Solutions to the Fall 2015 CAS Exam 8

(Incorporating what I found useful in the CAS Examiner's Report)

The solutions and comments are solely the responsibility of the author.
While some of the comments may seem critical of certain questions, this is intended solely to aid you in studying and in no way is intended as a criticism of the many volunteers who work extremely long and hard to produce quality exams.

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1. (2.5 points) An actuary is evaluating a merit rating plan for private passenger cars.
Given the following:

Number of Accident-Free Years	Earned Car Years	Number of Claims Incurred
2 or More	500,000	20,000
1	200,000	15,000
0	100,000	9,000
Total	800,000	44,000

- Frequency varies by territory.
- State law prohibits reflecting territory differences in rating.
- Annual claims for an individual driver follow a Poisson distribution.
- Claim cost distributions are similar across all drivers.

a. (0.5 point) Identify one potential issue with the exposure base used. Briefly explain whether or not earned premium would be a better choice for the exposure base.
b. (1.0 point) Calculate the credibility of one driver with one or more year's accident-free experience.
c. (1.0 point) Calculate the credibility of one driver with 0 Accident-Free years.

1. (a) Assume as in Bailey-Simon that this is data for one class.
Using car years may create maldistribution because some territories have higher frequency.
Using car years as the denominator of frequency, the credibility calculation would account for both "within territory differences" and "between territory differences". However, usually territory relativities already account for the between territory differences. We want Merit Rating to account for differences between cars not already accounted for by the class/territory relativities. Therefore using car years as the exposure base would double count territory differences, which usually would result in the credibility estimated for Merit Rating being too large.
However, since in this case state law prohibits reflecting territory differences in rating, using earned premium as the exposure base (dividing number of claims by earned premium) should work just as well as using earned exposures. Here using car years is appropriate due to the lack of territory differences in rating. Due to the rates not reflecting frequency differences between territory, the appropriate credibilities for Merit Rating are larger than they otherwise would be.
Alternately, premium may still be a stronger exposure base if nonterritorial factors are captured correctly, thereby reducing the maldistribution that exists using car years.
(b) Overall frequency is: 44/800 = 0.055.
Frequency of those with one or more years accident-free is:
(20 + 15) / (500 + 200) = 0.050.
Z = 1 - 0.05/0.055 = **9.09%**.
(c) Frequency of those with no years accident-free is: 9/100 = 9%.
9%/5.5% = M = Z / (1 - $e^{-0.055}$) + (1 - Z) (1). ⇒ 17.69Z = 0.6364. ⇒ Z = **3.60%**.
Comment: For part (c) we are using the alternate method discussed at page 160 in Bailey-Simon.
It uses the Poisson assumption. Let λ = the mean claim frequency (per exposure) for the class.
M = relative premium based frequency for risks with one or more claims in the past year.

Then, $M = Z / (1 - e^{-\lambda}) + (1 - Z)(1). \Rightarrow Z = \dfrac{M - 1}{1 / (1 - e^{-\lambda}) - 1} = (M - 1)(e^{\lambda} - 1)$.

The estimated credibilities in parts (b) and (c) are both for one year of data, and we would expect them to be more similar than they are here.
Bailey and Simon "have chosen to calculate Relative Claim Frequency on the basis of premium rather than car years. This avoids the maldistribution created by having higher claim frequency territories produce more X, Y, and B risks and also produce higher territorial premiums."

2. (2.75 points)
An actuary is modeling claim frequency for a portfolio with the following distribution of exposures.

	Vehicle Class			
Territory	Car	Van	Truck	Other
A	10,000	2,000	0	0
B	2,000	5,000	0	0
C	0	0	5,000	0
D	30	0	0	3,000

The actuary proposes a generalized linear model (GLM) with the following parameterization.

Territory		Vehicle Class	
Factor level	Parameter	Factor level	Parameter
A	β_1	Car	β_5
B	β_2	Van	β_6
C	β_3	Truck	β_7
D	β_4	Other	β_8

a. (1.0 points) Briefly discuss how intrinsic and extrinsic aliasing are present in this analysis using examples from the data. For each type of aliasing briefly explain the potential impact on the results.
b. (0.5 point) Provide one example of near aliasing in this analysis and briefly describe any potential impact on the modeling results.
c. (1.25 points) Propose an alternative GLM approach to avoid extrinsic, intrinsic, and near aliasing. Describe how many covariates would be required.

2. (a) Intrinsic aliasing is a linear dependency between covariates due to the definition.
There is intrinsic aliasing because 4 parameters are used for each of class and territory.
By definition $X_4 = 1 - X_1 - X_2 - X_3$, and $X_8 = 1 - X_5 - X_6 - X_7$.

For example, we could instead use an intercept and 3 parameters for each of class and territory, for a total of 7 rather than 8 parameters.
Alternately, we could use 4 parameters for class and only 3 for territory, or vice versa.
Extrinsic aliasing is a linear dependency between covariates that arises due to the particular values in the observed data rather than inherent properties of the covariates themselves.
Example of extrinsic aliasing:
all of the Trucks are in Territory C while all the vehicles in Territory C are Trucks.
In both cases, the GLM software should eliminate parameters to remove the effects of aliasing.
If parameters were not removed, then the model is overdetermined and one can not fit the GLM.
Alternately, these can lead to convergence issues or confusing results.
(b) Near aliasing occurs when there is strong correlation (but not perfect) between covariates.
Near aliasing: 99% of the vehicles in Territory D are Other (with 1% Cars).
This situation could lead to convergence problems, unstable parameter estimates, or confusing results when fitting the GLM.
(c) One way to define the covariates:
β_0 intercept
β_1 is 1 if Territory B
β_2 is 1 if Van
β_3 is 1 if Territory C and Truck
β_4 is 1 if Territory D and Car
β_5 is 1 if Territory D and Other

We would expect $\hat{\beta}_4$ to have a large standard error due to very limited data.

Another way to proceed is to remove the Cars in Territory D from the data used to fit the GLM:
β_0 intercept
β_1 is 1 if Territory B
β_2 is 1 if Van
β_3 is 1 if Territory C and Truck
β_4 is 1 if Territory D and Other
Alternately, removing the Cars in Territory D from the data used to fit the GLM:
β_1 is 1 if Territory A
β_2 is 1 if Territory B
β_3 is 1 if Van
β_4 is 1 if Territory C and Truck
β_5 is 1 if Territory D and Other
Comment: There are many acceptable answers to part (c).

3. (2.5 points) An actuary is considering using a generalized linear model to estimate the expected frequency of a recently introduced insurance product.
Given the following assumptions:
- The expected frequency for a risk is assumed to vary by state and gender.
- A log link function is used.
- A Poisson error structure is used.
- The likelihood function of a Poisson is

$$l(y; \mu) = \sum \ln f(y_i; \mu_i) = \sum \{-\mu_i + y_i \ln[\mu_i] - \ln[y_i!]\}$$

- β_1 is the effect of gender = Male.
- β_2 is the effect of gender = Female.
- β_3 is the effect of State = State A.

Claim Frequency

	State A	State B
Male	0.0920	0.0267
Female	0.1500	0.0500

Given that β_3 = 1.149, determine the expected frequency of a male risk in State A.

3. We have to assume equal exposures in each of the four cells.
The mean modeled frequencies are:

	State A	State B
Male	$\exp[\beta_1 + \beta_3]$	$\exp[\beta_1]$
Female	$\exp[\beta_2 + \beta_3]$	$\exp[\beta_2]$

The loglikelihood ignoring terms that do not depend on the betas is:

$-\exp[\beta_1 + \beta_3] + 0.0920\ (\beta_1 + \beta_3) - \exp[\beta_2 + \beta_3] + 0.1500\ (\beta_2 + \beta_3)$
$\quad - \exp[\beta_1] + 0.0267\ \beta_1 - \exp[\beta_2] + 0.0500\ \beta_2.$

Setting the partial derivative of the loglikelihood with respect to β_1 equal to zero:

$-\exp[\beta_1 + \beta_3] + 0.0920 - \exp[\beta_1] + 0.0267 = 0.$

Given $\beta_3 = 1.149$: $-\exp[\beta_1]\ e^{1.149} + 0.0920 - \exp[\beta_1] + 0.0267 = 0.$

$\Rightarrow \exp[\beta_1] = (0.0920 + 0.0267) / (1 + e^{1.149}) = 0.02857.$

$\Rightarrow \exp[\beta_1 + \beta_3] = 0.02857\ e^{1.149} =$ **0.0901** = expected frequency of a male risk in State A.

Comment: Similar to 8, 11/13, Q.2c.

What the exam questions calls "the likelihood function" is the loglikelihood function.

$\hat{\beta}_1 = \ln(0.02857) = -3.555.$

Setting the partial derivative of the loglikelihood with respect to β_2 equal to zero:

$-\exp[\beta_2 + \beta_3] + 0.1500 - \exp[\beta_2] + 0.0500.$

Given $\beta_3 = 1.149$: $-\exp[\beta_2]\ e^{1.149} + 0.1500 - \exp[\beta_2] + 0.0500 = 0.$

$\Rightarrow \exp[\beta_2] = (0.1500 + 0.0500) / (1 + e^{1.149}) = 0.04813. \Rightarrow \hat{\beta}_2 = -3.034.$

Using a computer, without being given β_3, the maximum likelihood fit is:

$\hat{\beta}_1 = -3.5555$, $\hat{\beta}_2 = -3.0338$, and $\hat{\beta}_3 = 1.1490$.

The mean modeled frequencies are:

	State A	State B
Male	exp[-3.5555 + 1.1490] = 9.01%	exp[-3.5555] = 2.86%
Female	exp[-3.0338 + 1.1490] = 15.19%	exp[-3.0338] = 4.81%

4. (2.25 points)
An actuary is reviewing an account that has been with the company for over ten years.
Given the following:

- The claim frequency for this account follows a Poisson distribution, with $\lambda = 0.012$
- The recorded frequency for the last five years is as follows:

Year	Exposures	Frequency
2010	9,500	0.011
2011	11,000	0.010
2012	13,000	0.013
2013	10,500	0.012
2014	12,000	0.010

- The critical value for the relevant Chi-squared distribution is 9.49

a. (1.5 points) Use the Chi-squared test to evaluate whether the claim frequency is shifting over time. Include the hypotheses, test statistic, and provide an interpretation of the result.
b. (0.75 points)
Fully describe another method for determining whether claim frequency is shifting over time.

4. (a) H_0: The expected frequency is 1.2% for each year.

H_1: Not H_0.

For 2011 the observed number is: (11,000)(0.010) = 110,
and the expected number is: (11,000)(0.012) = 132.

Contribution is: $(\text{Observed} - \text{Expected})^2 / \text{Expected} = (110 - 132)^2 / 132 = 3.6667$

Year	Exposures	Frequency	Observed	Expected	Chi-Square Contribution
2010	9,500	0.011	104.5	114	0.79167
2011	11,000	0.010	110	132	3.6667
2012	13,000	0.013	169	156	1.0833
2013	10,500	0.012	126	126	0
2014	12,000	0.010 .	120	144	4
					9.54

Since the Chi-Square statistic is 9.54 > 9.49, at the corresponding significance level we reject the null hypothesis. This is evidence that (expected) claim frequency is shifting over time.
(b) For a given risk, compute the correlations between pairs of different years of data.
Average the correlations for all pairs with the same number of years between them.
If these average correlations decline quickly towards zero as the distance between pairs of years increases, then parameters are shifting at a significant rate.
Comment: 9.49 is the 5% critical value for a Chi-Square Distribution with 4 degrees of freedom.

5. (2.5 points) An actuary estimated the loss cost for workers compensation insurance using a multidimensional credibility method.

Given the following:

- There were 2 classes in Hazard Group X.
- There were no major or minor permanent partial losses.
- Premium information was not available.
- Holdout sample of odd years was used as a proxy of the true mean.

Claim Count by Injury Type for Hazard Group X

	Even Year 1			Even Year 2		
Class	Fatal (F)	Permanent Total (PT)	Temporary Total (TT)	Fatal (F)	Permanent Total (PT)	Temporary Total (TT)
Class 1	2	10	1,000	1	12	1,000
Class 2	3	10	1,000	2	13	1,000
Total	5	20	2,000	3	25	2,000

Optimal Weights for Estimation of Permanent Total Injury Ratio

Fatal	Permanent Total
0.2	0.3

a. (1 point) Determine the ratio of permanent total injury to temporary total injury for Class 2 using a multi-dimensional credibility method.

b. (1 point)
Fully describe the steps involved in performing a quintile test to evaluate the actuary's work.

c. (0.5 point)
Briefly describe one shortcoming of the individual class sum of squared errors test and briefly describe why the quintiles test is a better way to evaluate the actuary's work.

5. (a) For the hazard group, fatal ratio is: E[V] = 8 / 4000 = 0.002.
For class 2, fatal ratio is: V_2 = 5 / 2000 = 0.0025.

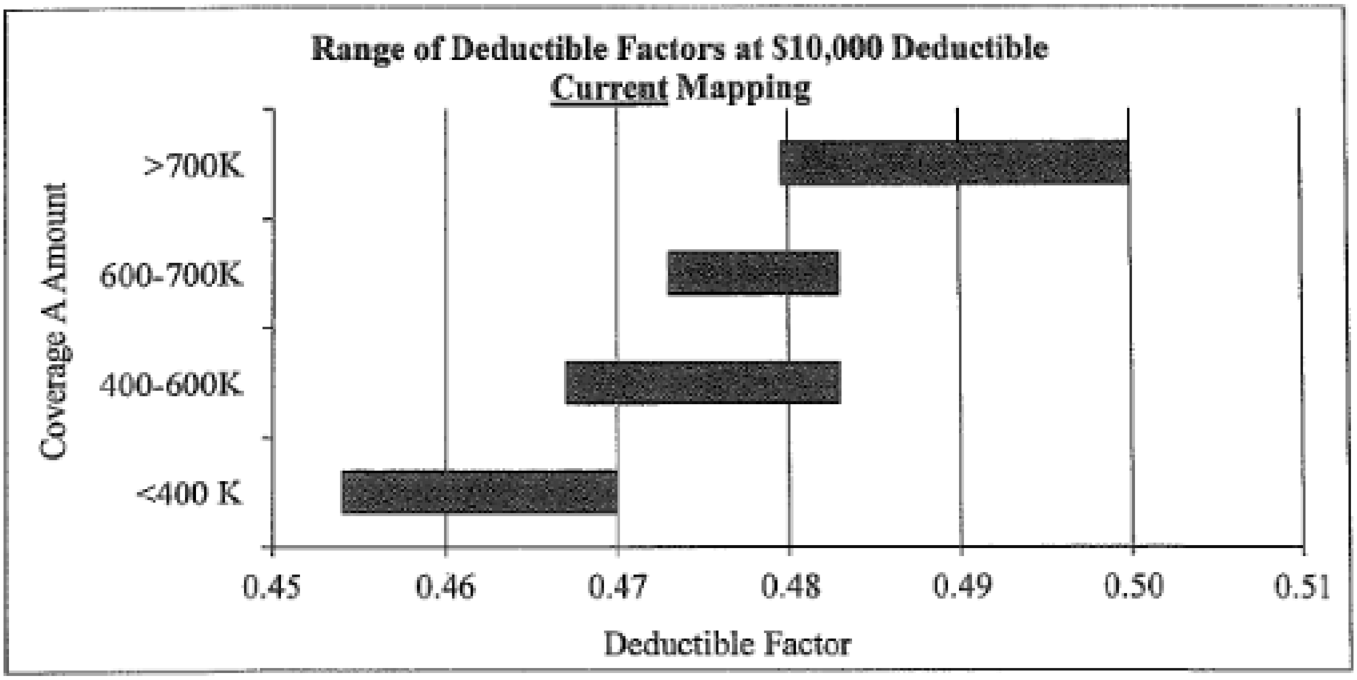
For the hazard group, P.T. ratio is: E[W] = 45 / 4000 = 0.01125.
For class 2, P.T. ratio is: W_2 = 23 / 2000 = 0.0115.

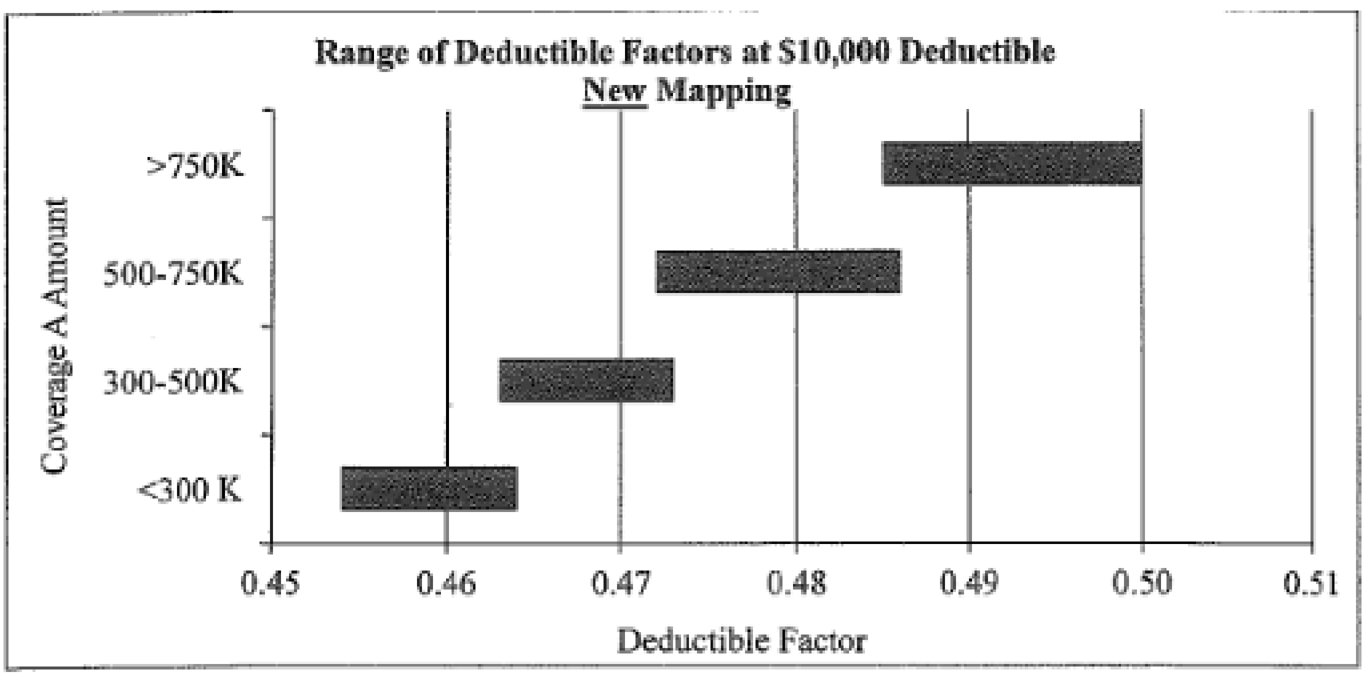
$\hat{W}_2 = E[W] + (0.2)(V_2 - E[V]) + (0.3)(W_2 - E[W])$

$= 0.01125 + (0.2)(0.0025 - 0.002) + (0.3)(0.0115 - 0.01125) =$ **0.011425**.

(b) Apply the test separately to each injury kind (V, W, X, Y) and to each hazard group.
First apply the estimator to the even reports for each class in the hazard group.
Then get an estimated relativity by dividing by the corresponding value for the hazard group.
Then based on these estimated relativities group the classes into 5 quintiles, from smallest estimate to largest. Each quintile should have about the same number of temporary total claims.
Then for each quintile compare the observed relativity for the hold out sample (odd reports) to the result of three estimators:
1. Prediction based solely on the hazard group, in other words a relativity of one.
2. Prediction based on the multi-dimensional credibility method.
3. Prediction based solely on the class data for that injury kind.
The sums of squares errors (SSE) compares these predictions to the observed relativities for the odd reports (holdout sample).
The smallest SSE is best; hopefully the credibility procedure has the smallest SSE of the three estimators.
(c) The individual ratios for a class are quite volatile. Therefore, improving the estimates of the class means might produce only a small improvement in the total sum of squared errors.
In other words, the improvement in squared error could be masked by the large predication errors caused by random fluctuation.
A more refined test is needed to evaluate the multi-dimensional credibility method. By grouping classes together by quintile, the quintiles minimizes the effects on the test of the random fluctuations of class observations.
Alternately, there is too much noise in the individual test. Grouping into quintiles diversifies away the class specific variation allowing one to see the effect of the credibility procedure.
Alternately, each class is relatively small compared to the hazard group and results can be volatile from class to class, grouping into quintile allows for a more credible evaluation of the results.
<u>Comment</u>: A typical hazard group would have 50 or 100 classes; two classes were used solely for simplicity. One would need at least 5 classes and preferably a lot more to perform a quintiles test.
This question assumed no major or minor permanent partial losses solely for simplicity.

6. (3 points) A company groups its homeowners' policies based on Coverage A amount for ratemaking. The company is proposing using a new method, k-means clustering, to group these policies. The graphs below show the range of deductible factors by Coverage A amount group for the current and proposed method:





<< QUESTION 6 CONTINUED ON NEXT PAGE >>

a. (1 point) Describe the steps in performing the k-means clustering method.
b. (0.5 point) Discuss whether the current or proposed method should be used to group homeowners' policies using the two graphs provided.
c. (1.5 point) Identify two operational considerations that would affect the decision to implement a change in policy grouping and explain how these considerations would apply to implementation of the new groups.

6. (a) In this case, one would presumably first group homes into amount of insurance brackets of width for example 50,000, and then determine the average observed loss elimination ratio and corresponding deductible factor for those homes.

In general, one has to choose a distance function to use; since this is a one dimensional application this should not make a difference.

For the k-means clustering algorithm, one has the choose the number of clusters, k, which is presumably 4 in this case. (In general, one can apply the algorithm for different values of k, and use some statistic(s) to decide how many clusters are optimal.)

The following steps are performed iteratively:

0. Some initial assignment to clusters is made.

In this case, one can use the current groupings.

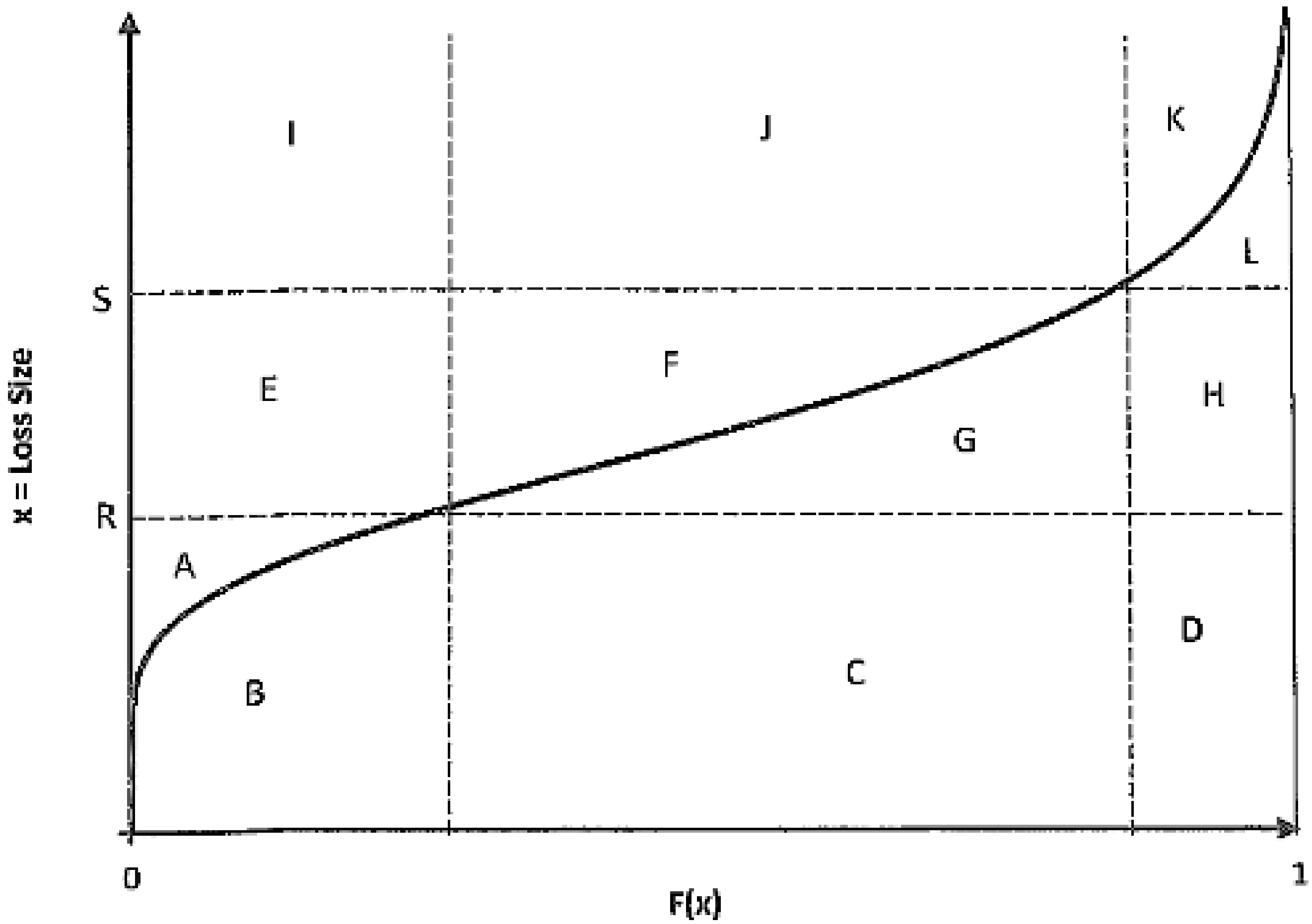
1. Compute the centroid of each cluster, in this case the average deductible factor.
2. Assign each amount of insurance interval to the closest centroid from step 1.
3. If step 2 results in any changes to the clusters, return to step 1.

(b) The **new groupings are preferable**, since there is less overlap between groups.

For the new groupings, the between variation in deductible factors is bigger (since there is less overlap) while the within variation in deductible factors is smaller (since the bars are shorter) than for the current groups.

(c) 1. Expense: There is no major expense involved in implementing a change in policy grouping.
2. Constancy: "It is desirable that the characteristics used in any risk classification system should be constant in their relationship to a particular risk. This constancy should prevail over the period covered by the insurance contract or, alternatively, over the period for which a class is assigned."
The Coverage A amount of insurance almost always stays the same for the policy period, so this criteria is met.
(One would need to update the groupings after several years for the effect of inflation, but this should not be problem.)
3. Availability of Coverage: Since the new groupings are somewhat more accurate, the insurer should be somewhat more willing to write houses with different amounts of insurance. This would increase the availability of coverage.
4. Avoidance of Extreme Discontinuities:
Since the difference in deductible factors is small between the groups, I do not see an effect.
5. Absence of Ambiguity:
There is no ambiguity, since the Coverage A amount of insurance is one of the items listed on the homeowners policy.
6. Manipulation: This is not an issue as far as amount of insurance relates to deductible credits; insureds would not report a higher Coverage A amount solely in order to get a bigger deductible credit. (It is important to have insureds maintain insurance to value as it directly relates to premium.)
7. Measurability: Not an issue, since amount of insurance is one of the items listed on the policy. The risk characteristic is conveniently and reliably measured.
<u>Comment</u>: Part (c) refers to AAA Risk Classification; this is a poor example to apply these ideas. I would have found it helpful if the question gave a little context.
Presumably, the insurer wants to give different percentage credits for a \$10,000 deductible to different groups of amount of insurance. For example, a home with Coverage A amount of \$200,000 might get a credit of 55% for a \$10,000 deductible, while a home with Coverage A amount of \$1,000,000 might get only a credit of 50%. (I am assuming the "deductible factor" is one minus the credit. I also do not see why the insurer would create 4 groups when the difference in deductible factors is so small.)
In order to help find good breakpoints to use to divide a continuous variable, such as amount of insurance, into discrete categories, I believe that it would be more common to use another technique than k-means clustering, such as MARS (Multivariate Adaptive Regression Spline). MARS, not on the syllabus, operates as a multiple piecewise linear regression.

7. (2 points) The following Lee diagram applies to a cumulative size of loss distribution F(x), where letters A through L represent the areas of the enclosed regions.



a. (0.25 point) Express the area G + H in integral form, using the layer method.
b, (0.25 point) Express the area B + C + D in integral form, using the size method.
c. (0.25 point) Assume R is the basic limit. Express the increased limits factor for limit S algebraically using the area labels provided in the graph.
d. (1.25 points) Describe the consistency test for increased limit factors. Use a graph to explain what the consistency test is evaluating. Label all relevant features of the graph.

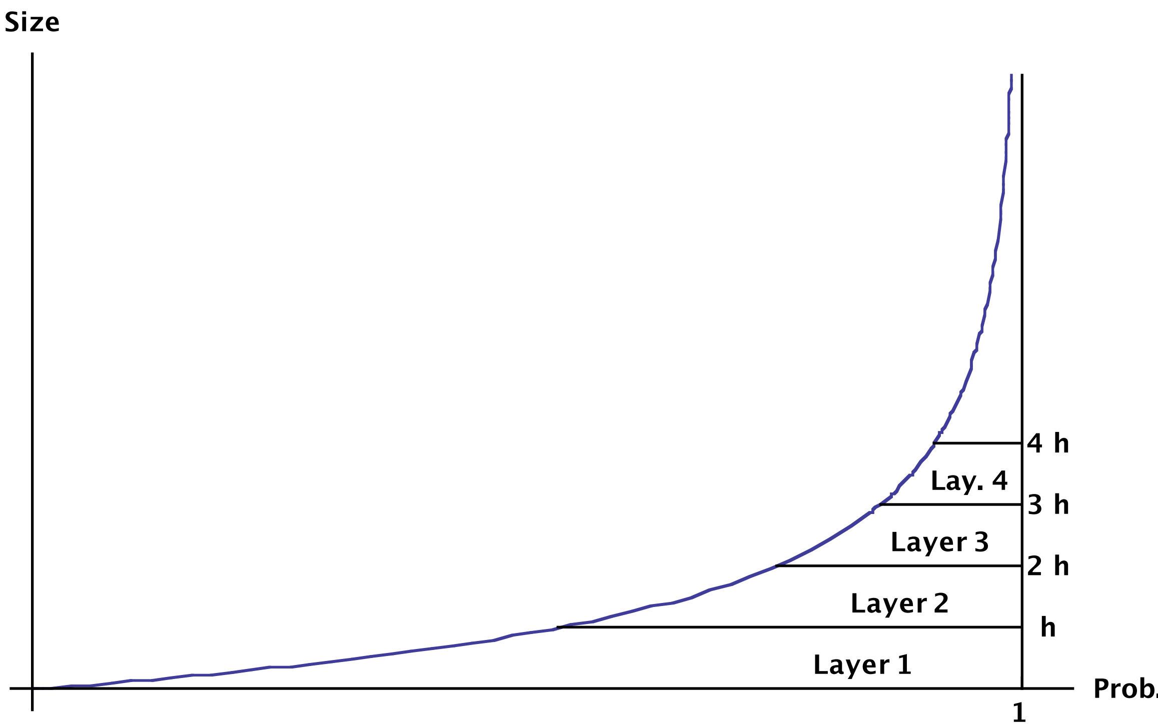
7. (a) $G + H = \int_R^S \{1 - F(x)\} \, dx$.

(b) $B + C + D = \int_0^R x \, f(x) \, dx + R \{1 - F(R)\}$.

(c) ILF = (B + C + D + G + H) / (B + C + D) = 1 + (G + H) / (B + C + D).

(d) The consistency test for increased limits factors is that the increased limits factor must increase at a decreasing rate as the limit increases. (Positive first derivative and negative second derivative.)

Here is a Lee Diagram showing four layers of equal width:



Since they have the same height, but decreasing widths: Layer 1 > Layer 2 > Layer 3 > Layer 4.

If h were the basic limit, then the increased limit factor for a limit of 2h would be:

1 + (Layer 2)/(Layer 1).

The increased limit factor for a limit of 3h would be: 1 + (Layer 2 + Layer 3)/(Layer 1).

The increase in the increased limit factor to go from 2h to 3h is: (Layer 3)/(Layer 1).

Similarly, the increase in the increased limit factor to go from 3h to 4h is: (Layer 4)/(Layer 1).

Therefore, the increase in the increased limit factor to go from 3h to 4h is less than the increase in the increased limit to go from 2h to 3h.

The increased limit factors increase at a decreasing rate.

For layers of different widths, the increase in the increased limit per additional amount of coverage decreases as the limit gets higher.

Comment: For part (d), the CAS also gave credit for discussing a graph showing ILFs increasing and concave downwards.

8. (2.5 points) An actuary is working with ground-up historical loss data and is considering fitting one continuous curve to this data to calculate ILFs for higher limits.
The last time such an analysis was conducted, empirical losses were used to determine ILFs directly without fitting a continuous curve to the data.
(a) (1 point) Provide two shortcomings of using empirical data to determine ILFs and briefly describe how curve fitting may overcome each of these shortcomings.
(b) (1.5 points) There is a concern that fitting one continuous curve to the entire distribution of losses will overstate losses over certain intervals and understate losses over other intervals.
Propose and fully describe a solution that addresses this concern while still incorporating an element of curve fitting in the solution.

8. (a) 1. The data at higher limits is usually sparse. Therefore, there is lots of random fluctuation.
Also empirical losses may not reach maximum policy limits for which one wants to calculate ILFs, so that using empirical data would result in a free cover.
Fitting a curve both reduces the effect of random fluctuation and allows the behavior of the smaller losses to be extrapolated to that of the larger losses.
2. The raw data included in the claim size distribution has gaps for certain intervals where no claims appear. Fitting a curve alleviates this problem because of its smooth nature.
3. The existence of “cluster points” (intervals where the number of claims drastically rises and immediately drops) magnify the discontinuity between intervals.
Fitting a curve alleviates this problem because of its smooth nature.
4. Each open claim has a probability distribution of its ultimate value. One has to somehow adjust for the effect of development on known claims and the effect of unreported claims which tend to be larger. Curve fitting can take loss development as well as the dispersion in development into consideration

(b) As per the paper by Mahler, one could use the empirical excess ratios to determine ILFs below a certain breakpoint, and fit a distribution or mixed distribution above that breakpoint. The distribution or mixed distribution is fit to the data truncated and shifted at the breakpoint.
For values above the breakpoint, the excess ratio is: R(x) =
(empirical excess ratio at breakpoint) (the fitted excess ratio at the entry ratio corresponding to x).
This allows us to rely on the actual data for the lower layers where there is a larger volume of data, less subject to random fluctuations. The empirical distribution and curve are joined smoothly together. The threshold above which curve fitting should be employed should be selected to permit the maximum reliance on reported data while still retaining enough data above the threshold to permit reasonable fitting of a loss distribution. This breakpoint should be a round number prior to the 'thinning out' of the data.
This method provides a smooth transition from relying on data for lower accident limits to relying on a fitted curve to provide some information at higher accident limits.
Comment: Data on individual losses usually come from different policies with different policy limits, causing a bias in the distribution. One can use the Kaplan-Meier or Nelson-Aalen techniques to combine empirical data from different policy limits.
One other advantage of using a theoretical distribution to represent the data is that it facilitates the computation of a variance at each policy limit, which can be used as a basis for risk adjustments.
I would have allowed two alternatives in part (b), but the Exam Committee gave these no credit:
1. One could fit a mixed distribution, that is a weighted average of two or more continuous curves. This allows us to capture the different behaviors of the data over different portions of the size distribution.
2. One could fit a two or more component splice. Over each interval the density is proportional to some size of loss distribution. This allows the capture of the different behaviors of the data over different intervals.

9. (3.25 points) Given the following Premises/Operations General Liability loss experience evaluated as of September 1, 2013:

Policy Effective Date	Policy Type	Total Ground-Up Incurred Loss	Total Ground-Up Incurred ALAE
March 1, 2010 to February 28, 2011	Occurrence	1,500,000	600,000
March 1, 2011 to February 29, 2012	Occurrence	400,000	400,000
March 1, 2012 to February 28, 2013	Occurrence	350,000	2,000,000
March 1, 2013 to February 28, 2014	Occurrence	150,000	20,000

- The insured has experienced the following ground-up large losses:

Accident Date	Incurred Loss	Incurred ALAE
June 30, 2010	700,000	500,000
December 31, 2011	150,000	200,000
April 5, 2012	55,000	60,000

- Annual Basic Limits Premium = $800,000.
- Expected Loss and ALAE Ratio = 80%.

A new policy will become effective March 1, 2014 to February 28, 2015 and will be written on an occurrence basis.

Using the ISO Commercial General Liability Experience and Schedule Rating Plan, calculate the experience modification factor used to price this policy.

9. Since all policies are occurrence, all the policy adjustment factors are one.
We use the experience of the policies written in 2010, 2011, and 2012.
The data for the 2012 policy is as of 18 months.
For example, (800,000)(0.8)(0.907) = 580,480.
(580,480)(0.995)(0.519) = 299,763.

	Premium	ELR	Detrend Factor	Subject Loss Cost
Latest	800,000	0.8	0.907	580,480
2nd Latest	800,000	0.8	0.864	552,960
3rd Latest	800,000	0.8	0.823	526,720
				1,660,160
Subject Loss Cost	EER	LDF	Expected Unreported	
580,480	0.995	0.519	299,763	
552,960	0.995	0.338	185,966	
526,720	0.995	0.198	103,769	
			589,498	

Based on the subject loss cost of \$1,660,160: Z = 0.85, EER = 0.995, and MSL = \$551,800.
Assume that the basic limit is \$100,000.
Limit each large loss to basic limits, then add in ALAE; then limit to the MSL.

Loss	ALAE	Basic Limit Loss & ALAE	Limited by MSL
700,000	500,000	600,000	551,800
150,000	200,000	300,000	300,000
55,000	60,000	115,000	115,000

Thus the Loss and ALAE entering the mod calculation is:
1.5M + 0.4M + 0.35M + 0.6M + 0.4M + 2M + (0.5518M + 0.3M) - (1.2M + 0.35M) = 4.5518M.
Adding in the expected unreported, the AER is: (4,551,800 + 589,498) / 1,660,160 = 3.097.
Mod = Z(AER - EER)/EER = (0.85)(3.097 - 0.995) / 0.995 = 1.796. ⇔ **179.6% debit**.
Modification factor is: 1 + 1.796 = 2.796.

10. (3 points) One common expression for the experience modification for a single-split plan is:

$$M = 1 + Z_p \frac{A_p - E_p}{E} + Z_e \frac{A_e - E_e}{E}.$$

where:

- M is the modification factor
- Z_p and Z_e are credibility constants
- A_p is the actual primary loss
- A_e is the actual excess loss
- E_p is the expected primary loss
- E_e is the expected excess loss
- E is the expected total loss.

a. (0.75 point) In the right-hand side of the equation above, there are three terms separated by '+' signs. Briefly describe the role that each term serves in computing the experience mod.

b. (0.5 point) Of the two credibility constants, Z_p and Z_e, identify which of the two is typically the larger in magnitude, and explain why.

c. (1.75 points) Determine the effectiveness of each of the following credibility functions and select which function is the most appropriate.

	Credibility			
Expected Loss	Function 1	Function 2	Function 3	Function 4
1,000	15%	65%	55%	80%
2,000	35%	75%	63%	75%
3,000	55%	85%	70%	62%
4,000	75%	95%	76%	53%
5,000	95%	105%	81%	40%

10. (a) 1 is the modification factor for an insured with average experience.
It reflects the manual rate as a starting point.

$\frac{A_p - E_p}{E}$ measures how the actual primary losses for this insured differ from the expected primary losses, and $Z_p \frac{A_p - E_p}{E}$ is the effect on the modification of that difference. If $A_p > E_p$ then this term increases the mod, while if $A_p < E_p$ then this term decreases the mod.

$\frac{A_e - E_e}{E}$ measures how the actual excess losses for this insured differ from the expected excess losses, and $Z_e \frac{A_e - E_e}{E}$ is the effect on the modification of that difference. If $A_e > E_e$ then this term increases the mod, while if $A_e < E_e$ then this term decreases the mod.

(b) $Z_p > Z_e$, since the primary losses have less random fluctuation than the excess losses, and thus have more informational content while the excess losses have more noise.
The primary losses are more reflective of future loss potential and thus are given greater credibility.
(c) We want $0 \le Z \le 1$.
All of the credibilities are between 0 and 1 inclusive, except for Function 2.

We want $\frac{dZ}{dE} \ge 0$.

Function 4 is a decreasing function of size, which is no good!

Finally we want $\frac{d\,(Z/E)}{dE} \le 0$. Here we test whether: $\frac{\Delta\,(Z/E)}{\Delta E} \le 0$.

Here, since the changes in E are all the same, this is equivalent to: $\Delta\,(Z/E) \le 0$.

E	Z1	Z1/E	Change	Z3	Z1/E	Change
1000	15%	0.0001500		55%	0.0005500	
2000	35%	0.0001750	0.0000250	63%	0.0003150	-0.0002350
3000	55%	0.0001833	0.0000083	70%	0.0002333	-0.0000817
4000	75%	0.0001875	0.0000042	76%	0.0001900	-0.0000433
5000	95%	0.0001900	0.0000025	81%	0.0001620	-0.0000280

$\Delta\,(Z/E) \le 0$ is true for Function 3, but not for Function 1.

Thus **Function 3 is the most appropriate**.

Comment: In the NCCI experience rating plan, $Z_e = W\,Z_p$, with $W < 1$.

11. (2.5 points) An underwriter and an actuary are discussing the effectiveness of the current experience rating plan.

The following table contains experience from five experience rated risks (all of similar size):

Risk	Manual Premium	Modified Premium	Actual Losses
1	400,000	360,000	300,000
2	600,000	840,000	690,000
3	800,000	560,000	440,000
4	900,000	1,080,000	860,000
5	1,000,000	800,000	650,000

a. (1.5 points) Evaluate whether the experience rating plan is effective or not and explain why.

b. (1 point) The underwriter argues that the modification factor for risk 4 is too high. Propose two additional pieces of information the actuary could request regarding risk 4 in order to support or disprove the underwriter's argument and explain why the information would be useful.

11. Assume the actual losses are the subsequent losses for the policy being experience rated.
a. For risk 1: Mod = 36/40 = 0.9, Manual LR = 30/40 = 75%, Modified LR = 30/36 = 83.3%.

Risk	Mod	Manual Loss Ratio	Modified Loss Ratio
3	0.7	55%	78.6%
5	0.8	65%	81.3%
1	0.9	75%	83.3%
4	1.2	95.6%	79.6%
2	1.4	115%	82.1%

The manual loss ratios increase with mod, thus the plan does a good job of identifying risk differences.
The modified loss ratios (loss ratios to standard) are close to level, thus the plan does a good job of correcting for risk differences.
Based solely on this data, the experience rating plan is effective.
b. 1. We could look at the individual losses that went into the experience rating of risk #4, to see whether the debit mod was largely due to one or two very large claims or due to a high frequency of claims. The former could be due to bad luck, while the latter would be more indicative of a worse than average risk.
2. We could look at a history of its previous experience mods and see how well the plan has done in predicting the losses of risk 4. If over this longer period of time the plan has tended to overestimate losses for risk 4, then perhaps there is some validity to the underwriter's claim.
3. We could see whether the insured has made any changes in the recent past, too recent to be reflected in its experience mod, that might qualify it for a schedule credit.
4. We could examine risk and class characteristics for risk 4, as this would indicate whether the mod was correcting for a poor class fit.
5. We could check whether the rate and thus the expected loss rate for the biggest class for risk 4 was somehow capped in the most recent rate change. In that case the expected loss rate is probably inadequate. This might result in a debit mod, even if risk 4 is average for its class.
6. We could look at the individual losses that went into its subsequent losses, to see whether the $860,000 was largely due to one or two very large claims or due to a high frequency of claims. The former could be do to bad luck, while the latter would be more indicative of a worse than average risk.
<u>Comment</u>: "all of similar size" adds nothing to the question, as we can see how big the risks are from their given manual premiums. Whether these risks are of similar size is a matter of opinion.
In part (b), it would have helped if we were told why the underwriter is arguing that the modification factor for risk 4 is too high. The fact that risk 4 has a subsequent modified loss ratio slightly lower than average indicates virtually nothing; given the random fluctuation one would expect in the subsequent losses, the subsequent modified loss ratios are amazingly similar to each other.

12. (1.5 points) An actuary is evaluating the effectiveness of an experience rating plan and has calculated the following values:

	Standard Loss Ratio		Sample Variance in Loss Ratios	
Risk size	Risks with credit mod	Risks with debit mod	Unmodified	Modified
Small	1.05	1.08	0.07	0.008
Medium	0.98	0.96	0.05	0.004
Large	0.99	1.00	0.04	0.004

a. (0.5 point) Evaluate whether this plan satisfies the necessary condition for proper credibility.
b. (0.5 point)
Determine which risk size has the most accurate experience rating based on the efficiency test.
c. (0.5 point) It has been determined that premiums are inadequate for small risks. Discuss whether premium inadequacy is better corrected by changing the manual rates or the experience rating plan.

12. (a) Dorweiler's necessary condition for proper experience rating credibility:
For each size category, debit and credit risks should have equal loss ratios to standard premium in the prospective period. In which case, insurers would find credit risks and debit risks equally desirable as insureds.
For small risks: 1.05 ≠ 1.08. For medium sized risks: 0.98 ≠ 0.96. For large risks: 0.99 ≠ 1.00.
Thus **this condition is not satisfied**.
(Based on the fact that 1.08 > 1.05, the credibility for small risks is too small.
0.98 > 0.96 implies that the credibility for medium sized risks is too big.
0.99 < 1.00 implies that the credibility for large risks is a little too small.)
On the other hand, the standard loss ratios are not that far apart. Depending on how many risks the actuary is examining, this could be due to random fluctuation.

(b) Test statistic = $\frac{\text{the sample variance of the modified loss ratios}}{\text{the sample variance of the unmodified loss ratios}}$.

Lower test statistic ⇔ Better.
Small: 0.008/0.07 = 0.114.
Medium: 0.004/0.05 = 0.080. **Best**.
Large: 0.004/0.04 = 0.100.
Medium sized risks have the most accurate experience rating based on the efficiency test.

(c) Altering the experience rating plan by giving more credibility to small risks, would alleviate the problem of inadequate rates for small risks (who are big enough to be eligible for experience rating), since now the premium for a small risk would depend more on its own experience. However, in order to have a substantial effect, the credibilities for small risks would have to be made much, much bigger. This would create very significant problems due to the standard premiums for small risks fluctuating widely from year to year. Thus altering the experience rating plan is <u>not</u> a good solution.
Experience rating is intended to adjust for <u>individual</u> cost differences.
Just changing the manual rates is <u>not</u> a solution to the problem, since this would affect risks of all sizes. Instead one should investigate why the rates for small risks are inadequate.
Then potential solutions are one or more of the following:
1. Introduce a loss constant or make an existing loss constant bigger.
2. introduce an expense constant or make an existing expense constant bigger.
3. Increase the loading in the manual rates for expenses and increase the premium discounts which are received by larger risks.

<u>Comment</u>: Dorweiler's sufficient condition: There should be no way to select credible subgroups of risks based on their experience that will produce significantly different loss ratios to standard premium in the prospective period.
This would involve comparing standard loss ratios between the size categories.
In part (b) we want the variance of modified loss ratios to be small and the variance of unmodified loss ratios to be big. This test would normally be used to compare different experience rating plans.
In part (c), the CAS Exam Committee did not seem to understand that manual rates apply to all sized risks, and thus that increasing manual rates is not a solution to the problem of inadequate rates for small insureds.

13. (3.25 points)
An actuary prices two loss-sensitive options for a workers compensation policy as follows:
Option 1: A large deductible plan with a per-occurrence deductible of $50,000
Option 2: An incurred retrospective rating plan with the following parameters:

Per Occurrence Limit	$50,000
Basic Premium	$150,000
Tax Multiplier	1.045
Loss Conversion Factor	1.100
Deposit Premium (paid at policy inception)	$1,000,000

For each of the options above, assume that no aggregate limits or maximum premiums apply and that the first adjustment will take place 18 months after policy inception.
Additionally, the actuary has developed the following assumptions for the insured:

	Unlimited	Limited to $50,000
Expected Loss	$650,000	$435,000
18-Ultimate Incurred LDF	4.25	3.75
18-Ultimate Paid LDF	8.80	6.55

a. (2.25 points) For each of the plans above, determine the expected cash flows between the insured and insurer 18 months after policy inception.
b. (1 point) The insured is contemplating a third option of purchasing an excess policy with a self-insured retention of $50,000.
i. Which of the three options would be least attractive to the insurer if they wish to minimize credit risk? Briefly explain your choice.
ii. Which of the three options would be least attractive to the insurer if they wish to minimize interest rate risk? Briefly explain your choice.

13. (a) The expected limited incurred losses at 18 months are: 435,000/2.75 = 116,000.
The expected limited paid losses at 18 months are: 435,000/6.55 = 66,412.
For the large deductible policy, through 18 months the expected primary losses for which the insurer expects to be reimbursed are **$66,412**.
(The insurer would have already collected the premium. The insurer would have been billing for reimbursements all along, perhaps every quarter.)
Excess Loss Premium is equal to expected excess times loss conversion factor:
(1.1)(650,000 - 435,000) = 236,500.
The expected retro premium at first adjustment (18 months) is:
(1.045) {150,000 + 236,500 + (1.1)(116,000)} = $537,235.
Then the insurer would owe the insured: $1,000,000 - $537,235 = **$462,766**.
(I have assumed that the payroll at final adult equals that estimated at policy inception.
I have also assumed that retrospective development premium does not apply.)
Alternately, if the insured and insurer have agreed to include the optional retrospective development premium in the plan, then at first adjustment it should be of size:
(1.1)($435,000) / (1 - 1/3.75) = $350,900.
Then the expected retro premium at first adjustment (18 months) is:
(1.045) {150,000 + 236,500 + 350,900 + (1.1)(116,000)} = $903,925 =
(1.045) {150,000 + (1.1)(650,000)}.
Thus, the insurer would owe the insured: $1,000,000 - $903,925 = **$96,075**.
(b) i. Under the large deductible plan, the insurer faces the credit risk of the insured not reimbursing it for the losses within the deductible. Thus the large deductible is the least desirable.
(Under the excess policy the insurer faces no credit risk. Under the retro plan the insurer faces the credit risk that the insured may not pay premium that may be owed at a retro adjustment.)
ii. Under the excess policy there is the longest average time between when the insurer gets its premium and when it has to pay (excess) losses, since expected losses make up a larger portion of the premium in this case than for LDD. Therefore, the excess policy has the largest opportunity to earn investment income, and therefore also the biggest interest rate risk for the insurer.
(The premiums for the incurred retro plan are based on incurred losses, paid plus case reserves, at each adjustment, while the LDD reimbursements are based on paid losses. Therefore, the insurer has more opportunity to earn investment income, and therefore also a bigger interest rate risk for the LDD than for the retro plan.)
<u>Comment</u>: The given loss development factors seem to be very big to me.
In part (a), I think we also need to assume no (specified) minimum premium or ignore the impact of the minimum premium, since otherwise the expected retro premium is not gotten by plugging in the average (limited) losses.
Page 430 of Teng: “The average loss and expense payout period for Excess WC is considerably later than for LDD. This is because most of the Excess WC premium covers the excess loss. which has a long average payout period (average to over 10 years). where as LDD premium is split roughly half in expense. which is paid out quickly, and half in excess loss. This implies a significant interest rate risk in Excess WC.”

14. (4 points) An insured has a large dollar deductible (LDD) policy. Total losses and ALAE limited to the deductible are distributed uniformly on the interval [0, 400,000], and total unlimited losses and ALAE are distributed uniformly on the interval [0, 800,000].

- The insured currently has an aggregate loss limit of $300,000.
- Credit risk is not contemplated in pricing.
- The deductible applies to both loss and ALAE.

The following expenses apply to this insured:

Expense Item	Value	Applies to
ULAE	7.5%	Loss & ALAE
Loss Based Assessments	5%	Loss & ALAE
Overhead	$45,000	Fixed
Acquisition	6%	Written Premium
Commission	12.5%	Written Premium
Premium Tax	4%	Written Premium
Profit and Contingency	-5%	Written Premium

a. (2 points) Calculate the LDD premium for this insured.

b. (2 points) It is later determined that, although the distribution of total unlimited losses and ALAE remains unchanged, the total losses and ALAE limited to the deductible actually follow the following distribution:

- 75% probability of loss and ALAE between $0 and $300,000
- 25% probability of loss and ALAE between $300,000 and $700,000
- Losses follow a uniform distribution within each range.

Use one or more Lee diagrams to demonstrate the impact to the premium for the LDD policy.

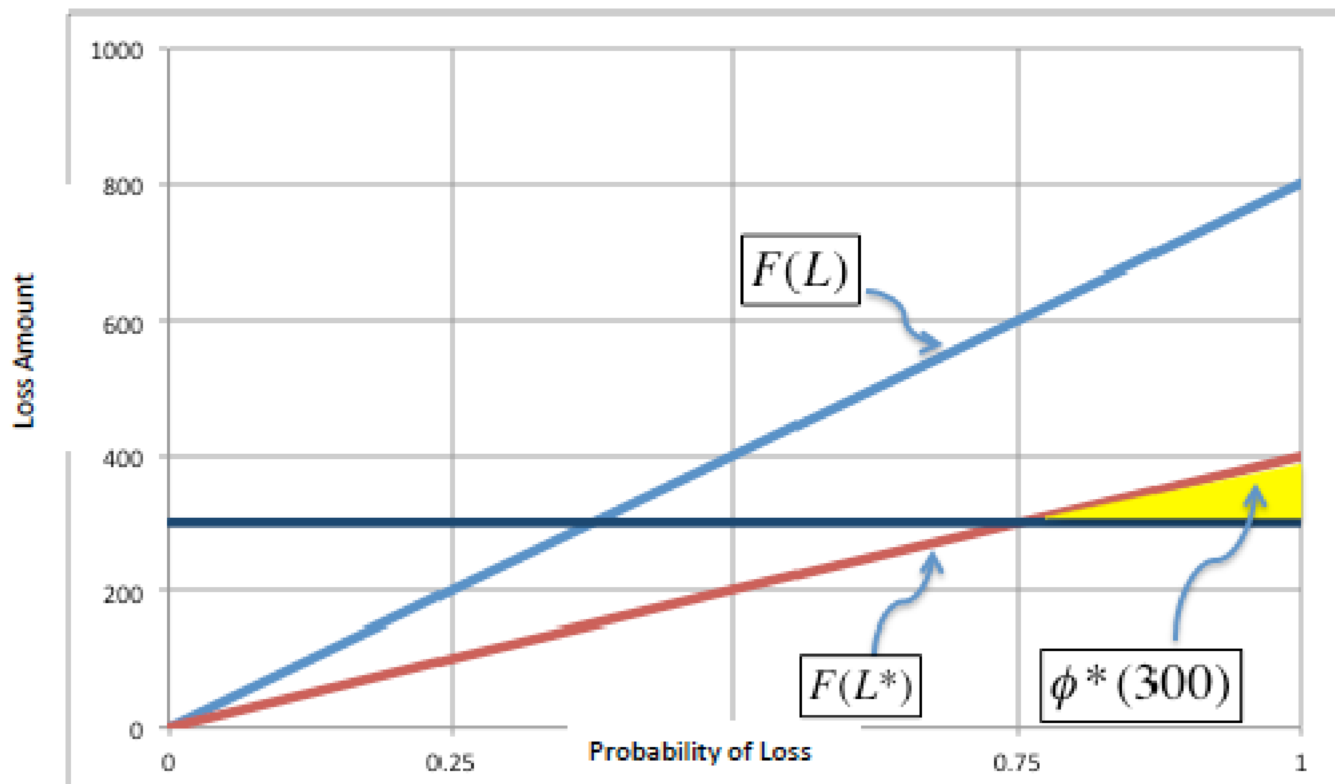
14. (a) The expected total loss and ALAE is: (0 + 800,000)/2 = 400,000.
ULAE + Loss Based Assessments = (5% + 7.5%)(400,000) = 50,000.
Taking into account the $300,000 aggregate limit, the reimbursements are uniform from 0 to 300,000 75% of the time, and $300,000 25% of the time.
Thus the average reimbursements are: (75%) (0 + 300,000) / 2 + (25%)(300,000) = 187,500.
Average losses paid by insurer net of reimbursements are: 400,000 - 187,500 = $212,500.

The LDD premium is: $\frac{212{,}500 + 50{,}000 + 45{,}000}{1 - (6\% + 12.5\% + 4\% - 5\%)}$ = **$372,727**.

Alternately, the expected total loss and ALAE is: (0 + 400,000)/2 = 200,000.
Thus the expected reimbursements without an aggregate limit are: 400,000 - 200,000 = $200,000.
The charge for the aggregate limit is:

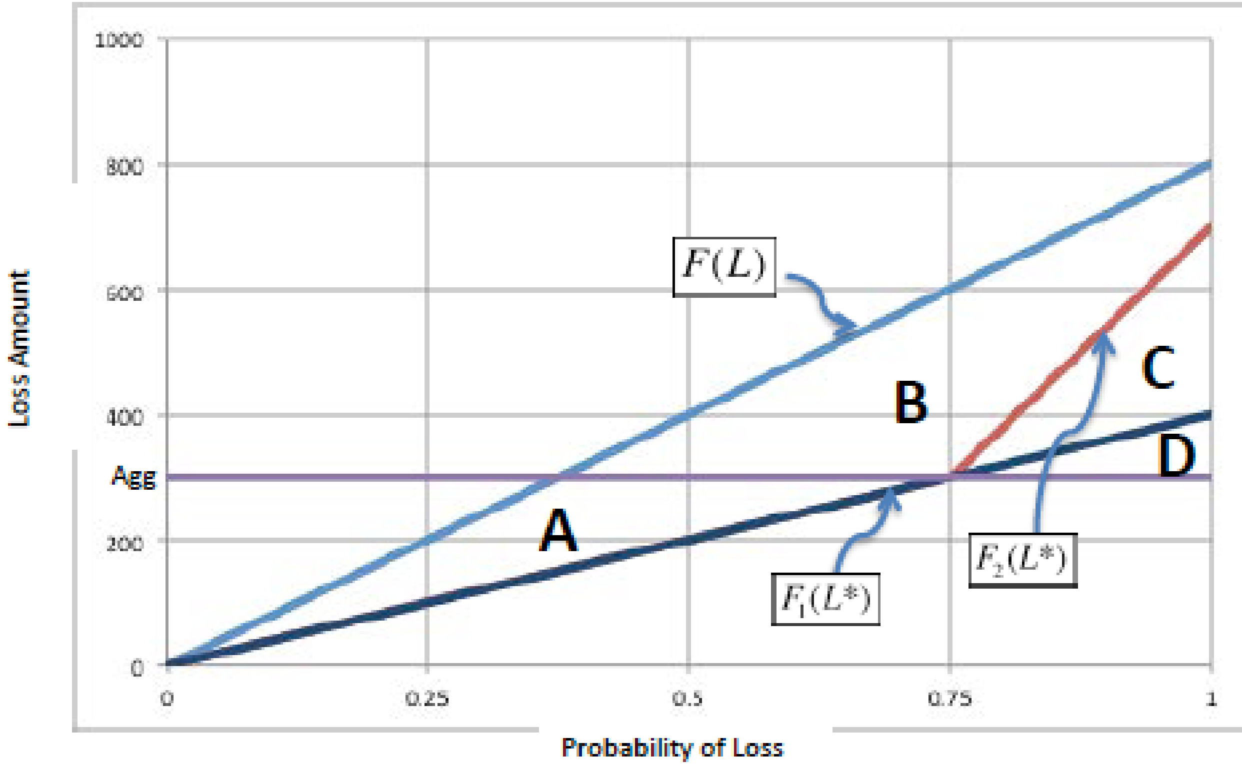
$$\int_{300{,}000}^{400{,}000} (1/400{,}000)\,(x - 300{,}000)\,dx = 100{,}000^2 / 800{,}000 = \$12{,}500.$$

Thus average losses paid by insurer net of reimbursements are: 200,000 + 12,500 = $212,500.
Proceed as before.
Alternately we can draw a Lee Diagram to get the insurance charge:



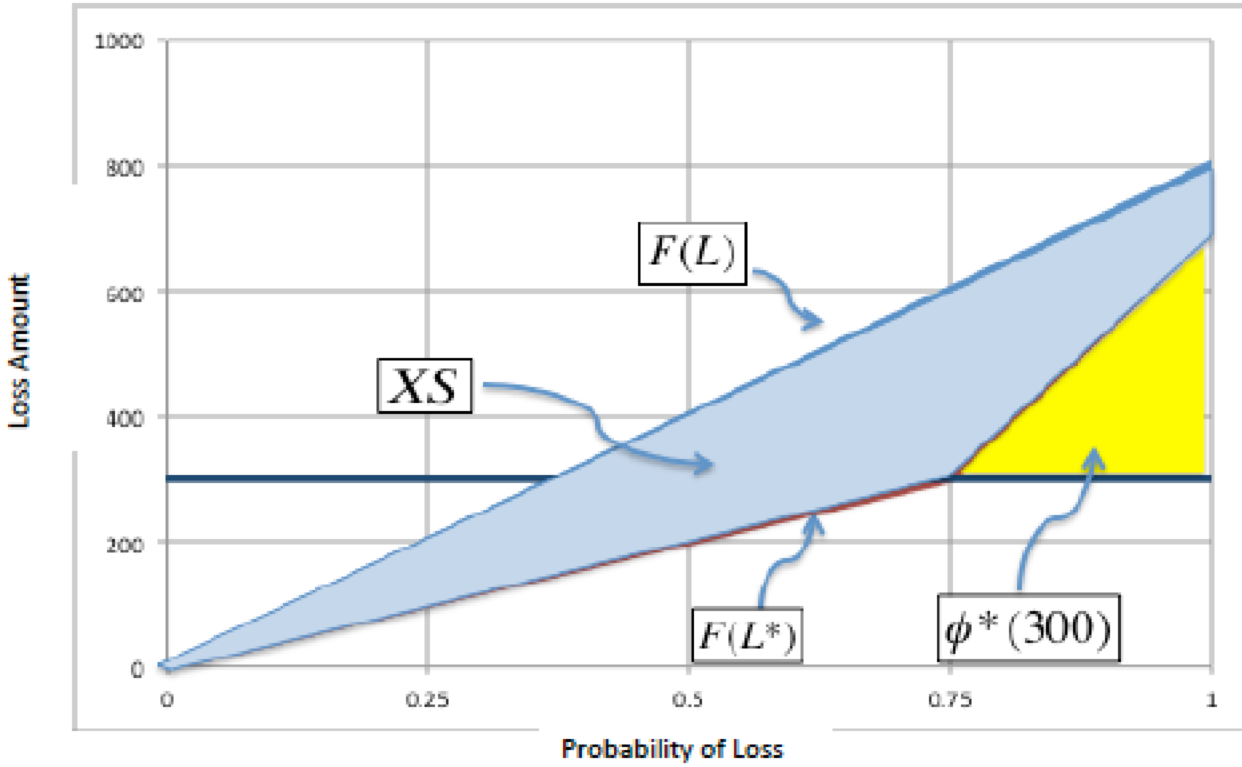
ϕ^*(300K) = (0.25)(100K)/2 = 12.5K.
Thus average losses paid by insurer net of reimbursements are: 200,000 + 12,500 = $212,500.
Proceed as before.

(b) A Lee Diagram, with $F_1(L^*)$ being the original distribution of limited losses, and $F_2(L^*)$ being the new distribution of limited losses:



Loss cost under old distribution = Excess Loss + Insurance Charge = (A+B+C) + D
Loss cost under new distribution = Excess Loss + Insurance Charge = (A+B) + (C+D).
Since loss costs are equal and expenses do not change, there is **no change** in LDD premium.

Alternately, from part (a) the expected loss cost is: $200,000 + $12,500 = $212,500.
A Lee Diagram for the new distribution of limited losses:



New insurance charge = (0.25)(700,000 - 300,000)/2 = 50,000.
Expected unlimited loss = 400,000.
New expected limited loss = (0.75)(300,000)/2 + (0.25)(300,000) + 50,000 = 237,500.
New area XS is: expected unlimited - expected limited
= 400,000 – 237,500 = $162,500.
New expected loss cost = expected excess + insurance charge
= 162,500 + 50,000 = $212,500.
Since loss costs are equal and expenses do not change, there is **no change** in LDD premium.

Comment: The Lee Diagram the CAS forced you to draw in part (b) is a big waste to time. Taking into account the $300,000 aggregate limit, the new reimbursements are uniform from 0 to 300,000 75% of the time, and $300,000 25% of the time, the same as before. Thus since loss costs are equal and expenses do not change, there is no change in LDD premium.

15. (2.5 points) An insured in a retrospectively-rated workers compensation plan currently pays a basic premium of $26,820. The following parameters apply to the insured's policy:

Standard Premium	$100,000
Expense Ratio (e)	$20,000
Expected Losses	$70,000
Tax Multiplier	1.00
Loss Conversion Factor	1.17
Entry Ratio @ G	1.00
Entry Ratio @ H	0.75

The insured believes that the insurance charge embedded in the current basic premium is unfair and cites the following unlimited loss ratios from five similarly-sized competitors doing business in the same industry:

Competitor	Loss Ratio
1	70.0%
2	105.0%
3	52.5%
4	87.5%
5	35.0%

a. (2 points) Compare the net insurance charge in the current basic premium for this policy to the net charge based on the provided competitor loss ratio experience.

b. (0.5 point) Discuss the appropriateness of using the basic premium derived from the competitor data to price this policy.

15. (a) The expenses in the basic are: 20,000 - (0.17)(70,000) = $8,100.
Thus the converted net insurance charge is: 26,820 - 8100 = $18,720.
Thus the net (unconverted) insurance charge is: 18,720/1.17 = $16,000.
(As a percent of expected losses this is: 16/70 = 22.86%.)
The maximum loss ratio is: (1)(70%) = 70%. The minimum loss ratio is: (0.75)(70%) = 52.5%.
Thus using the five competitors, the average loss ratio above the maximum is:
(35% + 17.5%) / 5 = 10.5%.
The average loss ratio below the minimum is: (52.5% - 35%)/5 = 17.5%/5 = 3.5%.
Net (unconverted) insurance charge is: (10.5% - 3.5%) ($100,000) = $7000.
(As a percent of expected losses this is: 7/70 = 10%.)
This $7000 is less than the $16,000 included in the retro plan.
(b) Loss ratios for individual insureds are subject to lots of random fluctuation. Thus five insured for one year each is too little data to be credible. Thus it would not be appropriate to use the basic premium derived from the competitor data to price this policy.
Any five insureds, even if we had many years of data, due to differences in certain risk characteristics might have distributions of annual aggregate losses around their expected value that differ from the average over the insurance industry for similar sized risks.
In any case, the past loss ratios depend on the rate adequacy in the past and the mix of business by state. Also to be meaningful, the data for the competitors would have to be at a later maturity such that there are no unreported claims, in which case the loss ratios would still depend on the adequacy of the case reserves.
Comment: From the CAS Exam Committee "Candidates needed to identify why either the expense component or net insurance charge imbedded within basic premium might vary between the insured and competitors." We are given no data on expenses for writing the other insureds, nor am I aware of the possibility of having data on the expense needs to write individual insureds (other than based on size of insured). So I have no idea why the CAS mentioned this and gave credit for "Not appropriate, basic premium includes expenses that could vary significantly from company to company." If they are referring to the fact that expenses vary from insurer to insurer, that is not relevant here. This insurer used the given $20,000 for expenses since it was thought to be appropriate for this sized insured and this insurer; nothing about the other insureds would change that.
The CAS also gave credit for "It may not be appropriate to use competitor data to price the policy due to differences in certain risk characteristics although the nature of business is the same. For example, there will be differences in operations, locations, safety programs, morale of employees which varies across companies. This will result in different loss distribution, and hence, produce different insurance charge." I think this misses the point to some extent. Table M is constructed using data from lots of different insureds, not just five, each with differences in certain risk characteristics. Thus the same argument could be made for not using the charge in the basic in the retro, which is presumably based on using Table M. Implicit in Table M is a typical distribution of aggregate losses for a risk of that standard premium. (The standard premium includes the impact of the experience rating plan which takes into account differences in risk characteristics.)

16. (2.5 points) An actuary prices a retrospectively rated policy with effective date January 1, 2015 using the NCCI Retrospective Rating Plan Manual. The following information applies:

Maximum premium	\$12,300,000
Expected loss	\$4,000,000
Expense provision (excluding taxes)	\$1,629,440
Loss conversion factor	1.2
Tax Multiplier	1.025
State Hazard Group Differential	0.98

The plan has no specified minimum premium and no per occurrence limit.

a. (2 points) Calculate the loss at maximum premium that balances the plan.

b. (0.5 point) Calculate the basic premium.

16. (a) When there is no specified minimum premium, in other words H = bT, $S_H = 0$:

$$S_G = \frac{G/T - e - E}{cE} = \frac{12.3/1.025 - 4 - 1.62944}{(1.2)(4)} = 1.3272.$$

LUGS = (0.98) (4M) = 3.92 million. $\Rightarrow$ ELG 26.

Looking in the correct column of Table M, we try entry ratios.

$\psi(2.26) = \phi(2.26) + 2.26 - 1 = 0.0577 + 1.26 = 1.3177.$

$\psi(2.27) = \phi(2.27) + 2.27 - 1 = 0.0572 + 1.27 = 1.3272.$ OK.

The corresponding loss at the maximum is: (2.27)(4 million) = **\$9.08 million**.

(b) Remembering that $S_H = 0$, the basic premium is:

$e - (c-1)E + cE\,X_G = 1.629440 - (0.2)(4) + (1.2)(4)\,(0.0572) =$ **\$1.104 million**.

<u>Comment</u>: See Equation 16 on page 36 in Part II of Gillam and Snader.

One can derive this relationship, by writing down the equation for the maximum premium:

$G = T\,(b + c\,E\,r_G) = T\{e - (c-1)E + cE(X_G - S_H) + c\,E\,r_G\}$.

However, $S_H = 0$ and $S_G = X_G + r_G - 1$.

Therefore, $G = T\{e + E - cE + c\,E\,X_G + c\,E\,r_G\} = T(e + E + c\,E\,S_G)$. $\Rightarrow$ $S_G = (G/T - e - E) / (cE)$.

17. (2 points) An actuary prices a retrospectively rated policy based on the assumption that aggregate losses follow a uniform distribution between $0 and $1,000,000.
The actuary determines that the following provisions result in a balanced plan:

Standard premium	$700,000
Loss at minimum premium	$80,000
Loss at maximum premium	$750,000
Basic premium	$83,660
Loss conversion factor	1.2

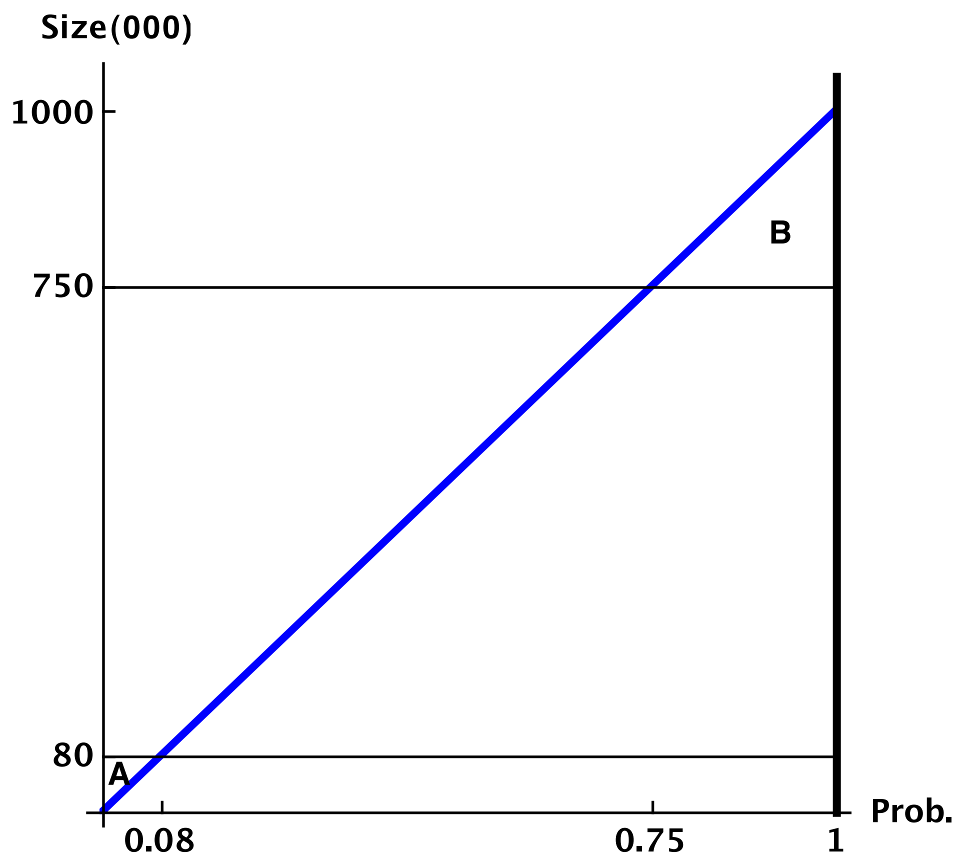
Assume there are no taxes.
a. (1.5 points) Calculate the implicit premium discount associated with the plan.
b. (0.5 point) Briefly describe how premium discount is treated in a retrospectively rated policy compared to a guaranteed cost policy.

17. (a) The expected losses are: (0 + 1 million)/2 = \$500,000.

Savings at the minimum: $\int_0^{80,000} (80,000 - x) / 1,000,000 \, dx = 80,000^2 / 2,000,000 = 3200.$

Charge at the maximum: $\int_{750,000}^{1,000,000} (x - 750,000) / 1,000,000 \, dx = 80,000^2 / 2,000,000 = 31,250.$

83,660 = b = e - (0.2)(500,000) + (1.2)(31,250 - 3200). ⇒ e = \$150,000.
Since there are no taxes, 700,000 = Standard Premium = 500,000 + (expense provision).
⇒ Expense provision (prior to premium discount) = \$200,000.
Premium Discount (for this sized insured) = 200,000 - 150,000 = **\$50,000**.
This premium discount is: 50/700 = **7.14%**.
Alternately, one can use the following Lee Diagram:



Savings at the minimum: A = (80,000)(0.08)/2 = 3200.
Charge at the maximum: B = (250,000)(0.25)/2 = 31,250. Proceed as before.
Alternately, if the distribution of aggregate losses is uniform from 0 to b, then at entry ratio r:
$\phi(r) = (2-r)^2 / 4, 0 \le r \le 2$, and $\psi(r) = r^2 / 4, 0 \le r \le 2$.

E = 500K. $\psi(80/500) = 0.16^2/4 = 0.0064$. E $\psi(0.16)$ = (500,000)(0.0064) = 3200.

$\phi(750/500) = (2-1.5)^2/4 = 0.0625$. E $\phi(1.5)$ = (500,000)(0.0625) = 31,250. Proceed as before.

(b) In a guaranteed cost policy, the insured receives the premium discount corresponding to its standard premium; the discount is subtracted from standard premium to get the net premium charged to the insured. A similarly sized insured who is retrospective rated has the same premium discount included in the retro plan; the expense allowance e, and thus the basic premium b, reflect the reduction in expenses underlying the premium discount given to guaranteed cost policies.

18. (2 points) An actuary is given the following claims experience for large dollar deductible workers compensation insurance for five identically sized risks. Each claim is a separate occurrence.

	Individual Claims Experience (Gross of Deductible)			
Risk #	Claim 1	Claim 2	Claim 3	Claim 4
1	70,000	80,000		
2	165,000	300,000	250,000	
3	150,000			
4	150,000	250,000	200,000	150,000
5	250,000			

Calculate the expected loss cost for a risk identical to the five above with
a per-occurrence deductible of $150,000 and an annual aggregate deductible of $450,000.

18. Insurer pays 0 for risk 1.
Insured pays: 150K + 150K + 150K = 450K for risk 2.
Insurer pays for risk 2: 165K + 300K + 250K - 450K = 265K.
Insurer pays 0 for risk 3.
Insured pays: 150K + 150K + 150K + 150K = 600K, limited to 450K for risk 4.
Insurer pays for risk 4: 150K + 250K + 200K + 150K - 450K = 300K.
Insurer pays for risk 5: 250K - 150K = 100K.
Loss cost is: (0 + 265K + 0 + 300K + 100K) / 5 = **$133,000**.
Comment: An insured needs to have at least 4 large occurrences in order to exceed the annual aggregate limit.

19. (1.5 points) An actuary has priced a large dollar deductible policy using the information given below and has indicated a premium of $273,500.
Assume no collateral is held on behalf of the insured.

Standard Premium	1,000,000
Ground-up Expected Loss and ALAE Ratio	65%
Excess Ratio (% of Total Loss and ALAE)	10%
ULAE Load (% of Total Loss and ALAE)	8%
General Expenses (% of Standard Premium)	5%
Credit Risk Load (% of Standard Premium)	X%
Acquisition Expenses (% of Net Premium)	5%
Taxes (% of Net Premium)	7%
Profit and Contingencies (% of Net Premium)	5%

a. (1 point) Calculate X, the implied Credit Risk Load in the actuary's indicated premium.
b. (0.5 points) Explain why profit loads can be higher for large dollar deductible policies compared to excess policies.

19. (a) Expected Excess Loss & ALAE: (10%)(65%)(1,000,000) = $65,000.
ULAE: (8%)(65%)(1,000,000) = $52,000.
General Expense: (5%)(1,000,000) = $50,000.

$$273{,}500 = \frac{65{,}000 + 52{,}000 + 50{,}000 + X\,(1 \text{ million})}{1 - 5\% - 7\% - 5\%}. \Rightarrow X = \mathbf{6.00\%}.$$

(b) The market for excess policies is even more competitive than that for LDD, since excess policies are only selling risk transfer, while a LDD also competes on the quality of its services such as claims handling and loss control.
Therefore, profit loads for excess policies tend to be lower than those for LDD.
In addition, since expected losses are a larger percentage of premium for excess policies than for LDD, there is a longer average payout period on excess policies. Therefore, there is more opportunity to earn investment income on excess policies than LDD, allowing for a lower profit load for excess policies than LDD.

20. (2.5 points) An actuary is using the following exposure curve to rate a non-proportional reinsurance treaty:

$$G(x) = \frac{1 - b^x}{1 - b}.$$

The actuary is also given the following information:

Maximum Possible Loss	\$5,000,000
Insured Value	\$5,000,000
Gross Premium	\$6,000
Expected Loss Ratio	60%
Retention of non-proportional reinsurance treaty	\$150,000
Expected Ceded Risk Premium	\$2,705

a. (0.5 point) Briefly describe a method to allocate gross premium for the non-proportional reinsurance treaty between the ceding company and the reinsurer.

b. (0.5 point) Given the probability of a total loss is 0.03, calculate the parameter b in the formula above.

c. (1.5 points)
Given the answer in part b. above, calculate the limit of the non-proportional reinsurance treaty.

20. (a) First estimate the total expected pure premium under the underlying business.
(In this case that is: (60%)(6000) = 3600.)
Then apportion the pure premium between the reinsurer and ceding company by using exposure curves.
The expected percent ceded will be: G(A + L) - G(L), where A is the retention and L is the limit.

(b) $G(x) = \frac{1 - b^x}{1 - b}$. $G'(x) = -\ln b\, b^x / (1-b)$.

$G'(0) = -\ln b / (1-b)$. $G'(1) = -\ln b\, b / (1-b)$.
0.03 = Probability of a total loss = $G'(1)/G'(0) = b$. ⇒ b = **0.03**.
Alternately, as shown in equation 3.3 in Bernegger, this is a special case of MBBEFD with bg = 1.
The probability of a total loss is 0.03. ⇒ g = 1/0.03. ⇒ b = **0.03**.
(c) The ceded premium divided by the total expected losses is: 2705 / 3600 = 75.14%.
Let the limit be L, then the ceded layer is from 150K to L + 150K.
Thus we have: 75.14% = G({L + 150K}/5000K) - G(150K/5000K)

$$= \frac{1 - 0.03^{(L + 150K)/5000K}}{1 - 0.03} - \frac{1 - 0.03^{(150K/5000K)}}{1 - 0.03} = \frac{1 - 0.03^{(L + 150K)/5000K}}{0.97} - 10.29\%.$$

$$\Rightarrow 85.43\% = \frac{1 - 0.03^{(L + 150K)/5000K}}{0.97}. \Rightarrow 0.03^{(L+150K)/5000K} = 0.1713.$$

⇒ (L + 150K)/5000K = ln(0.1713) / ln(0.03) = 0.5032. ⇒ L = **\$2.366 million**.

Comment: In part (a), "allocate gross premium for the non-proportional reinsurance treaty between the ceding company and the reinsurer" is language that is not used in the syllabus readings. Personally, it took me a while to figure out what the questioner was getting at.

21. (3.75 points)
A reinsurer is offering a ceding company a two-year aggregate stop loss with the following terms:
- The treaty is effective on 1/1/2016 and expires on 12/31/2017.
- The treaty will cover aggregate losses between a 65% and a 70% loss ratio for accident years 2016 and 2017 separately.
- Premium is paid at the beginning of each year and all losses are paid at the end of the year incurred.

Additionally, the reinsurance treaty has the following termination provisions:
- The ceding company has the option to terminate the contract at the end of 2016 and receive a profit commission from the reinsurer calculated as follows:
  Profit commission = [35% * 2016 Ceded Premium] - [2016 Ceded Loss],
  subject to a minimum of 0.
- If the contract is not terminated at the end of 2016, the ceding company will again have the option to terminate the contract at term expiration and receive a profit commission from the reinsurer calculated as follows:
  Profit commission = [35% * Full Term Ceded Premium] - [Full Term Ceded Loss],
  subject to a minimum of 0.
- The reinsurer cannot terminate the contract at any time.

The reinsurer has simulated five trials of the ceding company's loss ratio subject to the aggregate stop loss as follows (values in millions):

	Accident Year 2016		Accident Year 2017	
Simulation	Loss Ratio	Loss Amount	Loss Ratio	Loss Amount
1	73.9%	517	68.0%	510
2	64.8%	454	67.9%	509
3	66.8%	468	65.5%	491
4	65.6%	459	65.3%	490
5	65.1%	456	58.7%	440
Mean	67.2%	471	65.1%	488

	2016	2017
Subject Premium	700	750
Ceded Premium	9.8	10.5

For each simulation, assume that the ceding company will only terminate the contract if the profit commission payable is greater than zero.
a. (1 point) Calculate the expected profit commission payable at the end of 2016.
b. (2.75 points) Calculate the expected profit commission for the full term of the contract.

21.

Simulation	AY 2016 Loss Ratio	AY 2016 Ceded Loss	AY 2017 Loss Ratio	AY 2017 Ceded Loss
1	73.9%	(5%)(700) = 35	68.0%	(3%)(750) = 22.5
2	64.8%	0	67.9%	(2.9%)(750) = 21.75
3	66.8%	(1.8%)(700) = 12.6	65.5%	(0.5%)(750) = 3.75
4	65.6%	(0.6%)(700) = 4.2	65.3%	(0.3%)(750) = 2.25
5	65.1%	(0.1%)(700) = 0.7	58.7%	0

(a) For 2016, 35% of the ceded premium is: (35%)(9.8) = 3.43.
Thus there is a profit commission if there is a ceded loss of less than 3.43.
Thus only in simulations #2 and #5 does the insurer terminate the contract.
The profit commission for simulation #2 is: 3.43M - 0 = 3.43M.
The profit commission for simulation #5 is: 3.43M - 0.7 = 2.73M.
Expected profit commission payable at the end of 2016 is: (3.43 + 2.73)/5 = **1,232,000**.
(b) In simulation #1 the profit commission is: (35%)(9.8 + 10.5) - (35 + 22.5) = -50.395. ⇒ 0.
In simulation #3 the profit commission is: (35%)(9.8 + 10.5) - (12.6 + 3.75) = -9.245. ⇒ 0.
In simulation #4 the profit commission is: (35%)(9.8 + 10.5) - (4.2 + 2.25) = 0.655.
Expected profit commission for the full term of the contract is: (3.43 + 2.73 + 0.655)/5 = **1,363,000**.
Comment: I do not understand their division of points between parts (a) and (b).

22. (2.75 points) An insurance company is exposed to three independent catastrophic risks in three different regions in a given year. More than one event can occur in a year but each event can only occur once in a year. Events have the following size and probability:

Event	Loss Amount	Annual Probability of Occurrence
1	$10,000,000	0.10
2	$15,000,000	0.05
3	$35,000,000	0.02

a. (2.25 points) Calculate the Aggregate Exceedance Probabilities associated with the insurance company's exposure.
b. (0.5 points) Using a randomly generated number of 0.86, simulate the insured total loss.

22. (a) Probability of no loss: (0.9)(0.95)(0.98) = 83.79%.
Probability of 10,000,000 aggregate: (0.1)(0.95)(0.98) = 9.31%.
Probability of 15,000,000 aggregate: (0.9)(0.05)(0.98) = 4.41%
Probability of 35,000,000 aggregate: (0.9)(0.95)(0.02) = 1.71%
Probability of 10M + 15M = 25M aggregate: (0.1)(0.05)(0.98) = 0.49%
Probability of 10M + 35M = 45M aggregate: (0.1)(0.95)(0.02) = 0.19%
Probability of 15M + 35M = 50M aggregate: (0.9)(0.05)(0.02) = 0.09%
Probability of 10M + 15M + 35M = 60M aggregate: (0.1)(0.05)(0.02) = 0.01%

Probability[Aggregate > 60M] = 0.
Probability[Aggregate > 50M] = 0.01%.
Probability[Aggregate > 45M] = 0.01% + 0.09% = 0.10%.
Probability[Aggregate > 35M] = 0.10% + 0.19% = 0.29%.
Probability[Aggregate > 25M] = 0.29% + 1.71% = 2.00%.
Probability[Aggregate > 15M] = 2.00% + 0.49% = 2.49%.
Probability[Aggregate > 10M] = 2.49% + 4.41% = 6.90%.
Probability[Aggregate > 0M] = 6.90% + 9.31% = 16.21%.

(b) FInd where the discrete distribution of aggregate losses first exceeds 0.86.
The distribution function at 0 is 83.79%.
The distribution function at 10,000,000 is: 83.79% + 9.31% = 93.10%.
Thus the simulated aggregate loss is **$10,000,000**.
Alternately, 1 - 0.86 = 0.14 lies between 6.90% and 16.21% on the aggregate exceedance probabilities, so we simulate **$10 million** in aggregate loss.
Comment: For part (b), see Table 4.1 in Grossi and Kunreuther.
I have used the Exam 4 technique of simulating from a discrete distribution, which differs in minor detail from what Grossi and Kunreuther did.

23. (2.5 points) An actuary is pricing the $9,000,000 excess of $1,000,000 layer for an excess of loss policy. Total insured value of the properties is $10,000,000.
Historical information for this policy is as follows:

Accident Year	Excess Loss Development Factor	On-Level Trended Premium
2012	1.01	$5,200,000
2013	1.05	$5,700,000
2014	1.10	$5,900,000

Accident Date	Actual Ground- Up Loss	Trend Factor	Trended Losses
July 18, 2012	$3,500,000	1.10	$3,850,000
February 12, 2013	$2,000,000	1.08	$2,160,000
August 15, 2013	$1,000,000	1.07	$1,070,000
March 3, 2014	$3,000,000	1.04	$3,120,000
November 2, 2014	$920,000	1.01	$929,200

The exposure curve below applies to the insured risk:

% of Insured Value	Exposure Factor
10%	30%
20%	41%
30%	52%
40%	63%
50%	71%
60%	75%
70%	80%
80%	84%
90%	89%
100%	90%
120%	100%

Calculate the policy's loss cost as a percentage of premium.

23. We determine the trended losses in the reinsured layer:

<u>Accident Date</u>	<u>Trended Losses</u>	<u>Loss in Layer</u>
July 18, 2012	$3,850,000	2,850,000
February 12, 2013	$2,160,000	1,160,000
August 15, 2013	$1,070,000	70,000
March 3, 2014	$3,120,000	2,120,000
November 2, 2014	$929,200	0

Apply the Excess Loss Development Factor to each accident year:
(2,850,000)(1.01) = 2,878,500.
(1,160,000 + 70,000)(1.05) = 1,291,500.
(2,120,000 + 0)(1.10) = 2,332,000.

$$\text{Experience Rate} = \frac{2,878,500 + 1,291,500 + 2,332,000}{5,200,000 + 5,700,000 + 5,900,000} = 38.7\%.$$

However, there are no trended losses in the layer from 4M to 10M.
Therefore, we would be providing a <u>free cover</u>; to avoid that we use the exposure curve.
The exposure rate for the layer from 1M to 4M is:
(63% - 30%) (expected loss ratio) = (33%) (expected loss ratio).
The exposure rate for the layer from 4M to 10M is:
(90% - 63%) (expected loss ratio) = (27%) (expected loss ratio).
Estimate rate for 4M to 10M layer: (38.7%) (27%/33%) = 31.7%.
Thus the policy's loss cost as a percentage of premium is: 38.7% + 31.7% = **70.4%**.
<u>Comment</u>: “Candidates who selected prorated layers other than 3M xs 1M and 6M xs 4M were also given credit as long as selection was reasonable and/or justified.”