Solutions to the Fall 2018 CAS Exam 8

(Incorporating what I found useful in the CAS Examiner's Report)

The solutions and comments are solely the responsibility of the author.
While some of the comments may seem critical of certain questions, this is intended solely to aid you in studying and in no way is intended as a criticism of the many volunteers who work extremely long and hard to produce quality exams.

This exam had 17 questions totaling 52 points.
There were two integrated questions.

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1. (9 points) An insurance company is planning to expand into a new territory and has decided to review its historical loss experience in order to determine whether it will require additional capital to support the expansion.
The insurance company has engaged an actuarial consultant to provide insights into a prospective loss ratio for the new territory.
The following table outlines the insurance company's historical experience for two long-tailed lines of business (LOB):

	Earned Premiums		Ultimate Losses		Ultimate Claim Counts	
Accident Years	LOB 1	LOB 2	LOB 1	LOB 2	LOB 1	LOB 2
1991-1995	$12,033,000	$1,766,000	$2,329,000	$1,236,000	170	170
1996-2000	$13,812,000	$1,819,000	$2,762,000	$1,273,000	210	172
2001-2005	$13,985,000	$1,751,000	$2,797,000	$1,506,000	210	201
2006-2010	$16,444,000	$1,710,000	$3,288,000	$1,471 000	240	195
2011-2015	$17,507,000	$1,673,000	$3,350,000	$1,439,000	250	198
Total	$73,781,000	$8,719,000	$14,526,000	$6,925,000	1,080	936

a. (1.5 points) Conduct chi-square tests with an α value of 0.10 on actual vs. expected claim counts to confirm whether or not risk parameters have shifted over time.
Use the following table of critical values:

Degrees of Freedom	Critical Value $\alpha = 0.10$
1	2.706
2	4.605
3	6.251
4	7.779
5	9.236
6	10.645

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b. (1.5 points) To select an expected future claim frequency for LOB 2, the actuarial consultant has decided to assign equal weight (Z/2) to each of the most recent two groups of accident years and the remaining weight (1-Z) to the overall mean frequency.
Calculate the expected future claim frequency for LOB 2 by first using the mean-squared-error (MSE) criterion to determine the optimal value for Z from the following three choices:

Z value	MSE
0.10	Not provided
0.50	0.0190%
0.90	0.0164%

(The following information relates to parts c., d., e., f., and g. below)
The insurance company is planning to write $10,000,000 of new business for LOB 1 in the new territory. The following reinsurance treaty options are available to the insurance company to support its expansion:

<u>Option 1: 2,000,000 xs 6,000,000 Aggregate Excess of Loss</u>

- Rate on Line = 12.5%

<u>Option 2: 25% Aggregate Quota Share</u>

- Ceding Commission = 20%
- Target ceded profit of 20%, which specifies that if ceded premiums less ceding commission and ceded loss exceeds 20% of ceded premiums, the excess is paid back to the cedent through a profit commission
- Aggregate Ceded Loss Ratio Cap = 220%

c. (2.25 points) Calculate the ceded profit for each reinsurance option for the following gross loss amount scenarios:

Scenario	LOB 1 Gross Loss Amount
1	5,000,000
2	10,000,000
3	20,000,000
4	30,000,000

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d. (1.5 points) The actuarial consultant has decided that a prospective loss ratio of 20% for LOB 1 is appropriate, but that the gross capital required to support the expansion is $8,000,000 based on the insurance company's requirement to hold capital at a 1-in-100 year return period. Assume that aggregate LOB 1 ground-up losses are lognormally distributed.

The cumulative distribution function of a lognormal distribution is: $F(x) = \Phi\left(\frac{\ln(x) - \mu}{\sigma}\right)$, where Φ is the standard normal CDF, and μ and σ are the mean and standard deviation of the associated normal distribution.

The mean and variance of a lognormal distribution are given below, as well as a table of values from the standard normal distribution function:

$E(X) = \exp(\mu + \sigma^2 / 2)$ $\qquad$ $Var(X) = \{\exp(\sigma^2) - 1\} \exp(2\mu + \sigma^2)$

x	Φ(x)
1.751	96.00%
1.814	96.52%
1.881	97.00%
1.916	97.23%
2.054	98.00%
2.121	98.31%
2.326	99.00%
2.531	99.43%

Determine the coefficient of variation for LOB 1, assuming it is less than 100%.

e. (1 point) Calculate the probability of each of the following for LOB 1 assuming aggregate losses are lognormally distributed:

i. Attaching the 2,000,000 xs 6,000,000 Aggregate Excess of Loss treaty

ii. A negative ceded profit on the Aggregate Quota Share

f. (0.5 point) State one advantage and one disadvantage of the Aggregate Quote Share from the perspective of the reinsurer.

g. (0.75 point) Recommend and justify which reinsurance treaty option the insurance company should select for LOB 1.

Assume both reinsurance treaty options provide the same amount of capital relief.

1. (a) Work separately on each line of business.
Work with the ratio of claim counts to premium. (See my comment.)
For LOB 1, the overall frequency (per $million) is 1080/73.781 = 14.6379.
Thus the expected claim count for the first period is: (14.6379)(12.003) = 175.70.

Accident Years	Earned Premiums	Ultimate Claim Counts	Expected Claim Count	Chi-Square
1991-1995	$12,033,000	170	176.14	0.214
1996-2000	$13,812,000	210	202.18	0.303
2001-2005	$13,985,000	210	204.71	0.137
2006-2010	$16,444,000	240	240.71	0.002
2011-2015	$17,507,000	250	256.27	0.153
Total	$73,781,000	1,080		**0.808**

The contributions are: (observed - expected)^2 / expected.
We compare to the 10% critical value for 5 - 1 = 4 degrees of freedom: 7.779.
Since 0.808 < 7.779, there is no evidence of shifting risk parameters for LOB 1.
For LOB 2, the Ch-Square test statistic is

Accident Years	Earned Premiums	Ultimate Claim Counts	Expected Claim Count	Chi-Square
1991-1995	$1,766,000	170	189.58	2.023
1996-2000	$1,819,000	172	195.27	2.774
2001-2005	$1,751,000	201	187.97	0.903
2006-2010	$1,710,000	195	183.57	0.711
2011-2015	$1,673,000	198	179.60	1.885
Total	$8,719,000	936		**8.296**

Since 8.296 > 7.779, there is evidence of shifting risk parameters for LOB 2.

(b) Since MSE follows a U-shaped curve, Z = 0.9 must be the best of the three choices.
(The CAS Examiner's Report did not include this solution, which is based on the what is said in the paper by Mahler: "Generally the mean squared error can be written as a second order polynomial in the credibilities." "This in turn allows one to obtain linear equation(s) which can be solved for the least squares credibilities ..." Here with one Z, this implies a U-shaped curve.)
Continue to work with frequencies with respect to premiums ($ million).
For LOB 2: (45%)(195/1.710) + (45%)(198/1.673) + (1 - 90%)(936/8.719) =
115.3 claims per $ million.
Alternately, one can determine the MSE for Z = 0.1.
If we use frequencies per thousand dollars, then the prediction for 2001-2005 is:
(5%)(170/1,766) + (5%)(172/1,819) + (1 - 10%){(170 + 172)/(1,766 + 1,819)} = 0.0953988.
The prediction for 2006-2010 is:
(5%)(172/1,819) + (5%)(201/1,751)
+ (1 - 10%){(170 + 172 + 201)/(1,766 + 1,819 + 1,751)} = 0.102053.
The prediction for 2011-2015 is:
(5%)(201/1,751) + (5%)(195/1,710)
+ (1 - 10%){(170 + 172 + 201 + 195)/(1,766 + 1,819 + 1,751 + 1,710)} = 0.105708.
Thus the mean squared error is:
$\{(0.0953988 - 201/1{,}751)^2 + (0.102053 - 195/1{,}710)^2$
$+ (0.105708 - 198/1{,}673)^2\} / 3 = 0.0226\%$.
Thus Z = 90% has the lowest mean squared error.
(45%)(195/1710) + (45%)(198/1673) + (1 - 90%)(936/8719) = **0.1153** claims per $ thousand.
Alternately, if we use frequencies per dollar, then the prediction for 2001-2005 is:
(5%)(170/1,766,000) + (5%)(172/1,819,000) + (1 - 10%){(170 + 172)/(1,766,000 + 1,819,000)} = 0.0000953988.
The prediction for 2006-2010 is:
(5%)(172/1,819,000) + (5%)(201/1,751,000)
+ (1 - 10%){(170 + 172 + 201)/(1,766,000 + 1,819,000 + 1,751,000)} = 0.000102053.
The prediction for 2011-2015 is:
(5%)(201/1,751,000) + (5%)(195/1,710,000)
+ (1 - 10%){(170 + 172 + 201 + 195)/(1,766,000 + 1,819,000 + 1,751,000 + 1,710,000)} = 0.000105708.
Thus the mean squared error is:
$\{(0.0000953988 - 201/1{,}751{,}000)^2 + (0.000102053 - 195/1{,}710{,}000)^2$
$+ (0.000105708 - 198/1{,}673{,}000)^2\} / 3 = 2.3 \times 10^{-10}$.
Thus the MSE for Z = 10% is smallest. Using Z = 10%:
(5%)(195/1,710,000) + (5%)(198/1,673,000) + (1 - 10%)(936/8,719,000)
= **0.000108** claims per dollar.

(c) (i) The reinsurance premium is: (12.5%)(2 million) = $250,000.
For scenario 1 the ceded loss is zero, and the reinsurer's profit is **$250,000**.
For the other scenarios, the ceded loss is 2 million and the reinsurer's profit is:
$250,000 - $2,000,000 = **-$1,750,000**.
(ii) The ceded premium is: (25%)(10,000,000) = $2.5 million.
The ceding commission is: (20%)($2.5 million) = $500,000.
Thus net of the ceding commission, the reinsurer receives $2 million.
If the ground up losses are 6 million, then the ceded losses are (25%)(6) = 1.5 million, and the reinsurer's profit is: (2 million - 1.5 million) / 2.5 million = 20%.
Thus the profit commission is paid when the ground-up losses are less than $6 million; this is the case for scenario 1.
For scenario 1, (25%)(5M) = 1.25M in loss is ceded; the profit commission is:
2.5M - 0.5M - 1.25M - (20%)(2.5M) = 0.25M.
Scenario 4 exceeds the 220% aggregate ceded loss ratio cap.
Thus in this case, only (220%)(2.5 million) = $5,500,000 of loss is ceded to the reinsurer.

Scenario	Ceded Loss	Ceded Premium - Ceding Commission	Profit Commission	Profit
1	1,250,000	2,000,000	250,000	**$500,000**
2	2,500,000	2,000,000	0	**-$500,000**
3	5,000,000	2,000,000	0	**-$3,000,000**
4	5,500,000	2,000,000	0	**-$3,500,000**

(d) As per the intent of the examiners, assume that 8 million is the 99th percentile of the aggregate loss distribution. (See my comment.)

$\Rightarrow$ F(8 million) = $\Phi\left(\frac{\ln(8{,}000{,}000) - \mu}{\sigma}\right)$ = 0.99. $\Rightarrow$ $\frac{\ln(8{,}000{,}000) - \mu}{\sigma}$ = 2.326.

$\Rightarrow$ 15.895 - μ = 2.326 σ.

Also the mean aggregate losses are: (20%)(10 million) = 2 million.
$\Rightarrow$ $\exp(\mu + \sigma^2 / 2)$ = 2 million. $\Rightarrow$ $\mu + \sigma^2 / 2$ = 14.509.

Solve the two equations in two unknowns by eliminating μ.

$\sigma^2 - 4.652\,\sigma + 2.772 = 0. \Rightarrow \sigma = \frac{4.652 \pm \sqrt{4.652^2 - (4)(2.772)}}{2}$ = 0.7017 or 3.950.

$1 + CV^2 = E[X^2]/E[X]^2 = \exp(2\mu + 2\sigma^2) / \exp(\mu + \sigma^2 / 2)^2 = \exp(\sigma^2)$.
$\Rightarrow CV = \sqrt{\exp(\sigma^2) - 1}$. $\Rightarrow CV = \sqrt{\exp(0.7017^2) - 1}$ = **0.798**. (We want CV < 1.)

Alternately, μ = 15.895 - (2.326)(0.7017) = 14.263.
The variance is: $\{\exp(\sigma^2) - 1\} \exp(2\mu + \sigma^2) = \{\exp(0.7017^2) - 1\} \exp[(2)(14.263) + 0.7017^2]$ = 2.547×10^{12}.
CV = Standard Deviation / Mean = $\sqrt{2.547 \times 10^{12}}$ / 2 million = **0.798**.
Alternately, the insurer should be concerned if the aggregate annual loss exceeded the capital plus premium, or \$18 million. $\Rightarrow$ F(18 million) = $\Phi\left(\frac{\ln(18{,}000{,}000) - \mu}{\sigma}\right)$ = 0.99.

$\Rightarrow$ $\frac{\ln(18{,}000{,}000) - \mu}{\sigma}$ = 2.326. $\Rightarrow$ 16.706 - μ = 2.326 σ.

Also the mean aggregate losses are: (20%)(10 million) = 2 million.
$\Rightarrow$ $\exp(\mu + \sigma^2 / 2)$ = 2 million. $\Rightarrow$ $\mu + \sigma^2 / 2$ = 14.509.

Solve the two equations in two unknowns by eliminating μ.

$\sigma^2 - 4.652\,\sigma + 4.394 = 0. \Rightarrow \sigma = \frac{4.652 \pm \sqrt{4.652^2 - (4)(4.394)}}{2}$ = 1.318 or 3.334.

$\Rightarrow CV = \sqrt{\exp(\sigma^2) - 1} = \sqrt{\exp(1.318^2) - 1}$ = **2.164**. (Cannot have CV < 1.)

(e) Assume from part (d) that μ = 14.263 and σ = 0.7017.

i. $S(6 \text{ million}) = 1 - \Phi\left(\frac{\ln(6{,}000{,}000) - 14.263}{0.7017}\right) = 1 - \Phi[1.916] = 1 - 97.23\% =$ **2.77%**.

ii. For a negative profit on the quota share, the ceded losses have to be greater than 2 million, and therefore the direct losses have to greater than 8 million.

$S(8 \text{ million}) = 1 - \Phi\left(\frac{\ln(8{,}000{,}000) - 14.263}{0.7017}\right) = 1 - \Phi[2.326] = 1 - 99.00\% =$ **1.00%**.

(This also follows from having set the 99th percentile of the aggregate losses = 8 million.)
Alternately, if from part (d) μ = 13.640 and σ = 1.318.

i. $S(6 \text{ million}) = 1 - \Phi\left(\frac{\ln(6{,}000{,}000) - 13.640}{1.318}\right) = 1 - \Phi[1.493]$. Using a computer this is 6.78%.

ii. $S(8 \text{ million}) = 1 - \Phi\left(\frac{\ln(8{,}000{,}000) - 13.640}{1.318}\right) = 1 - \Phi[1.711]$. Using a computer this is 4.36%.

(f) Advantages of Quota Share from the point of view of the reinsurer:
- As determined in part c, the maximum profit for the reinsurer is greater for the Quota Share than the Aggregate Excess.
- Since under the quota share the insurer retains 75% of losses, the insurer will have a financial incentive for loss control. In contrast, once losses reach the covered layer for the Aggregate Excess, the insurer shares none of the losses in that layer.
- Reinsurer can share in the profits of the primary insurer.
- The $2 million the reinsurer gets net of ceding commission, can be used to earn investment income. (The reinsurer also takes on more total risk.)
- Easier to administer since the same percent is ceded on every loss.
- Less volatility compared to the Aggregate Excess treaty, since for the Quota Share a fixed percent of ground up losses are ceded.
- The reinsurer learns about losses on a first dollar basis, so it will have a shorter report lag and where necessary can try to help the insurer to control large losses.

Disadvantages of Quota Share from the point of view of the reinsurer:
- As determined in part c, the largest loss for the reinsurer in absolute value for the Quota Share is more than for the Aggregate Excess. (The ceded loss ratio cap of 220% does limit the downside of the reinsurer.)
- The profit commission limits the reinsurer's upside.
- There is a potential for a large loss for the reinsurer, if loss experience of the primary insurer is bad.

(g) I recommend the Quota Share treaty for the insurer.
The Quota Share will allow the insurer to feel the effects of its own loss ratio, while providing the desired capital relief. The Quota Share provides protection up to a 220% loss ratio.
With $10 million in direct premium, the given Aggregate Excess of Loss treaty covers the layer from a 60% to an 80% loss ratio. Thus this treaty would provide limited relief to the insurer in the case of a really bad year such as a 150% loss ratio. The insurer would be paying $250,000 for this very limited protection.
The Quota Share also has a profit commission which rewards the insurer for good experience.
Also with Quota Share, the reinsurer can provide underwriting guidance for this new territory.
Alternately, I recommend the Aggregate Excess of Loss treaty for the insurer for LOB 1.
The expected loss ratio is only 20% for LOB 1. The Quota Share would be ceding away profitable business. In contrast, the Aggregate Excess would be providing protection against bad years (loss ratios of more than 60%) for a fixed cost of $250,000.

<u>Comment</u>: A good example of why it is hard to write a good integrated question.
The observed and projected loss ratios for LOB 1 are extremely small.
Part (a) would make sense if they had provided you with exposures that are not inflation sensitive; in that case we would still need to be concerned about the effect of changes in mix of business. We would expect the ratio of claim counts to <u>premium</u> to decline very significantly over time, as inflation increases average severity over these extended periods of time. Thus using a “frequency” with respect to premiums, as intended by the examiners, demonstrates a lack of understanding. The student was expected to somehow read the examiner’s mind and then to mechanically put together a Chi-Square test that does <u>not</u> test whether risk parameters shift over time in any meaningful way. In the syllabus reading by Mahler at page 236, the technique is applied to games lost by 5 year period versus expected games lost.
It should be noted that one can calculate the average severities from the data given.
For LOB 1: 2,329,000/170 = $13,700, 13,152, 13,319, 13,700, 13,400.
For LOB 2: 1,236,000/170 = $7271, 7401, 7493, 7544, 7268.
These average severities are virtually constant over decades, which should make an actuary very suspicious of the given data.
In Part (b), one is expected to continue to use the ratio of claim counts to <u>premium</u>.
Applying the given credibility technique when one has only two periods, just applies equal weight to the two periods, regardless of Z; thus it makes no sense to include it in the calculation of MSE, but also does not harm as long as one is consistent. The calculated mean squared error depends on whether you include this first situation and what units you use as the denominator of the frequency; the given mean squared errors were calculated including the first situation and using thousands of dollars in the denominator.
It turns out the optimal credibility in this case is 100% rather than 90%.
In part (c), as usual on this exam, we ignore the reinsurer’s expenses and investment income in determining its profit,
In part (d), the insurer should be concerned if the aggregate annual loss exceeded the capital plus premium, or $18 million. See the equation at page 38 of Grossi and Kunreuther.
The examiner’s instead intended that the insurer should be concerned if the aggregate annual loss exceeded the $8 million in capital; in other words, the examiner’s assumed that solvency would be impaired if the insurer ran an 80% loss ratio.
Parts (f) and (g) are not really covered by the syllabus readings for this exam.
The ceding commission for the Quota Share is intended to cover the expenses of the primary insurer, and thus cannot be used by the primary insurer to earn investment income.

2. (9.5 points) A certain golf course holds a series of tournaments throughout the year, during which prizes are awarded if a player hits a hole-in-one. The prize amounts vary based on the difficulty of the hole, but are consistent across tournaments.
The course has approached an insurer to design a policy which would protect the course from large prize payouts.
The insurer would pay the full prize amount directly to the winning player, and seek recoveries of any applicable deductible from the golf course.
A single annual policy will cover all of the tournaments throughout a given year.
The total number of holes-in-one in a given year (n), across all tournaments, has the following probability distribution:

n	P(n)
0	0.80
1	0.15
2	0.05
3 or more	0.00

Given that a hole-in-one has occurred, the prize amount (x) has the following probability distribution:

x	P(x)
$100,000	0.40
$250,000	0.35
$500,000	0.24
$1,000,000	0.01

The expenses and profit for this policy are:

Fixed Expenses	None
General Expenses	8% of premium
Commission	10% of premium
Taxes	5% of premium
Underwriting Profit	5% of premium

The above assumptions apply to all program structures.
a. (1 point) Calculate the annual guaranteed cost premium for this policy.
b. (1 point)
Calculate the annual premium for this policy under a $100,000 per-occurrence deductible.

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c. (1.75 points) Calculate the annual premium for this policy under a $1,000,000 annual aggregate deductible and no per occurrence deductible.
d. (1 point) Calculate the annual premium for this policy under a $250,000 franchise deductible.
e. (1 point) Explain which of the deductible structures above would generate the greatest credit risk to the insurer, and estimate the magnitude of that risk.
f. (1.25 points) The insurer has allocated $45,000 of capital to support the guaranteed cost policy in part a. above.
Recommend and justify which of the three options below would be the most appropriate capital requirement to support the per-occurrence deductible policy in part b. above:
i. $45,000
ii. $32,000
iii. $19,000
g. (2.5 points) Based on the success of this program, the insurer wishes to expand this coverage to other golf courses.
The insurer will offer varying per occurrence limits with no aggregate limits, the prize amounts would remain the same, and the same distribution assumptions would apply.
Using $100,000 as a basic limit, use the variance method to calculate the risk-loaded ILF for a $500,000 policy limit. Assume $\rho(100,000) = 5,750$ and use $k = 0.0000005$.

2. (a) Average Frequency is: (0)(0.8) + (1)(0.15) + (2)(0.05) = 0.25.
Average Severity is: (0.4)(100,000) + (0.35)(250,000) + (0.24)(500,000) + (0.01)(1,000,000) = $257,500.
Expected Loss = (0.25)($257,500) = $64.375.
Premium = 64,375 / (1 - 28%) = **$89,410**.
(b) Average payment per loss is: 257,500 - 100,000 = 157,500.
Premium = (0.25)(157,500) / (1 - 28%) = **$54,688**.
Alternately, the average payment per loss is:
(0.4)(0) + (0.35)(150,000) + (0.24)(400,000) + (0.01)(900,000) = $157,500. Proceed as before.
(c) In order to have the aggregate loss be greater than 1 million, we need to have two claims, at least one of which is of size 1 million.
For example, given we have two claims, the probability that one is of size 100,000 and the other is of size 1 million is: (2)(0.4)(0.01).
In this case, the insurer would pay: 1.1million - 1 million = 100,000.
Expected Loss =
(0.05) ($1000) {(2)(0.4)(0.01)(100) + (2)(0.35)(0.01)(250) + (2)(0.24)(0.01)(500) + (0.01^2)(1000)} = 252.5.
Premium = 252.5 / (1 - 28%) = **$350.7**.
(d) The insurer pays nothing for a loss less than or equal to $250,000 and pays the whole loss for a loss of size greater than $250,000.
Average payment per loss is: (0.24)(500,000) + (0.01)(1,000,000) = $130,000.
Premium = (0.25)($130,000) / (1 - 28%) = **$45,139**.
(e) In all three cases the insurer pay the full prize amount directly to the winning player, and seeks recoveries of any applicable deductible from the golf course.
The $1,000,000 annual aggregate deductible has the largest expected deductible payments and thus the largest credit risk; the expected deductible payments are: 64,375 - 252.5 = **$64,122.5**.
(f) The insurer is taking on less risk with the per occurrence deductible and thus needs less capital than the $45,000 needed to support the full coverage policy.
The premium to capital ratio for the full coverage policy is: 89,410 / 45,000 = 1.99.
Applying the same ratio to per occurrence deductible policy: $54,688 / 1.99 = $27,481.
However, the per occurrence deductible policy has a larger coefficient of variation of aggregate loss than the full coverage policy, so I would want somewhat more capital than $27,481.
Of the given choices, **$32,000** makes sense.

(g) E[X ; 500,000] = (0.4)(100,000) + (0.35)(250,000) + (0.25)(500,000) = 252,500.
E[X^2 ; 500,000] = (0.4)($100,000^2$) + (0.35)($250,000^2$) + (0.25)($500,000^2$) = 88,375 million.
Var[X ; 500,000] = 88,375 million - $252,500^2$ = 24,619 million.
Second Moment of frequency is: (0^2)(0.8) + (1^2)(0.15) + (2^2)(0.05) = 0.35.
Variance of frequency is: 0.35 - 0.25^2 = 0.2875.
Var[Agg] = Mean[Freq] Var[Sev] + Var[Freq] Mean[Sev]^2 =
(0.25)(24,619 million) + (0.2875)($252,500^2$) = 24,485 million.
Thus the risk load for 500,000 is:
Var[Agg] k / E[N] = (24,485 million)(0.0000005) / 0.25 = 48,969.

$$\text{ILF}(500,000) = \frac{E[X\,;\,500,000] + \rho(500,000)}{E[X\,;\,100,000] + \rho(100,000)} = \frac{252,500 + 48,970}{100,000 + 5750} = \mathbf{2.851}.$$

Alternately, the risk load is proportional to the variance of the aggregate.
For a 100,000 limit, the severity is always 100,000.
Var[Agg for 100,000 limit] = (0.2875)($100,000^2$) = 2875 million.
Thus the risk load for 500,000 is:
ρ(500,000) = (24,485/2875) ρ(100,000) = (8,516522)(5750) = 48,970.
Proceed as before.
Alternately, as per Bahnemann, for the frequency distribution:
δ = VAR[N]/E[N] - 1 = 0.2875 / 0.25 - 1 = 0.15.
Then the variance of the aggregate loss is: E[N] {E[X^2 ; L] + δ E[X ; L]^2}.
For the 500,000 limit, the variance of aggregate loss is:
(0.25) {88,375 million + (0.15)($252,500^2$)} = 24,485 million. Proceed as before.
Alternately, ρ(500,000) = k {E[X^2 ; 500,000] + δ E[X ; 500,000]^2} =
(0.0000005) {88,375 million + (0.15)($252,500^2$)} = 48,970. Proceed as before.

<u>Comment</u>: In part (d), franchise deductibles would not be commonly used in this situation.
In part (e), one could multiply the expected deductible payments by an estimated chance of default, in order to get an amount to load in the premium in order to cover credit risk.
Part (f) is discussed in Section 9 of Chapter 2 of "Individual Risk Rating".
The insured retains the less risky primary losses, while the insurer covers the more risky excess losses. Thus as a percent of premium, the risk of the insurer is higher. Therefore, the reduction in capital is less than in proportion to the reduction in expected insured losses.
Also the introduction of credit risk, reduces the overall reduction in capital.
In part (g), one has to assume that all expenses (including ALAE) are proportional to expected losses by limit.

3. (2.75 points) An insurance company has a private passenger auto book of business with the following claims experience:

Group	Number of Accident-Free Years	Earned Premiums	Current Merit Rating Factor	Number of Claims Incurred
A	3 or more	216,000,000	0.60	25,000
X	2	135,000,000	0.75	18,000
Y	1	63,750,000	0.85	20,000
B	0	200,000,000	1.00	C
Total		614,750,000		63,000 + C

• Claim counts follow a Poisson distribution with parameter $\lambda = 0.05$.

• The credibility for the new policy period for an insured that has had no claim-free years is equal to 0.038.

a. (1.5 points) Calculate C, the number of claims incurred for Group B.

b. (0.75 point) Calculate the merit rating factor for an exposure that is accident-free for two or more years for the new policy period.

c. (0.5 point) Briefly explain two circumstances under which using earned premium as the exposure base would not correct for maldistribution.

3. (a) Looking at the experience of those insureds who were accident free for 0 years:
$M = Z / (1 - e^{-\lambda}) + (1 - Z)(1) = (0.038)/(1 - e^{-0.05}) + (1 - 0.038) = 1.741$.
We need to adjust premiums to the Group B rate:
216/0.6 = 360, 135/0.75 = 180, 63.750/0.85 = 75. 360 + 180 + 75 + 200 = 815 million.
M is ratio of the premium based frequency for those in Group B to that overall.

$$1.741 = M = \frac{C / 200}{(63{,}000 + C) / 815}. \Rightarrow 109{,}683 + 1.741C = 4.075C. \Rightarrow C = \mathbf{46{,}994}.$$

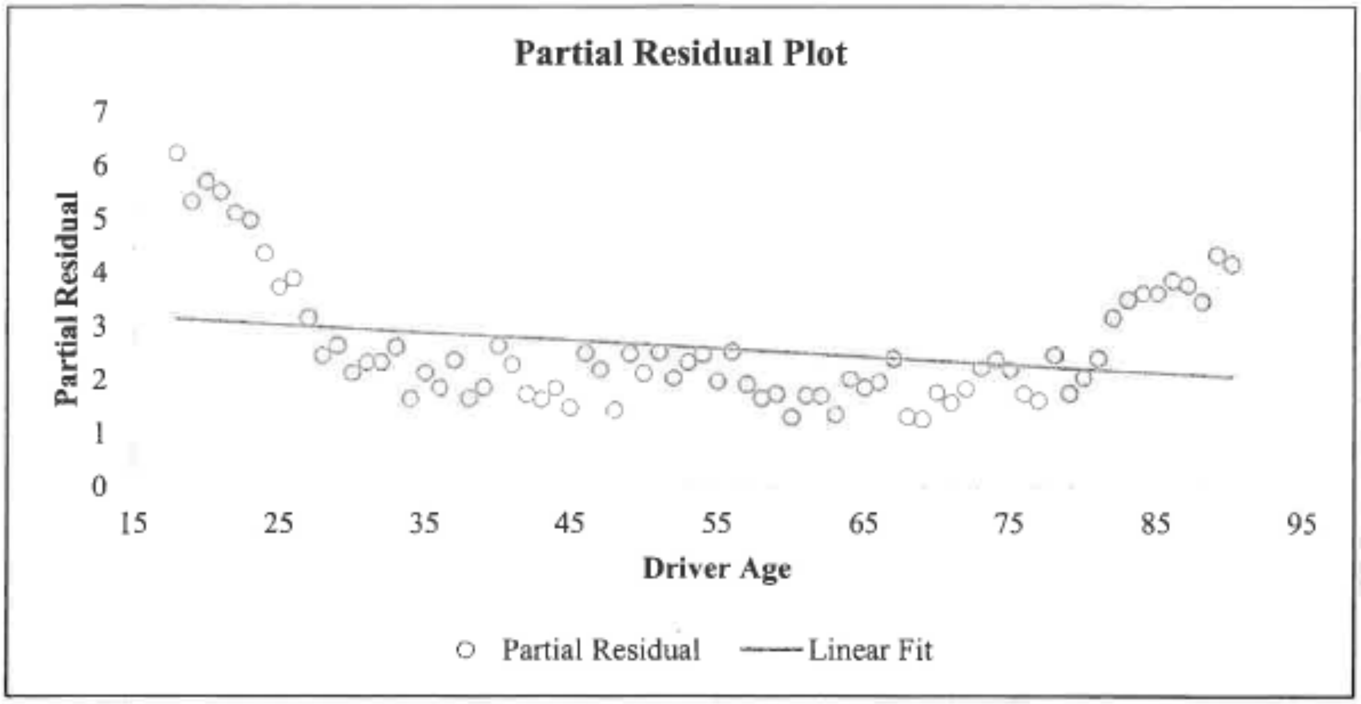
Alternately, $Z = (M - 1) / \{1/(1 - e^{-\lambda}) - 1\} = (M - 1)(e^{\lambda} - 1)$.
$\Rightarrow 0.038 = (M - 1)(e^{0.05} - 1). \Rightarrow M = 1.741$. Proceed as before.

(b) The premium based frequency for two or more years claims free is:
(25,000 + 18,000) / (360 + 180) = 79.629.
The premium based frequency for Group B is: 46,994 / 200 = 233.32.
Indicated Merit Rating Factor = 79.629/233.32 = **0.34**.
(c) A premium base eliminates maldistribution only:
(1) If high frequency territories are also high premium territories.
(2) If territorial differentials are proper.
Thus using earned premium as the exposure base would not correct for maldistribution if:
(1) High premium territories are not also high frequency territories.
(2) or if the current territory differentials are not (approximately) correct.
Comment: The second bullet should have read instead “The credibility for one year of data estimated by examining the experience of those insureds who were accident free for zero years is equal to 0.038.”
$\lambda = 0.05$ should be the mean frequency per exposure.
In Table 1 of Bailey-Simon, earned premiums have been adjusted to the Group B rate.
There is no educational value to making part (a) a backwards question.
In part (b), “candidates who calculated the mod relative frequency to total or the merit rating factor relative frequency to group B received full credit.” Nevertheless, the merit rating factor, which is what was asked for, is gotten by measuring with respect to Group B, and differs from the mod which is relative to overall.
The current merit rating factors are for 2 years claims free or 3 or more years claims free. There is no current merit rating factor for 2 or more years claims free.

4. (0.75 point) An insurance company is launching a new telematics program for their private passenger automobile book of business. Telematics devices record various attributes such as miles driven and braking practices. Management decided to give a 5% discount to all customers that participate in the program. The Department of Insurance questions the filing and wants the company to address the following potential concerns:
• Risk of adverse selection
• Relationship between risk and expected outcomes
• Practicality of monitoring the discount's effectiveness
Defend the use of the discount by briefly addressing each of the concerns in light of Actuarial Standard of Practice No. 12, Risk Classification.

4. Risk of adverse selection:
Drivers who know that they drive poorly are unlikely to submit to monitoring. Therefore, this program should attract less risky drivers. Therefore, adverse selection is not a concern. Instead, there is potential for favorable selection.
Relationship between risk and expected outcomes:
Drivers are more likely to drive safely if they know they are being monitored. Therefore, drivers who get a discount due to participating in this program, should have lower expected losses.
Practicality of monitoring the discount's effectiveness:
It is practical to monitor the discount's effectiveness; compare the loss ratios of those who participate (including the effect of the 5% discount) to those who do not.
<u>Comment</u>: Such programs usually offer the insured a chance for a discount; the size of the discount, if any, depends on miles driven and braking practices. Therefore, I found it difficult to do as asked and to defend the use of the discount in this question, which gives a 5% discount to anyone who participates.
I think a better question would have asked one to assess the use of the discount with respect to each of the three concerns.
Since the insurer is not using information on miles driven and braking practices to determine whether or not to give a discount, and how large of a discount, there is not a direct relationship between this risk classification and expected losses.
Since the insureds decide whether or not they want to participate in the program, and there is no direct relationship to expected costs, there is a possibility of adverse selection.

5. (1.5 points)
An actuary is analyzing a partial residual plot of the driver age variable, which is shown below:



a. (1 point)
Adding polynomial terms is one approach to address the non-linearity in the driver age variable. Briefly describe two other alternative approaches and how they can be used to improve the fit of the driver age variable shown above.
b. (0.5 point) Briefly describe a downside to each of the two alternative approaches recommended in part a. above.

5. (a) i. One can bin driver age into groups.
For this example three bins may work well: 18 to 25, 26 to 80, above 80.
ii. One can use hinge functions. A hinge function is of the form: $(X - c)_+ = \max(0, X - c)$.
This will result in a piecewise linear function, with a change in slope at each breakpoint c_i.
For this example, I would choose breakpoints at 25 and 80.
(b) i. With binning: Continuity is not guaranteed.
Variation within intervals is ignored.
There may not be enough data in each bin to be credible.
There could be non-intuitive results, such as reversals.
ii. Using hinge functions:
The breakpoints must be selected by the user.
<u>Comment</u>: In both cases, more parameters are added to the model; the principal of parsimony states that we prefer a simpler model with fewer parameters, all else being equal.

6. (2.5 points) An actuary is comparing the output of two generalized linear models to develop a new rating plan for personal auto. Model statistics are shown below:

	Saturated Model	Model A	Model B
Log-Likelihood	-1,000	-1,500	-1,465
Estimated Dispersion Parameter	1.75	1.75	1.75

- Model A is a nested model of Model B,
  where Model B has an additional variable for driver age.
- Driver age is fit using a second-order polynomial.
- The critical value to be used from the F-distribution is 19.5.

a. (2 points) Using two statistical tests, recommend whether or not driver age should be included in the rating plan.

b. (0.5 point) Describe why the deviance statistic alone should not be used to assess model fit.

6. (a) Assuming model A has an intercept, adding driver age using a second order polynomial adds two parameters.
i. Deviance = 2 {(loglikelihood for the saturated model) - (loglikelihood for the fitted model)}.
$D_A = (2)\{-1000 - (-1500)\} = 1000$. $D_B = (2)\{-1000 - (-1465)\} = 930$.

$$F = \frac{(D_A - D_B) / (\text{number of added parameters})}{\hat{\phi}_A} = \{(1000 - 930) / 2\} / 1.75 = 20.$$

We compare to the given critical value of 19.5.
Since 20 > 19.5, model B is significantly better than model A.
Driver age should be included in the rating plan.
ii. Let p be the number of fitted parameters for Model A.
$AIC_A = (-2)(-1500) + 2p = 3000 + 2p$.
$AIC_B = (-2)(-1465) + 2(p+2) = 2934 + 2p$.
Since $AIC_B < AIC_A$, model B is better than model A.
Driver age should be included in the rating plan.
iii. Let n be the number of data points (for each of the models).
$BIC_A = (-2)(-1500) + p \ln(n) = 3000 + p \ln(n)$.
$BIC_B = (-2)(-1465) + (p+2) \ln(n) = 2930 + (p+2) \ln(n)$.
$BIC_A - BIC_B = 70 - 2 \ln(n)$. This difference is positive for $n < e^{35} = 1.586 \times 10^{15}$.
Thus $BIC_B < BIC_A$, and model B is better than model A.
Driver age should be included in the rating plan.
(b) When parameters are added to a model, the deviance improves (gets smaller). Thus using deviance alone would lead to overfitting. The issue is whether the deviance gets <u>significantly</u> better. Using AIC or BIC is more appropriate, as they penalize for adding new parameters.
<u>Comment</u>: The degrees of freedom for Model A =
number of observations minus number of fitted parameters for model A.
The F-statistic has degrees of freedom equal that of Model A and 2.
The given critical value is the 5% critical value for 100 and 2 degrees of freedom.
According to the CAS Examiner's report, for part b common mistakes included: "Giving some of the limitations of deviance such as needing to have the same underlying dataset with the same distribution. This limitation is not restricted to deviance alone." Besides this limitation on using the deviance, Section 6.1.3 of "Generalized Linear Models for Insurance Rating" also says: "For any comparisons of models that use deviance, in addition to the caveat above, it is also necessary that the assumed distribution and the dispersion parameter (basically, everything other than the coefficients) must be identical as well." However, the question asked why the deviance statistic <u>alone</u> should not be used to assess model fit. As seen in the solution to part (a), deviance together with the difference in number of parameters and the estimated dispersion parameter can be used to assess model fit.

7. (2.5 points) An actuary is planning to add a credit-based insurance score to a model that estimates the probability of a policy having a claim. The actuary has decided to offset all of the current model variables before fitting the new variable.
Given the following:

- The current model (without the insurance score variable) is a logit-link binomial GLM (logistic regression).
- The logit link function is defined as $g(\mu) = \ln\left(\frac{\mu}{1 - \mu}\right)$
- The insurance score is a continuous variable having a value between 1 and 100.
- The current fitted values and insurance score for three policies as well as regression results from the fit of the insurance score variable are given below:

Policy Number	Fitted Probability Without Insurance Score	Insurance Score
1	1.3%	78
2	20.3%	92
3	2.5%	35

Variable	Parameter Estimate
Intercept	1.250
Insurance Score	-0.020

a. (0.5 point)
Calculate the offset term to be used in the regression for each of the three policies above.
b. (0.75 point)
Calculate the revised fitted probability of having a claim for each of the three policies above.
c. (0.5 point) Identify the range of:
i. the logit function
ii. the logistic function
d. (0.25 point) Briefly explain why logistic regression is often used to model probabilities.
e. (0.5 point) Identify and briefly describe one situation in which the use of an offset is preferable to (re)fitting all variables.

7. (a) As per Section 2.6 of “Generalized Linear Models for Insurance Rating”:

$g(\mu_i) = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + ... + \beta_p x_{ip}$ + offset.

Thus the offset for Policy 1 is: $\ln(\frac{0.013}{1 - 0.013})$ = **-4.330**.

The offset for Policy 2 is: $\ln(\frac{0.203}{1 - 0.203})$ = **-1.368**.

The offset for Policy 3 is: $\ln(\frac{0.025}{1 - 0.025})$ = **-3.664**.

(b) In each case we add the offset to the linear component from the insurance score.
For Policy 1: 1.250 + (-0.020)(78) - 4.330 = -4.640.
Probability of a claim is: $\frac{\exp[-4.640]}{1 + \exp[-4.640]}$ = **0.96%**.
For Policy 2: 1.250 + (-0.020)(92) - 1.368 = -1.958.
Probability of a claim is: $\frac{\exp[-1.958]}{1 + \exp[-1.958]}$ = **12.37%**.
For Policy 3: 1.250 + (-0.020)(35) - 3.664 = -3.114.
Probability of a claim is: $\frac{\exp[-3.114]}{1 + \exp[-3.114]}$ = **4.25%**.

(c) The logit function is: $\ln(\frac{x}{1 - x})$, for $0 < x < 1$.
The logit function has range from $-\infty$ to ∞.
The logistic function is: $\frac{e^x}{1 + e^x}$, for $-\infty < x < \infty$.
The logistic function has range from 0 to 1.
(d) Singe the range of the logistic function is 0 to 1, using its inverse the logit as a link function guarantees that the response is in the correct range for probabilities, zero to one.

(e) • For example, assume that deductible relativities are being estimated separately, by a method other than a GLM, then one would include the effect of the deductible relativities via an offset term in the GLM used to estimate classification relativities. (Deductible relativities are better estimated outside GLMs via analysis of loss elimination ratios, since a GLM often produces counterintuitive results due to effect of selection and correlation with variables outside model. This is the also case for other coverage-related options on a policy.)

• When introducing additional variables, but you do not want to change existing ones due to constraints like rate filing approval, IT system constraints, etc.

• Territory rating is impractical to use in a GLM since there are hundreds or even thousands of territories with no easy way to group them without losing signal. However, territory differences are significant so it's important that the rating plan be offset for territory rates. Thus it's best to include territory factors as an offset in a GLM.

• If you're creating a model on renewal business after having already made a model for new business only, you would likely use an offset for many of the variables. This would ensure consistency between the sets of business that you do not expect to change over time.

• When including the effect of a coverage limit in a pure premium model. Limits may be correlated with other covariates not being accounted for in the model and this might lead to inconsistent ILFs based on model results, so it's better to do loss elimination analysis outside of the modeling process and include the effect of a coverage limit as an offset.

<u>Comment</u>: I have taken many responses to part (e) from the CAS Examiner's Report.
From "Generalized Linear Models for Insurance Rating":
"For this reason it is recommended that factors for coverage options—deductible factors, ILFs, peril group factors and the like—be estimated outside the GLM, using traditional actuarial loss elimination techniques. The resulting factors should then be included in the GLM as an offset."
"Territories are not a good fit for the GLM framework. ... Since there are usually many complicated relationships between territory and other variables, your GLM should still consider territory. This is accomplished by including territory in your model as an offset."
In parts (a) and (b), a high insurance score is good and lowers the chance of having a claim.

8. (1.5 points) An actuary has been using risk-adjusted increased limit factors to account for riskiness in pricing. The actuary's coworker has suggested an alternate approach of using non-risk-adjusted increased limit factors and, instead, varying the profit and contingency load with the policy limit.

Compare and contrast the two methods with respect to each of the following:

- Accuracy
- Ease of calculation
- Clarity

8. i. Neither one is more inherently accurate than the other. For the risk-adjusted increased limit factors to be accurate, the choice of method of loading for risk has to be "correct" and the inputs by limit (such as variance) have to be accurately estimated. Also a parameter such as k would need to be appropriately selected. For different profit provisions by limit, the choice of profit model has to be "correct" and the inputs by limit (such as needed capital, the needed return, and length of cashflows) have to be accurately estimated.
ii. If using a relatively simple method of adjusting ILFs for risk, such as shown in Bahnemann, the risk adjusted ILFs would be easier to calculate than would different profit provision by limit. The later would involve changing the inputs to the underwriting profit model and rerunning the model for each limit.
iii. The use of different profit provisions by limit would be less clear. Underwriters and insureds are used to increased limit factors; if risk loads are included in the ILFs there is no need for these parties to know the details rather than the general idea. On the other hand, the different profit provisions would result in different base rates by limit for each class and territory. This would be much less clear than a table of increased limit factors.

Selected sample responses from the CAS Examiner's Report without my endorsement:

Accuracy – the risk adjusted ILF is more accurate than the varying profit and contingency because it is more explicitly calculating the risk load as limits increase. The profit provision varying is more arbitrary.
Accuracy – The variance and standard deviation approaches are much more rigorous and do a better job of calculating risk loads from an accuracy perspective. They can calculate loads more precisely and would not have to be bucketed like the proposed plan would.

Ease of Calculation – The risk load is much more computationally difficult. The profit can be more judgmentally selected.
Ease of Calculation – The proposed plan would be easier to calculate risk loads for. A system of varying profit and contingency could be as simple as you would like. On the other hand, the more complicated you make it, the closer you get to using risk-adjusted ILFs.
Ease of Calculation - Determining a risk load may require more work because the k constant will need to be calibrated to the portfolio. Profit and contingency needs to be set judgmentally for each limit, which may also take time but is far less technically rigorous.
Ease of Calculation - Calculating separate risk loads for each possible limit is much more time consuming and calculation intensive compared to having a variable percentage applied to calculate the premium.

Clarity – The risk load has a foundation in mathematics so would be more clear to a trained eye. But a lay person would likely better understand the variation of profit provisions.
Clarity – The risk-adjusted ILFs are clear in that the loading takes place behind the scenes (in the calculation of the ILF). The proposed method is presumably clear because the method of varying profit and contingency load would be explicitly laid out and simple to apply. In this sense, the proposed method is probably simpler.
Clarity - It's unclear how the variable profit provision load would be determined using the alternate method, while the current method is a defined and reasonable approach.

Comment: Personally I had a lot of trouble answering this question; the syllabus readings provided little help.
The question did not specify how the different profit provisions by limit would be determined; the determination of appropriate of profit provisions is not discussed on this exam.
The two techniques can be made to produce the same premiums.
For example, assume for a given increased limit, the ILF is 2 with no risk adjustment and 2.1 with risk adjustment. Further assume the profit provision (for the basic limit) is 5%, and the variable expenses are 20%. Let x be the equivalent risk adjusted profit provision for that limit.
Then in order to get the same premiums either way: 2 / (1 - 20% - x) = 2.1 / (1 - 20% - 5%).
⇒ 1.50 = 1.68 - 2.1x. ⇒ x = 8.6%. Of course one can go in the other direction as well. A risk adjusted profit provision of 8.6% would lead to a risk adjusted ILF of 2.1.

9. (4.25 points) Three insurers are using experience rating to determine premiums for a specific class of business. The same ten risks were rated using the experience plan of each insurer. Information related to each rating plan is given below:

Insurer 1's Plan

Risk #	Manual Premium	Loss	Mod	Standard Premium
1	$810	$750	0.97	$786
2	$900	$490	0.68	$612
3	$950	$1,075	1.13	$1,074
4	$975	$650	0.78	$761
5	$1,075	$850	0.88	$946
6	$1,100	$1,000	0.96	$1,056
7	$1,225	$1,300	1.06	$1,299
8	$1,300	$800	0.72	$936
9	$1,450	$1,175	0.90	$1,305
10	$1,500	$975	0.76	$1,140

Insurer 2's Plan

Quintile	Manual Loss Ratio	Standard Loss Ratio
1	58.6%	94.5%
2	65.7%	90.0%
3	80.2%	85.3%
4	91.6%	79.7%
5	109.2%	75.3%
Sample Variance	0.0411	0.0059

Insurer 3's Plan

Efficiency Test Statistic 0.0000

a. (2.75 points) Rank each of the insurers' rating plans from most to least equitable using the Efficiency Test as described by Fisher et al.

b. (1.5 points) Explain how adverse selection may affect each of the three insurers in a well-functioning insurance market.

Assume that no adjustments are made to the experience rating plans over time.

9. (a) The efficiency statistic is: $\frac{\text{the variance of the subsequent standard ratios}}{\text{the variance of the subsequent manual ratios}}$.

The smaller the efficiency statistic, the better the experience rating plan.
For insurer #1, we group the risks into quintiles using their mods:

Risk Numbers	Manual Premium	Loss	Manual L.R.	Standard Premium	Standard L.R.
2 ,8	$2200	$1290	58.6%	$1548	83.3%
10, 4	$2475	$1625	65.7%	$1901	85.5%
5, 9	$2525	$2025	80.2%	$2251	90.0%
6, 1	$1910	$1750	91.6%	$1842	95.0%
7, 3	$2175	$2375	109.2%	$2373	100.1%
Sample Variance			0.04110		0.00472

Efficiency statistic for Insurer #1 is: 0.00472/0.04110 = 0.1147.
Efficiency statistic for Insurer #2 is: 0.0059/0.0411 = 0.1436.
Efficiency statistic for Insurer #3 is given as: 0.0000.
Thus based on the efficiency test, **the third plan is best and the second plan is worst**.
(b) Insurer 3's plan has a sample variance of standard loss ratios of zero. In other words, their experience rating plan does a perfect job of correcting for risk differences, at least for these 10 risks for the period of time studied.
Therefore, the third insurer will not be affected by adverse selection.
Insurer 1's plan has increasing standard loss ratios by quintile. This indicates that the plan is correcting too little for risk differences; the credibilities are too low.
Insurer 2's plan has decreasing standard loss ratios by quintile. This indicates that the plan is correcting too much for risk differences; the credibilities are too high.
Therefore, **both the first and second insurer will be affected by adverse selection**.
The first insurer is overcharging good risks, who will tend to leave, and is undercharging bad risks who will be attracted.
The second insurer is overcharging bad risks, who will tend to leave, and is undercharging good risks who will be attracted.
In both cases, over time each insurer will attract risks it is charging too little and will lose risks it is charging too much. Therefore, each insurer's standard premiums will become inadequate. They will be forced to raise their manual rates, which will provide only a temporary fix, as this will drive away more insureds who are being overcharged.
Comment: The manual loss ratios by quintile for the first and second insurer will be equal if the same risks are grouped into each quintile; this appears to be the case here but need not be the case in general.
In the efficiency test, one can use either sample variances (dividing by n - 1) or population variances (dividing by n), as long as the numerator and denominator of the test statistic are consistent.
The given manual premiums are unrealistically low; such risks would be too small to be eligible for experience rating.

10. (2.25 points) An actuary is analyzing an experience rating plan using three years of experience that has the following information:

Policy effective date	January 1, 2019
Policy term	One Year
Annual loss trend	2%
Cap for individual claims	100,000
Credibility factor	0.60
Expected ultimate loss before modification	500,000

Reported Losses on Individual Claims by Policy Year as of June 30, 2018			
2015	2016	2017	2018
3,450	2,389	456	5,694
5,000	345	126,890	99,832
234	1,236,806	2,345	76,532
98,000		1,874	
324,789		690	
		26,986	

The actuary has determined limited loss development factors (LDFs) and limited expected values using aggregate data from similar policies as follows:

Maturity to Ultimate	Limited LDFs
18 months	1.50
30 months	1.23
42 months	1.20

Limit	Limited Expected Value
100,000	17,500
500,000	28,567
1,000,000	43,393
Unlimited	85,504

a. (1.25 points)
Calculate the expected reported losses for this plan for the three years of experience combined.
b. (1 point) Calculate the experience modification factor for this policy.

10. (a) Data from PY18 is at 6 months and is too immature to use in experience rating; we will use data from PY15, PY16, and PY17.
We need to multiply by 17,500 / 85,504 in order to account for the effect of the 100,000 limit.
We need to detrend. For example for PY17 the detrend factor is $1/1.02^2$.
We need to divide by the relevant LDF in order to get the expected reported losses.
For example for PY17, 1/1.50.
(500,000) (17,500 / 85,504) ($1.02^{-2}/1.50 + 1.02^{-3}/1.23 + 1.02^{-4}/1.20$) = **222,758**.
(b) The reported losses for PY15, PY16, and PY17 need to be capped at 100,000:
3450 + 5000 + 234 + 98,000 + 100,000 + 2389 + 345 + 100,000 + 456 + 100,000 + 2345 + 1874 + 690 + 26,986 = 441,769.
M = (60%) (441,769 / 222,7580) + (1 - 60%) = **1.59**.
Comment: Similar to step 3 of the case study in "Individual Risk Rating".

11. (1.25 points)
Given the following information for a construction insured's general liability policy that is subject to the ISO Commercial General Liability Experience and Schedule Rating Plan:

Company Subject Loss Cost	270,500
Actual Experience Ratio	0.85
Policy Effective Date	January 1, 2020
Policy Term	Annual

All major changes in the last five years for this insured:
- Upgraded all equipment in 2015
- Improved employee training in 2018
- Implemented a new safety program in 2018

a. (0.5 point) Calculate the experience modification for this policy.
b. (0.75 point) An underwriter has selected a credit of 10% for schedule rating.
Assess the reasonability of this selection.

11. (a) From Rule 16 in the ISO plan, for Company Subject Loss Cost of 270,500:
Z = 0.48, EER = 0.922, and MSL = 150,200.

$$M = \frac{\text{AER - EER}}{\text{EER}} Z = \frac{0.85 - 0.922}{0.922} (0.48) = -0.037$$, or a **3.7% credit mod**.

(b) Upgraded equipment would normally be worth up a 10% credit. However, this took place in 2015, and therefore should already be reflected in the experience used in the experience rating. (Data from 2016, 2017, and 2018 is being used to experience rate this insured.) Therefore, in order to avoid double counting, no schedule credit should be given for the upgraded equipment.
Improved employee training in 2018 is worth a credit of up to 6%.
Implemented a new safety program in 2018 is worth up to a credit of 2%.
Thus a schedule rating credit is appropriate, but it should be no more than 8%.
The selected credit of 10% for schedule rating is <u>not</u> reasonable.
Also the employee training and safety programs may already be partially reflected in the experience period, depending on when in 2018 they were implemented. If they affected all of 2018, then the maximum schedule credit should be: (8%)(2/3) = 5%.
<u>Comment</u>: For part (b) see Rule 9 of the ISO plan.

12. (3 points) An insured is considering an incurred loss retrospective rating plan and a paid loss deductible plan with the following characteristics:

Incurred Loss Retro	
Per Occurrence Limit	\$150,000
Maximum Ratable Loss	\$500,000
Paid Loss Deductible	
Per Occurrence Deductible	\$150,000
Aggregate Deductible	\$500,000
The following apply to both plans:	
Expected Ultimate Losses	\$450,000
Expected Ultimate Losses limited by occurrence	\$350,000
Expected Ultimate Losses limited by occurrence and aggregate	\$325,000
Fixed underwriting expenses including commission & profit	\$100,000
LAE as a percentage of Loss	8.0%
Tax Multiplier	1.030

a. (1.5 points) Compare the expected total cost of insurance of the two plans, taking into account both premium and deductible payments, from the perspective of the insured.
b. (0.5 point)
Recommend and briefly justify one of the two plans above from the perspective of the insured.
c. (1 point) Propose two changes to the recommended plan in part b. above and briefly explain why each would be beneficial for the insured.

12.(a) Expected Retro Premium = Guaranteed Cost Premium =
(1.03) {(1.08)($450,000) + $100,000} = $603,580.
Large Deductible Premium =
(1.03) {(450,000 - 325,000) + (8%)($450,000) + $100,000} = $268,830.
Expected Deductible Reimbursements = $325,000.
Sum of Deductible Premium and Reimbursements: $268,830 + $325,000 = $593,830.
The expected cost of the deductible plan is somewhat less than that of the retro plan.

(b) The insured (probably) pays a smaller upfront premium for the Large Deductible than for the Retro Plan. Also it takes longer for losses to be paid, than for losses to be incurred. Thus the paid loss deductible will provide a significant cashflow advantage to the insured.
Also the expected total cost of the deductible plan is a little less than the retro plan.
Thus I recommend the paid loss deductible.
A selected sample answer from the Examiner's Report:
The retro has much less credit risk for the insurer, so the insured has a lower cost of posting security/collateral. Therefore, I recommend the retro plan.

(c) Switch to a SIR (self-insured retention) plan with the same risk transfer structure.
The insured can hire a TPA (third party administrator) to handle claims, gaining more control over claims handling and maybe saving on loss adjustment expense. Hopefully, the insurer or reinsurer who sells the excess policy will require less profit and expenses due to the lack of credit risk and fewer services provided. (The excess policy is also not subject to any residual market assessments.)

Assuming the answer to part b was the paid loss deductible, selected sample answers from the Examiner's Report, without my endorsement:
• Increase the aggregate deductible: this would lower premium while not significantly increasing risk to the insured.
• Lower the aggregate deductible; this will provide more certainty to the insured as there will be less variation in the total cost.
• Increase the per-occurrence deductible. This would lower the premium and provide a bigger cash flow advantage to the insured.
• Lower the per-occurrence deductible. The insured would need to retain less of each large loss, leading to less uncertainty.
• Include LAE with loss and subject to deductible. This will reduce the insured's loss share.

Assuming the answer to part b was the retro plan, selected sample answers from the Examiner's Report, without my endorsement:
• Use a paid loss basis instead of incurred loss basis; this would give the insured a cashflow advantage.
• Adding a minimum premium would reduce swings in the retro premium and reduce the basic premium.
• Lowering the maximum ratable loss would put less pressure on the insured if they have extremely high losses.
• Can opt for a retro development factor in order to limit premium swings.
• Use a multi-year plan to minimize rate fluctuations year over year.

Comment: I did not understand part (c). It seems that the intent was to suggest two changes and in each case to explain how the insured would be better off while ignoring how they were also worse off. In other words, ignore the tradeoffs in making changes.
For example, an insured has a collision policy with a $500 deductible. If the insured changes to a $1000 deductible, they pay less premium, but also take on more risk. If instead the insured changes to a $250 deductible, they pay more premium, but also take on less risk. However, I do not see how either change would be "beneficial for the insured". (With more information on the individual insured, one might be able to give him good advice. If he is able, it might be best to drop collision coverage all together.)

13. (2.75 points) An insurance company observes the following ground-up claim experience for a book of business:
- Claim counts (N) follow a Poisson distribution with $\lambda = 5{,}000$
- Claim size (X) follows a Pareto distribution with $\alpha = 2$ and $\beta = 6{,}000$

The insurance company pays claims in excess of $10,000.
Given the following:
- For a Poisson distribution: $\Pr(N = n) = \dfrac{\lambda^n e^{-\lambda}}{n!}$.
- For a Pareto distribution:

$$f(x) = \frac{\alpha \beta^\alpha}{(x+\beta)^{\alpha+1}}. \qquad F(x) = 1 - \left(\frac{\beta}{x+\beta}\right)^\alpha. \qquad E[X; x] = \frac{\beta}{\alpha - 1}\left[1 - \left(\frac{\beta}{x+\beta}\right)^{\alpha-1}\right].$$

a. (0.75 point) Calculate the expected number of claims in excess of $10,000.
b. (2 points) Assume claim size is subject to a uniform annual inflation rate of 3%.
Calculate the rate at which ground-up claim counts must change in order for there to be no change in expected annual total aggregate excess losses.

13. (a) $S(10{,}000) = \left(\dfrac{6000}{10{,}000 + 6000}\right)^2 = 0.140625.$

(5000) S(10,000) = **703**.

(b) $E[X] = E[X; \infty] = \dfrac{\beta}{\alpha - 1}$.

Prior to inflation, the average payment per loss is:

$$E[X] - E[X; x] = \frac{\beta}{\alpha - 1}\left(\frac{\beta}{x+\beta}\right)^{\alpha-1} = 6000\left(\frac{6000}{10{,}000 + 6000}\right)^{2-1} = 2250.$$

After 3% inflation, the average payment per loss is:

$$(1.03)(6000)\left(\frac{6000}{10{,}000/1.03 + 6000}\right)^{2-1} = 2360.5.$$

Thus the ground-up claim counts would need to change by: 2250/2360.5 - 1 = **-4.68%**.
Alternately, prior to inflation the expected aggregate losses are: (5000)(2250) = 11,250,000.
In order to have the same expected aggregate losses after inflation, the expected number of claims would have to be: 11,250,000/2360.5 = 4765.9. 4765.9/5000 - 1 = **-4.68%**.
<u>Comment</u>: Bahnemann calls this a shifted Pareto Distribution.
After 3% inflation, severity follows a Pareto distribution with $\alpha = 2$
and $\beta = (1.03)(6{,}000)$.
Over a period of more than one year the answer to part (b) would change slightly.
For example after ten years of inflation, the average payment per loss is:

$$(1.03^{10})(6000)\left(\frac{6000}{10{,}000/1.03^{10} + 6000}\right)^{2-1} = 3599.5.$$

The ground-up claim counts would need to change at a rate of: $(2250/3599.5)^{1/10} - 1 =$ **-4.59%**.

14. (3 points) The balanced plan provisions for a workers' compensation risk in the state of Alabama are given below:

Standard Premium	\$9,000,000
Minimum Entry Ratio, r_H	0.28
Maximum Entry Ratio, r_G	2.55
Loss Conversion Factor	1.30
Tax Multiplier	1.05
State Hazard Group Differential	0.90
Adjusted Expected Loss	\$4,860,000

• Using the 2008 NCCI table of expected loss ranges,
the expected retrospective premium = \$9,493,205.

• There is no per-occurrence loss limit.

a. (1.25 points) Calculate the basic premium ratio to standard premium.

b. (1.25 points) The insured believes that the insurance charge embedded in the current basic premium is unfair and cites the following five years of loss experience the insured had with a prior carrier:

Year	Loss Ratio
1	40%
2	165%
3	15%
4	55%
5	25%

The insured's exposure base has remained stable over time.

Compare the net insurance charge in the current basic premium for this policy to the net insurance charge based on the prior loss experience using the plan provisions given above.

c. (0.5 point)

Discuss the appropriateness of using a basic premium derived from the prior loss experience.

14. (a) Look up the adjusted expected losses of 4,860,000 in the 2008 NCCI table of expected loss ranges and get Expected Loss Group 25.
The charge at r = 2.55 is 0.0379. The savings at r = 0.28 is 0.0053.
Adjusted Expected Loss is after multiplying by the State / Hazard Group relativity and the ICRLL adjustment factor. With no per occurrence limit, the ICRLL adjustment factor is one.
Thus Expected Losses = 4,860,000/0.9 = $5,400,000.
$9,493,205 = Expected retro premium = guaranteed cost premium =
(1.05) {(1.3)(5.4M) + Expenses in the basic}. ⇒ Expenses in the basic = $2,021,148.

Converted Net Insurance Charge = (1.3)(5,400,000)(0.0379 - 0.0053) = $228,852.
⇒ Basic Premium is: 2,021,148 + 228,852 = $2,250,000.

(Basic Premium) / (Standard Premium) = 2.250/9 = **25%**.
Alternately, $9,493,205 = T (E + e) = (1.05) (5.4M + e). ⇒ e = $3,641,148.

⇒ Expenses in the basic = e - (c-1)E = 3,641,148 - (0.3)(5,400,000) = $2,021,148.

Proceed as before.
(b) The average loss ratio for the five years is 60%.
(This matches that in part a: 5.4M / 9M = 60%.)
r = 2.55 corresponds to a loss ratio of: (60%)(2.55) = 153%.
r = 0.28 corresponds to a loss ratio of: (60%)(2.55) = 16.8%.
The insurance charge is: (165% - 153%)/5 = 2.4%.
The insurance savings are: (16.8% - 15%)/5 = 0.36%.
The net insurance charge is: 2.4% - 0.36% = **2.04%**.
The net insurance charge from Table M is: (0.6)(0.0379 - 0.0053) = 1.956%.
Thus the next insurance charge based on this insured's experience is somewhat higher.
(c) Five years of experience from one insured is too few on which to reliably base an insurance charge; there is too much variation in the annual loss ratios. For example, what if over the last five years the insured had had either no really bad year (165% loss ratio) or two such years, instead of one such year.
(Also, the insured's own past experience is already reflected in its experience mod.)
Thus it is <u>not</u> appropriate to use a basic premium derived from the prior loss experience.
Selected responses from the CAS Examiner's Report, without endorsement by me:
• Basic premium should not be derived using prior loss experience because prior carrier could have had completely different expense loads.
• If the max or min entry ratios have changed, it is not appropriate to use the prior carrier experience to calculate basic premium.
• The prior carrier could have a different mix of business than the current carrier so aggregate distributions may not be the same.
<u>Comment</u>: Part (a) is an example of a bad backwards question.
Expected retro premium = guaranteed cost premium. Unlike here, usually the guaranteed cost premium is less than the standard premium due to premium discounts.
In part (b), the comparison could have instead been done as a dollar amount or as a percent of expected losses, rather than as a percent of standard premium.
Past data used to develop Table M has to be sufficiently mature; then adjustments have to be made for the effect of loss development. These are other reasons why one would not use 5 past years of data from this insured. (The syllabus readings do not go into the practical details of constructing Table M.)

15. (1.75 points) A reinsurer has priced a quota share treaty to achieve an expected combined ratio of 90%. Expenses for the treaty are as follows:

Ceding Commission	20%
Brokerage Fees	5%
Administrative Expenses	1%
Unallocated Expenses	1%

The following table represents the expected loss ratio distribution for the primary insurer under the treaty:

Range of Loss Ratios	Average Ratio in Range	Probability of being in Range
0-40%	37.3%	0.03
40-60%	53.2%	0.21
60-80%	66.1%	0.55
80% or above	91.1%	0.21

The treaty includes a loss corridor from 60% to 80% loss ratio.
Calculate the percent of loss reassumed by the primary insurer in the loss corridor.

15. The reinsurer's expected loss ratio is: 90% - 20% - 5% - 1% - 1% = 63%.
The primary insurer's direct loss ratio is:
(37.3%)(0.03) + (53.2%)(0.21) + (66.1%)(0.55) + (91.1%)(0.21) = 67.777%.
Thus the loss corridor must have the primary insurer reassume: 67.777% - 63 = 4.777%.
If all the losses in the loss corridor were reassumed:
(66.1% - 60%)(0.55) + (20%)(0.21) = 7.555%.
Thus the percent of loss reassumed by the primary insurer in the loss corridor must be:
4.777%/7.555% = **63.2%.**
Comment: I see no educational value to making this a backwards question.

16. (1.75 points) A reinsurance company is evaluating whether or not to write a $50 million excess of $50 million catastrophe reinsurance contract with a primary insurer. The reinsurer is currently holding $850 million of capital and is required to hold enough capital to survive a 1-in-250 event. Without the new contract, the reinsurance company has a 1-in-250 probable maximum loss (PML) of $825 million which is solely driven by the hurricane peril.
Given the following:

- The primary insurer's PMLs are driven by the hurricane and earthquake perils only.
- The primary insurer's aggregate annual PMLs by return period are as follows:

Return Period (years)	PML ($000,000)
1000	125
500	105
200	95
100	70
50	50
25	30
20	25
10	20
5	15

- The largest hurricane event in the primary insurer's event catalog is $45,000,000.

a. (1 point) Calculate the ceded, and net,1-in-250 PMLs for this contract for the primary insurer.
b. (0.75 point) Evaluate whether the reinsurer should participate in this treaty.

16. (a) S(95 million) = 1/200 = 0.5%.
S(105 million) = 1/500 = 0.2%.
We want S(x) = 1/250 = 0.4%.
Using linear interpolation, x = (2/3(95) + (1/3)(105) = $98.33 million.
I estimate the one-in-250 year PML as about $98.33 million.
This corresponds to **ceded losses of $48.33 million**, and **net losses of $50 million**.

(b) Assume the table and treaty are both on a per occurrence basis. (See my comment.)
The Occurrence Exceedance Probability (OEP) is the probability that at least one loss exceeds the specified loss amount.
Since the largest hurricane of $45 million would not pierce the layer covered by the treaty, the treaty is (almost totally) exposed to earthquakes. Since the reinsurer's current portfolio is exposed to hurricanes and not earthquakes, the losses from this treaty would be uncorrelated with the reinsurer's current losses. Thus adding to this treaty would not significantly increase the reinsurer's current one-in-250 PML. Writing this treaty would diversify the reinsurer's portfolio.
There is no reason based on capital why the reinsurer should not write this treaty. (The potential profitability of this treaty compared to other possible new treaties would help determine whether to write this treaty.)

Alternately, assume that the table is on an (annual) aggregate basis as stated, and also assume the treaty is on an aggregate basis. (See my comment.)
The Aggregate Exceedance Probability (AEP) is the probability that the sum of all losses during a given period exceeds some amount.
If the primary insurers losses were perfectly correlated with the current portfolio of the reinsurer, then $48.3 million would be added to the one-in-250 year PML of the reinsurer.
$825 + 48.3 = $873.3 million would exceed the $850 million in current capital, and thus the reinsurer should not write the treaty.
However, since the reinsurer is currently only worried about the hurricane peril, and the primary insurer is worried about both earthquake and hurricane peril, the losses can not be perfectly correlated.
If the new treaty's losses are (largely) uncorrelated with the current portfolio of the reinsurer, then very little if anything would be added to the one-in-250 year PML of the reinsurer. (This can occur if the geographical regions covered are mostly distinct. Also earthquake and hurricane losses are uncorrelated with each other.) In this case, there is no reason based on capital why the reinsurer should not write the treaty. (The potential profitability of this treaty compared to other possible new treaties would help determine whether to write this treaty.)
The situation is probably somewhere between these two extremes, and there is insufficient information to determine whether the reinsurer has enough capital to write this treaty.

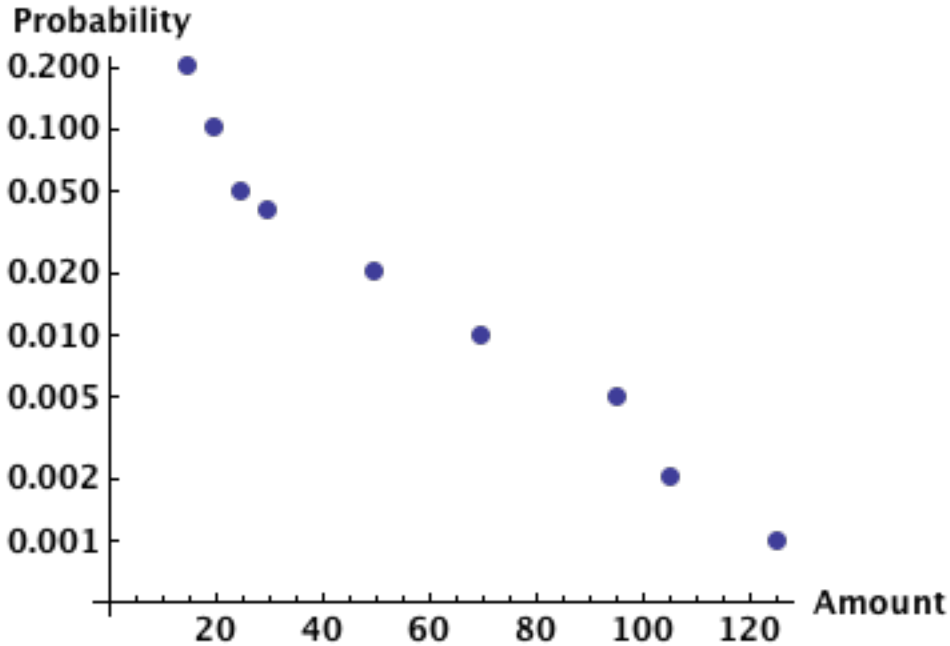
Selected sample responses to part (b) from the CAS Examiner's Report, edited by me:

• Reinsurer new PML = 825 + 48.33 = 873.33.
However, this PML of 873.33 is the maximum number; PML cannot be added.
Due to the diversification benefit from earthquake exposure of this new treaty, the PML may be reduced to an acceptable level. Therefore, the reinsurer should accept.

• The reinsurer's 1-in-250 PML is driven completely by hurricane, so it should participate in the treaty to diversify the perils it is exposed to. Since the largest hurricane event for the primary insurer is 45 million, the reinsurer will not increase its exposure to hurricane – only earthquake. This means the reinsurer's 1:250 should not grow by taking on additional EQ exposure. This is all, of course, on a modeling basis. There is potential for the model to be wrong, but the reinsurer should diversify.

• The reinsurer has 850 - 825 = 25 million of available capital. The 48.33 million PLM could seem too high, but it depends how this possible loss is correlated with the reinsurer's current book of business.
Since the current book is solely driven by hurricane peril and this proposed contract is driven by both hurricane peril and EQ peril, I would recommend to accept this new treaty. Furthermore, the largest hurricane event in the primary insurer's event catalog is $45,000,000, which is under the attachment point of the treaty.

Comment: In part (a), one should interpolate using the exceedance probabilities rather than the return periods. A graph of the given exceedance curve:



For part (b), from the CAS Examiner's Report: "Due to the ambiguity in outlining the type of reinsurance contract being offered and the primary insurer's PML table incorrectly being labeled "aggregate," this question was ruled to be defective.
This defect was addressed through the grading of the question, as discussed herein.
The intent of the question was for the PML table to be interpreted as occurrence. Most candidates answered the question as intended, as if it were an OEP."
"Part b becomes unanswerable with the assumption that the treaty is per-occurrence, combined with an aggregate PML table as we do not have enough information. Candidates who recognized this issue and explained why received full credit."
Catastrophe treaties are written on a per event basis, covering the total losses (net of other reinsurance) from one event such as a single hurricane.
Assuming that both the treaty and exceedance curve were on an occurrence basis, things are complicated by reinstatement clauses. For example, assume that there is an earthquake with $100 million is loss (net of other reinsurance), exhausting the treaty limits. Then the insurer can reinstate coverage by paying the reinstatement premium. Thus the reinsurer can be exposed to more than the treaty limit if there is more than one covered catastrophe in a year.
The effect of the reinsurance premium and any reinstatement premiums are being ignored; compared to a one-in-250 year PML, their effect would be relatively minor.

17. (2 points) A catastrophe modeler would like to incorporate a new construction technique into a catastrophe model. This new technique would theoretically reduce the amount of building damage sustained during hurricane force winds.
However, experts have not reached a consensus on the effectiveness of the new construction technique because it has not been exposed to an actual hurricane.
a. (0.5 point) Briefly describe which module(s) of the catastrophe model would need to be modified to account for the new information.
b. (0.5 point) Classify the uncertainty created with the new construction technique as either aleatory or epistemic and briefly justify the selection.
c. (1 point) Briefly describe and contrast two methods the modeler could use to incorporate uncertainty in this catastrophe model.

17. (a) This change would primarily affect the **vulnerability module** which relates to how susceptible different building types are to damage from a catastrophe. Here is where the assumed effectiveness of the new construction in reducing damage from hurricane force winds would have to be incorporated.
One would also have to update to the **inventory module** to make sure that any buildings in the insurer's portfolio that used this new construction technique are appropriately identified as such.
(b) This is **epistemic** uncertainty since it is parameter risk (or modeling risk) rather than the process risk inherent to the random nature of the catastrophe.
(c) • A Logic Tree assigns weight to parameter alternatives, in this case based on the differing opinions of the experts of the effectiveness of the new construction technique at reducing wind damage. Then for each of the different possibilities from the Logic Tree, the cat model is run. Using each of the model outputs and its corresponding probability from the Logic Tree, a weighted average is calculated to get the mean result. The different model outputs and corresponding probabilities can also be used to get the range of variability in results, in this case due to the uncertainty in the effectiveness of the new construction technique.
Logic Trees are relatively easy to document and communicate. However, they are not easy to use when there are a large number of possible scenarios, for example when there are many items varying in many possible ways.
• Simulation creates many randomly sampled alternatives from the probability distribution(s) of the parameter(s). Then for each such set of inputs, the cat model is run. The result is a large set of outputs, which can be used to get the mean result as well as quantify the variability.
Simulation can handle situations with a large number of possibilities. However, it is harder to document and communicate than Logic Trees. Simulation may appear to be a "black box" to non-actuaries. Simulation gives a better overall view of the uncertainty than Logic Trees, but is more computationally intensive.
• Credibility weight with and without effects of the new science to get a credibility weighted damage function, with the compliment of credibility being no inclusion of non-consensus science. This may be more stable, as it will have less major change year over year until the new science becomes more mainstream and generally accepted.
Comment: See Sections 4.2-4.4 of Grossi and Kunreuther.
The other two modules are the Hazard Module (possible hurricanes and probabilities), and the Loss Module (applies the provisions of the insurance policy to translate damage to insurance loss.)
Aleatory uncertainty relates to the inherent randomness in the risk process.
In the specific situation described, it seems like a Logic Tree would be a good approach.

END OF EXAMINATION