Solutions to the Fall 2014 CAS Exam 8

(Incorporating what I found useful in the CAS Examiner's Report)

The solutions and comments are solely the responsibility of the author.
While some of the comments may seem critical of certain questions, this is intended solely to aid you in studying and in no way is intended as a criticism of the many volunteers who work extremely long and hard to produce quality exams.

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1. (1.25 points) An actuary has devised a new method to assign credibility to observations of severity relativities by state. In order to test the validity of the method, the following quintile test has been prepared. The actuary has split the data into two distinct partitions: Test and Holdout.
Test data was used to predict the credibility-adjusted relativities of the holdout data.

Quintile	Holdout Relativity	Prediction Based on Countrywide Average	Prediction Based on Raw Test Data	Prediction Based on New Credibility Procedure
1	0.55	1.00	0.25	0.90
2	0.75	1.00	0.40	0.95
3	0.90	1.00	0.85	0.99
4	1.30	1.00	1.40	1.05
5	1.50	1.00	2.10	1.10
Mean	1.00	1.00	1.00	1.00
Sum of Squared Errors		0.6150	0.5850	0.3931

a. (0.75 point) Describe whether this new method overstates or understates the credibility of the state relativities.
b. (0.5 point) Discuss whether the new method or the countrywide average should be used to determine state relativities.

1. (a) The holdout relativities are further from unity than the predications based on the new credibility procedure. Thus the new method **understates** the credibility of the state relativities.
For example, for Quintile #1, we can infer that the state relativity was given credibility of about:
(1 - 0.9) / (1 - 0.25) = 0.13.
However, based on the holdout relativity, it would have been better to give the state relativity a credibility of about: (1 - 0.55) / (1 - 0.25) = 0.60.
Similarly, for Quintile #5, we can infer that the state relativity was given credibility of about:
(1.10 - 1) / (2.10 -1) = 0.09.
However, based on the holdout relativity, it would have been better to give the state relativity a credibility of about: (1.50 - 1) / (2.10 - 1) = 0.45.

Quintile	Prediction Based on Credibility Procedure / Holdout
1	1.636 = 0.90/0.55
2	1.267 = 0.95/0.75
3	1.100 = 0.99/0.90
4	0.808 = 1.05/1.30
5	0.733 = 1.10/1.50

Ideally the above ratios should all be the same, but there is a decreasing trend.
Thus the procedure is not responsive enough to the data, and the credibility is **understated**.
(b) The new method has a lower squared error and thus should be used rather than the countrywide average. (Based on part (a), we could improve the credibility method, but it is still better than totally ignoring the state data.)

2. (1 point) In their 2007 review of hazard group mappings, the National Council of Compensation Insurance (NCCI) chose to continue to use the following formula to calculate the credibility by class:

$$z = \min\left(\frac{n}{n+k} \times 1.5,\ 1\right)$$

where
n = number of claims in the class
k = average number of claims per class

The following table shows the distribution of classes by credibility range using the credibility formula above:

	Credibility Range	Claims per Year	Number of Classes	Percent of Premium
	0% ≤ z < 10%	0 - 237	355	1.2%
	10% ≤ z < 20%	238 - 511	89	1.3%
	20% ≤ z < 30%	512 - 831	61	1.6%
	30% ≤ z < 40%	832 - 1209	56	2.7%
	40% ≤ z < 50%	1210 - 1662	46	2.5%
	50% ≤ z < 60%	1663 - 2216	34	2.5%
	60% ≤ z< 70%	2217 - 2909	46	4.8%
	70% ≤ z < 80%	2910 - 3799	35	4.3%
	80% ≤ z < 90%	3800 - 4987	29	4.0%
	90% ≤ z < 100%	4988 - 6649	18	3.2%
	z = 100%	≥ 6650	101	71.8%
Total			870	100.0%

a. (0.5 point) Based on the table above, discuss one consideration when deciding whether to use this credibility formula.
b. (0.5 point)
Briefly describe two alternative methods that could be used to calculate credibility by class.

2. (a) The distribution of the number of claims per class is highly skewed. The large classes, representing 72% of the total premium, will have their own data on excess ratios be used for clustering. The smaller classes, representing 88% of the total classes, will have their own data weighted with that for their current hazard group for use in the clustering process; relying on their own data would lead to too much random fluctuation.
A class must have at least 6650 claims in order to be given full credibility.

(b) 1) $z = \min(\frac{n}{n+k} \times 1.5, 1)$, where n = number of claims in the class, and

k = median number of claims per class

2) $z = \min(\frac{n}{n+k} \times 1.5, 1)$, where n = number of lost time claims in the class, and

k = average number of lost time claims per class.

3) $z = \min(\frac{n}{n+k} \times 1.5, 1)$, where n = number of serious claims in the class, and

k = average number of serious claims per class.
Serious claims include: Fatal, Permanent Total, and Major Permanent Partial.

4) $z = \min(\frac{n}{n+k} \times 1.5, 1)$, where n = number of claims in the class, and

k is the average number of claims based on only those classes with some minimal number of claims.

5) $Z = \sqrt{\frac{n}{384}}$, where n = number of claims in the class.

6) $Z = \frac{N_F \sqrt{\frac{n_F}{384}} + N_M \sqrt{\frac{n_M}{384}} + N_m \sqrt{\frac{n_m}{384}}}{N_F + N_M + N_m}$, A weighted average of separate credibilities.

Where n_F = number of fatal claims for the class, N_F = number of fatal claims for all classes,
n_M = number of P.T. and Major P.P. claims for the class,
N_M = number of P.T. and Major P.P. claims for all classes,
n_m = number of T.T. and Minor P.P. claims for the class,
N_m = number of T.T. and Minor P.P. claims for all classes.

7) $Z = \frac{N_s \sqrt{\frac{n_s}{175}} + (N - N_s) \sqrt{\frac{n - n_s}{384}}}{N}$.

Where n_s = number of serious claims for the class, N_s = number of serious claims for all classes,
n = the total number of claims in the class, N = the total number of claims in all classes.
Comments: In part (b) only give two possibilities. Tests the details of a choice that made very little practical difference; however, Robertson spends some time discussing it in his paper in Section 2.1.

3. (2 points) The random component of a generalized linear model must come from the exponential family of distributions. The variance of a distribution from the exponential family can be expressed using the following formula: $\text{Var}(Y_i) = \frac{\phi\, V(\mu_i)}{\omega_i}$

a. (0.5 point) Define the parameters ϕ and ω_i in the formula above.

b. (1 point) For each of the data sets below, identify the error distribution that should be used to model the data. Briefly explain why that error distribution is appropriate.

i. Severity

ii, Policy Renewal Retention

c. (0.5 point) For each of the error distributions in part b. above, provide an example of how ω_i should be assigned for the type of data being modeled.

3. (a) ϕ is the scale or dispersion parameter, which scales the variance.

ω_i is a (prior) weight, representing the amount of data we have for observation i; the variance is inversely proportional to the volume of data.

(b) i. Gamma Distribution is most commonly used to model the error structure for severity; it works well in many situations based on diagnostics.

The Gamma is continuous with support from zero to infinity.

The gamma distribution also has an intuitively attractive property for modeling claim amounts since it is invariant to measures of currency. In other words measuring severities in dollars and measuring severities in cents will yield the same results using a gamma multiplicative GLM.

(This is not true of some other distributions such as Poisson, but would be for the Inverse Gaussian.)

For the Gamma: $V(\mu_i) = \mu_i^2$.

ii. For policy renewal a Bernoulli or Binomial is used, since policy renewal is a yes/no process.

For the Bernoulli: $V(\mu_i) = \mu_i\,(1 - \mu_i)$.

For the Binomial representing m trials (m policies): $V(\mu_i) = \mu_i\,(1 - \mu_i) / m$.

(c) 1. For severity, ω_i would be the number of claims, the measure of how much data we have.

2. For policy renewal, if using the Bernoulli, ω_i would be the number of policies.

If using the Binomial, $\omega_i = 1$.

<u>Comment</u>: See pages 14, 17, and 23 of Anderson, et. al.

4. (1.5 points) One approach for estimating excess ratios by individual class in workers compensation insurance is to use a multi-dimensional credibility technique.
According to each of the three statistical considerations listed below, explain whether this technique is an improvement over estimating excess ratios by hazard group:
i. Homogeneity
ii. Credibility
iii. Predictive Stability

4. i. Homogeneity: This has nothing to do with homogeneity, since we are not grouping insureds into classes or classes into hazard groups. In using the hazard groups, we assume that they are sufficiently homogeneous, in other words that the classes within them are sufficiently similar with respect to expected excess ratios.
In general, homogeneity is the desire to have the insureds grouped into a class to be similar to each other in expected costs. In the case of hazard groups, we desire the expected excess ratios to be similar for the classes grouped together. On the other hand, we want each hazard group to be big enough so that its data will be credible for estimating excess ratios.
From the CAS Examiner's Report: "Yes, the multi-dimensional credibility technique for individual classes could be viewed as an improvement over estimating excess ratios by hazard group from a homogeneity standpoint, because at a hazard group level there could be greater variance within the hazard group, whereas we would expect lower within variance at the class level and thus greater homogeneity. Hazard groups are made up of classes, so the classes themselves will be more homogenous than combined classes."
ii. Credibility: Since the technique presumably calculates the appropriate credibility to apply, it satisfies this criteria.
From the CAS Examiner's Report: "Each hazard group or class needs to be large enough to allow credible statistical predictions. Since hazard groups contain multiple classes, they are larger than the individual classes and their excess ratios will be more credible. Therefore, the class excess ratio technique is not an improvement over the hazard group excess ratios for credibility."
iii. Predictive Stability: Using credibility, the predictions should be more stable than relying solely on the data for each class, but less stable than relying on the excess ratio for its hazard group.
The predictions will be responsive to changes in the nature of insurance losses, yet stable in avoiding unwarranted abrupt changes in resulting prices.
Predictive stability is improved by incorporating information on the frequency of the more common minor injury types in order to estimate the expected frequency of the less frequent serious injuries.
The predictive stability will be improved because excess ratios will no longer be largely dependent on the observed number of the less frequent serious claims; the number of serious claims for a class is subject to a great deal of random fluctuation.
Comment: This question inappropriately attempts to apply criteria for establishing classes from the AAA reading on Risk Classification, in other words variables to be used to establish classes, to estimating excess ratios for an already existing set of classes and hazard groups. The Couret and Venter paper has nothing to do with either creating classes or grouping them into hazard groups.

5. (2.5 points) The following data shows the experience of a merit rating plan for a specific state.

Number of Accident-Free Years	Earned Car Years	Earned Premium ($000)	Number of Incurred Claims
3 or More	250,000	250,000	1,200
2	300,000	100,000	625
1	25,000	100,000	750
0	12,000	150,000	1,500
Total	587,000	600,000	4,075

The base rate is $1,000 per exposure. No other rating variables are applicable.

a. (0.5 point) The typical exposure base used to develop the merit rating plan is earned premium. Briefly discuss two assumptions in selecting this exposure base.

b. (1.5 points) Calculate the ratio of credibility for an exposure with two or more years accident-free experience to one or more years accident-free experience.

c. (0.5 point) Calculate the premium for an exposure that is accident free for two or more years.

5. (a) I will assume we are analyzing data separately for each class, as per Bailey and Simon.
(Here there do not seem to be any classes; "No other rating variables are applicable.")
Also I will assume as per Bailey and Simon that the premiums have been put on the current "Class B" rate level, in other words on the Merit Rating level of those with no years claims free; we need to remove the current impact of the Merit Rating Plan.
We assume that the current territory relativities are correct, and that differences in territory relativities are due to differences in expected frequency (per caryear) rather than expected severity.
According to the review by Hazam: "a premium base eliminates maldistribution only if (1) high frequency territories are also high premium territories and (2) if territorial differentials are proper."
(b) Overall, frequency with respect to premium ($ million) is: 4075/600 = 6.792.
For two or more years claims free, frequency with respect to premium ($ million) is:
(1200 + 625) / (250 + 100) = 5.214.
Thus for two or more years claims free, Z = 1 - 5.214/6.792 = 23.2%.
For one or more years claims free, frequency with respect to premium ($ million) is:
(1200 + 625 + 750) / (250 + 100 + 100) = 5.722.
Thus for one or more years claims free, Z = 1 - 5.722/6.792 = 15.8%.
The ratio of these two credibilities is: 23.2% / 15.8% = **1.47**.
(c) Assume that the base rate is to be applied to an exposure which has zero years claim free.
For exposures who are zero years claims free, frequency with respect to premium ($ million) is:
1500/150 = 10.
Thus we should charge an exposure that is accident free for two or more years:
(1000)(5.214/10) = **$521**.
Alternately, compared to average, we should give an exposure that is accident free for two or more years a discount of 23.2%.
Compared to average those with zero years claims free they should get a surcharge of:
10/6.792 - 1 = 47.2%.
Thus we should charge an exposure that is accident free for two or more years:
(1000/1.472)(1 - 23.2%) = $522.
Alternately, assuming that the base rate is the average rate, then we should charge an exposure that is accident free for two or more years: (1000) (1 - 23.2%) = **$768**.
Comment: In part (c), the examiners seem unaware that the base rate is for Merit Rating Class B, those who are zero years claims free. Rather they seem to assume that the base rate is the average rate, which is not how it is done in the real world. Bailey and Simon put all of their premiums on a Class B level; in other words they treat Merit Rating Class B as the base class.
In any case, the calculated mods are with respect to average.
The credibilities determined are unrealistically big.
The given data is very unusual and unrealistic, including the average premiums:

Number of Accident-Free Years	Earned Car Years	Earned Premium ($000)	Average Premiums
3 or More	250,000	250,000	$1000
2	300,000	100,000	$333
1	25,000	100,000	$4000
0	12,000	150,000	$12,500
Total	587,000	600,000	$1022

6. (3.5 points) Losses on a policy have the following distribution:

- 60% probability of a loss between $0 and $250,000
- 30% probability of a loss between $250,000 and $500,000
- 10% probability of a loss between $500,000 and $1 million

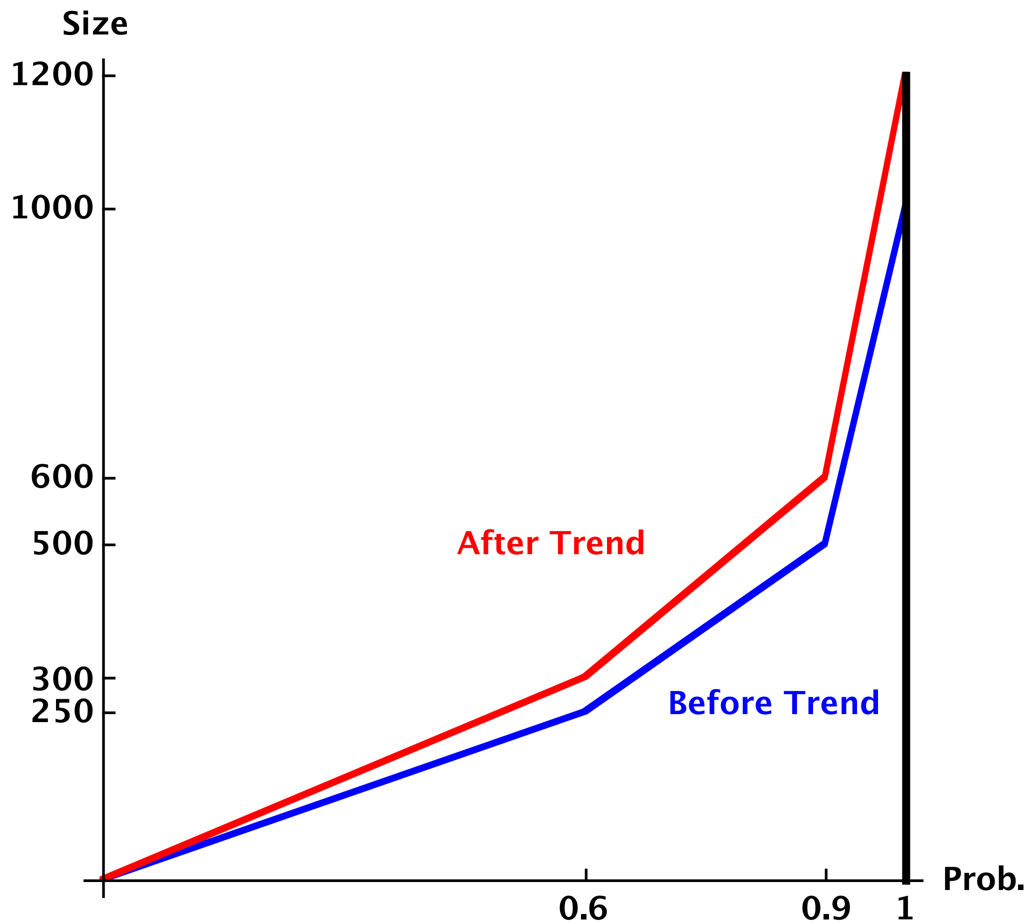
Losses are uniformly distributed within each range.

Assume a 20% trend is applied uniformly to all losses.

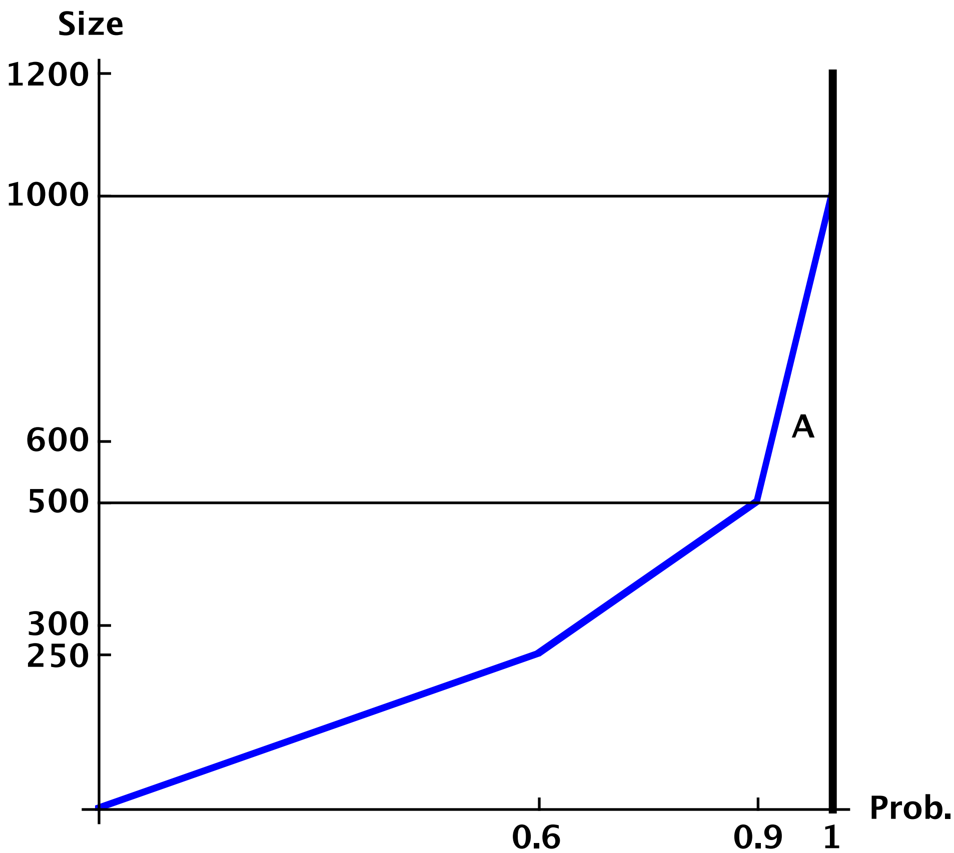
a. (1.5 points) Draw a diagram depicting the cumulative loss distribution described above before and after the 20% trend. Label all relevant features of the diagram.

b. (2 points) Calculate the implied trend for the layer $500,000 excess of $500,000.

6. (a) Subsequent to trend, we have three uniform distributions:
from 0 to 300,000; from 300,000 to 600,000; and from 600,000 to 1,200,000.
Here is a Lee Diagram, with size in thousands:

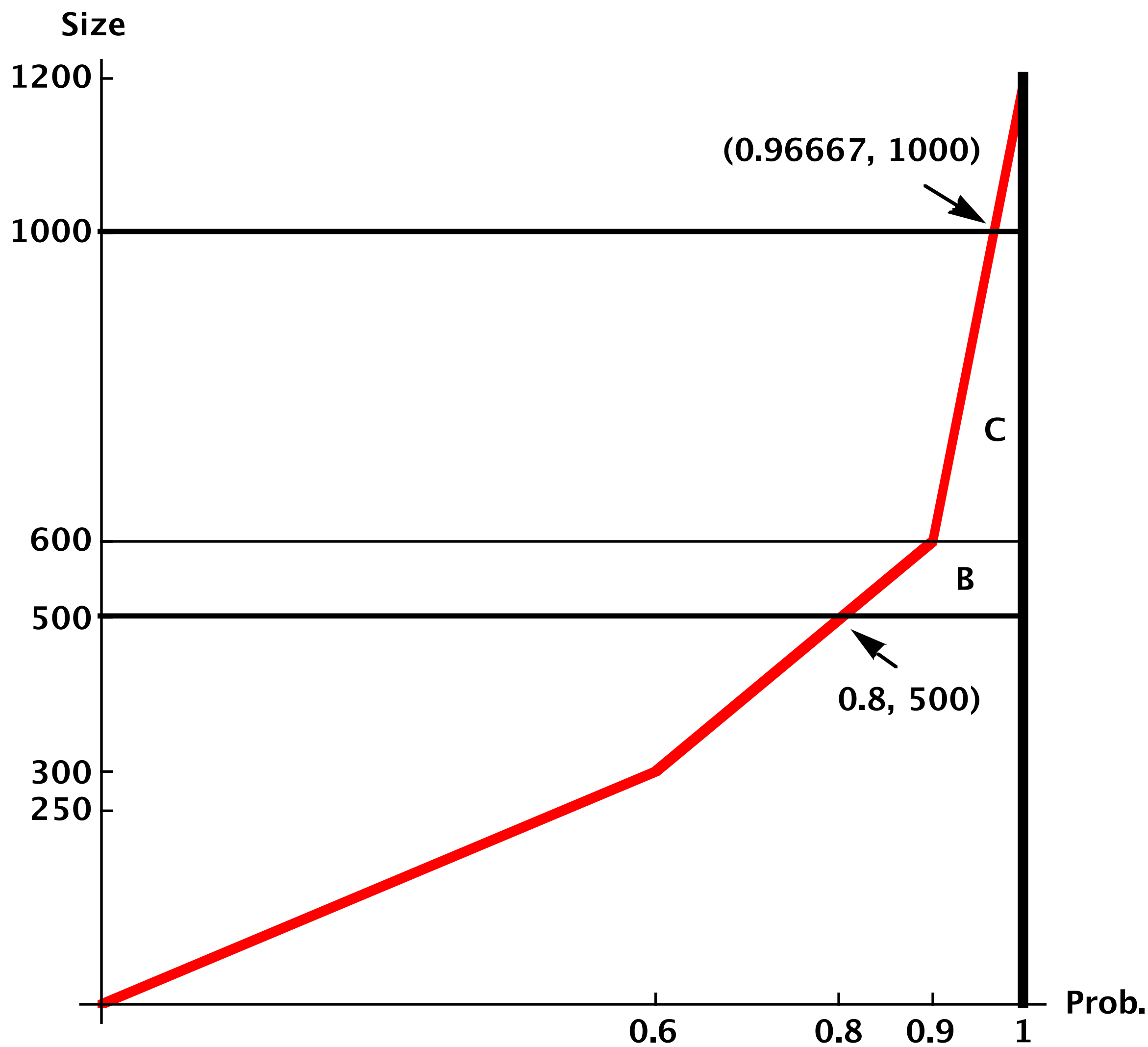


(b) Prior to trend, the layer from 500,000 to 1,000,000 is the area below the distribution and between horizontal lines at heights 500,000 and 1,000,000.
This is Area A in the following Lee Diagram.



Area A is a triangle, with base 0.1 and height 500, and thus area: (0.1)(500)/2 = 25 (thousand).

After trend the distribution function at 500 is: (1/3)(0.6) + (2/3)(0.9) = 0.8.
After trend the distribution function at 1000 is: (1/3)(0.9) + (2/3)(1) = 0.96667.
After trend the Lee Diagram is as follows, with the excess layer being the sum of Areas B and C:



Area B is a trapezoid with height 100 and widths 0.1 and 0.2, and thus area:
(100)(0.1 + 0.2)/2 = 15 (thousand).
Area C is a trapezoid with height 400 and widths 0.1 and 0.03333, and thus area:
(400)(0.1 + 0.03333)/2 = 26.667 (thousand).
Thus after trend the excess layer is: Area B + Area C = 15 + 26.667 = 41.667 (thousand).
The implied trend for the layer $500,000 excess of $500,000 is: 41.667/25 - 1 = **66.7%**.

Alternately, both prior and posterior to trend, the first interval contributes nothing to the excess layer.
Prior to trend, the second interval contributes nothing to the excess layer.
After trend, the second interval is uniform from 300,000 to 600,000.
After trend, the second interval contributes to the excess layer:

$$(1/300{,}000) \int_{500{,}000}^{600{,}000} (x - 500{,}000)\, dx = (1/300{,}000)\ (100{,}000^2 / 2) = 16{,}667.$$

Prior to trend, the third interval contributes to the excess layer:

$$(1/500{,}000) \int_{500{,}000}^{1{,}000{,}000} (x - 500{,}000)\, dx = (1/500{,}000)\ (500{,}000^2 / 2) = 250{,}000.$$

After trend, the third interval is uniform from 600,000 to 1,200,000,
and contributes to the excess layer:

$$(1/600{,}000) \int_{600{,}000}^{1{,}000{,}000} (x - 500{,}000)\, dx + (200{,}000/600{,}000)(500{,}000) =$$

$$(1/600{,}000)\ (500{,}000^2 / 2 - 100{,}000^2/2) + 166{,}667 = 366{,}667.$$

Thus prior to trend the expected losses in the excess layer are:
(60%)(0) + (30%)(0) + (10%)(250,000) = $25,000.
After to trend the expected losses in the excess layer are:
(60%)(0) + (30%)(16,667) + (10%)(366,667) = $41,667.
The implied trend for the layer $500,000 excess of $500,000 is: 41,667/25,000 - 1 = **66.7%**.
Alternately, prior to trend: $E[X \wedge 1 \text{ million}]$ = (0.6)(125K) + (0.3)(375K) + (0.1)(750K) = 262.5K.
$E[X \wedge 500K]$ = (0.6)(125K) + (0.3)(375K) + (0.1)(500K) = 237.5K.
$E[X \wedge 1 \text{ million}] - E[X \wedge 500K]$ = 262.5K - 237.5K = 25,000.
Subsequent to trend, we have three uniform distributions:
from 0 to 300,000; from 300,000 to 600,000; and from 600,000 to 1,200,000.
Break the last interval into 600K to 1000K and 1000K to 1200K.
$E[X \wedge 1 \text{ million}]$ = (0.6)(150K) + (0.3)(450K) + (0.1)(2/3)(800K) + (0.1)(1/3)(1000K) = 311,667.
Break the middle interval into 300K to 500K and 500K to 600K.
$E[X \wedge 500K]$ = (0.6)(150K) + (0.3)(2/3)(400K) + (0.3)(1/3)(500K) + (0.1)(500K) = 270,000.
$E[X \wedge 1 \text{ million}] - E[X \wedge 500K]$ = 311,667 - 270,000 = 41,667.
The implied trend for the layer $500,000 excess of $500,000 is: 41,667/25,000 - 1 = **66.7%**.
Comment: In part (a) it would have been helpful if the question specified whether or not they wanted a Lee Diagram.

7. (1.75 points)
An insurer currently offers the following coverage limits at actuarially sound premiums.

Limit	Premium
$100,000	$350
250,000	700

Next year, underwriting would like to offer additional coverage options.
The following premiums are under actuarial review.

Limit	Premium
$500,000	$1,000
1,000,000	1,800

Argue for and against the actuarial soundness of these new premiums as they relate to the lower-limit options.

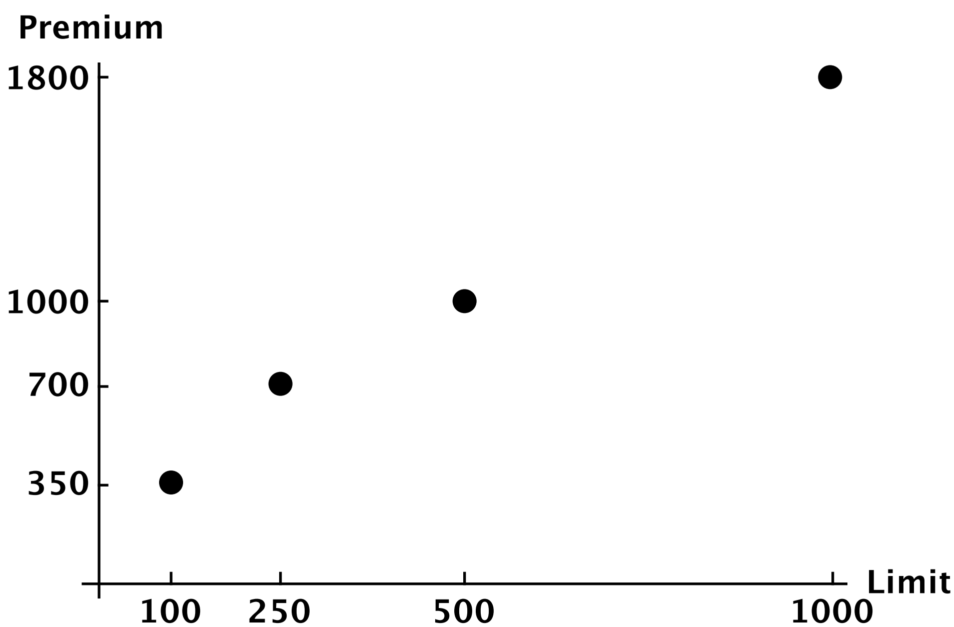
7.

Limit ($000)	Premium	Marginal Rate
100	350	
250	700	(700 - 350)/(250-100) = 2.33
500	1000	(1000 - 700)/(500-250) = 1.2
1000	1800	(1800-1000)/(1000-500) = 1.6

The marginal rates should decline.
The premium for $500,000 looks OK compared to the lower limits.
However, 1.6 > 1.2, so the rate for $1 million is <u>not</u> actuarially sound compared to all of the others.
Equivalently, one can look at increased limit factors.

Limit ($000)	Premium	ILF	Change in ILF Over Change in Limit
100	350	1.00	
250	700	2.00	(2.00 - 1.00) / (250 - 100) = 0.0067
500	1,000	2.86	(2.86 - 2.00) / (500-250) = 0.0034
1000	1,800	5.14	(5.14-2.86) / (1000-500) = 0.0046

The change in ILFs over changes in limit should decline.
The premium for $500,000 looks OK compared to the lower limits.
0.0046 > 0.0034, so the rate for $1 million is <u>not</u> actuarially sound compared to all of the others.
Equivalently, graph the premiums versus limits:



The curve should be increasing and concave downwards.
It is concave downwards if we exclude the $1 million limit.
The premium for $500,000 looks OK compared to the lower limits.
It is not concave downwards when we include the $1 million limit!
The rate for $1 million is <u>not</u> actuarially sound compared to all of the others.

Looking at just the lower limits and the $500K premium:

Limit ($000)	Premium	Marginal Rate
100	350	
250	700	(700 - 350)/(250-100) = 2.33
500	1000	(1000 - 700)/(500-250) = 1.2

The premium for $500,000 looks OK compared to the lower limits.
Looking at just the lower limits and the $1000K premium:

Limit ($000)	Premium	Marginal Rate
100	350	
250	700	(700 - 350)/(250-100) = 2.33
1000	1800	(1800-700)/(1000-250) = 1.467

The premium for $1,000,000 looks OK compared to the lower limits.
However, as seen above, for all four limits together the premiums are not actuarially sound.
Possible reasons for the actuarial soundness of these new premiums in spite of the above:
1. There may be adverse selection, where those with higher potential for large losses are more likely to buy the 1 million limit.
2. Legal settlements may depend on limits purchased, so that the size of loss distributions underlying the different limits are not the same.
3. If risk loads are included, the 1 million limit may have a much bigger risk load, justifying how high its premium is compared to the others. (GIven the actual values for the ILFs this seems unlikely.)
4. The 500K premium may be smaller than it otherwise would have been compared to the others due to favorable selection; those will lower expected losses may tend to choose the 500K limit.
5. From the Examiner's Report: "market conditions", which does not relate to actuarial soundness.
Comment: In my opinion, they should have made it clearer that they wanted you to argue both sides of the issue, by for example making part (a) against, and part (b) for.
One could get premiums that satisfy the consistency test by either adjusting the 500K premium up or the 1000K premium down.
For example, increasing the 500K premium:

Limit ($000)	Premium	Marginal Rate
100	350	
250	700	(700 - 350)/(250-100) = 2.33
500	1200	(1200 - 700)/(500-250) = 2.0
1000	1800	(1800-1200)/(1000-500) = 1.2

For example, decreasing the 1000K premium:

Limit ($000)	Premium	Marginal Rate
100	350	
250	700	(700 - 350)/(250-100) = 2.33
500	1000	(1000 - 700)/(500-250) = 1.2
1000	1300	(1300-1000)/(1000-500) = 0.6.

8. (2.5 points) An actuary is pricing a one-year commercial general liability occurrence policy. The following information is available:

- Renewal effective date is January 1, 2014
- Losses are evaluated as of September 1, 2013

Effective Date of Policy	Company Subject Loss Costs
January 1, 2013	$35,700
January 1, 2012	50,300
January 1, 2011	40,300
January 1, 2010	32,600
January 1, 2009	22,500

Date of Loss	Paid & Outstanding Loss Amounts	ALAE
July 21, 2013	$2,000	$0
September 7, 2012	10,000	0
April 1, 2011	100,000	20,000
November 13, 2010	40,000	0
February 14, 2010	70,000	0
May 5, 2009	12,000	0

Given that the experience modification is equal to 0.443, calculate the adjustment to reflect the ultimate level of loss.

8. In the rating we use the 2010, 2011, and 2012 policies.
Company subject loss costs are: 32,600 + 40,300 + 50,300 = \$123,200.
Therefore, looking at Rule 16 in the plan: Z = 0.30, EER = 0.853, and MSL = \$102,850.
Only the 2nd through 5th losses enter the rating.
The third loss plus ALAE of \$120,000 is limited to the MSL of \$102,850.
(The loss presumably has already limited to the basic limit of \$100,000.)
Includable Loss and ALAE is: 10,000 + 102,850 + 40,000 + 70,000 = \$222,850.
Let x = adjustment to reflect the ultimate level of loss.
Then AER = (222,850 + x)/123,200 = 1.80885 + x/123,200.

0.443 = M = (0.30) {(1.80885 + x/123,200)/0.853 - 1}. ⇒ x = **\$37,421**.

Alternately, M = Z (AER - EER)/EER. ⇒ 0.443 = (0.30)(AER / 0.853 - 1). ⇒ AER = 2.113.

⇒ 2.113 = (222,850 + x)/123,200. ⇒ x = **\$37,472**.

<u>Comment</u>: Backwards question!
Check. AER = (222,850 + 37,421)/123,200 = 2.113.
M = (0.30) (2.113 - 0.853) / 0.853 = 0.443, or a 44.3% debit.
As per Table 6.B.6 in the plan, without being given the mod, we would need to know additional information, including the division of the company subject loss costs for each year between prem/ops and products, in order to calculate the adjustment to reflect the ultimate level of loss.
For each year separately, for prem/ops and products separately, we would multiply the company subject loss costs by the EER and the expected percent unreported.
The average expected percent unreported is: 37,421 / {(0.853)(123,200)} = 35.6%.
It is unusual to have no ALAE for a claim, let alone for so many of the claims as here.

9. (2 points) An actuary is pricing an account that qualifies under a single-split experience rating plan. The account's actual losses during the experience rating period are:

Claim	Loss
1	$8,000
2	21,000
3	3,000
4	11,500

The following information is also available:
Split point: $10,000
Primary credibility: 0.80
Excess credibility: 0.20
Expected loss: 30,000
Loss-free modification: 0.60
Calculate the experience modification.
Note: The original exam question said "Loss and ALAE".

9. The given losses total: 43,500.

A_p = 8000 + 10,000 + 3000 + 10,000 = 31,000.

A_e = 43,500 - 31,000 = 12,500 = 0 + 11,000 + 0 + 1,500.

Mod = (loss-free mod) + $Z_pA_p/E + Z_eA_e/E$ =

0.60 + (0.8)(31,000)/30,000 + (0.2)(12,500)/30,000 = **1.51**.

Alternately, let E_p be the expected primary losses and E_e = the expected excess losses.

30,000 = $E_p + E_e$.

Then when we have no actual losses:

0.60 = M = 1 + (0.8)(0 - E_p)/E + (0.2)(0 - E_e)/E = 1 - 0.8E_p/30,000 - 0.2(30,000 - E_p)/30,000. ⇒

(0.8E_p - 0.2E_p) / 30,000 = 1 - 0.2 - 0.6. ⇒ E_p = (30,000)(0.2/0.6) = 10,000. ⇒ E_e = 20,000.

M = 1 + (0.8)(31,000 - 10,000)/30,000 + (0.2)(12,500 - 20,000)/30,000 = **1.51**.

Comment: Check. For no losses: M = 1 + (0.8)(0 - 10)/30 + (0.2)(0 - 20)/30 = 0.60.

10. (2.25 points) The National Council of Compensation Insurance (NCCI) has proposed making the following three changes to its Experience Rating Plan:

1. Increasing the effect of medical-only losses.
2. Giving zero credibility to excess losses.
3. Keeping the primary-excess split of actual losses at a constant value of $10,000 for the next 10 years.

a. (1.5 points)

Evaluate each of these changes with respect to the following goals of experience rating:

- Safety incentive
- Predictive accuracy

b. (0.75 points) Briefly discuss each of the proposed changes' effect on the experience modification of an insured that has historically experienced worse than class average claim frequency, but has never had a loss greater than $10,000.

10. (a) 1. This will make the plan more sensitive to the insured's own experience and thus increase safety incentives. The insured will be more likely to maintain a safe workplace in order to reduce its experience mod and thus it premium.
Alternately, since medical only losses are so small a part of total loses, this will have no significant impact on safety incentives.
Alternately, this change will induce many employers to not report medical only claims and as a result will hurt safety incentive because the insurer is more qualified to ensure quick and proper treatment for injured workers and effective management of the claim.
This change should increase predictive accuracy, since the number of claims, whether medical-only or lost-time, has significant predictive value.
Alternately, this change will induce many employers to not report medical only claims, thus reducing predictive accuracy.
Alternately, according to the NCCI, reducing the weight to medical onlys in 1998 increased predictive accuracy, so that increasing the weight to medical onlys is likely to decrease predictive accuracy.
(In any case, the effect of this change will depend on the magnitude of the change in weight on medical onlys. Are we going from 30% weight to 40% weight, or from 30% weight to 100% weight?)
2. Giving zero weight to excess losses will decrease safety incentives, since employer's premiums will no longer be affected by the severity of accidents.
Giving zero weight to excess losses will decrease predictive accuracy, since the quintiles tests performed by the NCCI indicated that the current significant weight to excess losses performed best. (Excess losses help to predict severity, while primary losses help to predict frequency.)
Alternately, giving zero weight to excess losses will have little if any effect on safety incentives, since severity is largely out of the control of the employer's safety efforts; the employer's premium will still be affected by frequency, which the employer can affect via safety efforts.
3. Keeping the primary-excess split of actual losses at a constant value of $10,000 for the next 10 years would decrease over time the percentage of losses that are considered primary.
Since $Z_p > Z_e$, this would lead to less weight to insured's experience over time.
This would lead to some decrease in safety incentives over time.
Assume the current Z_p and Z_e work well with the $10,000 split point. Over time the definition of what is primary is effectively changing. Using the current credibilities with this new definition will decrease predictive accuracy.
(b) 1. Their frequency is above average, presumably including above average frequency for medical onlys, and we will be giving medical onlys more weight. This will increase the experience mod of this insured.
2. Assuming the current split point is $10,000, this insured has had $A_e = 0$. Let us assume this continues to be the case going forward. With excess losses given positive weight, having $A_e = 0$ currently reduces this insured's mod compared to average. Giving instead zero-weight to excess losses, this insured would lose that benefit. Thus this insured's mod would increase.

3. Keeping the primary-excess split of actual losses at a constant value of $10,000 for the next 10 years would decrease over time the percentage of losses that are considered primary.
D-ratios should decline, so that in real terms E_p declines and E_e increases.
This insured has A_p worse than average and $A_e = 0$.
This insured's ratio of A_p/E_p should increase as E_p declines, while A_e/E_e remains zero or least very small. Thus assuming Z_p remains the same, this insured's mod should increase.
Alternately, inflation may cause some of this insured's claims to be greater than the 10,000 split point, shifting some of its losses from primary to excess. All other things being equal, this would decrease this insured's mod.
Comment: As far as I know, the NCCI has not proposed these changes.
The Examiner's Report stated that one could get full credit just writing down your conclusion for some of the many subparts, which could be only one or two words, provided of course your conclusion agreed with that of the examiners. The Examiner's Report pointed out that some students wrote much more than the examiners decided was needed for full credit on these subparts. I am not sure how someone taking the exam would know exactly how much was sufficient to write; as always just try to do the best you can.
I do not agree with the Examiner's Report: "The most commonly made mistake on this part is the evaluation of the change regarding increasing the effect of med-only losses with respect to safety incentive. The key concept here is that this change will discourage companies from reporting med-only losses and as a result will hurt safety incentive because the carrier is more qualified to ensure quick and proper treatment for injured workers and effective management of the claim. Many candidates predicted that companies will be less likely to report under this change, but not all were able to make the connection that this would hurt safety incentive." However, to most people "safety incentive" refers to an incentive for the employer to maintain a safer workplace. In my opinion, the examiner's mistake on this part was confusing whether or not the insurer is given the ability to help control costs with a direct incentive for the employer to have a safer workplace.

11. (2.5 points)
The following formula is used by the National Council of Compensation Insurance (NCCI) to calculate workers compensation experience modifications under its Experience Rating Plan:

$$M = \frac{A_p + WA_e + (1-W)E_e + B}{E + B}$$

where $W = \frac{E + B}{E + K}$ and B and K vary by size of risk.

a. (0.5 point)
Explain the assumptions behind the theory that B and K should be constant for all risk sizes.
b. (0.5 point) Critique the theory that B and K should be constant for all risk sizes.

Suppose a new experience rating plan is proposed such that B and K are constant for all risk sizes.
c. (1 point) Fully explain a valid method that can be used to assess the performance of the proposed plan as compared to the current NCCI plan.
d. (0.5 point) Assess the impact of implementing the newly proposed experience rating plan in a competitive market.

Note: The original exam question incorrectly had: $W = \frac{E - B}{E + K}$.

11. a. It is assumed that risks are the sum of independent identically distributed units and therefore for large insureds their process variance goes down as one over their size.
Also that the variance of the hypothetical means is independent of size
Also it is assumed that risk parameters for an insured do not shift over time, except as reflected in any changing mix of exposures by class.
b. Due to parameter uncertainty, certain changes affect large insureds to the same extent as small insureds, and thus the process variance of large insureds goes down more slowly than one over their size. Empirical tests show that the variance of loss ratios does not decrease as quickly as one over size.
Also, large risks are heterogeneous, and thus are not the sum of the sum of independent identically distributed units.
For these two reasons, the optimal credibilities are not based on B and K that are constant.
For the NCCI plan, B and K both increase with size. As E approaches infinity, the credibility approaches a value less than one.
From the syllabus readings, it is not clear whether risk parameters of an insured are likely to shift significantly over the time period of about 5 years including the experience rating period and the policy to be rated, but if they did this would be another reason why B and K would not be constant.
c. One could use the quintiles test. For a set of risk of similar size, one would experience rate them using each plan. Group the risks in a given size category into five groups by experience modification: the lowest mods, next lowest mods, average mods, next highest mods, and highest mods. Since the experience modifications differ for each plan, the risks in the quintiles will differ somewhat between the plans.
Then we examine the subsequent period loss ratios to manual premium and standard premium.
Unmodified loss ratio = loss ratio to manual premium.
Modified loss ratio = loss ratio to standard premium.

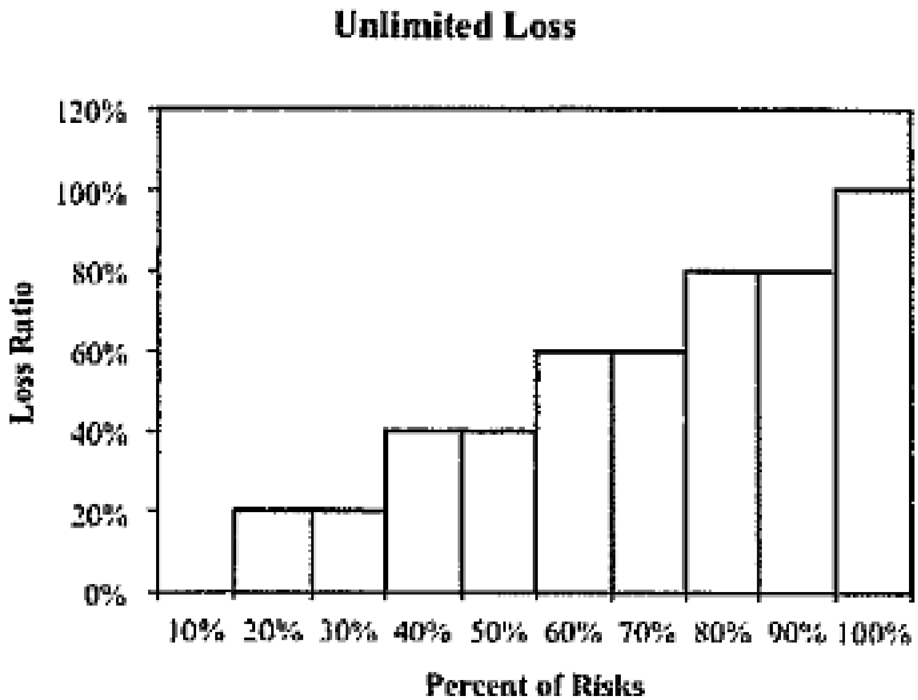
The quintiles test statistic is: $\dfrac{\text{the variance of the modified ratios}}{\text{the variance of the unmodified ratios}}$.

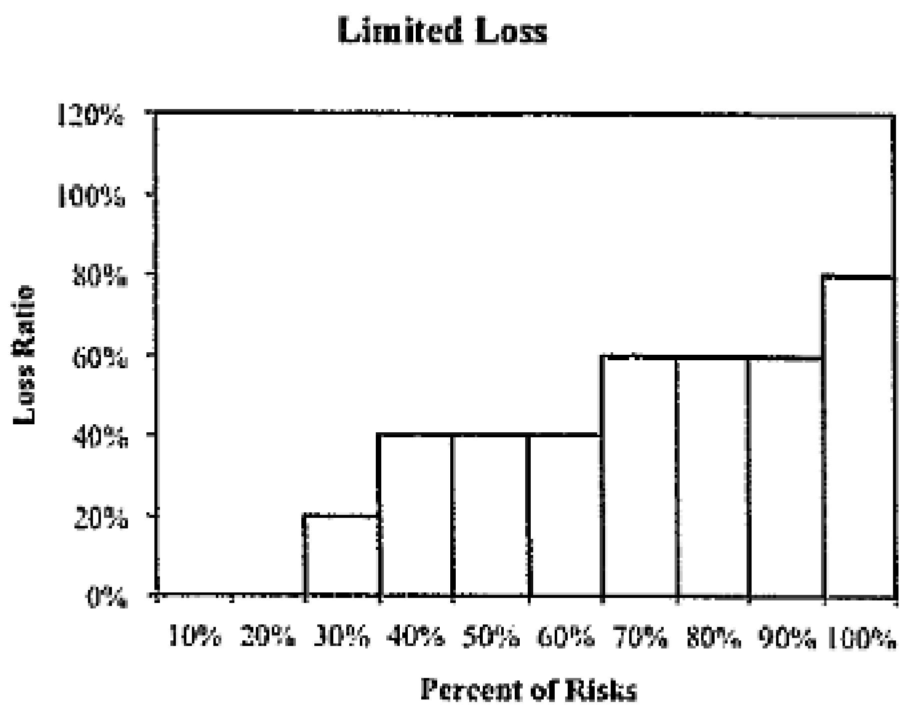
The smaller the quintiles statistic, the better the experience rating plan.
So if the quintiles statistic is lower for the NCCI plan than the proposed plan, that would indicate it is better than the proposed plan, at least for this size category.
One would repeat this quintiles test for many different size categories.
d. Assume that all insurers will have to use the new inferior experience rating plan.
The new plan would give too much weight to the experience of large risks.
(Z would approach one as size approaches infinity, while for the NCCI plan it approaches a value significantly less than one.)
Therefore, smart underwriters would be reluctant to write large insureds with good experience and thus credit mods, and might be eager to write large insureds with debit mods. It is likely that many large insureds would have trouble obtaining insurance. Underwriters may be able to overcome some of the problems caused by the new experience rating plan via creative schedule rating, retrospective rating or large deductible plans.
Also, small insureds will be given too much credibility, creating additional market disruptions.

Comment: Compared to prior to 1991, the current NCCI plan gives more credibility to small insured and less credibility to large insureds.
The Examiner's Report mistakenly states that if B and K were constant, that would lead to some risks being self-rated. In fact, $Z_p = E/(E+B) < 1$, for any fixed $B > 0$ and finite E.

Also the Examiner's Report states that "in a competitive market, rates will go down for those preferred risks." No explanation is made of how that could occur. Will an insurer have two sets of manual rates, one for large risks with debit mods and one for large risks with credit mods? Even within the large risks with debit mods, one would need several sets of rates depending on how far the insured's mod was from one and how big the large insured was, in order to make up for the problems created by the new experience rating plan. Totally impractical!

12. (3 points) The following Lee diagrams depict the loss experience of a group of 10 similar risks; one for unlimited losses and the other for limited losses.





a. (2.25 points) Calculate the Table L charges at loss ratios of 0% to 100% in 20% increments.
b. (0.75 point) Describe what the Table L savings at an entry ratio of 0.4 reflects, assuming an expected unlimited loss of \$500,000 and a per accident limit of \$100,000.

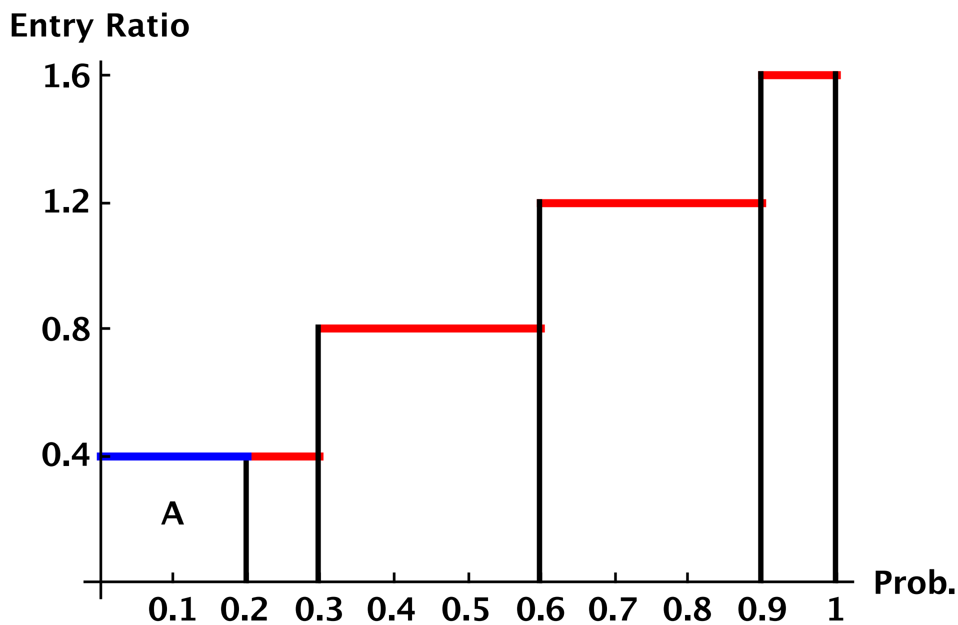
12. a. The total area under the unlimited curve is:
(10%)(0 + 20% + 20% + 40% + 40% + 60% + 60% + 80% + 80% + 100%) = 50%.
This is the expected unlimited loss ratio.
Thus multiplying all of the loss ratios by 1/50% = 2 would give entry ratios.
In terms of entry ratios, the total area under the limited loss curve is:
(2)(10%)(20% + 40% + 40% = 40% + 60% +60% + 60% + 80%) = 0.80.
Thus the loss elimination ratio (LER) = 1 - 0.80 = 0.20.
Then for each entry ratio, the Table L charge is the LER plus
the area in the limited graph with respect to entry ratios under the curve and above horizontal line.
Table L charge at loss ratio of 0% = 0.20 + 0.80 = **1**.
Table L charge at loss ratio of 20% = 0.20 + (2)(10%){(3)(20%) + (3)(40%) + 60%} = **0.68**.
Table L charge at loss ratio of 40% = 0.20 + (2)(10%){(3)(20%) + 40%} = **0.40**.
Table L charge at loss ratio of 60% = 0.20 + (2)(10%)(20%) = **0.24**.
Table L charge at loss ratio of 80% = 0.20 + 0 = **0.20**.
Table L charge at loss ratio of 100% = 0.20 + 0 = **0.20**.
Alternately, the total area under the unlimited curve is 50%, the expected unlimited loss ratio.
The total area under the limited curve is 40%, the expected limited loss ratio.
Thus, LER = 1 - 40%/50% = 0.20.

The Table L entry ratios are: $\dfrac{\textbf{limited}\text{ aggregate}}{\text{expected }\textbf{unlimited}\text{ aggregate}}$.

For the ten risks, the entry ratios are: 0, 0, 0.4, 0.8, 0.8, 0.8, 1.2, 1.2, 1.2, and 1.6.
In order to calculate the portion of the Table L charges prior to adding in the LER, we do a double summation upwards on the number of risks with given values of the entry ratios.
We divide the double summation by 25, the equivalent total for the unlimited loss ratios.
25 = 20/(1 - 20%), where 20% is the LER.
Then we add the LER of 20%. For example, 12/25 + 20% = 0.68.

Entry Ratio	# of Risks	Sum Up	Double Sum Up	Divided by 25	Charge	Savings
0.0	2	8	20	0.8000	**1.0000**	0.0000
0.4	1	7	12	0.4800	**0.6800**	0.0800
0.8	3	4	5	0.2000	**0.4000**	0.2000
1.2	3	1	1	0.0400	**0.2400**	0.4400
1.6	1	0	0	0.0000	**0.2000**	0.8000
2.0	0	0	0	0.0000	**0.2000**	1.2000

(b) An entry ratio of 0.4 corresponds to a loss ratio of 20%.
Table L savings are the area above the limited curve and below the horizontal line at 40% loss ratio: (2)(10%)(20% + 20%) = 0.08. Given unlimited expected losses of $500,000, this savings corresponds to: (0.08)(500,000) = $40,000.
Assuming a limited loss of ratio of 20% corresponds to the minimum premium, then this savings is the expected amount the insured will have to pay extra due to the effect of the minimum premium (together with the accident limit of $100,000); it will be multiplied by c and T.
In most years the minimum premium has no effect; this average includes such years as well as those years in which the retro premium would otherwise be below the minimum premium.
For $500,000 in expected losses an entry ratio of 0.4 corresponds to losses of: (0.4)(500,000) = $200,000.
The savings is the average difference between the loss amount corresponding to the entry ratio of 0.4, 200,000 and the actual losses limited to the per occurrence limit of 100,000.
A Lee Diagram of the limited curve with respect to entry ratios:

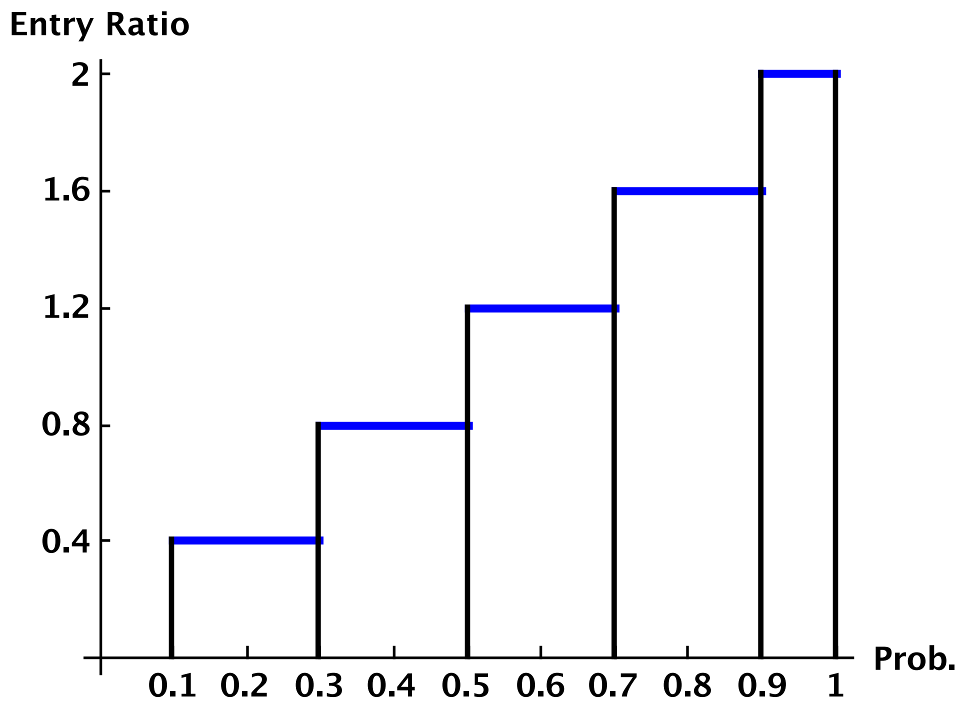


The savings (in percentage terms) for an entry ratio at 0.4, is Area A, below the horizontal line at 0.4 and above the limited loss curve.

$$\psi^*(0.4) = \int_0^{0.4} (0.4 - r)\, dF^*(r)$$, where F* is the distribution of limited losses.

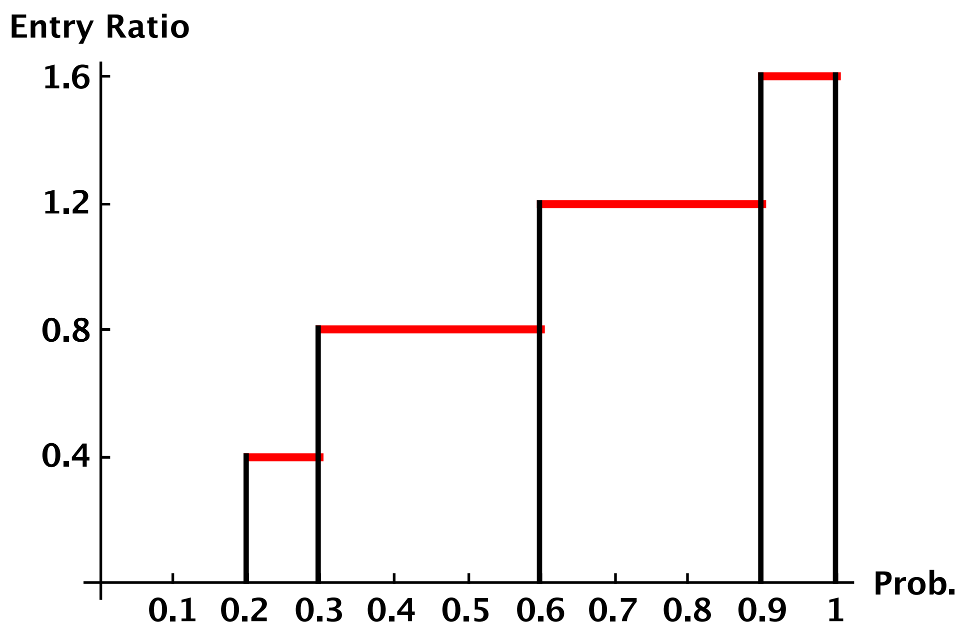
Comment: From the Examiner's Report: “Many candidates calculated the savings based on their part a results, while the question never indicated that the scenario in b applied to the data in a.”
This has been a pattern on recent exams; sometimes one part of a question is intended as being independent of the previous parts of a question. In those cases, one is expected to assume that unless stated otherwise the parts of a question are independent of each other. Personally, I am fooled by this poor exam construction every time. If the parts are independent, why not just ask separate questions?

A Lee Diagram of the unlimited curve with respect to entry ratios:



The total area under the unlimited curve is one.

A Lee Diagram of the limited curve with respect to entry ratios:



The total area under the limited curve is 0.8.

13. (2 points)
A Table M is constructed based on the experience of the following 10 similarly sized risks:

Risk	Aggregate Loss Ratio
1	10%
2	30%
3	35%
4	40%
5	60%
6	75%
7	X%
8	90%
9	110%
10	120%

X is the aggregate loss ratio for Risk 7.
Assume:

- $75\% \le X \le 90\%$.
- The Table M charge at entry ratio 1.5 is 0.05.

Calculate X.

13. The average loss ratio is: 57% + X/10.
Thus an entry ratio of 1.5, corresponds to a loss ratio of: (1.5)(57% + X/10) = 85.5% + 0.15X.
If X = 75%, then 85.5% + 0.15X = 96.75%.
And the Table M charge for r = 1.5 is:
(0.1) {(110% - 96.75%) + (120% - 96.75%)} / (57% + 75%/10) = 0.0566 > 0.05.
If X = 90%, then 85.5% + 0.15X = 99%.
And the Table M charge for r = 1.5 is:
(0.1) {(110% - 99%) + (120% - 99%)} / (57% + 90%/10) = 0.0485 < 0.05.
We want: 0.05 = (0.1) {(110% - 85.5% - 0.15X) + (120% - 85.5% - 0.15X} / (57% + X/10). ⇒

0.285 + 0.05 X = 0.59 - 0.3X. ⇒ X = 0.305 / 0.35 = **87.1%**

Comment: I am not sure what the point is of having a backwards question like this one.
For X = 87.1%, the average loss ratio is: 57% + 8.71% = 65.71%.
An entry ratio of 1.5 corresponds to a loss ratio of: (1.5)(65.71%) = 98.565%.
Thus the Table M charge at 1.5 is:

$$\frac{(110\% - 98.565\%)(1/10) + (120\% - 98.565\%)(1/10)}{65.71\%} = 0.050.$$

14. (3.75 points)

An actuary is pricing an excess workers compensation policy with the following characteristics:

- Excess loss pure premium factors are based on empirical data for losses and ALAE up to $250,000 and a fitted curve for losses greater than $250,000
- $1,000,000 attachment point
- No aggregate limit

Historical adjusted data for similarly sized risks:

Loss Amount	Probability
$20,000	70%
100,000	14%
250,000	8%
500,000	5%
750,000	2%
1,000,000	1%

Standard Premium:	$500,000
ULAE as Ratio of Loss+ALAE:	6.0%
General Expense:	2.0%
Acquisition Expense:	5.0%
Tax:	3.0%
Profit and Contingency Margin:	-10.0%

- Empirical data has been truncated and shifted at $250,000 and normalized to a unity mean.
- A mixed Exponential-Pareto curve has been fit to the resulting mean residual lives as described by the following parameters:

Distribution	Pareto	Exponential
Cumulative Function	$1 - \left(1 + \frac{x}{b}\right)^{-s}$	$1 - e^{-x/c}$
Mean	$\frac{b}{s-1}$	c
Variance	$\frac{b^2\, s}{(s-1)^2\,(s-2)}$	c^2
Excess Ratio	$\left(1 + \frac{x}{b}\right)^{1-s}$	$e^{-x/c}$
Mean Residual Life	$\frac{b+x}{s-1}$	c
Shape	4.0	n/a
Scale	12.0	0.8
Weight	0.050	0.950

Calculate the premium for this policy.

14. I assume the excess attachment point applies to loss plus ALAE by accident.
I assume that the given data is for loss plus ALAE by accident.
The average size of accident is: (70%)(20,000) + ... + (1%)(1 million) = $98,000.

The empirical excess ratio at 250,000 is: $\frac{(5\%)(250) + (2\%)(500) + (1\%)(750)}{98}$ = 30.612%.

The mean of the data truncated and shifted at $250,000 is:

$$1000 \ \frac{(5\%)(250) + (2\%)(500) + (1\%)(750)}{5\% + 2\% + 1\%} = 375{,}000.$$

Thus the attachment point of $1 million corresponds to an entry ratio of: $\frac{\$1 \text{ million} - \$250{,}000}{\$375{,}000}$ = 2.

The excess ratio for the Exponential at 2 is: $e^{-2/0.8}$ = 0.082085.

The excess ratio for the Pareto at 2 is: $(1 + 2/12)^{1-4}$ = 0.629738.

The mean of the Exponential is 0.8.
The mean of the Pareto is: 12/(4-1) = 4.
Thus the excess ratio at 2 for the mixed distribution is:

$$\frac{(0.95)(0.8)(0.082085) + (0.05)(4)(0.629738)}{(0.95)(0.8) + (0.050)(4)} = 0.196179.$$

Thus the estimated Excess Ratio at $1 million is: (0.196179)(30.612%) = 6.005%.
There is no way to determine the expected ground-up losses + ALAE from the given information.
I will assume for illustrative purposes an 80% expected loss and ALAE ratio,
which for $500,000 in Standard Premium, corresponds to $400,000 in Expected Loss + ALAE.
Therefore, the expected loss plus ALAE for the excess policy is: (6.005%)($400,000) = $24,020.
We need to load this for ULAE at 6%.
As per the reading by Teng, I will assume that the General Expense is 2% of standard premium,
while the acquisition, taxes, and profit are as a percent of net premium.

Premium for this excess policy is: $\frac{(1.06)(\$24{,}020) + (2\%)(\$500{,}000)}{1 - 5\% - 3\% + 10\%}$ = **$34,766**.

Alternately, as per the Examiner's Report, assume that the expected aggregate loss is equal to the average severity of $98,000. (This makes no sense and is inconsistent with a standard premium of $500,000.) Then plug into the formula from the reading by Teng: Excess WC Premium =

$$\frac{(EL)\,(XL)\,(1 + ULAE) + (SP)\,(GO)}{1 - A - T - P} = \frac{(\$98{,}000)(6.005\%)(1.06) + (\$500{,}000)(2\%)}{1 - 5\% - 3\% + 10\%} = \mathbf{\$15{,}920}.$$

Comment: We needed to be given the expected loss plus ALAE ratio to Standard Premium for a full coverage policy. The Examiner's Report solution of using the average severity as the expected aggregate loss in my opinion shows a fundamental lack of understanding of this material.
The expense assumptions should have been made clear in the question. Apparently, the examiners expected one to have memorized the formula from the reading by Teng and just plug in numbers. The examiners are apparently unaware that there are somewhat different expense assumptions made to price Excess Policies in the third part of Gillam and Snader, also on the syllabus. Unfortunately, by not stating their expense assumptions, the examiners penalized those students who stopped to try to figure out what was going on and/or knew the material better. In answering this poorly constructed question, the less thinking the better; this will not work for many other questions on your exam.
In the Mahler paper, the curve is not fit to mean residual lives.
In the Mahler paper, historical adjusted losses are for all sized risks in the given hazard group not just for similarly sized risks.
It is unclear whether the technique in the Mahler paper would work well applied to a distribution of size of accident for losses + ALAE rather than just losses, although there is no reason to assume it would not.

15. (2.75 points) The balanced plan provisions for a 2014 workers compensation risk are:

Standard Premium:	\$6,000,000
Expected Loss (Ratio to Standard Premium):	0.5
Minimum Entry Ratio, r_H:	0.15
Maximum Entry Ratio, r_G:	2.52
Basic Premium (Ratio to Standard Premium):	0.2766
Loss Conversion Factor:	1.25
Tax Multiplier:	1.08
State Hazard Group Differential:	0.95

Assume that in the following year the endpoints of the expected loss size ranges in the NCCI Retrospective Rating Manual are increased by 10% to reflect assumed inflation. Calculate the expected shortfall, as a percentage of 2015 expected retrospective premium, from failing to update the expected loss size ranges.

15. Expected losses = (0.5)(\$6 million) = \$3 million.
Assuming no accident limit, LUGS = (0.5)(6 million)(0.95) = \$2.85 million.
Using the 2008 Table of ELGs, this would be Expected Loss Group 28: 2,672,626 to 3,195,877.
If the edges of the Expected Loss Groups increase by 10%, ELG29 becomes:
(1.1)(2,248,334) = 2,473,167 to (1.1)(2,672,625) = 2,939,888.
Thus these expected losses should now be in ELG 29 instead.
For ELG 28: $\phi(2.52) = 0.0621$ and $\psi(0.15) = 0.0016$.
For ELG 29: $\phi(2.52) = 0.0700$ and $\psi(0.15) = 0.0020$.
The effect on the retro premium is:
{(0.0621 - 0.0700) - (0.0016 - 0.0020)} E c T =
(-0.0075)(\$3 million)(1.25)(1.08) = -\$30,375.
The expected losses entering the retro plan using correct ELG 29 are:
E(1 - 0.0700 + 0.0020) = (3 million) (0.932) = 2.796 million.
Expected Retro premium is: (1.08) {(2.796 million)(1.25) + (0.2766)(\$6 million)} = \$5.5670 million.
Percentage shortfall is: -\$30,375 / \$5.567 million = **-0.55%**.
Alternately, the losses that would enter the retro plan if the distribution followed ELG 28 are:
E(1 - 0.0621 + 0.0016) = (3 million) (0.9395) = 2.8185 million.
Thus the retro premium that was expected when the basic premium was calculated using incorrect ELG 28 is: (1.08) {(2.8185 million)(1.25) + (0.2766)(\$6 million)} = \$5.5973 million.
(This should be equal to the guaranteed cost premium.)
Percentage shortfall is: (5.5670 - 5.5973) / 5.5670 = **-0.54%**.
<u>Comment</u>: See the second example in Gillam's discussion of Skurnick. In a single year, we should calculate the basic premium for a retro plan using an outdated Table of Expected Loss Groups, and then figure out the expected retro premium using the correct Table of Expected Loss Groups.
Shortfall as a percent of 2015 guaranteed cost premium: (5.5670 - 5.5973) / 5.5973 = -0.54%.

16. (4 points) A company experiences an annual level of low-severity losses totaling $500,000 and periodic loss events as shown in the table below:

Period of Occurrence	Descriptor	Loss Amount
Once every Five years	Additional Low Severity Losses	$2,000,000
Once every Three years	Single Large Loss	$1,000,000

All loss events are independent of each other.

a. (1.25 points) The company and their insurer agree on a Large Dollar Deductible (LDD) policy with the following characteristics:

- minimizes effect of a single large loss
- guarantees reimbursable loss will not exceed $2,000,000
- results in expected annual reimbursable loss of $1,000,000

Design an LDD plan that meets the goals of the company. Note that the expected losses can be expressed as a function of the aggregate maximum and the per occurrence limit.

b. (1.75 points) Construct a Lee diagram showing the effect of the designed LDD plan structure on the loss profile of the company.

c. (1 point) The company is also considering a retrospective policy with the following characteristics:

- no per-occurrence limit
- same maximum entry ratio as in the LDD plan above

Assume the single large loss had an expected value of $500,000 instead of $1,000,000.
Describe the change to the Table M charge for the retrospective plan compared to the charge described in the LDD plan in a. above.

16. (a) In order to guarantee reimbursable loss will not exceed \$2,000,000,
we want a \$2 million aggregate limit.
There are four possible types of years:

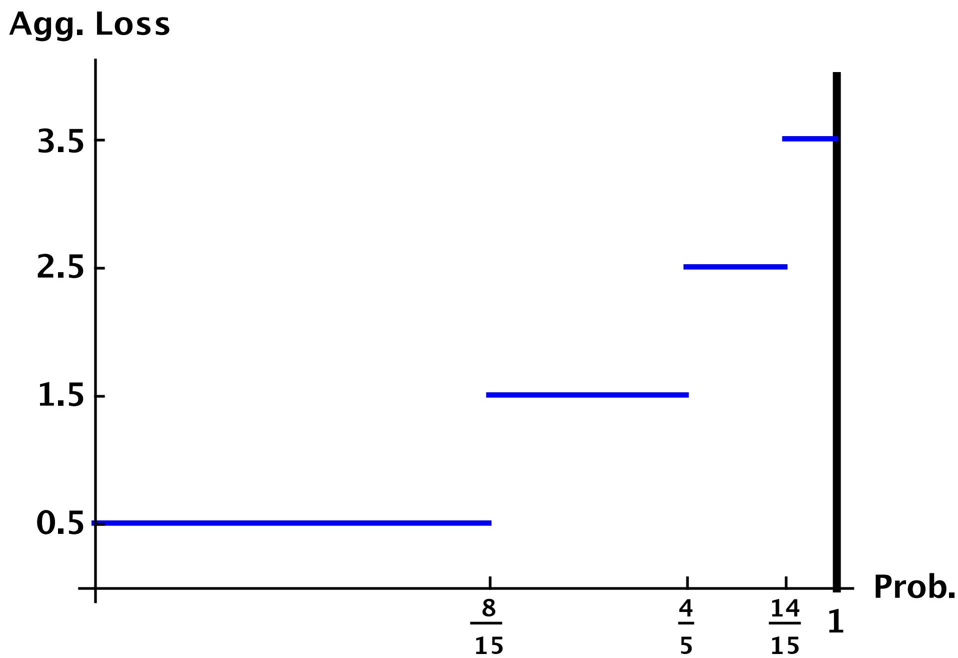
Probability	Low Severity Losses	\$1 Million Large Loss
(1/5)(1/3) = 1/15	2.5 million	Yes
(1/5)(2/3) = 2/15	2.5 million	No
(1/3)(4/5) = 4/15	0.5 million	Yes
(2/3)(4/5) = 8/15	0.5 million	No

Let us assume a per occurrence deductible of x.

Probability	Low Severity Losses	\$1 Million Large Loss	Reimbursements
(1/5)(1/3) = 1/15	2.5 million	Yes	2 million
(1/5)(2/3) = 2/15	2.5 million	No	2 million
(1/3)(4/5) = 4/15	0.5 million	Yes	0.5 million + x
(2/3)(4/5) = 8/15	0.5 million	No	0.5 million

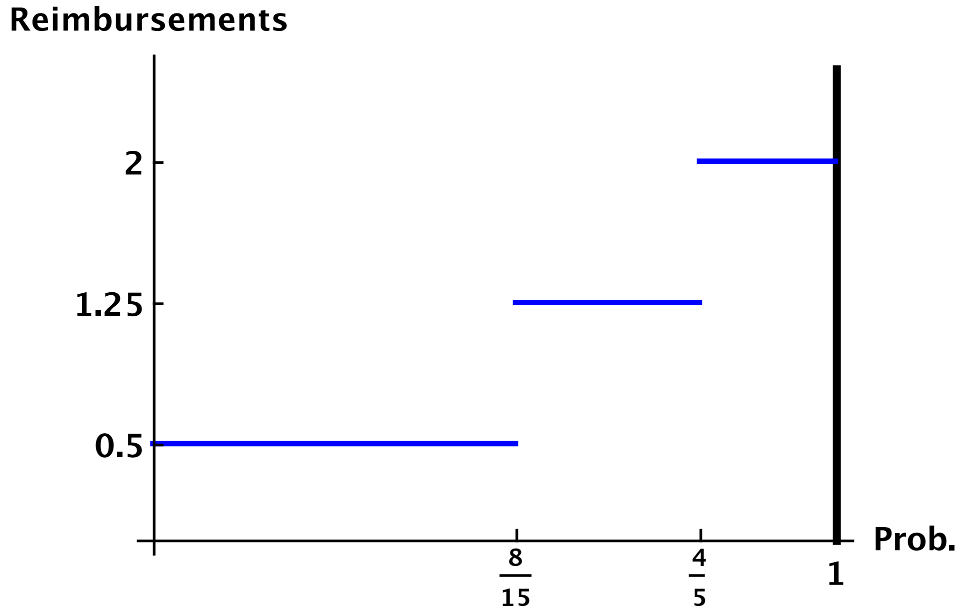
Thus: (1/15)(2) + (2/15)(2) + (4/15)(0.5 + x) + (8/15)(0.5) = 1. ⇒ x = \$**750,000**.

(b) Here is a Lee Diagram of the insured's annual aggregate losses (\$ million) prior to the LDD:

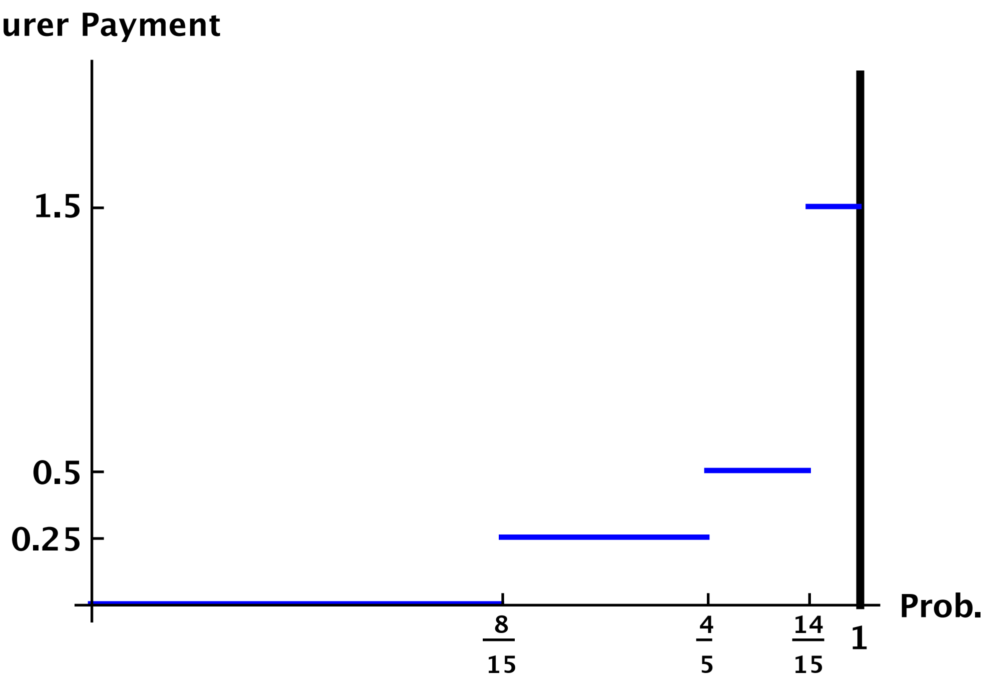


Probability	Low Severity Losses	$1 Million Large Loss	Reimbursements
(1/5)(1/3) = 1/15	2.5 million	Yes	2 million
(1/5)(2/3) = 2/15	2.5 million	No	2 million
(1/3)(4/5) = 4/15	0.5 million	Yes	1.25 million
(2/3)(4/5) = 8/15	0.5 million	No	0.5 million

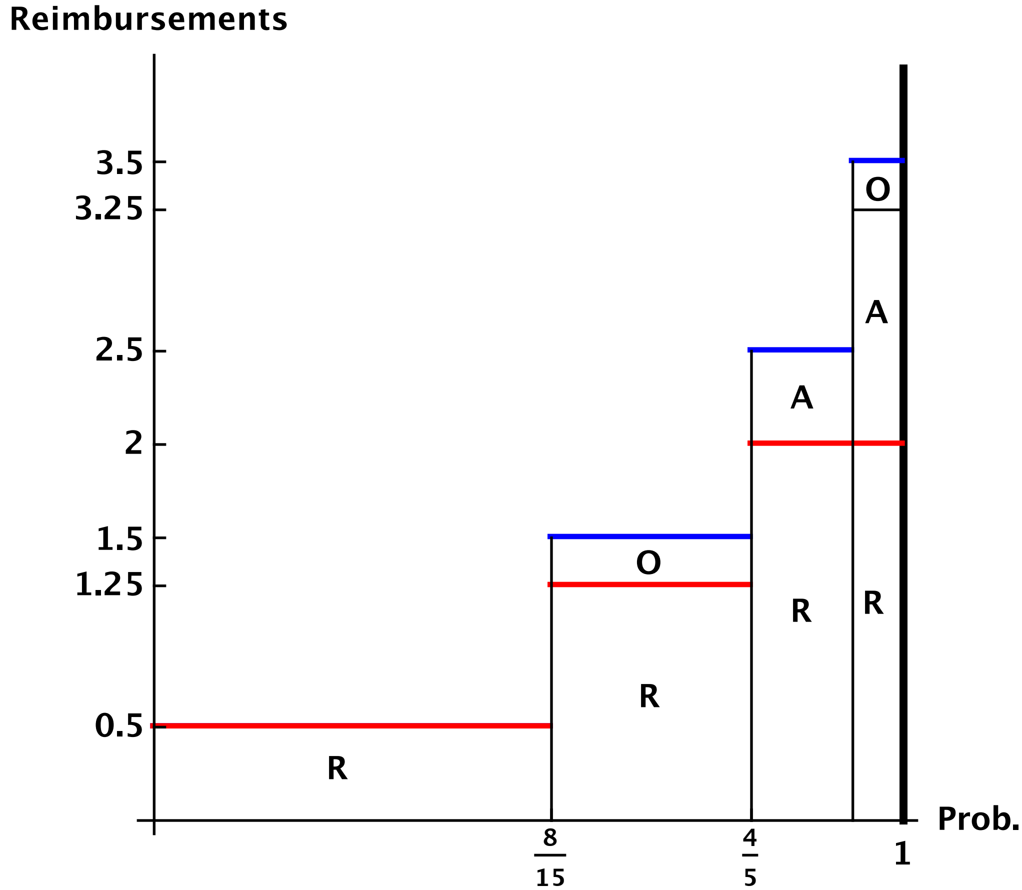
Here is a Lee Diagram of the insured's annual reimbursements ($ million) with the LDD:



Here is a Lee Diagram of the annual net amount the insurer pays ($ million) with the LDD:



Here is Lee Diagram, showing the losses prior to the LDD in blue, and the reimbursements by the insured with the LDD in red, both in millions of dollars:



Areas labeled R are reimbursements (retained losses), areas labeled O are eliminated from the point of view of the insured by the $750,000 per Occurrence limit, and areas labeled A are eliminated from the point of view of the insured by the $2 million Aggregate limit.
One could instead label the vertical axis using entry ratios. Using the $1.2333 million in expected aggregate losses, for example the entry ratio corresponding to 3.5 million is: 3.5/1.2333 = 2.838.

(c) For the retro policy, the decrease in the size of the large loss will decrease the volatility of the entry ratio distribution resulting in a smaller charge than if the large loss were $1 million. The effect of a decrease in the size of the large loss is in the same direction but of smaller magnitude on the LDD, since the occurrence limit already limits the impact of a single large loss.
Alternately, in the LDD, assume the “charge” is the expected excess of the $2 million aggregate limit after the per occurrence deductible. (Assume that the per occurrence deductible will be priced based on the size of occurrence distribution and then the amount to pay for the aggregate limit will be computed without overlap.)
After application of the occurrence deductible, in the two worst scenarios the amount the insured would retain in the absence of the aggregate limit are: 3.25 million and 2.5 million.
Then the ”charge” for the LDD Aggregate Limit is:
(1/15)(3.25 - 2) + (2/15)(2.5 - 2) = $0.15 million.

If one retains the same LDD, but the size of the large loss is $500,000 rather than $1 million, then:

Probability	Low Severity Losses	$500,000 Large Loss	Aggregate Loss
(1/5)(1/3) = 1/15	2.5 million	Yes	3 million
(1/5)(2/3) = 2/15	2.5 million	No	2.5 million
(1/3)(4/5) = 4/15	0.5 million	Yes	1 million
(2/3)(4/5) = 8/15	0.5 million	No	0.5 million

Now the $750,000 per occurrence limit has no effect, and in the two worst scenarios the amount the insured would retain in the absence of the aggregate limit are: 3 million and 2.5 million.
The "charge" for the LDD Aggregate Limit is now:
(1/15)(3 - 2) + (2/15)(2.5 - 2) = $0.1333 million.
The change in the size of the large loss has reduced the charge for the LDD Aggregate Limit by:
$0.15 million - $0.1333 million = **$0.01667 million**.
In the original situation, for the LDD the expected primary losses (losses retained by the insured) are: (8/15)(0.5) + (4/15)(1.25) + (1/5)(2) = 1 million.
Assume that the "maximum entry ratio in the LDD plan" means:
Aggregate Limit / Expected Primary Losses = $2 million / 1 = 2.
In the original situation, the expected aggregate loss is:
(1/15)(3.5) + (2/15)(2.5) + (4/15)(1.5) + (8/15)(0.5) = $1.233 million = 0.5 + 2/5 + 1/3.
A maximum entry ratio of 2 for the retro corresponds to: (2)(1.233) = $2.466 million.
With the original $1 million large loss, the charge at the maximum is:
(1/15)(3.5 - 2.466) + (2/15)(2.5 - 2.466) = $0.0735 million.
With instead the $500,000 large loss, the charge at the maximum is:
(1/15)(3.0 - 2.466) + (2/15)(2.5 - 2.466) = $0.0401 million.
The change in the size of the large loss has reduced the charge for the maximum in the retro by:
$0.0735 million - $0.0401 million = **$0.0334 million**.
The dollar effect on the charge of the change in the size of the large loss is larger for the retro plan than it was for the LDD.

Comment: Personally I found part (c) as worded to be almost indecipherable.
Apparently what they intended to ask was something like this:
"If the single large loss were of size of $500,000 instead of $1,000,000, this would affect both the impact of the maximum premium in the retro plan and the impact of the Aggregate Limit in the LDD.
In which direction would these effects be?
Without calculating these effects, which effect is larger in magnitude and why?"

17. (3.5 points) A large insured is considering a retrospectively rated policy for its workers compensation coverage with the following characteristics:

Standard premium:	$1,000,000
Unlimited expected loss ratio:	65%
Expense ratio:	20%
Loss conversion factor:	1.10
Premium tax rate:	4.0%
Maximum premium:	$1,200,000
Minimum premium:	$750,000

The actuary will use the following tables for rating:

Entry	Expected Loss Group (ELG)					
Ratio	31	30	29	28	27	26
0.70	0.4026	0.4000	0.3975	0.3949	0.3924	0.3898
0.80	0.3912	0.3826	0.3740	0.3656	0.3581	0.3489
0.90	0.3519	0.3426	0.3334	0.3242	0.3189	0.3060
1.00	0.3135	0.3037	0.2938	0.2839	0.2809	0.2642
1.10	0.2777	0.2673	0.2570	0.2464	0.2416	0.2254
1.20	0.2519	0.2413	0.2307	0.2200	0.2093	0.1986
1.30	0.2300	0.2194	0.2088	0.1981	0.1874	0.1766
1.40	0.2081	0.1975	0.1868	0.1761	0.1654	0.1547
1.50	0.1863	0.1756	0.1649	0.1542	0.1435	0.1327
1.60	0.1644	0.1537	0.1429	0.1322	0.1215	0.1107

ELG	Expected Loss Range
31	630,000 - 720,000
30	720,001 - 830,000
29	830,001 - 990,000
28	990,001 - 1,180,000
27	1,180,001 - 1,415,000
26	1,415,001 - 1,744,000

Per Occurrence Limit	Excess Loss Factor
$50,000	0.214

a. (0.5 point) Determine the guaranteed cost premium.

b. (2.25 points) Determine the basic premium for a $50,000 per occurrence limit.

c. (0.75 point) The insured's risk manager believes the retrospective premium can be reduced by selecting a higher per occurrence limit because the insured will assume a greater portion of the losses. Evaluate the risk manager's assertion.

17. (a) T = 1/(1 - 0.04) = 1/0.96.
Guaranteed cost premium as a percent of standard is: T(E + e) = (0.65 + 0.20)/0.96 = 88.5417%.
Guaranteed cost premium is: ($1 million)(88.5417%) = **$885,417**.
(b) LER = ELF/E = 0.214/0.65 = 0.329.
The ICRLL adjustment factor is: {1 + (0.8)(0.329)} / (1 - 0.329) = 1.883.
No State / Hazard Group relativity is mentioned, thus

LUGS = ($650,000)(1.883) = $1.224 million. ⇒ ELG 27.

(I have used the 2008 Table of Expected Loss Groups.)
Balance equations (with accident limit):

$$r_G - r_H = \frac{G - H}{c\,\hat{E}\,T} = \frac{1.20 - 0.75}{(1.1)(0.65 - 0.214)/0.96} = 0.901.$$

$$\hat{X}_H - \hat{X}_G = \frac{e + E - H/T}{c\,\hat{E}} = \frac{0.20 + 0.65 - (0.75)(0.96)}{(1.1)(0.65 - 0.214)} = 0.271.$$

Try $r_H = 0.70$, then $r_G = 0.70 + 0.90 = 1.60$.

$\hat{X}_G = 0.1215$. $\hat{X}_H = 0.3924$. $\hat{X}_H - \hat{X}_G = 0.3924 - 0.1251 = 0.2709$. OK.

$\hat{S}_H = 0.3924 + 0.70 - 1 = 0.0924$.

$b = c\,\hat{E}\,(\hat{X}_G - \hat{S}_H) + e - (c-1)E = (1.1)(0.65 - 0.214)(0.1215 - 0.0924) + 0.20 - (0.1)(0.65)$

$= 0.148956$.
Thus the basic premium is: (0.148956)($1 million) = **$148,956**.
(c) With a higher occurrence limit, the insured will pay for more of its own losses. More specifically when the insured has large occurrences usually more dollars will enter the retro calculation.
Thus in years in which the insured has several large occurrences, its retro premium would usually be more than it would have been with the lower occurrence limit in the first retro plan.
However, the ELF and thus the excess loss premium will be less than it would have been with a lower occurrence limit. Thus in years in which the insured has no or very few large occurrences, its retro premium would be less than it would have been with the first retro plan.
Thus which retro plan would produce the lower premium depends on the insured's experience, specifically how many large occurrences there turn out to be in the year being retro rated.
The new retro plan balances to the same guaranteed cost premium in part (a) as the plan in part (b).
Thus, on <u>average</u> the retro premium would be the same in both cases.

18. (2.5 points) An actuary prices a retrospectively rated policy with the following provisions resulting in a balanced plan:

Minimum premium: 17,500
Maximum premium: 80,000
Expense provision: 8,125
Loss conversion factor: 1.25

- Aggregate losses follow a uniform distribution between $0 and $100,000
- No taxes

After pricing the policy, the actuary discovers an error in the original loss distribution and determines that losses should instead follow a uniform distribution between $0 and $90,000.
The actuary decides to re-balance the plan based on the corrected distribution while still maintaining the same minimum and maximum premium.
Calculate the loss at minimum premium and loss at maximum premium that re-balance the plan.

18. We are given T = 1.

Losses follow a uniform distribution between \$0 and \$90,000. ⇒

E = 45,000 and the entry ratios are uniform from 0 to 2.
Balance equations (no accident limit):

$$r_G - r_H = \frac{G - H}{c\,E\,T} = \frac{80{,}000 - 17{,}500}{(1.25)(45{,}000)(1)} = 1.111.$$

$$X_H - X_G = \frac{e + E - H/T}{cE} = \frac{8215 + 45{,}000 - 17{,}500/1}{(1.25)(45{,}000)} = 0.6333.$$

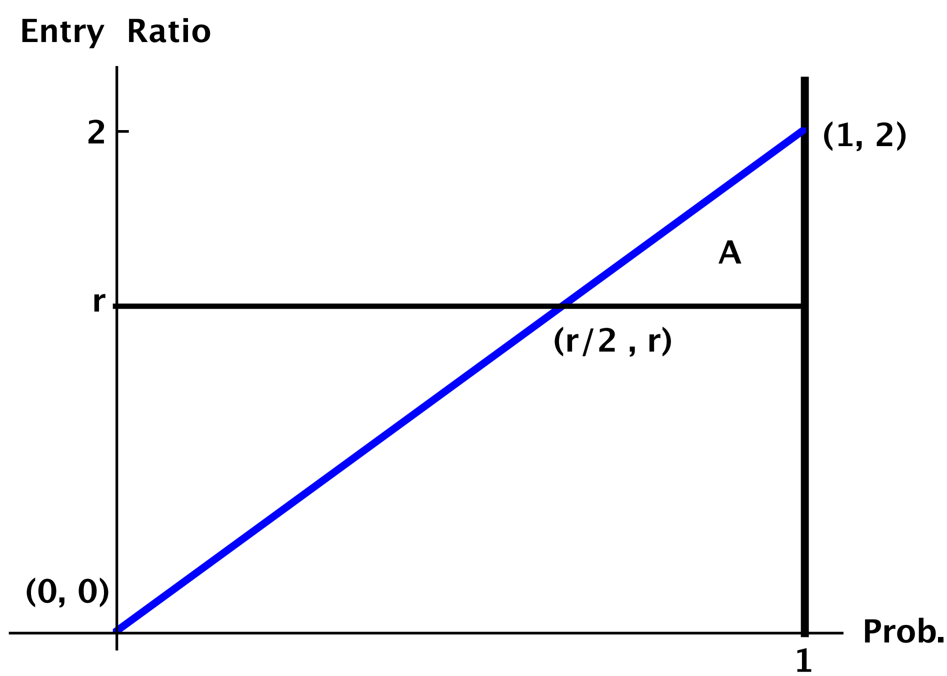
The insurance charge for an entry ratio of $r \le 2$ is: $\int_r^2 (1/2)\,(x - r)\,dx = (2 - r)^2 / 4.$

$0.6333 = X_H - X_G = (2 - r_H)^2 / 4 - (2 - r_G)^2 / 4 = (3.111 - r_G)^2 / 4 - (2 - r_G)^2 / 4.$ ⇒

$2.533 = 9.678 - 6.222 r_G + r_G^2 - (4 - 4r_G + r_G^2).$ ⇒

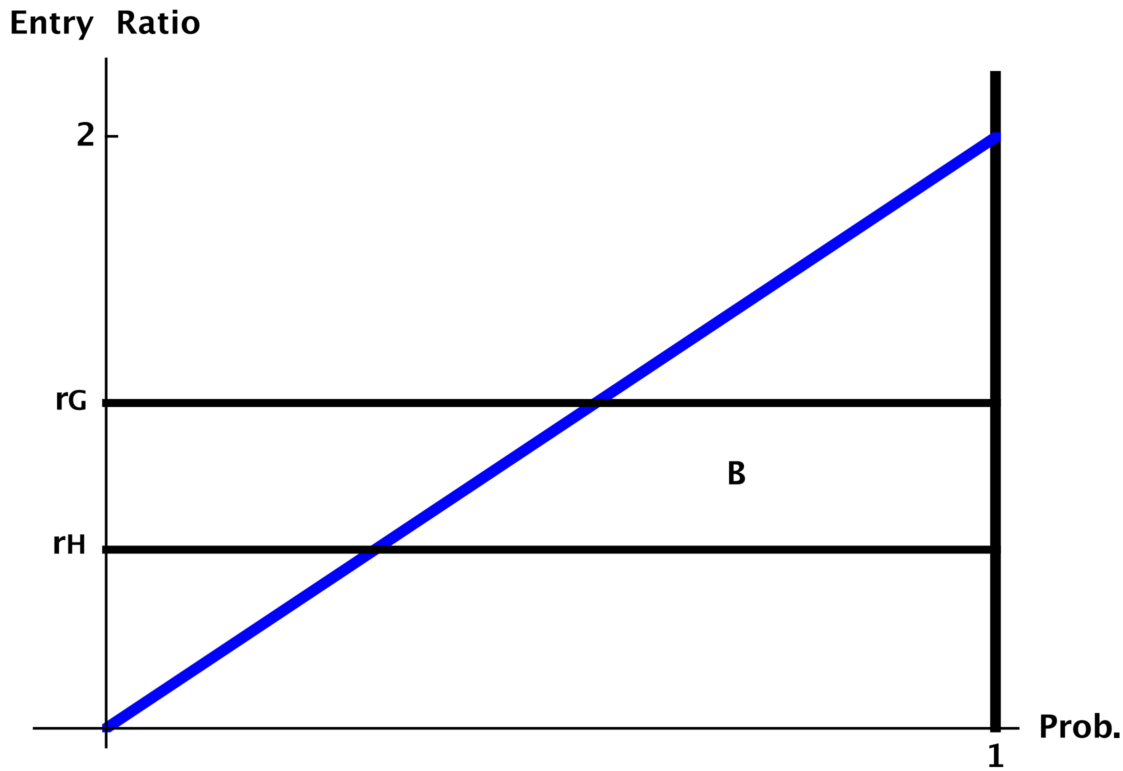
$r_G = 3.145/2.222 = 1.415.$ ⇒ $r_H = 1.415 - 1.111 = 0.304.$

The loss at the minimum premium is: (0.304)(\$45,000) = **\$13,680**.
The loss at the maximum premium is: (1.415)(\$45,000) = **\$63,675**.
Alternately, the insurance charge at r, is Area A in the following Lee Diagram:



Area A is a triangle with height 2 - r and width (2- r)/2, and thus area: (1/2)(2-r)(2-r)/2 = $(2 - r)^2 / 4$.
Note that the line connecting (0, 0) and (1, 2) has slope of 2. Proceed as before.

Alternately, in the following Lee Diagram, Area B is: $X_H - X_G$.



Area B is a trapezoid with height $r_G - r_H = 1.111$, and widths $1 - r_H/2$ and $1 - r_G/2$.

Thus Area B = $(1.111)(1 - r_H/2 + 1 - r_G/2)/2$. ⇒

$0.6333 = (1.111/2)(2 - r_H/2 - r_G/2)$. ⇒ $r_H + r_G = 1.720$.

Also $r_G - r_H = 1.111$. ⇒ $r_G = 1.415$. ⇒ $r_H = 0.304$.

The loss at the minimum premium is: (0.304)($45,000) = **$13,680**.
The loss at the maximum premium is: (1.415)($45,000) = **$63,675**.
Comment: One can ignore the fact that the actuary originally thought the losses were uniform from 0 to 100,000; this adds nothing but confusion to the question.
For the losses uniform from 0 to 100K, E = $50,000 and the entry ratios are uniform from 0 to 2.
Both the intended solution and I have used the given expense provision as e, which includes LAE; however, if one subtracts the LAE, then the remaining provision is negative:
8125 - (25%)(45,000) = -3125.
Also the expense provision including LAE should change as the expected losses change from 50,000 to 45,000 and thus the LAE changes from 12,500 to 11,250.
The examiners seemed to be unaware of both of these potential issues. In my opinion, a poorly constructed question that penalizes those more knowledgeable students who stop to try to figure out what is going on.

19. (2.75 points) A company is considering options for a workers compensation policy with the following parameters:

Item	Value	Applicable to:
Standard Premium	$1,000,000	
Expected Loss and ALAE	900,000	
Per Occurrence Deductible	250,000	Loss and ALAE
Aggregate Limit	1,000,000	Loss and ALAE
ULAE	7.0%	Loss and ALAE
Loss Based Assessment	5.0%	Loss and ALAE
General Overhead	5.0%	Standard Premium
Credit Risk	4.0%	Standard Premium
Acquisition Expense	5.0%	Net Premium
Profit and Contingency	2.5%	Net Premium
Tax and Assessment	8.0%	Net Premium

Limit	ELPPF
$250,000	0.20
500,000	0.09
750,000	0.06
1,000,000	0.03

Entry Ratio	Insurance Charge
1.11	0.18
1.25	0.13
1.39	0.09
1.50	0.06

a. (1 point) Calculate the premium for a large dollar deductible (LDD) policy.
b. (0.75 point) Identify three reasons why an employer might choose an LDD Plan.
c. (0.5 point)
Assume the following revised assumptions for an excess workers compensation (WC) policy:

- Profit and Contingency = -1.5%
- Tax and Assessment = 3%

Calculate the revised premium.
d. (0.5 point) Describe the difference in profit and tax assumptions for LDD and excess WC policies.

19. (a) General Expense: (5%)(1 million) = \$50,000.
Credit Risk: (4%)(1 million) = \$40,000.
Insurer settles all claims, so load total losses and ALAE for ULAE: (7%)(900,00) = \$63,000
The ELPPF for the \$250,000 per occurrence limit is: 0.20.
Thus the expected excess loss plus ALAE is: (0.20)(\$900,000) = \$180,000.
Primary loss and ALAE is: 900,000 - 180,000 = \$720,000.
Entry Ratio = Aggregate Limit / Expected Primary Loss & ALAE = \$1 million / \$720,000 = 1.39.
The Insurance Charge for an entry ratio of 1.39 is 0.09.
Thus the Charge for the Aggregate Limit is: (0.09)(\$720,000) = \$64,800.
Assume loss based assessments are to be paid on total loss not just excess loss.
The loss based assessments are: (5%)(900,000) = \$45,000.

LDD Premium is: $\dfrac{50{,}000 + 40{,}000 + 63{,}000 + 180{,}000 + 64{,}800 + 45{,}000}{1 - 0.05 - 0.025 - 0.08}$ = **\$524,024**.

(b) 1. One does not have to qualify for self-insurance, while getting some of the same advantages.
2. Gives employers more control over their primary loss layer (losses below the deductible).
3. Saves employers a sizable portion of the taxes and assessments associated with Workers Compensation insurance.
4. If converting from a fully insured plan, an LDD allows the employer to remain with an insurer whose operation and customer service it is already familiar.
5. An LDD plan is similar to a loss sensitive insurance plan (e.g. a paid loss retro plan) under which the employer may be currently insured.
6. Under an LDD plan the insurer still handles all claims. The insured can be comfortable with an experienced claims adjusters still handling all of the claims. “An insurer who is equipped with numerous years of experience in handling WC claims may be the best candidate to provide quality services to its customers.” (Under an excess policy or self-insurance the employer will commonly use a Third Party Administrator to handle claims.)
7. There are cashflow advantages compared to a full coverage policy. An employer will get to hold onto cash longer; the insurer will pay losses first and then seek reimbursement from the employer.
8. From the Examiner’s Report, “There is a tax incentive to the insured; the insured can deduct a tax liability for an unpaid deductible but not on a liability for a loss reserve.” Self-insurance reserves are not generally deductible.

(c) The general expenses would usually be lower for the excess policy than the LDD, but in the absence of any information I will use the same value as for the LDD.
There is no credit risk and the insurer will not have to pay loss based assessments.
In the absence of any information, I will assume the ULAE is the same % but only applied to excess losses & ALAE including the charge for the aggregate limit: (0.07)(180,000 + 64,800) = \$17,136.
In the absence of any information, I will assume the acquisition % is the same.

Excess Policy Premium is: $\dfrac{50{,}000 + 17{,}136 + 180{,}000 + 64{,}800}{1 - 0.05 + 0.015 - 0.03}$ = **\$333,621**.

(d) Unlike LDD, excess policies are not subject to Workers Compensation residual market assessments, loss based assessments, or taxes associated with administering the WC system. Excess WC premium is usually taxed as a General Liability product, rather than a WC product. The profit provision for LDD is often determined by the market. However, since the excess policy does not have a service component, the profit assumption is always determined by the market, which is competitive. "In Excess WC, however, there is very little service involved. Therefore, the insurers compete almost exclusively on price, and this usually drives down the profit margin." Also Excess Workers Compensation policies have a longer average cashflow than LDD. For these two reasons, the profit allowance in Excess Policies is usually smaller than for a similar LDD.

LDD Plan	Excess Policy
Profit is higher since insurer provides both service and excess coverage.	Profit is lower since the insurer provides excess coverage and competes solely on price.
Shorter average cashflow.	Longer average cashflow.
Tax rate is that for Workers Compensation.	Tax rate is that for General Liability.
Pays Residual Market and other assessments.	Does not pay assessments.

<u>Comment</u>: In part (a) it should really be an ELAEPPF rather than ELPPF since the limit is applicable to loss & ALAE; however, Teng uses an ELPPF here.
In parts (a) and (c), as per Fisher, I took the Insurance Charge to be as a percent of Expected <u>Primary</u> Loss and ALAE. According to the Examiner's Report: "Since it was not specified whether the insurance charge would apply to limited or unlimited losses, both answers were accepted."
In part (b) see page 418 of Teng. Gillam and Snader give four similar reasons at their page 50.

20. (1.5 points) An insurer is evaluating the experience of an annual umbrella policy that is renewing on April 1, 2015.

Attachment Point:	$3,000,000
Policy Limit:	10,000,000

- The insurer paid a $4,000,000 claim for a loss that occurred on July 1,2014.
- Trend is applied to the midpoint of a prospective policy year

a. (0.75 point) The insurer has trended the July 1, 2014 claim to $4,395,940.
Determine the insurer's annual trend factor.

b. (0.75 point) Explain how the upward drift of policy limits and attachment points on the underlying and umbrella policies can distort the trending of historical losses and if the trended claim would likely be overstated or understated had the loss occurred in 2004.

20. (a) Assume that this loss is <u>not</u> one for which the umbrella policy had to drop down in order to fill a coverage gap in the primary policy. Also assume that the attachment point of the umbrella policy that was in effect during 2014 was also $3 million (and the policy limit was at least 4 million.)
Then the ground up size of loss was: 3 + 4 = 7 million.
The trend to date is 6 months beyond April 1, 2015; thus the trend period is 1.25 years.
The trended ground up loss was: 3 million + 4,395,940 = 7,395,940.
$\Rightarrow (1+r)^{1.25}$ (7) = 7.395940. $\Rightarrow$ r = **4.5%**.

(b) Assume for simplicity that there is one underlying policy and that each year the limit of the underlying policy matches the attachment point of the umbrella policy.
Policy limits of the underlying policy and of the umbrella policy both tend to drift up over time with inflation; however, they also change due to other changes in the circumstances of the insured.
In any case, changes in policy limits usually occur at irregular intervals and do not match the continuous effects of inflation.
Let us for example assume 50% inflation between 2004 and 2014. Then if the loss had been in 2004 it would have been of ground up size: 7/1.5 = 4.666. Assuming an umbrella policy in 2004 with attachment point of 3/1.5 and limit 10/1.5, then the umbrella policy would have paid 7/1.5 - 3/1.5, and we could determine the ground up loss to be: 7/1.5 = 4.666. Then we could appropriately trend it to 2014 when it would have been 7.
Thus the changes in policy limits and attachment points should create no inherent problem if they keep up with inflation and provided we have complete information about the past.
Unfortunately, the policy limit in 2004 could have been anything, the umbrella policy limit could have been anything, this insurer may not have written an umbrella policy for the insured who had the loss, etc. The trended claim could be either overstated or understated had the loss occurred in 2004.
Alternately, if we mistakenly assume that the attachment point in 2004 was 3 million, the same as currently, when in fact it was 2 million, we would <u>overestimate</u> the trended loss. A $4 million payment in 2004 would be mistakenly assumed to come from a 3 + 4 = 7 million loss, while it in fact came from 6 million ground up loss; this would result in a trended loss that is bigger than it should be.
The upward drift of policy limits and attachment points can distort trending of historical losses if historical losses are mistakenly made subject to the <u>current</u> limits and attachment points before being trended to the prospective year.
<u>Comment</u>: While umbrella policies are mentioned at pages 31-33 of Clark in the context of pricing reinsurance, the details of umbrella polices are not covered in the syllabus readings.
"This procedure will still leave out losses from the underlying policy which historically did not exhaust the underlying limit, but which would have after the application of a trend factor."
In part (b), the alternate solution presented is that intended by the CAS, which personally would have required me to read the mind of the question writer in order to know what he or she had in mind.

21. (3.5 points) Company A is pricing an umbrella policy with an effective date of July 1, 2014 for a large commercial risk that is written above Company B's underlying layer. Company A is considering various options for the treatment of allocated loss adjustment expenses (ALAE).

a. (2 points)

Fully explain how the following options rank with respect to the relative cost of the umbrella policy:

1. ALAE is included within both the underlying and umbrella limits.
2. ALAE is in addition to both the underlying and umbrella limits.
3. ALAE is included within the underlying layer's limit, but is in addition to the umbrella's limit.

b. (1 point) The following loss history of the risk is provided by Company B below:

Date of Loss	Paid Loss	Paid ALAE	Reserved Loss	Reserved ALAE
September 16, 2009	$240,030	$324,235	$0	$0
December 14, 2009	43,658	8,750	0	0
March 1, 2010	2,000,000	140,000	0	0
March 14, 2010	50,000	891,320	0	0
August 11, 2010	0	75,500	25,000	174,500
January 2, 2011	1,257,902	124,870	0	0
March 14, 2012	200,000	45,040	1,800,000	55,960
July 1, 2012	32,320	175,340	0	0
November 30, 2012	1,000,000	22,430	1,000,000	250,000

Explain two reasons why the pricing actuary for Company A should be wary of using the historical data to determine the ALAE load.

c. (0.5 point) The pricing actuary has been asked to opine on the treatment of ALAE for a clash cover excess treaty. The actuary states that because penetration of high excess layers is infrequent, the calculation of ALAE is difficult and the loading is insignificant. Evaluate this statement.

21. (a) Assuming the underlying policy is CGL, ALAE is not included in the limit; to do so would be impractical and I do not know how it would work.
It is unclear what would be meant by including ALAE in the limit of the umbrella policy. The insurer who wrote the umbrella policy is not responsible for paying any portion of the ALAE incurred by the insurer who wrote the primary policy. The writer of the umbrella policy may choose to get involved in the settlement or trial relating to large claims and thus incur some its own ALAE. Any ALAE incurred by the writer of the umbrella policy either due to such situations or due to the umbrella policy dropping down would not be included in the limit; to do so would be impractical and I do not know how it would work. Option #2 is how things work in the real world; nevertheless I will try my best to answer this exam question.
The greatest cost would be option #3, because including ALAE in the limit of the underlying policy will mean more dollars will reach the umbrella layer, and for the umbrella policy ALAE is in addition to the umbrella limit.
For option #1, including ALAE in the limit of the underlying policy will mean that more dollars are reaching the umbrella layer than option #2; however, including ALAE in the limit of the umbrella policy caps the total loss & ALAE paid by the umbrella policy, resulting in less paid than option #3.
For option #2, not as many dollars will reach the umbrella layer than for the other two options. However, the umbrella policy could in theory pay more for loss plus ALAE than its limit. There should be no ALAE for the umbrella to pay from the underlying policy since the underlying policy has paid it all. Thus, option #2 is likely to be less expensive for the umbrella policy than option #1. However, if the umbrella carrier incurs a lot of its own ALAE, option #2 could be more expensive than option #1.
On average I would expect option #1 to be more expensive than option #2, since the effect of more losses reaching the umbrella layer for option #1 should be greater than the effect of the possibility of more ALAE being paid under option #2.
For example, let us assume that Company B writes a policy with a limit of $100K, and Company A writes an umbrella policy with a limit of $400K.
Let assume we have a large claim with $400K in loss.
Company B incurs $100K in ALAE and Company A incurs $50K in ALAE.
1. ALAE is included within both the underlying and umbrella limits.
B pays nothing to the third party since it has incurred 100K of ALAE, exhausting the limit.
A pays 350K to the third party, since it has incurred 50K of ALAE.
(Presumably the insured is responsible for the remaining 50K owed to the third party.)
2. ALAE is in addition to both the underlying and umbrella limits.
B pays 100K to the third party claimant as well as 100K for its ALAE.
A pays the remaining 300K to the third party claimant as well as 50K for its own ALAE.
3. ALAE is included within the underlying layer's limit, but is in addition to the umbrella's limit.
B pays nothing to the third party since it has incurred 100K of ALAE, exhausting the limit.
A pays 400K to the third party, as well as 50K for its own ALAE.
For this example, Option 3 costs the most for the umbrella policy, followed by Option 1, with Option 2 costing the umbrella policy the least.

(b) The umbrella policy will not be responsible for any of the ALAE incurred by Company B, and Company B will pay its own ALAE in addition to any policy limits. Thus there is in fact no need for Company A to pay any attention at all to the ALAE data supplied by Company B.
In any case, the data supplied by Company B is too sparse to be credible; the ratio of ALAE to loss is volatile.
Company A should be particularly concerned with its ALAE costs when the umbrella policy drops down. Such claims will be for perils not covered by the policy written by Company B, and thus the ALAE may have a significantly different relationship to loss.
"ALAE is not a constant percent of any given loss. For example, losses which close without an indemnity payment may still incur a large expense. In general, as the size of a loss increases, the ALAE as a percent of the loss will tend to decrease. The assumption that loss and ALAE are perfectly correlated will tend to result in an overstatement of expected amounts in the higher layers."
Thus since the umbrella policy is covering high layers of loss, Company A is only concerned with ALAE on large claims, which will be a smaller percent of losses on average than for all claims.
Also open claims will be affected by loss dispersion; in many cases the paid plus reserves will develop upwards. Thus as explained in Mahler's Discussion of "Retrospective Rating: 1997 Excess Loss Factors" the percent of losses and ALAE that will expose the umbrella treaty at ultimate are more on average more than at immature reports such as in the data from Company B.
From the Examiner's Report: "The underlying layer appears to have a limit of 2 million. It looks like losses may be capped, which distorts the ALAE ratio."
From the Examiner's Report: "There are multiple losses where paid ALAE exceeds paid loss. This could be evidence of extensive litigation and aggressive defense of claims that could inflate the ALAE ratio."
(c) Because penetration of high excess layers is infrequent, the calculation of ALAE is difficult.
If ALAE is included with loss for the attachment point of the clash treaty,then large ALAE may expose the clash layer. (This is sometimes called runaway ALAE.)
Clash covers are also penetrated when extra-contractual obligations (ECO) or rulings awarding damages in excess of policy limits (XPL) are determined in a settlement, both of which can have significant associated ALAE. **Thus there is no reason to assume the loading for ALAE is an insignificant part of the cost of the clash cover**; it is an empirical question.
Comment: The writer of this question and I disagree about how commercial liability policies and commercial umbrella policies work in the real world.
Part (b) of this question is not covered in the syllabus readings on this exam.
Part (c) is briefly referred to by Clark at pages 22 and 23 "Clash Covers: High layer attachment excess - typically a loss on a single policy will not penetrate the treaty layer. A clash cover will be penetrated due to multiple policies involved in a single occurrence, or when extra-contractual obligations (ECO) or rulings awarding damages in excess of policy limits (XPL) are determined in a settlement. The method for including allocated loss adjustment expenses in the treaty may also expose the clash layer." Prices for clash covers are driven by market conditions and are based on a lot of actuarial and underwriting judgement.

22. (2.5 points)
The following information is given for a one-year reinsurance treaty effective January 1, 2012.

Risk	Effective Date	Insured Value	Loss	Loss Date
1	June 1, 2011	$200,000	$100,000	March 1, 2012
2	January 1, 2012	500,000	400,000	June 1, 2012
3	June 1, 2012	1,500,000	1,500,000	September 1, 2012

a. (0.5 point) Calculate the reinsurer's loss liability for a 30% quota share agreement written on a risks attaching basis.
b. (1 point) Calculate the reinsurer's loss liability for a 5-line surplus share agreement written on a losses occurring basis with a retained line of $100,000.
c. (1 point) The primary insurance company purchased a treaty on a risks attaching basis for the time period from January 1, 2013 to December 31, 2013 time.
They then decided to purchase the same treaty but on a losses occurring basis for the time period from January 1, 2014 to December 31, 2014.
Describe a coverage issue that could arise with these two treaties.
Explain how the ceding company and reinsurer can structure the treaties to avoid this issue.

22. (a) On risks attaching basis, the reinsurer is essentially reinsuring policy year 2012,
so only risks 2 and 3 are covered by the treaty.
The reinsurer pays: (30%)($400,000 + 1,500,000) = **$570,000**.
(b) On losses occurring basis, the reinsurer is essentially reinsuring accident year 2012,
so all three losses are covered by the treaty.
The percent ceded depends on the insured value. The reinsurer pays:
($100,000)(200 - 100)/200 + ($400,000)(500 - 100)/500 + ($1,500,000)(500/1500) =
$870,000.
(c) The first treaty in essence covers PY2013 while the second treaty in essence covers AY2014.
However, some losses on policies written in 2013 will occur during 2014. Therefore, some losses would be paid for under each treaty. Not only would the insurer be reimbursed twice for the same loss, but there would be a morale hazard. The insurer would have an incentive to pay more for a claim so that after being reimbursed twice it would end up in better shape.
One could include in these treaties an “interlocking clause”, designed to equitably apportion losses that may be covered under more than one contract.
Alternately, the reinsurer may add a clause in the 2014 treaty to exclude claims that would be covered by the 2013 treaty.
Alternately, commute the risks attaching policy at 1/1/14 and replace it with the loss occurring treaty.

23. (2 points) While negotiating the terms of a quota share treaty, a reinsurer is considering an agreement with a sliding scale commission and an agreement with a loss corridor.
The terms of the sliding scale commission are as follows:

Provisional Commission:	20% at a 65% loss ratio
Sliding 1: 1 to a minimum:	10% at a 75% loss ratio
Sliding 0.5:1 to a maximum:	30% at a 45% loss ratio

The terms of the loss corridor are as follows:

Commission:	15%
Loss Corridor:	60% of 75% to 85% loss ratio

The expected loss ratio is 73%
a. (0.5 point) Assume that the ceding company is interested in optimizing the amount and the timing of cash flows. Identify and briefly describe one advantage of the sliding scale commission option when compared to the loss corridor option.
b. (1 point) The insurer and reinsurer agree that the performance of the underlying business is highly volatile. Describe two ways the insurer can stabilize its results for its sliding scale commission structure over time.
c. (0.5 point) Explain whether the smoothing mechanisms in part b. above should be used in the determination of an aggregate loss distribution model.

23. (a) Under the sliding scale commission, the ceding company gets a provisional 20% which is later adjusted up or down when the experience under the treaty becomes available.
Instead under the loss corridor option the ceding company gets a 15% commission when the treaty is written, which is later in essence reduced if the loss ratio under the treaty exceeds 75%.
Thus the ceding insurer is better off initially with the sliding scale commission option.
The ceding insurer will receive more cash up front with the sliding scale commission option, which they can invest; with the sliding scale commission the ceding insurer has a timing cashflow advantage compared to the loss corridor.
Under the sliding scale commission:

Loss Ratio	Commission
45%	30%
65%	20%
70%	15%
75%	10%
80%	10%
85%	10%
95%	10%

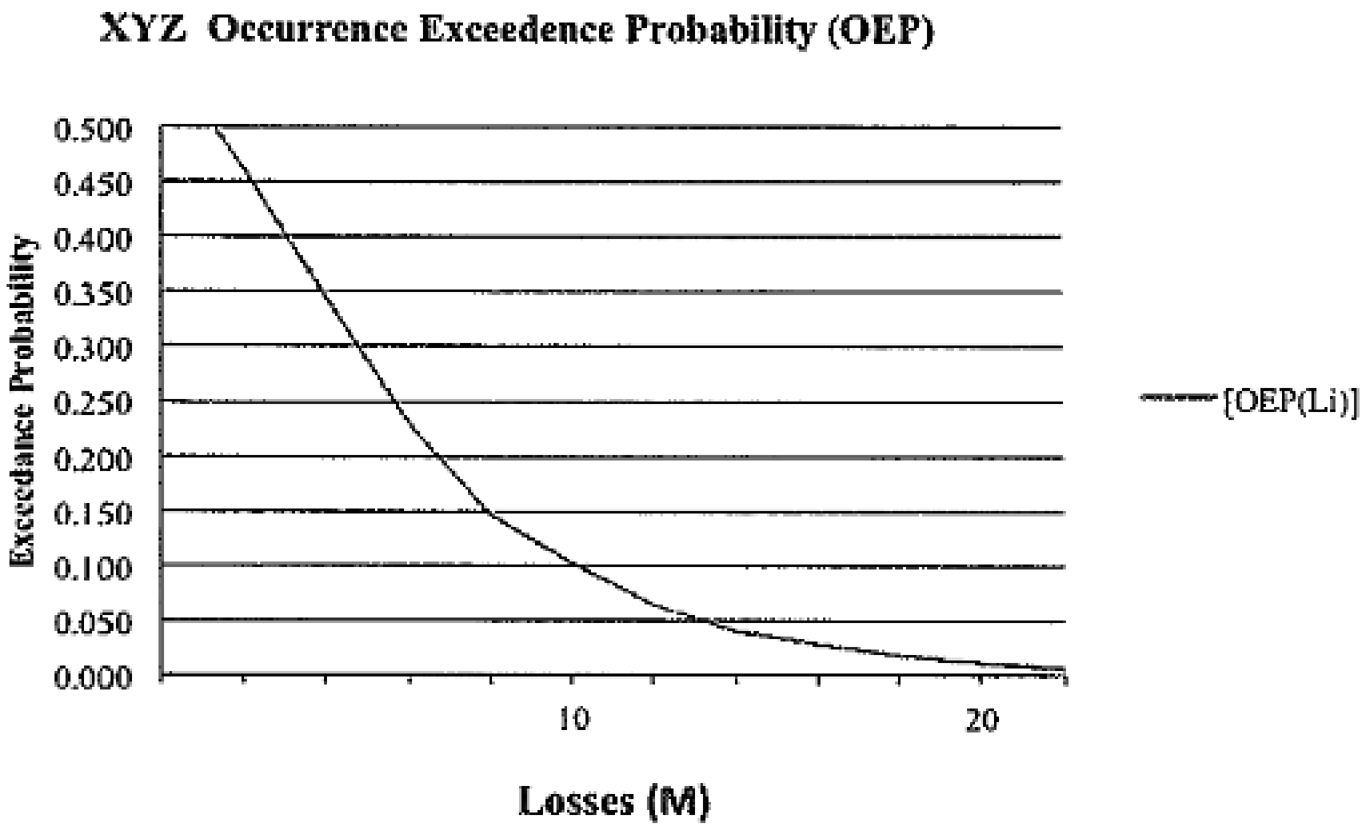
Under the Loss Corridor:

Loss Ratio	Commission	Loss Corridor Effect	Commission - Loss Corridor
45%	15%	0	15%
65%	15%	0	15%
70%	15%	0	15%
75%	15%	0	15%
80%	15%	(60%)(80% - 75%) = 3%	12%
85%	15%	(60%)(85% - 75%) = 6%	9%
95%	15%	6%	9%

Thus once the experience is known, for most loss ratios the ceding company is better off with the sliding scale option. However, at loss ratios between 70% and 83.3% the ceding company would be somewhat better off with the loss corridor option. There is no information on the distribution of loss ratios needed in order to calculate an average, but it is likely that on average the ceding company is also better off eventually with the sliding scale commission option.
(b) A carryforward provision allows that if the past loss ratios have been above the loss ratio corresponding to the minimum commission, then the excess loss amount can be included with the current year's loss in the estimate of the current year's commission. In the long run, this should help smooth the results for both the insurer and the reinsurer. One could also allow a case where the past loss ratios have been below the loss ratio corresponding to the maximum commission, then the credited loss amount can be included with the current year's loss in the estimate of the current year's commission.
Another way to stabilize results would be to reduce the maximum commission and increase the minimum commission in the schedule. Alternately, one could change the slopes; for example, sliding 0.5: 1 to a minimum: 10% at an <u>85%</u> loss ratio.

(c) Two approaches may be taken to pricing the impact of carryforward provisions. The first is to include any carryforward from past years and estimate the impact on the current year only. This amounts to shifting the slide by the amount of the carryforward. (It does not really affect the determination of an aggregate loss distribution model.) The problem with this approach is that it ignores the potential for carryforward beyond the current year. For example, in the first year of the program we would calculate the expected commission for the current year as though the program would be cancelled at the end of the year. The same price would result with or without the carryforward provision, which does not seem right because the benefit of the carryforward is ignored.
A second approach is to look at the "long run" of the contract. The sliding scale is modeled as applying to a longer block of years rather than just the single current year. The variance of the aggregate distribution would be reduced on the assumption that individual bad years would be smoothed by good experience on other years. The variance of the average loss ratio for a block of years should be significantly less than the variance of the loss ratio for a single year.
The first problem with this approach is that the method for reducing the variance is not obvious.
A second problem is that it ignores the fact that the contract may not renew the following year, potentially leaving the reinsured with no carryforward benefit. This issue is further complicated when a commission deficit can be carried forward but not a credit.
If the maximum commission, minimum commission, or slopes are changed this should be taken into account in pricing, although it does not affect the determination of an aggregate loss distribution model.
Comment: In my opinion, this question is poorly worded.
Part (c) does not flow smoothly from part (b).
“c. Explain whether the smoothing mechanisms in part b. above should be used in the determination of an aggregate loss distribution model.” However, part (b) does not specify that the mechanisms be “smoothing.” Also “ways the insurer can stabilize its results for its sliding scale commission structure over time” from part (b) do not directly translate into models of aggregate loss distributions used to quantify the effect of the commission structure. Rather the actuary has to make some adjustment to the model or his use of the model of aggregate loss distributions in order to try to quantify the impact of whatever change is made to the sliding scale commission structure in part (b).
Based on the Examiner’s Report, the examiners do not seem to realize that if one stabilizes the insurer’s results for its sliding scale commission structure over time, then one has also automatically stabilized the reinsurer’s results for its sliding scale commission structure over time, and vice-versa.
Part (a) says in part “the ceding company is interested in optimizing the amount and the timing of cash flows.” Yet based on the Examiner’s Report, the examiners seemed to ignore any comparison of the amount of the eventual cashflows under the two options, which of course would be a very important consideration for the ceding company.
In part (b), apparently the examiners intended the two ways the insurer can stabilize its results for its sliding scale commission structure over time to be “the carryforward provision and its extension in considering a block of years to model the carryforward.” I do not consider this two different ways; I believe that the examiners confused the provision in the treaty and different ways of pricing it.

24. (2.25 points) A reinsurer is offering a property catastrophe cover of $10,000,000 in excess of $10,000,000 per occurrence with one reinstatement to an insurer. The reinsurance broker produces a catastrophe model with the following output for losses in excess of $10,000,000.



a. (0.5 point) Calculate the pure premium using the payback approach.

b. (0.5 point) The treaty incepts on July 1, 2014 with a premium of $1,200,000. The insurer experiences a catastrophe loss on December 1, 2014 resulting in total loss amount of $15,000,000. Calculate the reinstatement premium given the reinstatement provision is 115% pro-rata as to amount.

c. (1.25 points) The insurer also purchases a 30% quota share treaty which inures to the benefit of the catastrophe treaty. Calculate the amount paid under each treaty and the insurer's net loss.

24. (a) OEP is the probability that at least one event exceeds the specified loss amount in a year.
There is about a 10% probability of at least one occurrence during a year exceeding $10 million.
Therefore, it should take 1/10% = 10 years to pay back. Thus the pure premium for the treaty is the full layer loss divided by the payback period: $10,000,000 / 10 = **$1,000,000**.
Alternately, assume the average payout in those cases is about 1/2 of the treaty limit of $10m.
Then the estimated pure premium is: (10%)($5 million) = $0.5 million.
Alternately, one can try to read the graph more precisely.
4% chance of between $10 and $12 million, 2% chance of between $12 million and $14 million,
3% chance of between $14 million and $20 million, and a 1% chance of exceeding $20 million.
Then the estimated pure premium is:
(4%)(1) + (2%)(3) + (3%)(7) + (1%)(10) = $0.41 million.

(b) Reinstatement premium is: $115\% \, \frac{20\text{M} - 15\text{M}}{20\text{M} - 10\text{M}}$ $1,200,000 = **$690,000**.

(c) The quota share treaty pays: (30%)(15M) = $4.5 million.
Without the cat treaty the insurer would pay: 15M - 4.5M = $10.5 million
Thus the cat treaty pays: 10.5 - 10 = $0.5 Million.
The insurer pays a net of $10 million.
Note that: $4.5M + $0.5M + $10M = $15 million.
<u>Comment</u>: In part (b) I have ignored the quota share treaty which is first mentioned in part (c).
In part (a), I did not like the reference to the "payback approach".
Clark at page 5: "In the past, reinsurers had priced catastrophe loads based on spreading large losses over expected payback periods. A 1-in-20-year event would be included as a loading of 5% of the loss amount. The payback approach may still be used for casualty events but is only referenced as a reasonability check for property."
Clark at page 44: "Before the widespread development of catastrophe models there had been few tools available to systematically price catastrophe covers. The most common method was known as the payback approach, in which premium was set so the offered limit was paid back over a given period of time. For the example above, the payback period is five years, meaning that the $2,000,000 of annual premium would cover a single total loss of $10,000,000 every five years. Catastrophe models are now the generally accepted approach for pricing of natural and some man-made events." Given the output from a catastrophe model such as in this question, Clark would <u>not</u> use the payback approach.
From the Examiner's Report, in part (a): "To get full credit, candidates were expected to either explicitly state the 10-year payback period and to divide the full layer loss by the payback period to get a pure premium, or to multiply the full layer loss by the relevant value from the exceedance probability curve."
In part (a), see CAS errata to Section 2.4 of Grossi & Kunreuther.
In the question: "the following output for losses in excess of $10,000,000" is not for losses exceeding $10 million, but rather also shows losses of size less than $10 million.

25. (1.5 points) An actuary is using exposure rating to calculate increased limit factors for an auto liability treaty. The actuary has selected a severity distribution for the exposures being considered. The expected value function of losses capped at L is:

$E[x; L] = 30 + 900\{1 - \ln(1000/L)\}$

Additionally, the actuary has the following information:

- All of the ceding company's underlying policy limits are $1,000,000
- The reinsurance treaty attachment point is $250,000
- The reinsurance treaty limit is $750,000

a. (1 point) Calculate the exposure factor.

b. (0.5 point)

Calculate the ground up expected loss if the estimated loss cost of the treaty layer is $243,500.

25. (a) $E[X ; 1 \text{ million}] = 30 + (900)\{1 - \ln(1/1000)\} = 7147$.

$E[X ; 250{,}000] = 30 + (900)\{1 - \ln(1/250)\} = 5899$.

Exposure factor is: $\frac{E[X ; 250K + 750K] - E[X ; 250K]}{E[X ; 1000K]} = \frac{7147 - 5899}{7147} = 17.46\%$.

(b) (17.46%) (ground-up expected loss) = $243,500.

$\Rightarrow$ ground-up expected loss = $243,500 / 0.1746 = **$1.395 million**.

<u>Comment</u>: I do not know what the purpose was of making part (b) backwards.

The given formula for the limited expected value is not from a well-known size of loss distribution, although it is increasing and concave downwards. However, as L gets small, the limited expected value becomes negative; for example E[X ; 300] = -154. Also as L approaches infinity, the limited expected value approaches infinity; therefore, there is no finite mean.

If the severity distribution is per claim, then we have to assume so is the treaty.

If the severity distribution is per occurrence, then we have to assume so is the treaty.

One does not "use exposure rating to calculate increased limit factors for an auto liability treaty."