Solutions to the
Fall 2013
CAS Exam 8

The solutions and comments are solely the responsibility of the author.
While some of the comments may seem critical of certain questions, this is intended solely to aid you in studying and in no way is intended as a criticism of the many volunteers who work extremely long and hard to produce quality exams.

prepared by
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1. (2.25 points) A company is considering introducing a risk accumulation surcharge into its homeowners rating plan. The surcharge is a territory factor beyond expected pure premium differences and will be applied to territories where the total number of homeowner's risks in the territory is beyond a selected threshold.
Construct an argument for or against the introduction of the risk accumulation surcharge and whether it accomplishes the three primary purposes of risk classification listed in the American Academy of Actuaries "Risk Classification Statement of Principles."

1. Primary Purposes of Risk Classification:
1. Protect the insurance system's financial soundness.
2. Be fair.
3. Permit economic incentives to operate and thus encourage widespread availability of coverage.

I will assume that the territories are subject to significant risk from catastrophes.

1. An insurer who writes too many homes in a concentrated area needs more reinsurance protection from catastrophes. For example, a single hurricane or earthquake can destroy thousands of insured homes. The proposed surcharge will provide money to buy catastrophe reinsurance, which will protect the insurance system's financial soundness. Otherwise, homes written in these territories could drain capital or cause insolvencies if a catastrophe were to occur.

Alternately, if the insurer insures too many homes in a territory, the accumulation surcharge will be big and this insurer will be charging a lot in that territory. This will cause better risks to seek out lower premiums from competitors. This results in adverse selection against this insurer, which would harm its financial soundness.

2. The surcharge is fair because those homes in territories that create a bigger risk for the insurer due to catastrophes are charged more. This is a fair way to spread the cost of needed catastrophe reinsurance.

Alternately, the surcharge is not fair since it is just a strategy for the insurer to make up for its error in writing too many homes that are located near one another. A homeowner will not understand why it should pay more just because the insurer decided to write more homes in its territory.

Alternately, the surcharge is not fair since it depends on the size of the different territories the insurer has chosen to define. If the insurer were to split one of its current territories, then the affected homes would pay less of a surcharge. Some better measure of concentration appropriate for each of the types of catastrophes that concern the insurer should be used. Also the surcharge is unfair since it does not take into account the differences in catastrophe risk between the territories. For example, a coastal territory would be more exposed to losses from hurricanes than would an inland territory.

3. The surcharge will allow this insurer to write more business in these territories than it would otherwise be able to write. (Some appropriate reflection of the risk of catastrophes in the premiums charged is needed for insurers to be willing to write insurance.) The same would be true of others insurers who also introduced a risk accumulation surcharge. This encourages the widespread availability of coverage.
Economic incentives will operate to prevent a single insurer writing too much business in a single territory, because with a large accumulation surcharge its resulting rates will be uncompetitive with those insurers who have a smaller accumulation in that territory. Insureds are free to shop around and see if they can get similar insurance at a lower premium from some other insurer. Economic incentives will be generated for new insurers to enter and for some existing insurers with low concentrations to expand their book of business in these territories, encouraging the widespread availability of coverage.

Alternately, if all insurers adopted similar surcharges, premiums would increase substantially in certain territories. This could lead to affordability problems, decreasing the availability of coverage.

Comment: It does not matter whether you for or against the surcharge; the important thing is you say something sensible and explain your reasoning. You could say the surcharge does satisfy one or two of the purposes while it does not fulfill the remaining purpose(s).
You need to do the best you can to answer this exam question in the absence of details on the surcharges that would be important in a practical application.

2. (3.5 points) An actuary at a private passenger auto insurance company wishes to use a generalized linear model to create an auto frequency model using the data below.

Number of Claims

Gender	Territory A	Territory B
Male	700	600
Female	400	420

Number of Exposures

Gender	Territory A	Territory B
Male	1,400	1,000
Female	1,000	1,200

The model will include three parameters: β_1, β_2, β_3, where β_1 is the average frequency for males, β_2 is the average frequency for Territory A, and β_3 is an intercept.

a. (0.5 point) Define the design matrix [X].

b. (0.25 point) Define the vector of responses [Y].

c. (2.25 points) Assuming $\beta_3 = 0.35$, solve a generalized linear model with a normal error structure and identity link function for β_1.

d. (0.5 point) The actuary determines that the analysis results would be improved by assuming a Poisson error structure with a log link function.
Identify two reasons this structure may better suit this data.

2. (a) The first column refers to β_1 whether or not we have a male,

the second column refers to β_2 whether or not we are in Territory A,

the third column refers to β_3 the intercept, and thus is all ones.

$$X = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 0 & 1 \end{pmatrix} \begin{matrix} \text{A, Male} \\ \text{A, Female} \\ \text{B, Male} \\ \text{B, Female} \end{matrix} \qquad \text{or } X = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} \begin{matrix} \text{A, Male} \\ \text{B, Male} \\ \text{A, Female} \\ \text{B, Female} \end{matrix}$$

$$\text{(b) } Y = \begin{pmatrix} 700/1400 \\ 400/1000 \\ 600/1000 \\ 420/1200 \end{pmatrix} = \begin{pmatrix} 0.50 \\ 0.40 \\ 0.60 \\ 0.35 \end{pmatrix} \begin{matrix} \text{A, Male} \\ \text{A, Female} \\ \text{B, Male} \\ \text{B, Female} \end{matrix} \qquad \text{or } Y = \begin{pmatrix} 0.50 \\ 0.60 \\ 0.40 \\ 0.35 \end{pmatrix} \begin{matrix} \text{A, Male} \\ \text{B, Male} \\ \text{A, Female} \\ \text{B, Female} \end{matrix}$$

(c) With a Normal error function and an identity link function, this is the same as a multiple regression.
Assuming $\beta_3 = 0.35$, then the squared error is:

$$1400\ (\beta_1 + \beta_2 + 0.35 - 0.5)^2 + 1000\ (\beta_2 + 0.35 - 0.4)^2$$

$$+ 1000\ (\beta_1 + 0.35 - 0.6)^2 + 1200\ (0.35 - 0.35)^2 =$$

$1400\ (\beta_1 + \beta_2 - 0.15)^2 + 1000\ (\beta_2 - 0.05)^2 + 1000\ (\beta_1 - 0.25)^2$.

Setting the partial derivative with respect to β_1 equal to zero:

$0 = 2800(\beta_1 + \beta_2 - 0.15) + 2000(\beta_1 - 0.25)$. $\Rightarrow 4800\ \beta_1 + 2800\ \beta_2 = 920$.

Setting the partial derivative with respect to β_2 equal to zero:

$0 = 2800(\beta_1 + \beta_2 - 0.15) + 2000(\beta_2 - 0.05)$. $\Rightarrow 2800\ \beta_1 + 4800\ \beta_2 = 520$.

$\Rightarrow \beta_2 = (520 - 2800\ \beta_1) / 4800$.

Plugging back into the first equation: $4800\ \beta_1 + 2800\ (520 - 2800\ \beta_1) / 4800 = 920$.

$\Rightarrow$ **$\beta_1 = 0.1947$**. $\Rightarrow$ **$\beta_2 = -0.0052$**.

Alternately, without taking into account exposures by cell, the squared error is:

$(\beta_1 + \beta_2 + 0.35 - 0.5)^2 + (\beta_2 + 0.35 - 0.4)^2 + (\beta_1 + 0.35 - 0.6)^2 + (0.35 - 0.35)^2 =$

$(\beta_1 + \beta_2 - 0.15)^2 + (\beta_2 - 0.05)^2 + (\beta_1 - 0.25)^2$.

Setting the partial derivative with respect to β_1 equal to zero:

$0 = 2(\beta_1 + \beta_2 - 0.15) + 2(\beta_1 - 0.25). \Rightarrow 4\ \beta_1 + 2\ \beta_2 = 0.8.$

Setting the partial derivative with respect to β_2 equal to zero:

$0 = 2(\beta_1 + \beta_2 - 0.15) + 2(\beta_2 - 0.05). \Rightarrow 2\ \beta_1 + 4\ \beta_2 = 0.4.$

$\Rightarrow \beta_2 = (0.4 - 2\ \beta_1) / 4 = 0.1 - 0.5\beta_1.$

Plugging back into the first equation: $4\ \beta_1 + 2\ (0.1 - 0.5\beta_1) = 0.8.$

$\Rightarrow$ **$\beta_1 = 0.2$**. $\Rightarrow$ **$\beta_2 = 0$**.

Alternately, in either case one can fit via maximum likelihood and get the same result as by minimizing the squared errors.

For the Normal Distribution, $f(x) = \dfrac{\exp[-\frac{(x-\mu)^2}{2\sigma^2}]}{\sigma\sqrt{2\pi}}$. $\ln[f(x)] = -\dfrac{(x-\mu)^2}{2\sigma^2} - \ln[\sigma] - \ln[2\pi]/2.$

Without taking info account exposures by cell, the loglikelihood is:

$- (\beta_1 + \beta_2 + 0.35 - 0.5)^2 / (2\sigma^2) - (\beta_1 + 0.35 - 0.4)^2 / (2\sigma^2) - (\beta_2 + 0.35 - 0.6)^2 / (2\sigma^2)$

$\qquad - (0.35 - 0.35)^2 / (2\sigma^2) - 4 \ln[\sigma] - \ln[2\pi]/2.$

Setting the partial derivative of the loglikelihood with respect to β_1 equal to zero:

$0 = -(\beta_1 + \beta_2 - 0.15) / \sigma^2 - (\beta_1 - 0.25) / \sigma^2. \Rightarrow 2\ \beta_1 + \beta_2 = 0.4.$

Setting the partial derivative with respect to β_2 equal to zero:

$0 = -(\beta_1 + \beta_2 - 0.15) / \sigma^2 - (\beta_2 - 0.05) / \sigma^2. \Rightarrow \beta_1 + 2\ \beta_2 = 0.2.$

Solving two equations in two unknowns: **$\beta_1 = 0.2$**, and **$\beta_2 = 0$**.

Setting the partial derivative with respect to σ equal to zero:

$0 = (\beta_1 + \beta_2 - 0.15)^2 / \sigma^3 - (\beta_1 - 0.05)^2 / \sigma^3 - (\beta_2 - 0.25)^2 / \sigma^3 - 4 /\sigma.$

$\Rightarrow \sigma^2 = \{(0.2 + 0 - 0.15)^2 + (0.2 + 0 - 0.05)^2 + (0.2 + 0 - 0.25)^2\} / 4 = 0.006875.$

(d) 1. The Normal Distribution allows negative values, while the Poisson Distribution does not. Since claim frequencies are never negative, the Poisson error structure is preferable here.

2. The Normal error structure assumes that the process variances of the frequency in the four cells are equal. In contrast, a Poisson error structure assumes that the process variances of the frequency in the four cells are equal to their means. I would expect that those cells with higher expected frequencies would have higher process variances than those with lower expected frequencies, and thus would prefer the Poisson error structure to the Normal.

3. The log link function would assume multiplicative relativities, while the identity link function assumes additive relativities. If the relationship is approximately multiplicative, then the log link function would do a better job than the identity link function

Comment: In parts (a) and (b), the order in which one lists the rows is arbitrary;
it would be a good idea to label what you did.
In part (c), if one includes the exposures in the sum of squared errors, that is equivalent to using exposures as the prior weights in the GLM, or using exposures in an offset term.
Including the exposures is equivalent to doing a weighted multiple regression.
The observed frequencies are:

Gender	Territory A	Territory B	Difference
Male	0.50	0.60	0.10
Female	0.40	0.35	-0.05
Difference	-0.10	-0.25	

The differences between territories are not similar for the two genders.
The differences between genders are not similar for the two territories.

Gender	Territory A	Territory B	Ratio
Male	0.50	0.60	1.2
Female	0.40	0.35	0.875
Ratio	0.80	0.583	

The ratios between territories are not similar for the two genders.
The ratios between genders are not similar for the two territories.
Thus perhaps neither an additive nor a multiplicative relationship is appropriate.

3. (1.75 points) An actuary is creating a workers compensation classification rating plan.
The actuary has access to frequency data from years 2002-2012 for fatal claims, which he believes may have low credibility.
After developing and testing a multivariate credibility procedure, the actuary finds the following results:

Sum of Squared Prediction Errors - Fatal Claims

Hazard Group	Prediction based on Hazard Group	Prediction based on raw data less the holdout sample	Prediction based on credibility procedure
A	100	90	80
B	110	70	105
C	90	115	75

a. (0.25 point) Briefly describe the purpose of a holdout sample.

b. (0.5 point) Justify an appropriate holdout sample from the available frequency data for the actuary to use in the classification analysis.

c. (0.5 point) Discuss what the actuary's predicted results imply about his credibility procedure.

d. (0.5 point) Suppose the actuary suspects that there may be an intrinsic downward trend in frequencies of fatal claims between 2002 and 2012 due to improved safety in his clients' workplaces. Propose a way for the actuary to test this theory.

3. (a) The holdout sample is a portion of the data which is not used to develop and fit the model. After one has developed what you think is a good model, you compare its predictions to what is seen in the holdout data set, in order to test the performance of this model and whether it is valid.

(b) One could use either all of the even or odd years as a holdout data set.
Either of these holdout sets would be affected similarly by any trend as would the data used to calibrate the model. Either of these holdout sets would be affected similarly by maturity as would the data used to calibrate the model.
(If 2012 is not sufficiently mature, one may not want to use it for either model calibration or as part of the holdout data set.)

(c) For Hazard Group A, the squared errors using the credibility procedure are less than either relying on the hazard group or the class data; this is good and indicates the credibility procedure is working.
For Hazard Group B, the squared errors using the credibility procedure are significantly more than relying on the class data; this is bad. This would indicate that the average credibility applied to fatal data is probably too low; one should probably on average be giving the fatal data by class in Hazard Group B more credibility. There may be something fundamentally wrong with the multivariate credibility procedure, at least as it is applied to Hazard Group B.
For Hazard Group C, the squared errors using the credibility procedure are less than either relying on the hazard group or the class data; this is good and indicates the credibility procedure is working.
In total the squared errors for the credibility procedure are 260, which is less than either the sum for the raw data 275, or relying on the Hazard Group 300. So over all of the hazard groups the credibility procedure is an improvement over either relying on the hazard group or the class data.

(d) As in a paper by Mahler, one could take the correlations between pairs of years of frequency data by class. If the correlations decline as the years get further apart, then this would indicate shifting risk parameters over time. If risk parameters are detected to be shifting, then one would have to look at the data to see whether frequency was decreasing over time. (This test might be adversely affected if the percent of exposures by class were changing significantly over time.)

Alternately, as in a paper by Mahler, one could apply a Chi-Square Test to each hazard group. The null hypothesis is that the mean frequency in all of the years is the same. If the Chi-Square statistic was large, this would indicate that not all of the years have the same mean frequency. If one rejected the null hypothesis, then one would have to look at the data to see whether frequency was decreasing over time. (This test might be adversely affected if the percent of exposures by class were changing significantly over time.)

Alternately, within a hazard group one could for each class rank the frequencies by year from smallest to largest. Then one could sum the ranks for each year, and see if there is pattern over time.
If the ranks by year decline, this would indicate that the mean frequency is decreasing.

Alternately, for each hazard group one could compute the weighted average frequencies for each year, using the exposures by class for the sum of the years as the weights. Then one could see whether the mean frequency is decreasing over time.

Alternately, one could calibrate two separate multivariate credibility models, one using five earlier years 2002 to 2006, and the other using five later years 2007 to 2011. One could then see whether the latter model predicts lower frequencies than the former model. (This would not work, if the inputs and outputs of the model are relativities with respect to average.)

Alternately, one could see whether the prediction errors for the multivariate credibility model for earlier years in the holdout set tend to be negative, while the prediction errors for later years in the holdout set tend to be positive. (This would not work, if the inputs and outputs of the model are relativities with respect to average.)

<u>Comment</u>: In part (d), it was not clear to me what the examiners intended; I believe that the CAS was expecting either of the two answers I gave first.

4. (3 points) An actuary is helping design a new internet liability product that would use industry as a rating factor. Different business types such as restaurants, auto manufacturers, and dairy farms would fall into different industry groups. The actuary wants to create several industry factors from a combination of insurance and demographic data, and use this to classify business types into industry groups.

a. (1 point) Describe two reasons that a generalized linear model might not be appropriate for developing industry factors.

b. (0.25 point) Describe a benefit that a principal component method would have over a generalized linear model for determining the industry factors.

c. (1 point) Briefly describe the major steps in using a cluster analysis to group the industry factors.

d. (0.75 points) Describe two test statistics that could be used to determine the optimal number of groups from the cluster analysis. Identify which statistic would be preferred when variables are correlated.

4. (a) 1. There are many possible error functions, link ratios, and possible interactions terms between variables that could be used in a GLM. Since this is not a situation with which most actuaries are familiar, it would take a lot of time and effort to sort out which of these possible GLMs worked best.
2. With a lot of variables there are likely to be significant correlations between them, creating intrinsic aliasing. This would harm the performance of the GLM. Also, some variables may not have a fitted parameter significantly different from zero. It would take a lot of time and effort to determine a reasonable subset of the original variables to use in a GLM.
3. The observations may not follow a member of the exponential family of distributions.
4. One could not fit a GLM unless one had observations for internet liability by type of business; since this is a new product we will not have such data (yet).

(b) A principal component analysis would identify the linear combination of inputs that accounted for the most variation between types of businesses. One could use the first principal component or the first two principal components, to combine similar businesses into industry groups. By identifying one or several linear combinations of inputs that account for most of the variation between businesses, principal component analysis simplifies the process of coming up with industry groups.
We could apply cluster analysis to the principal component(s), either in one dimension or in several dimensions, in order to group businesses into industry groups.
For the potential problems with using a GLM see the solutions to part (a).

(c) For the k-means clustering algorithm, after one has chosen the number of clusters, k, the following steps are performed iteratively:
0. Some initial assignment to clusters is made. This can be random.
1. Compute the centroid of each cluster,
in other words the vector of average factors.
2. Assign each business to the closest centroid from step 1.
3. If step 2 results in any changes to the clusters, return to step 1.

Alternately, for the general steps in using a cluster analysis to group the industry factors:
1. Gather appropriate data, a combination of insurance and demographic data.
2. Decide on the appropriate variables or indices (linear combinations of the initial variables) on which to perform the clustering.
For example, if one use the first two principal components, one would be clustering in two dimensions. (Robertson uses credibility weighted excess ratios at five different limits, with the complement of credibility going to the excess ratio for the current hazard group assignment. Thus Robertson had to decide on a credibility formula to use.)
3. Decide what algorithm to use, such as k-means clustering.
4. Decide what distance measure to use, such as squared distance or absolute distance.
5. Decide whether to use a weighted or unweighted algorithm.
6. Decide whether or not to standardize the different variables or indices used.
7. Use some statistic(s) to decide how many clusters are optimal.
8. If data is available, check the homogeneity of the resulting clusters. For example one could graph the pure premiums for the classes within each cluster and see how much overlap there is between the clusters. (Robertson graphs excess ratios.) One could do this for both all classes and/or just the larger classes.
9. Have insurance professionals such as underwriters check the reasonableness of the resulting clusters, particularly for types of businesses with less insurance data.

(d) Calinski-Harabasz statistic = $\frac{\text{trace(B)} \ / \ (\text{k-1})}{\text{trace(W)} \ / \ (\text{n-k})}$,

where n is the number of classes and k is the number of hazard groups,
B is the between cluster sum of squares and cross product matrix,
and W is the within cluster sum of squares and cross product matrix.
The Cubic Clustering Criterion (CCC) compares the amount of variance explained by a given set of clusters to that expected when clusters are formed at random based on data sampled from the multidimensional uniform distribution.
The Calinski-Harabasz statistic would be preferred when variables are correlated;
the CCC procedure performs less well when correlation is present.

Comment: In parts (a), (b), and (c) the examiners should have made it much clearer what they were looking for.

For part (a), the examiner's report said "A common response was that GLMs could not be used for grouping business types into industries. However, this question was not focused on that grouping task but rather on the deficiencies in GLM assumptions as they relate to a complex data set (industry and demographical data) when modeling response variables that differ from actuarial norms (frequency, severity, claims)."
They graded based on what they intended the question to mean rather than what it actually said.
The question writer did not seem to understand the purpose of giving information prior to the subparts. This context affects one's answer; one is going to use the industry factors for grouping.
For a new product, what would be the response variable to which we would fit the GLM?
The response variable can't be the industry factors, since they do not yet exist.
We don't have frequency, pure premium, or excess ratios for this new product.
Based on the limited information given in the question, I think that the GLM could not be used for the specified task.
For an existing product, one could fit the GLM with pure premium as the response variable and use this as the industry factor for a one-dimensional grouping.
In any case, since the response variable of a GLM is one-dimensional, its output could only be used to do a grouping in one dimension.
In my opinion, part (a) is just not well thought out.

In part (b), principal component analysis will find the best linear combination of the original variables, the first principal component. It will not identify one of the original variables to use. As mentioned, it is often the case that it will in addition be worthwhile to use the second principal component or even the third principal component.

In part (c), although this was not what was asked, apparently the CAS was looking for the steps in the k-means algorithm, the particular clustering algorithm used by Robertson, rather than the general steps discussed by Robertson in his grouping of classes into hazard groups.

5. (2.25 points)
The following increased limits factors (ILFs) are used to price a general liability policy:

Aggregate Limit (000)	Occurrence Limit (000) $25	$50	$100	$250	$500
$25	1.00				
$50	1.50	1.70			
$100	1.80	2.05	2.50		
$250	2.00	X	2.80	3.15	
$500	2.17	2.47	3.05	3.45	3.60

Calculate the range of possible values for the $50,000 occurrence / $250,000 aggregate ILF such that all the factors in the table pass the two-dimensional consistency test.

5. Going down the $50,000 occurrence limit column, we require that:

(2.05 - 1.70) / (100 - 50) ≥ (X - 2.05) / (250 - 100). ⇒ X ≤ 3.10.

(X - 2.05) / (250 - 100) ≥ (2.47 - X) / (500 - 250). ⇒ X ≥ 2.2075.

Going across the $250,000 aggregate limit row, we require that:

(X - 2.00) / (50 - 25) ≥ (2.80 - X) / (100 - 50). ⇒ X ≥ 2.2667.

(2.80 - X) / (100 - 50) ≥ (3.15 - 2.80) / (250 - 100). ⇒ X ≤ 2.683.

Comparing the $25,000 occurrence column to the $50,000 occurrence column, we require that:

2.05 - 1.80 ≤ X - 2.00 ≤ 2.47 - 2.17. ⇒ 2.25 ≤ X ≤ 2.30.

Comparing the $50,000 occurrence column to the $100,000 occurrence column, we require that:

2.50 - 2.05 ≤ 2.80 - X ≤ 3.05 - 2.47. ⇒ 2.22 ≤ X ≤ 2.35.

Comparing the $100,000 aggregate row to the $250,000 aggregate row, we require that:

2.00 - 1.80 ≤ X - 2.05 ≤ 2.80 - 2.50. ⇒ 2.25 ≤ X ≤ 2.35.

Comparing the $250,000 aggregate row to the $500,000 aggregate row, we require that:

2.17 - 2.00 ≤ 2.47 - X ≤ 3.05 - 2.80. ⇒ 2.22 ≤ X ≤ 2.30.

Combining all of the requirements: **2.2667 ≤ X ≤ 2.30**.

Comment: One can check that for example X = 2.28 passes all of the consistency tests.
Apparently the CAS gave full credit for either:
1. Going down the $50,000 occurrence limit column,
   and going across the $250,000 aggregate limit row; or
2. Comparing the $25,000 occurrence column to the $50,000 occurrence column,
   comparing the $50,000 occurrence column to the $100,000 occurrence column,
   comparing the $100,000 aggregate row to the $250,000 aggregate row, and
   comparing the $250,000 aggregate row to the $500,000 aggregate row.

In general, on the one hand comparing the $25,000 occurrence column to the $50,000 occurrence column and the $50,000 occurrence column to the $100,000 occurrence column, will produce the same restrictions in total as on the other hand comparing the $100,000 aggregate row to the $250,000 aggregate row and the $250,000 aggregate row to the $500,000 aggregate row.

6. (2 points) Given the following information:

Amount of Loss	Probability of Loss
$0	80%
$100,000	15%
$500,000	5%

- The basic limit is $200,000.
- The actuary selects 20% of the standard deviation as the risk load.
- Assume there are no expenses.

Calculate the risk-loaded increased limit factor for a policy limit of $400,000.

6. Solely for convenience, put everything in thousands of dollars.

$E[X \wedge 200] = (80\%)(0) + (15\%)(100) + (5\%)(200) = 25.$

$E[(X \wedge 200)^2] = (80\%)(0^2) + (15\%)(100^2) + (5\%)(200^2) = 3500.$

$Var[X \wedge 200] = 3500 - 25^2 = 2875.$

$E[X \wedge 400] = (80\%)(0) + (15\%)(100) + (5\%)(400) = 35.$

$E[(X \wedge 400)^2] = (80\%)(0^2) + (15\%)(100^2) + (5\%)(400^2) = 9500.$

$Var[X \wedge 400] = 9500 - 35^2 = 8275.$

The risk-loaded increased limit factor for a policy limit of $400,000 is:

$$\frac{35 + (20\%)\sqrt{8275}}{25 + (20\%)\sqrt{2875}} = \mathbf{1.489}.$$

<u>Comment</u>: Without the risk load, the increased limit factor would be: 35/25 = 1.400.

7. (2.5 points) An actuary is modeling the impact of dispersion on loss development and excess ratios. The actuary has assumed that undeveloped losses are uniformly distributed between $0 and $120,000.

a. (1 point) Calculate the excess ratio at $75,000.

b. (1 point) Assume a simple dispersion model such that each loss has an equal likelihood of developing by a multiplicative factor of 0.75, 1.00 or 1.25.

Given the following, calculate the excess ratio at $75,000 with simple dispersion.

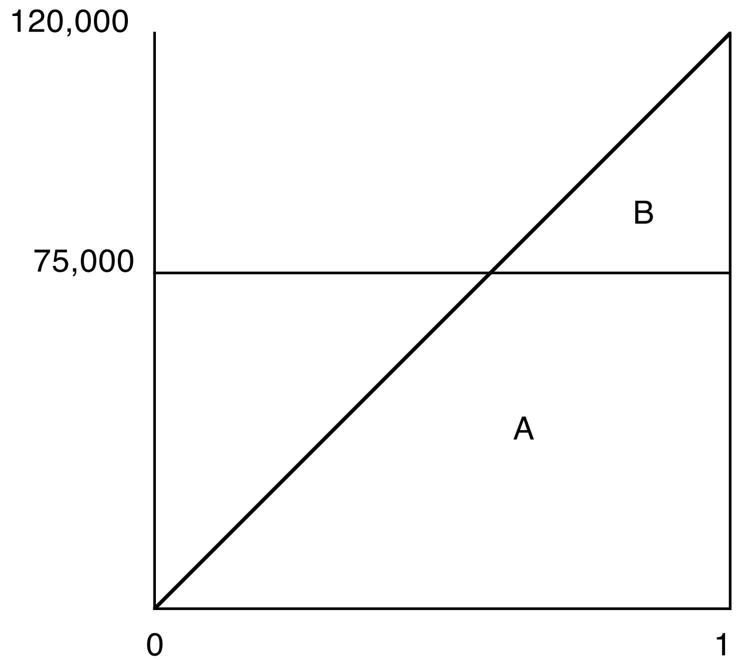
Loss	Excess Ratio
$50,000	0.3403
$56,250	0.2822
$60,000	0.2500
$93,750	0.0479
$100,000	0.0278
$110,000	0.0069

c. (0.5 point) Briefly explain two impacts that simple dispersion has on excess ratios.

7. (a) E[X] = 60,000. $E[X \wedge 75{,}000]$ = (75/120)(75,000/2) + (1 - 75/120)(75,000) = 51,563.
$R(75{,}000) = 1 - E[X \wedge 75{,}000] / E[X]$ = 1 - 51,563/60,000 = **14.06%.**
Alternately, the expected losses excess of 75,000 are:

$$\int_{75{,}000}^{120{,}000} (x - 75{,}000) / 120{,}000 \, dx = 8437.5.$$

Thus, R(75,000) = 8437.5/60,000 = **14.06%**.
Alternately, in the following Lee diagram of the uniform distribution from 0 to 120,000, the excess losses are area B, while the total losses are areas A + B.



R(75,000) = B / (A + B) = {(45,000)(45/120) / 2} / {(120,000)(1) / 2} = **14.06%**.

(b) Take the excess ratios from part (a) and the provided table at the "deflated" values:
75,000 / 0.75 = 100,000, 75,000/1 = 75,000, and 75,000 / 1.25 = 60,000.

After dispersion, $$\hat{R}(75{,}000) = \frac{(0.75)\,R(100{,}000) + (1)\,R(75{,}000) + (1.25)\,R(60{,}000)}{0.75 + 1 + 1.25}$$

$$= \frac{(0.75)(0.0278) + (1)(0.1406) + (1.25)(0.2500)}{0.75 + 1 + 1.25} = \mathbf{15.80\%}.$$

Alternately, with a multiplier of 0.75, the losses are uniform from 0 to 90.
E[X] = 45,000. $E[X \wedge 75{,}000]$ = (75/90)(75,000/2) + (1 - 75/90)(75,000) = 43,750.
With a multiplier of 1.25, the losses are uniform from 0 to 150.
E[X] = 75,000. $E[X \wedge 75{,}000]$ = (75/150)(75,000/2) + (1 - 75/150)(75,000) = 56,250.
Thus after dispersion:

$$\hat{R}(75{,}000) = 1 - \frac{(1/3)(43{,}750) + (1/3)(51{,}563) + (1/3)(56{,}250)}{(1/3)(45{,}000) + (1/3)(60{,}000) + (1/3)(75{,}000)} = \mathbf{15.80\%}.$$

(c) 1. By allowing for more extreme outcomes at ultimate than at earlier reports, simple dispersion increases the excess ratios. After dispersion the largest possible loss is bigger, so some limits where excess ratios were zero prior to dispersion will now have positive excess ratios after dispersion.
2. If as here and in the paper by Mahler the loss multipliers average to one, then the mean of the losses at ultimate is the same as at an earlier report. (Any overall average loss development is assumed to already have been applied to the losses at latest report.) Thus dispersion produces more excess losses without affecting total expected losses.
3. The higher the coefficient of variation of the loss multipliers, the greater the excess ratios at ultimate.
4. For the same coefficient of variation of loss multipliers, a Gamma Distribution of divisors (an Inverse Gamma Distribution of multipliers) will have a greater impact on excess ratios than the simple dispersion model. (See corrected Table 1 in the paper.)

Comment: In the syllabus reading by Mahler, loss divisors are used rather than multipliers.
On page 321, the paper reads: "As can be seen in Table 1, the dispersion effect raises the excess ratios for higher limits and alters those for lower limits." However, in corrected Table 1 in the errata, the excess ratios all increase, including those at lower limits.

8. (2.75 points) Given the following information for a general liability policy, determine the value of X that yields an experience modification of +4.5%.

- Effective period of the policy: January 1 to December 31, 2014.
- Expected loss ratio: 65.6%.
- Type of policy being rated: Claims-made.

Loss Experience

Policy period	Type of Policy	Includable Losses in Experience Period (Limited by MSL)
January 1,2012, to December 31,2012	Claims-made	X
January 1,2011, to December 31,2011	Occurrence	$72,234
January 1,2010, to December 31,2010	Occurrence	$30,484
Total		$102,718 +X

Policy Period	Coverage	Company Subject Loss Cost
Latest Policy Year	Prem/Ops	$32,160
	Products	$6,679
Prior Policy Year	Prem/Ops	$42,832
	Products	$14,137
Next Prior Policy Year	Prem/Ops	$38,695
	Products	$13,327
Total		$147,830

Subject Loss Cost	Credibility	Expected Experience Ratio	Maximum Single Loss
$138,763 - $145,183	0.33	0.866	$109,200
$145,184 - $151,800	0.34	0.870	$111,400
$151,801 - $158,621	0.35	0.874	$113,700
$158,622 - $165,658	0.36	0.878	$116,050
$165,659 - $172,920	0.37	0.882	$118,450

Loss Development Factors

Subline	Latest Policy Year	Prior Policy Year	Next Prior Policy Year
Prem/Ops	0.519	0.338	0.198
Products	0.766	0.637	0.528

8. For 147,830 in Company Subject Loss Costs, as shown in the table in the question:
Z = 0.34 and EER = 0.870.
In order to get the adjustment to reflect the ultimate level of losses, we multiply the given company subject loss costs by the Expected Experience Ratio and Loss Development Factors.
The LDFs are supplied in the question.
Since the most recent policy is claims-made, we use an LDF of 0 for it.

Policy Period	Subject Loss Cost	EER	LDF	Product
Latest Policy Year	$32,160	0.870	0	0
	$6,679	0.870	0	0
Prior Policy Year	$42,832	0.870	0.338	12,595
	$14,137	0.870	0.637	7835
Next Prior Policy Year	$38,695	0.870	0.198	6666
	$13,327	0.870	0.528	6122
Total	$147,830			33,218

We are given that includable losses in the experience period (limited by MSL) are: 102,718 + X.
Thus the Actual Experience Ratio is: (102,718 + X + 33,218) / 147,830.

Given mod of 4.5%. ⇒ 0.045 = M = Z (AER - EER) / EER = (0.34) (AER - 0.870) / 0.870.

⇒ AER = 0.98515 = (102,718 + X + 33,218) / 147,830. ⇒ X = **9698**.

Comment: Similar to the rating example in Rule 6 of the manual, except here we are already given the company subject loss cost which are shown in column 7 of Table 6.B.5.
The MSL is $111,400, but we do not need it since we are given the losses already limited by the MSL.
We do not use the given expected loss ratio since we do not need to calculate the subject loss costs ourselves.
The policy to be rated is a third year claims-made policy, which would be important if we needed to calculate the subject loss costs ourselves.

9. (4.5 points) Suppose that workers compensation risks are subject to a no-split experience rating plan under which credibility, as a function of expected loss, is calculated as follows:

$$Z = \frac{E}{E + 50{,}000}$$

During the experience rating period, a group of homogeneous risks had the following experience:

Risk	Actual Loss	Expected Loss
1	$130,000	$125,000
2	$60,000	$85,000
3	$160,000	$150,000
4	$200,000	$130,000
5	$100,000	$150,000
6	$250,000	$175,000

After the experience modifications were applied, the same group had the following experience:

Risk	Manual Premium	Actual Loss
1	$50,000	$35,000
2	$50,000	$25,000
3	$70,000	$60,000
4	$75,000	$50,000
5	$65,000	$40,000
6	$65,000	$100,000

a. (3.75 points) Assess how effectively the experience rating plan corrected for the differences it identified for these particular workers compensation risks. Group the risks as appropriate.
b. (0.75 point) Propose and justify a change to the plan that would improve its ability to correct the differences it identifies.

9. (a) In each case, M = (A/E)Z + 1 - Z.

Risk	A	E	Z	M	Man. Prem.	Stand. Prem.	Loss	L.R. to Stand.
1	130	125	71.4%	1.03	50	51.4	35	68.1%
2	60	85	63.0%	0.81	50	40.7	25	61.4%
3	160	150	75.0%	1.05	70	73.5	60	81.6%
4	200	130	72.2%	1.39	75	104.2	50	48.0%
5	100	150	75.0%	0.75	65	48.8	40	82.1%
6	250	175	77.8%	1.33	65	86.7	100	115.4%

With only 6 risks, due to the large random fluctuation in loss ratios, there is no advantage to grouping them for the analysis. If the experience rating plan did a good job of correcting for the differences it identified, then the loss ratios for credit and debit mods should be the same; the subsequent loss ratios with respect to standard premium should be uncorrelated with the modifications. (See the Meyers-Dorweiler criterion in the paper by Mahler; no grouping is required.)

Mod:	0.75	0.81	1.03	1.05	1.33	1.39
L. R. to Standard:	82.1%	61.4%	68.1%	81.6%	115.4%	48.0%

Given only six risks, it is hard to draw any conclusion. However, based on this limited data the subsequent loss ratios with respect to standard premium do not appear to be significantly correlated with the modifications; the correlation is 11%. The experience rating plan may be doing an adequate job of correcting for the differences it identified.

Alternately, group the risks into three groups by low, medium and high mods.

Group	Risks	Standard Premium	Losses	Loss Ratio to Standard
Low	2 and 5	40.7 + 48.8	25 + 40	72.6%
Medium	1 and 3	51.4 + 73.5	35 + 60	76.1%
High	4 and 6	104.2 + 86.7	50 + 100	78.6%

The loss ratios to standard are increasing with modification. This seems to indicate that the experience rating plan is not correcting enough for the differences it identifies; the credibility is too low. However, given only six risks, due to the large random fluctuation in loss ratios, it is hard to draw any conclusion.

Alternately, group the risks into two groups by credit and debit mods.
The credit risks are 2 and 5. Standard Premium is: 40.7 + 48.8 = 89.5.
Losses are: 25 + 40 = 65. Loss Ratio to Standard is: 65/89.5 = 72.6%.
The debit risks are 1, 3, 4, and 6. Standard Premium is: 51.4 + 73.5 + 104.2 + 86.7 = 315.8.
Losses are: 35 + 60 + 50 + 100 = 245. Loss Ratio to Standard is: 245/315.8 = 77.6%.
The loss ratios to standard is higher for the debit risks than for the credit risks. This seems to indicate that the experience rating plan is not correcting enough for the differences it identifies; the credibility is too low. However, given only six risks, due to the large random fluctuation in loss ratios, it is hard to draw any conclusion.

(b) 1. A split plan would allow one to give more weight to primary losses and less weight to excess losses. This more general plan usually works better for Workers Compensation Insurance due to its highly skewed unlimited loss distribution. Splitting losses into primary and excess, the distributions will be less skewed resulting in better predictive accuracy.
2. One could use a more general formula for credibility, such as is used by the NCCI Experience Rating Plan. The NCCI Experience Rating Plan uses the more general form: $Z = (E + I) / (JE + K)$, which would take into account the phenomena of risk heterogeneity (diversifying operations as size increases) and parameter uncertainty (changing conditions that affect large risks as much as small risks). Tests of performance have shown that this more general formula works better for Workers Compensation experience rating than does the Buhlmann credibility formula used in this question. Put another way, one can have the ballast value increase with size such that there are is a maximum credibility significantly less than one. (See the table in the NCCI experience rating plan.) Due to changing conditions that affect large risks as much as small risks, the variance of loss ratios decline slower than linear rate with size. Therefore, large risk should not be essentially self-rated as would be the case for the credibility formula in the question.
3. One could decrease the Buhlmann credibility parameter of 50,000, thus increasing the credibility. (Assuming in part (a) you concluded that the credibility was too low.)

Comment: If as stated in the question this group of risks were homogeneous, in other words none of the risks were better or worse than average, then there would be no reason to apply experience rating.

With only 6 risks, due to the large random fluctuation in loss ratios, there is no advantage to grouping them for the analysis. However, the CAS took the phrase "group risk as appropriate" to mean that one "needed to group the risks in order to appropriately assess the experience rating plan. The optimal grouping was to group the risks by mod (low, mid, high) as explained in the Venter (and) Gillam paper. Some candidates grouped credit versus debit risks, which was acceptable. Those who did not appropriately group the risks could only receive partial credit."
The Venter paper says no such thing. In Venter, a large number risks were grouped into five categories. (The use of five rather than four or six was a matter of judgement. Dorweiler in his original study had 10 groups, which he also grouped into two: credit and debit mods.) For the quintiles test it is necessary to group. However, one way to perform the type of test done in this question, is to compute a correlation as per the Meyers-Dorweiler criterion in the paper by Mahler; no grouping is required. In my opinion, unfortunately, the examiners are showing their limited understanding of this subject. Also they graded based on what they intended the question to mean rather than what it actually said.

In part (b), according to the CAS, "This question asked candidates to identify a change to the experience rating plan itself that would improve its ability to correct for the differences it identifies. Thus, this part was not directly related to the specific numerical example given in part a. Candidates who misunderstood the question and simply said that K should be lowered (to increase credibility) received partial credit, since it is true that if K is lowered, the standard loss ratios calculated in part a. become nearly flat across mod group."
Their second sentence does not follow from their first sentence. It would not be unreasonable to assume some connection between parts (a) and (b), particularly since part (b) refers to "the" plan that you had just tested in part (a). Again they graded based on what they intended the question to mean rather than what it actually said. Changing the parameter(s) of the credibility formula is one important way to change a plan; many of the quintiles tests by the NCCI were applied to exactly this issue.

In order to see whether the experience rating is doing a good job of identifying differences in insureds, we would look at the subsequent Manual Loss Ratios.

Risk	A	E	Z	M	Man. Prem.	Loss	L.R. to Man.
1	130	125	71.4%	1.03	50	35	70.0%
2	60	85	63.0%	0.81	50	25	50.0%
3	160	150	75.0%	1.05	70	60	85.7%
4	200	130	72.2%	1.39	75	50	66.7%
5	100	150	75.0%	0.75	65	40	61.5%
6	250	175	77.8%	1.33	65	100	153.8%

Mod:	0.75	0.81	1.03	1.05	1.33	1.39
L. R. to Manual:	61.5%	50.0%	70.0%	85.7%	153.8%	66.7%

Ideally, we would like the subsequent loss ratios with respect to manual premium to be highly positively correlated with the modifications. Based on this limited data the subsequent loss ratios with respect to manual premium appear to be correlated with the modifications; the correlation is 59%. The experience rating plan appears to be identifying some differences between risks.

In part (b), if the plan did not have an limit on the size of accident entering the plan, this would also be a worthwhile improvement.

10. (1.5 points) An actuarial analyst has experience rated five groups of policies under the current rating plan and two alternatives, Plan A and Plan B. The results are as follows:

		Manual Loss Ratio			Standard Loss Ratio		
Risks with	Manual Premium ($000)	Current Plan	Proposed Plan A	Proposed Plan B	Current Plan	Proposed Plan A	Proposed Plan B
Lowest mod	$50,000	0.70	0.70	0.70	1.03	0.97	0.89
Next lowest	$70,500	0.85	0.80	0.90	1.02	1.02	0.92
Middle	$98,000	1.05	1.05	1.05	0.98	0.96	0.93
Next highest	$150,000	1.20	1.15	1.15	0.97	1.02	1.06
Highest mod	$10,000	1.45	1.55	1.35	0.96	1.06	1.11

a. (0.75 point) The off-balance factor for the current plan is 1.05 and for proposed Plan A is 0.99. The analyst says that proposed Plan A performs better because the off-balance is less than 1. Critique this statement.

b. (0.75 point) The actuarial analyst recommends staying with the current plan because it has made the higher mod groups more attractive to write and it has the least standard loss ratio spread. Critique this reasoning.

10. (a) The off-balance is the ratio of standard premium to manual premium, or equivalently the weighted average modification using manual premium as the weights.

Ideally one would want the experience rating plan to be in balance, so that it neither increases nor decreases the total amount of revenue. However, the balance of the plan varies over the underwriting cycle; when rates are inadequate mods increase, while when rates are excessive mods decrease. Thus one would prefer an off balance to average to one over the course of an underwriting cycle rather than in any particular year.
Plan A is closer to being in balance over some period of unknown length than is the current plan. However, having an off-balance less than one is not a goal. Thus the analyst's conclusion is not valid.

In order to test whether the experience rating plan is doing a good job of differentiating between risks, look at the subsequent manual loss ratios. If the manual loss ratios increase with mod, then the plan is doing a good job; here Plan A seems to be doing the best job identifying differences between risks.
In order to test whether the experience rating plan is appropriately adjusting for the risk differences it has identified, we look at the subsequent standard loss ratios. If the standard standard ratios are flat with mod, then the plan is doing a good job; here Plan A is doing the best job adjusting for risk differences.
We would also like the variance of loss ratios to standard to be small. Both the current plan and plan A do a better job of this than plan B.

(b) Making the higher mod groups more attractive to write than they would be without experience rating is a goal. However, in this case, the current plan has made the higher mod groups more attractive to write than the lower mod groups, which is not a goal. Rather the plan should make the different groups equally attractive to write; we would like to see no correlation between mod and subsequent loss ratio to standard premium. Here instead they are negatively correlated, indicating that we have overcorrected for the differences the plan has identified; the credibility appears to be too big in the current plan.
Plan A does the best in this regard, since its loss ratios to standard premium are close to flat. Plan B has loss ratios to standard that increase with mod, indicating that the credibility used was too small.
The current plan would make underwriters less willing to write credit risks, while Plan B would make underwriters less likely to write debit risks; neither of these is desirable.

Comment: According to the CAS examiners report, in part (a):
"To receive full credit, the candidate must state that the off balance is not an indicator of plan performance, what the off balance is or represents and finally what metrics can be used to evaluate the performance of the experience rating plan."
While the off-balance is not the most important indicator of plan performance, or the one the examiners were looking for, the first phrase is not true.
It is always a good idea to define what you are talking about, in this case the off-balance, to demonstrate that knowledge to the examiners.
I do not know how you would know from the wording of the question to "state what metrics can be used to evaluate the performance of the experience rating plan." One might not have done so since part (a) is only worth 0.75 points and the previous question number 9 asked about this. In my opinion, it would have been much better for them to ask you what metrics were appropriate to use and ask you to apply them to the given data, if they wanted you to do this. Unfortunately, everyone taking the exam who is not a mind reader will both lose some credit somewhere due to not writing enough, and waste some time somewhere due to writing too much.

Usually we would group so that the different quintiles have approximately the same manual premium; this is not the case here. This would impact the performance of a quintiles test; nevertheless one could apply this test to each of the plans.
Quintiles Test Statistic is: (variance of modified loss ratios) / (variance of unmodified loss ratios).
For the current plan: 0.00097 / 0.08625 = 0.0112.
For plan A: 0.00168 / 0.11125 = 0.0151.
For plan B: 0.00937 / 0.06075 = 0.1542.
Based on the quintiles test the current plan is somewhat better than Plan A, with plan B performing badly. We note that the quintiles test looks at the variance of standard loss ratios, but not the pattern of standard loss ratios by mod.

The average modification for each quintile is the ratio of the manual loss ratio to the standard loss ratio. Thus in theory one could use the given information to compute the off-balance of each plan. However, we do not know if those risks too small to be experience rated are included in this data; they need to be included in the calculation of the off balance (as if they had a mod of one). Also the given data is presumably for only one size category of risks.

11. (2 points) Assume that workers compensation rate adequacy in a particular state has improved for several successive years.

a. (0.25 point) Briefly describe the impact this improvement will have on the statewide off-balance.

b. (0.5 point)
Discuss the effect that the off-balance impact will have on the state's indicated rate level.

c. (0.5 point) Discuss the effect on the off-balance factor and the state's premium adequacy over time if inadequate rates are approved.

d. (0.75 point) Fully explain why the experience rating off-balance is frequently a credit.

11. (a) The off-balance is the ratio of standard premium to manual premium, or equivalently the weighted average modification using manual premium as the weights.
As the rate adequacy improves, in experience rating the expected losses will increase compared to the actual losses, and thus the average mod will decrease. Therefore, the off-balance decreases.

(b) I assume that one is using standard premium, without any specific ratemaking adjustment to take into account the average historical experience modification.
The historical loss ratio to standard premium used in ratemaking includes the historical off-balance. Thus we are implicitly assuming that this historical off-balance will continue, in other words that the historical average experience modification will continue into the effective period.
However, if an indicated rate increase is approved, that will decrease the average modification during the effective period compared to the historical period. Thus less standard premium will be collected than assumed. Therefore, the rates will be inadequate; the indicated rate increase was smaller than was needed after taking into account the decrease in the off-balance.
Conversely, if a rate decrease was indicated, then the absolute value of the indicated rate decrease will be smaller than was needed after taking into account the increase in the off-balance.

(c) If inadequate rates are approved, then in experience rating the expected losses will be small compared to the actual losses. Thus the average mod will be greater than one; the off-balance will be greater than one. The standard premium will be greater than the manual premium. While premiums will be still be inadequate, on a standard premium basis they will be less inadequate than on a manual premium basis. The experience rating plan will partially compensate for the inadequacy of the rates.

(d) If overall the expected losses in the plan match the actual losses, then one would expect an average modification of one. (This is what one would expect if rates are adequate and the expected loss rates are properly calculated.) However, this ignores the differing credibilities and average experience of different sizes of insureds.
Larger risks have on average better experience than the overall average.
Thus the average modification for larger risks is less than the overall average modification.
Smaller risks have on average worse experience than the overall average.
Smaller risks also have less credibility or are not even eligible for experience rating.
Thus the better experience of larger risks gets more weight than the worse experience of smaller risks.
Thus the average modification tends to be a slight credit, in other words, the off-balance tends to be a slight credit, such as for example 0.99.

Comment: If excessive rates are approved, the experience rating plan will partially compensate for the excessiveness of the rates.

Some reasons why larger insureds have lower loss ratios on average than smaller insureds:
Since in the plan the experience of large firms receives greater credibility than the experience of small firms, large firms have greater incentives to reduce losses. Also very large insureds are likely to be on retros or large deductible plans, providing more incentives to control losses.
Safety programs require large fixed costs: installing guards on machines, replacing dangerous equipment, implementing safety programs, and hiring onsite medical personnel. The large expenditures required may be more cost-effective for large firms than for small firms.
Small risks may not incur severe injuries with sufficient frequency to warrant post-injury and back-to-work programs, both of which help decrease the severity of claims.

12. (2.5 points) Given the following loss ratios for a set of five identical risks:

Risk	Loss Ratio
1	40%
2	40%
3	80%
4	100%
5	140%

Assume that the sample loss ratio of 80% equals the expected loss ratio.
Construct a Table M showing the insurance charges for entry ratios from 0 to 2.0 in increments of 0.50.

12.

Risk	Loss Ratio	Entry Ratio
1	40%	40%/ 80% = 0.50
2	40%	0.50
3	80%	1.00
4	100%	1.25
5	140%	1.75

Calculate the double sum upwards, and normalize by dividing the double sum column by its value at zero, in this case 20. (The calculation shows increments of 0.25, although in this case we are only interested in charges at increments of 0.5.)

Entry Ratio	# of Risks	Sum Up	Double Sum up	Charge
0.00	0	5	20	**1.0000**
0.25	0	5	15	0.7500
0.50	2	3	10	**0.5000**
0.75	0	3	7	0.3500
1.00	1	2	4	**0.2000**
1.25	1	1	2	0.1000
1.50	0	1	1	**0.0500**
1.75	1	0	0	0.0000
2.00	0	0	0	**0.0000**
	5			

For example, 7 + 3 = 10. 10/20 = 0.50.

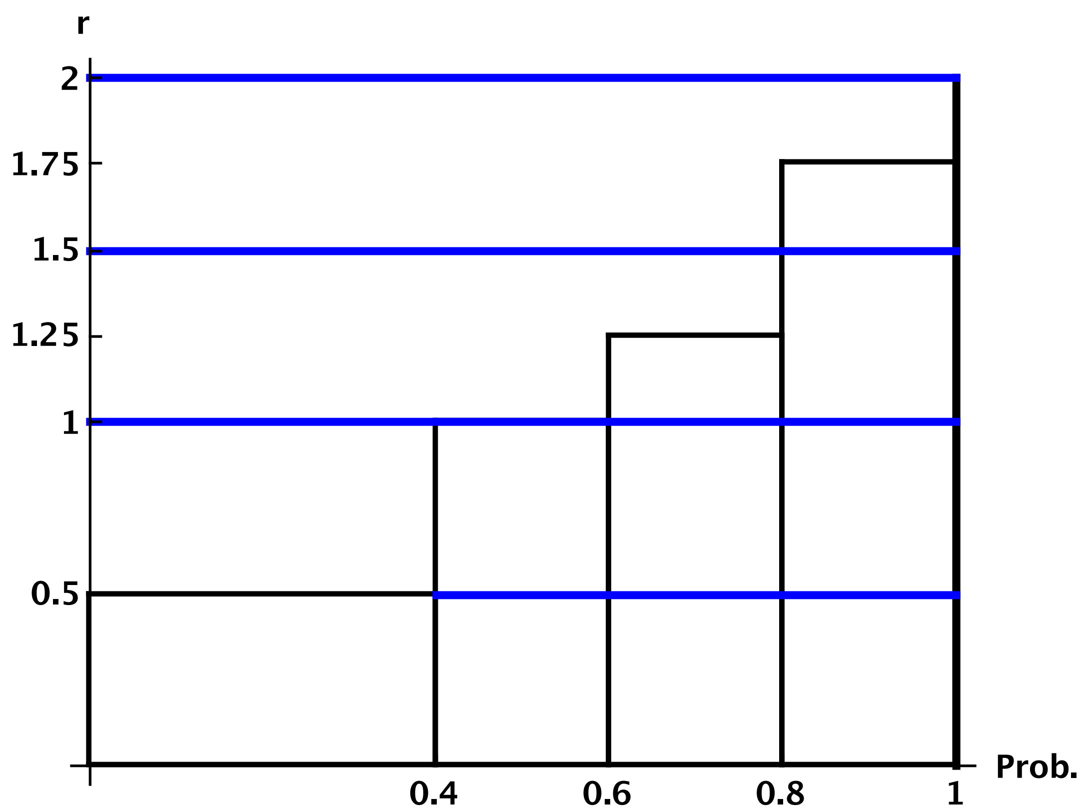
Alternately, $\phi(0) = 1$. $\phi(0.5) = (1/5)(1 - 0.5) + (1/5)(1.25 - 0.5) + (1/5)(1.75 - 0.5) = 0.5$.

$\phi(1) = (1/5)(1.25 - 1) + (1/5)(1.75 - 1) = 0.2$. $\phi(1.5) = (1/5)(1.75 - 1.5) = 0.05$. $\phi(2) = 0$.

Alternately, we can work with the percentage of risks above each entry ratio:

Entry Ratio	# Risks	# Risks above	% Risk above	Charge
0	0	5	100%	0.5 + (0.5)(100%) = 1
0.5	2	3	60%	0.2 + (0.5)(60%) = 0.5
1.0	1	2	40%	0.1 + (0.25)(40%) = 0.2
1.25	1	1	20%	0.05 + (0.25)(20%) = 0.1
1.50	0	1	20%	(0.25)(20%) = 0.05
1.75	1	0	0	0
2.00	0	0	0	0

Alternately, the following Lee Diagram might help you do the calculations.



$\phi(0)$ = area above horizontal line at 0 and within the rectangles = 1.

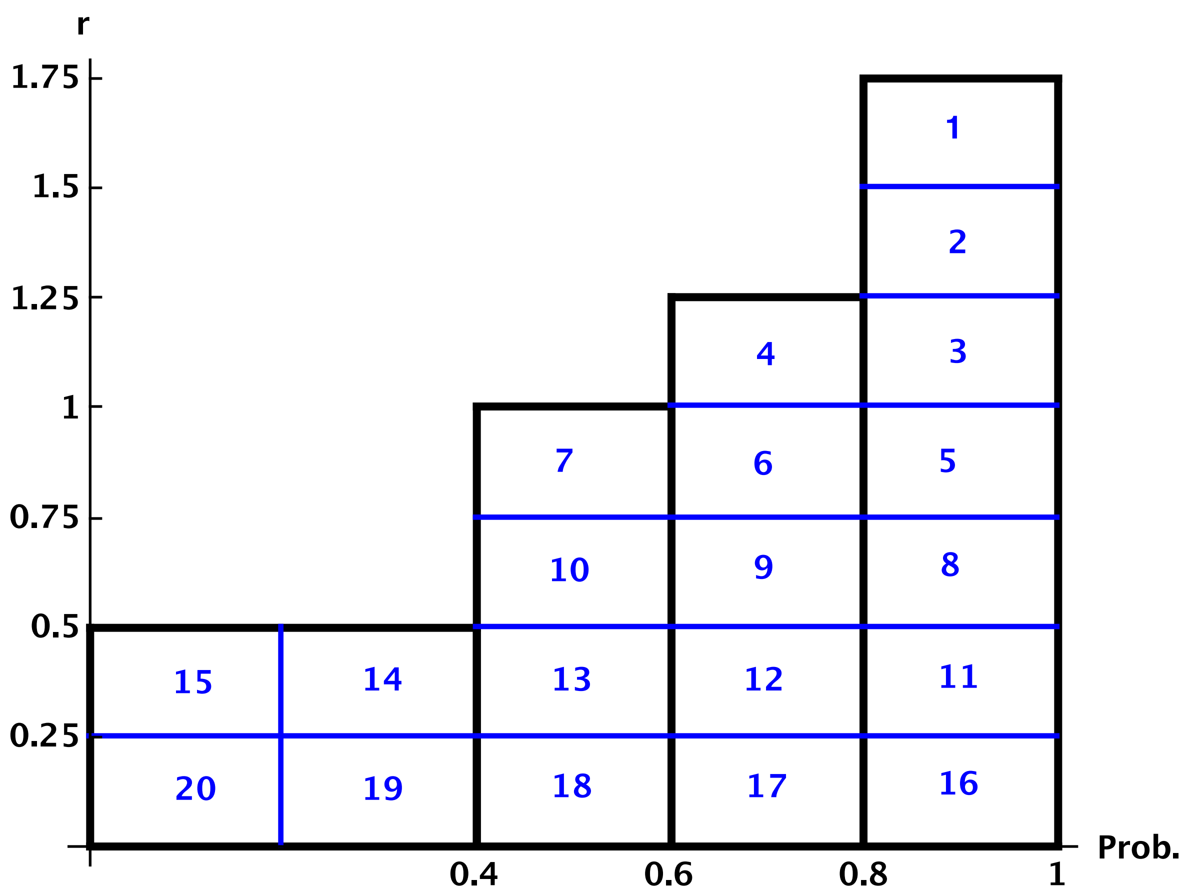
$\phi(0.5) = (0.6 - 0.4)(1 - 0.5) + (0.8 - 0.6)(1.25 - 0.5) + (1 - 0.8)(1.75 - 0.5) = 0.5.$

$\phi(1) = (0.8 - 0.6)(1.25 - 1) + (1 - 0.8)(1.75 - 1) = 0.2.$

$\phi(1.5) = (1 - 0.8)(1.75 - 1.5) = 0.05.$

$\phi(2) = 0.$

Alternately, in the following Lee Diagram I have divided the area under the broken line into 20 rectangles of equal area. (This is analogous to the double sum up method.)



$\phi(0)$ = (number of boxes above horizontal line at 0) / (total number of boxes) = 20/20 = 1.

$\phi(0.5)$ = 10/20 = 0.5. $\phi(1)$ = 4/20 = 0.2.

$\phi(1.5)$ = 1/20 = 0.05. $\phi(2)$ = 0/20 = 0.

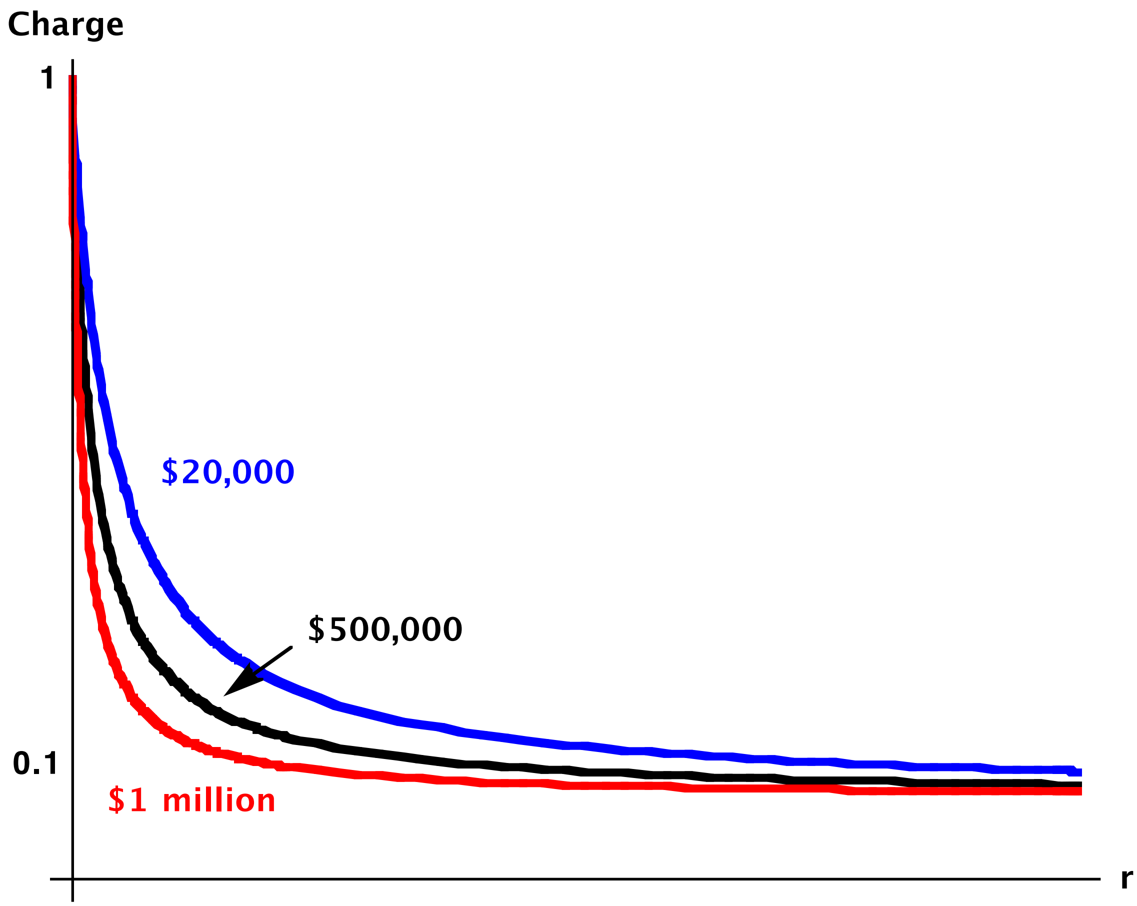
<u>Comment</u>: Since the sample loss ratio of 80% equals the expected loss ratio, there is no need to normalize as per the final page of the paper by Brosius.

13. (1.75 points) A cohort of policies has a loss elimination ratio of k = 0.1.

a. (1 point) Draw a graph with three curves showing the relationship between the Table L charge (y-axis) and the entry ratio (x -axis) for policies with premiums of $20,000; $500,000; and $1,000,000.

b. (0.75 point) Briefly explain three main features of the curves that describe their proper relationship to the axes and to each other.

13. (a) The curve for the \$1 million risk is lowest, while the curve for the \$20,000 risk is highest:



(b). 1. The larger risks have a smaller insurance charge, since there is less random fluctuation in their annual loss ratios than for a smaller risk.

2. $\phi^*(0) = 1$.

3. $\phi^*(\infty) = 0.1 = k$ = loss elimination ratio.

Even if there is no maximum premium, we still need to pay for the accident limitation via k.

4. The derivative is negative; $\phi^*(r)$ decreases as r increases.

The derivative is: $-S^*(r) \le 0$.

5. The second derivative is positive; the curve is concave upwards.

The second derivative is: $f^*(r) \ge 0$.

6. Limit of $\phi^*(r)$ as the premium size approaches infinity is: $\begin{cases} k & \text{if } r \ge 1 - k \\ 1-r & \text{if } r \le 1 - k \end{cases}$.

Comment: See Exhibit 6 in Skurnick.

We were asked to draw a diagram with entry ratios on the x-axis rather than a Lee Diagram.

14. (1.5 points)
A policy subject to a balanced retrospective rating plan is written with the following values:

- Expected loss ratio = 70% of standard premium
- Tax multiplier = 1.10
- Expense ratio = 20%
- Loss conversion factor = 1.125
- Maximum retrospective premium = 125% of standard premium
- Minimum retrospective premium = 75% of standard premium

The appropriate values from Table M for the current year are as follows:

- Insurance charge at entry ratio associated with maximum = 0.653
- Insurance savings at entry ratio associated with minimum = 0.031

Calculate the expected retrospective premium as a percentage of standard premium for the policy.

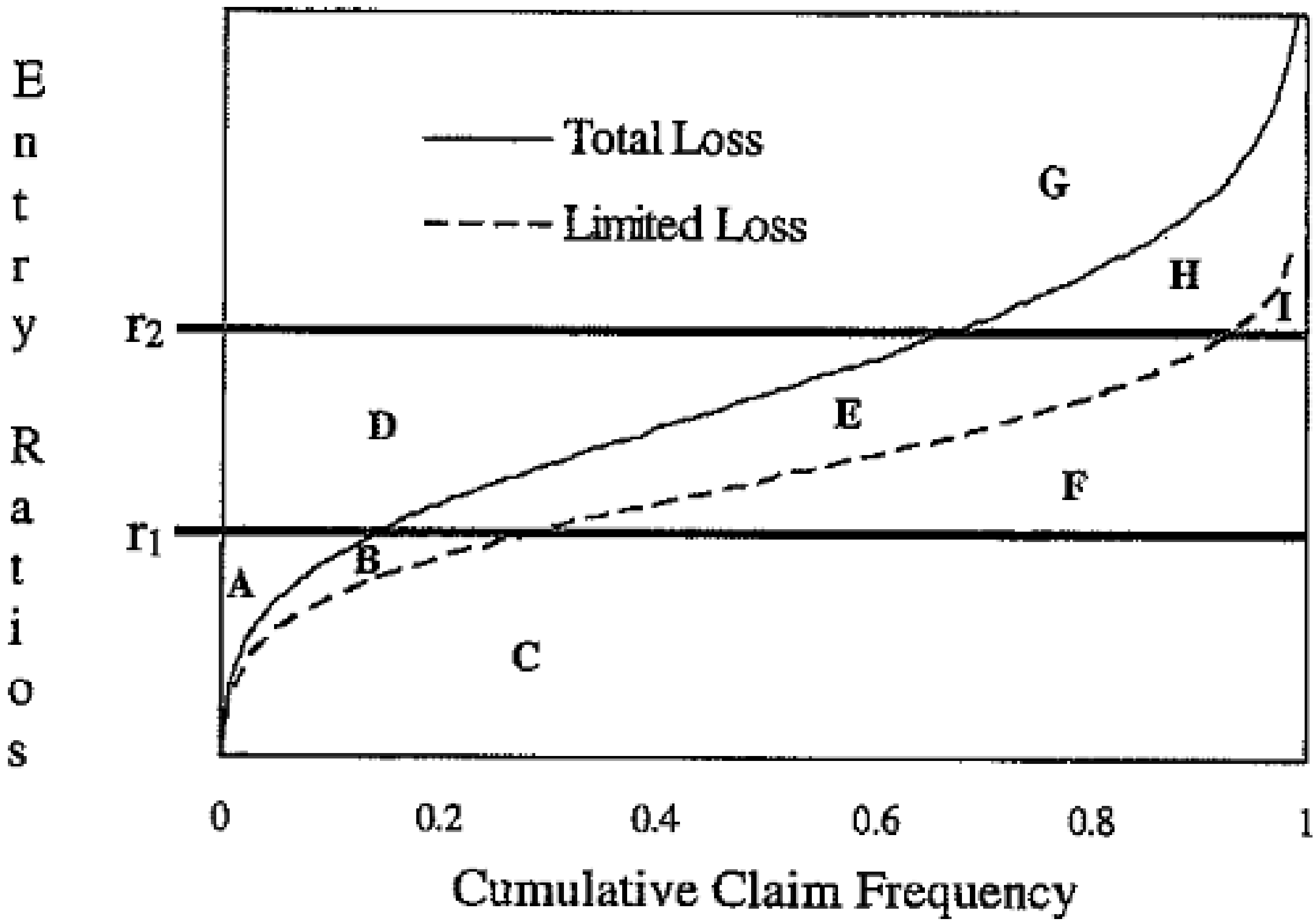
14. I assume that the expense ratio includes the LAE and reflects premium discounts.
For a balanced retro plan, the expected premium equals guaranteed cost premium.
T (e + E) = (1.1) (20% + 70%) = **99%**.

Alternately, the converted insurance charge is: (1.125)(70%)(0.653 - 0.031) = 0.4898.
Basic premium is: 0.4898 + 0.2 - (1.125 - 1)(0.70) = 0.6023.
The expected value of losses entering the retro calculation is:
$E - E(X_G - S_H)$ = 70% - (70%)(0.653 - 0.031) = 26.46% of standard premium.

⇒ Expected retro premium = (1.1) {0.6023 + (1.125)(0.2646)} = **0.990**.

Comment: I assume that there is no accident limitation, since none is mentioned.
According to the examiner's report, the CAS assumed that students would use the more complicated solution I gave second. The simpler solution that I gave first "can be used IF AND ONLY IF the candidate stated that it's a balanced plan."
While I see where the examiners are coming from, I do not think it is fair to require students to repeat something stated in the first sentence of the question. A good example of why it sometimes helps to explain a little of what you are doing in your solution, even if it should be obvious to someone who understands the material.
(The important definition of a balanced retro plan is that its expected premium is equal to guaranteed cost premium. Thus if you understand this material well, you could solve this question in your head.)

15. (3.25 points) The following diagram depicts a book of business for a retrospectively rated workers compensation plan:



The descriptions of the labels on the diagram are as follows:

- r_1 = Aggregate minimum.
- r_2 = Aggregate maximum.
- Total loss - Total aggregate losses with no per-accident limit.
- Limited loss - Total aggregate losses after application of a per-accident limit.

QUESTION 15 CONTINUED ON NEXT PAGE

a. (1.5 points) Using the letter labels above to represent portions of the graph, describe the following quantities:

1.) ϕ - The Table M insurance charge at r_2.

2.) ψ - The Table M savings at r_1.

3.) ϕ^* - The Table L insurance charge at r_2.

4.) ψ^* - The Table L savings at r_1.

5.) I - The amount expected to be paid by the insured
with an aggregate limit but no per-accident limit.

6.) I* - The amount expected to be paid by the insured in the presence of both
an aggregate and a per-accident limit.

b. (0.75 point) A change in relevant workers compensation law goes into effect that causes a significant increase in the most severe losses. Briefly explain what effect this is likely to have on the areas of E and H in the above diagram.

c. (1 point) Assume loss frequency and severity are independent, all individual losses come from the same distribution, and the only difference between large and small accounts is the number of expected claim counts. Determine whether the above diagram is accurate for both large and small accounts. Justify your answer.

15. (a) 1) H + I
2) A
3) I + (B + E + H)
4) A + B
5) Assuming that what was meant was I = net Table M insurance charge, then: H + I - A.
Alternately, assuming that what was meant was the amount paid by the insured other than due to the loss conversion factor, basic premium and the tax multiplier; in other words, what are the expected losses entering the retro calculation, then:
Area below F(r) and below r_2, however, including all of the area below r_1: E + F + A + B + C.
6) Assuming that what was meant was I* = net insurance Table L charge, then:
I + (B + E + H) - (A + B) = I + H + E - A.
Alternately, assuming that what was meant was the amount paid by the insured other than due to the loss conversion factor, basic premium and the tax multiplier; in other words, what are the expected losses entering the retro calculation, then:
Area below F*(r) and below r_2, however, including all of the area below r_1: F + A + B + C.

(b) B + E + H represents k, the loss elimination ratio due to the accident limit.
With more severe accidents, the loss elimination ratio will increase and therefore so will this area between F(r) and F*(r).
Therefore Areas E and H will increase.

Alternately, the large loses may already be above the accident limit. Therefore, an increase in the most severe losses will not have much effect on the limited loss curve F*(r), but will have a significant impact on the unlimited loss curve F(r), particularly for larger entry ratios which are more likely to include large losses. Therefore, the gap between F(r) and F*(r) will increase, and areas E and H will increase.

(c) The distributions of unlimited losses, F(r), would differ by size of insured.
There is more random fluctuation in loss ratios for smaller insureds.
Therefore, for smaller insureds the graph of F(r) should reflect more probability in both the lefthand and righthand tails, than the similar graph for larger insureds.
The resulting insurance charges would be bigger for small insureds than for large insureds.
Thus the diagrams for large and small insureds would be different.

Comment: r_2 is the entry ratio corresponding to the maximum premium,
while r_1 is the entry ratio corresponding to the minimum premium.

In part (b), The shape of F(r) will also change.
(The severity distribution has changed; there is a larger chance of a high loss ratio due to a severe accident. However, the area under F(r) has to remain one.)
Therefore, area H will increase and area E is also likely to increase.
Area B corresponds to low entry ratios. At really low entry ratios the aggregate losses are low and it is impossible to have a really severe accident. Thus area B may either increase slightly or not increase. In total H + E + B has to increase.

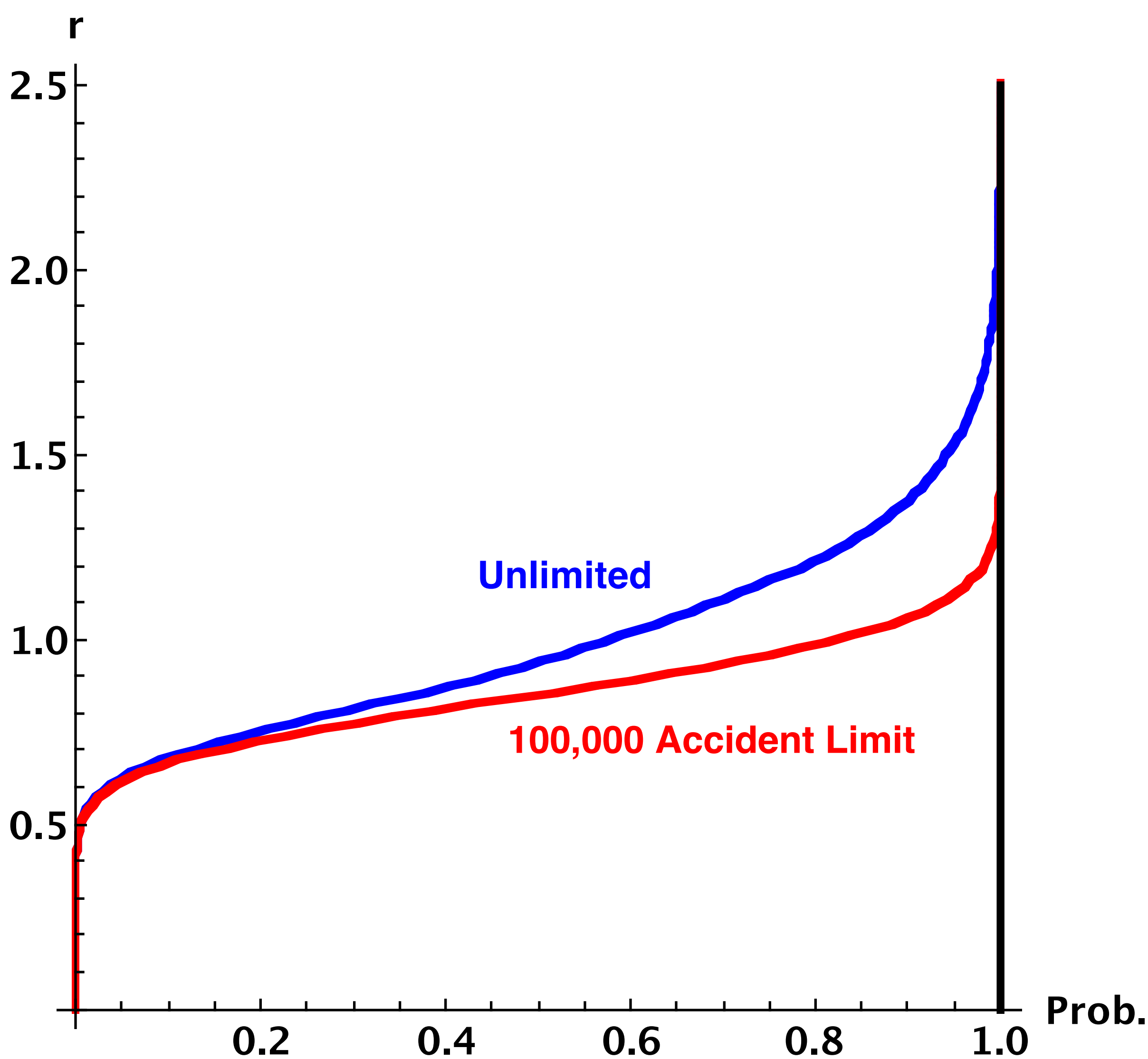
In part (c), the loss elimination ratio due to the accident limitation should be the same regardless of risk size. Thus the total area between F(r) and F*(r) would be the same regardless of risk size. However, the effect of the accident limit by entry ratio is different for very large and small insureds. For example, below the entry ratio corresponding to aggregate losses when there is just one claim of size equal to the accident limit, the accident limit can not have any effect. Thus below this entry ratio F(r) and F*(r) are the same. For a large insured with bigger expected losses, this entry ratio is smaller than for a smaller insured. This is another reason why the diagrams for large and small insureds would be different.

For a simple model, I applied the Panjer Algorithm to get the aggregate distribution.
(The Panjer algorithm is covered on Exam 4 and mentioned in the study note reading by Clark on reinsurance pricing.
I applied the Panjer Algorithm to a more complicated model in my section on Skurnick.)
Unlimited severity is $25,000 with probability 99% and $500,000 with probability 1%.

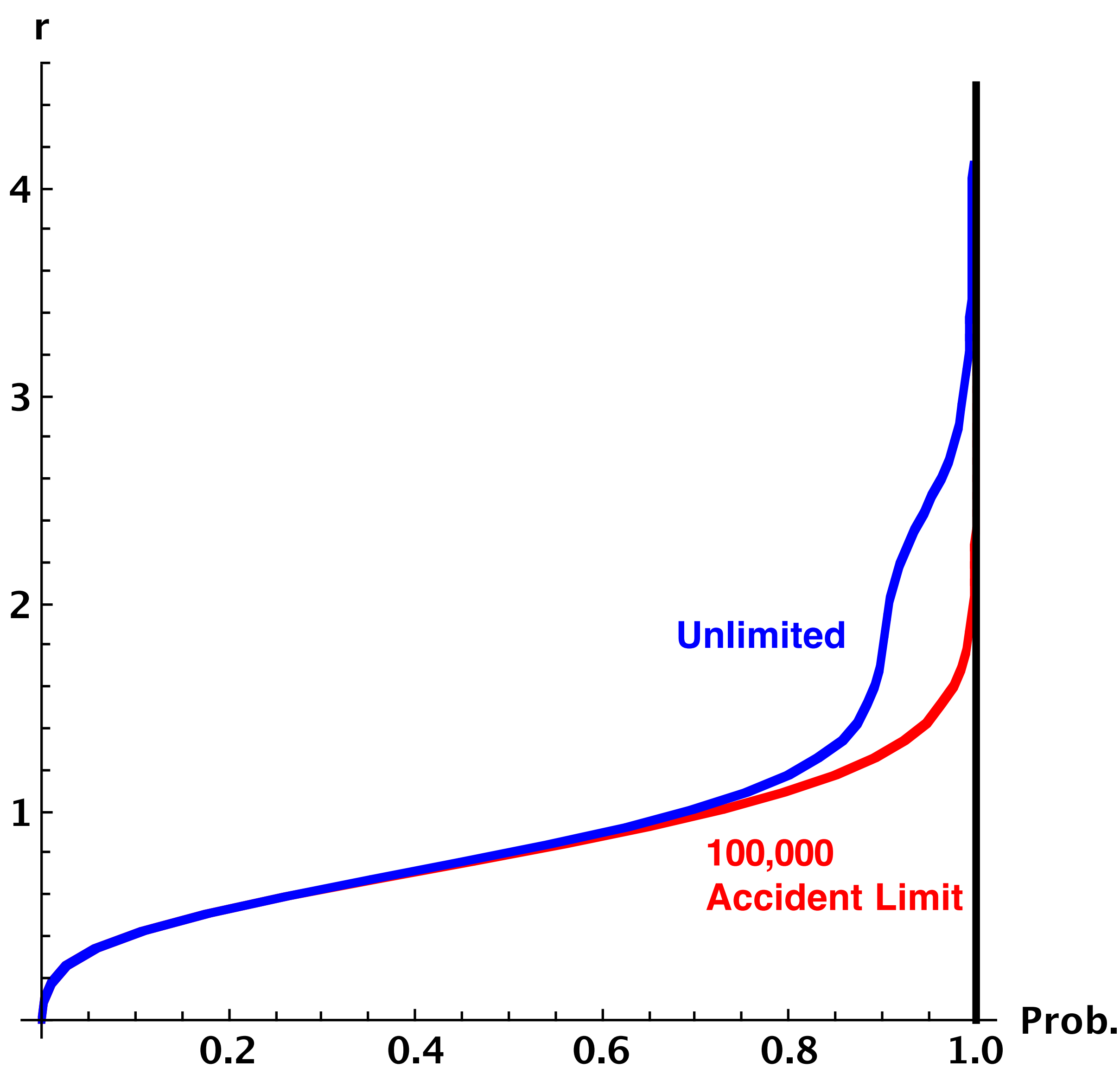
Frequency is Negative Binomial with $\beta = 0.5$.

For a “large” risk I chose r = 100, for a mean annual frequency of 50.

For such a large risk, here is a Lee Diagram of the aggregate distributions with either no accident limit or with a $100,000 accident limit:



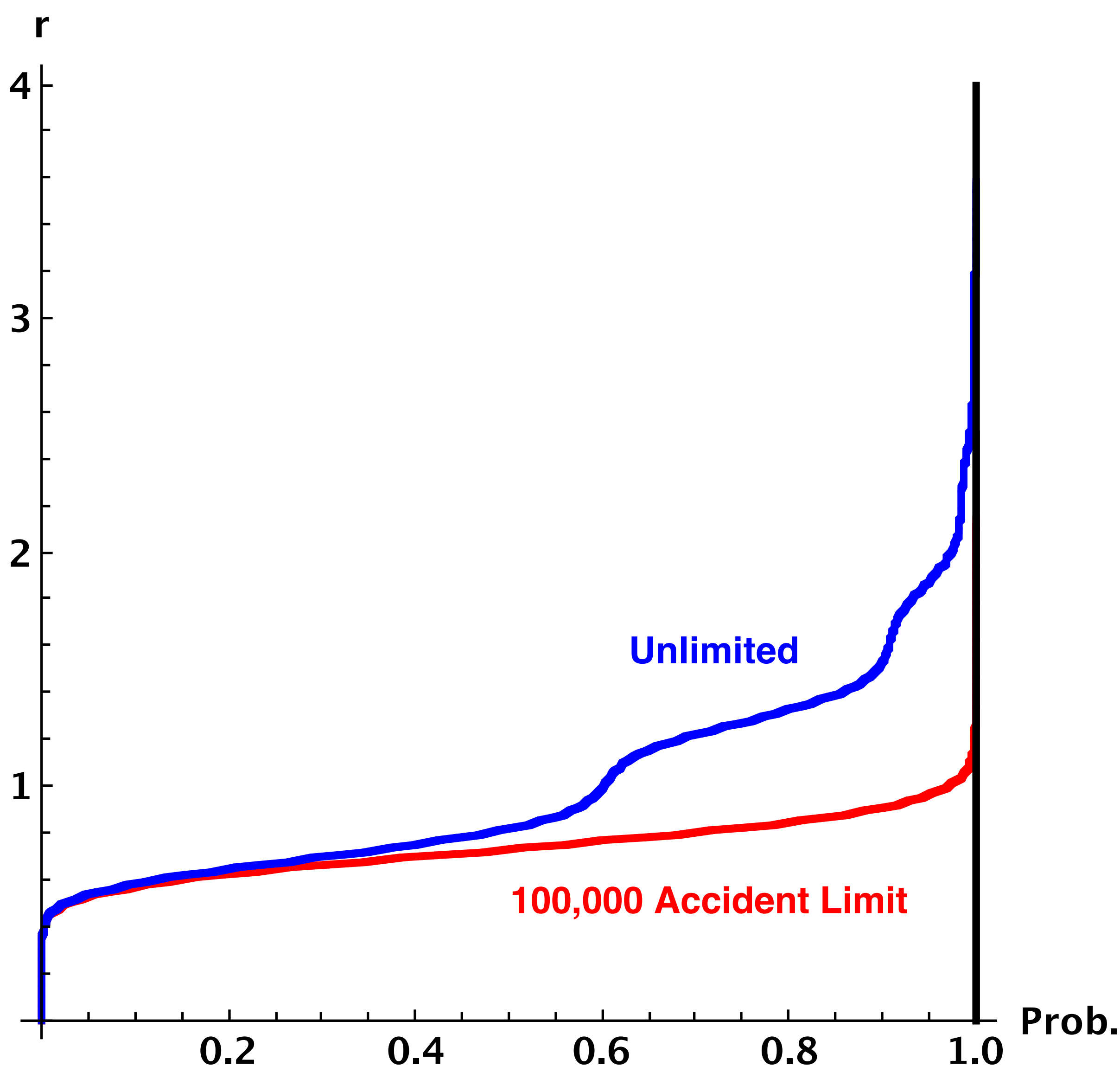
For a "small" risk I chose r = 20, for a mean annual frequency of 10. For such a small risk, here is a Lee Diagram of the aggregate distributions with no accident limit or with a $100,000 accident limit:



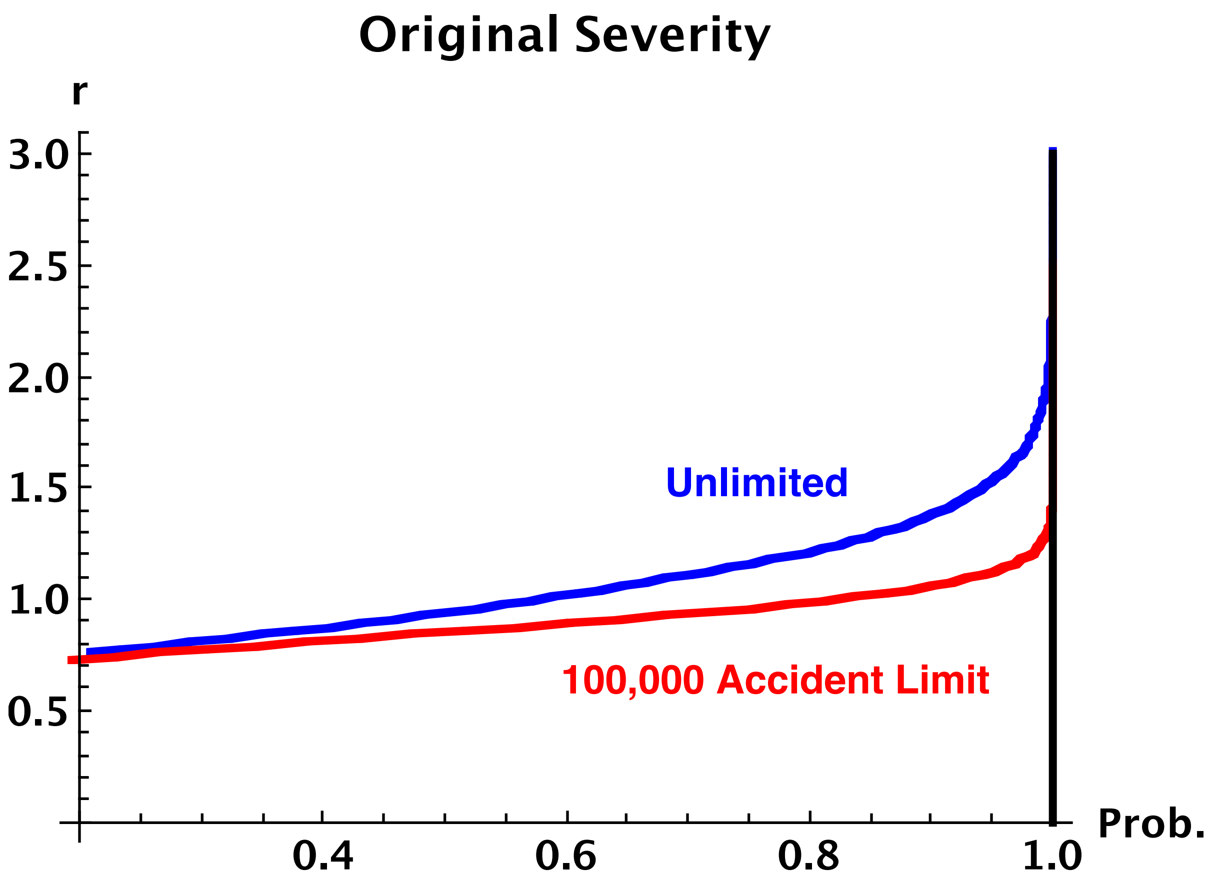
Verifying the solution to part (c), we can see that the diagrams for the larger and smaller risks are significantly different. However, the loss elimination ratio due to the accident limitation is the same regardless of risk size, thus the total area between F(r) and F*(r) is the same regardless of risk size. However, the shapes of the areas between F(r) and F*(r) are different in the two graphs.

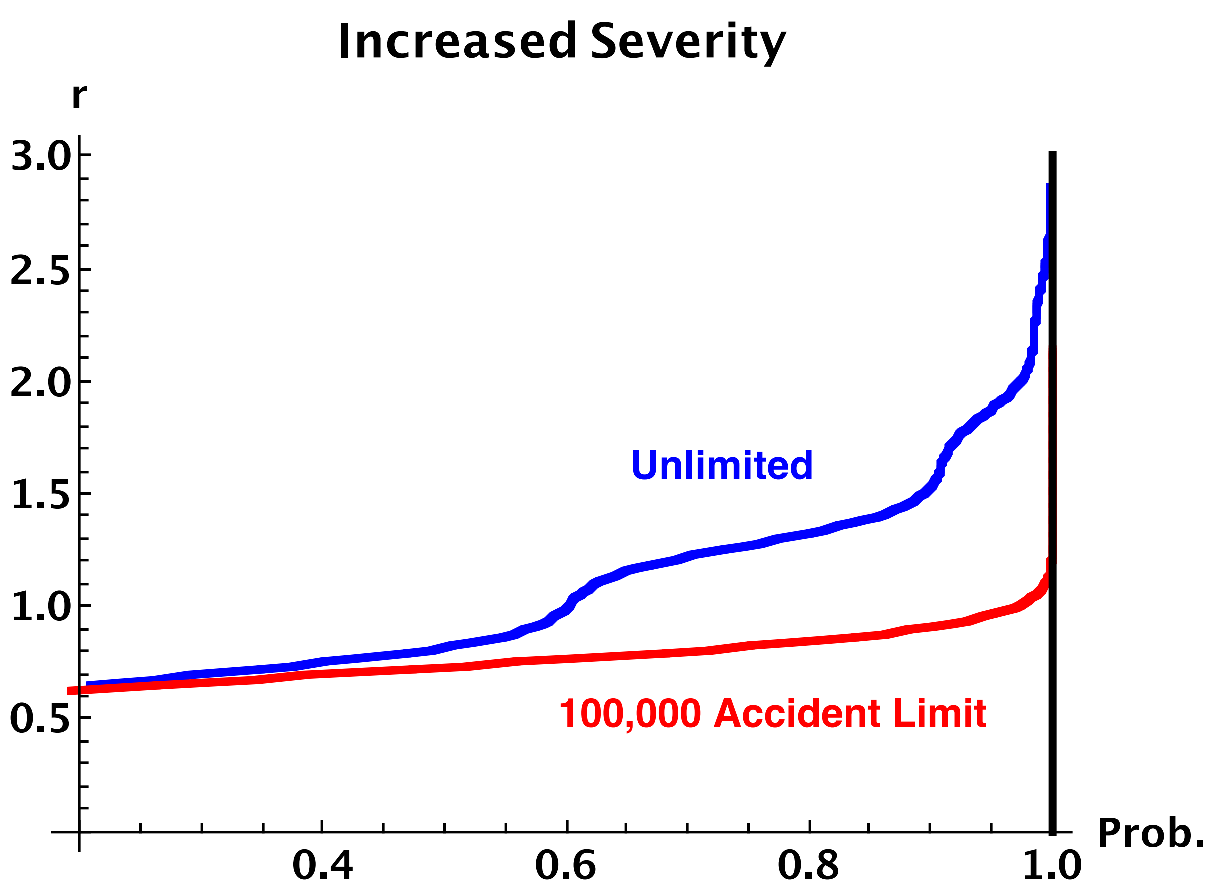
We see that for the smaller risk there is more probability of unusual loss ratios, in other words unusually small or large entry ratios. Also for the smaller risks F(r) and F*(r) are the same for more values of r than for is the case for the larger risks. (Although it is not clear from the graphs, for the smaller risks F(r) and F*(r) are the same for $r < 0.4$, while for the larger risks this is true for $r < 0.07$. In each case, these entry ratios corresponds to at most $100,000 in aggregate losses.)

Now let us change the unlimited severity to have instead \$25,000 with probability 99% and \$1,000,000 with probability 1%. This is along the lines of part (b), where "A change in relevant workers compensation law goes into effect that causes a significant increase in the most severe losses." For the new severity distribution, for a large risk with mean frequency of 50, here is a Lee Diagram of the aggregate distributions with no accident limit and with a \$100,000 accident limit:



Verifying, the solution to part (b), since the loss elimination ratio has increased the area between F(r) and F*(r) has increased as well as changed its shape. The equivalent of Areas E and H have each increased. It is unclear whether the equivalent of Area B increased. On the next page is a comparison of the original severity and the increased severity, both for the "large" risk with mean frequency of 50. It should be noted that the limited aggregate distributions are the same in dollar value, but since with the increased severity the mean unlimited aggregate is \$1.7375 million rather than \$1.4875 million, the dollar equivalent of the entry ratios differs between the two graphs.


Original Severity
r
3.0
2.5
2.0
1.5
1.0
0.5
Unlimited
100,000 Accident Limit
0.4
0.6
0.8
1.0
Prob.


Increased Severity
r
3.0
2.5
2.0
1.5
1.0
0.5
Unlimited
100,000 Accident Limit
0.4
0.6
0.8
1.0
Prob.

16. (1.25 points) An actuary calculates the insurance charges on an aggregate deductible for a general liability policy for house painters. All the losses in the historical data used in the analysis resulted from inadequate and/or sloppy paint jobs, which were relatively inexpensive to fix. Later, it is discovered that some paint contained a toxic substance and those painters are liable for very expensive remediation of the painted properties.
The new claims are 10% as common as the historical claims. For every 10 claims that would have been expected before, there are now 11, one of which is cleaning up toxic paint.
Had this been known, the expected cost of a policy would have been twice the cost the actuary used.
a. (0.75 point) At an entry ratio of 2.00, with no per-occurrence loss limit, explain whether the insurance charge would increase, decrease, or stay the same.
b. (0.5 point) Explain how a per-occurrence limit would affect the change in the insurance charge for the aggregate deductible.

16. (a) The coefficient of variation of the distribution of (unlimited) aggregate losses is increasing significantly. In other words, there will be more random fluctuation in loss ratios with the toxic claims than without them. Therefore, **the insurances charge at 2**, which is the percent of aggregate losses excess of twice the mean, **will increase**.

(b) With an occurrence limit, the amount that the insured retains is first limited for each occurrence. Therefore, the average amount the insured retains is reduced. Therefore, with an occurrence limit the Table L insurance charges would be greater than without an occurrence limit.
Some toxic claims that did not increase the amount the insurer paid without an occurrence limit, will with an occurrence limit increase the amount the insurance company pays, increasing significantly the Table L charge.
The occurrence limit is unlikely to affect any of the small sloppy paint claims. Thus with no toxic claims, there should be very little if any effect of the occurrence limit on the amount the insurer pays.
In other words, without toxic claims, the occurrence limit has very little if any effect on the Table L Insurance Charge.
Therefore, the increase in the Table L insurance charges due to large toxic claims will be <u>more</u> with an occurrence limit than without an occurrence limit.

Alternately, let us assume instead that the effect of the occurrence limit will be paid for by a separate ELF and then we will calculate an appropriate Table M insurance charge that will have no overlap with the ELF.
The occurrence limit will have little or no effect without the toxic claims. Thus, without toxic claims, the occurrence limit has very little if any effect on the Table M charge.
However, the toxic claims will be affected significantly by the occurrence limit. The effect of the occurrence limit on the toxic claims will be paid for by the large separate ELF, leaving less to be paid for by the Table M Charge. Therefore, with the toxic claims, the Table M charge should be less with the occurrence limit than without the occurrence limit.
Therefore, the change in the Table M insurance charges due to large toxic claims will be <u>less</u> with an occurrence limit than without an occurrence limit.

Comment: The aggregate limit on the large deductible is mathematically similar to the maximum premium for a retro policy. The occurrence limit for the large deductible policy is mathematically similar to the accident limit for a retro policy.

Mean aggregate loss = $\mu_{freq}\ \mu_{sev}$.

Variance of aggregate loss = $\mu_{freq}\ \sigma_{sev}^2 + \mu_{sev}^{\ 2}\ \sigma_{freq}^2$.

$\Rightarrow$ CV^2 of aggregate = (variance of aggregate) / (mean of aggregate)^2 = $CV_{sev}^2 / \mu_{freq} + CV_{freq}^2$.

With the introduction of large toxic paint claims, the first term increases a lot, while the second term decreases slightly, resulting in an increase in the coefficient of variation of aggregate losses.

Part (b) was difficult to reason out under exam conditions.

For a simple model, I applied the Panjer Algorithm to get the aggregate distribution.
(The Panjer algorithm is covered on Exam 4 and mentioned in the study note reading by Clark on reinsurance pricing.
I applied the Panjer Algorithm to a more complicated model in my section on Skurnick.)
I assumed the small sloppy painting claims were of size $25,000, while the toxic paint claims were ten times as large at $250,000.
Frequency is Poisson with mean 2 for the small claims and with mean 0.2 for the large claims.
(The overall frequency will affect the insurance charges, so this is just an illustrative example.)
The expected unlimited losses are $50,000 without the toxic claims and $100,000 with the toxic claims.

Thus if there are no toxic claims, then an entry ratio of 2 corresponds to an aggregate deductible of $100,000.
Without any occurrence limit the expected losses retained by the insured are $48,121.5.[1]
Choosing an occurrence limit of $100,000, there is no effect since the only occurrences are of size $25,000. Thus the expected losses retained by the insured are still $48,121.5.

If there are toxic claims, then an entry ratio of 2 corresponds to an aggregate deductible of $200,000.
Without any occurrence limit the expected losses retained by the insured are $77,184.4.[2]
Choosing again an occurrence limit of $100,000, there is an effect; the expected losses retained by the insured are now $68,639.5.[3]

[1] Based on the aggregate distribution calculated using the Panjer algorithm.
[2] Based on the aggregate distribution calculated using the Panjer algorithm.
[3] Based on the aggregate distribution calculated using the Panjer algorithm and a severity distribution limited to the occurrence limit of $100,000.

With no occurrence limit, the Insurance Charge in dollar terms is $50,000 - $48,121.5 = $1878.5 without toxic claims, and $100,000 - $77,184.4 = $22,815.6 with toxic claims. As a percent of expected losses these Insurance Charges are $1878.5/$50,000 = 3.8% without toxic claims, and $22,815.6/$100,000 = 22.8% with toxic claims. By either measurement, the insurance charge has increased with the addition of the toxic claims, as in the solution to part (a).

With a $100,000 occurrence limit, the Table L Insurance Charge in dollar terms is $50,000 - $48,121.5 = $1878.5 without toxic claims, and $100,000 - $68,639.5 = $31,361.5 with toxic claims. As a percent of expected losses these Insurance Charges are $1878.5/$50,000 = 3.8% without toxic claims, and $31,361.5/$100,000 = 31.4% with toxic claims. By either measurement, the Table L insurance charge has increased with the addition of the toxic claims, and the increase was more when we had the occurrence limit than when we did not, as in the solution to part (b).

Let us assume instead that the effect of the occurrence limit will be paid for by a separate ELF and then we will calculate an appropriate Table M insurance charge that will have no overlap with the ELF.

The occurrence limit has no effect without the toxic claims. With the toxic claims, the expected losses excess of the $100,000 occurrence limit (ignoring the aggregate limit) is: (0.2)(150,000) = $30,000.

Without toxic claims, the correct Table M insurance charge is what it was without the occurrence limit, $1878.5 or 3.8% of total expected losses. With toxic claims, the Table M Insurance Charge is without overlap: $31,361.5 - $30,000 = $1,361.5 or 1.4% of total expected losses. Thus, with the occurrence limit, the Table M insurance charge has decreased both in dollar amount and percentage terms with the addition of the toxic claims. Without the occurrence limit, the Table M Insurance Charge was $1878.5 or 3.8% without toxic claims, and $22,815.6 or 22.8% with toxic claims. The change in Table M insurance charge due to toxic claims was less with the occurrence limit than without an occurrence limit, as in the solution to part (b).

17. (1.5 points) In "Workers Compensation Excess Ratios: An Alternative Method of Estimation," Mahler discusses a method of estimating excess ratios using empirical data up to a certain truncation point and a fitted curve to handle larger loss sizes.
a. (0.5 point) Briefly discuss how to choose a truncation point for this method.
b. (1 point) Part of Mahler's method to handle larger loss sizes involves fitting a mixed Exponential-Pareto curve for estimating excess ratios. Evaluate the following alternative distributions in estimating excess ratios:

- Pareto curve only
- Mixed exponential-lognormal

17. (a) On the one hand, one wants to choose a truncation point as high as possible so as to rely on the empirical excess ratios as much as possible, without relying on empirical excess ratios that are subject to too much random fluctuation.
On the other hand, one wants to choose a truncation point that is low enough in order to leave enough data above it to normalize the data by hazard group so that it can be combined, and so that there is enough combined data to which to fit a distribution.
The truncation point chosen should be in the range where there is a reasonable balance between these offsetting goals.

(b) The mixed Exponential-Pareto curve was found by Mahler to fit the data well.

The Pareto distribution has a mean residual life that increases linearly, while the Exponential has a constant mean residual life. The Exponential portion of the mixture gets the vast majority of the weight and its behavior is significantly different than a Pareto. Thus the Pareto curve by itself has too heavy of a righthand tail and will not fit well. The Pareto Distribution used by itself would produce too large excess ratios and should <u>not</u> be used for estimating excess ratios.

The Pareto distribution has a mean residual life that increases linearly, while the LogNormal distribution has a mean residual life that increases slightly less than linearly. The Pareto has a somewhat heavier righthand tail than the LogNormal. Even though their behavior is different, both the LogNormal and Pareto have a lot of probability in their righthand tails. Thus a mixed Exponential-Lognormal might also fit the data well, and may be useful for estimating excess ratios. (One would have to work with the data in detail to determine whether a mixed Exponential-Pareto or mixed Exponential-LogNormal fit better.)
Very high limits would be modeled mostly by the LogNormal part of the mixture, and thus the excess ratios for very high limits would less for a mixed Exponential-Lognormal than for a mixed Exponential-Pareto.

18. (4 points) An actuary is pricing a Large Dollar Deductible (LDD) workers compensation policy. To price the excess loss portion, an actuary uses a blend of empirical data and a fitted curve to estimate excess loss pure premium factors. The cut-off for empirical data is $250,000.
Using the data below, calculate an LDD premium for a policy with a $500,000 deductible and no aggregate limit.

Historical adjusted loss and ALAE for similarly sized risks:

Loss and ALAE	Observed Percentage Of Claim Counts
8,000	80%
100,000	11%
250,000	6%
500,000	2%
1,000,000	0.7%
2,000,000	0.3%

Total expected losses per risk = 55,400

Excess ratios based on fitted mixed Exponential-Pareto distribution on losses Truncated and shifted at $250,000:

Entry Ratio	Excess Ratio
0.1	0.92
0.2	0.84
0.3	0.73
0.4	0.69
0.5	0.65
0.6	0.60
0.7	0.55
0.8	0.51
0.9	0.47
1.0	0.44

Standard premium	$100,000
Loss based assessment (% of loss and ALAE)	4%
ULAE (% of loss and ALAE)	8%
General expenses (% of standard premium)	5%
Credit risk (% of standard premium)	5%
Acquisition expense (% of net premium)	8%
Tax (% of net premium)	3%
Profit Load (% of net premium)	5%

18. I assume that the large deductible applies to loss and alae.
The average of the data truncated and shifted at $250,000 is:

$$1000 \ \frac{(500 - 250)(2\%) \ + \ (1000 - 250)(0.7\%) \ + \ (2000 - 250)(0.3\%)}{2\% \ + \ 0.7\% \ + \ 0.3\%} = 15{,}500 \ / \ 3\% = 516{,}667.$$

Thus the entry ratio corresponding to the $500,000 deductible is: $\frac{500{,}000 \ - \ 250{,}000}{516{,}667} = 0.484.$

Interpolating in the provided table, the excess ratio from the mixed distributions is: 0.656.
The empirical excess ratio at $250,000 is:

$$\frac{(500 - 250)(2\%) \ + \ (1000 - 250)(0.7\%) \ + \ (2000 - 250)(0.3\%)}{55.4} = 15.5 \ / \ 55.4 = 0.2798.$$

Therefore, the estimated excess ratio at $500,000 is: (0.656) (0.2798) = 18.35%.

I assume that the insurer pays all of the loss based assessments and all of ULAE.
Deductible premium is:

$$1000 \ \frac{(18.35\% \ + \ 4\% \ + \ 8\%)(55.4) \ + \ (5\% \ + \ 5\%)(100)}{1 \ - \ 8\% \ - \ 3\% \ - \ 5\%} = \mathbf{31{,}921}.$$

Comment: Somewhat different rounding and interpolation or lack thereof in using the provided table of excess ratios would result in somewhat different but acceptable modeled excess ratios than the 0.656 I calculated.
Using the given severity distribution one could calculate the given total expected losses per risk of 55,400.

19. (3.25 points) The following information is available for a LDD policy:

Standard premium $1,500,000
Expected ultimate loss ratio 0.75
State hazard group relativity 1.1
Deductible $200,000
Excess loss factor 0.21
Aggregate limit on deductible $1,000,000

The following Table M information is applicable to this policy:

Expected Loss Group	Range Rounded Values
30	$600,001 - $750,000
29	$750,001- $925,000
28	$925,001 - $1,100,000
27	$1,100,001 - $1,300,000
26	$1,300,001- $1,600,000
25	$1,600,001 - $1,950,000
24	$1,950,001- $2,200,000

Entry	Expected Loss Group						
Ratio	30	29	28	27	26	25	24
0.75	0.4069	0.3989	0.3911	0.3833	0.3755	0.3677	0.3599
0.81	0.3777	0.3690	0.3605	0.3521	0.3436	0.3352	0.3267
1.07	0.2764	0.2661	0.2557	0.2453	0.2349	0.2245	0.2141
1.15	0.2522	0.2417	0.2310	0.2203	0.2096	0.1989	0.1882
1.23	0.2347	0.2241	0.2134	0.2027	0.1920	0.1813	0.1706
1.53	0.1690	0.1583	0.1476	0.1369	0.1261	0.1154	0.1047

a. (2.25 points) Calculate the expected loss costs for the policy using the Insurance Charge Reflecting Loss Limitation (ICRLL) procedure.
b. (1 point) Explain why the ICRLL procedure produces reasonably accurate insurance charges.

19. a) Expected Excess Losses = (0.21)(1.5) = 0.315 million.
Expected Losses = (0.75)(1.5) = 1.125 million.
Expected Primary Losses = 1.125 - 0.315 = 0.81 million.
Entry Ratio = (1 million) / (0.81 million) = 1.235.
LER = 0.21 / 0.75 = 0.28.

$$\text{LUGS} = (1.125)\ (1.1)\ \frac{1\ +\ (0.8)(0.28)}{1\ -\ 0.28} = 2.103 \text{ million.} \Rightarrow \text{ELG 24.}$$

From the given table, the Insurance Charge is 0.1706. (I did not interpolate.)
The expected loss costs for the policy are:
(0.1706)(0.81 million) + 0.315 million = **$453,186**.

b) With an accident limit there is less random fluctuation in loss ratios than without an accident limit.
The smaller the accident limit, the more expected losses are excess of the accident limit., and the less random fluctuation there is in limited loss ratios. In the retro context the percent of expected losses excess of the accident limit is called the LER.
The ICRLL procedure shifts to a column of Table M that corresponds to larger insureds, by multiplying the expected losses for entering Table M by $\frac{1\ +\ 0.8\ \text{LER}}{1\ -\ \text{LER}}$. Larger insured have less random fluctuation in their (unlimited) loss ratios than smaller insureds.
The smaller the accident limit, the larger the LER, and the larger the adjustment factor. So the smaller the accident limit, the larger the insured to whose column we are shifted. This makes sense since both a smaller accident limit and a larger insured have less random fluctuation in loss ratios.
The particular formula $\frac{1\ +\ 0.8\ \text{LER}}{1\ -\ \text{LER}}$ was developed and tested by the NCCI so that it resulted in a reasonable approximation of removing any overlap between the insurance charge and the charge for the accident limit. (Testing was for sizes of insured typically retro rated and typical accident limits purchased.)

Comment: The ELF is the expected excess losses as a percent of standard premium.
I assumed that the given ELF was appropriate for the $200,000 deductible amount.
The ICRLL procedure has the advantage that it does not lead to any anomalous results as did its predecessor; once we apply the adjustment we are just using a different column of Table M.

20. (2 points) The aggregate loss experience of an insurer's book of business is described by the following distribution function:

$F(x) = x^{0.25}$ where $0 \le x \le 1$

a. (1 point) Derive an exposure curve from the above cumulative distribution function.

b. (1 point) Given that the maximum possible loss is \$2,000,000, use the derived exposure curve in part a. above to determine the ratio of pure risk premium in the layer \$1,000,000 excess of \$500,000.

20. (a) The exposure curve is the loss elimination ratio.

$S(x) = 1 - x^{0.25}$. $E[X \wedge x] = \int_0^x S(t)\,dt = x - 0.8\,x^{1.25}$. $E[X \wedge 1] = 0.2$.

Thus $G(x) = E[X \wedge x] / E[X \wedge 1] =$ $\mathbf{5x - 4x^{1.25}}$.

(b) We want the layer from $500,000 to $1,500,000.

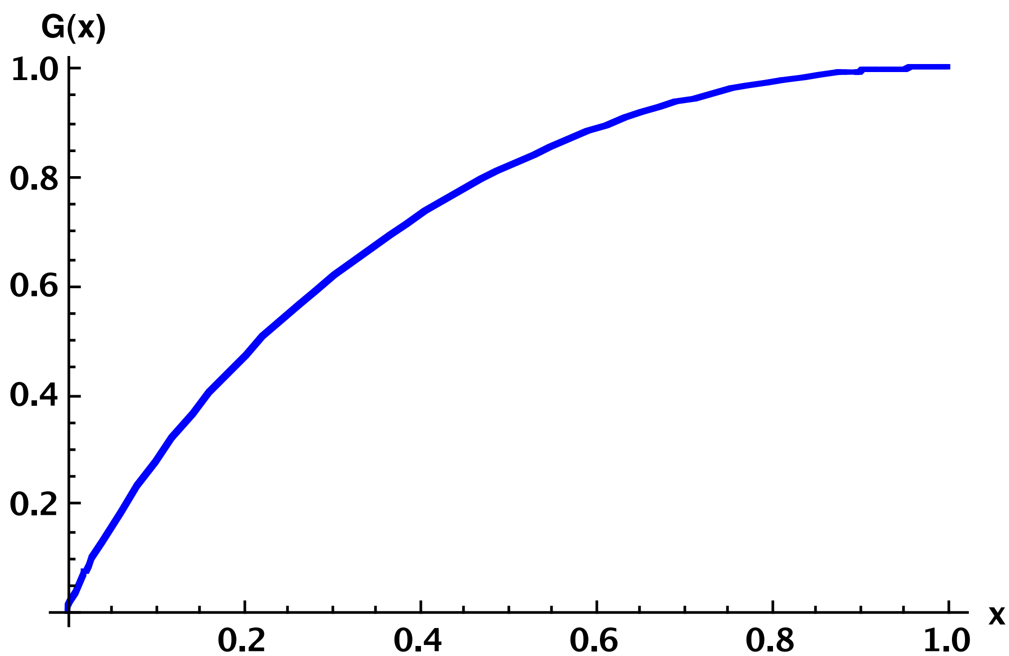
G(500,000 / 2,000,000) = G(0.25) = (5)(0.25) - (4) $(0.25^{1.25})$ = 0.5429.

G(1,500,000 / 2,000,000) = G(0.75) = (5)(0.75) - (4) $(0.75^{1.25})$ = 0.9582.

Thus the percent of expected losses in the layer is: 0.9582 - 0.5429 = **0.4153**.

<u>Comment</u>: Note that for the given distribution function, F(0) = 0 and F(1) = 1.

A graph of the exposure curve, $G(x) = 5x - 4x^{1.25}$, $0 \le x \le 1$:



21. (1.5 points) A property catastrophe treaty covers the layer $50,000,000 excess of $50,000,000 with an annual premium of $3,000,000 and a reinstatement provision that is 120% pro-rata as to amount with no limit on the number of reinstatements.
The treaty is issued for a one-year term effective January 1, 2013.
a. (0.75 point) Given the three following ground-up catastrophe losses during 2013:

- A loss of $65,000,000 on June 1.
- A loss of $85,000,000 on September 1.
- A loss of $115,000,000 on November 1.

Calculate how much the ceding company pays in reinstatement premiums during 2013.
b. (0.5 point) Calculate the annual total reinstatement premium with the same three losses as above if the reinstatement provision was pro-rata as to amount and pro-rata to time.
c. (0.25 point) Briefly explain why relatively few contracts include reinstatements pro-rata as to time.

21. (a) The catastrophe reinsurance covers the layer from 50M to 100M.
In each case we multiply by the percentage of the width of the layer that has been exhausted.

Date	Losses in Layer	Reinstatement Premium
June 1	15,000,000	($3m) (1.2) (15/50) = $1,080,000
Sept. 1	35,000,000	($3m) (1.2) (35/50) = $2,520,000
Nov. 1	50,000,000	($3m) (1.2) (50/50) = $3,600,000
Total		**$7,200,000**

(b) In each case we also multiply by the percentage of the year of coverage remaining.

Date	Losses in Layer	Reinstatement Premium
June 1	15,000,000	($3m) (1.2) (15/50) (7/12) = $630,000
Sept. 1	35,000,000	($3m) (1.2) (35/50) (4/12) = $840,000
Nov. 1	50,000,000	($3m) (1.2) (50/50) (2/12) = $600,000
Total		**$2,070,000**

(c) "Given the seasonal nature of some types of catastrophes (e.g. hurricanes), relatively few contracts include reinstatements pro-rata as to time."
Pro-rata as to time assumes instead of seasonality that the exposure to property catastrophes is constant throughout the year.

Comment: See page 41 of Clark. Similar to 6, 11/10, Q. 22.
We are implicitly assuming that there is no other reinsurance, which would inure to the benefit of the catastrophe reinsurance.
In the Northern Atlantic Ocean, a distinct hurricane season occurs from June 1 to November 30, sharply peaking from late August through September.
If the major peril we were concerned about was hurricanes in the United States, then after the June 1 catastrophe, almost all of the hurricane season would still be to left, so that a pro-rata as to time factor of 7/12 would be too low.
If instead the major peril we were concerned about was earthquakes, then reinstatements pro-rata as to time would make sense.

22. (1.5 points) An actuary for a reinsurer uses the following exposure curve to price a non-proportional treaty with the assumption that b = 0.1:

$$G(x) = \frac{1 - b^x}{1 - b}.$$

The maximum possible loss for the reinsurer is $50 million and the ratio of pure risk premium retained by the cedant is 65%.

Calculate the cedant's maximum retention under the treaty.

22. $0.65 = G(x) = \frac{1 - b^x}{1 - b} = (1 - 0.1^x) / 0.9. \Rightarrow x = \ln(0.415) / \ln(0.1) = 0.382.$

The reinsurer's maximum loss is: 50 million = M (1-x) = 0.618 M. ⇒ M = 80.9 million.

The cedant's maximum loss is: x M = 0.382 M = (0.382)(80.9) = **$30.9 million**.

Comment: G(x) is the loss elimination ratio at x, where is x is as a percent of the maximum possible loss M. G(x) is the percent of pure risk premium retained by the cedant if the reinsurer covers the layer from x M to the maximum possible loss M.

23. (2 points) A primary insurance company's actuary is evaluating the following three types of reinsurance contracts:

- 60% ceded quota share.
- Five-line surplus share treaty with retained line = $100,000.
- $400,000 xs $100,000 per-risk excess of loss.

In the most recent accident year, the company has experienced the following losses on its policies:

Risk	Insured Value	Loss
A	$250,000	$120,000
B	$1,000,000	$245,000
C	$85,000	$85,000
D	$1,250,000	$490,000
E	$400,000	$180,000
Total	$2,985,000	$1,120,000

Determine which reinsurance contract would result in the lowest retained losses for the insurance company.

23. Under the 60% ceded quota share, the insurer retains 40% of each loss and thus 40% of the total losses: (40%)($1,120,000) = **$448,000**.

Under the five-line surplus share treaty with retained line = $100,000:

Risk	Insured Value	Percent retained	Loss	Amount Retained
A	$250,000	100/250	$120,000	$48,000
B	$1,000,000	1 - 500/1000	$245,000	$122,500
C	$85,000	100%	$85,000	$85,000
D	$1,250,000	1 - 500/1250	$490,000	$294,000
E	$400,000	100/400	$180,000	$45,000
Total				**$594,500**

Under the $400,000 xs $100,000 per-risk excess of loss:

Risk	Loss	Amount Ceded	Amount Retained
A	$250,000	150,000	100,000
B	$245,000	145,000	100,000
C	$85,000	0	85,000
D	$490,000	390,000	100,000
E	$180,000	80,000	100,000
Total			**$485,000**

The least is retained by the insurer in total for the **60% ceded quota share**.

Comment: See pages 2 and 14-15 of Clark on Reinsurance Pricing.

One could instead compute the amounts ceded, and say what you are doing.
Then the most ceded corresponds to the least retained.

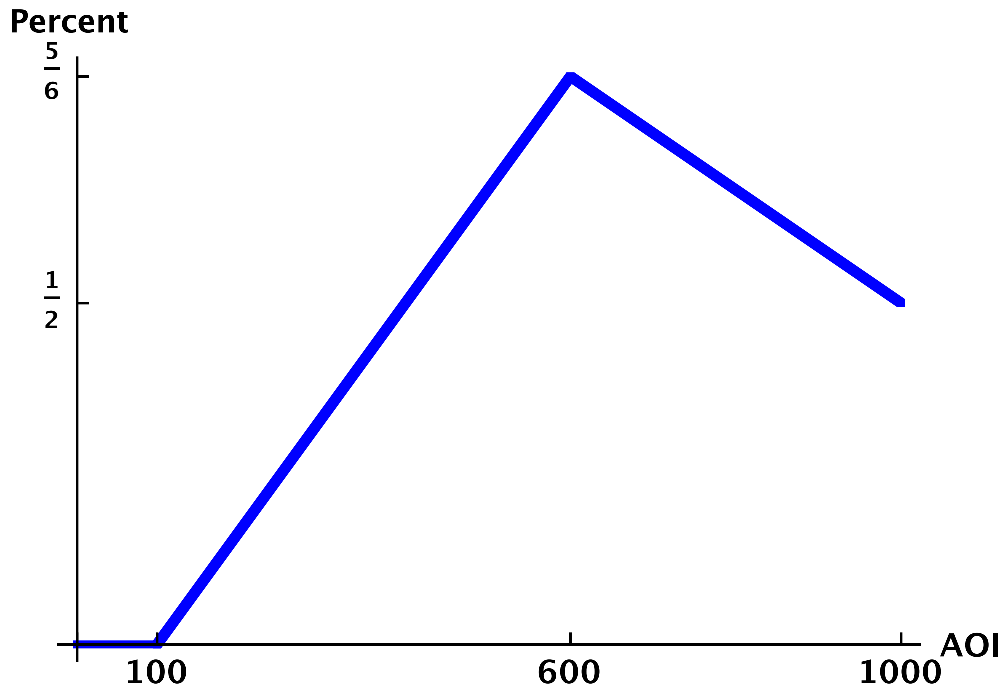
For the surplus share treaty, if X is the insured value, then the percent ceded is:

$$\begin{cases} 0 & \text{for } X < 100{,}000 \\ 1 - 100{,}000/X & \text{for } 100{,}000 \le X \le 600{,}000 \\ 500{,}000/X & \text{for } 600{,}000 < X \end{cases}$$

For the surplus share treaty, if X is the insured value, then the percent retained is:

$$\begin{cases} 1 & \text{for } X < 100{,}000 \\ 100{,}000/X & \text{for } 100{,}000 \le X \le 600{,}000 \\ 1 - 500{,}000/X & \text{for } 600{,}000 < X \end{cases}$$

Here is a graph of the percent ceded as a function of the Amount of Insurance ($000):



As the amount of insurance increases from $100,000 to $600,000, the percent ceded increases linearly from 0 to 5/6. As the amount of insurance increases beyond $600,000, the percent ceded decreases linearly; for X > $600,000, the percent ceded is $500,000/X.

24. (2.25 points) The following Occurrence Exceedance Probability curve is available for an insurance company's portfolio:

Return Period	Occurrence Exceedance Probability	Loss
10,000	0.0001	$200,000,000
500	0.0020	$50,000,000
200	0.0050	$20,000,000
100	0.0100	$12,000,000
50	0.0200	$7,000,000
33	0.0300	$3,500,000
25	0.0400	$1,500,000
20	0.0500	$500,000

a. (1 point) The insurer specifies that its acceptable risk level is 1-in-250 year PML.
Define PML and calculate the 1-in-250 year PML.
b. (1.25 points) The insurer decides to buy property catastrophe reinsurance protection up to the 1-in-500 year PML in the following treaties:
Quota share, where 30% is ceded up to a $40 million loss limit, which inures to the benefit of the following:

- 100% placed 1st layer property catastrophe excess of loss treaty $6 million xs $4 million
- 90% placed 2nd layer property catastrophe excess of loss treaty $10 million xs $10 million
- 75% placed 3rd layer property catastrophe excess of loss treaty $30 million xs $20 million

During the treaty year, the insurer suffers a $45 million earthquake loss.
Calculate the amount of loss ceded to each of the reinsurance treaties and the net retained loss by the primary insurer.

24. (a) The PML is the probable maximum loss, and is the largest occurrence amount the insurer can expect to have over a given period of time.
Alternately, the PML is the probable maximum loss, and it is the largest likely loss; the insurer wants to make sure it can withstand the PML if it occurs and remain solvent.

In this case, for a 1 in 250 year PML we are looking for an occurrence exceedance probability of 1/250 = 0.004. Interpolating linearly, this is **$30,000,000**.

(b) The quota share pays (30%)(40) = $12 million, which inures to the benefit of the excess of loss treaties.
Thus it is as if the loss were: 45 - 12 = $33 million.
Ceded to the 1st layer: (100%)(6) = $6 million.
Ceded to the 2nd layer: (90%)(10) = $9 million.
Ceded to the 3rd layer: (75%)(33 - 20) = $9.75 million.
Retained by the insurer: 45 - 12 - 6 - 9 - 9.75 = $8.25 million.

Comment: See Table 2.1 in Grossi and Kunreuther.

25. (1.25 points) An insurance company is considering a sliding-scale commission structure for its niche casualty excess of loss program, comprised of independent retailers. Using the program's loss history, two actuaries are tasked with calculating an aggregate loss distribution.
Given the following information:

- All policies have a $2,000,000 per occurrence insured retention.
- All policies have occurrence limits of either $1,000,000 or $2,000,000 in excess of the insured's retention.
- All policies have a $2,000,000 aggregate limit.
- The only claims that are reported to the insurer are those that exceed the insured's retention.

Policy Year	Aggregate Loss in Excess of $2,000,000	Reported Claim Count
2000	-	0
2001	$1,506,002	2
2002	$1,070,358	1
2003	-	0
2004	$977,602	1
2005	$2,490,714	2
2006	-	0
2007	$512,933	1
2008	-	0
2009	-	0

Actuary A wants to use the lognormal distribution to determine the aggregate loss distribution.
Actuary B prefers Panjer's recursive formula, using a Poisson frequency distribution.
Evaluate each actuary's selection and propose the more appropriate method.

25. I am assuming that the reinsurer is writing each year separate policies for several different self-insured independent retailers.

Disadvantages to the use of the lognormal distribution to model aggregate losses:

1) There is no allowance for the loss free scenario.
Since in this example we have 5 out 10 loss free years, and even in years with claims some retailers had no claims, there is a large probability of no reinsured loss from a retailer in a year. This argues very strongly against using the lognormal distribution here.

2) There is no easy way to reflect the impact of changing per occurrence limits on the aggregate losses.
Since in this example we have occurrence limits of either $1,000,000 or $2,000,000 in excess of the insured's retention this argues against using the lognormal distribution here.

The Panjer algorithm works well in low frequency situations, such as we have here.
In order to use the Panjer algorithm:

1) Since we have chosen to use a Poisson frequency, we would need to estimate its mean λ.

2) We would need to model severity (excess of the retention and prior to the impact of the occurrence and aggregate limits) as a discrete distribution
with support at evenly spaced points, such as for example $250,000.
(We would then apply the appropriate occurrence limit and aggregate limit.)

While **the Panjer algorithm is clearly the superior approach of the two in this example**, there will still be problems.
One could estimate the mean frequency per policy for an average independent retailer from the number of claims and the number of policies covered in each of the years. Unfortunately, with such a low frequency, there is considerable potential estimation error due to random fluctuation.
(A complication is that due to inflation, claims that were within the retention in the past will exceed the retention in the future.)
More importantly, with only 7 claims it will be difficult to model severity.
(One would want to take into account the effects of inflation.)
Perhaps some relevant data other than from this book of business can be obtained to help model severity.

<u>Comment</u>: See Section 4 of Clark on Reinsurance Pricing.
Panjer algorithm is discussed at pages 36-39 of Clark on Reinsurance Pricing, and also on Exam 4.