Solutions to the
Fall 2017
CAS Exam 8

(Incorporating what I found useful in the CAS Examiner's Report)

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While some of the comments may seem critical of certain questions, this is intended solely to aid you in studying and in no way is intended as a criticism of the many volunteers who work extremely long and hard to produce quality exams.

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1. (8.5 points) An actuary has constructed a pure premium model using the Tweedie distribution with parameter $1 < p < 2$ to determine manual rates for a Workers' Compensation book of business. The output of the model is pure premium per $100 of payroll.

The following variables were considered for inclusion in the model:

Variable	Description or Source of Data	p-value
Industry	Construction, Manufacturing, All Other	0.002
Return to Work Program	Yes or No	0.008
Employee Age	Average Age in Years	0.003
Employee Tenure	Average Number of Years of Employment	0.005
Location	State of Jurisdiction	0.080
Employee Morale	Based on Results of Annual Company Survey	0.150
Number of Back Injuries	Supplied by Employer	0.010

The actuary has decided to use the following variables in the model:
Industry, Employee Tenure, and Return to Work Program.

(a) (1.5 point) Discuss the statistical and non-statistical considerations of including each of the three variables (Industry, Employee Tenure and Return to Work Program) in the model.

(b) (2 points) Discuss the statistical and non-statistical considerations of excluding each of the remaining four variables (Employee Age, Location, Employee Morale and Number of Back Injuries) from the model.

(c) (1.5 points) The actuary fits the log link GLM model using the three selected variables. Given the fitted model parameters below and the following information for a Manufacturing Workers' Compensation risk, calculate the standard premium for this risk for an annual policy period.

Parameter	Coefficient
Intercept	-0.631
Employee Tenure	-0.040
Return to Work Program: Yes	-0.200
Industry Type: Construction	0.350
Industry Type: All Other	-0.550

Data for Manufacturing Risk	Value
Payroll	$1,000,000
Employee Tenure	5 Years
Return to Work Program	No
Actual Losses for Experience Rating	$12,500
Fixed Expenses	$1,500
Variable Expenses (as % of premium)	20%
Experience Rating Constant (K)	$10,000

QUESTION CONTINUED ON NEXT PAGE

d. (1.5 points) The actuary has graphed actual vs. modeled expected loss ratios by size of risk for the company's entire Workers' Compensation book, which is shown below.
In order to improve the model fit for larger risks, the actuary is considering incorporating the variable "Latest 3 Year Historical Losses" into the model.
Explain three reasons against doing so.

Actual vs. Modeled Expected Loss Ratio
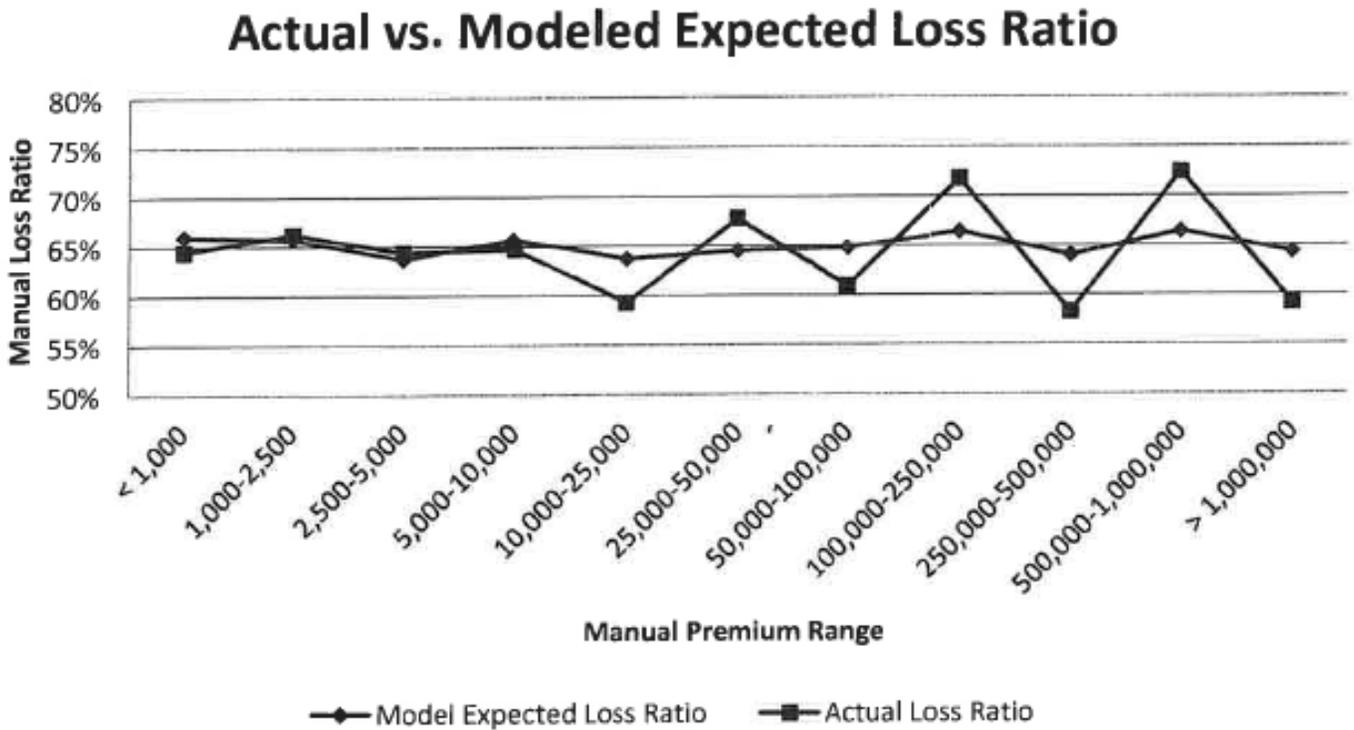
e. (2 points) The actuary is now developing a quote for a new Construction risk with $50,000,000 of payroll using this rating plan.
  i (1 point) Describe two potential issues in developing a premium for this risk under this rating plan.
  ii. (1 point) Provide an alternative rating approach for this risk and briefly discuss the advantages of this alternative for the insured as well as the insurance company.

1. (a) i. Industry has a p-value of 0.2%, so it is statistically significant at the 5% level.
Dividing classes into Industry Group is part of the common way to make Workers Compensation Rates; this follows insurance industry practice. Thus including Industry makes sense.
(The NCCI currently has five industry groups: Manufacturing, Contracting, Office & Clerical, Goods & Services, and Miscellaneous.)
Another reason to include this variable in the model: there is a connection between industry and expected pure premiums; for example, a construction company would be expected to have a larger pure premium than an actuarial consulting firm.
Also Industry is a variable that is practical; it is easy to collect and easy to verify via SIC code.
ii. Employee Tenure has a p-value of 0.5%, so it is statistically significant at the 5% level.
The longer the average employee is with a firm, the more safe the workplace should be all other things being equal; the more turnover in a workplace, the less safe it is all else being equal.
Thus since it is expected to have a relationship to losses, including Employee Tenure makes sense.
Alternately, employee tenure is directly connected to future loss activity – new employees may not be as safe as employees who know the building, the machinery, or have experience with other loss control method (i.e. know how to set safety guards, etc.)
iii. Return to Work Program has a p-value of 0.8%, so it is statistically significant at the 5% level.
Having a return to work program is expected to reduce the severity of some claims by getting the injured worker back to work in some capacity sooner; benefits are paid for a shorter period of time. Thus having a return to work program is expected to reduce losses, and thus including it in the model makes sense.
(Of course, there may be issues in deciding what is an acceptable/effective return to work program.)

(b) i. Employee Age has a p-value of 0.3%, so it is statistically significant at the 5% level.
When injured, an older worker takes on average more time to return to work; thus average age of worker is expected to be related to expected losses.
However, average age of employee is likely correlated with average employee tenure. This could create multicollinearity problems; the estimates of parameters may be unreliable if both employee age and tenure were included in the GLM. This would be a reason to exclude it.
Also, average age of employee may be prohibited for use as a rating variable by some regulators since it might induce some employers to fire older workers because of their age; discrimination in employment because of age is illegal in many countries. This would be a reason to exclude it.
ii. Location has a p-value of 8%, and thus is <u>not</u> significant at the 5% level.
(There are many levels to this categorical variable, which may be affecting the p-value.)
Also there are larger employers with workplaces in many different states, which may be affecting the usefulness of the Location (State of Jurisdiction) as it is being used in this model.
iii. Employee Morale has a p-value of 15% and thus is <u>not</u> significant at the 5% level.
It is based on the results of an annual company survey, and thus is <u>not</u> objective.
Thus I would <u>not</u> use Employee Morale.
iv. Number of Back Injuries has a p-value at 1%, and thus is significant at the 5% level.
However, the data is reported by the employer and thus is subject to employer manipulation.
Therefore, I would <u>not</u> use Number of Back Injuries.
(The insurer should be able to collect this information itself off of its claim reports.)

(c) Assume Manufacturing is the base class.
Assume that the given actual losses are for one year.
Exp[-0.631 + (5)(-0.04)] = 0.43561.
Expected Losses = (1 million / 100) (0.43561) = $4356.

Manual Premium = $\dfrac{4356 + 1500}{1 - 20\%}$ = $7320.

Assume that as in the ISO Experience Rating Plan: Z = E / (E + K).

$Z = \dfrac{4356}{4356 + 10{,}000}$ = 0.303.

Mod = (0.303) (12,500/4356) + (1 - 0.303)(1) = 1.566.
Standard Premium = (1.566)($7320) = **$11,463**.

Alternately, Mod = $\dfrac{A + K}{E + K}$ = $\dfrac{12{,}500 + 10{,}000}{4356 + 10{,}000}$ = 1.567.

Standard Premium = (1.567)($7320) = **$11,470**.
Alternately, assume instead that the given actual losses are for <u>three</u> years.
The expected losses for three years are: (3)(4356) = $13,068.

Mod = $\dfrac{A + K}{E + K}$ = $\dfrac{12{,}500 + 10{,}000}{13{,}068 + 10{,}000}$ = 0.975.

Standard Premium = (0.975)($7320) = **$7137**.

(d) • The latest 3 Year Historical Losses are already included in the experience rating plan, thus including them in the GLM would be redundant.

• 3 Year Historical Losses depends on the size of insured. Rather in this model to predict pure premium, one could use either reported pure premium or loss ratio.

• For small insureds, the latest 3 Year Historical Losses have a lot of random fluctuation. The GLM does not include an equivalent of credibility in order to limit the impact on small insureds; thus this would introduce too much random fluctuation in the manual premium of small insureds.

• The latest 3 Year Historical Losses could include one very large claim; one such very large claim could produce a very large manual premium for a small or medium sized insured. (The impact of one large claim is limited in experience rating.)

From the CAS Examiners Report, with my comments in parentheses:

• The bad fit on the higher range might be caused by sparse data of large premium insureds. So it is normal and maybe trying to improve the model fit would be a waste of time.
(This answer does make use of the given graph. However, this is <u>not</u> a reason not to add the historical pure premium if it would improve the model fit.)

• This variable will most likely be highly correlated with other variables which may lead to an unstable model. If multicollinearity is present, the model may not converge at all or lead to irrational outputs. (This general statement is a good way to try to get credit.)

• The latest 3 year losses are not fully developed. So, if the reporting lag varies by company it would be over or under predicting the loss experience. (I would <u>not</u> have given this answer credit, since it would argue against ever using experience rating plans.)

• 3 year historical losses does not separate frequency from severity. It may be better to use variables that separate these impacts. (This is the purpose of the primary/excess split in the NCCI Experience Rating Plan.)

• There may be recent changes to the risk's safety procedures or WC benefit levels that would not yet be fully reflected in 3 year historical loss. (But the current model also ignores these issues; I would <u>not</u> have given this credit.)

(e) i. Assume for example, an average tenure of 5 years and no return to work program.

Exp[-0.631 + 0.350 + (5)(-0.04)] = 0.61878.

Expected Losses = (50 million / 100) (0.61878) = \$309,390.

$$\text{Manual Premium} = \frac{309{,}390 + 1500}{1 - 20\%} = \$388{,}613.$$

From the graph in part (d), the GLM does not appear to do a good job of estimating expected losses for this size insured.

Also there are many different types of construction insureds which vary a lot from each other; for example electricians are much less risky than high steel workers. Thus such a simplified rating plan such as this GLM is unlikely to do a good job of estimating expected losses.

From the CAS Examiners Report, with my comments in parentheses:

• Since the risk is new, there will not be any actual loses with which to do experience rating, so it is difficult for the insurer to know whether manual premium is adequate and not excessive for this risk. (New in this context could be either new to this insurer or a new employer.

If the former, then the prior data of the insured would usually be used to experience rate it, even if it had been insured by a different insurer.)

• The experience rating constant K is likely too low for this risk as it is giving very high credibility to actual experience of a risk of \$50 million payroll. (The problem with the value of K applies to all sizes. The credibility assigned to the small insured in part (c) is unrealistically large. On the other hand, experience rating credibilities depend on the quality of the classification plan. Since the GLM being used is likely extremely bad at predicting pure premiums, perhaps the experience rating credibilities should be this large.)

• The experience rating plan does not seem to have a cap or apply a split between primary and excess losses which may lead to oversensitivity to large loss events. (Again this applies to insureds of all size, and would be even more of a problem for small insureds.)

• Large companies may have more effective return to work programs or other unique characteristics that are not contemplated by this model that doesn't consider variables for large size risks separately. (The model ignores unique characteristics of risks of all sizes.)

• The variable expenses of 20% may be appropriate for smaller risks but could be too high for a risk of this size and the plan does not include any expense discount to account for this. (There is no mention of how expenses actually vary by size, but it is reasonable to assume that for Workers Compensation they are a much smaller percent of expected losses for large insured than small insureds. How expenses would be loaded for this insured are not mentioned, although one could assume it would be the same as part c.)

• Assuming the question implies that the rate is set only using the GLM without an experience mod since the risk is new, then the risk has little incentive to control losses. (However, its losses would flow into its future experience ratings, affecting its premiums in future years.)

ii. I would recommend a retro rating plan, with an appropriately chosen maximum premium, minimum premium, and accident limit. This will have the advantage of limiting the insurer's risk of bad experience, while still allowing the insured protection against bad luck. It will also provide an incentive for the insured to cooperate with the insurer to control losses. The insured will be able to benefit from good experience, at the cost of taking on some more risk itself.

Alternately, I would recommend a large deductible plan, with an appropriately chosen deductible size, and aggregate limit. This will have the advantage of limiting the insurer's risk of bad experience, while still allowing the insured protection against bad luck. It will also provide an incentive for the insured to cooperate with the insurer to control losses. The insured will be able to benefit from good experience, at the cost of taking on some more risk itself.

(The insurer will still settle all claims.)

From the CAS Examiners Report, with my comments in parentheses:

• Base the experience rating component of pricing on the NCCI method of splitting primary and excess losses so that premium is not inappropriately impacted by large losses with less credibility. The advantage for the insured is that their premium will not see large swings year to year from large loss experience. The advantage for the insurer is that the experience mods from a split plan for WC are generally shown to be more accurate in estimating expected costs so they are less likely to underprice the risk.

(Write down something you know and hopefully get credit. Personally I did not see this as the thrust of this part of the question, but so what.)

• A new GLM could be developed to estimate pure premium that focuses on variables more relevant to construction risks. This could remove some of the premium uncertainly for larger construction policies. For the insurer, the model will provide more insights into drivers of expected loss. For the insured, the premium would be more likely to appropriately reflect their existing characteristics and therefore be more equitable.

(A very general response that requires no familiarity with the syllabus material.)

Comment: The first Integrative Question on an Exam 8; it illustrates the difficulty of doing a good job of writing such questions.

Personally, I had a lot of trouble figuring out what the questioners were getting at in many cases, perhaps due to my many years working on Workers Compensation ratemaking. It felt like I was being tested on whether I could read the examiner's mind.

The pure premiums from the GLM are unrealistically low for either Manufacturing or Construction. In any case, there is way too much variation between employers in broad industry groups in order rely on them to make Workers Compensation manual rates; that is why there are hundreds of different classes grouped into several different Industry Groups.

Since Industry has three categories, as shown there are two corresponding parameters; there should be a separate p-value for each of these parameters. Since Location has many categories, there are many corresponding parameters; there should be a separate p-value for each of these parameters.

The credibility parameter K is much smaller than in the ISO Experience Rating Plan, resulting in much larger credibilities.

In the graph, why should the expected manual loss ratios from the model vary in this pattern by size of insured? Should not the expected manual loss ratio just depend on the relationship between the expected losses and the fixed expenses of $15,000?

In the graph, why do the actual manual loss ratios vary so much for the large insureds, seemingly at random by size category, while the actual manual loss ratios vary so little for the small insureds, again seemingly at random by size category? Is this supposed to be based on the volume of business this insurer has by size of insured? For this to make sense, for example, the insurer would have to have at least 25,000 as many insureds in the $1000-2500 manual premium range as in the $100,000-$250,000 manual premium range; this would be an extremely unusual book of business. The CAS Examiner's report puts forward the explanation that the pattern in the graph is likely due to differences in risk characteristics that are not picked up by the GLM; however, why would this affect large risks so much and small risks so little? The provided graph makes no sense to me and was actually a hindrance in answering part (d).

In part (b), the workers compensation laws vary by State, and thus so do the benefits. Therefore, expected losses can vary significantly by State for otherwise similar workplaces. Therefore, one would usually include location (State of Jurisdiction) in such a model.

From the CAS Examiner's Report: "Location of the company would likely be an acceptable variable – companies that are located in areas with plenty of hospitals and doctors would likely have overall lower loss costs than companies in rural areas. However, the p-value for this variable is high indicating that it is not a good predictor of future losses." This is not quite right. Most States contain rural, suburban, and urban areas. Thus the proposed variable which uses State of Jurisdiction, would not pick up the stated reason for differences in costs.

In part (c), the CAS allowed one to interpret Manufacturing as either the base class or part of All Other. However, given an intercept plus parameters for both Industry Type: Construction, and Industry Type: All Other, Industry Type must have three categories.

In part (c), unlike in an actual experience rating plan, one does not adjust the current expected losses in order to match the level of trend and maturity of the reported historical losses.

In part (c), the CAS allowed one to take the Employee Tenure variable either directly or by taking the natural log; they allowed the latter since the syllabus reading recommends that when using the log link function in a GLM, you log your continuous predictor variables.

In part (c), the CAS allowed one to apply the experience modification factor either before or after expense loading; the syllabus readings apply the mod to the manual premium (which is after expense loading.)

From the CAS Examiner's Report with respect to part (d): "To provide an appropriate prediction for future loss, past 3 year losses may need to be developed and adjusted for changes in benefit levels or changes in the risk's size, operations, or safety practices over the three year period vs. prospective period." I disagree. Yes it would be good to adjust for any changes in operations and safety practices since the three year historical period. However, no experience rating plan does this; that is the purpose of schedule rating. One can use undeveloped losses (via a pure premium or loss ratio) in the GLM. Undeveloped pure premiums would have predictive value; the fitted coefficient in the GLM would compensate for the fact that all the pure premiums are undeveloped and at the same average level of maturity. The effect of recent benefit level changes are not included anywhere in the whole rating scheme in the question; their effects would have to be included in a separate step. Finally, using pure premiums (or loss ratios) in the GLM should make the predictions of future pure premiums robust to any changes in the size of the operations of the insured.

From the CAS Examiner's Report with respect to part (d):

"Although the question was designed to focus on concepts of experience rating, alternative responses related to knowledge of GLMs were also accepted. This could include:

• Adding more parameters or degrees of freedom to the model could lead to over-fitting.

• Correlation between variables in a model can lead to erratic coefficients and the prior loss could be highly correlated with in-model predictors.

• In practical application it is typical when fitting commercial models for there to be fewer risks in the large premium buckets so the lack of fit may still be within acceptable variation for actual results."

I do not know how reading the question as actually written one could know that part (d) was intended to be focused on concepts of experience rating. The first two are general statements that while they would have gotten credit, have nothing to do with the specifics of this question. The final statement is true of fitting GLMs to some commercial insurance data, but I am not sure that it is the case here. In any case, this does not imply that the addition of the historical pure premium would not (significantly) improve the model fit.

2. (2.0 points) An actuary wants to cluster five Workers' Compensation classes based on excess ratios at two limits: 500,000 and 1,000,000. The actuary decides to use a weighted k-means algorithm with two clusters. Given the following:

Class	On-Leveled Earned Premium ($ Thousands)	Normalized Excess Ratio at 500,000 Limit	Normalized Excess Ratio at 1,000,000 Limit	Initial Cluster
1	6,500	0.240	0.080	A
2	5,000	0.350	0.200	A
3	4,000	0.210	0.080	A
4	3,000	0.110	0.030	A
5	5,000	0.180	0.070	B

• Distance will be measured using the L^2 (Euclidean) norm.
• At the start of the algorithm, the actuary randomly assigns each class to a cluster.
a. (1.75 points)
Determine the cluster for each class after the first iteration of the weighted k-means algorithm.
b. (0.25 point)
Briefly describe one advantage of using the L^1 measure rather than L^2 when computing clusters.

2. (a) Take the weighted average of the excess ratios of the two initial clusters.
The first cluster has the first four of the five classes.

$$\frac{(6500)(0.240) + (5000)(0.350) + (4000)(0.210) + (3000)(0.110)}{6500 + 5000 + 4000 + 3000} = 0.2422.$$

$$\frac{(6500)(0.080) + (5000)(0.200) + (4000)(0.080) + (3000)(0.030)}{6500 + 5000 + 4000 + 3000} = 0.1043.$$

Thus the two weighted centroids are: (0.2422, 0.1043) and (0.180, 0.070).
Now for each class we determine to which centroid it is closest.
(Comparing the squared distances results in the same answer as comparing the distances.)
Class 1 squared distance to first centroid: $(0.240 - 0.2422)^2 + (0.080 - 0.1043)^2 = 0.000595$.
Class 1 squared distance to second centroid: $(0.240 - 0.180)^2 + (0.080 - 0.070)^2 = 0.0037$.
Class 1 is closer to the first centroid; assign Class 1 to cluster A.
Class 2 squared distance to first centroid: $(0.350 - 0.2422)^2 + (0.200 - 0.1043)^2 = 0.02078$.
Class 2 squared distance to second centroid: $(0.350 - 0.180)^2 + (0.200 - 0.070)^2 = 0.0458$.
Class 2 is closer to the first centroid; assign Class 2 to cluster A.
Class 3 squared distance to first centroid: $(0.210 - 0.2422)^2 + (0.080 - 0.1043)^2 = 0.00163$.
Class 3 squared distance to second centroid: $(0.210 - 0.180)^2 + (0.080 - 0.070)^2 = 0.001$.
Class 3 is closer to the second centroid; assign Class 3 to cluster B.
Class 4 squared distance to first centroid: $(0.110 - 0.2422)^2 + (0.030 - 0.1043)^2 = 0.0230$.
Class 4 squared distance to second centroid: $(0.110 - 0.180)^2 + (0.030 - 0.070)^2 = 0.0065$.
Class 4 is closer to the second centroid; assign Class 4 to cluster B.
Class 5 squared distance to first centroid: $(0.180 - 0.2422)^2 + (0.070 - 0.1043)^2 = 0.005045$.
Class 5 squared distance to second centroid is 0.
Class 5 is closer to the second centroid; assign Class 5 to cluster B.
Thus after one iteration, Cluster A is {1, 2}, and Cluster B is {3, 4, 5}.

(b) Using the L^1 measure, the distance between two points is the sum of the absolute
differences of their coordinates.
"The intuitive rationale for using this metric is that it minimizes the relative error in estimating
excess premium."
Alternately, using the L^1 measure, many small absolute errors would have the same effect as
one large error; this results in outliers having less of an impact on the result than if using the L^2
measure.

Comment: See pages 201 and 205 of Robertson.
Robertson did not normalize the excess ratios prior to using them to construct clusters.
A graph of the excess ratios for the five classes, also showing the first weighted centroid (the second centroid is at class 5):

Excess Ratio at 1M
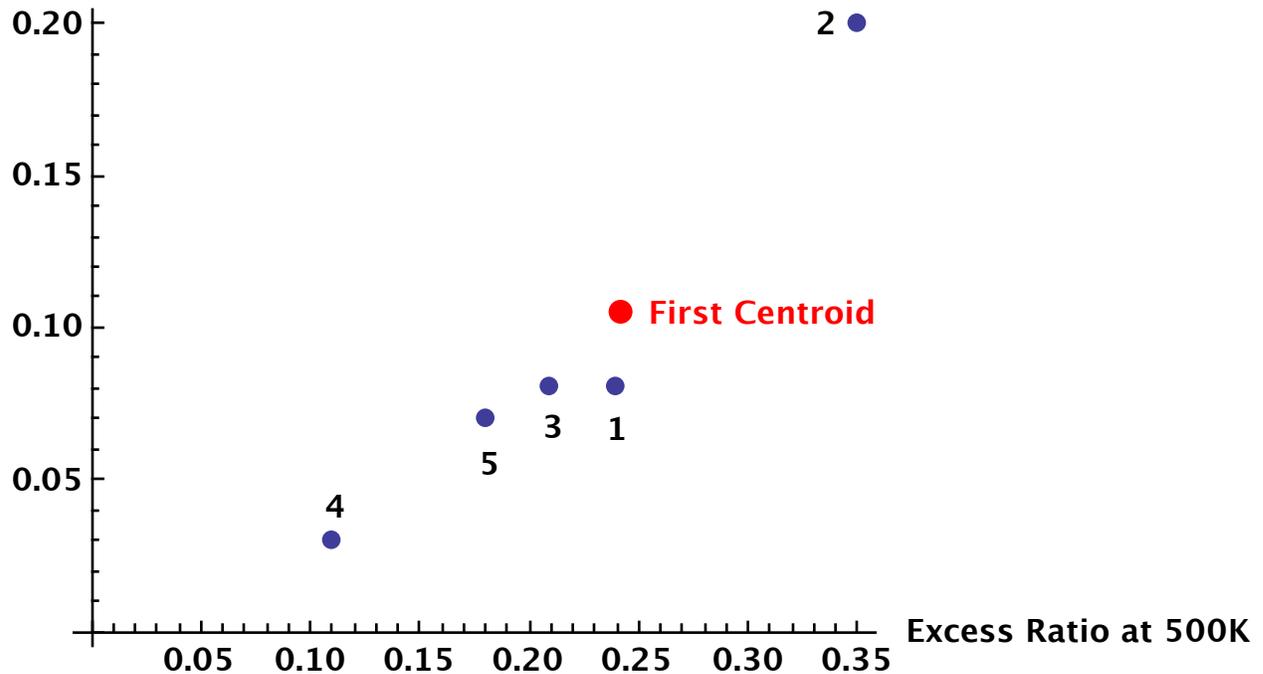
After one more iteration, the algorithm converges to Cluster A = {2}, and Cluster B = {1, 3, 4, 5}. With the same initial assignments, (unweighted) k-means would result in final clusters: Cluster A = {1, 2, 3}, and Cluster B = {4, 5}; however, with a different initial assignments (unweighted) k-means would result in different final clusters than these. In fact the optimal (unweighted) k-means clusters are Cluster A = {2}, and Cluster B = {1, 3, 4, 5}.

3. (1.5 points) The following data shows the experience of a merit rating plan for private passenger vehicles. The merit rating plan uses multiple rating variables, including territory.

Number of Accident-Free Years	Earned Car Years (000s)	Earned Premium ($000s)	Number of Incurred Claims
5 or More	250	500,000	15,000
3 and 4	100	90,000	13,500
1 and 2	80	60,000	8,000
0	70	50,000	10,500
Total	500	700,000	47,000

Territory	Frequency	Average Premium
A	0.05	1,500
B	0.10	2,000
C	0.15	1,250

a. (0.75 point) Recommend and justify an exposure base for this merit rating plan.
b. (0.75 point) Calculate the relative credibility of an exposure that has been three or more years accident-free using the exposure base from part (a) above.

3. (a) The use of premiums as the exposure base (as Bailey-Simon did) would make sense if the high rated territories are the high frequency territories. However, this is <u>not</u> the case here; territory C with the highest frequency has the lowest average premium.
(Different average severities seem to be responsible for a significant amount of the variation in premiums between territories.)
Thus I will use **earned car years as the exposure base**.
Note that in order to use premium as the exposure base to correct for maldistribution, one would also require that the territory differentials are properly priced; there is no way to determine whether or not that is the case here.

(b)
Number of Accident-Free Years	Car Years (000s)	Number of Claims	Frequency	Relative Freq.
3 or More	350	28,500	0.0814	0.866 = 814/940
1 or more	430	36,500	0.0849	0.903 = 849/940
Total	500	47,000	0.0940	1.000

Three year credibility is: 1 - 0.866 = 13.4%.
One year credibility is: 1 - 0.903 = 9.7%.
Three year credibility relative to the one year credibility: 13.4% / 9.7% = **1.38**.
Alternately, one can estimate the credibility for one year of data from the experience of those who were not claim free. The frequency per car year for those who are not claim free is:
10,500 / 70,000 = 0.1500. Relative frequency is: 0.1500/0.0940 = 1.596.
Assume a Poisson frequency with mean equal to the overall mean: λ = 0.0940.
Then the average frequency for those who are not claim free is: $\lambda / (1 - e^{-\lambda})$.
Thus the relative frequency of those who are not claim free is: $1 / (1 - e^{-\lambda}) = 1 / (1 - e^{-0.094})$.
$\Rightarrow 1.596 = M = Z/ (1 - e^{-0.094}) + (1-Z)(1)$. $\Rightarrow$ credibility for one year of data = Z = 5.9%.

Three year credibility relative to the one year credibility: 13.4% / 5.9% = **2.27**.

<u>Comment</u>: For part (b), see Tables 1 and 3 in Bailey-Simon. In Bailey-Simon, the premiums have been adjusted to remove the effect of any discounts from the (current) Merit Rating Plan.
In part (a), the CAS allowed arguing that "while the frequencies do not appear to be in-line with premiums by territory, that premium may still be a better choice as it addresses some maldistribution and should be still used as the exposure base."
In that case, in part (b), one should have gotten:

Number of Accident-Free Years	Premium ($million)	Number of Claims	Frequency	Relative Freq.
3 or More	590	28,500	48.31	0.720 = 48.31/67.14
1 or more	650	36,500	56.15	0.836 = 56.15/67.14
Total	700	47,000	67,14	1.000

Three year credibility is: 1 - 0.720 = 28.0%.
One year credibility is: 1 - 0.839 = 16.4%.
Three year credibility relative to the one year credibility: 28.0% / 16.4% = 1.71.

The merit rating plan uses the number of years an insured is claims free.
The merit rating plan does <u>not</u> "use multiple rating variables, including territory." Rather the rating plan upon which merit rating is superimposed, uses multiple rating variables, including territory. These other rating variables should be controlled for. This is why Bailey and Simon apply this technique to data from each class separately.
Part (b) is unclear; it should have said "three year credibility relative to the one year credibility."

4. (1.75 points) An actuary has split data into training and test groups for a model. The chart below shows the relationship between model performance and model complexity. Model performance is represented by model error and model complexity is represented by degrees of freedom.
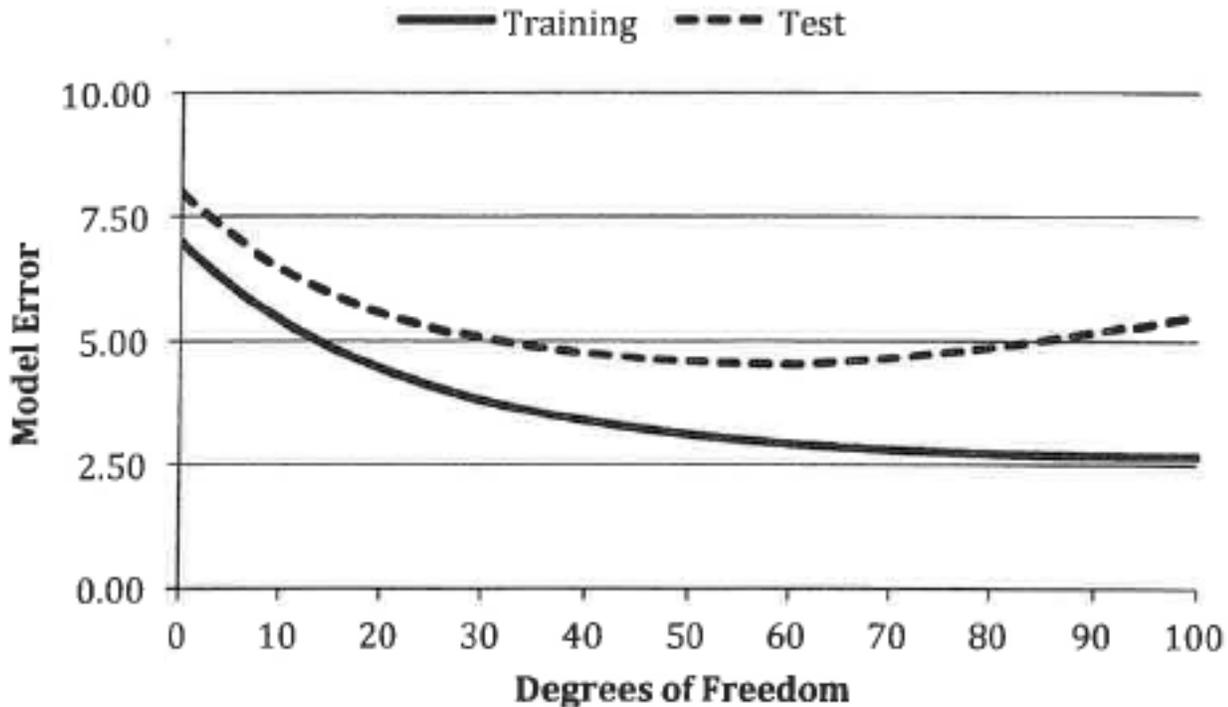
a. (0.5 point) Briefly describe two reasons for splitting modeling data into training
      and test groups.
b. (0.75 point) Briefly describe whether each of the following model iterations has an optimal
      balance of complexity and performance.
      i. Model iteration 1: 10 degrees of freedom
      ii. Model iteration 2: 60 degrees of freedom
      iii. Model iteration 3: 100 degrees of freedom
c. (0.5 points) Identify and briefly describe one situation where it is an advantage to split the data
      by time rather than by random assignment.

4. (a) 1. Attempting to test the performance of any model on the same set of data on which the model was built will produce overoptimistic results. Using the training data to compare this model to any model built on different data would give our model an unfair advantage.
2. As we increase the complexity of the model, the fit to the <u>training</u> data will always get better. In contrast, for data the model fitting process has <u>not</u> seen, additional complexity may not improve the performance of a model; as the model gets more complex its performance on the holdout data (test data) will eventually get worse, as shown in the figure in this question.
(b) Model 2 has the right balance, since it has the smallest test MSE.
Model 1 is too simple (fewer degrees of freedom than Model 2), while model 3 is too complex (more degrees of freedom than Model 2).
(c) "Out-of-time validation is especially important when modeling perils driven by common events that affect multiple policyholders at once. An example of this is the wind peril, for which a single storm will cause many incurred losses in the same area. If random sampling is used for the split, losses related to the same event will be present in both sets of data, and so the test set will not be true unseen data, since the model has already seen those events in the training set. This will result in overoptimistic validation results. Choosing a test set that covers different time periods than the training set will minimize such overlap and allow for better measures of how the model will perform on the completely unknown future."
Alternately, as in Couret and Venter, one may select either the even or odd years of data as the training set and the other as the holdout set, in order to be neutral with respect to trend and maturity.
<u>Comment</u>: See Section 4.3 of <u>Generalized Linear Models for Insurance Rating</u>.
The figure shown is very similar to Figure 7 in <u>Generalized Linear Models for Insurance Rating</u>. We are interested in how the GLM will perform at predicting the response variable on some future set of data rather than on the set of past data with which we are currently working.
Our goal in modeling is to find the right balance where we pick up as much of the signal as possible with minimal noise, represented in this case by Model 2.

5. (1.75 points) An analyst has fit several different variations of a logistic GLM to a dataset
containing 1,000 records of fraudulent claims and 9,000 records of legitimate claims.
For each model variation listed below, draw a quintile plot based on the training data.
Label the axes and identify each data series.
i. A saturated model
ii. A null model
iii. A model that could be used in practice

5. A simple quintile plot is a simple quantile plot with 5 buckets.
● Sort the dataset based on the model predicted fraud rate from smallest to largest.
● Group the data into 5 buckets with equal volume. (In this case 2000 claims in each.)
● Within each group, calculate the average predicted fraud rate based on the model,
       and the average actual fraud rate.
● Plot for each group, the actual fraud rate and the predicted fraud rate.

The saturated model has as many predictors as data points. Thus for the saturated model, the predictions exactly match the observations for each claim. In this case, 1000 of the claims involve fraud, and would all be placed in the last quintile. Thus the last quintile would consist of 1000 claims with fraud and 1000 claims without fraud.
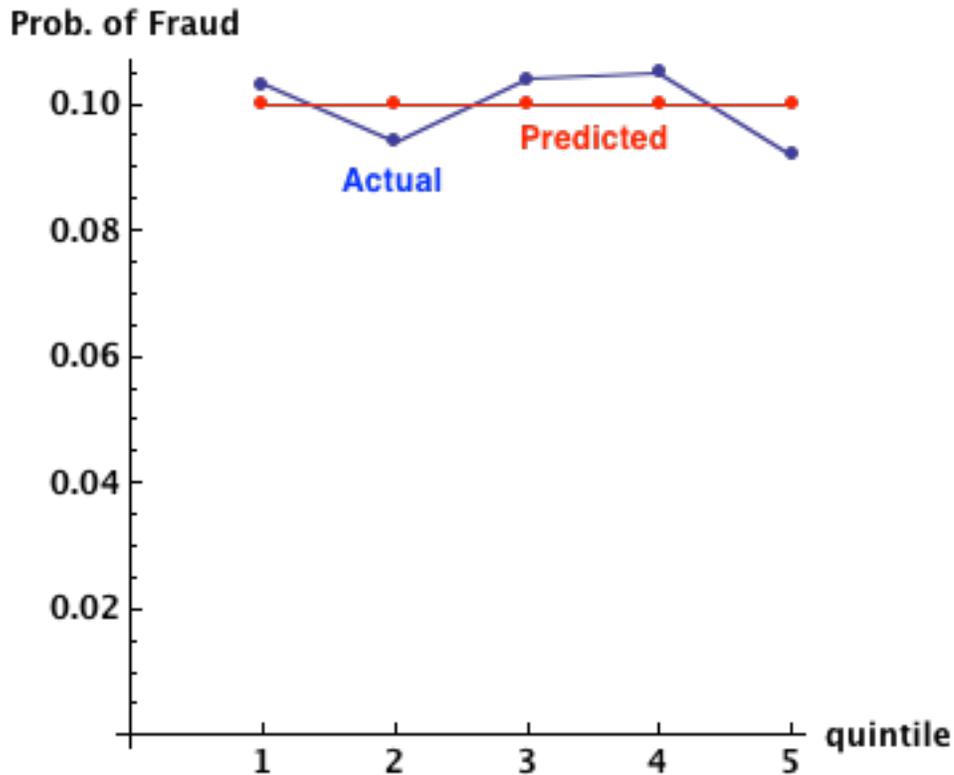The simple quintile plot:
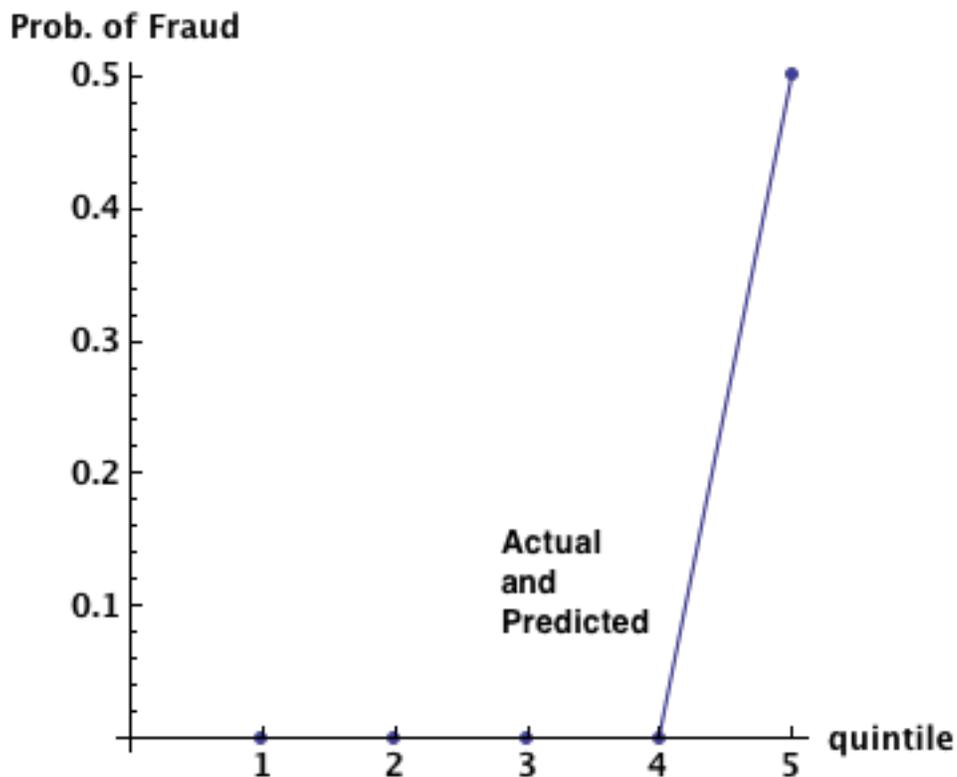
Prob. of Fraud

The null model, has no predictors, only an intercept. Thus for the null model the prediction is the same for every record: the grand mean.
In this case, the overall probability of fraud is: 1,000/10,000 = 10%.
Since every risk has the same prediction, one would assign them to buckets at random.
Thus all of the actuals by quintile should be close to the grand mean, with small differences due to the randomness of assignments. The simple quintile plot:

"A model that could be used in practice", would have the actuals increase monotonically, have good but not perfect predictive accuracy, and a reasonably large vertical distance between the actuals in the first and last quintiles. A simple quintile plot:
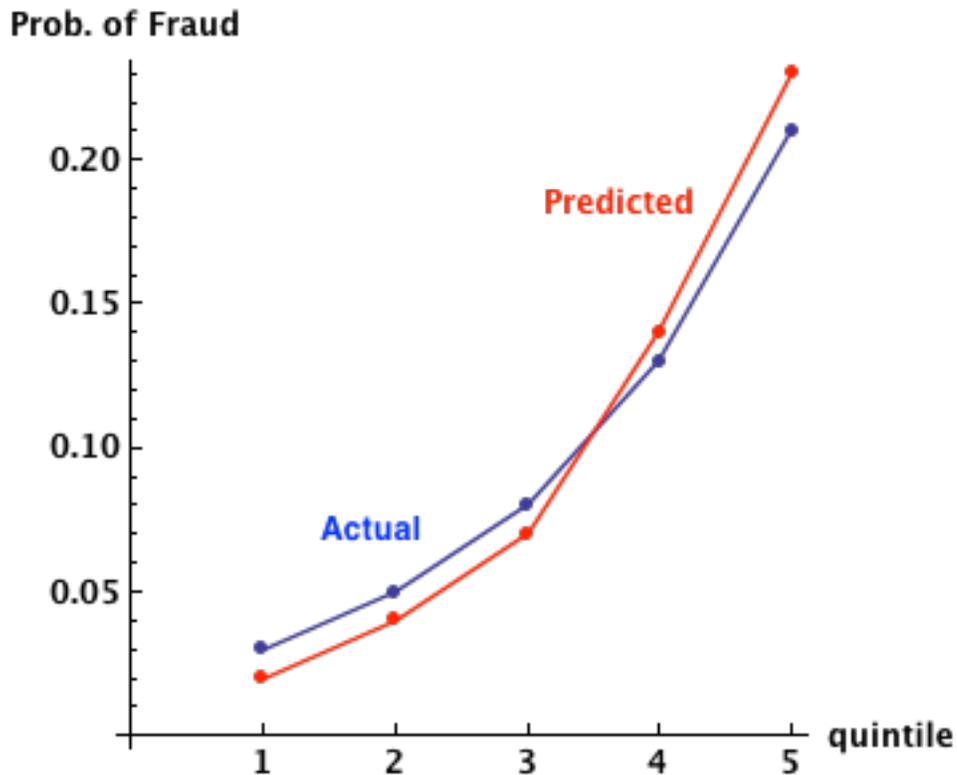
Comment: See Section 7.2.1 and page 59 of "GLMs for Insurance Rating."
Combines separate ideas in the syllabus reading.
There are many possible examples of the last plot.
Since the records are ordered by predicted values, the records in each bucket change for each graph. Thus, actuals are not the same for each graph.
Quintile plots are sorted by predicted values from smallest to largest value. Thus the predicted values must be monotonically increasing (or in the case of the null model equal). Actuals need not be monotonically increasing, although that is desirable.
In every graph, the average of the actuals should be the grand mean of 10%.
In the final plot, the average of the predicteds should be close to if not equal to 10%; the GLM may have a small bias.
In the final plot, the predicted and actuals for the final quintile should each be less than the 50% in the saturated model. In the final plot, the predicted and actuals for the final quintile should each be more than the 10% in the null model.

6. (3.5 points) A logistic model was built to predict the probability of a claim being fraudulent.
Consider the predicted probabilities for the 10 claims below to be a representative sample of the
total model.

Claim Number	Actual Fraud Indicator	Predicted Probability of Fraud
1	Y	11%
2	N	23%
3	N	15%
4	N	70%
5	Y	91%
6	Y	30%
7	N	11%
8	Y	75%
9	N	58%
10	N	27%

a. (1 point) Construct confusion matrices for discrimination thresholds of 0.50 and 0.25.
b. (1.5 points) Plot the Receiver Operating Characteristic (ROC) curve with the discrimination
thresholds of 0.50 and 0.25.
Label each axis and the coordinates and discrimination threshold of each point on the
curve.
c. (0.5 point) Describe an advantage and a disadvantage of selecting a discrimination threshold
of 0.25 instead of 0.50.
d. (0.5 point) Describe whether a discrimination threshold of 0.25 or 0.50 is more appropriate for
a line of business with low frequency and high severity.

6.   25% Threshold   50% Threshold

Claim #	Fraud	Predict.		Predict.	
1	Y	N	False Neg.	N	False Neg.
2	N	N	True Neg.	N	True Neg
3	N	N	True Neg.	N	True Neg.
4	N	Y	False Pos.	Y	False Pos.
5	Y	Y	True Pos.	Y	True Pos.
6	Y	Y	True Pos.	N	False Neg.
7	N	N	True Neg.	N	True Neg.
8	Y	Y	True Pos.	Y	True Pos.
9	N	Y	False Pos.	Y	False Pos.
10	N	Y	False Pos.	N	True Neg.

(a)   25% Threshold
                            Predicted

Actual	Fraud	No Fraud	Total
Fraud	true pos.: 3	false neg.: 1	4
No Fraud	false pos.: 3	true neg.: 3	6
Total	6	4	10

                            50% Threshold
                            Predicted

| Actual	Fraud	No Fraud	Total
Fraud	true pos.: 2	false neg.: 2	4
No Fraud	false pos.: 2	true neg.: 4	6
Total	6	4	10

(b) Sensitivity = $\dfrac{\text{True Positives}}{\text{Total Number of Events}}$ = $\dfrac{\text{Correct Predictions of Fraud}}{\text{Total Number of Fraudulent Claims}}$.

Specificity = $\dfrac{\text{True Negatives}}{\text{Total Number of Non-Events}}$ = $\dfrac{\text{Correct Predictons of No Fraud}}{\text{Total Number of Nonfraudulent Claims}}$.

25% threshold: sensitivity = 3/4, and specificity = 3/6 = 1/2.   Graph (1 - 1/2, 3/4).
50% threshold: sensitivity = 2/4 = 1/2, and specificity = 4/6 = 2/3.   Graph (1 - 2/3, 1/2).
The ROC Curve, plus the 45-degree comparison line:
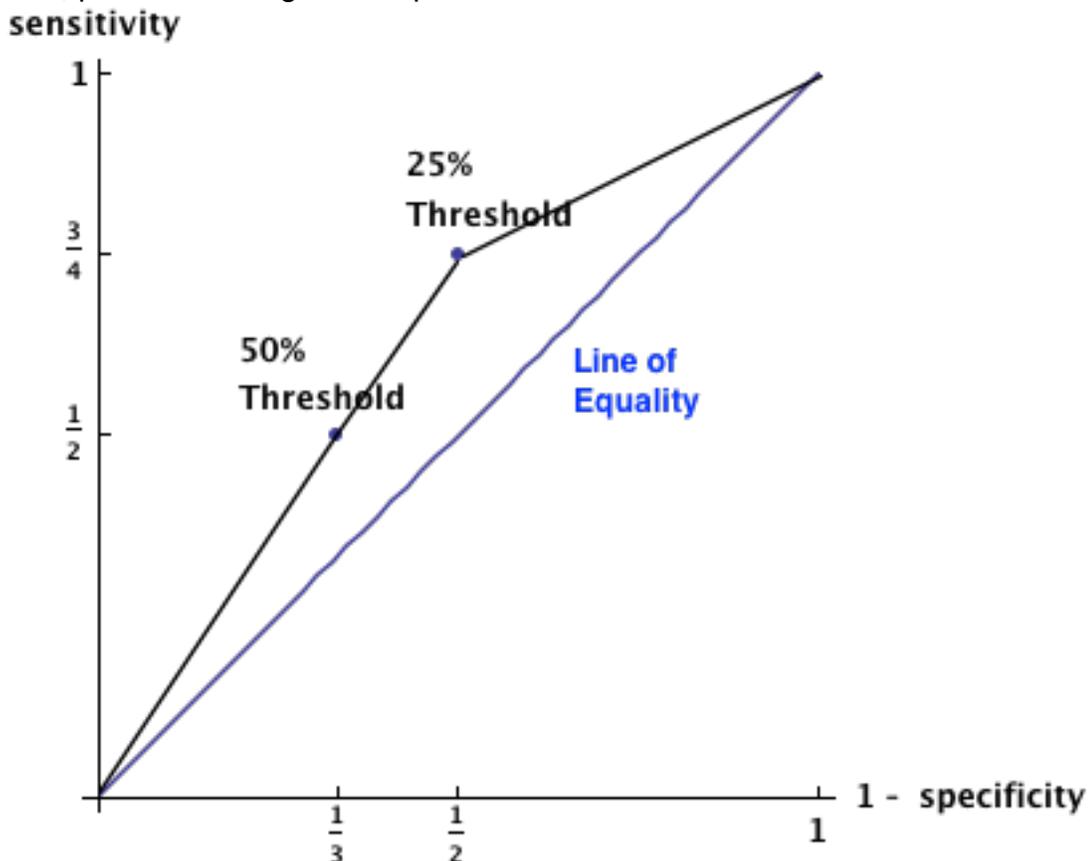
(c) Using a 25% threshold results in more predictions of fraud than using a 50% threshold.
Therefore, the 25% threshold has greater sensitivity, more true positives, which is good;
however, this is at the cost of lower specificity, more false positives, which is bad.
Alternately, Advantage: You will catch more actual fraud claims because you will have a higher
true positive rate. Disadvantage: You will have a higher false positive rate as well, which means
you will waste resources to review claims that are not fraudulent.

(d) There are few claims, but they are large. Thus we are very willing to spend money investigating claims for possible fraud; we do not want to miss any true positives and are willing to live with false positives. Therefore, we would prefer the lower threshold of 25%, which has greater sensitivity.

Alternately, a threshold of 0.25 is more appropriate. The high severity makes the cost of not investigating a fraudulent claim very high. The low frequency means that the number of additional claims that will need to be investigated is not very large. The cost of investigating these few additional claims is far less than the cost of potentially missing a few fraudulent claims at a higher discrimination threshold.

Comment: See Table 13 and Figure 22 in "GLMs for Insurance Rating."

According to the CAS Examiner's Report, in part (a) one was required to show a table similar to the one I have, showing the origin of the true positives, false positives, true negatives, and false negatives.

7. (1.75 points) Given the following information:
• Allocated loss adjustment expense is 15% of the indemnity amount.
• The variance method has been selected to include a risk load in the Increased Limits Factors
      (ILFs) with k = 0.000064 and δ = 0.

Limit, ℓ	E[X; ℓ]	E[X²: ℓ]
1,000	840	790,123
5,000	2,485	9,467,456

a. (1 point) Calculate the following:
     • The risk loads for each limit.
     • The ILF with and without risk load for the 5,000 limit.
b. (0.75 point) Assuming portfolio weights of 75% for a 1,000 limit and 25% for a 5,000 limit,
      determine the overall impact on premium by using the ILFs with the risk loads instead of
      the ILFs without risk loads.

7. Assume that 1000 is the basic limit.

(a) ILF = $\dfrac{(E[X \wedge \ell] + \varepsilon)(1+u) + k \{E[(X \wedge \ell)^2] + \delta\, E[X \wedge \ell]^2\}}{(E[X \wedge b] + \varepsilon)(1+u) + k \{E[(X \wedge b)^2] + \delta\, E[X \wedge b]^2\}}$.

With $\delta = \varepsilon = 0$, ILF = $\dfrac{E[X \wedge \ell]\,(1+u) + k\, E[(X \wedge \ell)^2]}{E[X \wedge b]\,(1+u) + k\, E[(X \wedge b)^2]}$.

The risk load for 1000 is: k E[X² ; 1000] = (0.000064) (790,123) = **50.6**.
The risk load for 5000 is: k E[X² ; 1000] = (0.000064) (9,467,456) = **605.9**.

ILF for the 5000 limit <u>without</u> risk load is: $\dfrac{(2,485)(1.15)}{(840)(1.15)}$ = **2.958**.

ILF for the 5000 limit <u>with</u> risk load is: $\dfrac{(2,485)(1.15) + (0.000064)\,(9,467,456)}{(840)(1.15) + (0.000064)\,(790,123)}$ = **3.407**.

(b) Assume that 75% of the policies are written with the 1000 basic limit and that the average basic limit premium for both types are the same.
Assume that while the risk load is included in the calculation of the ILF, it does not alter the basic limit premium from what it otherwise would have been.

$\dfrac{(75\%)(1) + (25\%)(3.407)}{(75\%)(1) + (25\%)(2.958)}$ = 1.075. ⇔ **7.5% increase** in premiums.

Alternately, assume that the risk load increases the basic limit premium from what it otherwise would have been.
Then without risk loads, loss & ALAE:
(75%)(840)(1.15) + (25%)(2485)(1.15) = 1439.
Adding in the risk loads: 1439 + (75%)(50.6) + (25%)(605.9) = 1628.
1628/1439 - 1 = **13.1% increase**.
<u>Comment</u>: Bahnemann defines: δ = VAR[N]/E[N] - 1. For a Poisson Frequency, δ = 0.
In Bahnemann, u represents ALAE that varies with loss, while ε represents ALAE that is fixed.
The CAS Examiner's Report did not include my second solution to part (b); I would have given my first solution, but I thought the question was not clear.

8. (1.75 points) An insurer sells coverage with an attachment point of 5,000 and a layer limit of 5,000. The following table represents the expected cumulative severity distribution and limited expected severity for the current year at various limits:

ℓ	$F(\ell)$	$E[X; \ell]$
4,545	0.842	1,807
5,000	0.859	1,875
5,500	0.875	1,941
9,091	0.938	2,256
10,000	0.947	2,308
11,000	0.954	2,357

The actuary expects a 10% severity increase for all claim sizes next year.
a. (0.75 point) Calculate the percent change in frequency of claims in the layer.
b. (1 point) Calculate the percent change in pure premiums in the layer.

8. (a) Prior to inflation, those claims of size at least 5000 will pierce the layer.
After inflation, those claims that were 5000/1.1 = 4545 will now pierce the layer.
S(5K/1.1) / S(5K) = (1 - 0.842) / (1 - 0.859) = 1.121. ⇔

12.1% increase in the number of claims piercing the layer.

(b) Pure premium trend factor $= 1.1 \dfrac{E[X \; ; \; 10,000/1.1] - E[X \; ; \; 5000/1.1]}{E[X \; ; \; 10,000] - E[X \; ; \; 5000]} = 1.1 \dfrac{2256 - 1807}{2308 - 1875} =$

1.141. ⇔ **14.1%** change in pure premium in the layer.

Alternately, the average size of claims piercing the layer prior to inflation is:
$$\dfrac{E[X \; ; \; 10K] - E[X \; ; \; 5K]}{S(5K)} = \dfrac{2308 - 1875}{1 - 0.859} = 3071.$$

The average size of claims piercing the layer after inflation is:
$$1.1 \dfrac{E[X \; ; \; 10K / 1.1] - E[X \; ; \; 5K / 1.1]}{S(5K / 1.1)} = 1.1 \dfrac{2256 - 1807}{1 - 0.842} = 3126.$$

⇒ The increase in severity is: 3126/3071 = 1.018.

⇒ Pure premium trend factor = (1.121)(1.018) = 1.141.

⇔ **14.1%** change in pure premium in the layer.

Comment: See Examples 5.10 to 5.12 in Bahnemann.
The CAS gave credit for 2 interpretations of "claims in the layer":
● All the losses above 5,000.
● Only the losses of sizes 5,000 to 10,000.
Thus in part (a), the CAS also allowed as a solution the change in the number of claims of sizes
between 5000 and 10,000: $\dfrac{F(10K / 1.1) - F(5K/1.1)}{F(10K) - F(5K)} = \dfrac{0.938 - 0.842}{0.947 - 0.859} = 1.091.$

⇔ 9.1% increase.

9. (3.25 points) Consider the following claims-made commercial general liability policy:
• The insurance contract was originally written January 1, 2011, and has been renewed
     annually as a claims-made policy.
• The annual Premises/Operations basic limits manual premium is $200,000 and there is
     no products exposure.
• The expected loss ratio is 70%.
• Loss experience is evaluated as of June 30, 2016.

Claim Number	Policy Year	Indemnity	ALAE
1	2011	$5,000	$5,000
2	2012	$15,000	$25,000
3	2013	$58,000	$0
4	2013	$20,000	$85,000
5	2014	$118,000	$82,000
6	2015	$8,000	$5,000

Calculate the experience modified premium for the policy effective January 1, 2017.

9. We use experience from the 2013, 2014, and 2015 policies.
Therefore, exclude claims #1 and #2.
The expected annual losses and ALAE are: (70%)($200,000) = $140,000.
From Rule 14 of the ISO manual, the detrend factors are: 0.907, 0.864, 0.823.
From Rule 13B of the ISO manual, since the policy being rated is a mature claims-made policy, the first policy adjustment factor is 1.03.
The 2013 policy is a third year policy, the 2014 policy is a fourth year policy, and the 2015 is a fifth year claims-made policy.
⇒ From Rule 13C of the ISO manual, the second policy adjustment factors are: 0.94, 0.91, 0.88.

Policy Period	Loss Costs	PAF1	PAF2	Detrend Factor	Subject Loss & ALAE
2013	$140,000	1.03	0.94	0.907	$122,942
2014	$140,000	1.03	0.91	0.864	$113,376
2015	$140,000	1.03	0.88	0.823	$104,435
Total					$340,753

For example, ($140,000)(1.03)(0.94)(0.907) = $122,942.
The company subject loss cost is $340,753.
⇒ From Rule 16 of the ISO manual, Z = 0.54, EER = 0.940, and MSL = 173,150.

I assume the basic limit is $100,000; so we limit the indemnity of the 5th claim to $100,000.
Then the $182,000 Loss & ALAE for the 5th claim is limited to the $173,150 MSL.
Loss and ALAE entering the rating is: ($1000) (58 + 105 + 173.15 + 13) = $349,150.
Since the historical policies are all claims-made, there is no provision for expected unreported losses.
Actual Experience Ratio (AER) = 349,150 / 340,753 = 1.025.
M = (0.54) (1.025 - 0.940) / 0.940 = 0.049 or a 4.9% debit mod.
⇒ Experience modified (basic limit) premium is: (1.049)($200,000) = **$209,800**.

Comment: The question should have specified the ISO Experience Rating Plan.
See the helpful example shown in Rule 6 of the ISO manual.

10. (1 point) An actuary is evaluating a risk to determine the appropriateness of applying any schedule credits or debits. In order to reduce expenses the insured removed a safety program 2 years ago and in the next month will be reducing its staff.

		Range of Modifications		
Risk Characteristic	Description	Credit		Debit
Cooperation	Safety Program	10%	to	10%
Employees	Selection, training, supervision, experience	5%	to	5%

Recommend and defend an appropriate schedule modification.

10. For concreteness assume that the current policy being rated is 2017.
Assume that the policy is experience rated, and that data from 2013, 2014, and 2015 are being used in the experience rating.
Then the effect of the removal of the safety program should already be reflected in the 2015 experience, but not in the other two historical years. Thus while one could give a debit for the removal of the safety program, it should be 2/3 of what it would otherwise be. I will choose a 6% debit for the removal of the safety program.
The reduction in staff is not reflected yet in the historical data. However, there is too little information provided in order to determine whether this would affect the supervision, level of training, or experience of the employees. Nevertheless, let us assume the reduction in staff would lead to a loss of institutional knowledge and thus make the risk a little worse than it was; I choose a small 2% debit.
Then in total, we have a schedule rating debit of: 6% + 2% = **8%**.
Alternately, the reduction in personnel will occur in the next month and is definitely not reflected in the 3 year experience window, but will impact prospective loss experience. Assuming that the insured reduces its staff by eliminating the most reckless ones, perhaps one can give a small credit of 2% for this,
Alternately, the upcoming change in staffing is not reflected in the historical data. This change could lead to understaffing or improper supervision. I recommend the full 5% debit in this category of Employees.
Alternately, I recommend a 5% credit for reduction in staff, assuming they will keep the most experienced and highly trained employees and let the less experienced employees with less training go. This will reduce future expected losses.
Alternately, reduction in staff does not mean remaining staff is any less experienced, needs more training or more supervision. Hence, I don't see a need or ability to schedule rate based solely on fewer employees. 0%.
Comment: There are many possible full credit answers. My alternative solutions are taken from the samples in the CAS Examiner's Report.
The question should have mentioned that the insured is experience rated; there are some insureds that are too small to be experience rated, but which can be schedule rated.
Schedule rating involves a lot of underwriting judgement, and is not discussed in detail in the syllabus readings.

11. (2.5 points) A workers' compensation insurer is facing an increasingly competitive market. Its management is concerned about customer retention, premium growth, and loss ratio deterioration. The insurer's actuaries have proposed an updated experience rating plan. After grouping insureds for the purposes of an efficiency test, the projected impact of this proposal is below (values in thousands):

<u>Current Plan</u>

Quintile	Manual Premium	Loss	Standard Premium	Loss Ratio to Manual	Loss Ratio to Standard
A	4,750	1,978	3,278	42%	60%
B	4,825	2,824	4,005	59%	71%
C	4,450	2,915	4,450	66%	66%
D	4,845	3,608	5,378	74%	67%
E	4,400	3,520	5,500	80%	64%
Total	23,270	14,845	22,610	64%	66%

<u>Proposed Plan</u>

Quintile	Manual Premium	Loss	Standard Premium	Loss Ratio to Manual	Loss Ratio to Standard
A	4,220	1,494	2,068	35%	72%
B	5,100	2,922	4,233	57%	69%
C	4,150	3,088	4,109	74%	75%
D	4,950	3,689	5,346	75%	69%
E	4,850	3,652	5,723	75%	64%
Total	23,270	14,845	21,478	64%	69%

It is estimated that the upfront cost to adopt the new rating plan will be $500,000.
a. (1.5 points) Perform an efficiency test and evaluate the proposed experience rating plan relative to the current plan.
b. (1 point) In light of management's concerns, evaluate the merits of adopting the new rating plan versus keeping the current plan, and provide a recommendation.

11. (a) For example, the sample variance for the manual loss ratios for the current plan is:

$$\frac{(42\% - 64.2\%)^2 + (59\% - 64.2\%)^2 + (66\% - 64.2\%)^2 + (74\% - 64.2\%)^2 + (80\% - 64.2\%)^2}{5 - 1} = 0.02172.$$

(It would also have been acceptable to use the given average manual loss ratio of 64% rather than the 64.2% I used.)

For the current plan: $\dfrac{\text{Sample Variance of Standard Loss Ratios}}{\text{Sample Variance of Manual Loss Ratios}} = \dfrac{0.00163}{0.02172} = 0.0750.$

For the proposed plan: $\dfrac{\text{Sample Variance of Standard Loss Ratios}}{\text{Sample Variance of Manual Loss Ratios}} = \dfrac{0.00167}{0.03072} = 0.0544.$

Smaller statistic is better; thus, based on the efficiency test the **proposed plan is preferred**.
(b) The manual premium is $23,270 million. 0.5/23.270 = 2.1% of annual premium.
So the one time upfront cost should not be a major consideration, if the other goals are met.
Based on the efficiency test, the proposed plan does a better job of identifying differences between insureds and adjusting for those differences than does the current plan. This should lead to an improvement in estimating future expected losses of individual insureds. This should give this insurer a competitive advantage (or reduce any current competitive disadvantage. The deteriorating loss ratios may be due to its major competitors doing a better job of pricing individual insureds.) The insurer should be able to retain and attract better insureds, and better price those worse than average insureds it does write. This should improve the loss ratios to standard premium. We would expect the introduction of the new plan to adversely affect some insureds, and favorably affect other insureds. Thus in the short run it may not improve customer retention; however, it should improve customer retention in the long run since this insurer is less likely to suffer from adverse selection. While this may not lead to premium growth it should prevent premium shrinkage.
Based on the above reasons, **I recommend using the proposed plan**.
Alternately, I am concerned that the new plan would result in $1.132 million less in standard premium per year (about 5% of manual premium) than the current plan. (The current plan's standard premium is 2.8% less than manual premium, which is not atypical for experience rating plans. However, the proposed plan's standard premium is 7.7% less than manual premium.)
This probably would overcome most if not all of the benefits from adopting the new plan.
Therefore, **I recommend that the actuaries go back and revise their proposed plan in order to get rid of this reduction in standard premium compared to that for the current plan, while retaining the improvements**.
A sample solution from the CAS Examiner's Report:
● Customer retention – changing plans may give big hikes to some customers, hurting retention
     as they go somewhere else.
● Premium growth – more efficient plan will grow premium healthily rather than by getting risks
     no one else wants.
● Loss ratio deterioration – more efficient plan $\Rightarrow$ less adverse selection

     $\Rightarrow$ less LRs deterioration.

Therefore, I recommend adopting the new plan as its cost is only about 2% of premium and it addresses more concerns.

Another sample solution from the CAS Examiner's Report:

Even though the proposed plan has a better efficiency test statistic, I would <u>not</u> choose the proposed plan. The proposed plan's separation from minimum to maximum manual LRs is only a slight improvement: 75% - 35% versus 80% - 42%, but at the cost of 3% on the standard LRs. Also the implementation of the proposed plan would be expensive.

<u>Comment</u>: In the case of both plans, the manual loss ratios increase with quintile, although the current plan is better in this regard; both plans identify risk differences. Both plans have somewhat level standard loss ratios with quintile, although the proposed plan is better; both plans adjust for risk differences.

In part (a), one could consistently use the biased estimator of the variance with 5 in the denominator, rather than the sample variance with 4 in the denominator; the denominators cancel in the calculation of the efficiency test statistic.

In a given state, often all Workers Compensation insurers will use the same experience rating plan, which was developed and filed by a rating bureau such as NCCI.

12. (3.25 points) A policy has a flat dollar deductible of M and a maximum payout on a loss by the insurer of N.

a. (1.0 point) Draw a Lee diagram representing the expected amount of loss incurred by this policy. Label the following:
- i. The axes
- ii, The deductible amount
- iii. The policy limit
- iv. The expected insured loss.

b. (0.5 point) Assume cumulative losses follow a distribution F(x). Write the formula for covered losses for this policy using:
- i. The layer method
- ii. The size method

c. (0.5 point) Briefly describe when the layer method may be preferred and when the size method may be preferred.

d. (1.25 points) Use a Lee diagram to demonstrate the consistency test of ILFs.

12. (a) We are pricing the layer from M to M+N.
The curve labeled F(x) is the size of loss distribution:
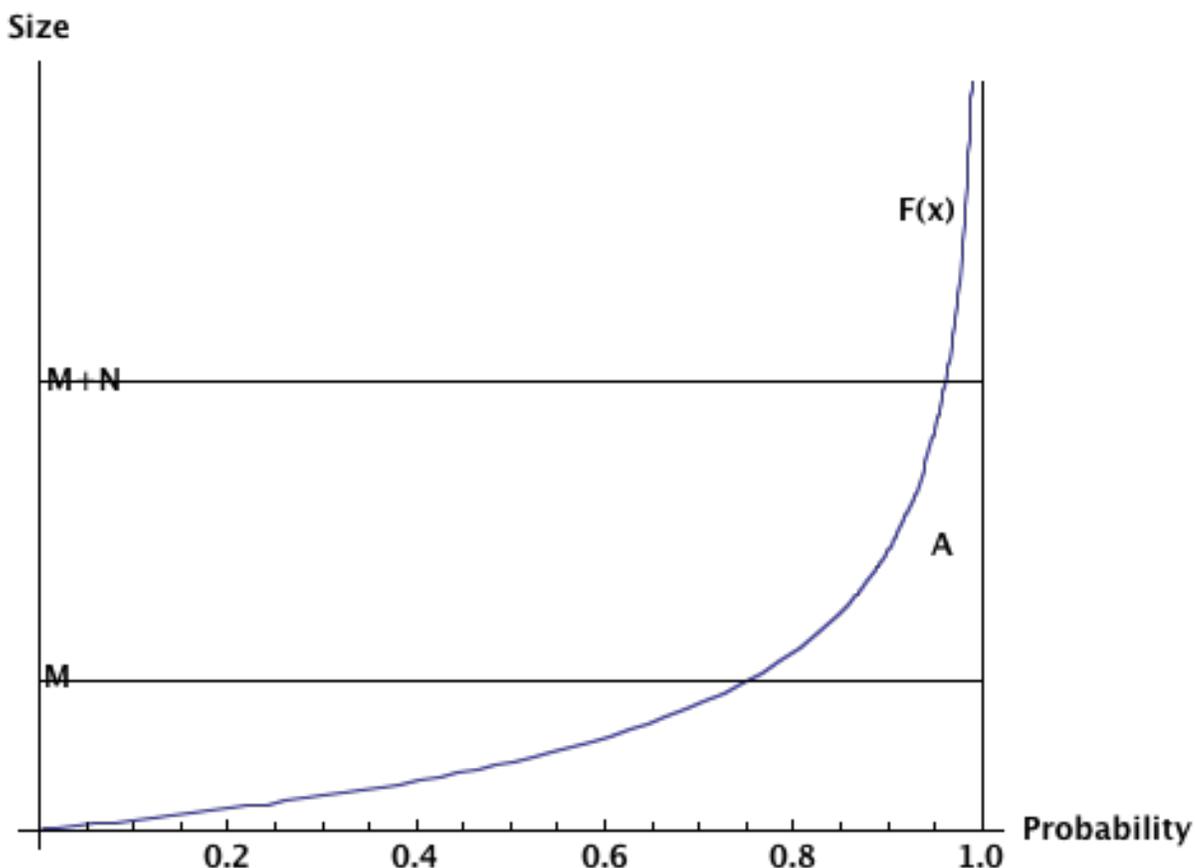
The expected loss in the layer is Area A,
below F(x) and between the horizontal lines at M and M+N.
(b) Where $S(x) = 1 - F(x)$.

i. Expected covered losses = $\displaystyle\int_{M}^{M+N} S(x)\ dx$. Adding up horizontal strips.

ii. Expected covered losses = $\displaystyle\int_{M}^{M+N} (x - M)\ f(x)\ dx\ +\ N\ S(M+N)$.   Adding up vertical strips.

Alternately, Expected covered losses = $\displaystyle\int_{M}^{M+N} x\ dF(x)\ +\ (M+N)\ S(M+N)\ -\ M\ S(M)$.

(c) The layer method would be preferred when S(x) is easier to integrate than x f(x), while the size method would be preferred when it is easier to integrate x f(x) than S(x).
Alternately, if working with empirical data, then the layer method would be preferred when one is trying to price many layers with different deductibles and maximum payouts, while the size method may be preferred when one is trying to price a single layer.

(d) In general, the consistency test is: for $x < y < z$, $\dfrac{ILF(y) - ILF(x)}{y - x} > \dfrac{ILF(z) - ILF(y)}{z - y}$.

Each increased limit factor is: $\dfrac{\text{Expected Losses Less Than the Limit}}{\text{Expected Basic Limit Losses}}$.

Thus $ILF(y) - ILF(x) = \dfrac{\text{Expected Losses in the Layer from x to y}}{\text{Expected Basic Limit Losses}}$.

Thus for purposes of the comparison, the Expected Basic Limit Losses drop out.
For simplicity let us take three limits spaced equally.
Then the comparison is between expected losses in different layers.
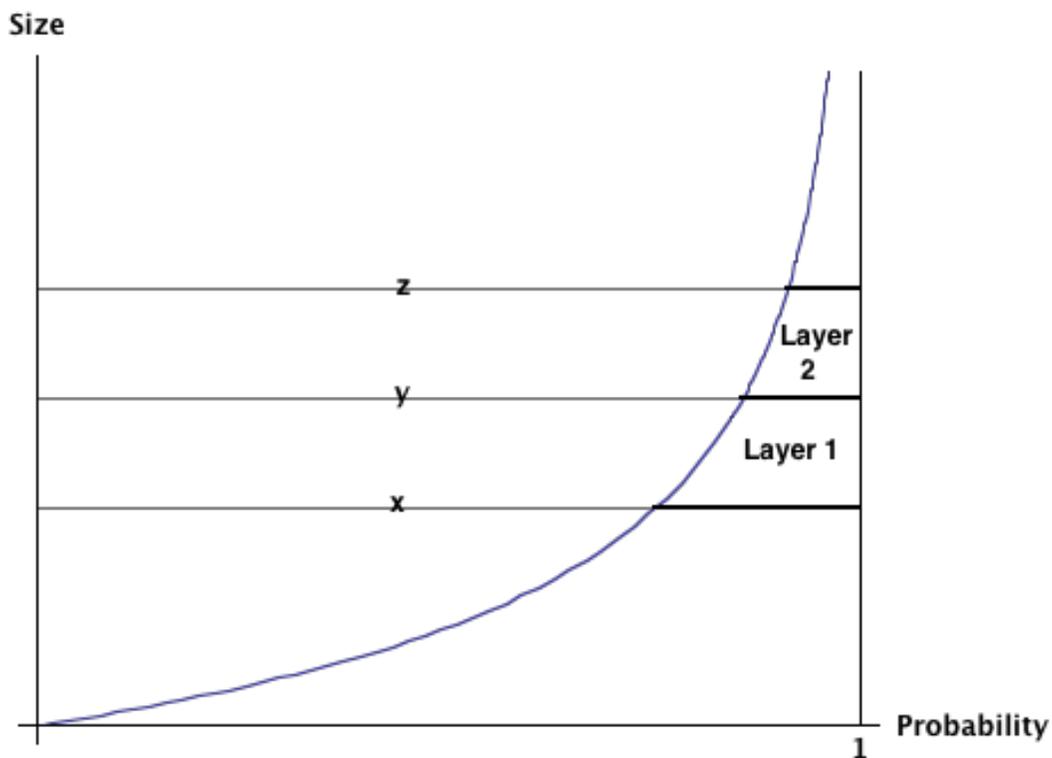
Layer 1 from x to y is greater than Layer 2 from y to z, because both layers have the same height and Layer 1 has a larger width. Thus demonstrating the consistency test.
<u>Comment</u>: See Section 2.1 of Chapter 3 of "Individual Risk Rating".
Part (c) is not fully discussed in this syllabus reading. In part (d), the consistency test for ILFs is discussed at page 170 of Bahnemann, but not in terms of Lee Diagrams.

$$\int_{M}^{M+N} (x - M)\, f(x)\, dx + N\, S(M+N) = \int_{M}^{M+N} x\, f(x)\, dx - \int_{M}^{M+N} M\, f(x)\, dx + N\, S(M+N) =$$

$$\int_{M}^{M+N} x\, dF(x) - M\, \{S(M) - S(M+N)\} + N\, S(M+N) = \int_{M}^{M+N} x\, dF(x) + (M+N)\, S(M+N) - M\, S(M).$$

Size
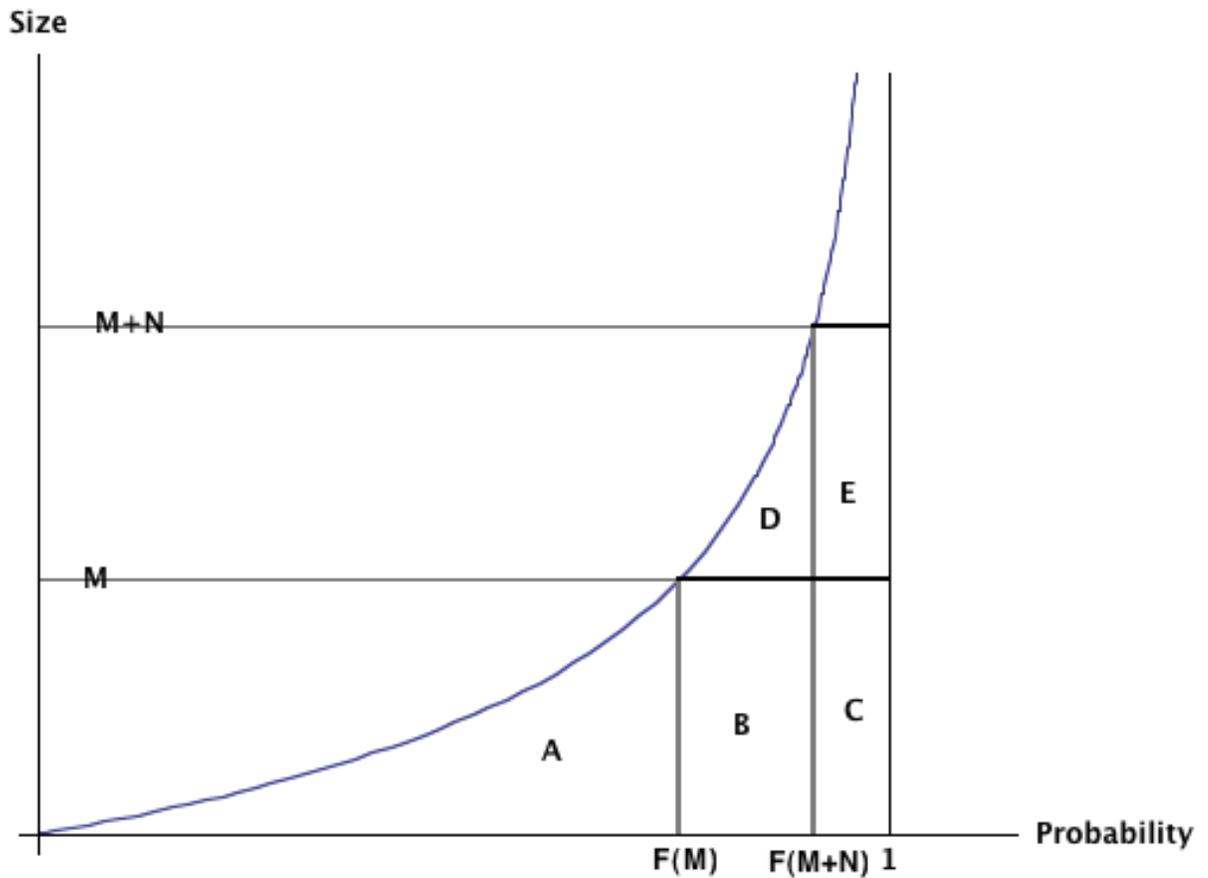
The layer from M to M+N is: D + E.

$$\int_{M}^{M+N} x \, dF(x) \text{ is: B + D.}$$

(M+N) S(M+N) = C + E.

M S(M) = B + C.

Thus, $\int_{M}^{M+N} x \, dF(x) + (M+N) \, S(M+N) - M \, S(M) = (B + D) + (C + E) - (B + C) = D + E$

= Layer from M to M+N.

13. (2.5 points)
A risk is written using a balanced retrospective rating plan with the following characteristics:
• Losses at the minimum premium = $50,000
• Losses at the maximum premium = $300,000
• Loss conversion factor = 1.05
• e = $10,000
The following table shows actual experience from a representative sample of risks that are
similar to the risk in question:

Risk	Actual Aggregate Loss	Risk	Actual Aggregate Loss
1	$25,000	6	$175,000
2	$50,000	7	$200,000
3	$100,000	8	$300,000
4	$100,000	9	$350,000
5	$150,000	10	$550,000

a. (2 points) Determine the maximum premium that the insured can be charged.
b. (0.5 point) An actuary has acquired actual aggregate data from a new book consisting of risks
    within the same industry.
    The loss experience of five representative risks from this book is shown below:

Risk	Actual Aggregate Loss
1	$275,000
2	$300,000
3	$500,000
4	$700,000
5	$800,000

The actuary proposes to combine the data from the two books, stating that the combined data
will result in a more accurate calculation of the insured's retrospective premium.
Assess the validity of the actuary's statement.

13. (a) Assume that there is no accident limit and no tax multiplier. Assume that the insurance charge and savings are based on the empirical experience given for the 10 similar risks.

E = (1000) (25 + 50 + 100 + 100 + 150 + 175 + 200 + 300 + 350 + 550) / 10 = $200,000.

Expense in basic premium = e - (c - 1)E = 10,000 - (1.05 - 1)(200,000) = 0.

Charge = {(350,000 – 300,000) + (550,000 – 300,000)} / 10 = $30,000.

Savings = (50,000 – 25,000) /10 = $2500.

Basic premium = (1.05)(30,000 - 2500) + 0 = $28,875.

Maximum Premium = 28,875 + (1.05)(300,000) = **$343,875**.

(b) The average aggregate loss for the 5 new risks is:

(1000) (275 + 300 + 500 + 700 + 800) = $515,000.

This is considerably bigger than the $200,000 average aggregate loss for 10 original risks. Therefore, the distribution of entry ratios for the new risks is expected to be different and have a smaller variance than the distribution of entry ratios for the original risks; this would result in inappropriate charges and savings for the original risks.

Thus the actuary's statement is <u>not</u> valid.

<u>Comment</u>: Part (a) should have said something like "assuming the insurance charge and savings are based on the empirical experience given for similar risks."

In part (a), there is no need to compute Table M. However, if one did the entry ratio corresponding to $300,000 in losses is: 300,000/200,000 = 1.5. ϕ(1.5) = 0.15.

The entry ratio corresponding to $50,000 in losses is: 50,000/200,000 = 0.25. ϕ(0.25) = 0.7625.

$\Rightarrow \psi$(0.25) = 0.7625 + 0.25 - 1 = 0.0125.

(200,000)(0.15 - 0.0125) = $27,500.

In part (b), while it is not possible to make a definitive conclusion based solely on only five observed values of aggregate loss, $515,000 is much bigger than $200,000.

14. (3.0 points)
A reinsurer has been supplied the following information from a large insurance company:

Claim Size Range	Expected Number of Claims	Expected Ultimate Losses (000s)
$0 to $1,000,000	19,000	$6,750,500
$1,000,001 to $2,000,000	359	$525,300
$2,000,001 to $3,000,000	230	$566,500
$3,000,001 to $4,000,000	147	$507,700
above $4,000,001	264	$1,650,000
TOTAL	20,000	$10,000,000

The reinsurer is entering into an excess of loss contract with the primary insurance company.
The reinsurer will pay all losses above a $5,000,000 per claim retention.
a. (1.5 points) Construct a graph of the excess severity function for claim sizes of
$1,000,000, $2,000,000, $3,000,000, and $4,000,000.
b. (1.5 points) Calculate the reinsurer's expected losses under the proposed contract.

14. (a) e(1M) = E[X - 1M | X > 1M] =
(525.3M + 566.5M + 507.7M + 1.650M) / (359 + 230 + 147 + 264) - 1M = 2,249,500.
e(2M) = (566.5M + 507.7M + 1.650M) / (230 + 147 + 264) - 2M = 2,249,920.
e(3M) = (507.7M + 1.650M) / (147 + 264) - 3M = 2,249,880.
e(4M) =1.650M / 264 - 4M = 2,250,000.
A graph of the excess severity function, with everything in millions of dollars:

Excess Severity
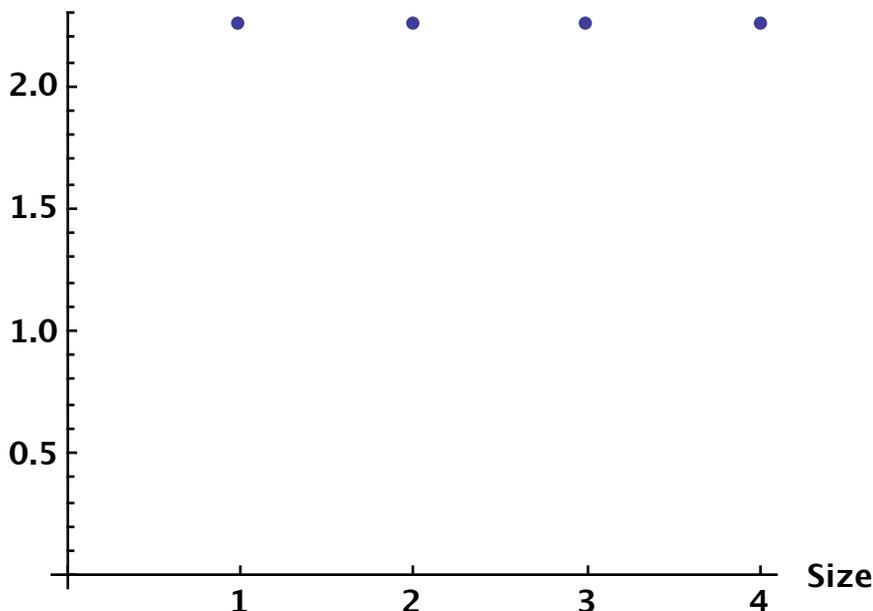
(b) Since the excess severity is constant, assume that the righthand tail acts like an Exponential.
Thus e(5 million) = 2.25 million.
Since the excess severities are 2.25 million, this is the mean of the Exponential.
Thus, Prob[X > 5 million | X > 1 million] = $e^{-(5-1)/2.25}$ = 0.1690.
Thus we estimate the number of claims greater than 5 million as:
(0.1690) (359 + 230 + 147 + 264) = 169.
Thus the dollars excess of $5 million are: (169)($2.25 million) = **$380 million**.
Alternately, examine the ratios of the number of claims over $1m, $2m, etc.:
(230 + 147 + 264) / (359 + 230 + 147 + 264) = 0.641.
(147 + 264) / (230 + 147 + 264) = 0.641.
264 / (147 + 264) = 0.642.
Thus infer that the number of claims greater than $5m is about: (0.641)(264) = 169.
Thus the dollars excess of $5 million are: (169)($2.25 million) = **$380 million**.
Comment: In part (a), one could instead use e(L) = (E[X] - E[X ; L]) / S(L).
See Figure 5.1 in Bahnemann.
In part (b), while up to $4 million the excess severity function is acting as it is from an
Exponential, this is no guarantee that the extreme righthand tail beyond $4 million is also
Exponential. Any sensible (reinsurance) actuary would check this out with additional detail; at a
minimum the actuary would also want data for the interval from $4 million to $5 million.

If in fact all of this data were from an Exponential Distribution with mean 2.25 million, then S(1M) = e$^{-1/2.25}$ = 64.1%. However, we observe that only 1000/20,000 = 5% of the claims are of size greater than 1 million. Therefore, while the righthand tail seems to behave like an Exponential Distribution, the smaller claims (of size less than $1 million) do not share this same behavior.

15. (3.25 points) The following applies to an incurred loss retrospectively rated policy effective January 1, 2017.

Per occurrence limit	$150,000
Maximum ratable loss	$600,000
ULAE as a percentage of loss & ALAE	8%
Expected Loss & ALAE limited to $150,000	$400,000
Basic Premium	$375,000
Tax Rate	3%

The expected loss & ALAE limited to $150,000 is used to determine the initial premium.
Retrospective rating adjustments start at 18 months and occur every twelve months thereafter.
The following loss experience occurs during the policy period.
All claims are closed at 42 months.

	Maturity (months)		
	18	**30**	**42**
Unlimited Incurred Loss & ALAE	$425,000	$650,000	$950,000
Incurred Loss & ALAE Limited to $150,000	$375,000	$475,000	$700,000
Incurred Loss & ALAE Excess of $150,000	$50,000	$175,000	$250,000

a. (1.75 points) Determine the amount and timing of each incremental cash flow payment made by the insured for this policy beginning with time 0.

b. (1.0 point) Propose and justify an alternative policy that would increase the insured's cash flow benefit without the insured retaining any additional excess loss risk.

c. (0.5 point) Identify and briefly describe one disadvantage to the insurer of the proposed plan in part b. compared to the existing retrospectively rated plan.

15. (a) Since no separate charge is given for the per occurrence limit, I will assume that it is included in the basic premium.

Initial Premium (January 1, 2017): {(1.08)($400,000) + 375,000}/(1- 3%) = $831,959.

Premium at 18 months (July 1, 2018): {(1.08)($375,000) + 375,000}/(1- 3%) = $804,124.

$831,959 - $804,124 = $27,835 from the insurer to the insured.

Premium at 30 months (July 1, 2019): {(1.08)($475,000) + 375,000}/(1- 3%) = $915,464.

$915,464 - $804,124 = $111,340 from the insured to the insurer.

The maximum rateable loss is $600,000.

Therefore, premium at 42 months (July 1, 2020):

{(1.08)($600,000) + 375,000}/(1- 3%) = $1,054,639.

$1,054,639 - $915,464 = $139,175 from the insured to the insurer.

(b) A paid loss retro with the same parameters. Losses would only enter the retro calculation when they are paid rather than when case reserves are set up. Thus the retro premium would be lower at each adjustment, than with an incurred loss retro.

Alternately, a Large Deductible Policy with similar parameters. ($150,000 per occurrence deductible and a $600,000 aggregate limit.) The insured is only responsible for reimbursing losses above the deductible as they are paid. Losses are paid after they are first incurred (when case reserves are set up). Thus there is a cashflow advantage for the insured compared to an incurred loss retro.

Alternately, a Self-Insured Retention (SIR) Policy with similar parameters. ($150,000 per occurrence retention and a $600,000 annual aggregate limit.) The insured is responsible for paying losses and then makes a claim with the insurer for any amounts above the retention. Thus there is a cashflow advantage to the insured compared to a incurred loss retro, where the insured is in essence charged for losses when they are incurred, which is sooner than when they are paid.

(c) Since the insured has a cashflow advantage, the insurer has a cashflow disadvantage.

The insurer would have less opportunity to earn investment income.

Alternately, the insurer would face greater credit risk for either the Paid Loss Retro or LDD. Under a paid loss retro, there is a chance that the insured would not pay its retro adjustments; the insured is expected to owe more money at adjustments than under a similar incurred loss retro. Under an LDD, there is a chance that the insured will not pay its loss reimbursements.

Alternately, under an SIR policy, usually the insured hires a third party administrator (who is not the insurer) to handle claims. Thus the insurer loses some ability to control losses. (What is at first a small claim, may eventually exceed the retention; it is only then that the insurer would get involved.)

Comment: It would have been better if the question had said that the basic premium includes the charge for the per occurrence limit.

In the question, for simplicity we ignore payroll audits. At about 15 months from policy inception, the final audited payroll would be used to adjust the standard premium, and thus the basic premium (and maximum premium). If the audited payroll is larger than the initial estimated payroll, then the premium would increase, if the audited payroll is smaller than the initial estimated payroll, then the premium would decrease.

In part (b) I think they should have said "briefly explain" rather than "justify".

16. (1.75 points) An insurance company generally uses two different methods to price excess layer insurance contracts:
> • Empirical construction of Table M; or
> • Approximating the distribution of aggregate losses
> > with a continuous approximation model.

a. (0.5 point)
Briefly describe two potential disadvantages of using a continuous approximation model.

b. (1.25 points) Fully describe the process of constructing an empirical Table M and estimating
> the aggregate loss cost of an excess layer using the table.

16. (a) Assume that the "continuous approximation model" is the single distribution approach from Clark (see my comment):
1. There is no allowance for the loss free scenario; in fact the lognormal is not defined for y = 0.
2. There is no easy way to reflect the impact of changing per occurrence limits on
        the aggregate losses.
From the CAS Examiner's Report with my comments in parentheses:
1. Sparse data makes it hard to accurately estimate the parameters of the distribution.
2. Computationally complex and intense. This computation can be time consuming.
(This is also true of practical applications of computing an empirical Table M, in which there are more steps than in the simplified discussion in the syllabus reading.)
3. Parameter risk; one needs to estimate parameters.
4. Since data is thin especially for the highest claim amounts, the charge for highest entry ratio
        will have high standard errors. (This also true for an empirical Table M.)
5. The fitted curve will depend a lot on highest few points, which are the most volatile so shape
        of curve may have significant bias.
(b) For many risks of similar size, get data on their annual aggregate losses.
For each risk, entry ratio = (aggregate losses) / (expected aggregate loss).
Order the risks by entry ratio.
For each desired entry ratio for Table M, determine the percent of risks whose entry ratio is above the desired entry ratio.
The charge for the largest entry ratio is 0.
Iteratively, one calculates charges for smaller entry ratios by successively adding:
        (increment in entry ratios) (percent of risks above this entry ratio).
If desired, the table M savings can be also included: $\psi(r) = \phi(r) + r - 1$.
Assume we want the aggregate layer excess of A. Let E be the expected aggregate loss.
Then the expected aggregate losses in the layer are: $E \phi(A/E)$.
If instead one wants the aggregate layer from b to t, then the expected aggregate losses in the layer are: $E \{\phi(b/E) - \phi(t/E)\}$. (See Comment).
Alternately, for each desired entry ratio r for Table M,

$$\phi(r) = \frac{\sum_{i=1}^{i=n}(r_i - r)_+}{n}$$, where we have n risks with entry ratios r_i, and X_+ is X if X ≥ 0 and 0 if X < 0.

Comment: I could not find "continuous approximation model" in any of the syllabus readings. Thus in my opinion, part (a) is a defective question.
Nor could I find in the syllabus readings any of the answers given for part (a) in the CAS Examiner's Report.
"Individual Risk Rating" in Section 3.3 of Chapter 3: "Once the underlying frequency and severity distributions have been selected, the aggregate loss distribution can be simulated or, in many cases, calculated using a variety of closed-form methods." "Or the aggregate loss distribution might be directly approximated using a lognormal distribution."
Did the Exam Committee mean the former or the latter?
In "Reinsurance Pricing" at pages 36-38 Clark talks about the single distribution approach, using for example a LogNormal Distribution. "The single distribution approach assumes that the aggregate of all losses to the treaty follows a known CDF form. This is in contrast to a collective risk model for which there is explicit modeling of frequency and severity distributions."

According to the CAS Examiner's Report common mistakes for part (a) include:
- Stating that the model doesn't allow for loss-free scenarios - but the continuous approximation model is a severity distribution.
- Stating that it would be difficult for the model to reflect changing or varying per occurrence limits - but this is a problem for both Table M and the continuous approximation model.

Their first bullet directly contradicts what is said by Clark. Their second bullet would be valid if their question had said "potential disadvantages compared to the empirical construction of Table M." Some of the sample answers they provided make this same "mistake", for example, sparsity of data would affect both methods.

Other common mistakes for part (a) according to the CAS Examiner's Report:
- Stating that the model does not split the impacts of frequency and severity - but the continuous approximation model is a severity distribution.
- Stating the need to select a distribution to use - but the question states that the company is already using the continuous approximation model.

Their first bullet shows a lack of understanding. If one uses for example a LogNormal Distribution to approximate the aggregate distribution, then one is <u>not</u> separately modeling frequency and severity as in a collective risk model. In this case, the mathematical LogNormal Distribution is <u>not</u> being used as a severity distribution.

Their second bullet is at best silly; just because an insurer is using a method does <u>not</u> mean that they have chosen an appropriate form of distribution to use. For example, the company may be using a LogNormal Distribution, while more appropriate for the particular application is a mixture of 5 Exponential Distributions.

I do not like questions like part (b); instead just ask one to construct Table M using some given data.

A common mistakes for part (b) according to the CAS Examiner's Report:
- Calculating layer loss cost using the charge at the top of the layer minus the charge (instead of the savings) at bottom.

This is all wrong! We are not asked to price a retro with a maximum and a minimum premium. For an aggregate layer from b to t, the insurer pays nothing for aggregate losses less than b, pays x - b for aggregate losses greater than b but less than t, and pays t - b for aggregate losses of t or more. For a layer we subtract the excess ratios in the reverse order; here the insurance charges are mathematically the same as excess ratios for aggregate losses. Thus for a layer of aggregate loss: (loss cost above b) - (loss cost above t) = φ(bottom/E) - φ(top/E), is correct, rather than φ(top/E) - ψ(bottom/E).

In the following Lee Diagram, the entry ratio b/E corresponds to the bottom of the aggregate layer, while the entry ratio t/E corresponds to the top of the aggregate layer.

Entry Ratio
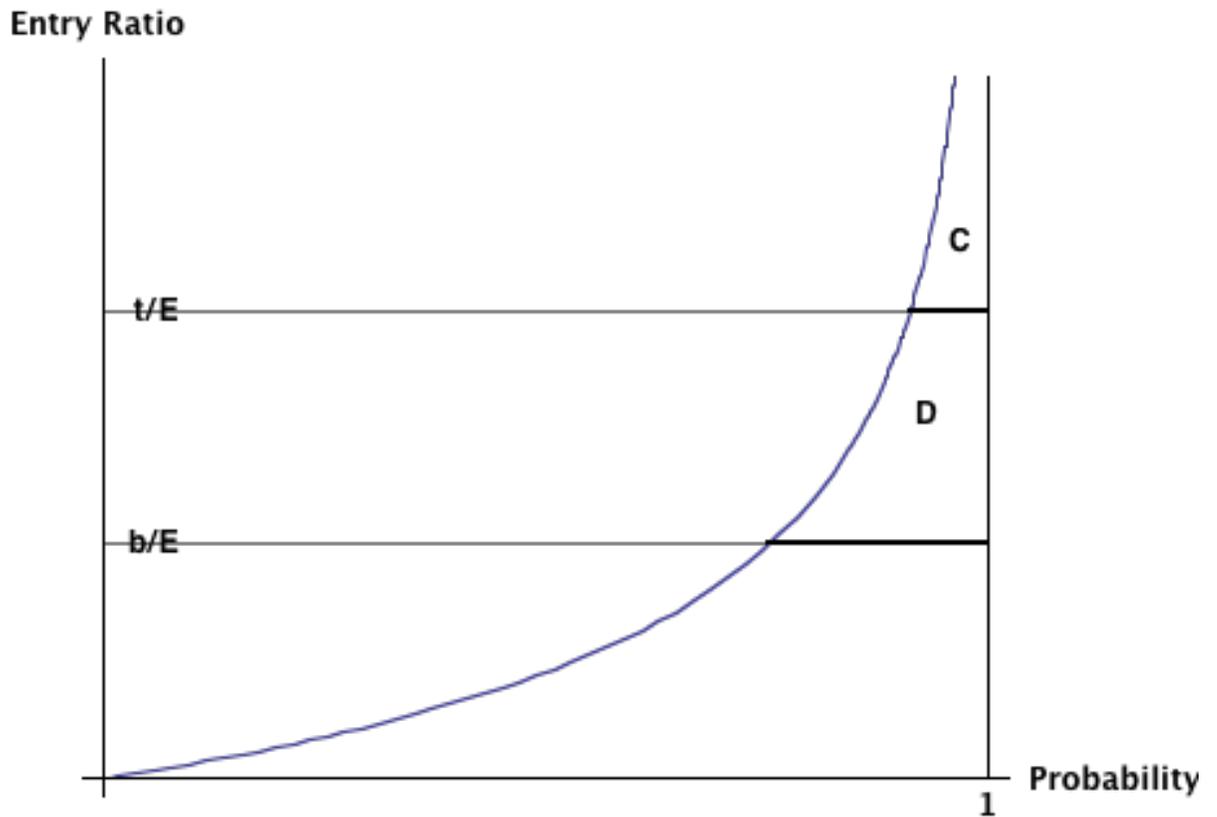
Area C = φ(t/E) = expected percent of aggregate losses excess of t.
Area C + Area D = φ(b/E) = expected percent of aggregate losses excess of b.
Area D = φ(b/E) - φ(t/E) = expected percent of losses in the aggregate layer from b to t.

17. (2.0 points) A workers' compensation policy has the following limits and expected losses:

Per-Occurrence Limit	$300,000
Expected Unlimited Aggregate Losses	$700,000
Expected Limited Aggregate Losses	$500,000
Aggregate Deductible Limit	$800,000
State Hazard Group Differential	0.95

Table of Expected Loss Groups

Expected Loss Group	Range of Values
31	630,000-720,000
30	720,001-830,000
29	830,001-990,000
28	990,001-1,180,000
27	1,180,001-1,415,000
26	1,415,001-1,744,000

Table of Insurance Charges
Expected Loss Group

Entry Ratio	31	30	29	28	27	26
0.43	0.5675	0.5626	0.5584	0.5543	0.5497	0.5456
0.60	0.4865	0.4799	0.4737	0.4676	0.4613	0.4553
1.14	0.2658	0.2552	0.2448	0.2341	0.2234	0.2128
1.60	0.1644	0.1537	0.1367	0.1322	0.1215	0.1107
2.67	0.0822	0.0769	0.0684	0.0661	0.0608	0.0554

a. (1.5 points) Calculate the total expected loss cost for this policy using
   the Insurance Charge Reflecting Loss Limitation (ICRLL) adjustment procedure.
b. (0.5 points) Describe how the ICRLL procedure is used to adjust expected losses for
   a workers' compensation policy.

17. (a) LER = 1 - 500/700 = 2/7.   ICRLL adjustment factor = $\dfrac{1 + (0.8)(2/7)}{1 - 2/7}$ = 1.72.

Including the state and hazard group relativity, LUGS = (0.95)(1.72)(700,000) = 1,143,800.
This implies Expected Loss Group 28.

The entry ratio = $\dfrac{\text{Aggregate Deductible Limit}}{\text{Expected Primary Losses}}$ = 800/500 = 1.6.

From the given Table, for ELG28, $\phi(1.6)$ = 0.1322.
To get the charge for the aggregate limit we multiply by the expected primary losses:
(0.1322)(500,000) = $66,100.
The expected losses excess of the occurrence limit is: 700,000 - 500,000 = $200,000.
The total expected loss cost for this policy is: $66,100 + $200,000 = **$266,100**.
(b) The presence of a per occurrence limit reduces the variance of the severity distribution, thus reducing the variance of the aggregate loss distribution. The ICRLL procedure approximates a limited table M by increasing the expected losses used to determine the Expected Loss Group, since policies with larger expected losses also have lower variance in aggregate loss distribution. This shifting of the column used in looking up the insurance charge in Table M approximates the impact of the per occurrence limit. See an example in the solution to part (a).
Comment: I did not understand what was the purpose of part (b), "Describe how the ICRLL procedure is used to adjust expected losses for a workers' compensation policy." given one just did an example of the ICRLL procedure in part (a). Based on the CAS Examiner's Report, the Exam Committee apparently wanted one to briefly describe the purpose and ideas behind the ICRLL procedure. The Exam Committee seemed to mistakenly believe that the state/hazard group adjustment is part of the ICRLL procedure. However, the Insurance Charge Reflecting Loss Limitations (ICRLL) is only used when there is a loss limit. The state and hazard group adjustment is used whether or not there is a loss limit. The state/hazard group relativity is based on the average claim cost, excluding medical only claims.
The relativity is the ratio: (ACC Countrywide) / (ACC for the state and hazard group).
By multiplying by the state/hazard group relativity, we adjust the expected losses to what they would have been for a similar insured in a state and hazard group that had the countrywide average severity. The Expected Loss Groups ranges used to enter Table M were designed to be used with such an insured.

18. (3.25 points) A homogenous group of property risks exhibit the following risk profile:

Exposure Distribution	Losses as a percent of the Maximum Possible Loss (MPL)
80%	0%
6%	25%
8%	50%
4%	75%
2%	100%

a. (2 points) Plot the exposure curve, G(x). Label the axes and the points on the curve.

b. (0.75 point) Determine the parameters b and g for the 2 parameter MBBEFD distribution
    that fits this exposure curve using the following information:

Calculated value of parameter b

Parameter				μ					
g	40.0%	42.5%	45.0%	47.5%	50.0%	52.5%	55.0%	57.5%	60.0%
50.0	0.0023	0.0017	0.0013	0.0009	0.0005	0.0003	0.0002	0.0001	0.0001
25.0	0.0096	0.0073	0.0055	0.0041	0.0022	0.0015	0.0010	0.0006	0.0004
16.7	0.0239	0.0181	0.0137	0.0103	0.0057	0.0039	0.0026	0.0017	0.0010
12.5	0.0483	0.0365	0.0276	0.0209	0.0118	0.0081	0.0055	0.0036	0.0023
10.0	0.0877	0.0659	0.0497	0.0375	0.0212	0.0147	0.0100	0.0067	0.0044
8.3	0.1498	0.1116	0.0836	0.0628	0.0354	0.0245	0.0168	0.0113	0.0075
7.1	0.2470	0.]817	0.1347	0.1004	0.0561	0.0389	0.0267	0.0181	0.0121
6.3	0.4000	0.2892	0.2116	0.1561	0.0861	0.0594	0.0408	0.0277	0.0185
5.6	0.6446	0.4557	0.3277	0.2385	0.1291	0.0885	0.0605	0.0411	0.0276
5.0	1.0469	0.7i90	0.5055	0.3616	0.1910	0.1297	0.0882	0.0597	0.0400

c. (0.5 point) A reinsurer uses this exposure curve to price a property excess of loss treaty where
    the underlying limit is $100 million. Calculate the proportion of total losses in the layer
    $25 million excess of $25 million.

18. (a) We want the limited expected values; we only look at the 20% of the time there is a loss.

E[X; 25%] = (20%)(25%) / 20% = 25%.

E[X; 50%] = {(6%)(25%) + (14%)(50%)} / 20% = 42.5%.

E[X; 75%] = {(6%)(25%) + (8%)(50%) + (6%)(75%)} / 20% = 50%.
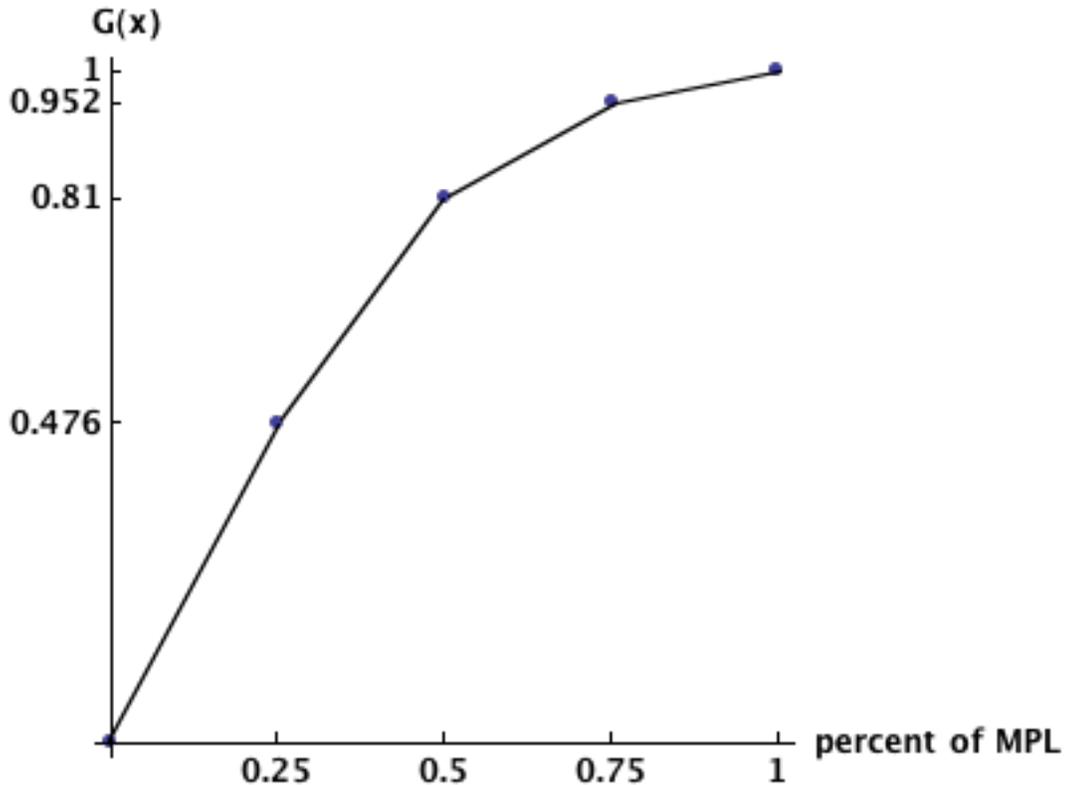
E[X] = {(6%)(25%) + (8%)(50%) + (4%)(75%) + (2%)(100%) } / 20% = 52.5%.

Exposure curve is a graph of loss elimination ratios.

G(0) = 0.   G(25%) = E[X ; 25%] / E[X] = 25/52.5 = 0.476. G(50%) = 42.5/52.5 = 0.810.

G(75%) = 50/52.5 = 0.952.   G(100%) = 1.

The Exposure Curve:

(b) g = 1 / Prob[MPL I there is a loss] = 1 / (2%/20%) = **10**.

µ = mean = 0.525.

Reading off the table, b = **0.0147**.

(c) We want the layer from 25% to 50% of the MPL.

$$\frac{E[X; 50\%] - E[X; 25\%]}{E[X]} = \frac{42.5\% - 25\%}{52.5\%} = \textbf{1/3}.$$

Alternately, G(50%) - G(25%) = 0.810 - 0.476 = **0.334**.

<u>Comment</u>: G'(0) = $\frac{0.476 - 0}{0.25 - 0}$ = 1.904. ⇒ µ = 1/G'(0) = 1/1.904 = 0.525.

G'(1) = $\frac{1 - 0.952}{1 - 0.75}$ = 0.192. ⇒ p = G'(1) / G'(0) = 0.192 / 1.904 = 0.10. ⇒ g = 1/p = 10.

19. (3.0 points) A primary insurer wants to institute a swing plan with the following terms:
• Layer: $500,000 xs $500,000 per claim
• Premium = Layer Losses * 1.3
• Minimum Premium = $100,000
• Maximum Premium = $325,000
The reinsurer has analyzed the insurer's experience.
All claims are closed and trend has been applied.

Accident Year	On-Level Premium	Claim #1	Claim #2	Claim #3	Claim #4
2012	400,000	465,899	230,567	512,017	
2013	420,000	240,785	125,766	80,154	470,666
2014	1,200,000	704,895	106,528		
2015	2,000,000	45,988	349,145	376,011	1,246,852
2016	2,200,000	437,100	654,158	250,000	156,750

a. (1.5 points) Calculate the expected loss ratio for the reinsurer based on the empirical
      distribution model using equal weights for each year.
b. (0.5 point) Determine if an empirical distribution model is appropriate for this account and give
      one reason in support of your answer.
c. (1 point) The use of a collective risk model is an alternative approach.
      Explain two concerns when using this type of model.

19. (a) 2012: (12,017)(1.3) = 15,622. So premium would be the minimum $100,000.
Loss Ratio = 12,017/100,000 = 12.017%.
2013: (0)(1.3) = 0. So premium would be the minimum $100,000. Loss ratio = 0%.
2014: (204,895)(1.3) = 266,364. This is the premium.
Loss ratio is: 204,895/266,364 = 76.923%.
2015: (500,000)(1.3) = 650,000. So the premium would be the maximum $325,000.
Loss ratio is: 500,000/325,000 = 153.846%.
2016: (154,158)(1.3) = 200,405. This is the premium.
Loss ratio is: 154,158/200,405 = 76.923%.
I assume that "using equal weights for each year" means averaging the five loss ratios:
(12.017% + 0 + 76.923% + 153.846% + 76.923%)/5 = **63.94%**.
(b) I would <u>not</u> rely on the empirical distribution to price this account. There are only 4 claims in
five years that pierce the layer; thus there is a lot of uncertainly in the estimate of the frequency
of such large claims. Also only one of the claims exhausts the layer, so again there is much
uncertainty in the estimate of how often this would be expected to happen in the future. The data
is <u>not</u> credible.
Alternately, I would <u>not</u> rely on the empirical distribution to price this account.
The on-level premiums of the primary insurer have increased tremendously over the historical
period. I assume this is due to the expected annual frequency increasing tremendously
(presumably due to increased exposures.) Therefore, one can not combine these years in order
to usefully predict the future
(c) 1. One would have determine the frequency distribution to use, both the form and the
parameters. One would have to determine the severity distribution to use, both the form and the
parameters. This is often difficult to do with limited data, particularly when we are interested in
the righthand tail. (In this case we only have 17 claims in 5 years.)
2. Most models assume that each occurrence is independent of the others and that the
frequency and severity distributions are independent of each other. This may be a reasonable
assumption in many cases, but could be false in others.
3. The aggregate distribution reflects the process variance of losses but does <u>not</u> reflect the full
parameter variance, let alone the risk of whether one has picked the correct model.
4. The complexity of the calculations can lead to a "black box" mentality - assuming the numbers
must be right because of the accuracy of the computer. Be sure to check results for
reasonableness.
5. Some collective risk models use numerical methods with a large error term for low frequency
scenarios. Check the output of the model; the expected error term should be given.
6. Panjer's Recursive Method is an example of a collective risk model. For higher expected
frequencies, the calculation is inconvenient because all the probabilities up to the desired level
must be calculated.
7. Panjer's Recursive Method is an example of a collective risk model. Only a single severity
distribution can be used in the analysis.

<u>Comment</u>: For part (c), see pages 41-42 of Clark.
If the on-level premiums had not grown so rapidly during the historical data period, I think one could make a case that it is okay to use the empirical distribution to price this account. The reinsured layer is a working layer that is relatively narrow. Also the swing in the plan is relatively small. Therefore, the reinsurer would get a reasonable estimate of the different types of years and the premiums it can expect in the future for this account. What other information/technique would the actuary use instead that would be superior? Of course the actuary would not have to rely on the estimate from a single technique.

20. (3 points) An earthquake model produces the following damage function for a $1,000,000 home in California (intensity on the horizontal axis and Damage Ratio on the vertical axis):
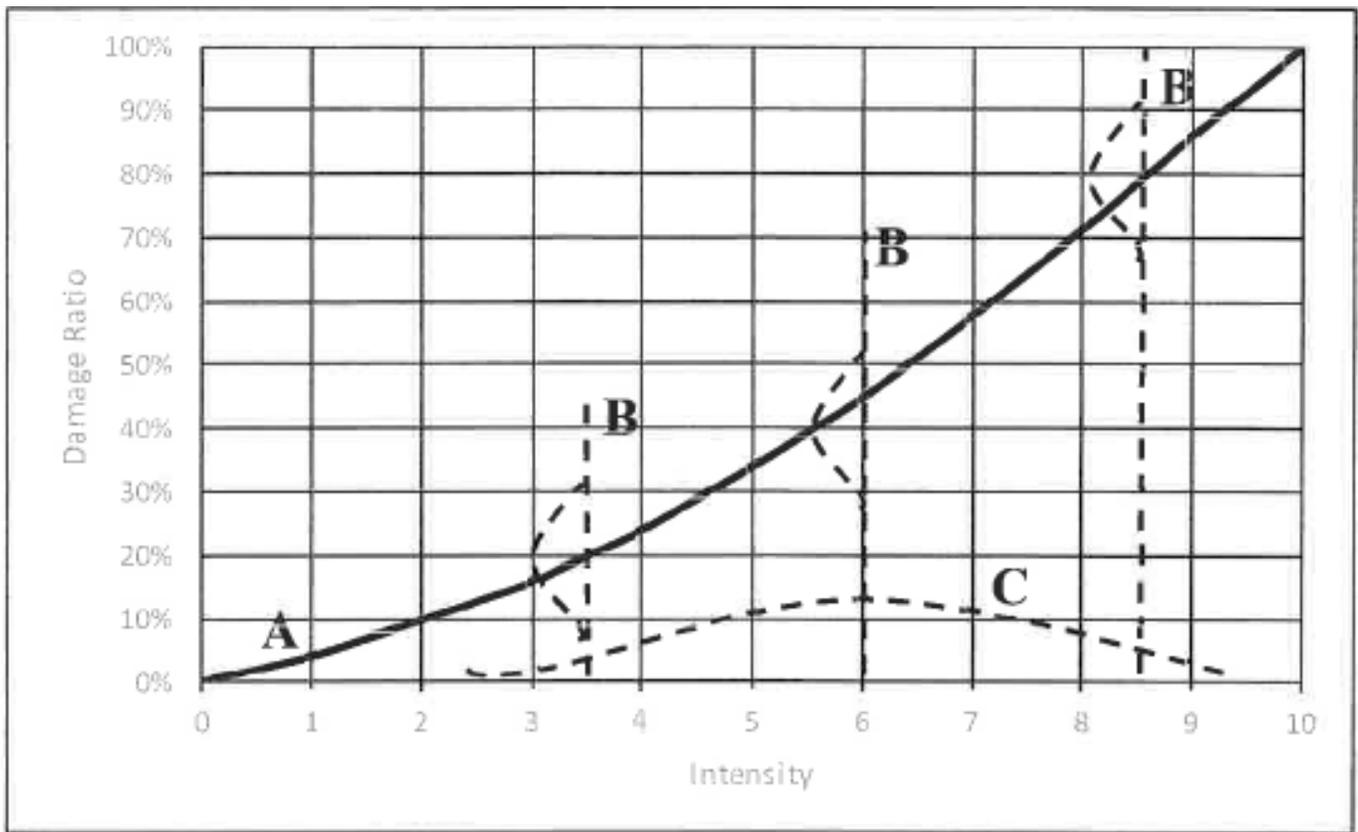
The probability of an event occurring with a given intensity is:

Intensity	Probability
3.5	10%
6.0	5%
8.5	2%

a. (0.75 points) Identify what A, B, and C represent in the above graph.

b. (1.75 points) Determine the premium for this home given the following information:

• The insurer's expense load is 20% of premium

• The risk load is set to 8% of the standard deviation of the loss

20. (a) The damage ratio = (repair costs) / (replacement cost of the building).
Curve A is the damage function, and represents the expected damage ratio for a given intensity earthquake.
Curve B is the damage-state distribution, and represents for a given intensity the distribution of the damage ratio around its expected value.
C is the intensity distribution, and represents the distribution of the intensity of external excitation for a given modeled earthquake.
(b) Assume that the stated intensities are the only possible intensities.
Assume that the probabilities are those for one year.
Reading the damage ratios as best as I can off of the graph:

Intensity	Probability	Damage Ratio
3.5	10%	20%
6.0	5%	45%
8.5	2%	80%

Expected loss: ($1,000,000) {(10%)(20%) + (5%)(45%) + (2%)(80%)} = $58,500.
The second moment of the distribution of expected damage by intensity is:
($1,000,000^2) {(10%)(20%^{2}) + (5%)(45%^{2}) + (2%)(80%^{2})} = 2.6925 x 10^{10}.
The variance, ignoring the distribution of damage ratios around their mean:
2.6925 x 10^{10} - 58,500^2 = 2.350 x 10^{10}.
Trying to read the standard deviations of the distributions of damage ratios from curves B, assuming most of the probability is within ±2 standard deviations: 5%, 4%, and 5%
Thus the variance in damage ratio around their mean is:
(10%)(0.05^2) + (5%)(0.04^2) + (2%)(0.05^2) = 0.00038.
Thus the standard deviation of damage due to the distribution of damage ratios is:
($1,000,000) $\sqrt{0.00038}$ = $19,494.

Thus the total variance is: 2.350 x 10^{10} + 19,494^2 = 2.388 x 10^{10}.

The standard deviation of the loss is: $\sqrt{2.388 \times 10^{10}}$ = 154,540.

The premium is: $\dfrac{58,500 + (8\%)(154,540)}{1 - 20\%}$ = **$88,579**.

Comment: See Figure 3.7 in Grossi and Kunreuther, which is not thoroughly explained.
In the CAS Examiner's report, they ignored the contribution to the total variance from the distribution for a given intensity of the damage ratios around their mean; in this case that contribution is <u>not</u> significant. Without that contribution, the standard deviation of loss would be instead $153,306, and instead the premium would be $88,456.
The total of the given probabilities is: 10% + 5% + 2% = 17%; there is a chance that there will not be an earthquake that affects this home next year.
We have to assume that the policy is only covering damage from earthquakes, as would be the case for the California Earthquake Authority (CEA).
See Tables 5.1 and 5.2 in Grossi and Kunreuther, where the risk load is taken as half of the standard deviation.