Solutions to the Fall 2016 CAS Exam 8

(Incorporating what I found useful in the CAS Examiner's Report)

The solutions and comments are solely the responsibility of the author.
While some of the comments may seem critical of certain questions, this is intended solely to aid you in studying and in no way is intended as a criticism of the many volunteers who work extremely long and hard to produce quality exams.

For 2017, the CAS is substituting a Study Note and part of a monograph "Distributions for Actuaries," by David Bahnemann **for the readings previously in Section B of the syllabus: Excess, Deductible, and Individual Risk Rating.**
Also ASOP 12 will replace AAA Risk Classifications Statement of Principles.

Therefore, some of these past exam question may not be on the syllabus for 2017, or may have to be rewritten to match the new syllabus.
Possibly affected questions: 2c, 3, 8, 9, 11-17.

prepared by
Howard C. Mahler, FCAS

Howard Mahler
hmahler@mac.com
www.howardmahler.com/Teaching/

1. (2.75 points) A group of insureds have different expected claim frequencies.
The number of insureds claim-free for the past t years is as follows:

Expected Claim Frequency	t=0	t=1	t=2	t=3
0.05	50,000	47,500	45,000	44,000
0.10	50,000	45,000	43,000	36,000
0.20	25,000	20,500	16,500	14,000
Total	125,000	113,000	104,500	94,000

Determine whether the variation of an individual insured's chance for an accident changes over time.

1. Here is the best solution I could come up with, expanding on the ideas in Appendix II of Bailey-Simon.
Let us assume that each insured is Poisson with mean λ, with the lambdas varying across the portfolio. Assume that over several years each insured has a constant expected frequency λ.

Then the probability of being claim free for zero years is: $1 - e^{-\lambda}$.

The probability of being claim free for at least one year is: $e^{-\lambda}$.

The probability of being claim free for at least two years is: $e^{-2\lambda}$.

Thus the probability of being claim free for exactly one year is: $e^{-\lambda} - e^{-2\lambda}$.

The probability of being claim free for at least two years is: $e^{-3\lambda}$.

Thus the probability of being claim free for exactly two years is: $e^{-2\lambda} - e^{-3\lambda}$.

Similarly, the probability of being claim free for exactly three years is: $e^{-3\lambda} - e^{-4\lambda}$.

Then for a subset of insureds with the same lambda:

$$\frac{\text{expected number claim free for exactly one year}}{\text{expected number not claim free}} = (e^{-\lambda} - e^{-2\lambda}) / (1 - e^{-\lambda}) = e^{-\lambda}.$$

$$\frac{\text{expected number claim free for exactly two years}}{\text{expected number claim free for exactly one year}} = (e^{-2\lambda} - e^{-3\lambda}) / (e^{-\lambda} - e^{-2\lambda}) = e^{-\lambda}.$$

$$\frac{\text{expected number claim free for exactly three years}}{\text{expected number claim free for exactly two years}} = (e^{-3\lambda} - e^{-4\lambda}) / (e^{-2\lambda} - e^{-3\lambda}) = e^{-\lambda}.$$

Thus within each of the given rows, if the assumptions are correct, we would expect these observed ratios to be close to equal. (Ignore the issue of how would one know the expected claim frequencies for the different rows of insureds.)
For the first row, the observed ratios are: 47,500/50,000 = 0.95, 45,000/47,500 = 0.947, and 44,000/45,000 = 0.978. The last ratio is dissimilar from the other two.
For the second row, the observed ratios are: 45,000/50,000 = 0.90, 43,000/45,000 = 0.956, and 36,000/43,000 = 0.837. These ratios are not similar to each other!
For the third row, the observed ratios are: 20,500/25,000 = 0.82, 16,500/20,500 = 0.805, and 14,000/16,500 = 0.849. These ratios are dissimilar from each other.
We do not see what we would expect; therefore something is wrong with the assumptions.
One or more of the following are true: individuals risk parameters are shifting over time, the frequency process is not Poisson, or insureds are entering and leaving the data base over the period of time studied.

Here is a sample solution from the CAS Examiner's Report that attempts to apply the ideas from Bailey-Simon (but fails to do so correctly, see my comments below):
Total insureds: 125,000 + 113,000 + 104,5000 + 94,000 = 436,500.
Insureds claims free for at least one year (in fact for those claims free for exactly 1, 2 or 3 years):
113,000 + 104,5000 + 94,000 = 311,500.
Insureds claims free for at least two years (in fact for those claims free for exactly 2 or 3 years):
104,5000 + 94,000 = 198,500.
Insureds claims free for at least three years (in fact for those claims free for exactly 3 years): 94,000.
Total expected claims: (186,500)(0.05) + (174,000)(0.10) + (76,000)(0.20) = 41,925.
Expected claims for those claims free exactly one year:
(47,500)(0.05) + (45,000)(0.10) + (20,500)(0.20) = 10,975.
Expected claims for those claims free exactly two years:
(45,000)(0.05) + (43,000)(0.10) + (16,500)(0.20) = 9,850.
Expected claims for those claims free exactly three years:
(44,000)(0.05) + (36,000)(0.10) + (14,000)(0.20) = 8,600.
Expected claims for insureds claims free for at least one year (in fact for those claims free for exactly 1, 2 or 3 years): 10,975 + 9,850 + 8,600 = 29,425.
Expected claims for insureds claims free for at least two years (in fact for those claims free for exactly 2 or 3 years): 9,850 + 8,600 = 18,450.
Expected claims for insureds claims free for at least three years (in fact for those claims free for exactly 3 years): 8,600.
Then for example, the expected frequency for those claims free for at least three years (in fact for those claims free for exactly 3 years): 8600/94,000 = 0.0915. Then, 0.0915/0.0960 = 0.9525.

n	# Claim free n or more years	Expected Claims	Expected Frequency	Relative Exp. Freq.	"Credibility"
3	94,000	8,600	0.0915	0.9525	0.0475
2	198,500	18,450	0.0929	0.9677	0.0323
1	311,500	29,425	0.0945	0.9835	0.0165
Total	436,500	41,925	0.0960	1	

For example, the "credibility" for three years is: 1 - 0.9525 = 0.0475.
If the variation of an insured's chance for an accident is not changing over time, then
$\frac{\text{3 year credibility}}{\text{1 year credibility}}$ will be approximately equal to 3, and $\frac{\text{2 year credibility}}{\text{1 year credibility}}$ will be approximately equal to 2.

$\frac{\text{3 year credibility}}{\text{1 year credibility}} = 0.0475 / 0.0165 = 2.88.$ $\frac{\text{2 year credibility}}{\text{1 year credibility}} = 0.0323 / 0.0165 = 1.96.$

The ratios are approximately 3 and 2, and therefore the chance for an accident is stable.

Here is a second sample solution from the CAS Examiner's Report that is a parody of the calculations in Bailey-Simon, demonstrating a lack of understanding of the ideas in Bailey-Simon.
Expected claims at t = 0 (actually for those not claims free):
(50,500)(0.05) + (50,000)(0.10) + (25,000)(0.20) = 12,500.
Expected claims at t = 1 (actually for those claims free exactly one year):
(47,500)(0.05) + (45,000)(0.10) + (20,500)(0.20) = 10,975.
Expected claims at t = 2 (actually for those claims free exactly two years):
(45,000)(0.05) + (43,000)(0.10) + (16,500)(0.20) = 9,850.
Expected claims at t = 3 (actually for those claims free exactly three years):
(44,000)(0.05) + (36,000)(0.10) + (14,000)(0.20) = 8,600.
Then the "frequency at t = 0": 12,000/125,000 = 0.1000.
The "frequency at t = 1": 10,975/113,000 = 0.09712.
The "frequency at t = 2": 9850/104,500 = 0.09426.
The "frequency at t = 3": 8600/94,000 = 0.09149.
The "frequency at t = 1 relative to t = 0": 0.09712/0.1000 = 0.9712.
⇒ One year "credibility": 1 - 0.9712 = 2.88%.
The "frequency at t = 2 relative to t = 0": 0.09426/0.1000 = 0.9426.
⇒ Two year "credibility": 1 - 0.9426 = 5.74%.
The "frequency at t = 3 relative to t = 0": 0.09149/0.1000 = 0.9149.
⇒ Three year "credibility": 1 - 0.9149 = 8.51%.
(Note this is <u>not</u> how Bailey-Simon calculates credibilities. Within a rating class, they compare for example the observed subsequent (premium based) frequency for those who are claims free for 2 years or more, to the overall observed subsequent (premium based) frequency.
Then Bailey-Simon are backing out the credibility for 2 years of data based on an observed credit appropriate for 2 or more years claims free.)
If the variation of an insured's chance for an accident is not changing over time, then

$\frac{\text{3 year credibility}}{\text{1 year credibility}}$ will be approximately equal to 3, and $\frac{\text{2 year credibility}}{\text{1 year credibility}}$ will be approximately equal to 2.

$\frac{\text{3 year credibility}}{\text{1 year credibility}}$ = 8.51% / 2.88% = 2.95. $\frac{\text{2 year credibility}}{\text{1 year credibility}}$ = 5.74% / 2.88% = 1.99.

The ratios are approximately 3 and 2, and therefore the chance for an accident is stable.

Here is a third sample solution from the CAS Examiner's Report that is a parody of the calculations in the paper by Mahler, demonstrating a lack of understanding of the ideas in that paper.
Determine the percent of the insureds in each column that are in each of the three rows.

t=0	t=1	t=2	t=3
50/125 = 40%	47.5/113 = 42.03%.	45/104.5 = 43.06%	44/94 = 46.81%
50/125 = 40%	45/113 = 39.82%	43/104.5 = 41.15%	36/94 = 38.30%
25/125 = 20%	20.5/113 = 18.14%	16.5/104.5 = 15.79%	14/94 = 14.89%

Now calculate the correlations between the various columns:

"lag 1"	t=0 vs t=1: 0.9965	t=1 vs t=2: 0.9998	t=2 vs t=3: 0.9806	AVG: 0.9923.
"lag 2"	t=0 vs t=2: 0.9980	t=1 vs t=3: 0.9845		AVG: 0.9913
"lag 3"	t=0 vs t=3: 0.9663			AVG: 0.9663

Since the correlations are decreasing with lag, this indicates that parameters are shifting over time.
Here is a fourth sample solution from the CAS Examiner's Report that is another parody of the calculations in the paper by Mahler, demonstrating a lack of understanding of the ideas in that paper.
Determine the expected claims for each entry in the rows and columns.

t=0	t=1	t=2	t=3
(0.05)(50,000) = 2500	2375	2250	2200
(0.10)(50,000) = 5000	4500	4300	3600
(0.20)(25,000) = 5000	4100	3300	2800

Then compute the correlations between the different columns.
"lag 1" r(0,1) = 0.9842, r(1,2) = 0.9456, r(2,3) = 0.9954. Average = 0.9750.
"lag 2" r(0,2) = 0.8730, r(1,3) = 0.9909. Average = 0.8914.
"lag 3" r(0,3) = 0.8220. Average = 0.8220.
Downward trending average correlation as lag increases. ⇒ Risk parameters are shifting.
Here is a fifth sample solution from the CAS Examiner's Report that is another parody of the calculations in the paper by Mahler, demonstrating a lack of understanding of the ideas in that paper.
Determine the ratios of the number of insureds in adjacent columns in each of the three rows.
For the first row, the observed ratios are: 47,500/50,000 = 0.95, 45,000/47,500 = 0.9474, and 44,000/45,000 = 0.9778.
For the second row, the observed ratios are: 45,000/50,000 = 0.90, 43,000/45,000 = 0.9556, and 36,000/43,000 = 0.8372.
For the third row, the observed ratios are: 20,500/25,000 = 0.82, 16,500/20,500 = 0.8049, and 14,000/16,500 = 0.8485.
Then take the correlations between these sets of ratios:
corr[{0.95, 0.90, 0.82}, {0.9474, 0.9556, 0.8049}] = 0.9049,
corr[{0.9474, 0.9556, 0.8049}, {0.9778, 0.8372, 0.8485}] = 0.3920.
corr[[{0.95, 0.90, 0.82}, {0.9778, 0.8372, 0.8485}]] = 0.748.
Average of correlations for "lag 1": (0.9049 + 0.3920)/2 = 0.6485.
Average of correlations for "lag 2": 0.748.
These correlations are not declining with increase in lags.
Thus there is no evidence that parameters are shifting over time.

Comment: This question does not follow any of the syllabus readings. Although this question bears a similarity to ideas in Bailey-Simon and to the shifting risk parameters paper by Mahler, the information needed to properly apply the ideas in those syllabus readings is not provided.
In my opinion, this is a terrible exam question, which demonstrates the lack of understanding of this material by its writer. I suspect those with a better understanding of this material did worse in attempting to somehow answer this exam question. For study purposes, I think this question has negative educational value. Of course, you might want to know how to mechanically reproduce one of the sample solutions in case this exact same form of question is repeated.
Therefore, I have given the sample answers from the CAS Examiner's Report.
My commentary on the question and sample solutions follows.
How would one know the expected claim frequencies for the different subsets of insureds?
If an insured's individual chance of an accident changes over time, what could it mean to be in a given row? If an insured's individual chance of an accident changes over time, the insureds in a given row can not have the same expected claim frequency over several years.
Although we are not shown the information, aren't there insureds who are claims-free for exactly four years, exactly five years, etc.? Thus, we do not know for example how many insureds were claims-free for 3 or more years.
In the first sample solution I showed, expected claims are calculated "at time t", by multiplying the number of insureds by the expected claim frequency. What does this mean? Yes if we have 50,000 insureds with an expected claim frequency of 0.05 then we would expect 2500 claims. However, these 50,000 insureds in the first column were not claim free, so they each had at least one claim. Perhaps this means we would expect 2500 claims the following year from these insureds; however; this would ignore the fact that those in a given (heterogeneous) group who are not claim free have higher than average expected future claim frequency compared to the group (the idea behind using credibility) and also that insureds claim propensity may change over time.
Rather as per Bailey-Simon, what we want to know is for a class of insureds the subsequent actual experience of those who were not claim-free, those who were claim free for at least one year, those who were claim free for at least two years, etc. Here we not given this vital information. The solution compares "expected" frequencies rather than as it should observed actual subsequent frequencies.
There is a comparison of the data for those claims free for exactly 1 to 3 years, those claims free for exactly 2 or 3 years, and those who are claims free for exactly 3. The correct comparisons would be between those claims free for at least one year, those who are claims free for at least 2 years, and those who were claims free for at least 3 years; we do not have that information.
Having performed a bunch of arithmetic, "credibilities" supposedly for one, two and three years of data are determined, which are not in fact credibilities in any meaningful sense. However, the conclusion drawn from these "credibilities" is correct. If risk parameters were shifting significantly over time, then the credibilities for one, two, and three years should increase significantly less than linearly.
In the paper by Mahler, the correlations are between different years of actual experience for a set of individual risks. After doing some arithmetic, the sample solutions compute correlations. However, these are not the type of correlations one would use to answer the question of whether we have shifting risk parameters.

The third sample solution works with correlations of the percent of insureds who are claims-free for exactly t years. There is no reason to assume that if risk parameters are constant, that this type of correlation will be independent of the differences in t. If these types of correlations decline as the difference in t (which is not the lag between different years of data) increases, this does not demonstrate that parameters are shifting.
The fourth sample solution and fifth sample solutions are also invalid.
Partial credit was also given for a Chi-Square approach, which is not shown. The Examiner's Report does not explain how one would know the expected number of insureds claims-free for exactly t years, to compare to the actual number. Nor does the Examiner's Report explain exactly how this has any relation to whether or not risk parameters shift.

2. (2 points)

a. (0.5 point) Describe the purpose of clustering as it pertains to risk classification.

b. (0.5 point) Briefly contrast hierarchical and non-hierarchical clustering methods.

c. (1 point) An insurer wants to increase the number of groupings for a particular class within their classification plan. According to the American Academy of Actuaries "Risk Classification Statement of Principles," describe two considerations in using a hierarchical or non-hierarchical clustering method to determine the groupings.

2. (a) Clustering is used in order to group similar risks into classes. Clustering techniques can be used to produce the most homogeneous classes. In order to apply clustering techniques one would need an index or set of indices that relate to the expected pure premium of each risk.
Clustering produces groups in a way such that the variance between groups is maximized and the variance within groups is minimized.
(b) In hierarchical clustering, one would first for example determine the best set of 3 groups.
Then one would determine the best set of 4 groups such that they can be combined to produce the best set of 3 groups. One would proceed in a similar manner to get the best set of 5 groups that can be combined to produce the previously determined set of 4 groups, etc.
In contrast in nonhierarchical clustering, we just get the best set of groups (the most homogeneous groups) for each number of groups without considering what the algorithm produced for a fewer number of groups.
“Non-hierarchical cluster analysis simply seeks the best partition for any given number of clusters.
In hierarchical cluster analysis the partition with k +1 clusters is related to the partition with k clusters in that one of the k clusters is simply subdivided to get the k +1 element partition. Thus if two objects are in different clusters in the k cluster partition then they will be in different clusters in all partitions with more than k elements. This places a restriction on the clusters that can be sensible in some contexts.”
Alternately, for hierarchical clustering, to go from k to k+1 groups you must break one of the k groups into two. In other words, each set of groups is a nested version of the previous set. Nonhierarchical clustering does not have this constraint on how the groups are formed.

(c) Based on the CAS Examiner's Report, apparently they meant to ask "describe two considerations related to increasing the number of groupings." The sample answers shown have nothing specifically to do with using a clustering algorithm, or the difference between hierarchical or nonhierarchical clustering methods.

• Homogeneity: Classes should have similar risk characteristics and have similar expected costs. We want to subdivide groups in a way that makes them more similar and homogeneous

• Credibility: Will the resulting groups be credible? As you add groups, each one becomes smaller. While this could help improve homogeneity within classes, you don't want to sacrifice too much credibility; one needs balance homogeneity versus credibility.

• Avoidance of Extreme Discontinuities: Increasing the numbers of groupings will create a more continuous spectrum of rates which will make it easier to avoid extreme discontinuities.

• Expense: There may be extra expense related to having more groups. Any cost of increasing the number of groupings should not outweigh the benefit of the new groupings

• Availability of Coverage: A greater number of groups should allow the actuary to estimate class rates that more closely match the expected costs of individual insureds.

This should make good risks and bad risks equally profitable to insurers, ensuring that underwriters will want to provide coverage.

Alternately, due to the wording of the question, I assumed they were asking about whether to use a hierarchical or nonhierarchical clustering method.

1. Nonhierarchical clustering will produce more homogeneous classes than hierarchical clustering, since we are not constraining the groups at each number of groups to relate to the previous number of groups. Thus nonhierarchical clustering is preferred.

2. The system should be acceptable to the public. A nonhierarchical clustering method is slightly simpler and thus easier to explain to the public and regulators. Thus nonhierarchical clustering is preferred.

<u>Comment</u>: According the CAS Examiner's report, in part a

"Candidates were expected to respond with one of the following concepts:

• Clustering creates homogeneous and credible groupings for a risk classification system; or

• Clustering will maximize the variance between classes and minimize the variance within classes.

A common mistake was mentioning homogeneity <u>but failing to mention credibility</u>."

However, clustering may or may not produce credible groupings. (Robertson used a version of the clustering algorithm in which a class with twice as much premium is counted twice as much. This weighted version is more likely to produce groupings that are credible than would the unweighted version of the clustering algorithm.)

In any case, Robertson nowhere states that clustering creates credible groupings.

3. (2 points) A private passenger automobile insurer sells policies through two distribution channels: independent insurance agents and directly to consumers via the internet.
The company's rates already incorporate expense differentials between the two channels but the head of sales has asked their actuary to file different pure premium factors for the two groups.
The actuary wishes to evaluate the acceptability of the request against the American Academy of Actuaries "Risk Classification Statement of Principles."

a. (1 point) Identify and briefly describe two considerations supporting inclusion of distribution channel in the pure premium factors.

b. (1 point) Identify and briefly describe two considerations against inclusion of distribution channel in the pure premium factors.

3. a. 1. The proposal divides the insureds in an <u>objective</u> way. The insurer already divides its insureds in this manner. There is an absence of ambiguity.
2. The system is <u>practical</u> and <u>cost-effective</u>. The proposal should involve little extra expense, since the insurer already has its computer systems set up to deal with the two different distribution channels and already charges different rates due to expense considerations.
3. Each classification is likely large enough to produce <u>credible</u> statistical predictions; there are enough policies sold through each distribution channel.
b. 1. The system should reflect expected cost differences and should distinguish among risks on the basis of relevant cost-related factors. Assuming there are <u>not</u> significantly different pure premiums between the two distribution channels for otherwise similar risks, then this argues against the proposal.
2. It is hard to justify a causal relationship and thus the proposal will not be acceptable to some insurance regulators. (In many states private passenger automobile insurance rates are highly regulated.)
3. Varying rates by distribution channel in no way promotes insureds to mitigate their exposure to hazard.
4. A consumer one year could go to an agent and then the next year go online, so there is not <u>constancy</u> in measure.
5. Using distribution channels is more prone to insured's <u>manipulation</u>. They can price through different channels and select the lowest price. (However, which distribution channel is not subject to misrepresentation by the insured.)

4. (3 points) An actuary is conducting a generalized linear model (GLM) analysis on historical personal automobile data in order to develop a rating plan.

a. (1.5 points)

Argue against the following factors being included as predictors in the actuary's GLM analysis:

i. Limit of liability.

ii. Number of coverage changes during the current policy period.

iii. ZIP code of the garaging location of the automobile.

b. (1 point) The actuary is modeling pure premium with a log-link function and a Tweedie error distribution ($1 < p < 2$). Provide two arguments against the inclusion of deductible as a predictor in the actuary's GLM analysis.

c. (0.5 point) Other than including deductible as a predictor in the GLM, describe how to determine deductible relativities and how such relativities can be incorporated in a GLM.

4. i. Where the variable in question relates to a policy option selected by the insured, having its factor reflect anything other than the excess losses due to higher limit is not a good idea. One can get counterintuitive results such as charging less for more coverage.
Even if the indicated result is not counterintuitive, to the extent that the factor differs from the pure effect on loss potential, it will affect the way insureds choose coverage options in the future. Thus, the selection dynamic will change and the past results would not be expected to replicate for new policies. For this reason it is recommended that factors for coverage options such as increased limit factors be estimated outside the GLM, using traditional actuarial techniques. (The resulting factors should then be included in the GLM as an offset.)
ii. I assume what is intended is that the number of coverage changes during the current policy period will be used to help rate the policy during its next policy period. (We are not given any information on whether the number of coverage changes in a policy period is related to the insurance costs the following period compared to otherwise similar insureds.)
The number of changes during a given policy period is not a good classification variable.
It is something that is likely to be zero for many policy periods, and vary somewhat randomly over time. If those with more coverage changes are changed more it is unlikely to be acceptable to insurance regulators and the public. If those with more coverage changes are changed more, then it will give insureds less incentive to make necessary coverage changes during a policy period; some of these coverage changes would have resulted in additional premiums for the insurer.
Alternately, the information will not be available for new business since we are building a GLM for the prospective period.
Alternately, the number of coverage changes is likely to change from what it is in the current policy period and thereafter year by year.
iii. Territories are not a good fit for the GLM framework. You may have thousands of zipcodes to consider and aggregating them to a manageable level will cause you to lose a great deal of important signal. If one does not aggregate the large number of zipcodes, then there are too many parameters which can lead to overfitting.
Using a spatial smoothing technique would be a more appropriate technique; one would then include the value determined for ZIP code as an offset term in the GLM.
(b) 1. One can get counterintuitive results such as charging more for less coverage.
2. Even if the indicated result is not counterintuitive, to the extent that the factor differs from the pure effect on loss potential, it will affect the way insureds choose coverage options in the future. Thus, the selection dynamic will change and the past results would not be expected to replicate for new policies.
3. Deductibles should lower frequency (small losses below deductible not reported) but usually increase severity (since claims that do get reported are higher average cost). This violates the assumption for the Tweedie Distribution, that a lower pure premium is due to both a lower frequency and a lower severity.

(c) One can calculate deductible relativities from loss elimination ratios.
Deductible Relativity = (1 - LER for chosen deductible) / (1 - LER for Base Deductible).
Loss elimination ratios can be estimated from size of loss data.
Loss elimination ratio = (Limited Expected Value at Deducible Amount) / Mean.
In the GLM, one would then include an offset of ln[deductible relativity].
Comment: While the average size of non-zero payment, equal to the mean residual life, usually increases as the size of deductible increases, this is not always the case.
Deductible factors may produce higher relativities at higher deductibles due to factors other than pure losses elimination:
1. Insureds at high loss potential and high premiums may be more likely to elect high deductibles in order to reduce their premium.
2. Underwriters may force high deductibles on riskier insureds.

5. (2.25 points) A GLM has been used to develop an insurance rating plan.
The results are given below:

Risk	Model Predicted Loss	Actual Loss
1	2,000	2,050
2	500	220
3	1,500	1,480
4	800	850
5	200	400

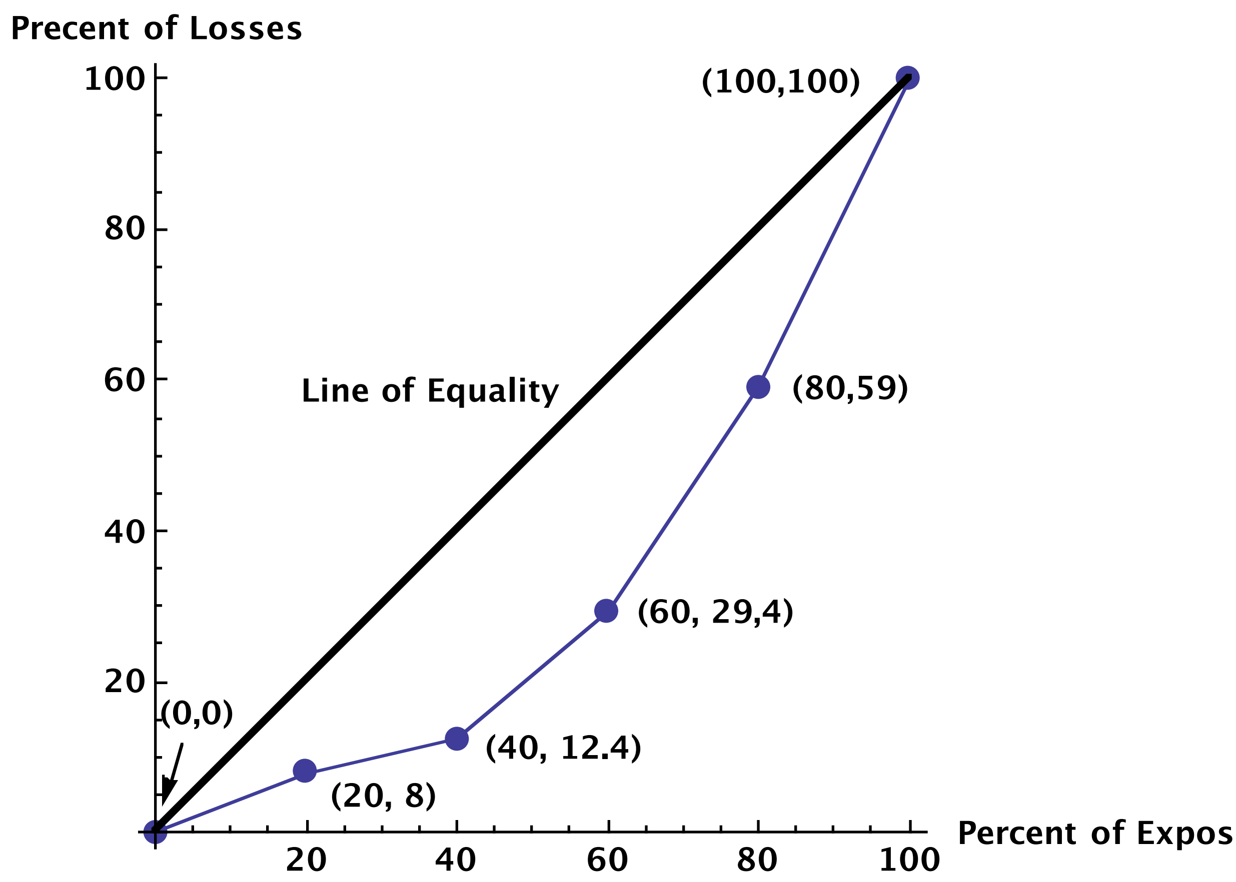
a. (1.75 points) Plot the Lorenz curve for this rating plan.
Label each axis and the coordinates of each point on the curve.

b. (0.5 point) Briefly describe how the Gini index is calculated and what the Gini index measures when applied to an insurance rating program. Do not calculate the Gini index.

5. a. Sort the risks from best to worst based on the model predicted loss.

Risk	Model Predicted Loss	Actual Loss	Cumulative Losses	% of Losses
5	200	400	400	8.0%
2	500	220	620	12.4%
4	800	850	1470	29.4%
3	1,500	1,480	2950	59.0%
1	2,000	2,050	5000	100.0%
Total		5000		

On the x-axis, plot the cumulative percentage of exposures.
I will assume that each risk has the same number of exposures.
On the y-axis, plot the cumulative percentage of actual losses.



b. The Gini index is twice the area between the Lorenz Curve and the line of equality.
The higher the Gini Index, the better the rating plan is at identifying risk differences, in other words the rating plan has more lift.
"The Gini index can also be used to measure the lift of an insurance rating plan by quantifying its ability to segment the population into the best and worst risks."
Comment: See Section 7.2.4 including Figure 21 in Goldburd, Khare, and Tevet.
The Gini index is between zero and one. Usually, one would be working with thousands of risks.

6. (2.5 points) An actuary has constructed a three-variable Tweedie GLM with a log-link function to estimate loss ratios for commercial property new business. The actuary wants to create a second model for renewal business that will include all of the variables from the new business model, plus a variable for the prior year claim count. The actuary requires that the coefficients of the variables: Average Building Age, log(Manual Premium), and Location Count, are consistent between the new and renewal models. The fitted new business model parameters are as follows:

Variable	Name	Estimate
	intercept	0.910
Average Building Age (Years)	age	0.013
log(Manual Premium)	logprem	-0.187
Location Count	loccnt	0.062

a. (0.75 point) Calculate the modeled loss ratio for a new business policy with a manual premium of $25,000, an average building age of four years, and having eight locations.

b. (0.75 point) Briefly describe how to produce the renewal business model, and specify the resulting equation for the renewal business modeled loss ratio.

c. (1 point) Identify and briefly describe two techniques that the actuary can use to assess the stability of the new variable in the renewal business model.

6. (a) exp[0.910 + (3)(0.013) + ln[25,000] (-0.187) + (8)(0.062)] = $e^{-0.4357}$ = **64.7%**.

(b) One could take the coefficients of the new business model as a given, other than β_0, which will be re-estimated.

Let the prior year claim count be x for renewal business.

Then the renewal business model is:

μ = exp[β_0 + 0.013 age + (-0.187) logprem + 0.62 locont + β_4 x].

We would fit the model via maximum likelihood to the data for renewal business, taking into account the form of density for the Tweedie Distribution.

Alternately, one can fit a single model to the data for new and renewal business.

Let the prior year claim count be x for renewal business

Let D = 0 if new business and 1 if renewal business.

Then the combined model is: μ = exp[β_0 + β_1 age + β_2 logprem + β_3 locont + D β_4 + D β_5 x].

We would fit the model via maximum likelihood to the combined data, taking into account the form of density for the Tweedie Distribution.

(c) 1. Time-consistency. One can fit the model to the data for separate years and compare the coefficients. If the fitted coefficients are similar, that indicates stability over time.

Alternately, one could introduces dummy variables into the model for the various years of data.

For example, if we have data from 2012, 2013 and 2104,

then we could take 2012 = base year, x_5 = 1 if 2013, x_6 =1 if 2014.

Then test whether the coefficients of these variables are significantly different from zero. If one or more of the fitted coefficients are significantly different than zero, that indicates instability over time.

2. Bootstrapping. Create multiple datasets from the initial dataset by sampling with replacement. Run the model on each sampled set. Assess stability of estimates of coefficients by comparing the results from each run.

3. Cross-Validation. Split the data into k parts and run the model on the (k-1) parts, then validate the result on the remaining part. Compare how similar the estimates are from the k iterations to assess variable stability.

4. Validation on Holdout Dataset. Split the data into two subsets, training and holdout. Determine the best model on the training set. Ideally, this model should fit well the holdout data.

5. Cook's Distance. Sort the observations based on their Cook's Distance value (higher distance = more influence on the model.) Remove one or more of the most influential observations and rerun the model on this new set of data to see the effect on estimated parameters.

7. (1.5 points) A company is considering modifying its rating plan to include factors by age group. Below are statistics for the base model and for the new model.

Statistic	Base Model	New Model
Loglikelihood	-750	-737.5
Deviance	500	475
Parameters	10	15
Data points	1,000,000	1,000,000

a. (1 point) Calculate the Akaike Information Criterion (AIC) and the Bayesian Information Criterion (BIC) for both models.
b. (0.25 point) Explain whether AIC or BIC is a more reliable test statistic as an indicator of whether to adopt the new model.
c. (0.25 point) Recommend and briefly justify whether to adopt the new model.

7. (a) For the base model:
AIC = (-2)(-750) + (2)(10) = **1520**.
BIC = (-2)(-750) + 10 ln[1,000,000] = **1638.2**.
For the new model:
AIC = (-2)(-737.5) + (2)(15) = **1505**.
BIC = (-2)(-737.5) + 15 ln[1,000,000] = **1682.2**.
(b) AIC is preferable. As here, most actuarial models involve a lot of data points. Therefore, the penalty for more parameters is very large for the BIC. Using BIC will tend to result in too simple models. In contrast, AIC does not depend on the number of data points.
(c) Based on part (b), I will rely on AIC.
Smaller AIC is better, so I will recommend the new model.
Comment: See Section 6.2.2 in Goldburd, Khare, and Tevet.
If one instead relied on BIC, the base model would be preferred.
Deviance = 2 (loglikelihood of saturated model - loglikelihood of model).
Thus equivalently to using AIC, one could compare models using: Deviance + 2p.
For the base model, Deviance + 2p = 500 + (2)(10) = 520.
For the new model, Deviance + 2p = 475 + (2)(15) = 505.
Since 505 < 530, we prefer the new model based on this criterion.
Equivalently to using BIC, one could compare models using: Deviance + p ln[N]..
For the base model, Deviance + 2p = 500 + 10 ln[1 million] = 638.16.
For the new model, Deviance + 2p = 475 + 15 ln[1 million] = 682.23.
Since 638.16 < 682.23, we prefer the base model based on this criterion.

8. (2.75 points) Given the following average severity, an actuary wants to validate if anti-selection impacts its Increased Limits Factors for a given insurance coverage:

Severity limited to:	Policy Limit = $25,000	Policy Limit = $50,000
$10,000	$4,000	$6,000
$25,000	$6,500	$8,000
$50,000	$9,000	$10,500

50% of the policies have a $25,000 policy limit and 50% of the policies have a $50,000 policy limit.
a. (1.25 points) Demonstrate if anti-selection impacts the ILFs.
b. (1.5 points) Identify and briefly describe two possible forms of anti-selection for ILFs and give one example for each.

8. (a) The ILFs based solely on the data for $25,000 policy limits:
25K limit: 6500/4000 = 1.625. 50K limit: 9000/4000 = 2.25.
The ILFs based solely on the data for $50,000 policy limits:
25K limit: 8000/6000 = 1.333. 50K limit: 10,500/6000 = 1.75.
Since the two sets of increased limits factors are significantly different, this indicates that there is anti-selection.
(b) Adverse selection, occurs when purchasing higher limits is associated with worse loss experience. The mirror image of adverse selection is favorable selection, in which the insureds with the highest policy limits show the best loss experience.
Adverse selection, can occur for two different reasons:
1. Insureds who can expect higher loss potential could be more inclined to purchase higher limits.
2. Liability law suits or settlements may be influenced by the policy limit. Thus the same accident may result in higher losses for the insured that purchased higher policy limits.
Favorable selection, can occur for two different reasons:
1. Financially secure insureds may be better risks. Yet since they have more assets to protect, they will be inclined to purchase higher limits.
2. Insurance companies, knowing that these are the better risks, would be more willing to insure them at higher limits.
<u>Comment</u>: I wonder how we got the information on $E[X \wedge 50,000]$ for the 25,000 policy limits.
In part (b), I assume they meant “give one reason of each”.

9. (1.5 points)
An actuary and an underwriter are discussing a Worker's Compensation book of business.

a. (0.5 point) The underwriter is considering not renewing a risk in the book because the experience modifier of the account has increased.
The underwriter notes that the expected loss ratio (ELR) for the predominant class code on the account has decreased over the last three years.
Discuss whether the underwriter should non-renew the risk given the change in ELR.

b. (0.5 point) A different account had large losses last year and now has a debit modification.
The underwriter wants to renew this account since the debit mod will help recoup losses.
Evaluate the underwriter's reasoning to renew the account.

c. (0.5 point) The underwriter points out that the book of business has an overall experience modifier below 1.0 and believes this is due to superior risk selection.
Provide two reasons why this may not be the case.

9. (a) I assume they meant to say "expected loss rate."
(Expected loss ratios are not determined separately by class code. Of course there are observed loss ratios by class; one needs to carefully distinguish between loss ratios to manual premium and loss ratios to standard premium.)
A lower expected loss rate will produce lower expected losses to which to compare the insured's actual losses. Thus all else being equal, this will result in a higher experience mod. There is no reason to believe that the current higher modification is not a good prediction of this insured's future experience compared to the average for similar risks. Thus there is no reason to non-renew the risk, based solely on the information given.
(The expected loss rate probably went down due to a decrease in the manual rate. Thus this insureds manual premium is probably lower than it would have been, while its experience modification is higher.)
(b) The experience modification is intended to predict the insured's future experience compared to average. It is not intended to recoup past losses. The underwriter's reasoning is not valid.
If the experience rating plan is well designed, underwriters should be equally willing to write credit and debit risks.
(c) 1. The average mod tends to be a slight credit. (This is because larger insureds have better experience on average than smaller insureds, and larger insureds have larger experience rating credibilities.)
2. When manual rates are excessive, the average experience modification is a credit.
Comment: The CAS Examiner's Report seemed incorrectly to believe that expected loss ratios are determined separately by class.
Also the CAS Examiner's Report for part (a) "Candidates were expected to identify the potential for poor class fit and explain that the experience mod is intended to correct for this." This does not follow from the given information, has nothing to do with either decreasing expected loss rates or ratios for the predominant class code, nor does it explain why the experience modification increased. A risk that is a poor fit for its class may therefore have an average modification over many years that is consistently either high or low; however, this does not explain an increase in its modification from one year to the next. "Graders also accepted what that the increasing experience mod could be due to random large losses, if the candidate explained their response." While this is better, this still ignores the information on the either decreasing expected loss rates or ratios for the predominant class code. (On several questions on this exam, the examiners seemed to want generic answers that ignored much of the information they provided in a question. Students with a better understanding of the material were at a disadvantage.) Also the CAS Examiner's Report for part (a) showed ignorance of the relationship of expected loss rates, experience mods, and rate adequacy. The expected loss ratio is only used in the calculation of the expected loss rates to determine the loss portion of the manual rates. if the expected loss ratio that produced the manual rates is used in the calculation of the expected loss rates, then the size of the expected loss ratio does not affect the calculated expected loss rates. Thus unless used incorrectly, the expected loss ratio has no affect on the calculated expected loss rates or other experience rating plan parameters, and thus has no affect on experience mods.

10. (2.5 points) An insured is subject to experience rating under the National Council on Compensation Insurance (NCCI)'s Experience Rating Plan Manual for Workers Compensation and Employers Liability Insurance. The following information about the insured is given:

Payroll (Experience Period)	$5,000,000
State	Alabama
Class	7705

The following claims apply to the experience period.
Each claim involves only one person, and none are disease claims:

Claim Number	Type	Loss
1	Indemnity	29,000
2	Medical	30,500
3	Indemnity	90,000
4	Indemnity	1,500
5	Medical	45,000

Calculate the experience modification for this insured.

10. Looking in manual, for 2011 for Alabama for Class 7705, the Expected Loss Rate = 2.02 and D-Ratio = 0.17. Thus Expected Losses = (2.02) ($5,000,000 / 100) = $101,000.
The expected primary losses = (0.17)(101,000) = $17,170.
The expected excess losses = $101,000 - $17,170 = $83,830.
E = $101,000. ⇒ W = 0.14, and B = 28,000.
The state per accident limit is 175,500, which does not affect any of the observed claims.

Type	Loss	Primary	Excess
Indemnity	29,000	5000	29,000 - 5000 = 24,000
Medical	30,500	(0.3)(5000) = 1500	(0.3)(30,5000 - 5000) = 7650
Indemnity	90,000	5000	85,000
Indemnity	1,500	1500	0
Medical	45,000.	(0.3)(5000) = 1500	(0.3)(40,000) = 12,000
Total		14,500	128,650.

$$M = \frac{A_p + W A_e + (1 - W) E_e + B}{E + B} =$$

$$\frac{14{,}500 + (0.14)(128{,}650) + (1 - 0.14)(83{,}830) + 28{,}000}{101{,}000 + 28{,}000} = \mathbf{1.03}.$$

Alternately, $Z_p = E/(E + B) = 101{,}000 / (101{,}000 + 28{,}000) = 0.7829$.

$Z_e = w\ Zp = (0.14)(0.7829) = 0.1100$.

$M = 1 + Z_p\ (A_p + E_p)/E + Z_e\ (A_e - E_e)/E =$

1 + (0.7829) (14,500 - 17,170) / 101,000 + (0.1100) (128,650 - 83,830) / 101,000 = **1.03**.

<u>Comment</u>: The result depends on which vintage of the rating plan manual one uses.

11. (3.75 points) Given the following information about 10 risks for a manufacturing class:

Risk	Type of Manufacturing	Manual Premium	Losses	Current Modification	Proposed Modification
1	Uses Robots	1,000	500	0.65	0.50
2	Uses Robots	1,000	600	0.75	0.55
3	Made by hand	1,000	700	0.70	0.70
4	Uses Robots	1,000	800	1.00	0.75
5	Uses Robots	1,000	900	0.90	0.95
6	Made by hand	1,000	1,100	1.15	1.05
7	Uses Robots	1,000	1,200	1.10	1.25
8	Made by hand	1,000	1,500	1.25	1.50
9	Made by hand	1,000	1,600	1.20	1.75
10	Made by hand	1,000	1,800	1.30	2.00

a. (3 points) Evaluate the proposed experience rating plan compared to the current plan.
b. (0.75 points) Evaluate the classification plan for this class.

11. (a) I will use the quintiles test.
For the current plan, group based on the current modifications to get 5 quintiles:

Risk	Manual Premium	Losses	Current Mod.	Manual Loss Ratio	Standard Loss Ratio
1	1,000	500	0.65	1200/2000 = 60%	1200/1350 = 88.9%
3	1,000	700	0.70		
2	1,000	600	0.75	1500/2000 = 75%	1500/1650 = 90.9%
5	1,000	900	0.90		
4	1,000	800	1.00	2000/2000 = 100%	2000/2100 = 95.2%
7	1,000	1,200	1.10		
6	1,000	1,100	1.15	2700/2000 = 135%	2700/2350 = 114.9%
9	1,000	1,600	1.20		
8	1,000	1,500	1.25	3300/2000 = 165%	3300/2550 = 129.4%
10	1,000	1,800	1.30		

Sample variance of manual loss ratios: 0.18575.
Sample variance of standard loss ratios: 0.03102.
Quintiles statistic for the current plan is: 0.03102 / 0.18575 = 0.1670.

For the proposed plan, group based on the proposed modifications to get 5 quintiles:

Risk	Manual Premium	Losses	Proposed Mod.	Manual Loss Ratio	Standard Loss Ratio
1	1,000	500	0.50	1100/2000 = 55%	1100/1050 = 104.8%
2	1,000	600	0.55		
3	1,000	700	0.70	1500/2000 = 75%	1500/1450 = 103.4%
4	1,000	800	0.75		
5	1,000	900	0.95	2000/2000 = 100%	2000/2000 = 100%
6	1,000	1,100	1.05		
7	1,000	1,200	1.25	2700/2000 = 135%	2700/2750 = 98.2%
8	1,000	1,500	1.50		
9	1,000	1,600	1.75	3400/2000 = 170%	3400/3750 = 90.7%
10	1,000	1,800	2.00		

Sample variance of manual loss ratios: 0.21325.
Sample variance of standard loss ratios: 0.003066.
Quintiles statistic for the current plan is: 0.003066 / 0.21325 = 0.0144.

The smaller the quintiles statistic, the better the experience rating plan.
Thus the proposed plan is better.

(b) The risks in this class that use robots tend to have lower loss ratios to manual premium than those that do not.
The total (manual) loss ratio for using robots is: 4300/5000 = 86%.
The total (manual) loss ratio for those not using robots is: 6700/5000 = 134%.
So based on the limited information provided, this class is heterogeneous.
One could make the class more homogeneous by splitting onto two: robots vs. not robots.
Comment: In part (a), one could instead use the Meyers efficiency test which is similar but does not group the risks into quintiles.
For the proposed plan, the standard loss ratios are decreasing. $\Rightarrow$ The credibility is too large.
If this is a total of $10,000 in premium, one would need more data in order to make a solid conclusion! Perhaps this is a total of $10 million in premium.

12. (2.5 points) An actuary is given the following sample of experience from a grouping of five similarly-sized risks:

Risk	Actual Loss
1	$85,000
2	$127,500
3	$42,500
4	$63,750
5	$106,250

a. (1.5 points) Construct a Table M of insurance charges and savings at entry ratios of 0 to 1.50 in multiples of 0.25.
b. (0.25 points) Briefly describe what the insurance charge at an entry ratio of 1.25 reflects.
c. (0.75 points) Suppose the Table M constructed above is used to price a book of Worker's Compensation retrospectively rated business. The following table of actual losses reflects the experience of this book:

Risk	Actual Loss
1	$12,000
2	$42,500
3	$63,750
4	$106,250
5	$275,000

Evaluate the appropriateness of using the Table M constructed in part a. above.
Provide two reasons in support of the conclusion.

12. (a) The average loss is: 425.000/5 = 85,000.

Risk	Actual Loss	Entry Ratio
3	\$42,500	0.50 = 42,500/85,000
4	\$63,750	0.75
1	\$85,000	1.00
5	\$106,250	1.25
2	\$127,500	1.50

Entry Ratio	# of Risks	Sum Up	Double Sum up	Charge	Savings
0.00	0	5	20	**1.0000**	**0.0000**
0.25	0	5	15	**0.7500**	**0.0000**
0.50	1	4	10	**0.5000**	**0.0000**
0.75	1	3	6	**0.3000**	**0.0500**
1.00	1	2	3	**0.1500**	**0.1500**
1.25	1	1	1	**0.0500**	**0.3000**
1.50	1	0	0	**0.0000**	**0.5000**

$\psi(r) = \phi(r) + r - 1$.

Alternately, one can just compute each charge separately.

For example, $\phi(0.75) = (0 + 0 + 0.25 + 0.50 + 0.75) / 5 = 0.3$.

(b) The charge at 1.25 represents the expected amount by which the entry ratios exceed 1.25.

$$\phi(1.25) = \int_{1.25}^{\infty} (r - 1.25)\, dF(r).$$

In this case, r = 1.25 corresponds to aggregate losses of 106,250. $\phi(1.25)$ represents the average amount by which the aggregate losses exceed 106,250 as a percent of expected total losses.

In a Lee Diagram with entry ratios on the vertical axis, $\phi(1.25)$ is the area above the horizontal line at 1.25 and below the curve.

(c) 1. The average loss is: 499.5/5 = 99,900.

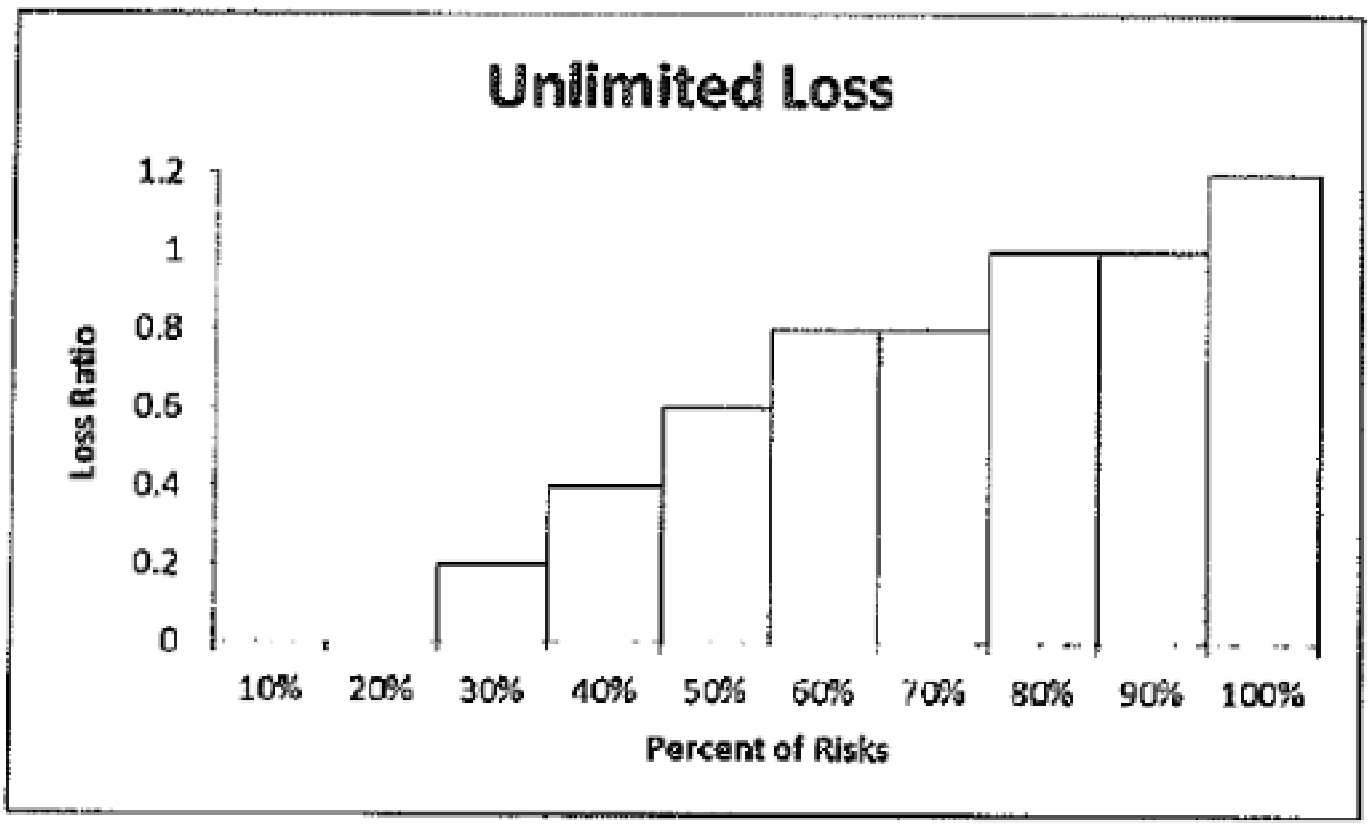
Risk	Actual Loss	Entry Ratio
1	$12,000	0.12
2	$42,500	0.43
3	$63,750	0.64
4	$106,250	1.06
5	$275,000	2.75

The variance of the entry ratios (which since the entry ratio average to one is the same as the coefficient of variation) is much larger than that underlying the Table M.
Therefore, the computed charges are incorrect. (The charges depend on the shape of the distribution of entry ratios, which can be described via the coefficient of variation, skewness, kurtosis, etc.) Thus, if one used the Table M from part (a), for this set of risks, a retro plan would not balance to a guaranteed cost policy.
Thus the Table M from part (a) is not appropriate.
2. The expected losses are different in part (c) than in the original data. This could imply a different shape of the distribution of aggregate losses. (In the NCCI retro manual, the expected losses are used to get an expected loss group; as expected losses increase, we would expect smaller variance of the distribution of entry ratios.)
Comment: In part (c), I had no idea what they meant by two reasons. The second reason I gave in part (c) is based on the CAS Examiner's Report. I would not have concluded based on two sets of five observed aggregate losses that the expected aggregate losses are different. In any case, the second set has a larger coefficient of variation than does the first set, which is the reverse of what we would expect based on the second set having a larger mean loss.
The experience of five risks for one year is way too few from which to construct a reliable Table M. It is also way too little data to use to test the appropriateness of a Table M.
In part (c), according to the CAS Examiner's Report "Common mistakes include: Not stating in some way that the retro plan would be out of balance with an identical guaranteed cost book. Stating that Table M, from Part a, lacked credibility because it was only based on 5 data points. This was not an accepted justification in this case." (However, I believe on appeal candidates were given credit for the lack of credibility because it was only based on 5 data points.)

13. (2.5 points) A risk is written using a retrospective rating plan with the following characteristics:

Standard Premium	$10,000,000
Expected Loss Ratio	60%
Loss Ratio at Maximum Premium	80%
Loss Ratio at Minimum Premium	20%
Loss Conversion Factor	1.085
Provision for Losses and Total Expenses Exclusive of Taxes	0.97

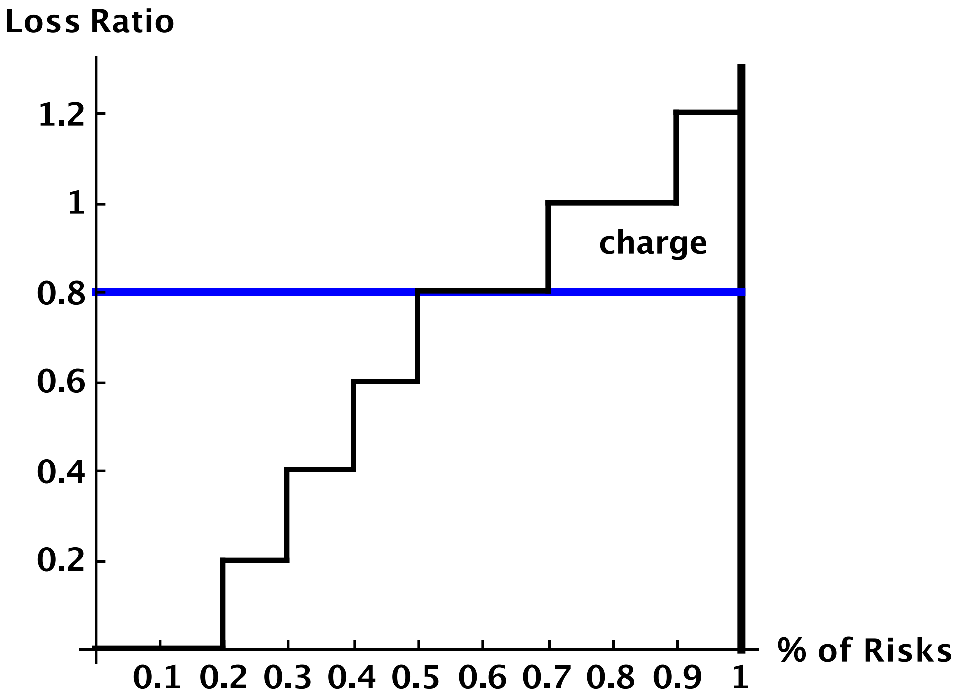
The following Lee Diagram depicts actual experience from a sample of similarly-sized risks and similar to the risk in question:



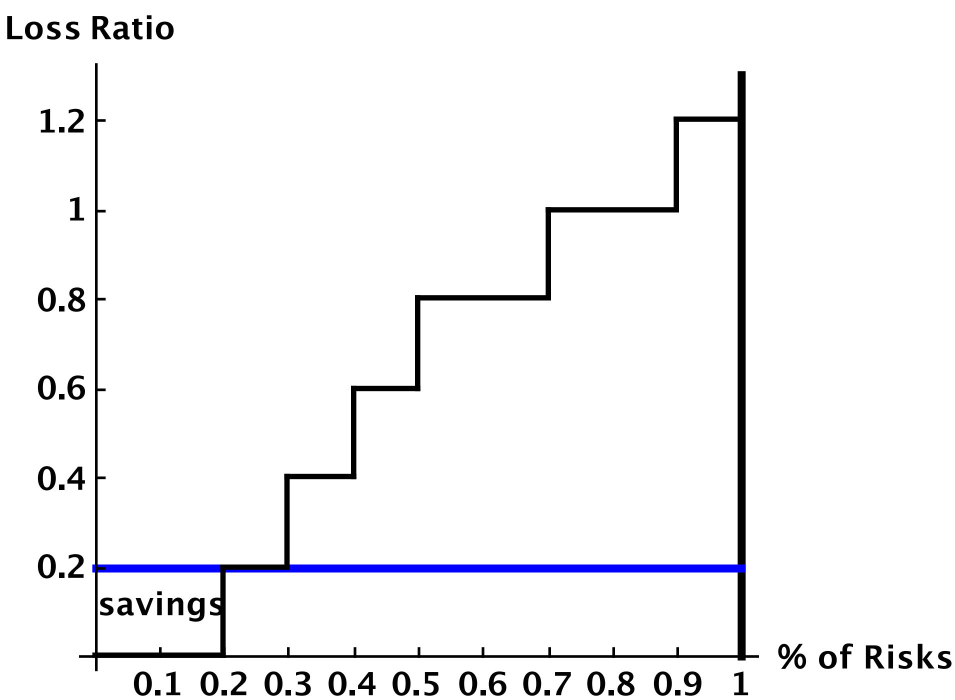
a. (1.25 points) Determine the converted insurance charge for this plan.

b. (1.25 points) The insured's actual ultimate losses are $8,700,000 and the final retrospective premium is $12,500,000. Determine the tax multiplier that was used in the rating of this plan.

13. (a) The average loss ratio in the Lee Diagram is:
(20% + 40% + 60% + 80% + 80% + 100% + 100% + 120%) / 10 = 60%.



The charge at the maximum loss ratio of 80% is the portion of the total area above the horizontal line at 80%: {(0.2)(20%) + (0.1)(40%)} / 60% = 8%/60%.



The savings at the minimum loss ratio of 20% is the portion of the total area between the distribution and the horizontal line at 20%: (0.2)(20%) / 60% = 4%/60%.
The converted net insurance charge is: (1.085) (60%) (8%/60% - 4%/60%) = 4.34%.
As a dollar amount the converted net insurance charge is: (4.34%)(10 million) = $434,000.

(b) Provision for Losses and Total Expenses Exclusive of Taxes: 0.97.
Expected Loss and LAE Ratio: (1.085)(60%) = 0.651.
Provision for Expenses exclusive of taxes and LAE: 0.97 - 0.651 = 0.319.
(This is very high for a risk of this size!)
Thus the basic premium is: (0.319 + 4.34%) (10 million) = $3,624,000.
The ultimate losses are greater than the 80% maximum loss ratio, thus the retro premium is:
T {(1.085) (8 M) + 3.624M} = T (12.304M).
Setting 12.5M = T (12.304M). ⇒ T = **1.016**.

14. (1.75 points) A retrospective rated policy has both a loss limitation and a maximum premium.

a. (0.5 point)

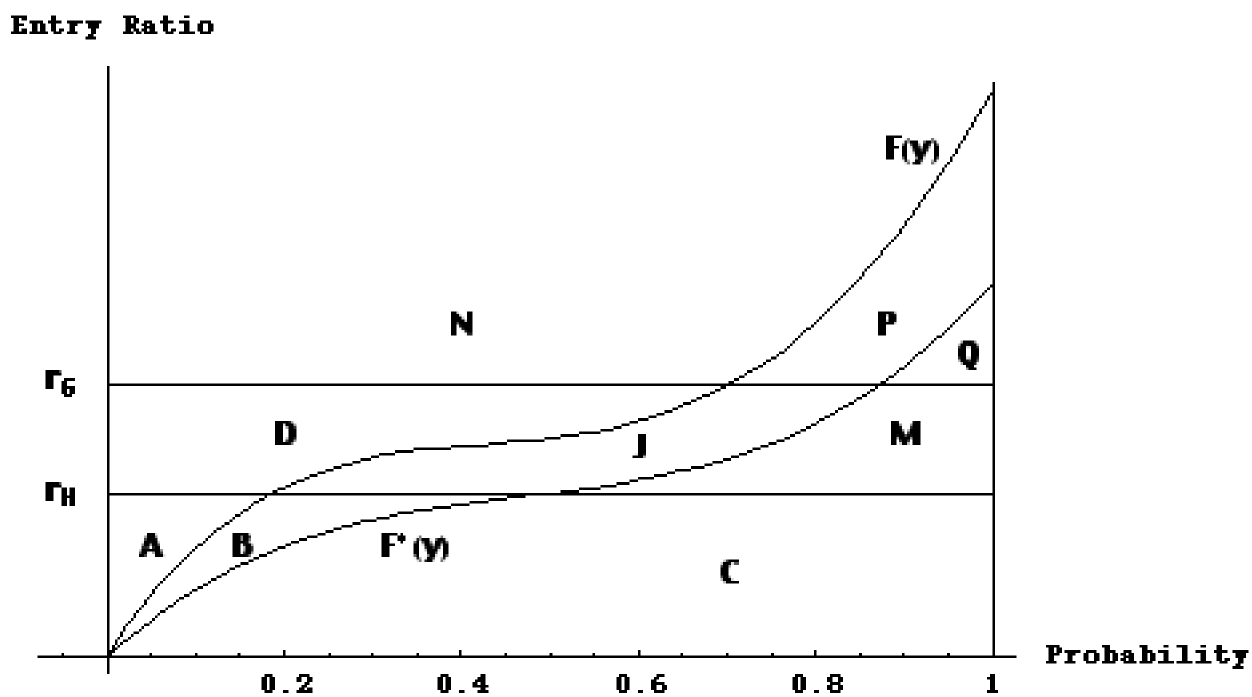
Demonstrate how the charges for the loss limitation and the maximum premium overlap.

b. (0.5 point) Explain how the overlap is handled differently when using Table M versus Table L.

c. (0.75 point)

An actuary is pricing a large worker's compensation policy with a fixed loss limit of $100,000. Recommend a table of insurance charges, Table M or Table L, that the actuary should use and provide two reasons supporting the recommendation.

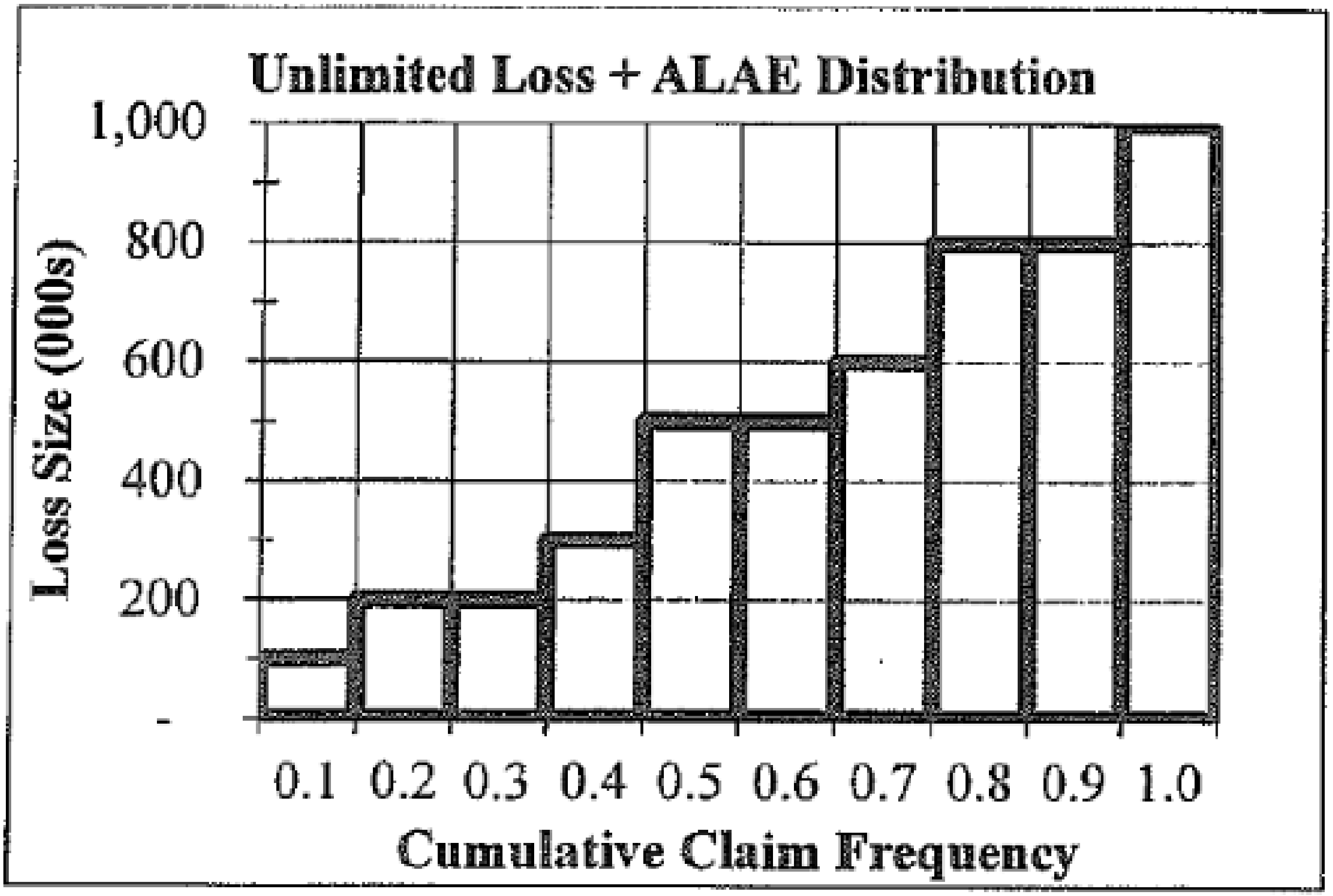
14. a) There are cases, where the loss limitation and maximum premium are in essence removing the same dollars of loss from the retro calculation. For example, the preliminary retro premium may exceed the maximum premium even after the application of the loss limitation to one or more large losses. In that case, the loss limitation provided the insured no benefit beyond that provided by the maximum premium. Thus the charges for the loss limitation and the maximum premium overlap.
In the following Lee Diagram, the Table M charge at the maximum is: P + Q.



The charge for the loss limitation is: P + J + B. Area P represents the overlap between the separate charges for the loss limitation and the maximum premium
Alternately, when there is an occurrence limit as well as an aggregate limit, the occurrence limit makes it less likely that the aggregate limit will be hit. Thus, the charge for the aggregate limit should be reduced, otherwise there will be overlap.
b) The Table L charge includes the affect of the maximum on the <u>limited</u> aggregate loss distribution plus the loss elimination ratio for the loss limitation. In the above Lee Diagram, the Table L charge is calculated in two pieces: Q + (P + J + B).
One can instead use Table M with the ICRLL procedure. In entering Table M, the column is determined by the size of insured. However when using ICRLL, the expected losses for the insured are first multiplied by a factor of: (1 + 0.8 LER) / (1 - LER). This column shift is an approximation to the effect of the loss limitation on the distribution of aggregate losses.
Then the Table M charge from the ICRLL procedure is added to the charge for the loss limitation.

c) I would use Table L, since it is not based on an hoc approximation such as the ICRLL procedure.
Since there is a fixed loss limit there is not a need for a large number of tables to accommodate changing limits.
Also, there is a Table L published for a $100,000 loss limit, avoiding having to interpolate between two Tables L.
Alternately, I would use Table M, since it is based on countrywide data and thus is more credible than Table L which is based on California data.
Table L has California taxes built in, so it is not appropriate for use in other states.
Also, Table M is more flexible for changing loss limits from year to year as you do not need a separate table for each limit as in Table L.
Comment: In part (c), I had trouble coming up with two reasons.

15. (1.75 points) Given the following information about a worker's compensation book of business:



Standard premium	$840,000
Loss based assessment factor	2.0%
Ratio of ULAE to Loss	5.0%
General overhead expense	2.0%
Risk load for credit risk	4.0%
Acquisition expense	5.0%
Taxes	3.0%
Profit	-8.8%

a. (1.25 points) Calculate the deductible for an excess workers compensation policy that minimizes the insured's loss retention with a maximum premium of $100,000.

b. (0.5 point) Contrast the profit load in a large dollar deductible (LDD) policy to the profit load in an excess workers compensation policy.

15. (a) Assume we are talking about an excess policy with an aggregate maximum and infinite per loss deductible, in other words, we have aggregate excess reinsurance or stop loss insurance.
Assume that the given Lee Diagram is for annual aggregate losses.
I assume that ULAE is 5% of excess loss and ALAE.
I assume that the loss based assessments are paid by the self-insured and not by the writer of the excess policy. Thus I will ignore the given loss based assessment factor.
Assume that the general expense loading is 2% of standard premium or \$16,800.
For an excess policy there is no credit risk, thus I will ignore the given credit risk load.
I assume acquisition expense, taxes, and profit are all loaded as a percent of net premiums.
Let us assume that the aggregate limit produces expected excess loss & ALAE is x.
Then we want:
\$100,000 = {\$16,800 + 1.05 x } / (1 - 5% - 3% + 8.8%). ⇒ x = \$80,000.
For an aggregate maximum of 600,000, the expected excess loss & ALAE is:
(10%)(400,000) + (20%)(200,000) = \$80,000.
Thus we want an aggregate maximum of \$600,000.
(A lower aggregate maximum would result in a larger premium than the desired \$100,000.)
(b) "In Excess WC since little service is involved the competition is almost exclusively in price. This tends to drive down the profit margin. The LDD market is usually not as price-competitive as the Excess WC market, because much of what the insurer sells in an LDD plan is the quality of the service."
Also, "The average loss and expense payout period for Excess WC is considerably later than for LDD. This is because most of the Excess WC premium covers the excess loss which has a long average payout period (average to over 10 years). where as LDD premium is split roughly half in expense. which is paid out quickly, and half in excess loss." Therefore, the Excess WC has relatively more opportunity to earn investment income, which would result in a lower (more negative) profit loading than LDD.
Comment: Part (a) is a very poorly constructed question, with lots of missing information.
You are required to figure out what the examiner intended to say.
Should have said: "An Excess Policy with a self-insured retention on an aggregate basis."
In part (a), one could instead solve for the desired excess ratio, which is 16%, which would result in the same aggregate maximum of \$600,000.
If one instead assumed that we want to determine a size of the deductible, with no aggregate limit, that results in a premium of \$100,000, then one has to assume that the Lee Diagram is for severity. However, then there is not enough information provided to determine the total expected losses and alae. One could just make up a number such as \$550,000 for the total expected loss and ALAE, and proceed to answer the question.

16. (3.75 points) An actuary is given the following expiring policy information for a Workers' Compensation Large Dollar Deductible policy:

Expected Total Loss & ALAE	$500,000
Deductible	$100,000
Percentage of Loss & ALAE Excess of $100,000	40%
Percentage of Loss & ALAE Excess of $200,000	20%
ULAE	5%
Loss Based Assessment Factor	3%
Profit and Variable Expenses	17%
Fixed Expense	$15,000
Aggregate Deductible	$300,000

There are no changes to these expenses and profit provision.
The table of Insurance Charges is displayed below:

Modified Table M for Similarly Sized Policies

	Deductible			
Entry Ratio	$100,000	$200.000	$300.000	$400,000
0.5	0.450	0.480	0.495	0.505
1.0	0.330	0.350	0.365	0.370
1.5	0.270	0.303	0.325	0.350
2.0	0.185	0.240	0.260	0.286

The insured would like to retain more risk and requests the price of the following options:
Option 1: Deductible of $200,000 with an aggregate deductible of $400,000.
Option 2: Excess Policy with self-insured retention of $200,000 on a per occurrence basis.

a. (2.75 points) Calculate the difference in price between Option 1 and the expiring structure.
Use linear interpolation as needed.

b. (1 point) The insurer observed an upward trend of ground up losses in the insured's industry for the most recent year, but is unsure if this trend will continue in the future.
Based on this observation, recommend one of the two options for the insurer and fully support the recommendation.

16. (a) For the current option, the expected primary loss and ALAE is:
(1 - 40%)($500,000) = $300,000.
⇒ Expected excess losses = $200,000.
For an aggregate deductible of $300,000, the entry ratio is 300K / 300K = 1.
Thus the charge from the Modified Table M is 0.330.
Thus the charge for the aggregate deductible is: (0.330)($300,000) = $99,000.
I assume, that the ULAE is as a ratio to total loss and ALAE.
I assume as per Teng, that the loss based assessments have to be paid on total losses;
I assume that the given 3% is as a ratio to loss and ALAE.

LDD premium is: $\frac{\$200{,}000 + \$99{,}000 + (\$500{,}000)(5\% + 3\%) + \$15{,}000}{1 - 17\%} = \$426{,}506.$

For Option 1, the expected primary loss and ALAE is: (1 - 20%)($500,000) = $400,000.
⇒ Expected excess losses = $100,000.
For an aggregate deductible of $400,000, the entry ratio is 400K / 400K = 1.
Thus the charge from the Modified Table M is 0.350.
Thus the charge for the aggregate deductible is: (0.350)($400,000) = $140,000.

LDD premium is: $\frac{\$100{,}000 + \$140{,}000 + (\$500{,}000)(5\% + 3\%) + \$15{,}000}{1 - 17\%} = \$355{,}422.$

Difference in premium between Option 1 and current is: $355,422 - $426,506 = **-$71,084**.
Alternately, the difference in premiums is: {(100 + 140) - (200 + 99)} / (1 - 0.17) = **-$71,084**.
(b) If the insurer is concerned about increasing losses, then we would prefer that the insured to do more cost bearing. The first option has the same per occurrence retention as the second option, but limits the insured's annual retention. Thus, under the second option, the insured bears more of the cost on average.
Also, in the second option the insurer will not have to adjust losses below the limit. However, in the first option the insurer will have to adjust all losses. If the expected number of losses is in fact increasing, this would result in more unexpected extra loss adjustment expense in option one than in option two.
Also in option one but not option two, the insurer faces credit risk. If the losses are unexpectedly high, then it is more likely that the insured will not be able to pay its owed reimbursements, which is only a potential problem in option one.
Thus the insurer should choose the second option rather than the first option.
Comment: In part (b), either option could be chosen as long as it was justified.
For part (b), from the CAS Examiner's Report: "Candidates were expected to provide 2 advantages of the option which they selected (either Option 1 or Option 2) and provide full reasoning for each advantage. Alternatively, candidates could have provided an advantage of the option which they selected and a disadvantage of the option which they didn't select."

17. (2.5 points)
The following table contains historical adjusted Worker's Compensation loss experience:

Loss Size	Number of Claims
<$100,000	62
$100,000	23
$200,000	7
$300,000	5
$500,000	2
$1,000,000	1

Mean claim size for losses less than $100,000: $38,000.
Excess ratios were calculated directly from data for limits of $100,000 or less.
For higher limits a mixed Pareto-Exponential curve was fit to losses truncated and shifted at $100,000, then normalized to mean unity.
Selected values of the fitted curve of these excess ratios are shown in the table below.

Entry Ratio	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2
Excess Ratio	0.92	0.83	0.76	0.69	0.63	0.58	0.53	0.49	0.45	0.41	0.38	0.35

a. (1.5 points) Calculate an excess ratio for a limit of $250,000.
b. (1 point) An actuary is pricing for Excess Worker's Compensation Coverage.
Evaluate the impact to the excess premium under the following alternative methodologies:
- Changing the selected truncation point from $100,000 to $300,000.
- Fitting losses using a Pareto Curve.

17. a. First calculate the empirical excess ratio at the 100,000 truncation point.

$$E[X] = \frac{1000\{(62)(38) + (23)(100) + (7)(200) + (5)(300) + (2)(500) + (1)(1000)\}}{100} = 95{,}560.$$

$$E[X \wedge 100{,}000] = \frac{1000\{(62)(38) + (38)(100)\}}{100} = 61{,}560.$$

R(100,000) = 1 - E[X ∧ 100,000]/E[X] = 1 - 61,560/95,560 = 35.6%.

The average value of the data truncated and shifted at 100,000 is:

$$\frac{1000\{(7)(200 - 100) + (5)(300 - 100) + (2)(500 - 100) + (1)(1000 - 100)\}}{15} = 226{,}667.$$

Thus the entry ratio corresponding to the \$250,000 limit is: $\frac{250{,}000 - 100{,}000}{226{,}667} = 0.66.$

Interpolating in the given table: $\hat{R}(0.66) = 0.55$.

Thus the estimated excess ratio at \$250,000 is: (35.6%)(0.55) = **19.6%**.

b. (i) Since the new truncation point of \$300,000 is greater than \$250,000, we would now rely on the empirical excess ratio at \$250,000.

$$E[X \wedge 250{,}000] = \frac{1000\{(62)(38) + (23)(100) + (7)(200) + (8)(250)\}}{100} = 80{,}560.$$

R(250,000) = 1 - E[X ∧ 250,000]/E[X] = 1 - 80,560/95,560 = **15.7%**.

This is lower than the previously calculated 19.6%, resulting in a **lower** excess premium.

(ii) I assume they meant instead fitting a Pareto to the data truncated and shifted at 100,000.

The Pareto has a heavier righthand tail than the Exponential.

Thus I would expect that the fitted Pareto would have a higher probability of large values than the fitted mixed Pareto-Exponential. Thus the excess ratios for the fitted Pareto would be larger.

Thus I would expect that the resulting estimate of the excess ratio at 250,000 would be higher, resulting in a **higher** excess premium.

Comment: There are only 15 claims of size greater than \$100,000. Thus there are only 15 values truncated and shifted at \$100,000 to which to fit a mixed Pareto-Exponential curve. This is far too few values to get reliable results when fitting a heavy-tailed curve with 4 parameters. Thus the exam question would have been much better if all of the number of claims had been multiplied by for example 100.

In general, if one changes the truncation point, then one has a different set of data to which to fit a mixed Pareto-Exponential curve; you can not do the fitting without a computer. (In this case, since we only have 3 claims of size greater than \$300,000, it would be impossible to fit such a curve.). This new curve combined with the empirical excess ratio at the new truncation point, would produce somewhat different estimates of excess ratios at very high limits than would be obtained using the original truncation point.

18. (2 points) Catastrophe models were built to assist the insurance industry in quantifying the risk of natural disasters.

a. (1 point) For any two of the four basic modules of a catastrophe model, provide an example of epistemic risk.

b. (1 point) For any two of the four basic modules of a catastrophe model, provide an example of aleatory risk.

18. (a) Epistemic Uncertainty ⇔ Parameter Risk and Modeling Risk.

1. In hurricane models, the hazard module includes a large number of possible hurricanes with different paths and different categories (magnitudes). We are uncertain of the actual future probabilities of hurricanes with certain paths and/or magnitudes.
In earthquake models, the hazard module includes fault lines. An example of epistemic uncertainty is whether all of the relevant fault lines have been included; there can be unknown fault lines.
In earthquake models, the hazard module uses recurrence models of earthquakes along faults; we are uncertain of what are the correct models and parameters.
2. The vulnerability module contains for each type of building a probability distribution of the building damage that would result from a given external force. We are uncertain as to what exactly is the appropriate probability distribution.
3. In the inventory module, examples of epistemic uncertainty would be incomplete information regarding construction quality, and replacement costs of buildings.
Another source of epistemic uncertainty is less than perfect information on the exact soil type where the structure is located, which is important for earthquake models.
Another source of epistemic uncertainty would be outdated information on the value of contents.
Another source of epistemic uncertainty would be lack of information on building codes.
4. The deficiency of information about repair costs and business interruption costs adds epistemic uncertainty to the loss module.

(b) Aleatory Uncertainty ⇔ Process Risk.

1. In hurricane models, the hazard module includes a large number of possible hurricanes with different paths and different categories (magnitudes). There is an inherent randomness as to how many and what hurricanes there will be in a given year. This produces aleatory uncertainty due to the inherent randomness of the process, similar to the roll of a die.
2. The vulnerability module contains for each type of building a probability distribution of the building damage that would result from a given external force. Thus there is aleatory uncertainty (randomness) in the amount of damage for a given external force.
Comment: There are many possible full credit answers.
The four modules of a catastrophe model are: hazard, inventory, vulnerability, and loss.

19. (3 points) In order for an insurance company to increase its return on capital, two reinsurance options are being considered:

I. $5 million excess of $5 million per risk:
- ALAE pro-rata
- Rate = 18% of premium

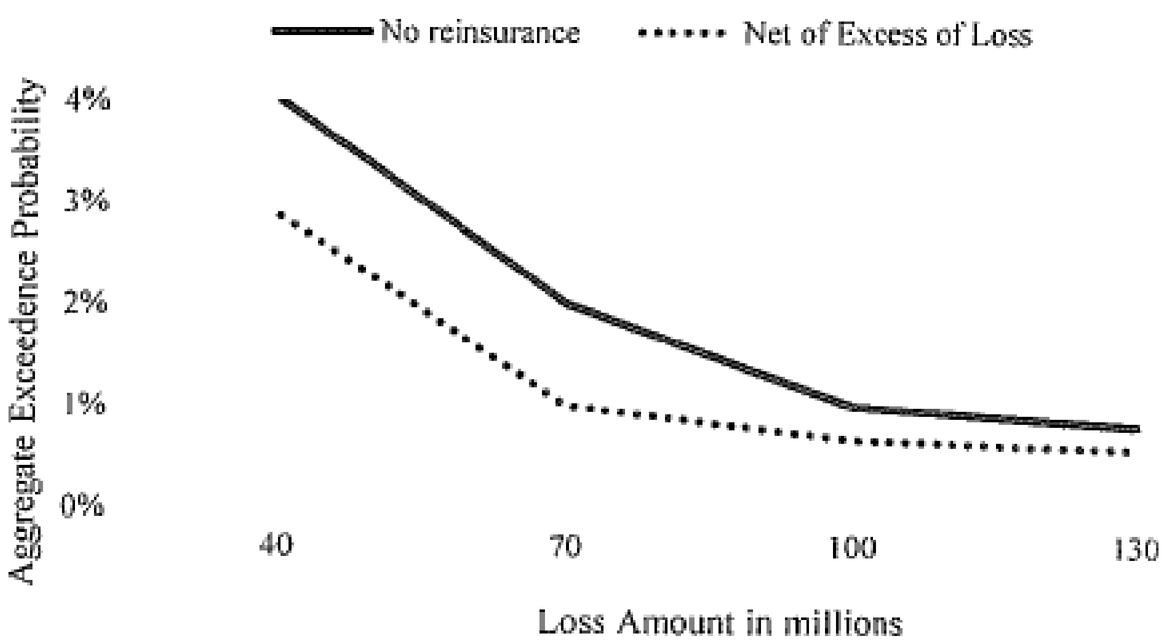
II. 20% Quota Share:
- Ceding commission of 30%
- Maximum ceded loss ratio of 150%

The insurance company must hold capital to support a 1-in-100 year Probable Maximum Loss. Additionally, the following information regarding the insurance company's performance last year is given:

- Premium: $50,000,000
- Expenses: $15,000,000
- Total Loss & ALAE: $30,000,000
- Return on Capital: 5%

	Claims greater than $5,000,000	
	Loss	ALAE
Claim 1	$7,500,000	$1,500,000
Claim 2	$10,000,000	$500,000

The Aggregate Exceedance Probability curve for the insurance company is shown below:



Determine the impact each reinsurance option would have had on last year's return on capital. Ignore investment income and taxes.

19. From the graph, the needed capital with the excess reinsurance is about 70M.
The ceded loss and ALAE are:

	Loss	Ceded Loss	ALAE	Ceded ALAE
Claim 1	$7,500,000	$2,500,000	$1,500,000	(1/3)($1,500,000)
Claim 2	$10,000,000	$5,000,000	$500,000	(1/2)($500,000)

Thus the total ceded loss and ALAE is: 8.25M.
Retained loss and ALAE is: 30M - 8.25M = 21.75M.
Reinsurance premium is: (18%)(50M) = 9M.
Underwriting profit is: 50M - 15M - 21.75M - 9M = 4.25M.
With excess reinsurance, return on capital is: 4.25/70 = **6.07%**, compared to 5% without reinsurance.

From the graph, without reinsurance the 1-in-100 year Probable Maximum Loss is about 100M.
With the quota share, the ceded premium is: (20%)(50M) = 10M.
The ceding commission is: (30%)(10M) = 3M.
Due to the maximum ceded loss ratio, the maximum ceded loss is: (150%)(10M) = 15M.
Thus for the 1-in-100 year Probable Maximum Loss, the insurer would have net losses of:
100M - 15M = 85M. Thus 85M is the needed capital with the quota share treaty.
For the given experience, the ceded losses and ALAE are: (20%)(30M) = 6M.
Underwriting profit is: 50M - 15M - (30M - 6M) - (10M - 3M) = 4M.
With quota share reinsurance, return on capital is:
4/85 = **4.71%**, compared to 5% without reinsurance.
Comment: Return on capital is not covered in the syllabus of this exam.
However, all you really need to know is that when we ignore investment income and income taxes, return on capital is: (underwriting profit) / capital.
For the current situation without reinsurance, the needed capital is 100M.
The underwriting profit is: 50 - 15 - 30 = 5M.
Return on capital is: 5/100 = 5%, matching what was given in the question.
.

20. (3 points) A primary insurer has entered into property catastrophe excess of loss treaties with three reinsurers. The terms of the treaties are as follows:

Reinsurer A: 100% of $100 excess of $200 written at a nominal rate on line of 10%
Reinsurer B: 100% of $100 excess of $300 written at a nominal rate on line of 7%
Reinsurer C: 100% of $100 excess of $400 written at a nominal rate on line of 4%

Each treaty specifies a single mandatory reinstatement that is 100% pro rata as to amount.
The primary insurer incurs three covered loss events during the contract period:

Loss Number	Loss Amount
1	$380
2	$260
3	$600

a. (0.75 point) For each loss event, calculate the amount of loss retained by the primary insurer.
b. (2.25 points) For each loss event, calculate the amount of reinstatement premium owed by the primary insurer to each of its reinsurers.

20. (a) For the first loss, A pays 100 and B pays 80. The insurer retains 200.
For the second loss, A pays 60, and the insurer retains 200.
For the third loss, since there was only one reinstatement, A pays only 100 - 60 = 40;
B pays 100 and C pays 100. Thus the insurer retains: 600 - 40 - 100 - 100 = 360.
(b) The premium for A is: (10%)(100) = 10. The premium for B is: (7%)(100) = 7.
The premium for C is: (4%)(100) = 4.
After the first loss, the insurer pays reinstatement premiums of (100/100)(10) = 10 to A, and (80/100)(7) = 5.6 to B.
After the second loss, the insurer pays no reinstatement premium to A, since there is only one reinstatement.
After the third loss, the insurer pays no reinstatement premium to A, since there is only one reinstatement and the coverage is exhausted. Reinsurer B receives a second reinstatement premium on loss event 3 as they had not fully exhausted their limit after loss event 1 and therefore only a partial reinstatement premium was paid after that loss event; the insurer pays B a reinstatement premium of: 7 - 5.6 = 1.4 = (20/100)(7).
The insurer pays a reinstatement premium to C of: (100/100)(4) = 4.
Comment: Presumably all of the figures are actually in millions of dollars.
Beyond the level of detail covered in Clark on reinstatement clauses.
I have assumed that A does not inure to the benefit of B, nor do A and B inure to the benefit of C; this is the way an insurer would set up a cat coverage, with the layer from 200 to 500 covered.
Cat treaties often have annual aggregate limits; here with only one reinstatement each reinsurer's payment during the year is capped at twice its pr event limit.
Reinsurer B would pay up to (2)(100) - 80 - 100 = 20 for a fourth catastrophe.

21. (4 points) An insurance company insures high value homes and plans to increase the maximum property value they will insure next year. The company is considering purchasing a new $4,000,000 excess of $4,000,000 reinsurance treaty. The reinsurer is given the following limit profile:

Insured Value Range	Experience Period On-Level Premium	Treaty Subject Premium
$1,000,000 to $4,000,000	$100,000,000	$25,000,000
$4,000,001 to $8,000,000	$0	$ 5,000,000

A reinsurance actuary has trended and developed the portfolio's historical losses for the Experience Period:

Loss Size	Number of Claims	Experience Period Ground-up Loss
Less than $1,000,000	200	$22,000,000
Greater than $1,000,000	10	$18,000,000
Total	210	$40,000,000

The actuary is unsure if a Swiss Re Y_3 or Y_4 exposure curve is a better fit.

The following MBBEFD exposure curve formulas are available along with the following information:

$$G(x) = \frac{\ln\left[\frac{(g-1)b + (1 - gb)\, b^x}{1-b}\right]}{\ln[gb]}$$

$$b(c) = \exp[3.1 - 0.15c(1+c)]$$
$$g(c) = \exp[c\,(0.78 + 0.12c)]$$

% of Insured Value	% of Cumulative Loss Y_3	Y_4
25%	60%	73%
40%	72%	82%
100%	100%	100%

Given a proposed treaty rate of 1% of total subject premium, calculate the expected ceded loss ratio for the new treaty.

21. The trended loss ratio during the experience period is: 40/100 = 40%.
I will assume this is the expected loss ratio in the future.
For the first interval of insured values, the middle is 2.5M.
Thus during the experience period, 1M is 1/2.5 = 40% of insurance to value.
During the experience period, the percent of cumulative losses in the layer from 0 to 1M is:
{22M + (10)(1M)} / 40M = 80%.
This more closely matches Swiss Re Y_4 exposure curve, G(40%) = 82%, so I will use Y_4.
For Y_4, c = 4. b = exp[3.1 - (0.15)(4)(1+4)] = 1.105. g = exp[(4){0.78 + (0.12)(4)}] = 154.47.
During the future treaty period, the second interval of insured value has a midpoint of 6M.
Thus the 4M xs 4M treaty covers from 4/6 = 2/3 to 1.

$$G(2/3) = \frac{\ln\left[\frac{(154.47 - 1)1.105 + (1 - (154.47)(1.105))\, 1.105^{2/3}}{1 - 1.105}\right]}{\ln[(154.47)(1.105)]}$$

= ln[112.2] / ln[170.7] = 0.918.
G(1) - G(2/3) = 1 - 91.8% = 8.2%.
The expected ceded losses are: (8.2%)(40%)(5M) = $164,000.
The ceded premium is: (1%)($30M) = $300,000.
Thus the expected ceded loss ratio is: 164/300 = **54.7%**.
<u>Comment</u>: You need to remember that c = 3 for Y_3 and c = 4 for Y_4.
For Y_3, G(2/3) = 86.2%.
One could interpolate between Y_3 and Y_4, giving more weight to Y_4.
I believe which curve is selected would often be based on the type of risks being covered and underwriting judgement, rather than one empirical exposure factor based on limited data, particularly when one is specifically going to make major changes to the book of business.
Y_3 is usually used for Commercial Lines (medium scale), and Y_4 is usually used for Industrial (not large scale) and Commercial Lines(large scale). Thus I am not sure why we are looking at these curves for use with homes, although these are high value homes.
Y_1 is usually used for Personal Lines. Y_2 is usually used for Commercial Lines (small scale).