] Taken from Appendix A of "Homeowners Ratemaking Revisited (Use of Computer Models to Estimate Catastrophe Loss Costs)," by Michael A. Walters and Francois Morin, PCAS 1997, formerly on the syllabus. Note that the terminology differs somewhat from that used in Grossi and Kunreuther.

Time Pattern of Earthquakes:[179]

One modeling issue is the pattern over time of the occurrence of large earthquakes along a section of a major fault. Assume for concreteness that the average return time is 100 years. One model would be that we get an event at regular intervals of 100 years; this is not what we observe for major earthquakes. Another model would be a Poisson Process; the chance of a major earthquake is 1% every year regardless of the history of past major earthquakes.[180] This is also not what we observe.

While geologist are not able to predict earthquakes, they have observed that as the time since the last major earthquake increases, the chance of a major earthquake also increases. This is due to stress building up along a fault line.[181] So for example, in this example, if it has been 150 years since the last major earthquake the probability of a major earthquake next year is higher than if it had been only 50 years since the last major earthquake.

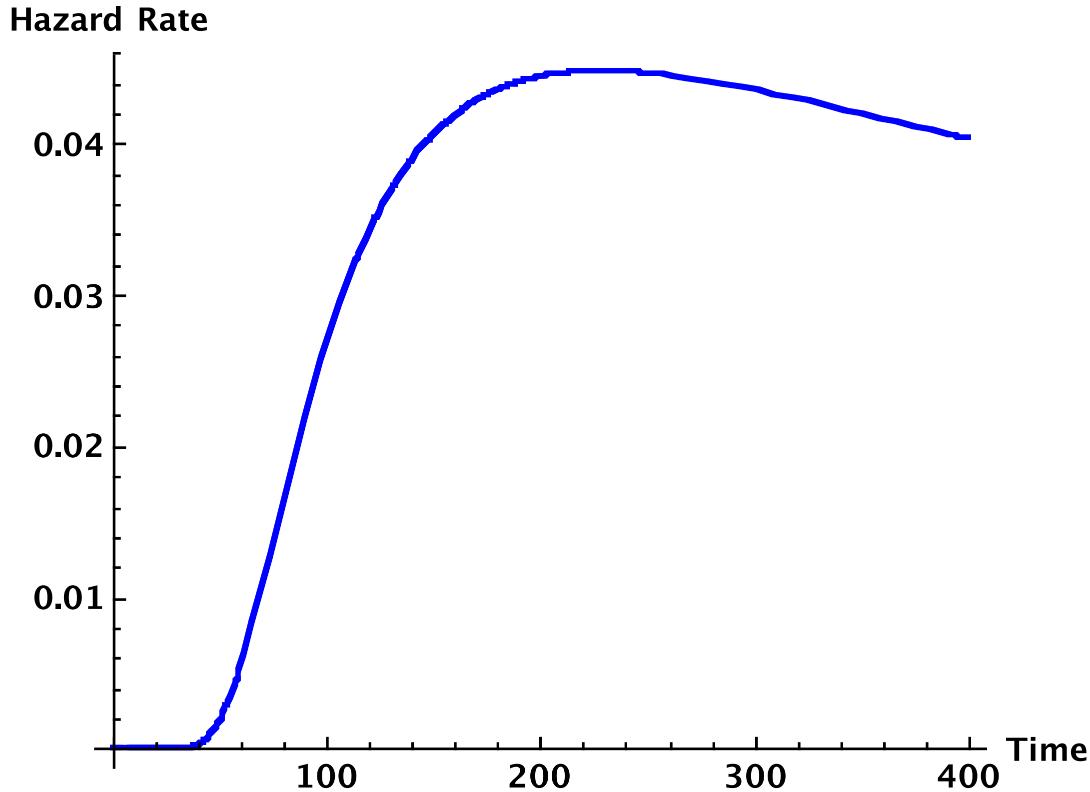
One simple model used is a LogNormal Distribution.[182] Different faults have different mean return times and therefore we would use different LogNormals to model different major faults. For example, here is a graph of the probability density function for the time in years between events for a LogNormal with $\mu = 4.6$ and $\sigma = 0.3$:[183]



[179] See pages 111-112 of Grossi and Kunreuther.
[180] For a Poisson Process, the time between events follows an Exponential Distribution.
[181] The state of stress on a fault can be influenced by the rupture of adjacent faults or even the occurrence of large earthquakes on distant faults. See page 52 of Grossi and Kunreuther.
[182] Not discussed in Grossi and Kunreuther. For practical applications a more complicated model would be used.
[183] This has mean: $\exp[4.6+0.3^2/2] = 104$, second moment: $\exp[(2)(4.6)+(2)(0.3^2)] = 11,849$, and standard deviation of 32.

Exercise: For this LogNormal model with μ = 4.6 and σ = 0.3, conditional on the last earthquake occurred 150 years ago, what is the probability of an earthquake over the next year?
[Solution: f(150) / S(150) = 0.00347422 / 0.0855334 = 4.06%.
Comment: Well beyond what you should be asked on your exam.
I used a computer to do the LogNormal calculations.]

In general, given the last earthquake took place t years ago, the probability of an earthquake the next year is the hazard rate: f(t) / S(t).[184]
Here is a graph of the hazard rate for the LogNormal with μ = 4.6 and σ = 0.3:



As can be seen, the hazard rate increases rapidly until its has been about 200 years since the last earthquake, and then starts to decline.[185] Up until 200 years, this is the general type of behavior we would expect for earthquakes; as the time since the last quake increases, the probability of a quake next year increases.[186]

[184] Assuming a person has lived to age t, the probability that they will die over the next year is approximately the hazard rate or force of mortality: f(t) / S(t).
[185] In general, for a LogNormal Distribution, the hazard goes to zero as x approaches infinity.
[186] This simple model is only a first approximation, and one should not infer that the behavior beyond 200 years for this simple model matches that of actual earthquakes.

As discussed previously, one way to estimate the effect of epistemic uncertainty in a catastrophe model is to use Logic Trees. Assuming this simple model for this one aspect of a catastrophe model for earthquakes, we could vary the parameters of the LogNormal. For example, we could use three equally likely sets of parameters in a Logic Tree.[187] In addition to varying parameters, one could include in the Logic Tree two or three different (realistic) models for the time pattern of earthquakes.

Types of Earthquakes:

The forces that create normal faults are pulling the sides apart, or extensional.
The forces creating reverse faults and thrust faults are compressional, pushing the sides together. Strike-slip faults have walls that move sideways with respect to each other.

"Earthquakes with thrust and reserve faulting mechanisms are in general observed to produce higher levels of ground motion than with strike-slip and normal faulting mechanisms."[188]

[187] The sets could have different mean return times and well as somewhat different behaviors of the hazard rate.
[188] See page 56 of Grossi and Kunreuther.

Wildfires:[189]

From the 1960s to about 1990, wildfires were not that big a deal to the U.S. insurance industry, but then something changed. Over the past decade we've had an incredible increase in severe events. There are at least three major reasons for the increase in catastrophic wildfires, particularly in California and the western United States: increased exposure,
20th century fire suppression tactics, and climate change.

There are many simple ways to help protect buildings from wildfires, including creating noncombustible "defensible space" around a structure; cleaning debris from the roof and gutters; and covering vents with mesh. Costlier measures include installing noncombustible siding,
a fire-rated roof, and multi-pane tempered glass windows.

Effective wildfire mitigation requires a holistic effort, including the implementation of community and land use planning that takes into account the elevated fire risks. Regulators, legislators and community members all contribute to these efforts. And insurers can create products to protect communities and to incentivize safe behavior, such as premium discounts for home mitigation.

The recent catastrophic wildfires have spurred many insurers to develop catastrophe modeling for wildfires. The newest frontier for catastrophe modeling is now wildfire risk.

A wildfire is a complicated risk. There are several hazards, including the flames themselves, flying embers, smoke and urban conflagrations. You can not model wildfires on a ZIP-code level. How a fire burns and spreads is highly dependent on the natural and built environment, including building construction and mitigation features.

Wildfires happen infrequently. Models will continue to improve as more data is collected.

How different states regulate insurance, particularly ratemaking, can impact how insurers can make use of catastrophe modeling for wildfires.

"The past several wildfire seasons in the U.S. have been some of the most destructive and costly on record. The wildfires of 2017 and 2018 were the costliest on record, primarily as a result of wildfires in California. High wildfire activity across many Western states continued in 2020, particularly in California and Oregon, where about 11,500 and 3,800 structures were destroyed by wildfire, respectively. California also had a record year by number of acres burned. To date in 2021, activity has been similar by acres burned when compared to the prior 10 years, and historically large fires burned in California and Oregon.

[189] Not discussed on the syllabus.
See "Wildfires: Could They Be Manageable Catastrophes?" by Lucian McMahon,
in the July-August 2019 Actuarial Review.
Based on presentations at the Spring 2019 CAS Meeting by Chris Folkman, John Rollins, and Cody Webb.

Several factors have led to the increase in wildfire events over the past few years. The impacts of climate change along with population shift toward the wildland-urban interface (WUI) areas have contributed to an increase in the frequency and severity of wildfires. On the other hand, more focus is being placed on ways that consumers can prevent or mitigate damages to their homes when these events do occur. Mitigation and prevention measures, both on an individual and community basis, have been shown to reduce wildfire risk substantially.

In order to assist insurers and reinsurers in effectively pricing for and managing the risk from wildfires, catastrophe modeling firms have developed wildfire catastrophe models. These models are developed to reflect the latest science underlying the peril from the ignition point to the spread and suppression of the wildfire. These models also consider the impacts from mitigation measures and have started to contemplate the impacts of climate change. However, regulators have been generally hesitant to accept these models due to concerns such as transparency, requiring insurers to use unreliable and volatile historical experience to set rates. Insurers in California are also disallowed from including the cost of reinsurance in their rates. These issues have led to fears of rate inadequacy in areas with substantial wildfire risk for many insurers and an insurance availability crisis in California.

The recent significant wildfire seasons have highlighted the need to continue researching and understanding this peril. It is critical to the future of the insurance market that insurers and regulators find ways to work together to appropriately price and assess this risk in order to provide an insurance market with stable rates, robust coverages, incentives to mitigate, and increased consumer choice.”[190]

“Wildfire mitigation is an emerging subject of great importance in the insurance industry and in society as a whole. The results show that individual mitigation actions related to installing fire-resistant roofs and maintaining defensible space can lead to actuarially indicated homeowners insurance premium discounts and a significant reduction in risk, particularly in high-risk areas. Aggressive community-level mitigation activities around fuel reduction can also substantially reduce risk and increase actuarially indicated insurance premium discounts.

The most impactful actions homeowners can take to reduce risk: clearing flammable materials from around the house (known as maintaining defensible space) and upgrading roofing to a fire-resistive type. In addition to individual property mitigation efforts, communities may engage in broader scale projects that could include management of the understory of forested areas surrounding the community. Beyond [homeowners insurance] credits, communities must be able to evaluate trade-offs between various mitigation actions, which can be achieved by calculating total benefits at scale.”[191]

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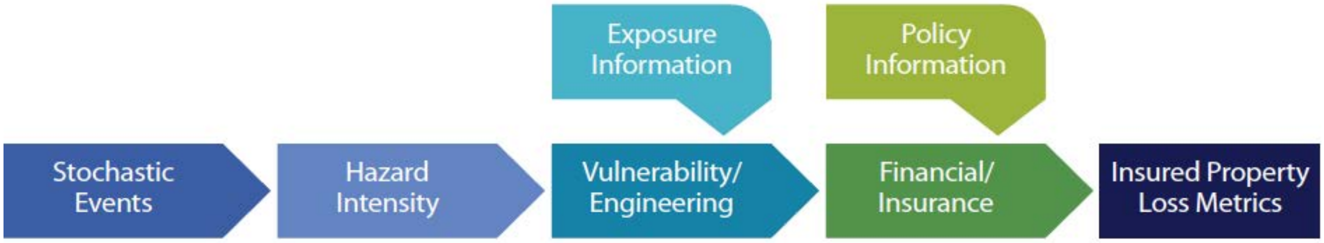
[190] “Wildfire, An Issue Paper,” American Academy of Actuaries, January 2022. Emphasis added.

[191] “Catastrophe Models for Wildfire Mitigation: Quantifying Credits and Benefits to Homeowners and Communities”, a 2022 Casualty Actuarial Society Research Paper.

AAA Monograph "Uses of Catastrophe Model Output":[192]

A flowchart illustrating how the components of a catastrophe model interact:



Advantages of Using Catastrophe Models - Catastrophe models overcome the limitations of the historical records in several ways:

Catastrophe models simulate significantly more realistically plausible events than are contained in the historical record. Catastrophe simulation models use a database of scenario events that are designed to be comprehensive and realistic. The frequency of each event is calibrated to reflect the scientific view of the likelihood of that event. For example, if a coastal segment has experienced more Category 3 storms than category 4 or 5 storms, then the event database will take this into account. Category 3 storms would make up a bigger portion of the storms affecting the area in the model analysis. These event parameters are smoothed to minimize the gaps in the historical records. Similar scientific knowledge is incorporated into each of the model modules as appropriate.

Catastrophe models allow users to import and analyze the current exposure and settlement terms, therefore avoiding the pitfalls in adjusting historical experience to reflect changes in the number, types, and values of structures exposed to the hazard. The models can also account for changes in building practices, building code, and loss-mitigation features.

Catastrophe models are updated regularly and often. This enables catastrophe models to incorporate the most advanced science in meteorology, hydrology, seismology, statistics, and structural engineering into the models. Catastrophe models incorporate the most current information on land use/land cover, surface roughness, soil type, flood defense, flood control measures, ZIP code boundary, etc.

Catastrophe models allow the insurance industry to develop forward-looking views. It allows users to analyze "what if" scenarios to assess the impact of certain catastrophe risk management strategies.

Catastrophe models encourage sensitivity testing, which leads to more frequent and thorough testing. These analyses can provide valuable information about characteristics to investigate more thoroughly, provide additional viewpoints to consider, and stress-test scenarios.

There are several catastrophe models available to the insurance industry. Having several viewpoints can provide additional, valuable information related to risk management.

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[192] Published by the American Academy of Actuaries in July 2018, not on the syllabus.
I have included a few things that you might find helpful for your exam.

Limitations of Catastrophe Models:

There are significant uncertainties around model estimates and large ranges of output values among different models. Many assumptions are involved in creating catastrophe models. A large range of output does not mean that any model is inaccurate or unreliable. The uncertainty is, to a large degree, expected, and its sources understood by actuaries. Uncertainties in alternate methods of estimating catastrophe damage are likely to be even larger and more difficult or impossible to quantify. However, a wide range of model output can cause concerns with consumers, regulators, and executives.

Collecting important building characteristics is not an easy task for an insurance company and may require a substantial financial output before any benefit is realized.

There may be damage or causes of loss that happen due to or concurrent with a catastrophic event that are not included in model output. These need to be treated separately. This is not usually problematic, but does emphasize the importance of understanding what the model assumptions are.

Model changes with software update can cause stability concerns. As science continues to evolve, and more data becomes available, modeling vendors have opportunities to incorporate new sciences and learnings into the models. As a result, the industry may experience large swings in the estimates from year to year. However, these changes are far smaller than what could happen when relying on historical experience.

Given the complexity of catastrophe models, using models requires either reliance on a company's reinsurance broker or other third party, or significant investment in training, software, and hardware to develop and maintain internal expertise.

While the technical documentation of the models is available to users for their general knowledge, some core assumptions are considered proprietary and are not readily accessible to users. A catastrophe model is developed by a group of scientists (meteorologist, seismologist, hydrologist, statisticians, engineers, actuaries, computer scientist, etc.) with specialized knowledge in different fields. It is not an easy task for model users to develop even a basic understanding of the model, as required by U.S. actuaries' standards of practice.

Catastrophe models are tools to help insurers assess and understand catastrophe risks. Like other tools, catastrophe models have limitations. Due to the uncertainties discussed above, it is impossible and unrealistic to expect a catastrophe model to produce perfect answers. However, this is not a reason to discredit a modeling approach, as relying solely on historical records is less reliable.

There is presented illustrative output from catastrophe models for four different perils:
Hurricane Wind, Tornado/Straight-Line Wind, Inland Flood, and Storm Surge.
Here are some examples.[193]

For nine selected counties and the whole state of Florida, for homeowners hurricane wind loss, a catastrophe model was used to estimate the average annual loss and its standard deviation. A risk load is added to the loss cost; the risk load was chosen for illustrative purposes to be equal to the standard deviation. Then variable expenses of 27% are loaded by dividing by 0.73. The Coverage A amount is assumed to be $207,500.

Hurricane Wind Rate and Premium Example

County	Modeled Gross Hurricane Wind Loss$ Per $1000 Cov A	Selected Risk Load (Standard Deviation)	Expense Load $	Hurricane Wind Premium $ Per $1000 Cov A	Hurricane Wind Premium $ for $207.5K Cov A Home
(A)	(B)	(C)	(D) = ((B)+(C))/.73-((B)+(C))	(E) = (B)+(C)+(D)	(F) = (E) * 207.5
Monroe	13.82	27.65	15.34	56.81	11,788.23
Broward	5.54	11.08	6.15	22.77	4,723.82
Palm Beach	5.26	10.51	5.83	21.60	4,482.44
Miami-Dade	7.60	15.21	8.44	31.25	6,484.54
Hillsborough	0.75	1.51	0.83	3.09	641.70
Orange	0.36	0.72	0.40	1.48	306.28
Okeechobee	1.91	3.81	2.11	7.83	1,624.67
Duval	0.25	0.49	0.27	1.01	209.96
Sarasota	1.74	3.48	1.93	7.14	1,481.68
Statewide	2.64	5.29	2.93	10.86	2,253.96

For example, for Monroe county: (207.5) (13.82 + 27.65) / 0.73 = $11,788.
We see that the calculated premiums vary widely across the state.[194]

[193] "We distributed 100,000 single-residential policies geographically throughout the state of Florida, representing approximately 1 percent of the market's policy count. The 100,000 policies were assigned to ZIP codes in proportion to the population of that ZIP. Random parcels within the ZIP were assigned to each policy that had been allocated to that ZIP. The building value for each structure is $207,500. Appurtenant structure values were 10 percent of building value ($20,750); Contents coverage value was set to 50 percent of building value ($103,750); and Additional Living Expense was 20 percent of building coverage, or $41,500. Each policy had a 2 percent blanket deductible (2 percent of the sum of all coverages combined, applied against losses from all coverages combined). Note that Florida requires 2 percent of building value be offered, and that choice is virtually universal in the admitted market in that state."
[194] In Florida, hurricane rates are often based on smaller geographical units than counties.

The catastrophe model was used to estimate loss elimination ratios for a 2% deductible applied to hurricane wind losses.[195]

Hurricane Wind Deductible Loss Elimination Ratio

County	Avg Hurricane Wind Ground Up AAL $	Avg Hurricane Wind Gross AAL $ @2% Deductible	2% Deductible Hurricane Wind Loss Elimination Ratio
(A)	(B)	(C)	(D) = 1-(C)/(B)
Monroe	3,577.20	2,868.47	19.8%
Broward	1,704.98	1,149.46	32.6%
Palm Beach	1,636.70	1,090.73	33.4%
Miami-Dade	2,190.53	1,577.90	28.0%
Hillsborough	365.76	156.15	57.3%
Orange	274.57	74.53	72.9%
Okeechobee	796.42	395.34	50.4%
Duval	182.22	51.09	72.0%
Sarasota	629.12	360.54	42.7%
Statewide	885.65	548.46	38.1%

We see that the loss elimination ratio varies considerably across the state. In certain parts of the state such as Orange County, the severity of hurricane wind losses is less than in other parts of the state such as Monroe County. Thus the deductible eliminates a larger percentage of loss dollars in Orange County than in Monroe County. Thus the appropriate Hurricane Wind deductible credit would also vary significantly across the state.

[195] I believe that this Hurricane Wind deductible is 2% of the Coverage A amount, but I am not sure.

The 100-year Probable Maximum Loss has a 1/100 chance of being exceeded in any given year. The 250-year Probable Maximum Loss has a 1/250 chance of being exceeded in any given year. The PMLs are in millions of dollars.[196] We note that the PMLs differ a lot by peril.

PML Amounts in $ millions by Peril

Return Period	Probability	Hurricane Wind	Flood Inland	Flood Storm Surge	Tornado/ SLW	All Causes Combined
(A)	(B)	(C)	(D)	(E)	(F)	(G)
100-year	1.0%	1,315	202	97	37	1,458
250-year	0.4%	1,902	384	157	52	2,031

Although Average Annual Losses are additive, PMLs are not.[197] Note that the PML for All Causes Combined is less than the sum of the PMLs from each cause of loss.

The following model output would be useful for example if an insurer bought a reinsurance treaty covering the layer from the 100-year PML to the 250 year PML.[198]

Layer Statistics for 100- to 250- and 250- to 500-year PMLs

	Hurricane Wind	Flood Inland	Flood Storm Surge	Tornado/ SLW	All Causes Combined
(A)	(B)	(C)	(D)	(E)	(F)
AAL in layer 100-year to 250-year	3,412	248	161	0	3,821
Standard deviation in layer 100-year to 250-year	39,649	8,385	2,652	0	43,441
Coefficient of Variation in layer 100-year to 250-year	11.6	33.8	16.5	na	11.4
AAL in layer 250-year to 500-year	1,348	35	64	0	1,448
Standard deviation in layer 250-year to 500-year	23,863	1,808	1,548	0	25,331
Coefficient of Variation in layer 250-year to 500-year	17.7	51.7	24.2	na	17.5

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[196] For 100,000 single-residential policies geographically throughout the state of Florida, representing approximately 1 percent of the market's policy count.

[197] The probability that all causes have a one in 100-year event in the same year is much less than 1 percent; therefore, the sum of the one in 100-year PMLs is associated with a longer return period.

[198] A reinsurer may decide to sell coverage for a loss of at least $1,315M up to $1,902M to a primary company for wind damage from hurricanes. This layer can be evaluated based on the AALs and standard deviations.

Supplemental Material:

I have a file containing some supplemental material on catastrophes on my webpage
www.howardmahler.com/Teaching

You my find this information interesting or useful, particularly if you learn best with concrete examples. Among the things discussed are:
More Detail on the Florida Hurricane Catastrophe Fund
How Home Construction Affects Expected Losses from Hurricanes
From the California Geological Survey, Earthquake Hazard Across the State
Cripple Walls
Risk Measures
AIR Severe Thunderstorm Model
New England's Hurricane Risk, Present and Future
Coastal Earthquakes in Northwest United States and British Columbia.

Problems:

1. (1.5 points) The vulnerability module is one part of a catastrophe model.
State its purpose and briefly discuss how the vulnerability module is developed.

2. (1 point) The results of three catastrophe models applied to the same portfolio of insured properties were submitted to a state commission on hurricane loss projection methodology:

Exceedance Probability	Estimated Loss ($ millions)		
	Model A	Model B	Model C
0.5%	452	381	400
1%	330	287	294
5%	158	160	189
10%	75	94	101

Combine the results of these different models into a single exceedance curve.

3. (2 points) List and discuss the four basic components of a catastrophe model.
Include a drawing showing how the components fit together.

4. (1 point) In a catastrophe model, why is it important to model the distribution of possible damage to a particular type of building as opposed to just modeling the mean?

5. (1 point) Your insurance company has used a computer model of hurricanes for the first time. The model is from a vendor who has been doing hurricane modeling for many years. The model indicates a large increase in the rates for homes on Cape Mackerel, which is on the coast of the Atlantic Ocean.
The state insurance commissioner questions the large increase in rates you are proposing. Members of the state legislature who represent this area point out that Cape Mackerel has not been hit by a hurricane in many decades.
Briefly respond.

6. (2 points) Define the damage ratio and damage function.
Discuss the uncertainties encountered in determining the damage function.
What is the key element in determining the building damage?
What is the key element in determining the contents damage?
What is the key element in determining the time-element (business interruption or additional living expenses) costs?

7. (3 points) Given the following:
● An insurance company is exposed to three independent catastrophic events:

Event	Size ($ billion)	Probability
1	1	6%
2	2	4%
3	3	2%

● While each event can only happen once, the total number of events per year is not limited to one.

Plot the aggregate exceedance probability curve associated with the insurance company's exposure and label the x and y axes.

8. (1 point) Name some items that would be used to determine the premium for earthquake insurance for a particular residence.

9. (1 point) List four sources of epistemic uncertainty in catastrophe modeling.

10. (1 point) For both hurricanes and earthquakes, there are local pockets in which all structures suffer more severe damage as a group as a group than similar buildings in neighboring areas. Explain why.

11. (1 point) Actuarial Standard of Practice number 38, Using Models Outside the Actuary's Area of Expertise (Property and Casualty), deals with situations in performing actuarial work, that an actuary finds it appropriate to use models that incorporate specialized knowledge outside of the actuary's own area of expertise. When using such a model, such as a catastrophe model, list the things the actuary should do.

12. (1 point) Grossi and Kunreuther describe a comparison of earthquake models applied to South Carolina. Briefly discuss this study.

13. (2 points) Slippery Rock Insurance Company writes only homeowners insurance.
Slippery Rock's expected annual non-catastrophe losses are $200 million;
assume these are independent of its possible losses from catastrophes.
A catastrophe model is run using the portfolio of homes insured by Slippery Rock.
The model covers all catastrophe perils to which Slippery Rock is subject.
The model produces the following annual Aggregate Exceedance Probabilities (AEPs):

Loss ($ million)	AEP
700	0.02%
600	0.11%
500	0.27%
400	0.49%
300	1.56%
250	2.25%
200	3.00%

Fully discuss how the management of Slippery Rock could make use of this output.

14. (1 point) Briefly discuss the loss module, one part of a catastrophe model.

15. (2 points) An insurer buys the following three reinsurance treaties:
- Quota share, where 25% is ceded up to a $20 million loss limit.
- 50% property catastrophe excess of loss treaty $5 million xs $15 million.
- 75% property catastrophe excess of loss treaty $10 million xs $20 million.

In each case draw a loss diagram, labeling all relevant features, illustrating the coverage provided.
(a) None of the reinsurance treaties inure to the benefit of the others.
(b) The quota share inures to the benefit of the excess of loss treaties.

16. (1 point) Define and briefly discuss three different types of exceedance probability curves.

17. (2.25 points) When the California Earthquake Authority first started writing policies, there was a regulatory hearing. Briefly discuss three issues raised in the hearing and how they were resolved.

18. (1 point) Discuss the conditions necessary for the formation of a hurricane (cyclone).

19. (1 point) Two of the most critical factors in differentiating risk for rate setting are the structural attributes and location attributes of a portfolio. Briefly discuss each.

20. (1 point) Discuss why the errors in damage estimates from a catastrophe model may be correlated between the different insured structures.

21. (1 point) Discuss things that can affect the amount of wind at a given building site due to a hurricane.

22. (1.5 points) An insurance company has three portfolios that are exposed to the same four catastrophic events. The details for each event are as follows, with losses in $ millions:

Event (E_j)	Probability (p_j)	Loss for Portfolio 1	Loss for Portfolio 2	Loss for Portfolio 3
1	2%	200	50	50
2	2%	100	100	100
3	2%	50	100	150
4	2%	50	150	100

The standard deviations of loss for the three portfolios, including the catastrophic losses, are:
Portfolio 1: $150 million
Portfolio 2: $125 million
Portfolio 3: $100 million

The company has decided to minimize risk.
Explain which portfolio of the three the insurer should eliminate.

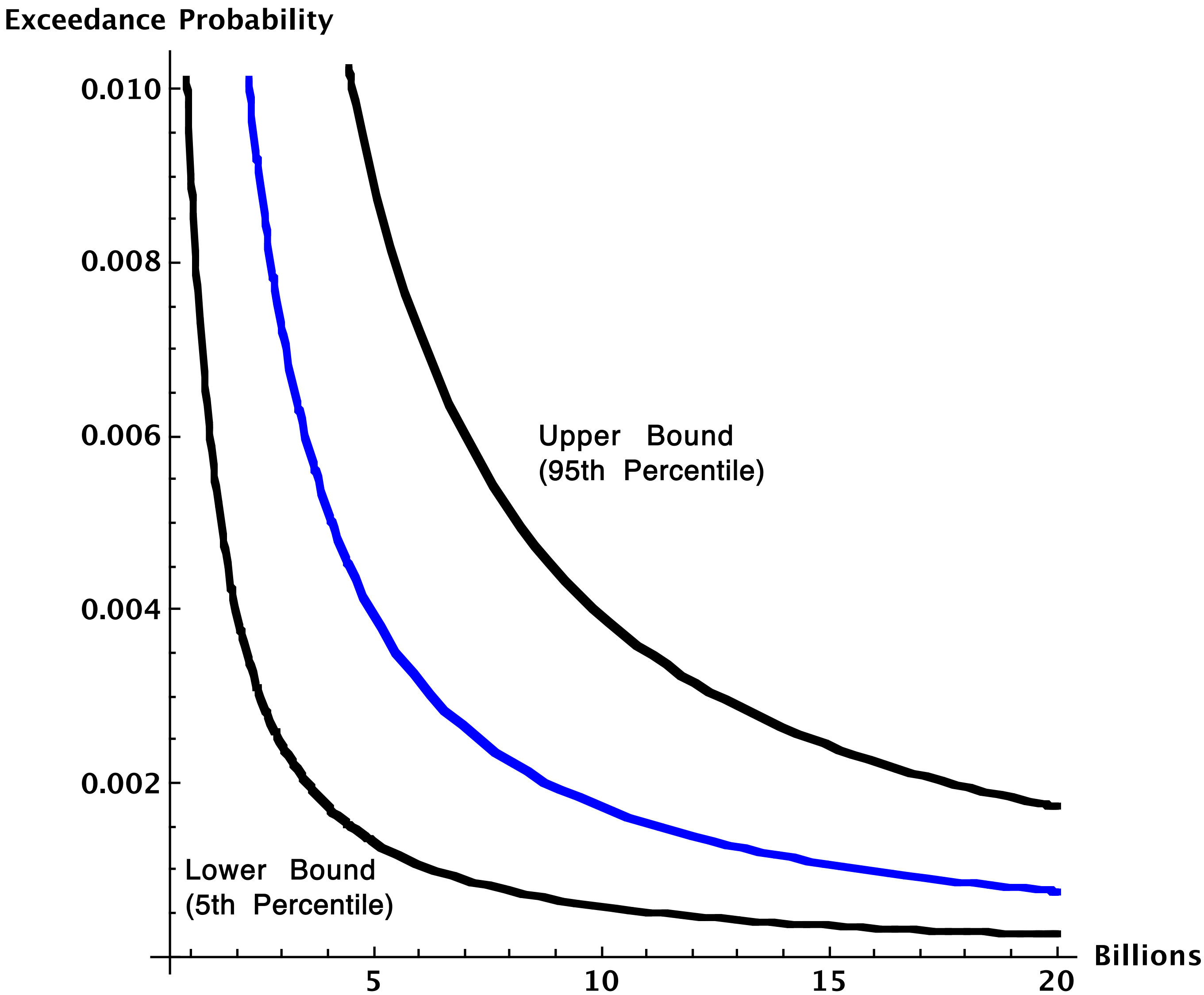
23. (1.5 points) The Eureka Insurance Company only writes homeowners insurance.
Colossal Reinsurance reinsures the hurricane exposure of the Eureka Insurance Company.
The per occurrence treaty pays Eureka 90% of the layer $180 million excess of $20 million.
Briefly describe how Colossal Reinsurance could use a computer model of hurricanes to help to price this reinsurance treaty. Ignore the possibility of more than one large hurricane in a year.

24. (1 point) In constructing a catastrophe model, discuss the evaluation of the performance of a building when it is hit by a catastrophe.

25. (2.5 points) Insurers use catastrophe models as part of their rate filings.
(a) Describe some difficulties that insurance regulators have faced in attempting to approve filings which use catastrophe models.
(b) Fully discuss how the use of hurricane models was handled in the state of Florida.

26. (1.5 points) The hazard module is one part of a catastrophe model.
State its purpose and briefly discuss its elements.

27. (2 points) Sandy is an actuary. For a given portfolio of insured properties, Sandy uses a hurricane model to produce an aggregate exceedance probability curve.
Using logic trees and varying the modeling assumptions, Sandy also estimates the 5th percentile of the exceedance probabilities and the 95th percentile of the exceedance probabilities:



(a) (1 point) Determine a 90% confidence interval for the one-in-200 year event.
(b) (1 point) Determine a 90% confidence interval for the return time for $10 billion.

28. (1 point) The inventory module is one part of a catastrophe model. Discuss it.

29. (1.5 points) The Sunshine Insurance Company writes homeowners insurance in only one state. This state has a significant potential for loss from hurricanes. Sunshine currently maintains sufficient surplus to withstand a one-in-200 year loss from a single hurricane. Sunshine would like to write additional business in this state, but does not wish to significantly increase its surplus. Describe how a catastrophe model can help Sunshine to both write additional business and maintain sufficient surplus to withstand a one-in-200 year loss from a single hurricane.

30. (2 points) Given the following:
● An owner insures a large office building worth $1 billion for its value.
● There is a $100 million deductible.
● The insurer buys excess of loss reinsurance which pays 80% of $600M excess of $300M.
● This office building is exposed to three independent catastrophic events:

Event	Size ($ million)	Probability
1	250	10%
2	500	5%
3	1000	2%

● The total number of events per year is limited to one.
● There are no other losses than due to these events.
For each of the three parties, the owner, the insurer, and the reinsurer, determine the mean and standard deviation of its annual loss.

31. (1 point) Discuss things that can affect the amount of ground motion at a given building site due to an earthquake.

32. (1 point) The CAS Principles of Ratemaking states:
1. A rate is an estimate of the expected value of future costs.
2. A rate provides for all costs associated with the transfer of risk.
3. A rate provides for the costs associated with an individual risk transfer.
4. A rate is reasonable and not excessive, inadequate, or unfairly discriminatory if it is an actuarially sound estimate of the expected value of all future costs associated with an individual risk transfer.

How does the use of catastrophe modeling help satisfy these principles?

33. (1 point) Write the equation for the survival constraint that insurers must satisfy.
Define all terms used.

34. (1 point) "A portfolio manager faces two critical questions with regard to dealing with catastrophic risks." List and briefly discuss these two critical questions.

35. (2 points) Six rate-setting considerations were stipulated in the creation of the California Earthquake Authority. Briefly describe four of them.

36. (1 point) Define the law of large numbers.
Discuss the law of large numbers as it relates to insurance losses from catastrophes and other than catastrophes.

37. (5 points) Two insurers both have similar sized homeowners insurance books of business confined to the state of Catawba.
Other than due to hurricanes, each of them has expected annual losses of $90 million with a standard deviation of $7 million. Their annual losses due to hurricanes are independent of their other losses.
A computer model was run for hurricanes hitting Catawba.
Ten different hurricanes were modeled, each with a 1% probability per year.
For simplicity ignore the possibility of more than one hurricane in a year.
The insured losses ($100 million) from the computer model for each insurer are:

Hurricane	Insurer A	Insurer B
1	1	2
2	2	5
3	3	4
4	4	5
5	5	1
6	1	2
7	2	1
8	3	1
9	1	1
10	1	3

A reinsurer sells each of these insurers catastrophe treaties, 400 million xs 100 million.
Calculate the coefficient of variation of each insurer's annual losses both prior to and after reinsurance.
Calculate the coefficient of variation of the reinsurer's annual losses both from each insurer separately and combined.

38. (1 point) Discuss attenuation equations for earthquakes.

39. (1 point) On a section of a major fault, you observe an annual average of 0.7 earthquakes of magnitude more than 4.
Using the Gutenberg-Richter law with b = 1.2, estimate the return time for earthquakes of magnitude more than 6.

40. (2 points) “In the last five to ten years, the use of catastrophe models for insurance rate making has become common practice in states with the potential for severe catastrophic insured losses. There are still a set of open questions that need to be resolved with respect to their use in rate making decisions.”
List and briefly discuss two of these issues mentioned by Grossi and Kunreuther.

41. (1 point) Compare and contrast the two broad risk management strategies.

42. (1 point) List items entering into the decision of a property insurance underwriter whether to write a new account.

43. (1 point) “All catastrophe models must address three basic issues regarding the source parameters of the hazard.” List and briefly discuss these three basic issues.

44. (1 point) Discuss the pattern over time of the occurrence of large earthquakes along a section of a major fault.

45. (2 points) The hearings associated with the California Earthquake Authority (CEA) raised a number of questions that require future research. Discuss two of them.

46. (1.5 points) You are given the following table of output from a catastrophe model.

Probability of Not Exceeding	Average Return Time (Years)	AEP ($ million)	OEP ($ million)
99.9%	1000	431	420
99.8%	500	318	308
99.5%	200	188	177
99.0%	100	127	116
98.0%	50	87	78
96.0%	25	59	53
95.0%	20	51	46

Briefly discuss the reason for the differences between the columns labeled AEP and OEP.

47. (1 point) Which of the following statements about hurricanes is <u>false</u>?
A. The tropics supply the essential ingredients for a hurricane.
B. Hurricanes consist of high speed winds blowing circularly around a low-pressure center.
C. The higher the central pressure in the eye of a hurricane, the stronger the storm.
D. Hurricanes are placed in Saffir/Simpson categories based on their sustained wind speed.
E. The strongest hurricanes are placed in Saffir/Simpson category 5.

48. (2 points) Using a logic tree approach, you come up with 20 equally likely versions of a catastrophe model. For a given portfolio of buildings you run each of these versions of the model.The 20 resulting estimates of the 99th percentile of annual losses are in $ million: 478, 462, 542, 546, 509, 525, 518, 481, 395, 492, 465, 479, 510, 510, 535, 495, 522, 451, 520, 538.
Determine an 80% confidence interval for the 99th percentile of annual losses.

49. (4 points) Assume that during a given year no more than one hurricane can occur.
The table below shows the probability of each category of hurricane and the associated loss that would be incurred.

Event (E_i)	Description	Annual probability of occurrence (p_i)	Loss (L_i)
1	Category 5	0.002	$10,000,000
2	Category 4	0.005	$5,000,000
3	Category 3	0.010	$3,000,000
4	Category 2	0.020	$2,000,000
5	Category 1	0.030	$1,000,000

(a) What is the probability that no hurricane occurs?
(b) What is the Average Annual Loss?
(c) What is the Aggregate Exceedance Probability at each level of loss ($ million):
0.5, 1, 2, 3, 5, 10.
(d) Draw the Aggregate Exceedance probability curve.
(e) Draw the Occurrence Exceedance probability curve.
(f) Determine the 1-in-100 year PML (Probable Maximum Loss).
(g) Determine the 1-in-250 year PML (Probable Maximum Loss).

50. (1 point) A building is insured against earthquake damage.
There is a deductible of 15% of the insured value.
The building is damaged 40% of its value due to a simulated earthquake.
Compare the amount paid by the insurance company if the building is insured for its true value of $500,000 or only insured for $300,000.

51. (1 point) There are five possible damage states for a building:
None, Minor, Moderate, Severe, and Collapse.
The probabilities are: 5%, 24%, 48%, 16%, and 7%.
Given a uniform random number from (0, 1) of 0.3361, simulate the amount of damage.

52. (1 point) Abandoning years of official skepticism, Oklahoma's government on April 21, 2015 embraced a scientific consensus that earthquakes rocking the state are largely caused by the underground disposal of billions of barrels of wastewater from oil and gas wells. The most intense seismic activity is occurring over a large area, about 15 percent of the area of Oklahoma, that has experienced significant increase in wastewater disposal volumes over the last several years.
Briefly discuss this in the context of catastrophe modeling.

53. (1.5 points) For a certain modeled catastrophe, the estimated damage to a insured building has the following distribution of damage as a ratio to the replacement cost.

Damage	Probability	Damage	Probability
2%	12%	12%	7%
4%	21%	14%	4%
6%	22%	16%	3%
8%	17%	18%	1%
10%	12%	20%	1%

If the insurance policy includes a deductible of 5% of the replacement value of the building, determine the amount expected to be paid by each of the insured and insurer for this catastrophe.

54. (1 point) List four ways in which hurricanes inflict property damage.

55. (3 points) You are attempting to quantify the epistemic uncertainty of a catastrophe model for earthquakes. Construct a Logic Tree based on the following:
- Two different attenuation equations. The first equation is given 30% weight.
- Two different damage functions. The first damage function is given 60% weight.
- Two different maps of the expected level of seismic activity, to be given equal weight.

Briefly discuss how you would then use this Logic Tree.

56. (1 point) For a county, you are given the following output from a catastrophe model:

Probability	Annual Loss per $1000 of value
95%	$0
2%	$50
2%	$100
1%	$300

Determine the rate prior to including expenses, which is calculated as Average Annual Loss (AAL) plus Risk Load, where the Risk Load is half of the standard deviation.

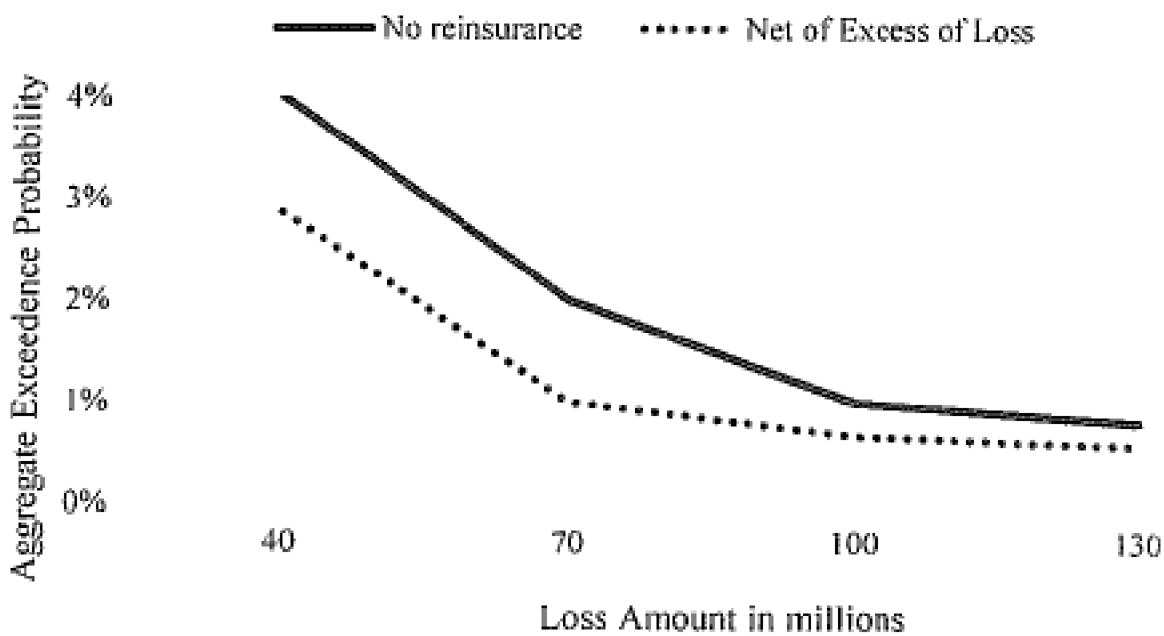
57. (1 point) You are helping to develop a catastrophe model for flood insurance.
One geographical variable that will be included in the model is elevation above sea level.
List two other geographical variables that would be useful to include.

58. (3 points) In order for an insurance company to increase its return on capital, two reinsurance options are being considered:
I. $5 million excess of $5 million per risk:
● ALAE pro-rata
● Rate = 17% of premium
II. 40% Quota Share:
● Ceding commission of 25%

The insurance company must hold capital to support a 1-in-50 year Probable Maximum Loss. Additionally, the following information regarding the insurance company's performance last year is given:
● Premium: $60,000,000
● Expenses: $15,000,000
● Total Loss & ALAE: $40,000,000

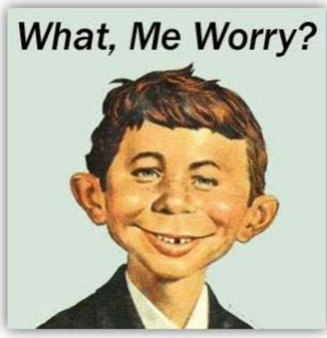
	Claims greater than $5,000,000	
	Loss	ALAE
Claim 1	$6,000,000	$900,000
Claim 2	$13,000,000	$2,600,000

The Aggregate Exceedance Probability curve for the insurance company is shown below:

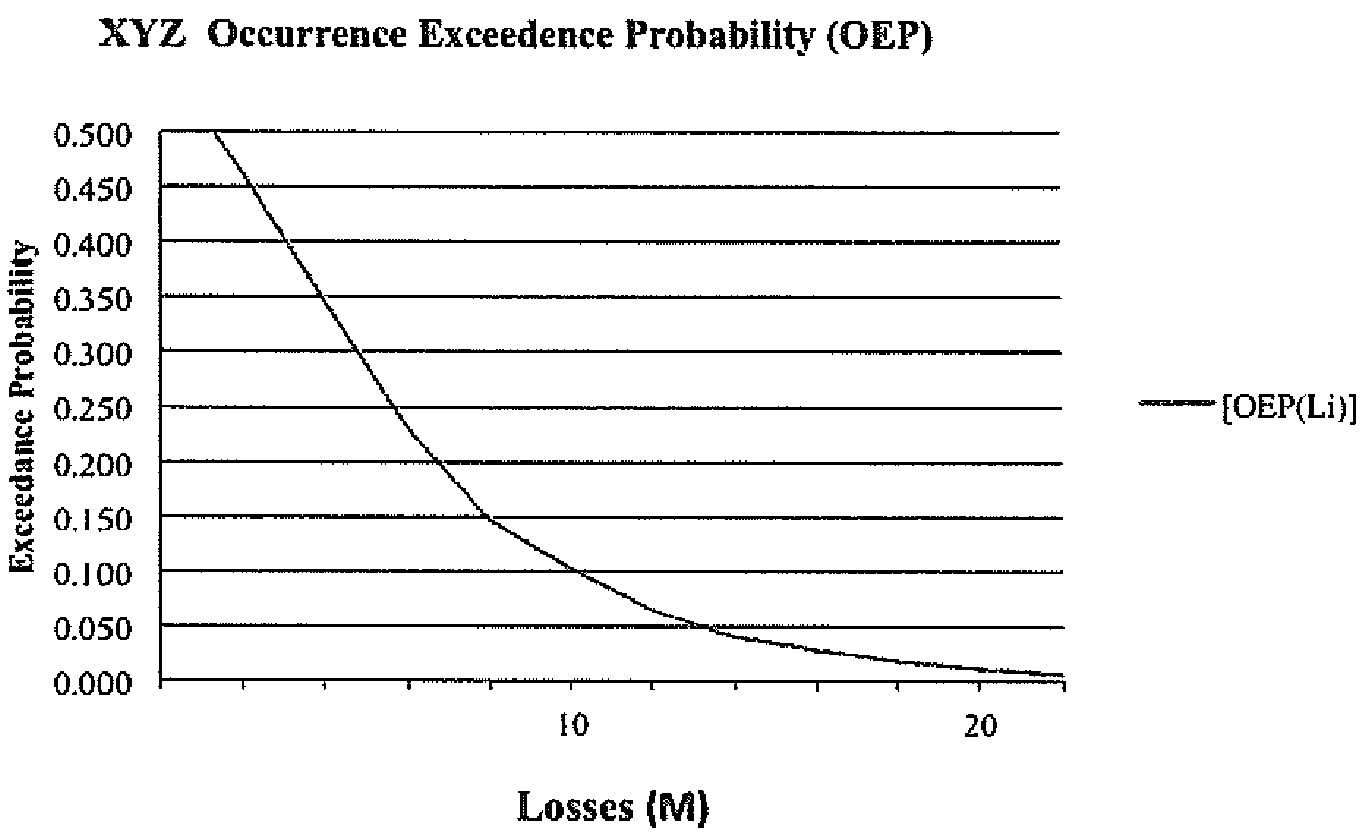


What would have been last year's return on capital if each reinsurance option had been in place?
Ignore investment income and taxes.

59. (1 point) Alfred E. Neuman, a resident of Houston, was rebuilding in the wake of Hurricane Harvey in 2017. Alfred was installing a set of glass doors that clearly were not up to code and would never withstand another hurricane. When asked why he was doing this (aside from saving money, of course), Alfred replied that since Hurricane Harvey was a one-in-25-year storm, and the doors were designed to last for maybe 20 years, there really wasn't anything to worry about.
Briefly discuss the problem with Alfred's reasoning.



60. (1 point) A reinsurer is offering a property catastrophe cover of $15,000,000 in excess of $5,000,000 per occurrence.
Estimate the pure premium, using the following output from a catastrophe model.



61. (1 point) Different versions of a catastrophe model, with corresponding probabilities, have been selected at random from a larger set of possible models. Each of these versions of the catastrophe model has been run to produce an aggregate exceedance probability curve.
The different values of S($200 million) have been ranked from smallest to largest:
0.0323 with probability 0.5%
0.0341 with probability 2%
0.0345 with probability 2.5%
⋮
0.0370 with probability 2.5%
0.0377 with probability 2%
0.0392 with probability 0.5%

Use this information to quantify the epistemic uncertainty of the probability of exceeding $200 million in aggregate losses that is estimated by this catastrophe model.

62. (0.5 point) The following Aggregate Exceedance Probabilities are associated with an insurance company's exposure.

Probability of Exceedance	Annual Aggregate Losses (million)
0.01%	736
0.05%	540
0.10%	431
0.20%	318
0.40%	215
0.50%	188
1.00%	127
2.00%	87
4.00%	58
5.00%	51
10.00%	29

Using a randomly generated number of 0.9926, simulate the insured total loss.

63. (1.5 points) A reinsurance company is evaluating whether or not to write a $100 million excess of $100 million catastrophe reinsurance treaty with a primary insurer.
The reinsurer is currently holding $700 million of capital and is required to hold enough capital to survive a 1-in-200 event.
Without the treaty, the reinsurance company has a 1-in-200 probable maximum loss (PML) of $670 million.
The primary insurer's 1-in-200 probable maximum loss (PML) is $180 million, net of other reinsurance.
Discuss how the reinsurer should determine whether it has enough capital to participate in this treaty.

64. (1.75 points) Geologists have very recently found a new earthquake fault which they have named the Bueller fault. There is yet no consensus on what the largest magnitude earthquake is likely to be along the Bueller fault. Nor is there yet any consensus on the mean return time of large earthquakes along the Bueller fault. Finally, the geologists are not yet certain of the precise location or extent of the Bueller fault.
Ferris is a catastrophe modeler who would like to incorporate the discovery of the Bueller fault into a catastrophe model that includes earthquakes.
a. (0.5 point) Briefly describe which module(s) of the catastrophe model would need to be modified to account for the new information.
b. (0.5 point) Classify the uncertainty created with the new earthquake fault as either aleatory or epistemic and briefly justify the selection.
c. (0.75 points) Briefly describe one method Ferris could use to incorporate this uncertainty in his catastrophe model.

65. (1 point) From a catastrophe model, you are given the following return times for an insurer:

Return Period (years)	PML ($000,000)
1000	360
500	240
200	150
100	60
50	40
20	20
10	10

(a) Estimate the 1-in-25 PML for this insurer.
(b) Estimate the 1-in-400 PML for this insurer.

66. (4 points) An insurance company is exposed to three independent catastrophic risks in three different regions in a given year. More than one event can occur in a year but each event can only occur once in a year. Events have the following size and probability:

Event	Loss Amount	Annual Probability of Occurrence
1	$20,000,000	0.08
2	$30,000,000	0.06
3	$40,000,000	0.04

(a) (2 points) Calculate the Aggregate Exceedance Probabilities associated with the insurance company's exposure.
(b) (1 point) Draw the Aggregate Exceedance probability curve. Label the axes.
(c) (0.5 points) Draw the Occurrence Exceedance probability curve. Label the axes.
(d) (0.5 points) Determine the 1-in-20 year PML (Probable Maximum Loss).

67. (1 point) For a county, you are given the following output from a catastrophe model:

Probability	Annual Loss per $1000 of value
95%	$0
3%	$100
2%	$200

Determine the rate prior to including expenses, which is calculated as Average Annual Loss (AAL) plus Risk Load, where the Risk Load is half of the standard deviation.

68. (1.5 points) An insurance company has three portfolios that are exposed to the same five catastrophic events. The details for each event are as follows, with losses in $ millions:

Event (E_j)	Probability (p_j)	Loss for Portfolio 1	Loss for Portfolio 2	Loss for Portfolio 3
1	1%	100	150	200
2	1%	150	100	100
3	1%	200	200	100
4	1%	100	150	100
5	1%	150	100	200

The standard deviations of loss for the three portfolios, including the catastrophic losses, are:
Portfolio 1: $130 million
Portfolio 2: $110 million
Portfolio 3: $150 million

The company has decided to minimize risk.
Explain which portfolio of the three the insurer should eliminate.

69. (2 points) Weasley Insurance and Granger Insurance are two similar sized companies.
Each insurer has a 100-year PML of 2 billion.
The two insurers are merging.
What is the size of the 100-year PML for the merged insurer? Explain your reasoning.

70. (2 points) Three of the many homeowners insurance causes of loss are:
earthquake, fire, and tornado.
Rank these causes from most likely to least likely to benefit from the use of a catastrophe model in ratemaking. Explain your reasoning.

71. (5, 5/05, Q.52) *(3 points) In determining loss costs in states with significant hurricane exposure, there are several limitations of traditional loss smoothing approaches. State and describe three of these limitations.*

72. (5, 5/06, Q.45) (1 point)
The actuary of an insurance company develops a hurricane provision for the company's prospective rates by averaging the company's historical losses for all states in which it has hurricane exposure, and then applying this provision evenly across these states.
State four potential disadvantages of this methodology.

73. (5, 5/08, Q.37) *(1 point) Describe two advantages to separating property rates into hurricane and non-hurricane components.*

74. (8, 11/11, Q.5) (3 points) Given the following:

- An insurance company is exposed to four independent catastrophic events:

Event	Size
1	\$5,000,000
2	\$10,000,000
3	\$20,000,000
4	\$40,000,000

- The annual probability of occurrence of a catastrophic event is given by
  p(x) = 500,000 / x ; where x is the amount of loss.
- While each event can only happen once, the total number of events per year is not limited to one.

Plot the aggregate exceedance probability curve associated with the insurance company's exposure and label the x and y axes.
Note: I have slightly reworded this past exam question.

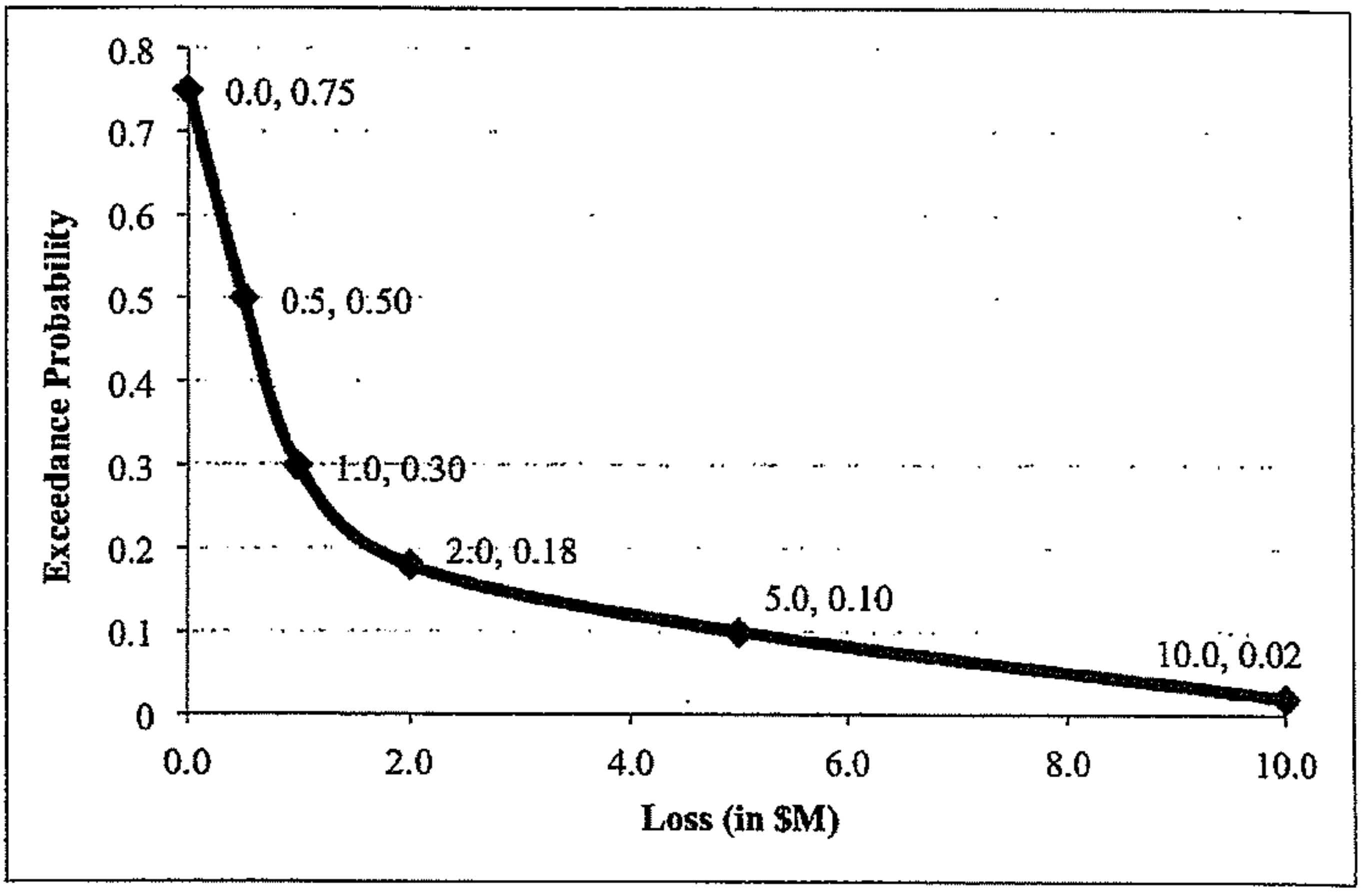
75. (8, 11/11, Q.6) (2.5 points) An insurance company is planning to introduce catastrophe modeling for portfolio management and ratemaking purposes. The company has three portfolios that are exposed to the same five catastrophic events. The details for each event are as follows:

Event (E_j)	Probability (p_j)	Loss for Portfolio 1	Loss for Portfolio 2	Loss for Portfolio 3
1	1.0%	$85,000	$40,000	$90,000
2	1.0%	$40,000	$90,000	$40,000
3	2.0%	$85,000	$50,000	$80,000
4	2.0%	$55,000	$85,000	$55,000
5	1.0%	$45.000	$50,000	$50,000

The standard deviations of loss for the three portfolios, including the catastrophic losses, are as follows:
Portfolio 1: $50,000
Portfolio 2: $125,000
Portfolio 3: $100,000

a. (1.5 points) The company has decided to minimize risk.
Explain which portfolio the insurer should eliminate.
b. (1 point) Describe two potential problems with using catastrophe models for ratemaking.

76. (8, 11/12, Q.9) (1.5 points)
The following exceedance probability curve is available for an insurer's portfolio:



a. (0.25 point) Briefly explain what an exceedance probability curve represents.
b. (0.5 point) The insurer wants to hold capital to support a 1 in 25 year Probable Maximum Loss (PML). Determine the loss level associated with this PML implied by the exceedance probability curve above.
c. (0.75 point) Briefly discuss three common uses for exceedance probability curves.

77. (8, 11/13, Q.24) (2.25 points) The following Occurrence Exceedance Probability curve is available for an insurance company's portfolio:

Return Period	Occurrence Exceedance Probability	Loss
10,000	0.0001	$200,000,000
500	0.0020	$50,000,000
200	0.0050	$20,000,000
100	0.0100	$12,000,000
50	0.0200	$7,000,000
33	0.0300	$3,500,000
25	0.0400	$1,500,000
20	0.0500	$500,000

a. (1 point) The insurer specifies that its acceptable risk level is 1-in-250 year PML.
Define PML and calculate the 1-in-250 year PML.
b. (1.25 points) The insurer decides to buy property catastrophe reinsurance protection up to the 1-in-500 year PML in the following treaties:
Quota share, where 30% is ceded up to a $40 million loss limit, which inures to the benefit of the following:

- 100% placed 1st layer property catastrophe excess of loss treaty $6 million xs $4 million
- 90% placed 2nd layer property catastrophe excess of loss treaty $10 million xs $10 million
- 75% placed 3rd layer property catastrophe excess of loss treaty $30 million xs $20 million

During the treaty year, the insurer suffers a $45 million earthquake loss.
Calculate the amount of loss ceded to each of the reinsurance treaties and the net retained loss by the primary insurer.

78. (SOA GIIRR Exam, 11/13, Q.7) (5 points) Grossi and Kunreuther define two types of uncertainty, aleatory and epistemic. The definitions are:

> "Aleatory uncertainty is the inherent randomness associated with natural hazard events …"
> "… epistemic uncertainty is the uncertainty due to lack of information or knowledge of the hazard."

(a) (2.5 points) Provide an example of each type of uncertainty with regard to earthquake models and explain why each example reflects that type of uncertainty.
(b) (1.25 point) Describe which of these types of uncertainty can be reduced by collecting more data, and illustrate your response using your example from part (a).
(c) (1.25 point) Explain how logic trees can be used to reflect epistemic uncertainty in the construction of exceedance probability curves.

79. (SOA GIIRR Exam, 5/14, Q.19) (4 points) The city of Faultline is in an earthquake zone.
CommCo insurance company covers over one-half of the commercial properties in Faultline, but does not offer homeowners insurance.
HomeCo insurance company covers over one-half of the single family homes in Faultline, but does not offer commercial property insurance.
Both companies' policies provide comprehensive coverage of all perils.
Three of the components of a catastrophe model are:

- Hazard module
- Inventory module
- Vulnerability module

(a) (1.5 points) Describe each of the components.
(b) (1.5 points) Indicate similarities (if any) and differences (if any) between the CommCo and HomeCo implementations of an earthquake model for each component.
Justify each of your answers.
(c) (1 point) Compare the analyses required to establish the claims loading for hurricane and non-hurricane weather claims for HomeCo.

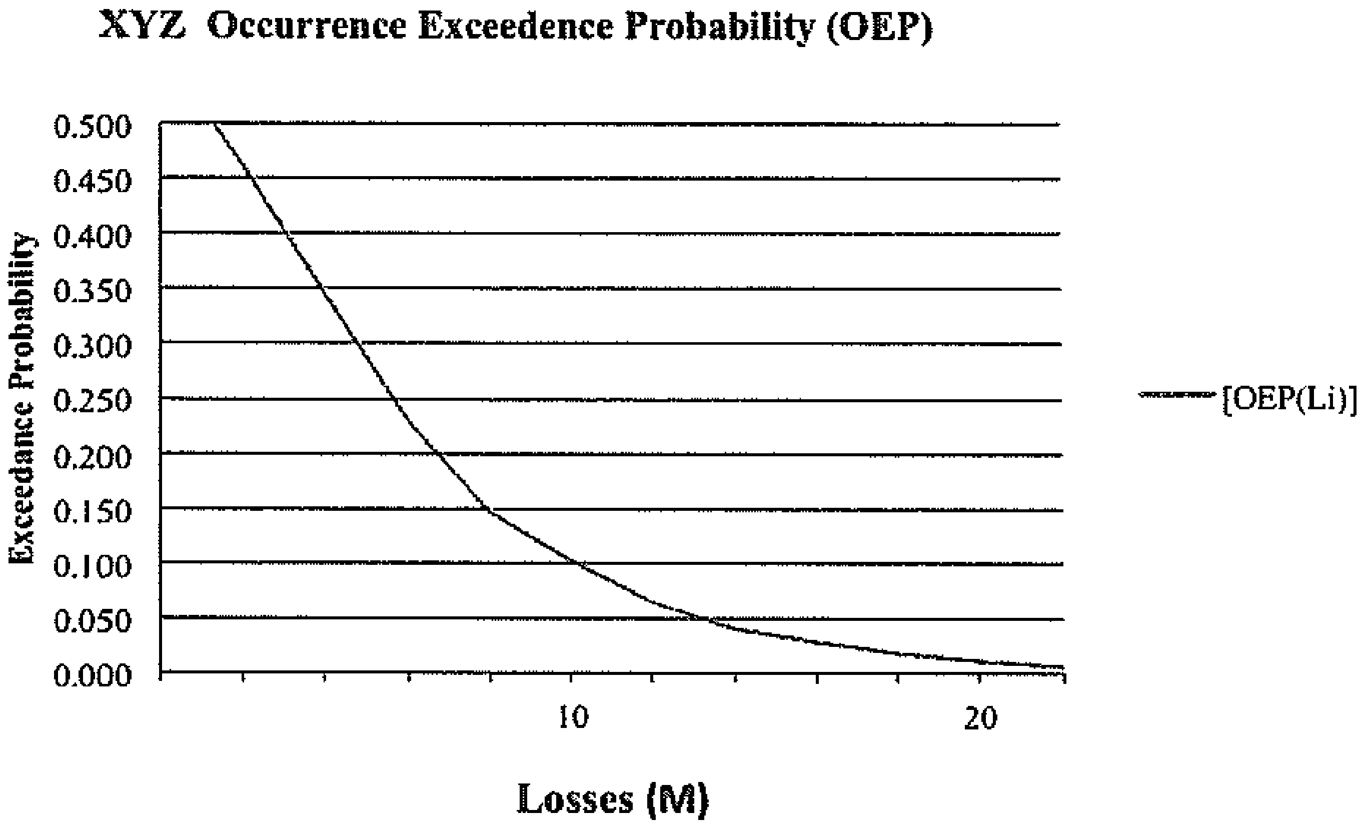
80. (SOA GIIRR Exam, 11/14, Q.4) (4 points) You are a regulator leading a group that is attempting to combine results from three commercial catastrophe models. Each model has produced three exceedance probability curves for a standard set of exposures. One of the curves is the model's best estimate of the probabilities and the other two form a 90% confidence interval. Your group's task is to produce three curves, one representing an average of the three models and two representing a confidence interval. The following table displays output from the three models.
Exceedance probabilities relate to a loss of 10 billion.

	Exceedance Probability				Return (in Years)		
Model	5%	Best Est.	95%		5%	Best Est.	95%
A	0.0022	0.0072	0.0114		455	139	88
B	0.0026	0.0080	0.0109		385	125	92
C	0.0031	0.0092	0.0117		323	109	85

For the best estimate, the combined value is a weighted average, with 50% weight on the middle of the three best estimates and 25% weight on the other two estimates.
(a) (1 point) Calculate the combined best estimate for both the probability and the return period.
(b) (1 point) You have observed that the period calculated in (a) is not the reciprocal of the calculated probability. You are considering two approaches for your final recommendation. They are:
● Approach 1: Estimate the return period as the reciprocal of the calculated probability.
● Approach 2: Retain the return period estimate from (a).
Recommend one of the two approaches to calculate the best estimate of the return based on the combined models. Justify your choice.
(c) (1 point) You are considering three options for combining the confidence interval estimates for the exceedance probability. They are:
● Option I: Use the same weights by model as used for combining the best estimates.
● Option II: Apply the 50% weight to the middle value regardless of which model is in the middle.
● Option III: Use the most extreme value from the three models.
Calculate the interval using each of the three options.
(d) (1 point) State the option that produces an interval that can most accurately be described as a 90% confidence interval. Justify your choice.

81. (SOA GIIRR Exam, 11/14, Q.20) (4 points) You are part of a team at Cool Breeze Insurance Company that is setting rates for homeowners insurance catastrophe coverage with respect to hurricane losses in southern Florida.
Grossi and Kunreuther suggest that the base premium for a given county comprises three components.
(a) (2 points) Describe each of the three components and indicate how each may be determined for Cool Breeze.
(b) (1 point) Within a given county, Cool Breeze's rates may be adjusted based on attributes of the insured dwelling. Grossi and Kunreuther discuss two attributes that are of primary importance. Describe each of the two attributes.
(c) (1 point) When estimating expected claims from a catastrophe it is important to account for post-event inflation. Define demand surge and provide an example that relates to hurricane losses.
Note: I have rewritten this past exam question to match the syllabus of your exam.

82. (8, 11/14, Q.24) (2.25 points) A reinsurer is offering a property catastrophe cover of $10,000,000 in excess of $10,000,000 per occurrence with one reinstatement to an insurer. The reinsurance broker produces a catastrophe model with the following output for losses in excess of $10,000,000.



a. (0.5 point) Calculate the pure premium using the payback approach.
b. (0.5 point) The treaty incepts on July 1, 2014 with a premium of $1,200,000. The insurer experiences a catastrophe loss on December 1, 2014 resulting in total loss amount of $15,000,000. Calculate the reinstatement premium given the reinstatement provision is 115% pro-rata as to amount.
c. (1.25 points) The insurer also purchases a 30% quota share treaty which inures to the benefit of the catastrophe treaty. Calculate the amount paid under each treaty and the insurer's net loss.

***83.* (SOA GIIRR Exam, 5/15, Q.16)** (4 points) In 1996, the California Earthquake Authority (CEA) articulated the following four factors that must be considered in rate setting:
1. Location of the insured property and its proximity to earthquake faults, and to other geological factors that affect the risk of earthquakes or damage from earthquakes
2. Soil type on which the insured dwelling is built
3. Construction type and features of the insured dwelling
4. Age of the insured dwelling

A catastrophe model has four modules: Hazard, Inventory, Vulnerability, and Loss.
(a) (2 points) Indicate, for each of the four factors listed by the CEA, which module or modules use that particular factor. Support your selections.
(b) (2 points) The CEA may select factors beyond the four listed above.
Propose two additional factors that might be considered. Support your proposal.

84. (8, 11/15, Q.22) (2.75 points) An insurance company is exposed to three independent catastrophic risks in three different regions in a given year. More than one event can occur in a year but each event can only occur once in a year. Events have the following size and probability:

Event	Loss Amount	Annual Probability of Occurrence
1	$10,000,000	0.10
2	$15,000,000	0.05
3	$35,000,000	0.02

a. (2.25 points) Calculate the Aggregate Exceedance Probabilities associated with the insurance company's exposure.
b. (0.5 points) Using a randomly generated number of 0.86, simulate the insured total loss.

85. (SOA GIIRR Exam, 5/16, Q.16) (5 points) Generic Insurance Company (GIC) is considering adding a portfolio of commercial property insurance coverage for regional hamburger chains. It is currently in discussions with three such chains:

- ABC Burgers has its outlets in California with significant earthquake risk.
- FGH Burgers has its outlets in Kansas with significant tornado risk.
- XYZ Burgers has its outlets in Florida with significant hurricane risk.

Each chain sells its hamburgers at freestanding buildings (as opposed to, for example, at shopping centers or strip malls). For a given chain, the buildings are identical with regard to design and construction. Each chain has the same number of outlets with the same total value.
As part of its decision process, GIC assumes that its current portfolio of commercial property policies has no significant catastrophe risk.
As GIC negotiates with each chain, it may or may not come to an agreement with all of them. For example, it may be able to insure ABC and XYZ but not FGH.
(a) (2 points) Rank the following portfolios from least to most catastrophe risk from GIC's perspective, with the possibility that some may be roughly equal in risk.
Justify your ranking.

I. ABC only
II. FGH only
III. XYZ only
IV. ABC and FGH
V. ABC and XYZ
VI. FGH and XYZ
VII. ABC, FGH, and XYZ

(b) (0.5 points) GIC has decided to offer coverage to ABC Burgers. Describe an action GIC may take to improve its underwriting to account for ABC's earthquake risk.
(c) (1 point) Describe coverage modifications GIC may use to reduce its earthquake risk.
(d) (1 point) Describe how a catastrophe model could be used to set the coverage modifications from part (c).
(e) (0.5 points) Describe an action other than using coverage modifications that GIC may use to reduce its earthquake risk.

86. (SOA GIIRR Exam, 11/16, Q.17) (5 points) Grossi and Kunreuther define epistemic uncertainty as "the uncertainty due to lack of information or knowledge of the hazard."
Two ways of reducing epistemic uncertainty are (1) collecting more data
and (2) having more accurate data or models.
A catastrophe model comprises four modules: hazard, inventory, vulnerability, and loss.
(a) (2.5 points) Provide, for each module, either:
(i) An explanation of why it may not be possible to reduce epistemic uncertainty by collecting more data, or
(ii) A hurricane or earthquake example of how collecting more data may reduce epistemic uncertainty.
(b) (2.5 points) Provide, for each module, either:
(i) An explanation of why it may not be possible to reduce epistemic uncertainty by improving the accuracy of the data or model, or
(ii) A hurricane or earthquake example of how improving the accuracy of the data or model may reduce epistemic uncertainty.

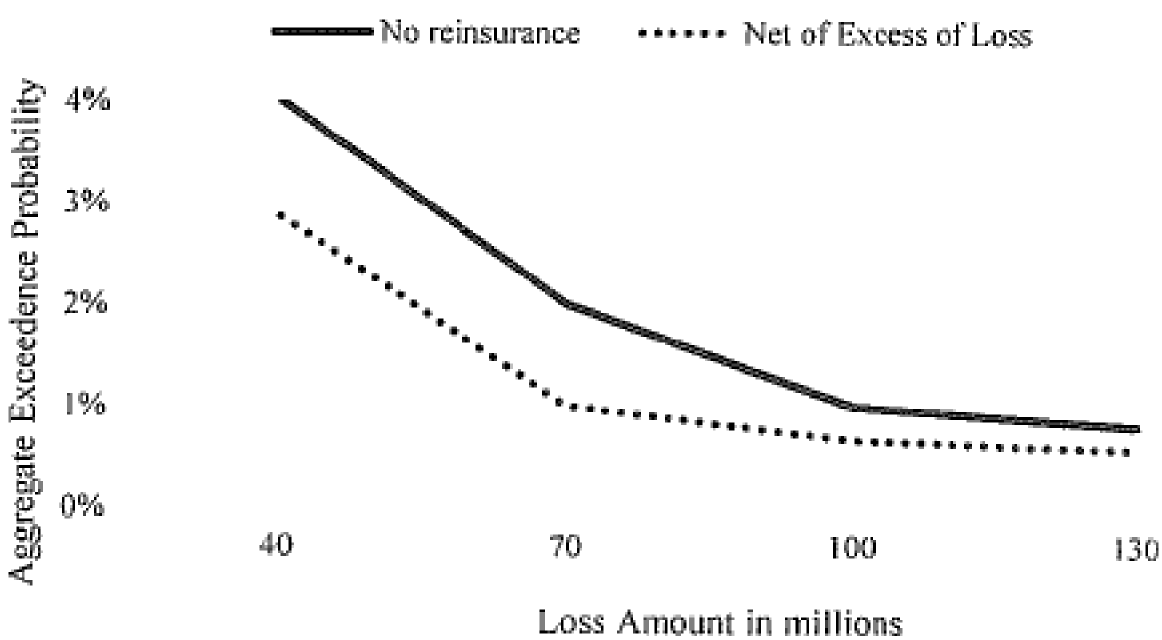
87. (8, 11/16, Q.18) (2 points) Catastrophe models were built to assist the insurance industry in quantifying the risk of natural disasters.
a. (1 point) For any two of the four basic modules of a catastrophe model, provide an example of <u>epistemic</u> risk.
b. (1 point) For any two of the four basic modules of a catastrophe model, provide an example of <u>aleatory</u> risk.

88. (8, 11/16, Q.19) (3 points) In order for an insurance company to increase its return on capital, two reinsurance options are being considered:
I. $5 million excess of $5 million per risk:
● ALAE pro-rata
● Rate = 18% of premium
II. 20% Quota Share:
● Ceding commission of 30%
● Maximum ceded loss ratio of 150%

The insurance company must hold capital to support a 1-in-100 year Probable Maximum Loss. Additionally, the following information regarding the insurance company's performance last year is given:
● Premium: $50,000,000
● Expenses: $15,000,000
● Total Loss & ALAE: $30,000,000
● Return on Capital: 5%

	Claims greater than $5,000,000	
	Loss	ALAE
Claim 1	$7,500,000	$1,500,000
Claim 2	$10,000,000	$500,000

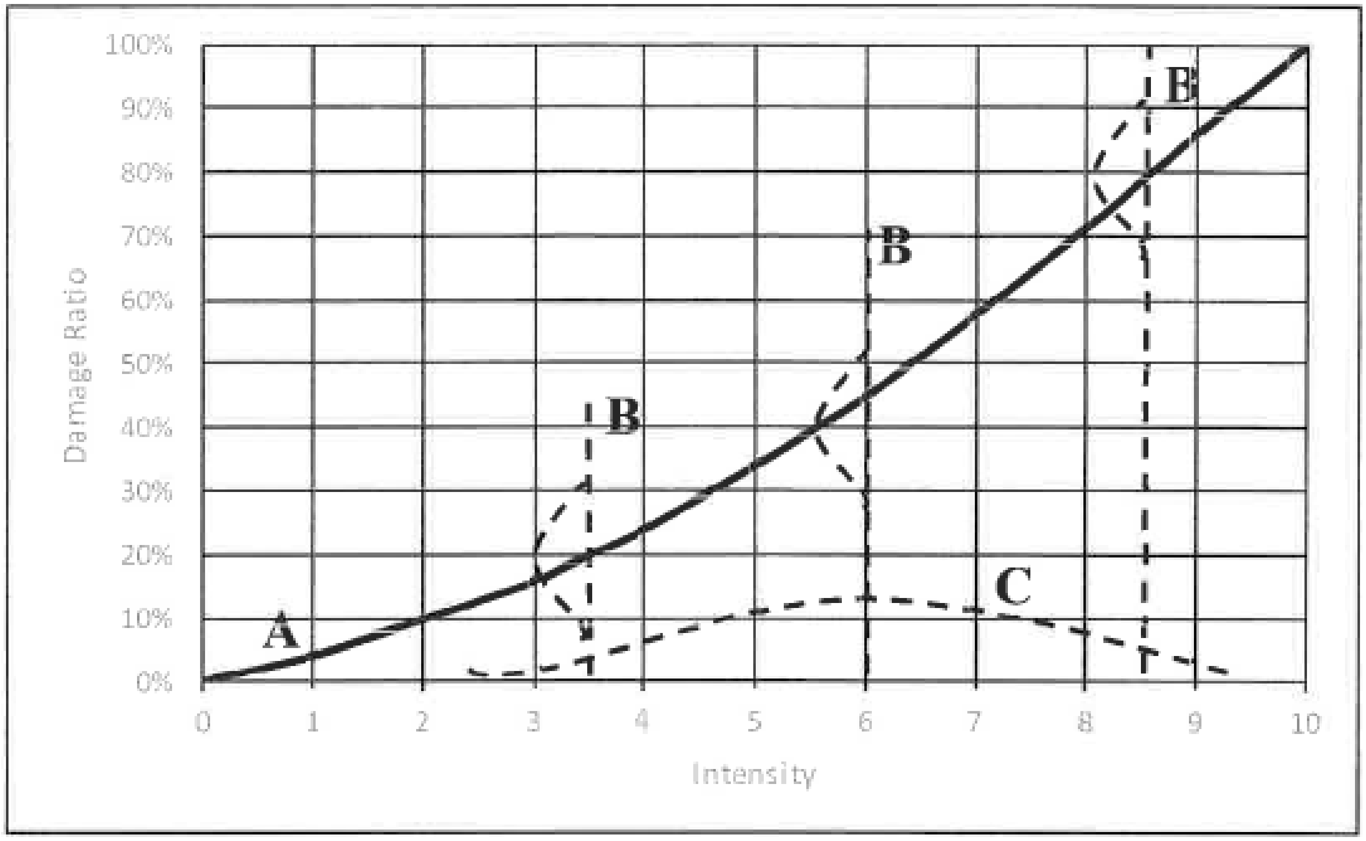
The Aggregate Exceedance Probability curve for the insurance company is shown below:



Determine the impact each reinsurance option would have had on last year's return on capital. Ignore investment income and taxes.

89. (SOA GIIRR Exam, 5/17, Q.18) (5 points) Cardinale Insurance Company (CIC) has two product lines. One is private passenger automobile insurance that is sold in all jurisdictions in the United States. The other is homeowners hurricane coverage in a hurricane prone area.
(a) (2 points) Explain why the hurricane coverage product line has higher risk than the automobile product line with respect to each of the following risk characteristics:
(i) Availability of data
(ii) Uncertainty of loss
(iii) Correlation between claims
(iv) Insurer capacity
(b) (1 point) Explain the relationship between insurer capacity and customer demand with respect to catastrophe coverages.
(c) (0.5 points) Grossi and Kunreuther describe a survival constraint with respect to insurability of catastrophe coverage. Define survival constraint.
(d) (0.5 points) Explain how an exceedance probability curve can be used as a tool in assessing an insurer's survival constraint.
(e) (0.5 points) Identify one regulatory constraint that could threaten an insurer's survival constraint.
(f) (0.5 points) State two actions CIC may be able to take to meet the survival constraint when offering hurricane coverage.

90. (8, 11/17, Q.20) (3 points)
An earthquake model produces the following damage function for a $1,000,000 home in California (intensity on the horizontal axis and Damage Ratio on the vertical axis):



The probability of an event occurring with a given intensity is:

Intensity	Probability
3.5	10%
6.0	5%
8.5	2%

a. (0.75 points) Identify what A, B, and C represent in the above graph.
b. (1.75 points) Determine the premium for this home given the following information:
● The insurer's expense load is 20% of premium
● The risk load is set to 8% of the standard deviation of the loss

91. (SOA GIIRR Exam, 5/18, Q.9) (5 points) IFTEM Insurance Company is working with King Consultants (King) to build a catastrophe model for its exposure to earthquake losses.
You are an actuary working for King and are reviewing a draft report by IFTEM containing the statements below.
Explain why each of the following statements is either correct or incorrect.
(i) In developing the model domain (the geographic extent of the region to be modeled) it is sufficient to know the location and likely magnitude of future earthquakes.
(ii) The return period (time until the next earthquake) and magnitude of the earthquake can be separately modeled.
(iii) To model IFTEM's specific portfolio for the inventory module, detailed information will be required for each building in the portfolio.
(iv) To model IFTEM's specific portfolio for the vulnerability module, detailed information will be required for each building in the portfolio.
(v) The total loss from an event (prior to applying the terms of the insurance coverage) could come from either the vulnerability module or the loss module.

92. (SOA GIIRR Exam, 11/18, Q.8) (4 points)
Grossi and Kunreuther identify three special issues with respect to portfolio risk:
Data Quality, Uncertainty Modeling, and the Impact of Correlation.
(a) (1.5 points) Describe two examples where data quality issues could arise in the inventory module of the catastrophe model.
(b) (0.5 points) Provide the reason why inventory is the component of the catastrophe model that requires the most attention with respect to data quality.
(c) (1 point) The mean damage ratio is the ratio of dollar loss to replacement value of the structure. Provide an example that illustrates why using the mean damage ratio is insufficient when determining the insurer's expected loss.
(d) (1 point) One way that a catastrophe model can reflect correlation is if the same simulated event is applied to all insured properties. Describe two additional ways that a catastrophe model can reflect correlation.

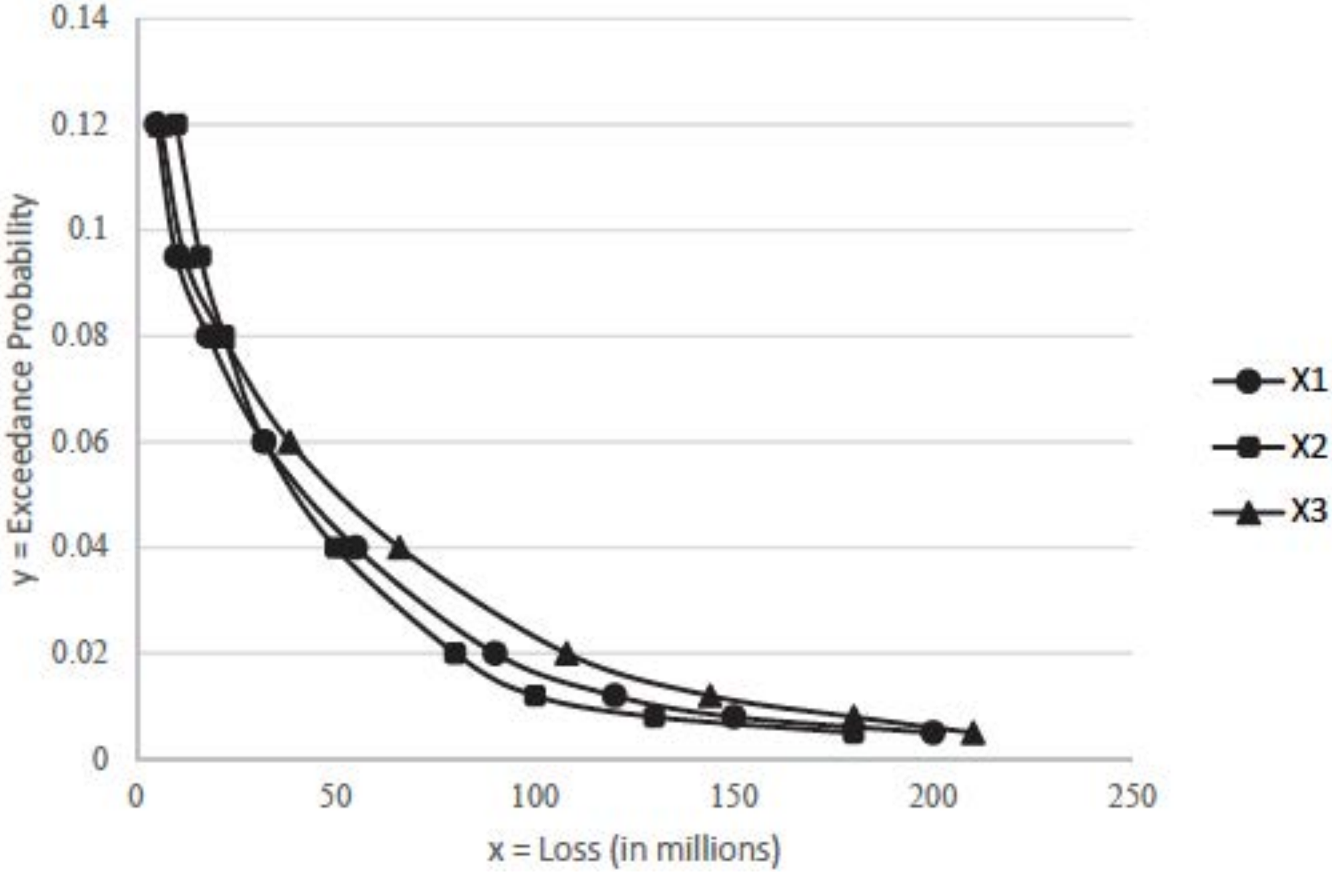
93. (8, 11/18, Q.16) (1.75 points) A reinsurance company is evaluating whether or not to write a $50 million excess of $50 million catastrophe reinsurance contract with a primary insurer. The reinsurer is currently holding $850 million of capital and is required to hold enough capital to survive a
1-in-250 event. Without the new contract, the reinsurance company has a 1-in-250 probable maximum loss (PML) of $825 million which is solely driven by the hurricane peril.
Given the following:
● The primary insurer's PMLs are driven by the hurricane and earthquake perils only.
● The primary insurer's aggregate annual PMLs by return period are as follows:

Return Period (years)	PML ($000,000)
1000	125
500	105
200	95
100	70
50	50
25	30
20	25
10	20
5	15

● The largest hurricane event in the primary insurer's event catalog is $45,000,000.
a. (1 point) Calculate the ceded, and net,1-in-250 PMLs for this contract for the primary insurer.
b. (0.75 point) Evaluate whether the reinsurer should participate in this treaty.

94. (8, 11/18, Q.17) (2 points) A catastrophe modeler would like to incorporate a new construction technique into a catastrophe model. This new technique would theoretically reduce the amount of building damage sustained during hurricane force winds.
However, experts have not reached a consensus on the effectiveness of the new construction technique because it has not been exposed to an actual hurricane.
a. (0.5 point) Briefly describe which module(s) of the catastrophe model would need to be modified to account for the new information.
b. (0.5 point) Classify the uncertainty created with the new construction technique as either aleatory or epistemic and briefly justify the selection.
c. (1 point) Briefly describe and contrast two methods the modeler could use to incorporate uncertainty in this catastrophe model.

95. (SOA GIRR Exam, 5/19, Q.5) (4 points) You are evaluating a property book of business for NAN Insurance Company, which is subject to catastrophe losses from hurricane, earthquake, flood, and tornadoes. Your evaluation considers the results from three different model vendors for the same book of business, with the following results:



(a) (2 points) Explain how each module of the catastrophe models could contribute to the difference in the modeling results from each vendor for the same book of business, for each of the following modules.

(i) Hazard (ii) Inventory (iii) Vulnerability (iv) Loss

(b) (0.5 points) Determine the 1-in-25 year probable maximum loss (PML) for Model X2.

(c) (1 point) The following modeling results are provided:

Model	Annual Average Loss (AAL) in millions
X1	12.55
X2	13.14
X3	14.91

Recommend an approach for selecting or deriving an appropriate AAL from the three estimates. Justify your recommendation.

(d) (0.5 points) The initial runs (shown above) used the actual policy information. The Management asked each cat modeler to re-run the same portfolios with construction type set to "unknown." The re-runs with "unknown" construction type all produced a smaller AAL.
The actuary decided to present the re-runs to a prospective reinsurer. Explain one implication of replacing the original model runs with the re-runs.

96. (SOA GIRR Exam, 11/19, Q.8) (5 points) You are a pricing actuary for a U.S. general insurance company working on catastrophe modeling for commercial property coverage. You are pricing hurricane coverage for two accounts X and Y. A catastrophe modeler has provided the following modeled annual losses for the hurricane wind peril:

Events	Occurrence Probability	Expected Gross Loss for Account X	Expected Gross Loss for Account Y
100	0.01	125,000	150,000
99	0.01	90,000	100,000
98	0.01	65,000	70,000
97	0.01	48,000	50,000
96	0.01	35,000	32,000
95	0.01	26,000	20,000
94	0.01	20,000	10,000
93	0.01	15,000	5,000
92	0.01	10,000	3,000
91	0.01	8,000	2,000
90	0.01	0	0
…	…	…	…
1	0.01	0	0
Total	1.00	442,000	442,000
Variance of Modeled Losses		309,903,600	395,083,600

- The expected loss for both X and Y is zero for events 1 through 90.
- The expense load factor is 27%.
- The risk load is 10% of the standard deviation of modeled losses.
- Accounts X and Y each have a coverage amount of 10,000.

(c) (2 points) Calculate the premium for each of account X and account Y.

(d) (1 point) Your company views risk at the 100-year return period and uses the following two risk metrics:

(i) AAL/TIV = (Average Annual Loss)/(Total Insured Value)

(ii) PML/TIV = (Probable Maximum Loss)/(Total Insured Value)

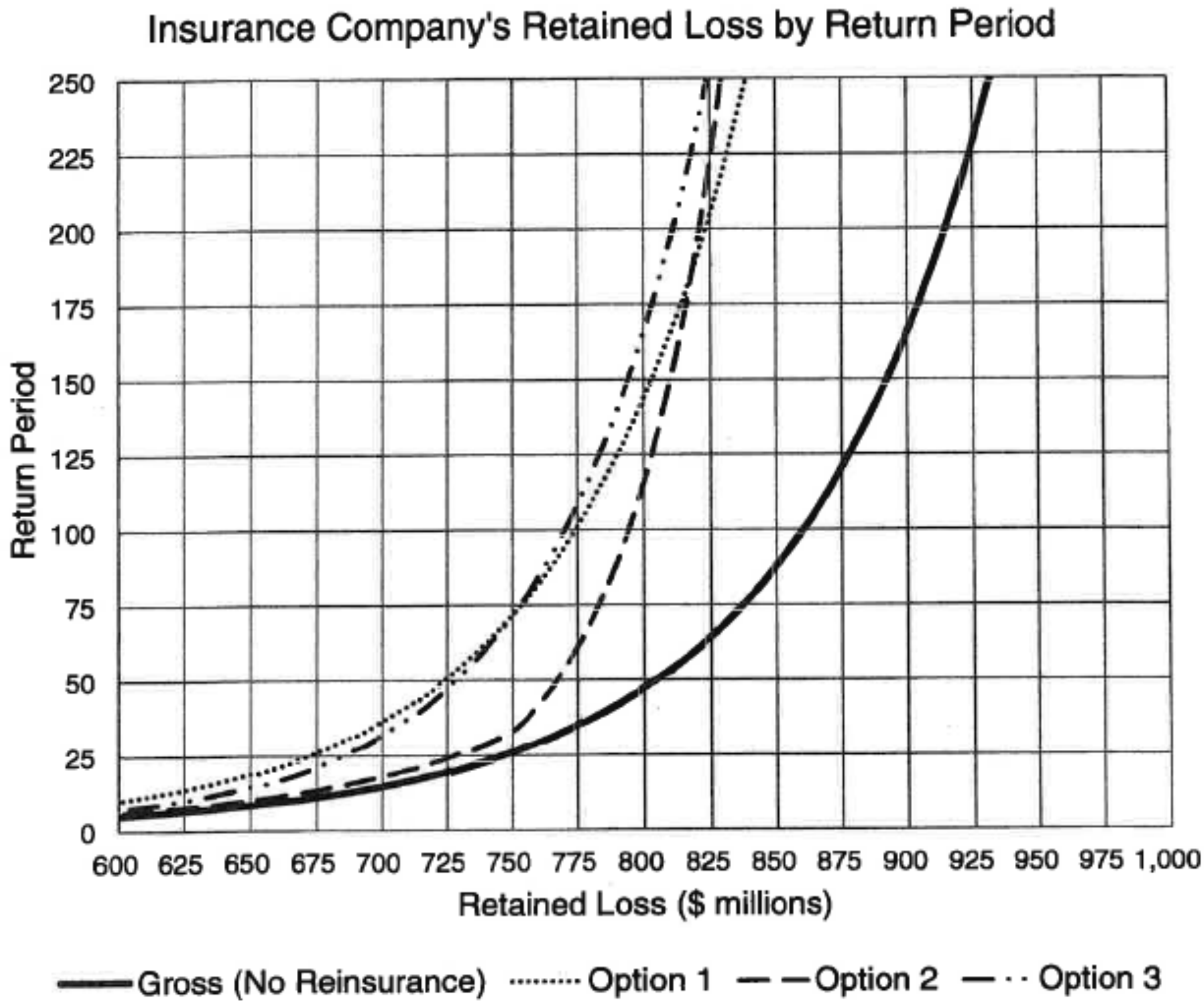
Describe the purpose of each risk metric.

(e) (1 point) Calculate each risk metric for each of account X and account Y.

(f) (1 point) Interpret the results from part (e).

Notes: Parts (a) and (b) of this past exam question are not on the syllabus of this exam.

97. (8, 11/19, Q.19) (1.75 points)
An insurance company is deciding between three reinsurance treaty options from the same reinsurance company based on the following output from a catastrophe model:



Insurance Company	Gross (No Reinsurance)	Option 1	Option 2	Option 3
Retained AAL	$500 million	$450 million	$486 million	$488 million
Coefficient of Variation of Retained Loss	25%	25%	24%	22%
Coefficient of Variation of Ceded Loss	n/a	25%	76%	169%

CONTINUED ON NEXT PAGE

a. (0.5 point) Calculate the probability that the insurance company retains more than $750 million of loss on a gross basis.
b. (0.75 point) One of the options represents a Quota Share reinsurance treaty. Justify which of the three options it is, and state the Quota Share percentage.
c. (0.5 point) Describe a reason why the reinsurance premium for Option 3 may be higher compared to Option 1.

98. (SOA GIIRR Exam, 11/20, Q.12) (4 points)
(a) (1 point) State four applications of catastrophe modeling for insurance.
(b) (1 point) You insure a small book of property portfolios in the state of Florida.
You receive two new requests (portfolio X and portfolio Y) for pricing quotes and you decide to add only one of these portfolios to the book.
You are given the following information:

	Average Annual Loss (AAL)	100-Year Probable Maximum Loss (PML)
Current Book	50,000	750,000
Current Book + Portfolio X	50,000 + 5,000	850,000
Current Book + Portfolio Y	50,000 + 6,000	770,000

Recommend which portfolio you would add to the book. Justify your recommendation.
(c) (1 point) Management decided to write the portfolio you didn't recommend in part (b).
The risk potential of the portfolio could be reduced by 13.7% if hurricane shutters are installed as a risk mitigation strategy. The expense load factor is 27%. The selected risk load is 440.
Calculate the premium for this other portfolio assuming hurricane shutters are installed on all properties in the portfolio.
(d) (0.5 points) Provide a consideration in the selection of a risk load in this situation.
(e) (0.5 points) You are concerned about the close geographical proximity of your existing book of business to the portfolio that management wants to add.
Recommend a way this risk could be managed.

Solutions:

1. The vulnerability module estimates the level of building damage expected from different levels of the external forces imposed, for example high winds in the case of a hurricane.
There are two main approaches, based on judgement and based on engineering studies. The latter is preferred and has two major steps:
1. Identification and definition of typical buildings. (A definition of as many building classes as is practical so as to divide the whole portfolio into subsets that will act reasonably similarly under the external force such as earthquake ground motion. Useful subsets will differ somewhat depending on what catastrophe is being modeled.)
2. Analysis of the vulnerability of these typical buildings to the external forces of different strengths. (Engineering studies and/or the observation of damage sustained in actual catastrophes.)
Comment: See Section 3.4 of Grossi and Kunreuther.

2. In each case I will weight the middle value 50% and the other two values 25%.
For example: (50%)(400) + (25%)(452) + (25%)(381) = 408.

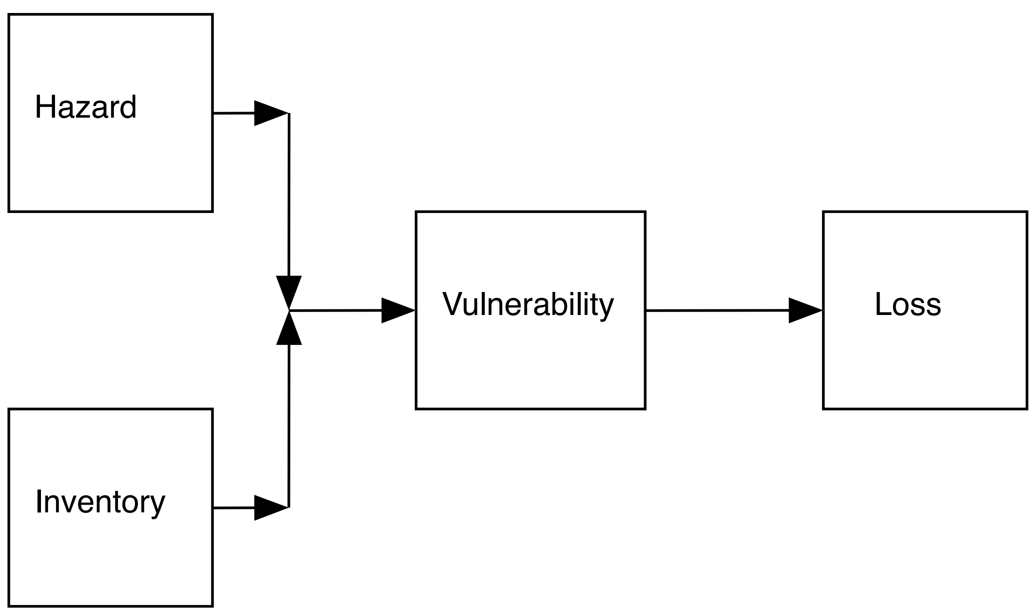
Exceedance Probability	Estimated Loss ($ millions)			
	Model A	Model B	Model C	Combination
0.5%	452	381	400	408
1%	330	287	294	301
5%	162	158	179	165
10%	75	94	101	91

Alternately, one could take a straight average of the values:

Exceedance Probability	Estimated Loss ($ millions)			
	Model A	Model B	Model C	Combination
0.5%	452	381	400	411
1%	330	287	294	304
5%	162	158	179	166
10%	75	94	101	90

Comment: See Table 4.2 in Grossi and Kunreuther.

3. The four components are: hazard, inventory, vulnerability, and loss.
The hazard component specifies the risk from whatever catastrophe is being insured against.
So for example, in an earthquake model we would specify the epicenter and magnitude of a quake.
The inventory component specifies the properties that are being insured, with detailed information on each property in the portfolio, such as its location, value, construction, age, etc.
The vulnerability component brings together the hazard and inventory, to determine the damage that results to the insured properties.
For example, for a given earthquake and a given property at a certain location, there could be a distribution of the percentage of damage to the building, contents, etc.
The loss component translates a given level of damage into an amount of insurance loss, based on the coverage features of the relevant insurance policy.
For example, an earthquake policy might have a 15% deductible that has to be taken into account.

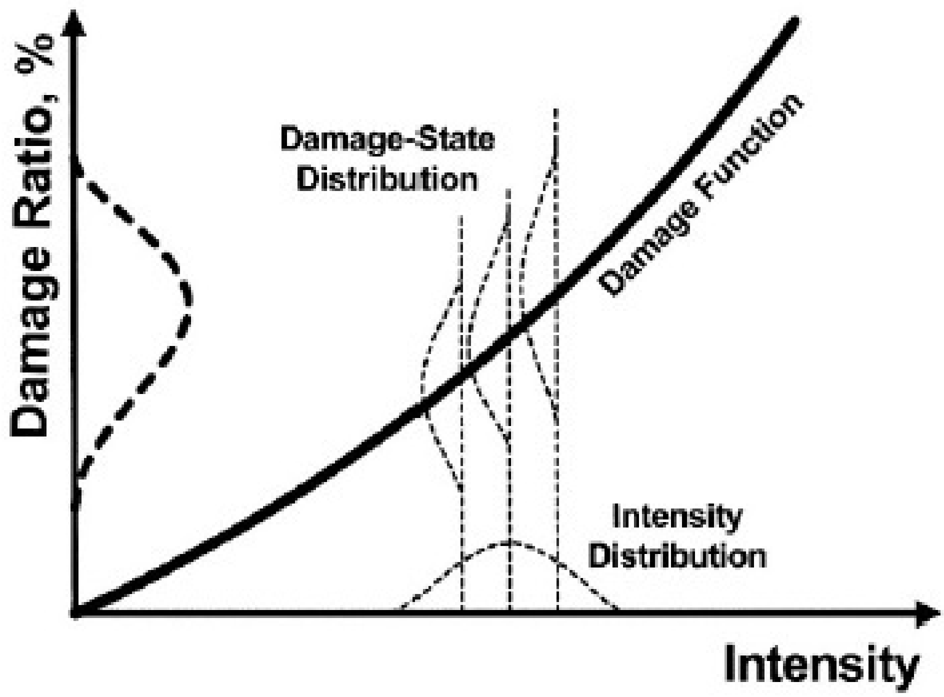


Comment: See pages 26 to 27 of Grossi and Kunreuther.

4. First of all one would be underestimating the process variance of outcomes, which is an important output of the catastrophe model. Also the mean damage could result in a financial loss less than the deductible of an insurance policy, but the average payment would not be zero. For example, the mean loss may be 6% of the value of the home, but there may be probability of 25% damage to the home. In this this situation the average payment with a 10% deductible is positive, even though the average damage would result in no payment.
Comment: See Section 6.4.2 of Grossi and Kunreuther.

5. On average between one and two hurricanes hit the United States per year, affecting only a small portion of the whole coastline. Thus recent historical information for a given area, even several decades, is not sufficient to capture the potential of loss from a hurricane.
The model includes a very large number of possible hurricane tracks, based in part on past historical tracks. This attempts to include everything that may happen with an appropriate probability attached.
There is no doubt, that with many homes near the ocean, a hurricane would cause lots of insurance losses if it hit Cape Mackerel. We are just taking into account this very real possibility.
Comment: The insurer might want to compare the estimate from this vendor's model to that from some other vendor's model.
Also an interesting question for this insurer is how do its proposed rates compare to those of its competitors.
If the insurance commissioner does not allow insurers to raise rates to take into account this loss potential from hurricanes, there could be availability problems with many homeowners being unable to get insurance (in the voluntary market).

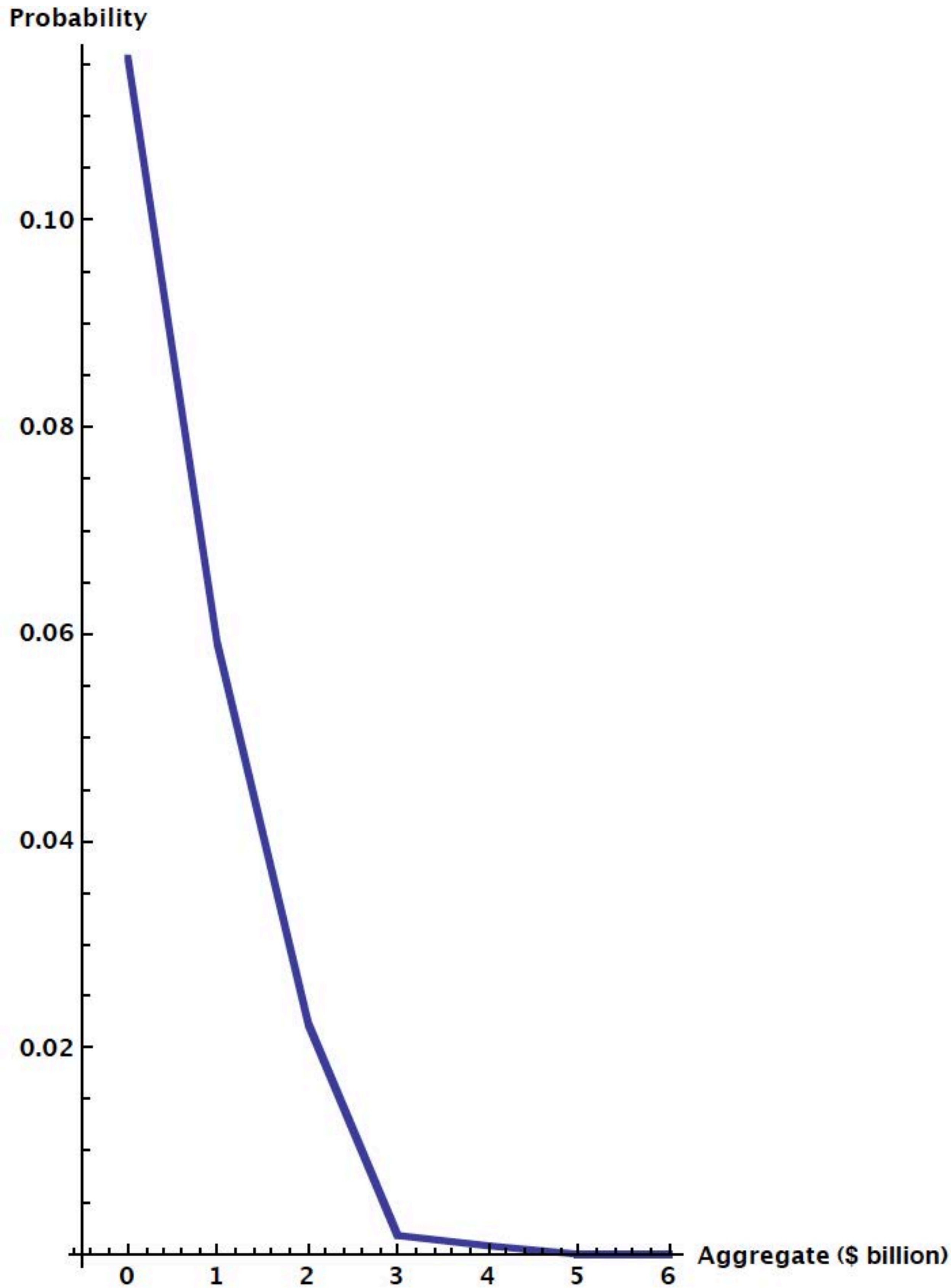
6. The damage ratio is the ratio of repair cost to replacement cost of a building. The damage ratio can range from 0% to 100%.
The damage function is an equation that relates the expected structural damage state of the entire building to the intensity of catastrophic event.
The coefficient of variation measures the uncertainty in the prediction of damage.
Both the intensity of external excitation and the level of damage given the level of excitation have a distribution associated with them. Therefore the damage function is an uncertain quantity as well.
The structure type is the key element in determining a building damage state. The occupancy type is a key element in determining the contents damage state and the time-element damage state.
Comment: See pages 62 to 64 of Grossi and Kunreuther, including Figure 3.7:



7.

Events #s Occurring	Aggregate ($ billion)	Probability
None	0	(0.94)(0.96)(0.98) = 0.884352.
1	1	(0.06)(0.96)(0.98) = 0.056448.
2	2	(0.94)(0.04)(0.98) = 0.036848.
3	3	(0.94)(0.96)(0.02) = 0.018048.
1, 2	3	(0.06)(0.04)(0.98) = 0.002352.
1, 3	4	(0.06)(0.96)(0.02) = 0.001152
2, 3	5	(0.94)(0.04)(0.02) = 0.000752.
1, 2, 3	6	(0.06)(0.04)(0.02) = 0.000048.

Now we need to compute the probability of the aggregate loss being greater than a certain values. S(6) = 0. S(5) = 0.000048. S(4) = S(5) + 0.000752 = 0.0008.
S(3) = S(4) + 0.001152 = 0.001952. S(2) = S(3) + 0.018048 + 0.002352 = 0.022352.
S(1) = S(2) + 0.036848 = 0.0592. S(0) = S(1) + 0.056448 = 0.115648.
S(0) + 0.884352 = 1. Check.



<u>Comment</u>: Similar 8, 11/11, Q.5.

8. Location, value of the home, deductible size, personal property coverage limit, loss of use coverage limit, year built, construction type (wood, masonry, etc.), type of foundation (slab, raised, other), and number of stories.
Mitigation efforts that reduce expected losses include: bolting walls to the house foundation, cripple wall bracing, and water heater tie-down.
Comment: Tying down the water heater, decreases the chance of fire following the earthquake.

9. 1. Limited scientific knowledge.
2. Limited historic data.
3. Cross disciplinary nature of catastrophe modeling leads to complexity.
4. Lack of available data to create the Geographic Information System (GIS) databases.
5. Uncertainty about the possible effects of demand surge.
6. Lack of detailed information on a structure's characteristics.
7. Lack of accurate information on the market value of structures.
8. Laboratory testing and expert opinions used to develop the vulnerability module.
Comment: See Section 4.3 of Grossi and Kunreuther.

10. For earthquakes, this high correlation is usually due to local conditions that can focus ground motion. For hurricanes such effects are observed with the presence of localized tornados.
Comment: See page 131 of Grossi and Kunreuther.

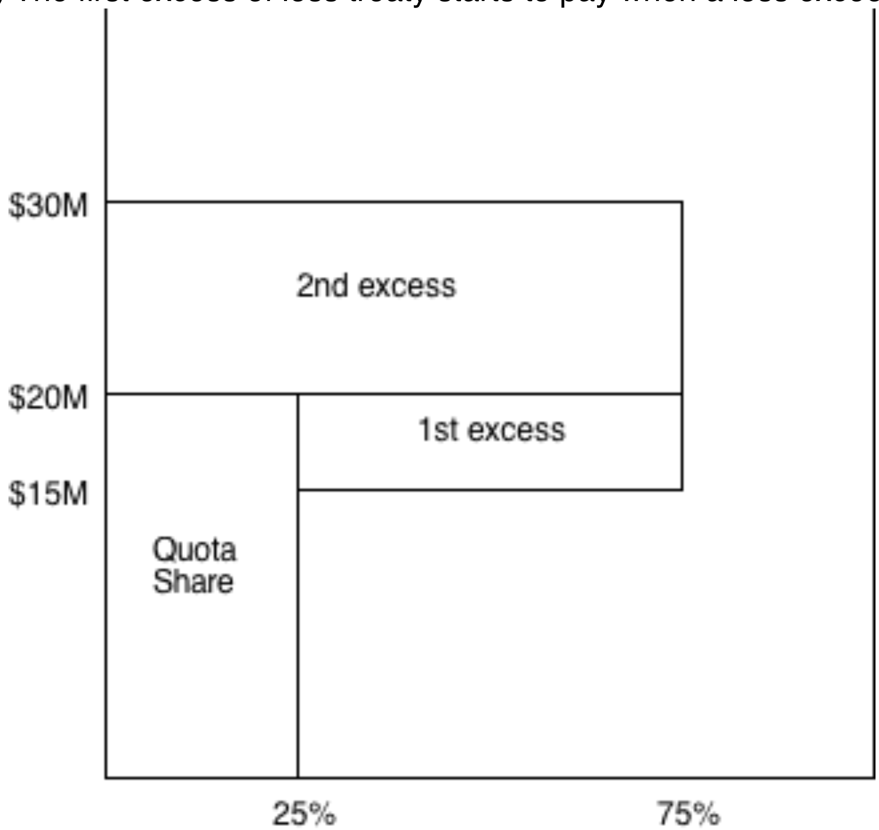
11. The actuary should do all of the following:
a. determine appropriate reliance on experts;
b. have a basic understanding of the model;
c. evaluate whether the model is appropriate for the intended application;
d. determine that appropriate validation has occurred; and
e. determine the appropriate use of the model.
Comment: See page 116 of Grossi and Kunreuther.
www.actuarialstandardsboard.org/pdf/asops/asop038_155.pdf
"The actuary's level of effort in understanding and evaluating a model should be consistent with the intended use of the model and its materiality to the results of the actuarial analysis."
"Computer simulation models are now widely used by actuaries for calculating expected losses due to hurricane and earthquake perils. The accuracy of these models is heavily dependent on the accuracy of meteorological, seismological, or engineering assumptions, areas clearly outside the expertise of most actuaries.
Although the development of this standard originated with the problem of providing accurate actuarial analysis of hurricane and earthquake exposure, the standard applies to any model."

12. Each of several model firm applied their model to a common portfolio of structures. In addition Federal Emergency Management Agency's HAZUS model was applied. (In each case, damage to the buildings was modeled, without considering insurance coverage provisions.) For each model separately, exceedance probability curves were produced, as well as a symmetric 90% confidence intervals around the mean loss.
The results for the four different models were compared. A composite exceedance probability curve plus 90% confidence interval was produced.
Comment: See Section 4.5.2 of Grossi and Kunreuther.

13. Management could compare the Probable Maximum Loss to its surplus.
The majority of Slippery Rock's annual premium is used to pay for non-catastrophe losses and expenses. Thus for example, Slippery Rock might want to have surplus equal to the one-in-100 PML from catastrophes, which in this case is about $350 million.
Management could use this output to help review its reinsurance program.
For example, it might want to adjust the attachment point and/or limits of its catastrophic reinsurance treaty or treaties.
If for example, its combination of current surplus and affordable catastrophe reinsurance does not provide sufficient financial stability, Slippery Rock may have to write less business, particularly in areas exposed to catastrophes in which it has a high market penetration. (More detailed output from the model, and additional trial runs of the model would help determine where writing less business would do the most good.)
Comment: There are many possible full credit answers.

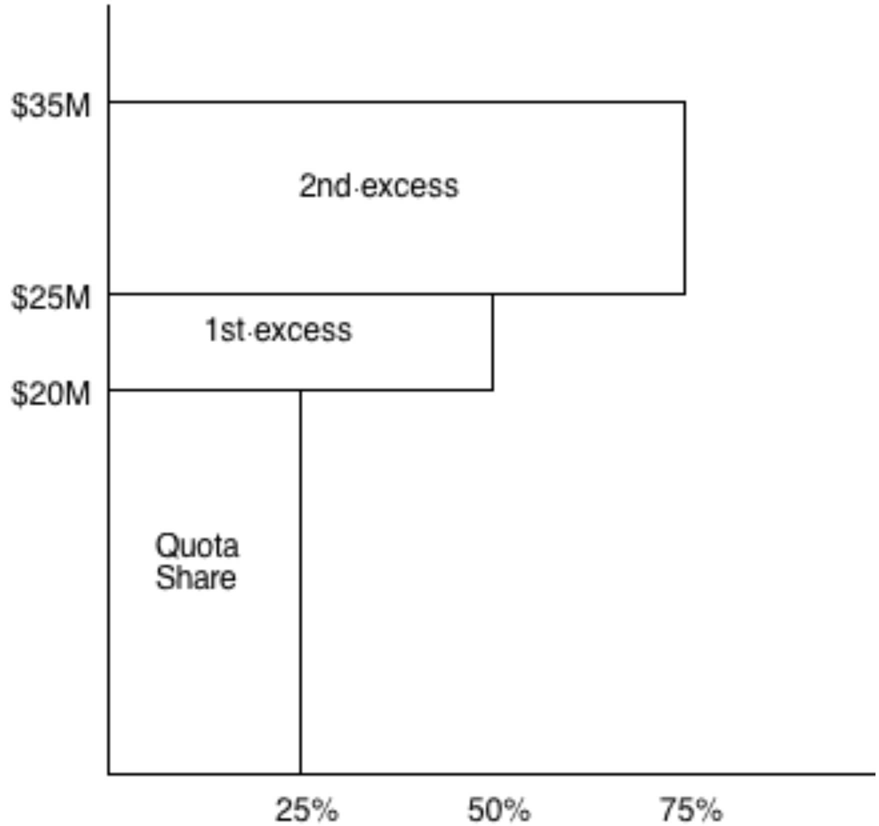
14. The loss module translates estimates of physical damage to a building, from the vulnerability module, into monetary costs to repair and replace damaged parts of the building. Consideration is also included of inspection costs, setup costs, and debris removal. Then the monetary costs are converted to insurance losses based on the coverage provisions of the insurance policy.
Comment: See Section 3.5 of Grossi and Kunreuther.

15. (a) The first excess of loss treaty starts to pay when a loss exceeds $15 million.



For example, for a loss of $18M, the quota share treaty pays: (25%)(18M) = $4.5M,
the first excess treaty pays: (50%)(18 - 15) = $1.5M, and the insured retains $12M.

(b) For loss of size \$20M, the insurer retain \$15M; the first excess of loss treaty starts to pay when a loss exceeds \$20 million.



For example, for a loss of \$24M, the quota share treaty pays: (25%)(20M) = \$5M,
the first excess treaty pays: (50%)(24 - 5 - 15) = \$2M, and the insured retains \$17M.
<u>Comment</u>: See page 125 of Grossi and Kunreuther.

16. Occurrence Exceedance Probability (OEP):
The OEP is the probability that at least one loss exceeds the specified loss amount.
For example, there might be a 10% chance that during a year at least one occurrence will exceed \$250 million.
Aggregate Exceedance Probability (AEP):
The AEP is the probability that the sum of all losses during a given period exceeds some amount.
For example, there might be a 0.5% chance that during a year the aggregate losses will exceed \$2 billion. In which case, a one-in-200 year Probable Maximum Loss (PML) would be \$2 billion.
Conditional Exceedance Probability (CEP):
The CEP is the probability that the amount on a single event exceeds a specified loss amount; this is the survival function of the size of event.
For example, given that there is an event, the probability that it exceeds size \$50 million might be 20%; in other words S(\$50 million) = 20%.
<u>Comment</u>: See the CAS errata to Section 2.4 of Grossi and Kunreuther.

17. 1. Earthquake recurrence rates: Since historical data in California is limited to 150 years, determining the long-term rate of earthquakes is critical. One of the most significant finds was that the models produced frequencies that were more than twice the historical record. This finding challenged the acceptability of the CEA model's frequency. It was ultimately determined to be valid by comparing that the model compared favorably to models produced by the California Division of Mines and Geology.

2. Damage Estimates: Model-based damage estimates are derived by associating a given level of ground shaking severity at a site with the vulnerability to shaking damage for a specific class of structure defined by age, type of construction, number of stories, etc. Prior to the Northridge Earthquake, earthquake damage curves were based on engineering opinions and judgments published by the Applied Technology Council (ATC). However, the model based its curves on over 50,000 claims from the Northridge quake. It was argued that the ATC-13 curves, which were in the public domain since 1985, should be relied upon as opposed to model-based proprietary curves, which were derived from principally one event. Testimony from representatives of the ATC itself supported the use of claims-based curves as the best available source of information for the link between shaking intensity and damage.

3. Underinsurance Factor: Model-based damage estimates are expressed as a percent of the building's value. Accordingly, if the value used is less than the replacement cost, damage and loss estimates are understated. In addition, the policy deductible is likely to be understated since it is typically defined as a percent of the policy limit. Because of inflation and lack of accurate valuation, the insurance to value ratio for most buildings is usually less than 1.0. In other words, most buildings are underinsured. Since the residential insurers in California did not readily have an estimate of the degree of underinsurance, consumer groups challenged the initial model assumptions of 13% derived from surveys of insurance actuaries in the state. They claimed that there was 0% underinsurance and that the properties were fully insured. Ultimately, a 6% underinsurance figure was agreed to and rates were lowered from the initial projections to reflect this compromise.

4. Demand Surge: The CEA testified that insurers estimated a 20% impact for demand surge following the Northridge earthquake. Since the vulnerability curves were based on Northridge data, the curves used were adjusted and initially reduced by 20% to eliminate the demand surge effect. Then the curves were increased by an adjustable factor, relating demand surge to the size of loss from each stochastic event in the model's probabilistic database. Although interveners argued that demand surge does not exist, the CEA actuarial group's testimony was accepted as reasonable even though little empirical data exists to support this assertion.

5. Policy Sublimits: Although a catastrophe model was used to establish the loss costs through such risk factors as location, soil conditions, age and type of structure, the model could not determine the contribution to losses from certain CEA policy features such as sublimits on masonry chimney damage, walkways, awnings, etc., because insurance claims data do not identify sources of loss from these categories. Hence, actuaries had to reduce the modeled loss costs to account for the specific CEA policy sublimits which were not reflected in the claims data used in the damage estimates produced by the model.

6. Rating Plan-Deviation: The CEA capped rates in two territories because of affordability issues and spread the capped costs to other territories. The commissioner ruled that a rating plan does not have to base premiums on risk in view of affordability issues, and that the plan was actuarially sound and not unfairly discriminatory.

7. Retrofit discount: The CEA offered discounts for three mitigation measures: bolting walls to the house foundation, cripple wall bracing, and water heater tie-down. Based on conversations with structural engineers, the CEA concluded that these measures would reduce losses; the statutory minimum of 5% was used as the discount. The Commissioner ruled the discount was appropriate, even though there was no empirical scientific guidance on the loss reduction.

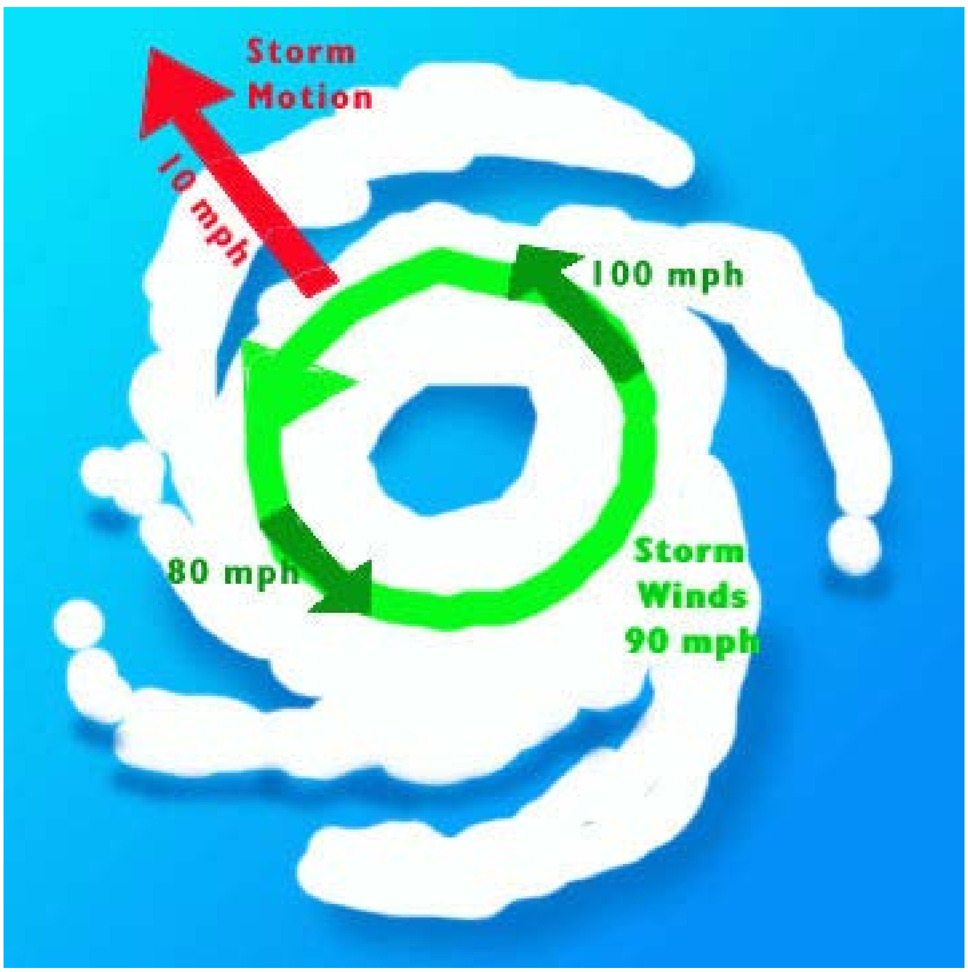
8. Changing Deductibles and Coverage Limits: The CEA proposed a 15% deductible, a $5000 limit on contents and $1500 limit on additional living expenses. Testimony supported that the insurance costs would be halved from what they were with the previous 10% deductible with much higher limits for contents and additional living expenses.
Comment: See Section 5.5.2 in Grossi and Kunreuther.

18. Need a large expanse of warm ocean water (generally above 80 degrees Fahrenheit).
Also need the absence of vertical shear (winds that change appreciably in either magnitude or direction with height.)
The further from the equator, the less the chance of sufficiently warm water and the greater the likelihood of vertical shear. However, hurricanes will not form too near the equator because of the absence of the Coriolis Force, which is required for the spiraling circulation of surface winds.
The most active hurricane months are when the oceans are warmest: August and September in the Northern Hemisphere and January and February in the Southern Hemisphere
Comment: See page 54 of Grossi and Kunreuther.
Coriolis force results from the rotation of the Earth, and causes moving objects in the Northern Hemisphere to be deflected to the right and moving objects in the Southern Hemisphere to be deflected to the left.

19. Examples of structural attributes of a building include: building materials, building codes, year of construction, and occupancy. Structural attributes help determine the performance of the building when hit by a catastrophe.
Where a building is located determines the forces it will be subjected to by a given catastrophe.
For example, in the case of earthquake: distance from an earthquake fault, and local soil conditions.
For example, in the case of hurricanes: distance from the coast, surface roughness and topography.
Comment: See Section 5.3.2 of Grossi and Kunreuther.

20. If for example, the damage ratio for a particular building class is underestimated in the model, then the estimated losses will be underestimated for all members of that class.
For a group of buildings all the same distance from an earthquake, if the attenuation model is incorrect, the errors of the damage for these buildings are likely to be in the same direction.
Comment: See page 131 of Grossi and Kunreuther.

21. The category of the hurricane; for example all else being equal a category 4 will have higher winds than a category 3.
Distance from the eye of hurricane; all else being equal the closer the eye of the hurricane passes to the site, the higher the maximum wind speed.
Different hurricanes have different sizes; some hurricanes spread over much wider areas.
In general in the Northern Hemisphere, the strongest winds in a hurricane are found on the right side of the storm because the motion of the hurricane also contributes to its swirling winds. For example, if the storm is moving north, sites to the east of the storm track will experience higher winds than those to the west of the storm track, all else being equal.
A hurricane usually loses strength as it moves over colder rather than warmer water.
A hurricane loses strength as it moves over land rather than water.
Differences in surface terrain can have a significant effect on wind speeds; the addition of obstacles, such as buildings or trees, degrades wind speed.
Comment: See page 57 in Grossi and Kunreuther.
If a storm stays in one area for a long time, it will do more damage than if it passes over an area quickly. Hurricanes can spawn local tornadoes with very high wind speeds.
Why in the Northern Hemisphere the strongest winds are to the right of the path of the hurricane:



22. (a) The expected value of losses for each of the portfolios are each equal to $8 million.
Portfolios 2 and 3 are positively correlated with each other.
Portfolio 1 is negatively correlated with each of portfolios 2 and 3.
So we want to keep portfolio #1, and eliminate one of the others.
Portfolio #2 has a larger standard deviation than portfolio #3.
Thus to reduce its risk, the insurer should eliminate **portfolio #2**.
Comment: Similar to 8, 11/11, Q.6.
Since it is negatively correlated with the others, writing more of portfolio #1 and less of the other two would reduce the insurer's risk.
Purchasing (more) catastrophe reinsurance would also reduce the insurer's risk, net of reinsurance.
Here the means of the portfolios are equal as in Table 6.2 of Grossi and Kunreuther.
The actuary is interested in the heaviness of the righthand tail of the aggregate loss distribution.
Here the coefficient of variation (standard deviation divided by the mean) would be one measure of how heavy is the righthand tail of the aggregate loss distribution; the larger the CV the heavier the righthand tail and the more likely an usually large annual aggregate loss.

23. One would run the model against a large number of possible hurricanes and Eureka Insurance Company's current book of exposures.
For each hurricane one would see whether Eureka's aggregate losses were greater than $20 million. If so, Colossal would pay 90% of the covered layer.
For example, if Eureka suffers $63 million in losses, then Colossal pays:
(90%)($43 million) = $38.7 million.
If instead Eureka suffers $200 million in losses, then Colossal pays:
(90%)($180 million) = $162 million.
Using the probability of each large hurricane, we could get a distribution of the annual amounts paid by Colossal. For example, the hurricane with $63 million in losses might have a probability of 0.01% per year. Then there would be a 0.01% probability of Colossal paying $38.7 million.
This distribution would help to determine a risk load.
To get the expected annual losses paid by Colossal, we would sum the product of the losses paid by Colossal for each large hurricane, by the probability of such a hurricane.
Comment: Under a reinstatement clause, when the amount of reinsurance coverage provided under a treaty is reduced by the payment of a reinsurance loss as the result of one catastrophe, the reinsurance cover is automatically reinstated usually by the payment of a reinstatement premium. Thus taking into account the possibility of more than one large hurricane in a year, one would have to consider any reinstatement clause.

24. For a typical structure in a class, one needs to establish a damage function. A damage function is an equation that relates the expected structural damage state of the entire building to the intensity of the catastrophic event. For financial analysis, building damage is expressed as a damage ratio, the ratio of repair costs to the replacement cost of the building.
Comment: See Section 3.4.2 of Grossi and Kunreuther.

25. (a) Catastrophe models are complex products that require specialized expertise to evaluate thoroughly and, for competitive reasons, modeling firms usually want to protect proprietary aspects of their models. This creates difficulties in reviewing models, particularly true for states with sunshine laws that require government documents be publicly available. Differences in model assumptions can cause loss results to vary considerably between modeling firms. Therefore regulators have faced difficulty in both understanding the models and being able to get enough information to properly evaluate the models.
(b) Florida has developed a review process by which modeling firms must show that their products meet technical criteria. The Florida Commission on Hurricane Loss Projection Methodology (FCHLPM) consists of independent experts.
To be certified for use in establishing residential rates, a catastrophe model undergoes a rigorous yearly review by FCHLPM. A professional team conducts an onsite audit; the team consists of a statistician, actuary, computer scientist, civil engineer, and meteorologist.
As part of the certification process of a model, the modeling firm must submit an exceedance probability curve for a specified portfolio of residential structures in Florida.
If an insurer submits a rate filing that relies on one or more of the approved catastrophe models, then the findings of the FCHLPM are admissible as evidence at any rate hearing.
Comment: See pages 80 and 107 of Grossi and Kunreuther.

26. The hazard module estimates the probability that the physical components of the hazard (wind speed or ground acceleration) will have various intensities.
One has to specify the likely locations of potential future events.
(For example, earthquake faults.)
Then one has to specify the frequency of occurrence of these events.
(For example, the return time for earthquakes of a given magnitude for a given fault.)
Next one has to parametrize the severity at the hazard source.
(For example, for hurricanes: central barometric pressure, forward speed, radius of maximum winds, and track angle at landfall.)
Finally, one must estimate the parameters of the hazard at the sites of buildings.
(For example, for earthquakes one can use attenuation equations. For example, for hurricanes one simulates the storm's movement along a track, and estimates wind speeds for building sites near that track.)
Comment: See Section 3.2 of Grossi and Kunreuther.

27. (a) The one-in-200 year event corresponds to a probability of 0.005. Reading off the graph, the 5th percentile is about $2 billion and the 95th percentile is about $8 billion.
Thus a 90% confidence interval for the one-in-200 year event is about **$2 to $8 billion**.
(b) Reading off the graph, at $10 billion the 5th percentile is about 0.0006
and the 95th percentile is about 0.004.
Thus a 90% confidence interval for the return time for $10 billion is about **250 to 1700 years**.
Comment: See Figure 4.5 in Grossi and Kunreuther.

28. One has to determine the portfolio of buildings (and contents) that can be damaged by the catastrophe being modeled. (For the entire insurance industry or for an individual insurer.) One would want detailed information about each building: location, value, occupancy, construction, etc. One would also want to know the line of business (residential, commercial, industrial) and details of the insurance coverage provided.
Comment: See Section 3.3 in Grossi and Kunreuther.
A large and valuable building may be subjected to an onsite inspection in order to get details about its construction.

29. In the output of the catastrophe model, there will be zipcodes within the state with the smallest current contribution to the large events that help to determine its current one-in-200 year PML. Sunshine can rerun the model and see what the effect would be of adding business in those zipcodes. Hopefully the one-in-200 year PML will not increase significantly. Sunshine should be able to figure out some types of homes in some zipcodes that it can try to write as new business. (Trial runs of the cat model may also help Sunshine identify zipcodes and/or types of homes that if it does not renew will significantly lower its PML.)
Alternately, Sunshine can use the model to investigate the effect of writing more business on its one-in-200 year PML, in order to determine how much (more) catastrophe reinsurance it would need to buy, so that its surplus would be equal to its one-in-200 year PML net of reinsurance.

30. For each event, divide the loss up between the three parties.

Event	Size ($ million)	Insurer		Reinsurer
1	250	100	150	0
2	500	100	300 + (20%)(100) = 320	80
3	1000	100	300 + (20%)(600) = 420	480

For the owner, 17% chance of retaining \$100M and 83% chance of paying nothing.
Mean is: (17%)(100) = \$17 million.
Variance is: $(17\%)(100^2) - 17^2 = 1411$. Standard Deviation is: \$37.6 million.
For the insurer, 10% chance of paying 150M, 5% chance of paying 320M, 2% chance of paying 420M, and 83% chance of paying nothing.
Mean is: (10%)(150) + (5%)(320) + (2%)(420) = \$39.4 million.
Variance is: $(10\%)(150^2) + (5\%)(320^2) + (2\%)(420^2) - 39.4^2 = 9346$.
Standard Deviation is: \$96.7 million.
For the reinsurer, 5% chance of paying 80M, 2% chance of paying 480M, and 93% chance of paying nothing.
Mean is: (5%)(80) + (2%)(480) = \$13.6 million.
Variance is: $(5\%)(80^2) + (2\%)(480^2) - 13.6^2 = 4743$. Standard Deviation is: \$68.9 million.

31. The magnitude of the earthquake; all else being equal, the larger the magnitude the more the ground motion.
The type of earthquake. (Earthquakes with thrust and reserve faulting mechanisms are in general observed to produce higher levels of ground motion than with strike-slip and normal faulting mechanisms.)
The distance from the earthquake to the site; all else being equal, the further the distance the less the ground motion.
The characteristics of the intervening geological materials determine the attenuation of seismic waves. (Certain geological materials will decrease the amplitude of seismic waves more than others.)
The local soil conditions at the site. (For example, soft soil materials that lie within a large bowl-like structure of underlying bedrock characterize can trap seismic waves and create very complex amplification and deamplification patterns for low frequency ground motions.)
Comment: See page 56 of Grossi and Kunreuther.
High frequency waves attenuate faster than low frequency waves.

32. Catastrophe models support this principle by providing increased accuracy in projecting expected future cost of a risk transfer compared to the prior methods used. Catastrophe models provide better estimates of the effect on losses of location, structural attributes, occupancy, etc. This enables better classification of risks and thus increased equity in rates between insureds. Catastrophe models provide more accurate estimates of the expected value of all future costs associated with an individual risk transfer.
Comment: See Section 5.2 of Grossi and Kunreuther, where they refer to the attachment of the CAS Principles of Ratemaking to the former Actuarial Standard of Practice number 9.

33. Prob[Total Loss > n z + A] < p,
where p is selected probability of insolvency such as 1%,
n is the number of policies written,
z is the average premium per policy,
and A is the surplus.
Comment: See page 38 of Grossi and Kunreuther.
Actually, it should be: Prob[Total Losses + Total Expenses > Premium + Surplus] < p.

34. 1. What is the Average Annual Loss?
This is used to help set the premium rate.
2. What is the likelihood that the insurer may become insolvent?
The insurer wants the probability of insolvency to be small.
It is important to adequately model the righthand tail of the aggregate exceedance curve.
Comment: See page 125 of Grossi and Kunreuther.

35. 1. Rates established by the Authority shall be actuarially sound so as to not be excessive inadequate, or unfairly discriminatory. Rates shall be based on the best available scientific data. Rates shall be equivalent for equivalent risks. Factors the Board shall consider in adopting rates include but are not limited to: location, soil type, construction type, and age of dwelling.
2. If scientific information is used, such information must be consistent with available geophysical data and state of the art knowledge.
3. Scientific information that is used to establish different rates between the most populous rating territories in northern and southern California cannot be used unless that information is analyzed and approved by experts.
4. The legislature does not intend to mandate a uniform statewide flat rate for residential policies.
5. Rates established shall not be adjusted to provide rates lower than are justified for classifications of high risk of loss or higher than are justified for classifications of low risk of loss.
6. Policyholders who have retrofitted homes to withstand earthquake shake damage shall receive a 5% premium discount, as long as it is determined to be actuarially sound.
<u>Comment</u>: See page 110 of Grossi and Kunreuther.

36. Law of large numbers: For the average of independent and identically distributed random variables, the sample mean converges to the expected value as the number of variables increases. As applied to insurance, the coefficient of variation of the aggregate loss should decrease as the insurer writes more similar insureds. This is what we would expect for example when writing private passenger auto liability insurance.
Losses from catastrophes do not follow the law of large numbers since they are not independent.
For example, a single hurricane can destroy thousands of homes written by a single insurer. (Writing homes that are spread far apart geographically would decrease this possibility.)
<u>Comment</u>: See page 37 in Grossi and Kunreuther.

37. For convenience, do all calculations in \$100 million.
For Insurer A, for hurricanes prior to reinsurance:
mean = (90%)(0) + (4%)(1) + (2%)(2) + (2%)(3) + (1%)(4) + (1%)(5) = 0.23.
2nd moment = (90%)(0^2) + (4%)(1^2) + (2%)(2^2) + (2%)(3^2) + (1%)(4^2) + (1%)(5^2) = 0.71.
Variance = 0.71 - 0.23^2 = 0.6571.
Thus including non-hurricanes, the mean is: 0.90 + 0.23 = 1.13,
and the variance is: 0.6571 + 0.07^2 = 0.662.
CV = $\sqrt{0.662}$ / 1.13 = 0.720.

For Insurer A, for hurricanes after reinsurance:
mean = (90%)(0) + (10%)(1) = 0.1.
2nd moment = (90%)(0^2) + (10%)(1^2) = 0.1.
Variance = 0.1 - 0.1^2 = 0.09.
Thus including non-hurricanes, the mean is: 0.90 + 0.1 = 1,
and the variance is: 0.09 + 0.07^2 = 0.0949.
CV = $\sqrt{0.0949}$ / 1 = 0.308.

For Insurer B, for hurricanes prior to reinsurance:
mean = (90%)(0) + (4%)(1) + (2%)(2) + (1%)(3) + (1%)(4) + (2%)(5) = 0.25.
2nd moment = (90%)(0^2) + (4%)(1^2) + (2%)(2^2) + (1%)(3^2) + (1%)(4^2) + (2%)(5^2) = 0.87.
Variance = 0.87 - 0.25^2 = 0.8075.
Thus including non-hurricanes, the mean is: 0.90 + 0.25 = 1.15,
and the variance is: 0.8075 + 0.07^2 = 0.8124.
CV = $\sqrt{0.8124}$ / 1.15 = 0.784.

For Insurer B, for hurricanes after reinsurance:
mean = (90%)(0) + (10%)(1) = 0.1.
2nd moment = (90%)(0^2) + (10%)(1^2) = 0.1.
Variance = 0.1 - 0.1^2 = 0.09.
Thus including non-hurricanes, the mean is: 0.90 + 0.1 = 1,
and the variance is: 0.09 + 0.07^2 = 0.0949.
CV = $\sqrt{0.0949}$ / 1 = 0.308.

Ceded from Insurer A:
mean = (94%)(0) + (2%)(1) + (2%)(2) + (1%)(3) + (1%)(4) = 0.13.
2nd moment = (94%)(0^2) + (2%)(1^2) + (2%)(2^2) + (1%)(3^2) + (1%)(4^2) = 0.35.
$1 + CV^2 = E[X^2] / E[X]^2 = 0.35 / 0.13^2 = 20.71.$ ⇒ CV = 4.43.

Ceded from Insurer B:
mean = (94%)(0) + (2%)(1) + (1%)(2) + (1%)(3) + (2%)(4) = 0.15.
2nd moment = (94%)(0^2) + (2%)(1^2) + (1%)(2^2) + (1%)(3^2) + (2%)(4^2) = 0.47.
$1 + CV^2 = E[X^2] / E[X]^2 = 0.47 / 0.15^2 = 20.89.$ ⇒ CV = 4.46.

Ceded from Insurer A plus Insurer B:

Hurricane	Insurer A	Insurer B	Ceded A	Ceded B	Ceded Both
1	1	2	0	1	1
2	2	5	1	4	5
3	3	4	2	3	5
4	4	5	3	4	7
5	5	1	4	0	4
6	1	2	0	1	1
7	2	1	1	0	1
8	3	1	2	0	2
9	1	1	0	0	0
10	1	3	0	2	2

mean = (91%)(0) + (3%)(1) + (2%)(2) + (1%)(4) + (2%)(5) + (1%)(7) = 0.28.
2nd moment = (91%)(0^2) + (3%)(1^2) + (2%)(2^2) + (1%)(4^2) + (2%)(5^2) + (1%)(7^2) = 1.26.
$1 + CV^2 = E[X^2] / E[X]^2 = 1.26 / 0.28^2 = 16.07.$ ⇒ CV = 3.88.

Comment: The losses for each insurer from each hurricane differ mostly due to the location of their insured homes within the state, but also due to the value and other characteristics of those homes.
Each insurer decreased its coefficient of variation by buying reinsurance.
For the reinsurer, its coefficient of variation for writing both cat treaties is less than for either one separately. The reinsurer could get more diversification by writing treaties for many different insurers who each wrote in different states or countries.

38. For catastrophe modeling of earthquakes one uses an attenuation equation, which describes mathematically the rate at which the amplitude of the seismic waves decreases as the wave propagates outward from the source of a rupture.
A typical attenuation equation: Y = F(f, M, r, Source, Site),
where Y is ground motion amplitude for frequency f, M is the magnitude of the earthquake, and r is the distance from the source to the site. The term Source reflects the rupture mechanisms of the earthquake. The term Site reflects the effects of soil conditions at the site.
High frequency waves decay more rapidly. The rate of decay is a function of the types of ground materials between the source and the site.
Comment: See page 56 and Figure 3.6 in Grossi and Kunreuther.
A seismic wave loses energy as it propagates through the earth, referred to as attenuation. Attenuation is tied in to the dispersion of the seismic energy with the distance. There are two types of dissipated energy: geometric dispersion caused by distribution of the seismic energy to greater volumes, and dispersion as heat.
Quoting from the U.S. Geological Survey website:
"Geophysical attenuation models are mathematical descriptions of how seismic waves ought to attenuate in the earth's crust, given the known properties of the crust. Portions of these mathematical descriptions can be adapted to be used in attenuation relations.
Attenuation relations present the results of analyzing strong motion data in showing how large the ground motions are expected to be for a certain earthquake magnitude and a certain distance from the earthquake.
Usually the attenuation relations are obtained by a statistical process called regression. Given a specified mathematical equation, regression determines parameters for that equation. In some cases, regression is used to determine the remaining parameters when the other parameters are given by geophysical attenuation models.
Then, given a magnitude, a distance, and a geologic site condition, the equation given the average value of the ground motion expected.
For a future earthquake, the actual ground motion will not be that average value, but rather a value in some uncertainty range around that average value. The regression also gives an estimate of that uncertainty range. The adjustment for geological site condition is sometimes determined by regression, but also sometimes determined by physical models of the soil column effect."

39. $\text{Log}_{10}(N) = a - b\ M$, or $N = 10^a / 10^{bM}$.
Thus for each increase of one in magnitude the number of earthquakes goes down by $10^b = 10^{1.2}$.
Thus the annual average frequency of an earthquakes of magnitude more than 6 is:
$0.7 / 10^{(2)(1.2)} = 0.002787$.
Thus the average return time is: 1/0.002787 = **359 years**.

40. 1. Regulatory Acceptance
"Proprietary sophisticated models create a problem for regulators who are unlikely to have the technical expertise to judge the reasonableness of the inputs, assumptions and outputs. Some states, such as Florida, have created independent commissions consisting of technical experts who certify models for use in insurance rate-setting situations in Florida. However, the State insurance commissioner has publicly criticized all models as biased a favor of the insurers.
In California, the Insurance Commission relied on the rate hearing process with experts provided by the interveners as well as by the Department of Insurance to examine model details and assumptions. However, regulators have a dual responsibility in setting rates. Rates need to be acceptable and affordable to the general public, but also actuarially sound to preserve the financial integrity of the insurers."
2. Public Acceptance
"As expected public acceptance of models has been low, principally because their use resulted in substantial increases in wind or earthquake rates. No one likes a rate increase. The problem is that previous rate making approaches based on historical experience fail to capture potential earthquake severity and frequency of these loss events. Rate estimates from models are not precise due to the uncertainty in the science, but they provide considerably more insight than extrapolations based on past loss experience."
3. Actuarial Acceptance
"Rate filings are usually the responsibility of a casualty actuary, who needs to comply with actuarial practice and principles. The catastrophe model is a tool that can be be used by an actuary in meeting his/her obligation to determine the fair and equitable rates to charge an insured. Since the models are outside the actuary's usual professional expertise, it is necessary for them to become familiar with the model components.
More recently, the Actuarial Standards Board has published Standard of Practice No. 38 that requires actuaries to (a) determine appropriate reliance on experts, (b) have a basic understanding of the model, (c) evaluate whether the model is appropriate for the intended application, (d) determine that appropriate model validation has occurred, and
(e) determine the appropriate use of the model."
4. Model-to-Model Variance
"Given the inherent uncertainty in catastrophe loss estimates, significant differences in loss estimates from one model to another do occur. Often models are dismissed for this reason, with claims that model are good only if they agree with each other. However, models are based on inputs from varying scientific data and engineering information, which may differ because of uncertainty in the understanding of hazards. The modeler is required to use one or more sub-models of hazard or severity defined by a reputable scientific researcher, which may result in different loss results. That is the inherent nature of the modeling process. Risk, and uncertainty in estimating the risk of loss, derives not only from the randomness of the event occurrence but also from the limits in knowledge and different interpretations by experts. It is unlikely that science will provide us all the answers, thus leading to continued differences in model results."
<u>Comment</u>: See pages 116-117 of Grossi and Kunreuther. No need to quote the reading.
In Florida, California, and other states have developed regulatory methods of allowing insurers to include "approved" models in rate filings.
Often insurers will rely upon the average of the results of two computer models from different vendors, thus reducing parameter risk.
There are actuaries who have spent many years working on catastrophe models, gaining a lot of expertise in the details of how the different modules work and fit together.

41. Risk reduction involves reducing the risk.
For example, a homeowner might install storm shutters or more securely attach the roof of his home in order to reduce the expected loss from a hurricane.
For example, a locality could implement well enforced updated building codes to reduce the expected losses from catastrophes.
Risk transfer involves transferring some of the risk to someone else.
For example, an insurer can buy reinsurance; the total expected loss remains the same, but the expected amount paid by the insurer as well as the variance have been decreased.
Comment: See page 39 of Grossi and Kunreuther.

42. The magnitude of the risk, its correlation with the existing portfolio, the highest acceptable price the insured is willing to pay, what is being insured, the location of a property, and any constraints (competitive or regulatory) on what can be charged.
Comment: See page 121 of Grossi and Kunreuther.

43. Location, frequency, and severity.
For example, earthquakes occur along fault lines, so one would want to determine all of the fault lines, and specify where an earthquake occurs along a given fault line.
Then we would want to specify how often earthquakes of a given magnitude occur along a given fault line or portion of a given fault line.
For example, we would need to specify the path of a given hurricane, how its strength varies as it moves along that path, and the probability of all of this occurring.

44. For example, assume for concreteness that the average return time is 200 years.
One model would be that we get an event at regular intervals of 200 years; this is not what we observe for major earthquakes.
Another model would be a Poisson Process; the chance of a major earthquake is 0.5% every year regardless of the history of past major earthquakes. This is not what we observe.
While geologist are not able to predict earthquakes, they have observed that as the time since the last major earthquake increases, the chance of a major earthquake also increases. (This is due to stress building up along a fault line.) So for example, in this case, if it has been 300 years since the last major earthquake the probability of a major earthquake next year is higher than if it had been only 100 years since the last major earthquake.
Comment: Some geologist are hopeful that In the future they will make progress on predicting earthquakes, while others feel that large earthquakes are inherently unpredictable.

45. 1. “The hearing highlighted the significant disagreement among earth scientists on frequency estimates, maximum magnitude and time dependent calculations. Given the high level of seismic research undertaken by academics and researchers in government agencies such as the USGS, and the inherent uncertainty in the estimation process, disagreements are likely to persist. The challenge is to select credible and representative research, and in some cases to include more than one methodology in the catastrophe models.”
2. “Insurance claims data from catastrophic loss are by nature very limited. Yet, it is the single best source from which to estimate future losses. Insurers need to capture and preserve loss data and portfolio exposures for each loss event. Because of the legal and commercial aspects, release of this data to third parties needs to be carefully managed to protect the insurance companies' interests.”
3. “Mitigating future catastrophic losses via structural retrofits, with commensurate insurance premium reductions, is strongly desired by politicians and the public. Models have the ability to quantify the various wind or earthquake mitigation applications, but are hampered by the lack of detailed loss data, since insurers typically do not distinguish losses by structural component, such as roof, chimney, foundations, or non-load bearing walls. The states of Florida, California, and Hawaii are encouraging research and studies to assist in estimating such benefits. Results of these efforts will undoubtedly find their way into model analysis in the development of actuarially sound retrofit discount programs.”
4. Demand surge increases repair and building costs after a major catastrophe; one wishes to better quantify this effect.
“Increases in settlement costs following a major catastrophe have been noted in Hurricane Andrew, the Northridge Earthquake and Typhoon Mirielle (1991) in Japan. Actuaries estimated a 20% increase in Northridge with similar levels for the hurricane events. Unfortunately, little research has been conducted to identify the sources of these losses and what size events would evidence such behavior.”
Comment: See page 115 of Grossi and Kunreuther. No need to quote the reading.

46. Aggregate Exceedance Probability (AEP) is the probability that the sum of all losses during a year exceeds some amount.
Occurrence Exceedance Probability (OEP) is the probability that during a year at least one event exceeds the specified loss amount.
If for example in a year at least one event exceeds \$100 million, then the aggregate losses in a year must also exceed \$100 million. The total for a year is greater than or equal to the largest event for a year. Thus the AEP for a given return time is greater than the corresponding OEP.
We note that the percentage difference between the AEP and OEP gets less as the return time increases. This is due to the fact that when we have for example an event of size \$400 million, that very large event is almost certainly most of the aggregate loss for that year. If instead we have an event of size \$50 million, even if it is the largest event during a year, it is on average a smaller percent of the aggregate loss for that year.
Comment: Adapted from the CAS Case Study on Catastrophe Modeling.

47. C. The lower the central pressure in the eye of a hurricane, in other words the larger the difference from normal, the stronger the storm.
Comment: For an brief discussion of hurricanes, see for example http://www.ihc.fiu.edu/about_us/meteorology.htm

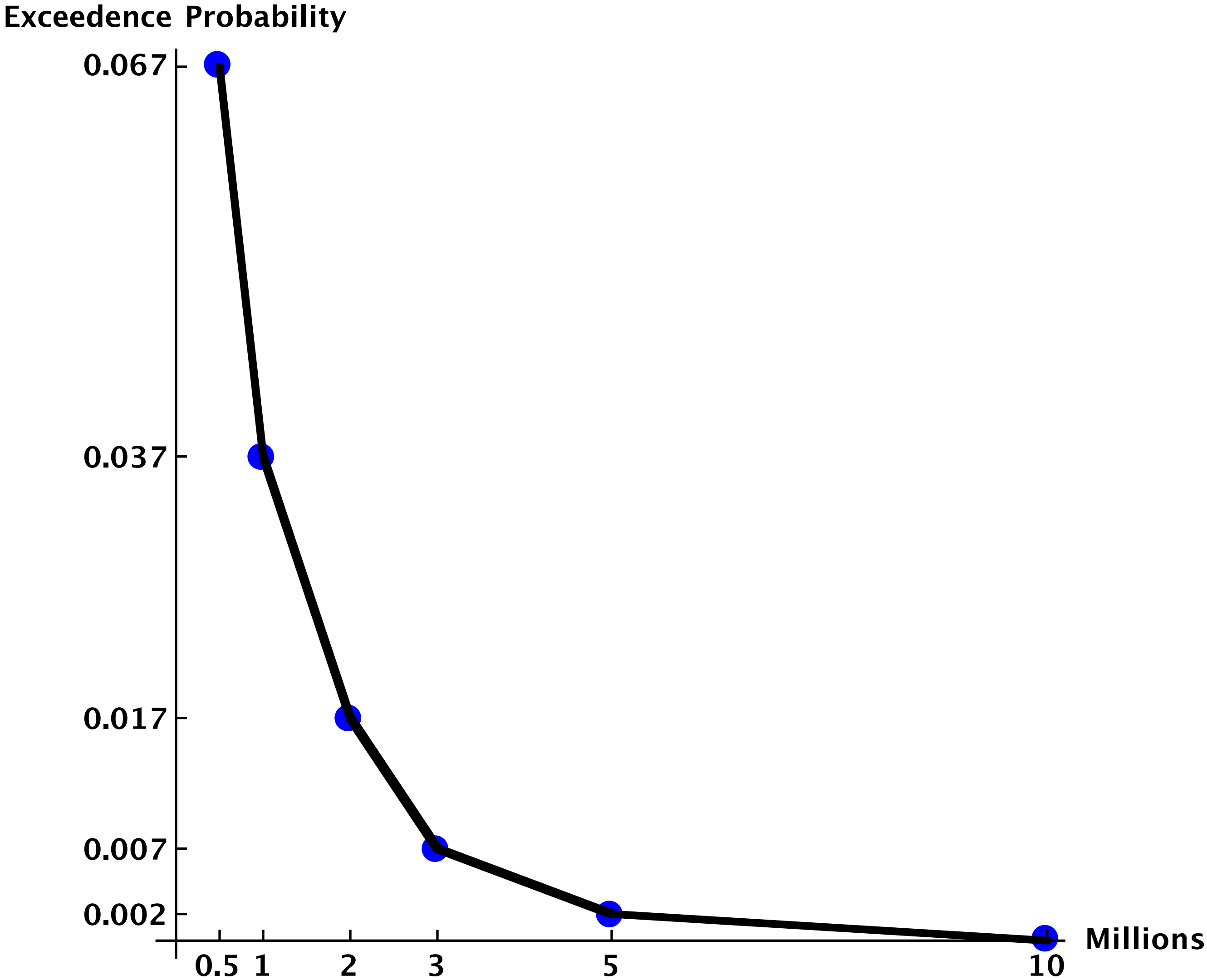
48. A 80% confidence interval would have 10% probability in each tail.
Sorting the estimates from smallest to largest: 395, 451, 462, 465, 478, 479, 481, 492, 495, 509, 510, 510, 518, 520, 522, 525, 535, 538, 542, 546.
An 80% confidence interval is from about 451 to 542.

Alternately, the smoothed empirical estimate of the 10^{th} percentile is (0.1)(20 + 1) = 2.1^{th} value from smallest to largest: (451)(0.9) + (462)(0.1) = 452.1.

The smoothed empirical estimate of the 90^{th} percentile is (0.9)(20 + 1) = 18.9^{th} value from smallest to largest: (542)(0.9) + (538)(0.1) = 541.6.
An 80% confidence interval is from about 452 to 542.
Comment: See Section 4.4.3 of Grossi and Kunreuther. If instead one had many more than 20 possible models, then it would be possible to get for example a 95% confidence interval.

49. (a) Since there is at most one event per year:
1 - (0.002 + 0.005 + 0.010 + 0.020 + 0.030) = **93.3%**.
(b) (1 million) {(0.002)(10) + (0.005)(5) + (0.010)(3) + (0.020)(2) + (0.030)(1)} = $**145,000**.
(c) Aggregate Exceedance Probability (AEP):
The AEP is the probability that the sum of all losses during a given period exceeds some amount.
Since there is at most one event per year:
Prob[Agg > 500K] = 3% + 2% + 1% + 0.5% + 0.2% = 6.7%.
Prob[Agg > 1 million] = 2% + 1% + 0.5% + 0.2% = 3.7%.
Prob[Agg > 2 million] = 1% + 0.5% + 0.2% = 1.7%.
Prob[Agg > 3 million] = 0.5% + 0.2% = 0.7%.
Prob[Agg > 5 million] = 0.2%.
Prob[Agg > 10 million] = 0.

(d) Aggregate Exceedance probability curve:



(e) Occurrence Exceedance Probability (OEP):
The OEP is the probability that at least one loss exceeds the specified loss amount.
Since there is at most one event per year, the OEP is the same as the AEP; the graph is the same.
(f) For the 1-in-100 year PML we want an Occurrence Exceedance Probability of 1%.
Linearly interpolating, on the graph of the OEP (which is the same as the graph of the AEP):
(0.3)(2) + (0.7)(3) = $**2.7 million**.
(g) For the 1-in-200 year PML we want an Occurrence Exceedance Probability of 0.5%.
Linearly interpolating, on the graph of the OEP (which is the same as the graph of the AEP):
(3/5)(3) + (2/5)(5) = $**3.8 million**.
Comment: Adapted from the CAS Case Study on Catastrophe Modeling.
The fact that not more than one event can occur each year makes things much simpler.

50. The damage is: (40%)($500,000) = $200,000.
If insured for $500,000, the deductible is: (15%)($500,000) = $75,000.
The insurer pays: $200,000 - $75,000 = $125,000.
If insured for $300,000, the deductible is: (15%)($300,000) = $45,000.
The insurer pays: $200,000 - $45,000 = $155,000.
Comment: See pages 72 and 73 of Grossi and Kunreuther. An example of why it is important to have accurate information on insurance to value when running a catastrophe model.

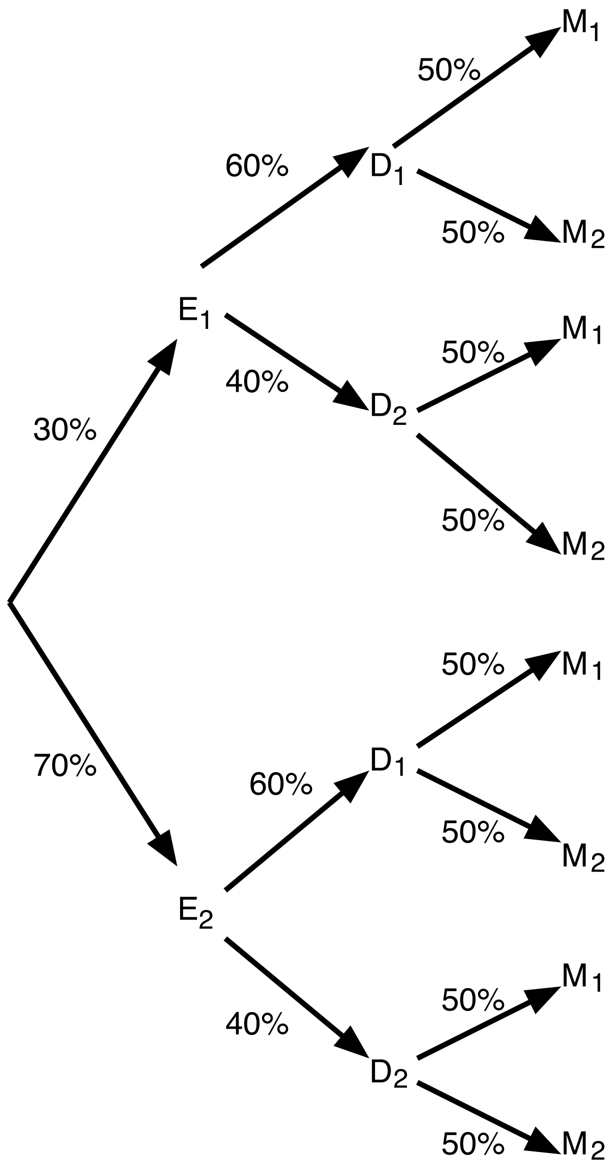
51. The cumulative probabilities are : 5%, 29%, 77%, 93%, and 100%.
Since 29% < 0.3361 ≤ 77%, the simulated damage is Moderate.
Comment: Not something I expect to be asked. See Table 4.1 in Grossi and Kunreuther.

52. The underground disposal of billions of barrels of wastewater from oil and gas wells is a relatively recent phenomena, producing a significantly increased hazard due to earthquakes over a portion of this state. A model from a decade ago would not have included this possibility. This is an example of how a catastrophe modeler must be diligent about updating the model for the latest information, including scientific findings. This also illustrates the parameter uncertainty inherent in catastrophe models; it is difficult to anticipate such phenomena as this, as well as quantify its future expected impact.
Comment: The U.S. Geological Survey released a report on human activity increasing the number of earthquakes "Incorporating Induced Seismicity in the 2014 United States National Seismic Hazard Model - Results of 2014 Workshop and Sensitivity Studies." Seventeen areas in eight states have seen more man-made earthquakes since 2009. "As part of the process of incorporating induced seismicity into the seismic hazard model, we evaluate the sensitivity of the seismic hazard from induced seismicity to five parts of the hazard model: (1) the earthquake catalog, (2) earthquake rates, (3) earthquake locations, (4) earthquake maximum magnitude, and (5) earthquake ground motions."

53. Expected amount paid by the insured: (2%)(12%) + (4%)(21%) + (5%)(67%) = **4.43%**.
Expected amount paid by the insurer: (1%)(22%) + (3%)(17%) + (5%)(12%) + (7%)(7%) + (9%)(4%) + (11%)(3%) + (13%)(1%) + (15%)(1%) = **2.79%**.
Comment: See Figure 6.5 in Grossi and Kunreuther.
4.43% + 2.79% = 7.22% = mean.

54. 1. High wind speeds
2. Intense rain
3. Projected missiles
4. High Water (flooding and storm surge)
Comment: See for example http://www.ihc.fiu.edu/about_us/hurricane_hazards.htm

55. Let E_1 be the first attenuation equation. Let D_1 be the first damage function.
Let M_1 be the first seismic map.



We have 8 different combinations of choices, corresponding to the different paths through the tree.
For a portfolio of proprieties, we could run each of these 8 different models.
Then we would have eight different values of for example S($1 billion) for the annual losses; each has a corresponding probability.
For example, combination E_1, D_1, M_1 has probability: (30%)(60%)(50%) = 9%.
While combination E_2, D_1, M_1 has probability: (70%)(60%)(50%) = 21%.
From these eight values and corresponding probabilities, we can construct a distribution of the estimated S($1 billion), illustrating the epistemic uncertainty of our estimate of S($1 billion).
Comment: See Figures 4.1 and 4.3 in Grossi and Kunreuther.

56. AAL = (2%)(50) + (2%)(100) + (1%)(300) = \$6 per \$1000 of value.
Second moment is: (2%)(50^2) + (2%)(100^2) + (1%)(300^2) = 1150.
Standard Deviation = $\sqrt{1150 - 6^2}$ = 33.38.
Ignoring expenses, the rate would be: 6 + 33.38/2 = **\$22.69** per \$1000 of value.
Comment: See the output in Tables 5.1 and 5.2 in Grossi and Kunreuther.

57. ● Relative elevation compared to the surrounding area (within a certain radius).
● Distance to the coast.
● Distance to rivers.
Comment: Not covered in the syllabus reading.

58. From the graph, the needed capital with the excess reinsurance is about 56M.
The ceded loss and ALAE are:

	Loss	Ceded Loss	ALAE	Ceded ALAE
Claim 1	\$6,000,000	\$1,000,000	\$900,000	(1/6)(\$900,000)
Claim 2	\$13,000,000	\$5,000,000	\$2,600,000	(5/13)(\$2,600,000)

Thus the total ceded loss and ALAE is: 1M + 5M + 0.15M + 1M = 7.15M.
Retained loss and ALAE is: 40M - 7.15M = 32.85M.
Reinsurance premium is: (17%)(60M) = 10.2M.
Underwriting profit is: 60M - 15M - 32.85M - 10.2M = 1.95M.
With excess reinsurance, return on capital is: 1.95/56 = **3.48%**.
From the graph, without reinsurance the 1-in-50 year Probable Maximum Loss is about 72M.
With the quota share the ceded premium is: (40%)(60M) = 24M.
The ceding commission is: (25%)(24M) = 6M.
For the 1-in-50 year Probable Maximum Loss, the insurer would have net losses of:
(72M)(60%) = 43.2M. Thus 43.2M is the needed capital with the quota share treaty.
For the given experience, the ceded losses and ALAE are: (40%)(40M) = 16M.
Underwriting profit is: 60M - 15M - (40M - 16M) - (24M - 6M) = 3M.
With quota share reinsurance, return on capital is: 3/43.2 = **6.94%**.
Comment: Similar to 8, 11/16 , Q.19.
Return on capital is not covered in the syllabus of this exam.
However, all you really need to know is that when we ignore investment income and income taxes, return on capital is: (underwriting profit) / capital.
For the current situation without reinsurance, the needed capital is about 72M.
The underwriting profit is: 60 - 15 - 40 = 5M.
Return on capital is: 5/72 = 6.94%.
One would not normally base such a decision on looking at only one year of past experience.

59. A one-in-25-year event is a label used for an event such that an event of at least the same size has a 4% probability of occurring each year. While the average time until the next such hurricane is 25 years, a hurricane this large or larger has a 4% chance of happening next year, a 4% chance for the year after, etc.
Comment: The one-in-25-year event represents the 96th percentile of the occurrence distribution.
Large earthquakes along a major fault do not have this independence from year to year.

60. Try to read the graph as precisely as possible:
10% probability of between \$5 million and \$8 million, 5% probability between 8 and 10 million, 4% chance of between \$10 and \$12 million, 2% chance of between \$12 million and \$14 million, 3% chance of between \$14 million and \$20 million, and a 1% chance of exceeding \$20 million.
Then the estimated pure premium is:
(10%)(6.5 - 5) + (5%)(4) + (4%)(6) + (2%)(8) + (3%)(12) + (1%)(15) = **\$1.26 million**.
Comment: The graph is taken from 8, 11/14, Q.24.
OEP is the probability that at least one event exceeds the specified loss amount in a year.
I have ignored the possibility of more than one large occurrence per year, which would involve taking into account reinstatement premiums as discussed in Clark.

61. A 90% confidence interval for the aggregate exceedance probability at \$200 million is from 0.0345 to 0.0370. (This leaves 10% probability in the tails including 0.0345 and 0.0370.)
Alternately, a 95% confidence interval for the aggregate exceedance probability at \$200 million is from 0.0345 to 0.0370. (This leaves 5% probability in the tails excluding 0.0345 and 0.0370.)
Alternately, a 95% confidence interval for the aggregate exceedance probability at \$200 million is from 0.0341 to 0.0377. (This leaves 5% probability in the tails including 0.0341 and 0.0377.)
Alternately, a 99% confidence interval for the aggregate exceedance probability at \$200 million is from 0.0341 to 0.0377. (This leaves 1% probability in the tails excluding 0.0341 and 0.0377.)
Alternately, a 99% confidence interval for the aggregate exceedance probability at \$200 million is from 0.0323 to 0.0392. (This leaves 1% probability in the tails including 0.0323 and 0.0392.)
Comment: See Section 4.4.3 of Grossi and Kunreuther.

62. Assume the given distribution is discrete.
Find where the discrete distribution of aggregate losses first exceeds 0.9926.
The distribution function at 127M is 99%.
The distribution function at 188M is: 99.5%.
Thus the simulated aggregate loss is **188 million**.
Alternately, assume the given distribution is continuous.
Linearly interpolate between 127M and 188M: (24/50)(127M) + (26/50)(188M) = **159 million**.
Comment: Similar to 8, 11/15, 22b.

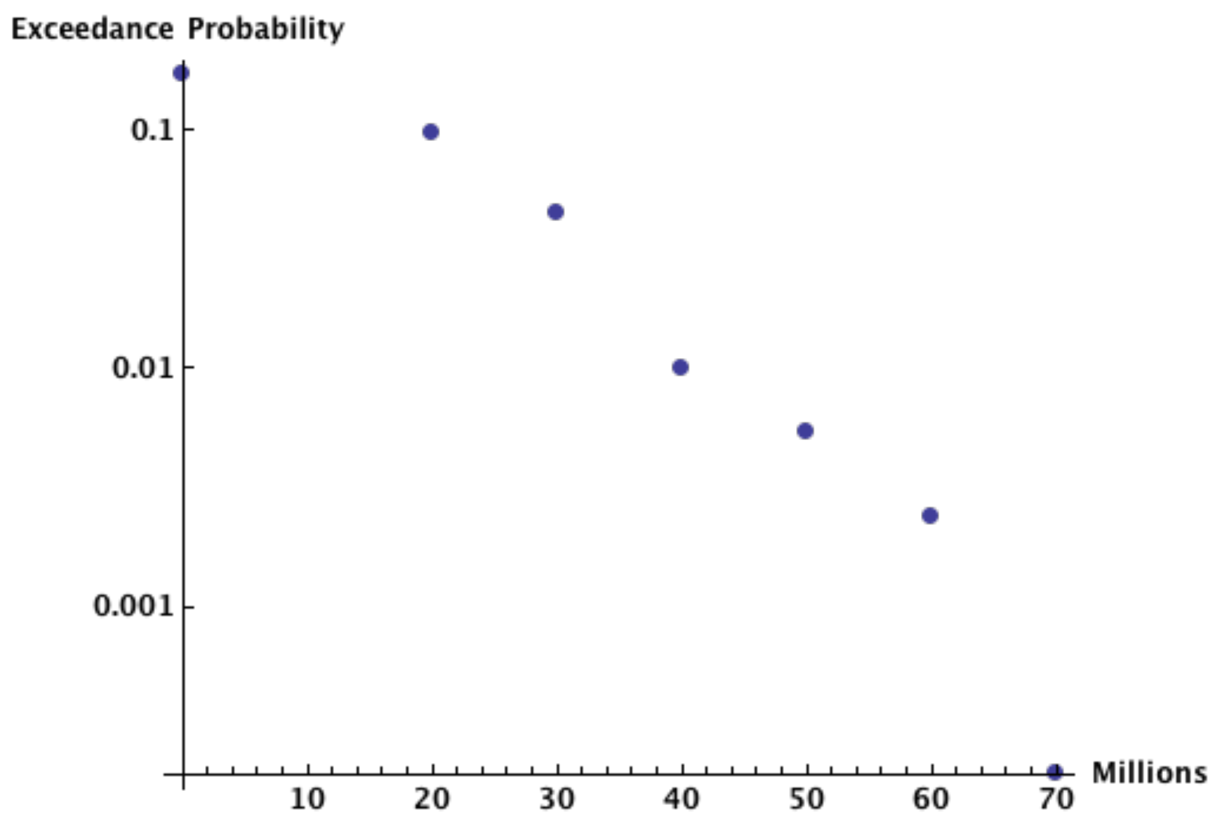
63. The insurer's 1-in-200 probable maximum ceded losses is: 180 - 100 = $80 million.
$670 + $80 = $750 million > $700 million. Thus there is a possibility that adding this treaty will violate the rule for required capital for the reinsurer.
The key is how correlated the losses of the insurer are with the current portfolio of the reinsurer. This would depend on among other things: geographical regions covered, catastrophe perils covered, etc.
For example, if the primary insurer writes mostly on the west coast of the United States and the reinsurer's current portfolio includes relatively little exposure there, then writing this treaty would diversify the reinsurer's portfolio and not significantly increase the 1-in-200 PML.
If for example the reinsurer is heavily exposed to tropical storms (hurricanes, etc.) and the primary insurer is not, then writing this treaty would diversify the reinsurer's portfolio and not significantly increase the 1-in-200 PML.
The reinsurer should run a catastrophe model both with and without this possible new treaty, and see what the effect is on the reinsurer's 1-in-200 PML. If with the new treaty the 1-in-200 PML exceeds $700 million, then the reinsurer should not participate in this new treaty.
Comment: Similar to 8, 11/18, Q.16b.
If capital requirements do not prevent the reinsurer from writing this treaty, then the potential profitability of this treaty compared to other possible new treaties would help determine whether to write this treaty.
Deciding whether or not to write a particular treaty, particularly a catastrophe treaty, is complicated and depends on many different items as well as judgment.
In the simplified situation in the question, we are only interested the capital requirement to survive a 1-200 event. The results of running the catastrophe model will depend on the treaty as well as the current book of business.
If for example, the 1-200 event from the proposed treaty were only 5 million, then no problem.
If for example, the 1-200 event from the proposed treaty were 50 million, if this treaty is not significantly correlated with current book of business, then there is probably not a problem.
If for example, the 1-200 event from the proposed treaty were 50 million, if this treaty is significantly correlated with current book of business, then there is probably a problem.

64. (a) This would affect the **Hazard Module**, which includes the possible earthquakes and their probabilities.
I assume that the inventory module already includes sufficient detail on the location and soil types of the buildings in the insurer's portfolio, so that one can know how far they are from the Bueller fault, and so that the damage from an earthquake can be modeled.
(b) This is **epistemic** uncertainty since it is parameter risk (or modeling risk) rather than the process risk inherent to the random nature of the catastrophe.
(c) ● A Logic Tree assigns weight to parameter alternatives, in this case based on the differing opinions of the experts on: the largest likely magnitude, the mean return time, and the precise location and extent of the fault. Ferris can choose various combinations of values of these different inputs, that are within the range of the different expert opinions; Ferris would also assign probabilities to each of these sets of inputs.
Then for each of the different possibilities from the Logic Tree, the cat model is run. Using each of the model outputs and its corresponding probability from the Logic Tree, a weighted average is calculated to get the mean result. The different model outputs and corresponding probabilities can also be used to get the range of variability in results, in this case due to the uncertainty related to the discovery of this new fault.
Logic Trees are relatively easy to document and communicate. However, they are not easy to use when there are a large number of possible scenarios, for example when there are many items varying in many possible ways.
● Simulation creates many randomly sampled alternatives from the probability distribution(s) of the parameter(s). (Here we would have to be careful to consider whether some items like mean return time and likely maximum magnitude are dependent variables.) Then for each such set of inputs, the cat model is run. The result is a large set of outputs, which can be used to get the mean result as well as quantify the variability.
Simulation can handle situations with a large number of possibilities. However, it is harder to document and communicate than Logic Trees. Simulation may appear to be a "black box" to non-actuaries. Simulation gives a better overall view of the uncertainty than Logic Trees, but is more computationally intensive.
Comment: Similar to 8, 11/18, Q.17.
For part (b):
Aleatory risk: we are rolling 2 six-sided dice and the result is random.
Epistemic risk: we are rolling an unknown number of six-sided dice; we are uncertain of the parameters / model.
For natural disasters there is always aleatory risk. For example, let us assume we have the best known and studied earthquake fault. In 2100 they now have an excellent model of the probabilities of earthquakes of different magnitudes over the coming year. Even if their model is perfect, we still have aleatory risk since there is only a probability distribution.
(Since no model of earthquakes is ever perfect, in reality we still have some epistemic risk.)
The other modules are the Vulnerability Module which relates to how susceptible different building types are to damage from a catastrophe, and the Loss Module which applies the provisions of the insurance policy to translate damage to insurance loss.

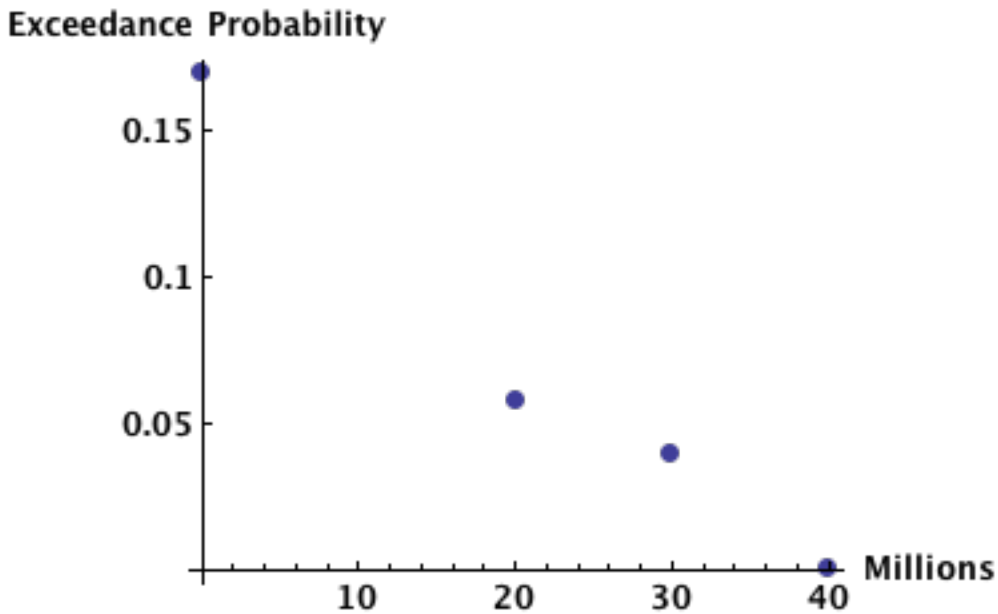
65. (a) S(20 million) = 1/20 = 5%.
S(40 million) = 1/50 = 2%.
We want S(x) = 1/25 = 4%.
Using linear interpolation, x = (2/3)(20) + (1/3)(40) = **$26.67** million.
(b) S(150 million) = 1/200 = 0.5%.
S(240 million) = 1/500 = 0.2%.
We want S(x) = 1/400 = 0.25%.
Using linear interpolation, x = (1/6)(150) + (5/6)(240) = **$225** million.
Comment: Similar to 8, 11/18, Q. 16a.
One should interpolate using the exceedance probabilities rather than the return periods.

66. (a) Probability of no loss: (0.92)(0.94)(0.96) = 83.02%.
Probability of 20,000,000 aggregate: (0.08)(0.94)(0.96) = 7.22%.
Probability of 30,000,000 aggregate: (0.92)(0.06)(0.96) = 5.30%
Probability of 40,000,000 aggregate: (0.92)(0.94)(0.04) = 3.46%
Probability of 20M + 30M = 50M aggregate: (0.08)(0.06)(0.96) = 0.46%
Probability of 20M + 40M = 60M aggregate: (0.08)(0.94)(0.04) = 0.30%
Probability of 30M + 40M = 70M aggregate: (0.92)(0.06)(0.04) = 0.22%
Probability of 20M + 30M + 40M = 90M aggregate: (0.08)(0.06)(0.04) = 0.02%
Probability[Aggregate > 90M] = 0.
Probability[Aggregate > 70M] = 0.02%.
Probability[Aggregate > 60M] = 0.02% + 0.22% = 0.24%.
Probability[Aggregate > 50M] = 0.24% + 0.30% = 0.54%.
Probability[Aggregate > 40M] = 0.54% + 0.46% = 1.00%.
Probability[Aggregate > 30M] = 1.00% + 3.46% = 4.46%.
Probability[Aggregate > 20M] = 4.46% + 5.30% = 9.76%.
Probability[Aggregate > 0M] = 9.76% + 7.22% = 16.98%.

(b) Aggregate Exceedance probability curve (with probability on a log scale):



(c) Occurrence Exceedance Probability (OEP):
The OEP is the probability that at least one loss exceeds the specified loss amount.
Prob[Maximum Occurrence > 0] = 1 - (92%)(94%)(96%) = 16.98%.
Prob[Maximum Occurrence > 20 million] = 1 - (94%)(96%) = 9.76%.
Prob[Maximum Occurrence > 30 million] = 4%.
Prob[Maximum Occurrence > 40 million] = 0%.



(d) For the 1-in-20 year PML we want an Occurrence Exceedance Probability of 5%.
Linearly interpolating on the graph of the OEP between (20, 9.76%) and (30, 4%):
(1/5.76)(20) + (4.76/5.76)(30) = $**28.3 million**.
Comment: Similar to 8, 11/15, Q.22a.

67. AAL = (3%)(100) + (2%)(200) = $7 per $1000 of value.
Second moment is: (3%)(100^2) + (2%)(200^2) = 1100.
Standard Deviation = $\sqrt{1100 - 7^2}$ = 32.42.
Ignoring expenses, the rate would be: 7 + 32.42/2 = **$23.21** per $1000 of value.

68. (a) The expected value of losses for each of the portfolios are each equal to $7 million.
Portfolios 1 and 2 are positively correlated with each other.
Portfolio 3 is negatively correlated with each of portfolios 1 and 2.
So we want to keep portfolio #3, and eliminate one of the others.
Portfolio #1 has a larger standard deviation than portfolio #2.
Thus to reduce its risk, the insurer should eliminate **portfolio #1**.
Comment: Similar to 8, 11/11, Q.6.

69. There is a 1% chance that the annual aggregate loss for Weasley Insurance is at least 2 billion.
There is a 1% chance that the annual aggregate loss for Granger Insurance is at least 2 billion.
In the 1% of years in which Weasley Insurance has at least 2 billion in loss, Granger has some loss, and thus the merged company has even more losses than just Weasley.
Therefore, the 100-year PML for the merged insurer is more than 2 billion.
For example, a year in which Granger had 2 billion in aggregate loss is unlikely to be a year in which Weasley has at least 2 billion in loss; thus the combined insurer has less than 4 billion.
Unless the annual losses of the two insurers are perfectly correlated, the 100-year PML of the merged insurer is less than 4 billion.
The PML for the merged insurer is between 2 billion and 4 billion.
Comment: In general, the probable maximum loss (PML) for two insurers combined is less than the sum of the individual PMLs.

70. Most likely to benefit: earthquake. Least likely to benefit: fire.
Earthquakes can be very severe catastrophes; large earthquakes on a fault are infrequent and occur at irregular intervals. Two or three decades of historical losses from earthquakes are insufficient to predict future expected losses.
While wildfires can be catastrophic, most fires involve one or perhaps two homes;
the annual losses due to fire are much more predictable than the other causes.
While a tornado or set of tornadoes can be catastrophic, the maximum severity of earthquakes is larger. Also in areas prone to tornadoes they are more frequent than are severe earthquakes in areas prone to earthquakes. A decade of historical losses from tornadoes would probably help to predict future expected tornado losses. A catastrophe model would also help to predict future expected tornado losses.
Comment: I am assuming that we are discussing geographical areas where earthquakes and tornadoes are prevalent.

71. 1. Not enough historical insurance data.
Even over half of a century, there are relatively few hurricanes that hit a given state.
Therefore, random fluctuations make it very difficult to estimate the expected value.
Also the frequency and severity of hurricanes are subject to cycles decades in length.
2. Over reliance on long-term premium and loss information.
Many things have changed since the historical period which stretches back several decades, among them: land use, construction techniques, construction materials, engineering techniques, building codes, and most importantly population density near the coast.
3. Grouping states into regions can mask hurricane potential.
Even states next to one another can vary tremendously in their hurricane potential.
4. Individual storms can have a disproportionate impact.
The size of loss distribution from hurricanes is very heavy-tailed. A single large hurricane can represent a significant portion of the total losses from hurricanes for a decade. These large random fluctuations in severity make it even more difficult to estimate the expected value.
5. Not all portions of a state are equally exposed to hurricanes. Coastal and low lying regions are much more exposed to hurricanes. In order to have enough data, traditional smoothing techniques are applied to whole states, and therefore do not allow one to take this into account the significant variation of exposure within the state.
<u>Comment</u>: Give only three limitations, and make your descriptions short.

72. 1. There is too little historical insurance hurricane experience to be credible.
2. The expected losses due to hurricanes varies widely by state, as well as within a state.
For example, the exposures to hurricanes is much higher near the coast.
3. The exposures to hurricanes has increased due to increased building near the coast.
Historical experience will not reflect the current exposures.
4. A single large hurricane can have a disproportionate impact on the insurance data, particularly that for a single insurer.
5. Exposure to hurricane loss changes over time as new buildings are built according to the then applicable building codes as well as the enforcement of those codes.
<u>Comment</u>: Give only four disadvantages.
One assumes the actuary would have taken into account inflation on both exposures and losses, by applying appropriate trend factors to pure premiums from old years of data.

73. 1. One can have separate territories for hurricane. The geographical variation in expected losses from hurricane differs from the other homeowners perils.
2. One can have a separate classes for hurricanes. The features of a home that affect expected hurricane losses differ from the other homeowners perils.
3. One can determine the hurricane and nonhurricane portions of the rate separately by different techniques. The hurricane rate would be determined using the output of the computer model and the pure premium method. The non-hurricane rate would be determined using the insurers past losses and the loss ratio method.
4. The wind coverage, including the coverage for hurricanes, does not vary by policy form. Thus appropriate policy form relativities can be applied to only the non-hurricane portion of the rate.
Comment: Give only two advantages.

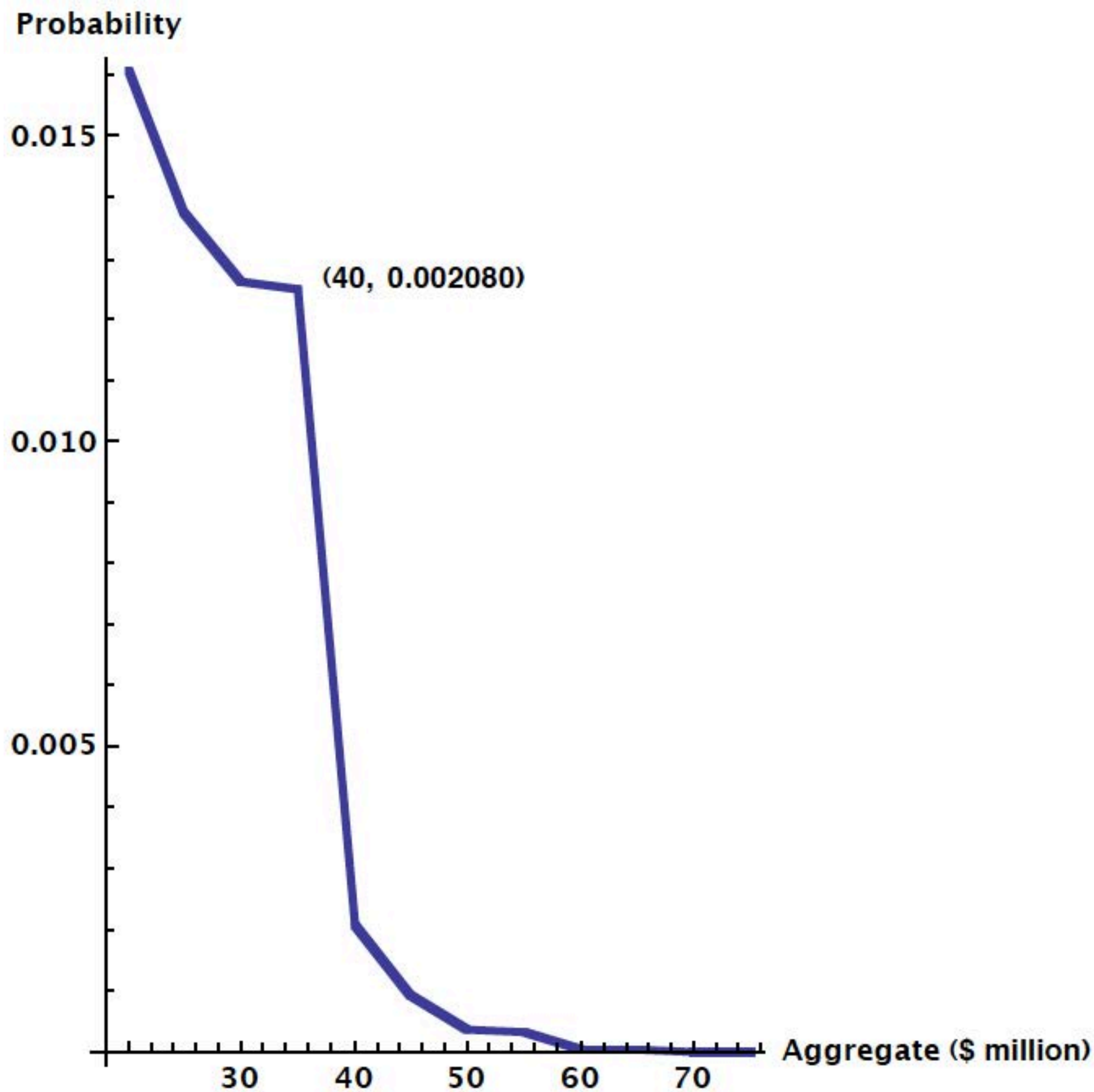
74. The probabilities of the events are: 0.5 / 5 = 10%, 5%, 2.5%, and 1.25%.

Events #s Occurring	Aggregate ($ million)	Probability
None	0	(0.9)(0.95)(0.975)(0.9875) = 0.823205.
1	5	(0.1)(0.95)(0.975)(0.9875) = 0.091467.
2	10	(0.9)(0.05)(0.975)(0.9875) = 0.043327.
3	20	(0.9)(0.95)(0.025)(0.9875) = 0.021108.
4	40	(0.9)(0.95)(0.975)(0.0125) = 0.010420.
1, 2	15	(0.1)(0.05)(0.975)(0.9875) = 0.004814.
1, 3	25	(0.1)(0.95)(0.025)(0.9875) = 0.002345
1, 4	45	(0.1)(0.95)(0.975)(0.0125) = 0.001158.
2, 3	30	(0.9)(0.05)(0.025)(0.9875) = 0.001111.
2, 4	50	(0.9)(0.05)(0.975)(0.0125) = 0.000548.
3, 4	60	(0.9)(0.95)(0.025)(0.0125) = 0.000267.
1, 2, 3	35	(0.1)(0.05)(0.025)(0.9875) = 0.000123.
1, 3, 4	65	(0.1)(0.95)(0.025)(0.0125) = 0.000030
1, 2, 4	55	(0.1)(0.05)(0.975)(0.0125) = 0.000061
2, 3, 4	70	(0.9)(0.05)(0.025)(0.0125) = 0.000014.
1, 2, 3, 4	75	(0.1)(0.05)(0.025)(0.0125) = 0.000002.

Now we need to compute the probability of the aggregate loss being greater than a certain values. S(75) = 0. S(70) = 0.000002. S(65) = S(70) + 0.000014 = 0.000016.
S(60) = S(65) + 0.000030 = 0.000046. S(55) = S(60) + 0.000267 = 0.000313.
S(50) = S(55) + 0.000061 = 0.000374. S(45) = S(50) + 0.000548 = 0.000922.
S(40) = S(45) + 0.001158 = 0.002080. S(35) = S(40) + 0.010420 = 0.012500.
S(30) = S(35) + 0.000123 = 0.012623. S(25) = S(30) + 0.001111 = 0.013734.
S(20) = S(25) + 0.002345 = 0.016079. S(15) = S(20) + 0.021108 = 0.037187.
S(10) = S(15) + 0.004814 = 0.042001. S(5) = S(10) + 0.043327 = 0.085328.
S(0) = S(10) + 0.091467 = 0.176795. S(0) + 0.823205 = 1. Check.

The original exam question did not specify which exceedance probability curve to plot.

Here is a graph of the aggregate exceedance curve, starting at $20 million, showing the probabilities of aggregate being greater than certain values:

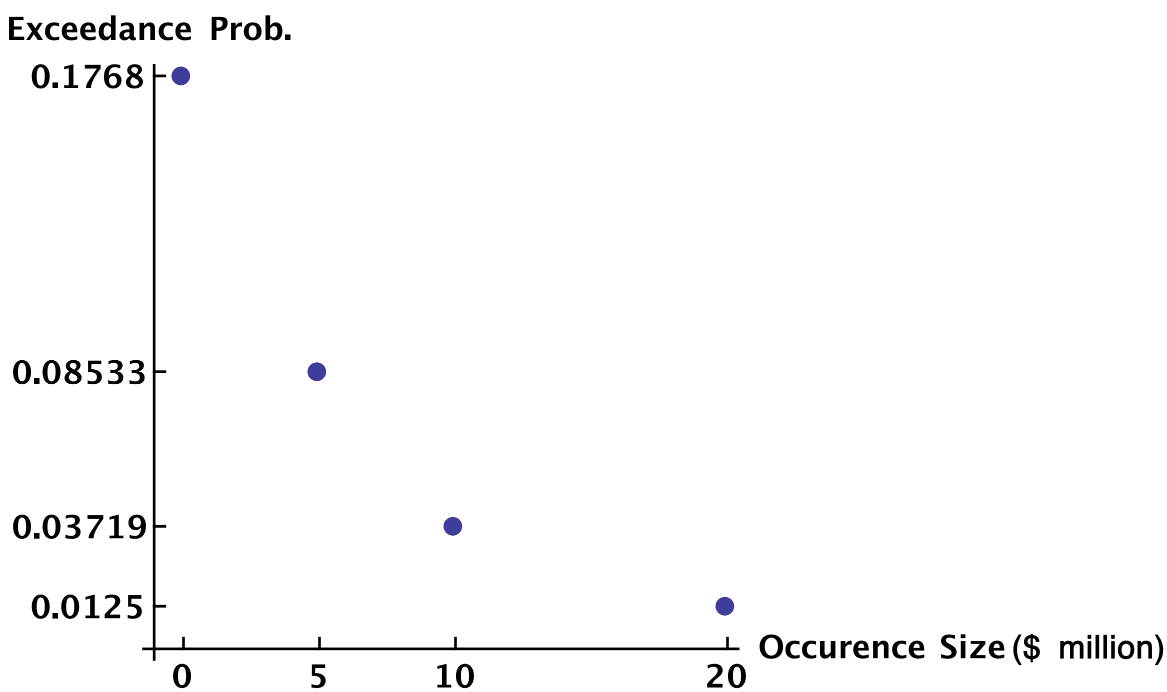


Alternately, one can compute the occurrence exceedance probability curve.

Event	Size ($million)	Probability
1	40	0.0125
2	20	0.025
3	10	0.05
4	5	0.1

Since the total number of events per year is not limited to one:

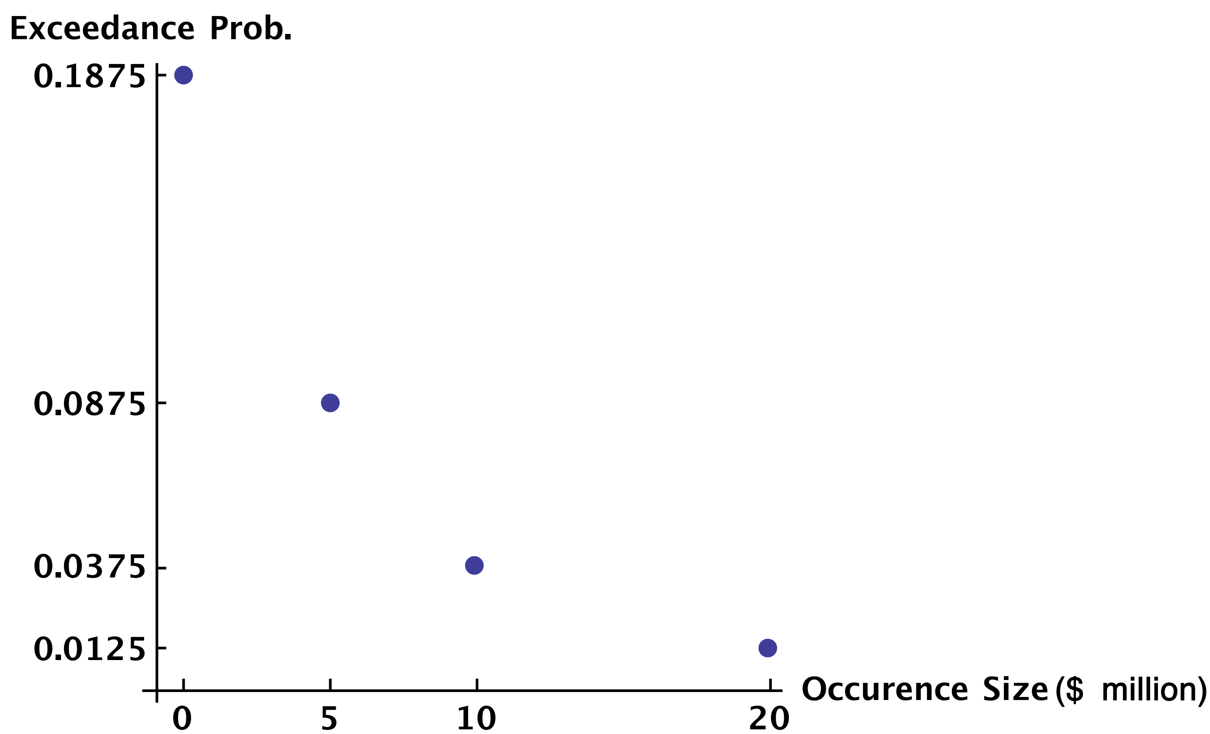
Size ($million)	Probability of At Least One Event Exceeding
20	0.0125
10	1 - (1 - 0.0125)(1 - 0.025) = 0.03719
5	1 - (1 - 0.0125)(1 - 0.025)(1 - 0.05) = 0.08533
0	1 - (1 - 0.0125)(1 - 0.025)(1 - 0.05)(1 - 0.1) = 0.17680



Comment: It is a lot of arithmetic to compute the aggregate exceedance probability curve! On your exam they should specify whether they want you to work with the aggregate or the occurrence exceedance probabilities. The difference is clarified in the errata from the CAS.

Assuming that unlike as specified in the question, at most one event can occur per year, one can compute the occurrence exceedance probability curve, as per Table 2.1 in Grossi and Kunreuther (as corrected in the errata from the CAS.)

Event	Size ($million)	Probability	Exceedance Probability
1	40	0.0125	0
2	20	0.025	0.0125
3	10	0.05	0.0125 + 0.025 = 0.0375
4	5	0.1	0.0375 + 0.05 = 0.0875
	0		0.0875 + 0.1 = 0.1875



75. (a) The expected value of losses for each of the portfolios are each equal to 4,500.
While portfolio 2 has the largest standard deviation, portfolio 3 is positively correlated with portfolio 1.
Portfolio 2 is instead negatively correlated with portfolio 1.
Thus the variance of portfolios 1 plus 2 will be less than portfolios 1 plus 3.
Thus to reduce its risk, the insurer should eliminate **portfolio #3**.
(b) 1. Getting regulatory approval. Some insurance regulators are concerned about approving rate increases based in part on a complicated computer model, which would require a lot of expertise to validate.
2. Parameter risk. It is hard to estimate the various parameters that go into the modules that make up a catastrophe model. Therefore, the result from two different computer models can differ significantly.
3. Catastrophe models require the input from areas with which most actuaries are unfamiliar, such as: meteorology, geology, structural engineering, etc.
4. It can be hard to get public acceptance for the use of a complicated "black box", particularly when used to significantly increase homeowners insurance rates in certain geographical areas.
<u>Comment</u>: For the sum of two random variables:
$\text{VAR}[X + Y] = \text{Var}[X] + \text{Var}[Y] + 2\,\text{Cov}[X,Y] = \sigma_X^2 + \sigma_Y^2 + 2\,\text{Corr}[X, Y]\,\sigma_X\,\sigma_Y$.
Here the means of the portfolios are equal as in Table 6.2 of Grossi and Kunreuther.
The actuary is interested in the heaviness of the righthand tail of the aggregate loss distribution.
Here the coefficient of variation (standard deviation divided by the mean) would be one measure of how heavy is the righthand tail of the aggregate loss distribution; the larger the CV the heavier the righthand tail and the more likely an usually large annual aggregate loss.

76. (a) The exceedance curve is a survival function; the aggregate exceedance curve shows the probability of the annual aggregate loss from an insurer's portfolio being greater than various values.

(b) We want to find where the survival function is 1/25 = 4%.
Linearly interpolating: 5 + (10 - 5)(10% - 4%) / (10% - 2%) = $8.75 million.

(c) 1. To determine the Probable Maximum Loss (PML) for a certain return period.
2. To help price insurance or reinsurance from catastrophes.
3. To help design a reinsurance program.
4. To help decide whether the insurer's capital is sufficient to support its current book of business.

77. (a) The PML is the probable maximum loss, and is the largest occurrence amount the insurer can expect to have over a given period of time.
Alternately, the PML is the probable maximum loss, and it is the largest likely loss; the insurer wants to make sure it can withstand the PML if it occurs and remain solvent.

In this case, for a one-in-250 year PML we are looking for an occurrence exceedance probability of 1/250 = 0.004. Interpolating linearly, this is **$30,000,000**.

(b) The quota share pays (30%)(40) = $12 million, which inures to the benefit of the excess of loss treaties.
Thus it is as if the loss were: 45 - 12 = $33 million.
Ceded to the 1st layer: (100%)(6) = $6 million.
Ceded to the 2nd layer: (90%)(10) = $9 million.
Ceded to the 3rd layer: (75%)(33 - 20) = $9.75 million.
Retained by the insurer: 45 - 12 - 6 - 9 - 9.75 = $8.25 million.

<u>Comment</u>: See Table 2.1 in Grossi and Kunreuther.

78. (a) In earthquake models, aleatory uncertainty is reflected in the probability distributions of: the location, magnitude, and amount of damage caused given the above. In each case, the uncertainty is due to the inherent randomness of the process reflected in the probability distribution.
In earthquake models, examples of epistemic uncertainty would be incomplete information regarding the topography, soil composition, construction quality, and replacement costs of buildings.
In earthquake models, another example of epistemic uncertainty is whether the correct model is being used to relate frequency and magnitude of earthquakes.
In earthquake models, another example of epistemic uncertainty is whether all of the relevant fault lines have been included; there can be unknown fault lines.
In earthquake models, another example of epistemic uncertainty is whether the correct relationship has been used between the force of shaking and the probability distribution of damage that will result based on different construction materials and types of buildings.

(b) Epistemic uncertainty can be reduced by collecting more data.
In earthquake models, examples of epistemic uncertainty would be incomplete information regarding the topography, soil composition, construction quality, and replacement costs of buildings.
As a result, the parameters in models are likely to be wrong, but this uncertainty could be reduced by collecting more information.
In the other cases in part (a), uncertainty could also be reduced by collecting more information.
Further fundamental research on earthquakes and more years of data should produce better models of the relationship of frequency and magnitude of earthquakes.
More years of data from more years of earthquakes should improve damage estimates.
More years of data will help uncover any currently unknown fault limes.

(c) Vary the assumptions of the various aspects of the model, resulting in many different sets of assumptions. For a given set of assumptions, run the model and develop an exceedance probability (EP) curve. Weight each set of assumptions with subjective probabilities. An average EP curve can then be constructed using these weights. For each given size, order the exceedance probability from smallest to largest. Then looking at the fifth and ninety-fifth percentiles provides a ninety percent confidence interval for the EP curve.

Comment: See Sections 4.2 to 4.4 of Grossi and Kunreuther.
"Therefore, developers of catastrophe models do not necessarily distinguish between these two types of uncertainty; instead model developers concentrate on not ignoring or double counting uncertainties and clearly documenting the process in which they represent and quantify uncertainties."

79. (a) Hazard module: The module assigns probabilities of an event by location. It also provides a model for the severity and propagation of an event.
Inventory module: The nature of the buildings in the area as a detailed census. Construction type is the most important characteristic.
Vulnerability module: Estimates the level of building damage expected for differing severities of event.
(b). Hazard module:
No difference. The events (earthquakes) are identical regardless of the insurance coverage.
Inventory module:
Completely different. One is an inventory of homes and the other of businesses.
Vulnerability module:
There will be many similarities as the module is based on construction types and other building characteristics.
(c) Non-hurricane weather claims loading:
Analyze historical claims data to estimate loading
Hurricane claims loading:
Rely on estimates from a catastrophe model (i.e. simulation model).
Models simulate the event, translate into a damage ratio and damage ratios are applied to current or projected amounts of insurance and produce the expected catastrophe loss estimate.

80. (a) (25%)(0.0072) + (50%)(0.0080) + (25%)(0.0092) = **0.0081**.
(25%)(139) + (50%)(125) + (25%)(109) = **124.5 years**.
(b) Results should be consistent; otherwise risk management strategies that employ different measures (probability versus return) will not align.
Thus I would estimate the return time as: 1/ 0.0081 = 123.5 years.
(c) Option I: (0.25)(0.0022) + (0.50)(0.0026) + (0.25)(0.0031) = 0.002625
to (0.25)(0.0114) + (0.50)(0.0109) + (0.25)(0.0117) = 0.011225
Option II: (0.25)(0.0022) + (0.50)(0.0026) + (0.25)(0.0031) = 0.002625
to (0.25)(0.0109) + (0.50)(0.0114) + (0.25)(0.0117) = 0.011350
Option III: minimum of 0.0022 to maximum of 0.0117
(d) Only Option I provides a 90% confidence interval as it is the weighted average of such intervals. By varying the weights, Option II distorts the results.
Option III is conservative and thus provides more than a 90% confidence interval.

81. (a) One would use a hurricane model.
Annual Average Loss (AAL) for each county is obtained by taking a sum of the (modeled) loss for that county from each possible event times the probability of that event.
The risk load reflects the standard deviation of the distribution of losses for that county.
The expense load is intended to cover for the insurer:
general expenses, acquisition expenses, fees, taxes, underwriting profit.
Base rate = AAL + Risk Load + Expense Load.
(b) Structural attributes and location attributes.
Examples of structural attributes of a building include: building materials (masonry is superior to wood for hurricanes), building codes, year of construction, and occupancy. Structural attributes help determine the performance of the building when hit by a catastrophe. For hurricanes, the type of roof and how it is attached to the rest of the building are of particular importance.
Where a building is located determines the forces it will be subjected to by a given catastrophe. For example, in the case of hurricanes: distance from the coast, surface roughness and topography.
(c) Post-event inflation is a sudden and temporary increase in the cost of materials, services, and labor due to increased demand following a catastrophe; also called demand surge. After a large disaster such as a large hurricane, construction material and labor can temporarily be in short supply, for example due to destruction of warehouses, retail outlets, and roads. Due to increased demand and short supply, construction costs are inflated. Because this is the time in which insured losses are adjusted, these increased costs have led to larger insured losses than otherwise expected. The larger the impact of the event on the local economy, the larger this effect.
Also there are only a limited number of trained claims adjusters. Even if some claims adjusters are brought in from other states, they may be overwhelmed by the tremendous number of claims that can be generated by a large hurricane. This can lead to excessive claim settlements.
Comment: See Section 5.3 and page 113 of Grossi and Kunreuther.

82.

(a) The given exceedance probability curve was stated from the perspective of the catastrophe reinsurer; the horizontal axis is the amount excess of $10 million.
OEP is the probability that at least one event exceeds the specified loss amount in a year.
There is about a 10% probability of at least one occurrence during a year exceeding $10 million in losses excess of $10 million, in other words of exhausting the treaty layer.
Therefore, it should take 1/10% = 10 years to pay back. Thus the pure premium for the treaty is the full layer loss divided by the payback period: $10,000,000 / 10 = **$1,000,000**.

(b) Reinstatement premium is: 115% $\frac{20\text{M} - 15\text{M}}{20\text{M} - 10\text{M}}$ \$1,200,000 = **$690,000**.

(c) The quota share treaty pays: (30%)(15M) = $4.5 million.
Without the cat treaty the insurer would pay: 15M - 4.5M = $10.5 million
Thus the cat treaty pays: 10.5 - 10 = $0.5 Million.
The insurer pays a net of $10 million.
Note that: $4.5M + $0.5M + $10M = $15 million.

<u>Comment</u>: In part (b) I have ignored the quota share treaty which is first mentioned in part (c).
In part (a), I did not like the reference to the "payback approach".
Clark at page 5: "In the past, reinsurers had priced catastrophe loads based on spreading large losses over expected payback periods. A 1-in-20-year event would be included as a loading of 5% of the loss amount. The payback approach may still be used for casualty events but is only referenced as a reasonability check for property."
Clark at page 44: "Before the widespread development of catastrophe models there had been few tools available to systematically price catastrophe covers. The most common method was known as the payback approach, in which premium was set so the offered limit was paid back over a given period of time. For the example above, the payback period is five years, meaning that the $2,000,000 of annual premium would cover a single total loss of $10,000,000 every five years. Catastrophe models are now the generally accepted approach for pricing of natural and some man-made events." Given the output from a catastrophe model such as in this question, Clark would <u>not</u> use the payback approach.
From the Examiner's Report, in part (a): "To get full credit, candidates were expected to either explicitly state the 10-year payback period and to divide the full layer loss by the payback period to get a pure premium, or to multiply the full layer loss by the relevant value from the exceedance probability curve."
In part (a), see CAS errata to Section 2.4 of Grossi and Kunreuther.

83. (a) 1. Location: Hazard and Inventory
2. Soil: Hazard
3. Construction: Inventory and Vulnerability and Loss
4. Age: Inventory and Vulnerability and Loss
Explanations:
● Hazard module identifies the location of faults and the risk associated with each, including soil type.
● Inventory module contains location, construction, and age (among others), but not soil type.
● Vulnerability module measures building damage given the event and is related to construction and age
● Loss module translates the event into money which depends on construction and age.
(b) Any two of the following are acceptable:
● Presence of retrofitting
● Building occupancy
● Building codes at time of construction

84. (a) Probability of no loss: (0.9)(0.95)(0.98) = 83.79%.
Probability of 10,000,000 aggregate: (0.1)(0.95)(0.98) = 9.31%.
Probability of 15,000,000 aggregate: (0.9)(0.05)(0.98) = 4.41%
Probability of 35,000,000 aggregate: (0.9)(0.95)(0.02) = 1.71%
Probability of 10M + 15M = 25M aggregate: (0.1)(0.05)(0.98) = 0.49%
Probability of 10M + 35M = 45M aggregate: (0.1)(0.95)(0.02) = 0.19%
Probability of 15M + 35M = 50M aggregate: (0.9)(0.05)(0.02) = 0.09%
Probability of 10M + 15M + 35M = 60M aggregate: (0.1)(0.05)(0.02) = 0.01%
Probability[Aggregate > 60M] = 0.
Probability[Aggregate > 50M] = 0.01%.
Probability[Aggregate > 45M] = 0.01% + 0.09% = 0.10%.
Probability[Aggregate > 35M] = 0.10% + 0.19% = 0.29%.
Probability[Aggregate > 25M] = 0.29% + 1.71% = 2.00%.
Probability[Aggregate > 15M] = 2.00% + 0.49% = 2.49%.
Probability[Aggregate > 10M] = 2.49% + 4.41% = 6.90%.
Probability[Aggregate > 0M] = 6.90% + 9.31% = 16.21%.
(b) Find where the discrete distribution of aggregate losses first exceeds 0.86.
The distribution function at 0 is 83.79%.
The distribution function at 10,000,000 is: 83.79% + 9.31% = 93.10%.
Thus the simulated aggregate loss is **$10,000,000**.
Alternately, 1 - 0.86 = 0.14 lies between 6.90% and 16.21% on the aggregate exceedance probabilities, so we simulate **$10 million** in aggregate loss.
<u>Comment</u>: For part (b), see Table 4.1 in Grossi and Kunreuther.
I have used the usual technique of simulating from a discrete distribution, which differs in minor detail from what Grossi and Kunreuther did.

85. (a) Combining portfolios brings diversification and thus reduces risk.
(Here I am measuring risk as a percent of expected losses, rather than as a pure dollar amount.)
Also FGH is probably less risky than ABC and XYZ, since an individual tornado does less widespread damage than either a hurricane or an earthquake. It is unlikely that more than a few of FGH Burgers outlets would be damaged by a single tornado.
In the absence of any additional information, I believe that ABC and XYZ are about equally risky.
{VII. ABC, FGH, and XYZ} <
{V. ABC and XYZ} = {VI. FGH and XYZ}
< {IV. ABC and FGH}
< {II. FGH only}
< {I. ABC only} = {III. XYZ only}.
(b) GIC could obtain detailed underwriting information about each property, perhaps including inspection by an engineer.
It would be concerned about the soil type on which each structure is built.
Also GIC would want to know for each location the distance to any (known) earthquake faults.
(c) GIC should consider coverage limits for each location as well as an aggregate limit.
GIC can apply a deductible to each location (either as a dollar amount or as a percent of value.)
(d) The model can be run incorporating various combinations of deductibles and limits. For each set of deductibles and limits the model produces a corresponding exceedance curve as well as an expected annual payment by GIC. Hopefully there will be a set of limits and deductible that is acceptable to ABC (less coverage but also less premium), and that also produces an exceedance curve that is acceptable to GIC.
(GIC can also include the effect of possible reinsurance treaties in the runs.)
(e) Any one of the following:
- Purchase catastrophe reinsurance.
- Work with ABC to improve the resistance to damage from earthquakes of the insured structures.
- Securitization; sell catastrophe bonds.
- Insure additional independent risks to add diversification.

Comment: If GIC just writes ABC Burgers in California, that is likely to be too small of a portfolio in order to make selling catastrophe bonds practical.

86. (a) Hazard: Not possible. For a given region historical data is likely available and there is no more additional data.
Hazard: Possible. One can get more information about geology, topography, and soil for earthquake models.
Inventory: Not possible. The structures are set, so there cannot be any more structures.
Inventory: Possible. One can add structures if the current model is incomplete.
Vulnerability: Possible. More data can be obtained relating damage to structure and hazard.
Loss: Possible. More information can be collected with regard to repair and business interruption costs.
(b) Hazard: Possible. There are several ways to use the data for earthquake models, such as magnitude-frequency and characteristic models. There is always room for improvement; for example, one can improve accuracy and modeling of geology, topography, and soil for earthquake models.
Inventory: Possible. More accurate information can be obtained on construction type, age, height, value, and location.
Vulnerability: Possible. There is not full understanding of how the nature of the hurricane/earthquake combines with the structure details to produce damage. There can be better models constructed.
Loss: Possible. Better models can be constructed to estimate demand surge and indirect costs related to business interruption.
Comment: The understanding of how hurricanes and earthquakes affect insurance losses is much greater than in 1980; however, there is always more to learn.

87. (a) Epistemic Uncertainty ⇔ Parameter Risk and Modeling Risk.
1. In hurricane models, the hazard module includes a large number of possible hurricanes with different paths and different categories (magnitudes). We are uncertain of the actual future probabilities of hurricanes with certain paths and/or magnitudes.
In earthquake models, the hazard module includes fault lines. An example of epistemic uncertainty is whether all of the relevant fault lines have been included; there can be unknown fault lines.
In earthquake models, the hazard module uses recurrence models of earthquakes along faults; we are uncertain of what are the correct models and parameters.
2. The vulnerability module contains for each type of building a probability distribution of the building damage that would result from a given external force. We are uncertain as to what exactly is the appropriate probability distribution.
3. In the inventory module, examples of epistemic uncertainty would be incomplete information regarding construction quality, and replacement costs of buildings.
Another source of epistemic uncertainty is less than perfect information on the exact soil type where the structure is located, which is important for earthquake models.
Another source of epistemic uncertainty would be outdated information on the value of contents.
Another source of epistemic uncertainty would be lack of information on building codes.
4. The deficiency of information about repair costs and business interruption costs adds epistemic uncertainty to the loss module.
(b) Aleatory Uncertainty ⇔ Process Risk.
1. In hurricane models, the hazard module includes a large number of possible hurricanes with different paths and different categories (magnitudes). There is an inherent randomness as to how many and what hurricanes there will be in a given year. This produces aleatory uncertainty due to the inherent randomness of the process, similar to the roll of a die.
2. The vulnerability module contains for each type of building a probability distribution of the building damage that would result from a given external force. Thus there is aleatory uncertainty (randomness) in the amount of damage for a given external force.
Comment: There are many possible full credit answers.
The four modules of a catastrophe model are: hazard, inventory, vulnerability, and loss.

88. From the graph, the needed capital with the excess reinsurance is about 70M.
The ceded loss and ALAE are:

	Loss	Ceded Loss	ALAE	Ceded ALAE
Claim 1	$7,500,000	$2,500,000	$1,500,000	(1/3)($1,500,000)
Claim 2	$10,000,000	$5,000,000	$500,000	(1/2)($500,000)

Thus the total ceded loss and ALAE is: 8.25M.
Retained loss and ALAE is: 30M - 8.25M = 21.75M.
Reinsurance premium is: (18%)(50M) = 9M.
Underwriting profit is: 50M - 15M - 21.75M - 9M = 4.25M.
With excess reinsurance, return on capital is: 4.25/70 = **6.07%**, compared to 5% without reinsurance.

From the graph, without reinsurance the 1-in-100 year Probable Maximum Loss is about 100M.
With the quota share, the ceded premium is: (20%)(50M) = 10M.
The ceding commission is: (30%)(10M) = 3M.
Due to the maximum ceded loss ratio, the maximum ceded loss is: (150%)(10M) = 15M.
Thus for the 1-in-100 year Probable Maximum Loss, the insurer would have net losses of:
100M - 15M = 85M. Thus 85M is the needed capital with the quota share treaty.
For the given experience, the ceded losses and ALAE are: (20%)(30M) = 6M.
Underwriting profit is: 50M - 15M - (30M - 6M) - (10M - 3M) = 4M.
With quota share reinsurance, return on capital is:
4/85 = **4.71%**, compared to 5% without reinsurance.
<u>Comment</u>: Return on capital is not covered in the syllabus of this exam.
However, all you really need to know is that when we ignore investment income and income taxes, return on capital is: (underwriting profit) / capital.
For the current situation without reinsurance, the needed capital is 100M.
The underwriting profit is: 50 - 15 - 30 = 5M.
Return on capital is: 5/100 = 5%, matching what was given in the question.
One would <u>not</u> normally base such a decision on looking at only one year of past experience.

89. (a) (i) Availability of data – Less data will be available for homeowners due to lower frequency. Also catastrophes are uncommon.
(ii) Uncertainty of loss – Hurricane coverage has more variability (uncertainty) because it is a higher severity coverage. There are years with zero loss and years with a very large aggregate loss from hurricanes.
(iii) Correlation between claims – Auto claims are largely uncorrelated while catastrophe claims are correlated due to concentration.
(iv) Insurer capacity – Due to the correlation between claims, catastrophe coverages are more likely to threaten insurer capacity.
(b) Catastrophe coverages are subject to large correlated losses. These losses can result in reduced capacity if the losses cause insurer insolvencies. The insolvencies, as well as heightened customer awareness following a large industry loss, can increase the demand for insurance further constraining capacity.
(c) The survival constraint states that pricing and selling decisions require that the probability of insolvency be less than a specified constraint, for example 1%.
(d) The insurer can use an exceedance probability curve to examine the probability that losses exceed a certain amount. For example, an aggregate loss of $100 million might lead to the insurer exhausting its capital. If there is a 1% probability of exceeding $100 million in aggregate loss, then there is a 1% chance of insolvency.
(e) ● Requirement to offer coverage (For example, requiring that one offers wind coverage.)
● Regulatory cap on rates (For example, a cap on rates that can charged for hurricane insurance near the coast.)
(f) • Decrease the coverage (e.g., lower limits, higher deductibles)
● Increase the premium
● Transfer some of the risk to others (e.g. catastrophe reinsurance)
● Re-underwrite its portfolio to make it more diverse geographically
<u>Comment</u>: See Chapter 2 of Grossi and Kunreuther.

90. (a) The damage ratio = (repair costs) / (replacement cost of the building).
Curve A is the damage function, and represents the expected damage ratio for a given intensity earthquake.
Curve B is the damage-state distribution, and represents for a given intensity the distribution of the damage ratio around its expected value.
C is the intensity distribution, and represents the distribution of the intensity of external excitation for a given modeled earthquake.
(b) Assume that the stated intensities are the only possible intensities.
Assume that the probabilities are those for one year.
Reading the damage ratios as best as I can off of the graph:

Intensity	Probability	Damage Ratio
3.5	10%	20%
6.0	5%	45%
8.5	2%	80%

Expected loss: (\$1,000,000) {(10%)(20%) + (5%)(45%) + (2%)(80%)} = \$58,500.
The second moment of the distribution of expected damage by intensity is:
$(\$1{,}000{,}000^2)\ \{(10\%)(20\%^2) + (5\%)(45\%^2) + (2\%)(80\%^2)\} = 2.6925 \times 10^{10}$.
The variance, ignoring the distribution of damage ratios around their mean:
$2.6925 \times 10^{10} - 58{,}500^2 = 2.350 \times 10^{10}$.
Trying to read the standard deviations of the distributions of damage ratios from curves B, assuming most of the probability is within ±2 standard deviations: 5%, 4%, and 5%
Thus the variance in damage ratio around their mean is:
$(10\%)(0.05^2) + (5\%)(0.04^2) + (2\%)(0.05^2) = 0.00038$.
Thus the standard deviation of damage due to the distribution of damage ratios is:
$(\$1{,}000{,}000)\ \sqrt{0.00038} = \$19{,}494$.

Thus the total variance is: $2.350 \times 10^{10} + 19{,}494^2 = 2.388 \times 10^{10}$.
The standard deviation of the loss is: $\sqrt{2.388 \times 10^{10}} = 154{,}540$.

The premium is: $\dfrac{58{,}500 + (8\%)(154{,}540)}{1 - 20\%} = \mathbf{\$88{,}579}$.

<u>Comment</u>: See Figure 3.7 in Grossi and Kunreuther, which is not thoroughly explained.
In the CAS Examiner’s report, they ignored the contribution to the total variance from the distribution for a given intensity of the damage ratios around their mean; in this case that contribution is <u>not</u> significant. Without that contribution, the standard deviation of loss would be instead \$153,306, and instead the premium would be \$88,456.
The total of the given probabilities is: 10% + 5% + 2% = 17%; there is a chance that there will not be an earthquake that affects this home next year.
We have to assume that the policy is only covering damage from earthquakes, as would be the case for the California Earthquake Authority (CEA).
See Tables 5.1 and 5.2 in Grossi and Kunreuther, where the risk load is taken as half of the standard deviation.

91. (i) This is incorrect. It is also essential to know the regions geological features and how an earthquake will propagate through the regions soil structure. (Sometimes an area far from the epicenter will sustain more damage than an area closer to the epicenter, due to their soil types.)
(ii) This is incorrect. The Gutenberg-Richter relationship indicates that the longer the time until the next earthquake, the higher the magnitude.
(iii) This is correct. While public data may be sufficient for aggregate estimates it is essential that risk-specific structural details be known.
(iv) This is incorrect. It is too difficult to do this and have a model that works with individual buildings. Instead, buildings are divided into classes for the vulnerability module, and one typical building is analyzed for each class.
(v) This is correct. One approach is to link the results in the vulnerability module directly to a monetary loss. A second option is to use the loss module to perform the translation of physical loss to monetary loss. In either case, the loss module applies the provisions of the insurance policy to get the insurance loss.
Comment: See Chapter 3 of Grossi and Kunreuther.
Part (ii) is a good example of why a modeler has to be careful of correlations between variables.

92. (a) Type of construction.
Age of building: This relates to building codes at the time.
The geology of the site is important for earthquakes.
The geography of the site is important for hurricanes.
The value of the covered property.
(b) The inventory module describes the portfolio of the insurer. If the inputted data does not accurately describe what is being insured, then the model output will not be appropriate for the given insurer.
In defining as accurately as possible the composition for their portfolio, insurers can reduce the degree of epistemic uncertainty.
(c) The mean damage ratio cannot properly reflect the impact of deductibles. For example, assume we have a 10% deductible. Also assume for simplicity that there is no loss 95% of the time, and a total loss 5% of the time. Then the mean damage ratio is 5%. While 5% < 10%, there are in fact losses excess of the 10% deductible.
(d) ● If buildings in a portfolio are all located in an area with common geology or geography, there will be a higher correlation of loss.
● Model errors will propagate across properties. For example, if the vulnerability model is off for a given class of building, all of that class will be simultaneously affected.
Comment: See Chapter 6 of Grossi and Kunreuther.

93. (a) S(95 million) = 1/200 = 0.5%.
S(105 million) = 1/500 = 0.2%.
We want S(x) = 1/250 = 0.4%.
Using linear interpolation, x = (2/3(95) + (1/3)(105) = $98.33 million.
I estimate the one-in-250 year PML as about $98.33 million.
This corresponds to **ceded losses of $48.33 million**, and **net losses of $50 million**.

(b) Assume the table and treaty are both on a per occurrence basis. (See my comment.)
The Occurrence Exceedance Probability (OEP) is the probability that at least one loss exceeds the specified loss amount.
Since the largest hurricane of $45 million would not pierce the layer covered by the treaty, the treaty is (almost totally) exposed to earthquakes. Since the reinsurer's current portfolio is exposed to hurricanes and not earthquakes, the losses from this treaty would be uncorrelated with the reinsurer's current losses. Thus adding to this treaty would not significantly increase the reinsurer's current one-in-250 PML. Writing this treaty would diversify the reinsurer's portfolio. There is no reason based on capital why the reinsurer should not write this treaty. (The potential profitability of this treaty compared to other possible new treaties would help determine whether to write this treaty.)

Alternately, assume that the table is on an (annual) aggregate basis as stated, and also assume the treaty is on an aggregate basis. (See my comment.)
The Aggregate Exceedance Probability (AEP) is the probability that the sum of all losses during a given period exceeds some amount.
If the primary insurers losses were perfectly correlated with the current portfolio of the reinsurer, then $48.3 million would be added to the one-in-250 year PML of the reinsurer.
$825 + 48.3 = $873.3 million would exceed the $850 million in current capital, and thus the reinsurer should not write the treaty.
However, since the reinsurer is currently only worried about the hurricane peril, and the primary insurer is worried about both earthquake and hurricane peril, the losses can not be perfectly correlated.
If the new treaty's losses are (largely) uncorrelated with the current portfolio of the reinsurer, then very little if anything would be added to the one-in-250 year PML of the reinsurer. (This can occur if the geographical regions covered are mostly distinct. Also earthquake and hurricane losses are uncorrelated with each other.) In this case, there is no reason based on capital why the reinsurer should not write the treaty. (The potential profitability of this treaty compared to other possible new treaties would help determine whether to write this treaty.)
The situation is probably somewhere between these two extremes, and there is insufficient information to determine whether the reinsurer has enough capital to write this treaty.

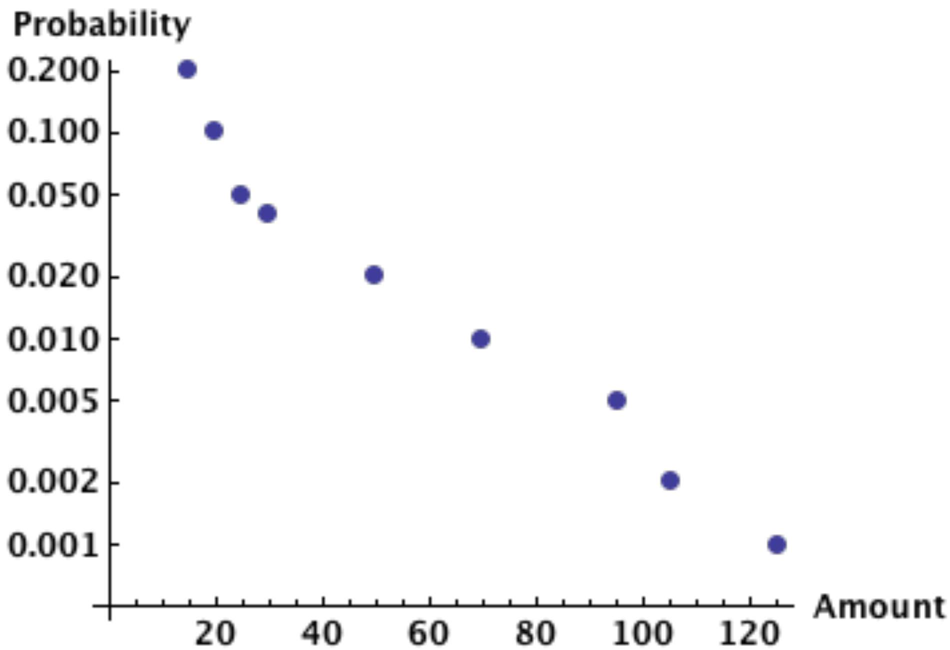
Selected sample responses to part (b) from the CAS Examiner's Report, edited by me:

● Reinsurer new PML = 825 + 48.33 = 873.33.
However, this PML of 873.33 is the maximum number; PML cannot be added.
Due to the diversification benefit from earthquake exposure of this new treaty, the PML may be reduced to an acceptable level. Therefore, the reinsurer should accept.

● The reinsurer's 1-in-250 PML is driven completely by hurricane, so it should participate in the treaty to diversify the perils it is exposed to. Since the largest hurricane event for the primary insurer is 45 million, the reinsurer will not increase its exposure to hurricane – only earthquake. This means the reinsurer's 1:250 should not grow by taking on additional EQ exposure. This is all, of course, on a modeling basis. There is potential for the model to be wrong, but the reinsurer should diversify.

● The reinsurer has 850 - 825 = 25 million of available capital. The 48.33 million PLM could seem too high, but it depends how this possible loss is correlated with the reinsurer's current book of business.
Since the current book is solely driven by hurricane peril and this proposed contract is driven by both hurricane peril and EQ peril, I would recommend to accept this new treaty. Furthermore, the largest hurricane event in the primary insurer's event catalog is $45,000,000, which is under the attachment point of the treaty.

Comment: In part (a), one should interpolate using the exceedance probabilities rather than the return periods. A graph of the given exceedance curve:



For part (b), from the CAS Examiner's Report: "Due to the ambiguity in outlining the type of reinsurance contract being offered and the primary insurer's PML table incorrectly being labeled "aggregate," this question was ruled to be defective.
This defect was addressed through the grading of the question, as discussed herein.
The intent of the question was for the PML table to be interpreted as occurrence. Most candidates answered the question as intended, as if it were an OEP."
"Part b becomes unanswerable with the assumption that the treaty is per-occurrence, combined with an aggregate PML table as we do not have enough information. Candidates who recognized this issue and explained why received full credit."
Catastrophe treaties are written on a per event basis, covering the total losses (net of other reinsurance) from one event such as a single hurricane.
Assuming that both the treaty and exceedance curve were on an occurrence basis, things are complicated by reinstatement clauses. For example, assume that there is an earthquake with \$100 million is loss (net of other reinsurance), exhausting the treaty limits. Then the insurer can reinstate coverage by paying the reinstatement premium. Thus the reinsurer can be exposed to more than the treaty limit if there is more than one covered catastrophe in a year.
The effect of the reinsurance premium and any reinstatement premiums are being ignored; compared to a one-in-250 year PML, their effect would be relatively minor.

94. (a) This change would primarily affect the **vulnerability module** which relates to how susceptible different building types are to damage from a catastrophe. Here is where the assumed effectiveness of the new construction in reducing damage from hurricane force winds would have to be incorporated.
One would also have to update to the **inventory module** to make sure that any buildings in the insurer's portfolio that used this new construction technique are appropriately identified as such.
(b) This is **epistemic** uncertainty since it is parameter risk (or modeling risk) rather than the process risk inherent to the random nature of the catastrophe.
(c) ● A Logic Tree assigns weight to parameter alternatives, in this case based on the differing opinions of the experts of the effectiveness of the new construction technique at reducing wind damage. Then for each of the different possibilities from the Logic Tree, the cat model is run. Using each of the model outputs and its corresponding probability from the Logic Tree, a weighted average is calculated to get the mean result. The different model outputs and corresponding probabilities can also be used to get the range of variability in results, in this case due to the uncertainty in the effectiveness of the new construction technique.
Logic Trees are relatively easy to document and communicate. However, they are not easy to use when there are a large number of possible scenarios, for example when there are many items varying in many possible ways.
● Simulation creates many randomly sampled alternatives from the probability distribution(s) of the parameter(s). Then for each such set of inputs, the cat model is run. The result is a large set of outputs, which can be used to get the mean result as well as quantify the variability.
Simulation can handle situations with a large number of possibilities. However, it is harder to document and communicate than Logic Trees. Simulation may appear to be a "black box" to non-actuaries. Simulation gives a better overall view of the uncertainty than Logic Trees, but is more computationally intensive.
● Credibility weight with and without effects of the new science to get a credibility weighted damage function, with the compliment of credibility being no inclusion of non-consensus science. This may be more stable, as it will have less major change year over year until the new science becomes more mainstream and generally accepted.
Comment: See Sections 4.2-4.4 of Grossi and Kunreuther.
The other two modules are the Hazard Module (possible hurricanes and probabilities), and the Loss Module (applies the provisions of the insurance policy to translate damage to insurance loss.)
Aleatory uncertainty relates to the inherent randomness in the risk process.
In the specific situation described, it seems like a Logic Tree would be a good approach.

95. (a)
(i) The hazard module simulates the events to be applied to the risks. The stochastic models used may differ, with somewhat different sets of possible events and/or somewhat different probabilities assigned to each possible event. When simulated events are different, modeled results will be different.
(ii) The inventory module contains the individual policy and risk information of the underlying portfolio. It should be same for all models and should not contribute to the difference among modeling results.
(iii) The vulnerability module estimates the level of building damage expected for different levels of severity of the oncoming external forces imposed. When damage ratios/damage curves (part of the model) are different, modeled results will be different.
(iv) The loss module converts the building damage, either direct or indirect, to a monetary loss amount. It is is applying damage ratios to the insured value and policy provisions; this logic should be the same across all models. Thus this module should not contribute to the difference among modeling results.
(b) 1/25 = 0.04 On the graph for X2, 0.04 is at **50 million**.
(c) Possible answers:
- The three modeling results are not materially different, therefore, all three should be considered.
- The actuary should use a (weighted) average of the three modeling results.
- The actuary should have an understanding of the differences in the models.
- For the line of business and perils being modeled for NAN insurance, identify which model is more robust, then it will be given more weight. Identify which model is less robust and give that model less weight.

(d) Presenting a lower AAL can result in lower (cheaper) reinsurance premium from the reinsurer. However, this is knowingly understating the exposure that the reinsurer is covering. By providing misleading information to the reinsurer, the insurer has not entered into this contract negotiation in good faith.
<u>Comment</u>: In part (a), each modeler simulated a large but finite number of years, which would result in somewhat different results even if the models were the same.
In part (d), an insurer who gets a reputation of doing such things, will have a hard time obtaining reinsurance in the future.

96. (c) Average Annual Loss (AAL) for each of X and Y is: (442,000)(1%) = 4,420.
Premium = (AAL + 0.10 × STDev) / (1 - 0.27).

For X: $\frac{4{,}420 + 0.1\sqrt{309{,}903{,}600}}{1 - 0.27}$ = **8466**. For Y: $\frac{4{,}420 + 0.1\sqrt{395{,}083{,}600}}{1 - 0.27}$ = **8778**.

(d) AAL/TIV ratio: The ratio of the AAL to the Total Insured Value (TIV) provides a metric that shows the long-term risk at a location. This can be useful in evaluating how similar properties that are close geographically can have significantly different expected losses AAL. For example, for hurricanes, the AAL/TIV ratio can vary significantly between two adjacent zipcodes based on their distances from the coast.
PML/TIV ratio: The ratio of a PML at a specified return period, to the TIV gives an indication of the possible severity at a location. A higher ratio suggests a higher loss potential from extreme events
(e) AAL/TIV for X and Y = 4,420/10,000 = **0.44**.
Using the stated 100-year return period to determine the PML:
PML/TIV for X = 125,000/10,000 = **12.5**.
PML/TIV for Y = 150,000/10,000 = **15**.
(f) X and Y have similar loss potential on average.
However, Y will contribute more tail risk and volatility than X.
Comment: Most of this question is not explicitly covered by Grossi and Kunreuther.
For an example of the geographical variation in risk, see the maps from the Florida Hurricane Catastrophe Fund in my supplemental material on my webpage.

97. (a) From the graph, $750 million in retained losses on a gross basis corresponds to a return period of 25 years. The probability is 1/25 = **4%**.
(b) From the graph, for Option 1, for each return period the retained losses are about 90% of the gross losses. (For example for a return period of 225 year, the gross losses are $925 million and the retained losses are about $830 million to $835 million.)
Thus **Option 1 is a 10% Quota Share treaty**.
Alternately, under Quota Share, the retained losses and ceded losses are each a constant percentage of the gross losses. Multiplying by a constant does not change the coefficient of variation. The CV is the same for gross, retained, and ceded losses for Option 1, which of the three options is therefore the Quota Share. For the one in 25 year return time, the net losses are 675 million, while the gross losses are 750 million. 1 - 675/750 = 10%.
Thus **Option 1 is a 10% Quota Share treaty**.
(c) Option 3 has a much larger coefficient of variation of ceded losses than Option 1.
Thus the ceded losses under Option 3 are more volatile than those under Option 1; Option 3 is more risky for the reinsurer than Option 1.
Therefore, the reinsurer will want a much higher risk load for Option 3 than Option 1.
This may lead to a higher reinsurance premium for Option 3 than Option 1.
Alternately, the reinsurer is taking on more righthand tail exposure in Option 3 compared to Option 1. Thus the reinsurer has to hold more capital, which drives up the needed return on premium. Therefore there is a higher reinsurance premium for Option 3 than Option 1.
<u>Comment</u>: The mean ceded losses for Option 3 are $12 million compared to $50 for Option 1. So which one has a higher reinsurance premium would depend on the magnitude of the reinsurer's risk load.
One can read other probabilities off of the graph. For example, for Option 2 the return time for $825 million on a net basis is 225 years; the probability is 1/225 = 0.44%.
The CV of an excess layer is usually higher than that of limited losses. However, if the layer has a top as well as bottom, then the CV decreases as the width of the layer decreases. For a sufficiently narrow width, the CV of a layer can be smaller than that of limited losses.

98. (a) Any four of the following are acceptable:
- Ratemaking
- Underwriting and Risk Selection
- Loss Mitigation
- Catastrophe Reinsurance
- State and federal public policymakers use catastrophe models to address public policy issues.
- Capital adequacy (sensitivity) testing
- For loss reserving purposes

(b) I would recommend Account Y. While Y has a higher AAL than Account X, it does not increase the combined book's PML very much.
(Account Y could consist of risks in an area with low concentration in the current book)
(c) For account X: {(5,000) (1 - 0.137) + 440} / (1 - 0.27) = **6,514**.
(d) The variability in losses, as measured by for example either the standard deviation or the coefficient of variation.
(e) The company could manage this exposure by buying reinsurance.
(One could buy a reinsurance program that includes catastrophe insurance.)
Alternately, one could manage the catastrophe risk by selling catastrophe bonds.