Mahler's Guide to

Advanced Ratemaking
Revision of NCCI Retro Plan Section
For the CAS 2024 Study Kit

CAS Exam 8

prepared by
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Study Aid 2024-**8Revised**

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Section 14, NCCI Retrospective Rating[1]

For 2024, the CAS updated the CAS8 study kit.[2] [3] [4]

The National Council on Compensation Insurance (NCCI) made major changes to its Retrospective Rating Plan effective January 1, 2019:[5]

- Changes in the methodology NCCI uses to determine tabulated Insurance Charges.
- Several different (sub)Tables of Insurance Charges;
  which one you should use depends on the policy excess ratio.
- One now determines which column of a table to use based on
  the policy expected claim count.
- Changes in terminology.
- A new computer based product, Aggregate Loss Factors on Demand (ALFs on Demand),
  that insurers can use rather than consulting published Tables of Insurance Charges.

An extract from the Retrospective Rating Plan Manual will be attached to your exam.

Have it with you when you do past exam questions.

Be sure to look through it all several different times during your studying.

The first part contains rules for the NCCI Retro Plan.

Note the useful Table of Contents in the front of the manual.

Appendix D, contains **"Basic Premium Factor Calculation Example,"**
to be discussed subsequently in detail.

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[1] Be sure to first read my section on Retrospective Rating.

[2] National Council on Compensation Insurance, Retrospective Rating Plan Manual for Workers Compensation and Employers Liability Insurance, 2022, selected pages are included in the Study Kit.
The manual consists mostly of three tables in Appendices A, B, and C:
- Tables of Expected Loss Ranges
- Tables of Aggregate Loss Factors
- Tables of Expense Ratios.

Only a sample of each of these tables will be included in the manual.
Candidates are not required to memorize the details but will be expected to be able to use them on the examination. The edited manual will be provided both in the study kit and on the exam.

[3] Despite what the CAS Content Outline says, the 2024 study kit does not contain
Tables of Aggregate Loss Factors.

[4] The 2023 Study Kit:
National Council on Compensation Insurance, Retrospective Rating Plan Manual for Workers Compensation and Employers Liability Insurance, Circular CIF-2018-28, 06/21/2018. The selected pages are included in the Study Kit and the exam. The manual consists mostly of two tables (the following tables have not been included in the study kit nor the exam):
• pp. 9-728: "Tables of Aggregate Loss Factors – sub tables 1 through 18" and
• pp. 735-882: tables for "Sample Values for a Policy with No Loss Limit and All Exposure in Single Hazard Group" by state.
Candidates are not required to memorize the details but will be expected to be able to use them on the examination.

[5] See for example NCCI Circular Letter CIF-2018-28, not on the current syllabus..

Unfortunately, other than rules the rest of the 2024 study kit contains a mixture of elements that apply to the old and new NCCI Retro Plans:
- Table of Expected Loss Ranges, used in the old plan.
- Table of Policy Excess Ratios Ranges, used in the new plan.
- Table of Expected Claim Count Groups, used in the new plan.
- Table of Insurance Charges, used in the old plan.[6]
- Table of Expense Ratios, used in both plans.
- Basic Premium Factor Calculation Example, for the new plan.[7]

Despite what the CAS Content Outline says, the 2024 study kit does not contain Tables of Aggregate Loss Factors.

This should limit the types of questions the CAS can ask on your exam.

Taking into Account Loss Limits in Retro Rating:[8]

In retrospective rating, when there is an loss limit, there is an overlap between the premium charge for a loss limit and the insurance charge for the maximum premium. In other words, if one calculate the effects of the loss limit and the maximum premium separately, then the average premium for a retrospectively rated policy with a loss limit would be too high.

NCCI used to deal with this issue via the ICRLL procedure, in which in the presence of a loss limit one shifts which column one uses in Table M. Instead, the new NCCI methodology is based on computing an aggregate distribution that reflects the effect of any loss limit.

The Interaction of Maximums and Loss Limits:

Let us assume a retrospectively rated insured had a basic premium of $30,000, an excess loss premium of $10,000, a loss conversion factor of 1.1, a tax multiplier of 1.05, a loss limit of $100,000, and a maximum premium of $250,000.

Exercise: If the insured has small losses totaling to $150,000 in year, what is the retro premium?
[Solution: {40,000 + (1.1)(150,000)} (1.05) = 215,250.
Comment: The insured benefited from neither the maximum premium nor the loss limit.]

Exercise: If the insured has small losses totaling to $200,000 in year, what is the retro premium?
[Solution: {40,000 + (1.1)(200,000)} (1.05) = 273,000. Limited to the maximum of $250,000.
Comment: The insured benefited from the maximum premium.]

Exercise: If the insured has one large loss of $150,000 in year, what is the retro premium?
[Solution: {40,000 + (1.1)(100,000)} (1.05) = 157,500.
Comment: The insured benefited from the loss limit.]

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[6] A portion of Table M.
[7] Same example as shown in the 2023 Study Kit.
[8] The NCCI considers both limits that apply per claim and per occurrence, although this is not on the syllabus.

Exercise: If the insured has one large loss of $150,000 in year plus $100,000 in small losses, what is the retro premium?
[Solution: {40,000 + (1.1)(200,000)} (1.05) = 273,000. Limited to the maximum of $250,000.
Comment: The loss limit decreased the losses entering the calculation, but the insured ended up paying the maximum premium anyway.]

The last case, is an example of the "overlap" between the effects of the maximum premium and the loss limit. In some years, even though there are large events, the loss limit will not provide any additional benefit to the insured beyond that provided by the maximum premium. In other words, for large events the loss limit and the maximum premium overlap.

Therefore, calculating independently additional amounts to charge an insured for the maximum premium and for the loss limit would overcharge the insured. This is the problem the NCCI is avoiding via its new methodology.

The Interaction of Minimums and Loss Limits:

Let us assume a retrospectively rated insured had a basic premium of $300,000, an excess loss premium of $100,000, a loss conversion factor of 1.1, a tax multiplier of 1.05, a loss limit of $100,000, and a minimum premium of $650,000.

Exercise: If the insured has small losses totaling to $150,000 in year, what is the retro premium?
[Solution: {400,000 + (1.1)(150,000)}(1.05) = 593,250.
The insured pays the minimum of premium of $650,000.]

Exercise: If the insured has one large loss of $150,000 in year, what is the retro premium?
[Solution: {400,000 + (1.1)(100,000)}(1.05) = 535,500.
The insured pays the minimum of premium of $650,000.]

The last case, is an example of the "underlap" between the effects of the minimum premium and the loss limit. In some years, even though there are large events, the loss limit will not provide any benefit to the insured due to the minimum premium. This "underlap" has a relatively small overall impact.

Eligibility for Retrospective Rating:[9]

For the one year plan, the insured must have an annual standard premium of at least $25,000.
For the three year plan, the insured must have an annual standard premium of at least $75,000.

[9] See Rules 2C and 2D of the NCCI Retrospective Rating Plan Manual.

Retrospective Premium:

R = (b + cL)T, subject to a minimum of H and a maximum of G. **H ≤ R ≤ G.**

R = Retrospective Premium
b = basic premium = (basic premium factor) (standard premium).
c = the loss conversion factor
L = reported losses subject to any applicable limitation[10]
T = Tax Multiplier
H = minimum premium = (minimum premium factor) (standard premium).
G = maximum premium = (maximum premium factor) (standard premium).

Basic Premium:

e = expenses plus profit but excluding taxes (includes loss adjustment expense).
c = Loss Conversion Factor (to include loss adjustment expense).
E = expected unlimited loss ratio.
Net Aggregate Loss Factor =
cE(Aggregate Excess Loss Factor @ Max. - Aggregate Minimum Loss Factor @Min.).

e - (c-1)E is called the expenses in the basic premium. It does not cover the LAE included in c, nor does it cover taxes and assessments included in the tax multiplier T.

basic premium factor = b = e - (c-1)E + Net Aggregate Loss Factor.
The basic premium = basic premium factor times standard premium.

Optional Features of the NCCI Retrospective Rating Plan:

The insured and the insurer can agree to have a loss limitation.
The insured and insurer can agree to use retrospective development factors.

R = (b + cL + Excess Loss Premium + Retro. Development Premium) T,
subject to a minimum of H and a maximum of G.

Loss Limitations:

With a loss limitation, the reported losses are limited to an agreed upon amount per loss.[11]
This lessens the impact on the insureds retrospective premium of large losses.
The expected cost of layer above the loss limitation, as well as the related ALAE and ULAE, are collected through the excess loss premium.[12]

Excess Loss Premium = c (Standard Premium) (Excess Loss Factor).[13] [14]

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[10] Reported losses = paid losses plus case reserves.
[11] The loss limitation can apply either per occurrence or per claim as agreed to by the insurer and insured.
[12] Without a loss limit, the Excess Loss Premium is zero.
[13] Excess Loss Factor = (Excess Ratio) (Expected Loss Ratio).
[14] One must be careful to avoid any overlap for the charge for the effect of the maximum premium included in the basic premium and the charge for the loss limit contained in the excess loss premium.

Retrospective Development Factors:

"Retrospective development premium (RDP) is an elective element that varies by state. It stabilizes premium adjustments for an employer written under a retrospective rating plan by anticipating future changes in losses. The retrospective development factor anticipates a pattern of increasing valuation of losses after the policy is expired. The retrospective development factor is included in the first three calculations of the retrospective premium."[15]

Retrospective Development Premium = c(Standard Premium)(Retro. Development Factor).[16]

"Because reported limited losses tend to develop over time upwards to the ultimate limited losses, the first retrospective adjustment is likely to result in the insurer returning premium to the insured. Successive retrospective adjustments will probably result in most of, if not all of or more than, this amount being returned by the insured to the insurer. To smooth out these back and forth payments, some insureds opt to use the retrospective development premium, which attempts to offset this process."[17]

Allocated Loss Adjustment Expense Option:

Unless stated otherwise, a retro plan applies to only losses, in which case the provision for ALAE is included in the loss conversion factor, c.[18] However, if agreed upon by the insured and insurer, ALAE may be included with losses for purposes of the retro plan.[19] This is called the ALAE Option. For the ALAE Option, E would be replaced by the expected loss and ALAE ratio, a different set of expense ratios is used, and the loss conversion factor would be smaller.[20]

Large Risk Alternative Rating Option:[21]

"The Large Risk Alternative Rating Option (LRARO) provides that a risk may be retrospectively rated as mutually agreed upon by carrier and insured. It is an available option for risks with an estimated annual standard premium in excess of $1,000,000 individually or in combination with General Liability, Hospital Professional Liability, Commercial Automobile, Crime, Glass or Workers Compensation. A different premium eligibility level may be used if filed by an individual insurance carrier, subject to regulatory approval."

If an insured is big enough to qualify and the insurer and insured agree, then all retrospective rating factors may be changed. Maximum premium factor, minimum premium factor, loss conversion factor, loss limit, may each be changed. The basic premium factor will change if any of these other factors are changed.

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[15] See Page 4 of Rule 1 of the NCCI Retrospective Rating Plan Manual.
[16] The Retrospective Development Premium is zero beyond the first three retrospective adjustments. The Retrospective Development Premium decreases from first to second to third adjustment.
[17] Quoted from Margaret Tiller Sherwood's Chapter 4 of Foundations of Casualty Actuarial Science.
[18] Examples of costs included in ALAE: Attorney Fees, Court Costs, Medical Evaluation, Autopsies Stenographic, Laboratory and X-Rays, Witness and Summonses, and making Copies.
[19] Regardless, for Employers Liability coverage, ALAE is reported and included with losses.
[20] If the ALAE option is elected as part of incurred losses, the loss conversion factor must be adjusted to exclude ALAE. See the second Table of Expense Ratios in the extract from the NCCI Retrospective Rating Plan.
[21] See Page 1 of Rule 2 in the NCCI Retro Plan and Section 2.4 of "Individual Risk Rating." This is also part of the ISO Retrospective Rating Plan, which is not on the syllabus.

Together, the loss conversion factor and the expense portion of the basic premium should pay for expenses including expected LAE. When c is larger, more of the expenses (including expected LAE) are being recouped via the Loss Conversion Factor. Therefore, when c is larger, the expense portion of the basic premium factor should be smaller.[22]

A paid loss retro plan can be written under the Large Risk Alternative Rating Option.

For example, maximum and minimum ratable loss amounts can be set directly, rather than indirectly through maximum and minimum premium amounts.

For example, the basic premium factor and/or the maximum and minimum ratable loss amounts can be based on exposures instead of standard premium.

"A key assumption underlying LRARO is that large risks are knowledgeable and sophisticated enough to negotiate with insurers their retrospective rating parameters.
Although LRARO allows for pricing flexibility, pricing still must comply with regulatory principles and not be inadequate, excessive, or unfairly discriminatory."[23]

Excess Loss Factors:[24]

Excess Loss Factor = ELF = (Excess Ratio) (Expected Loss Ratio).

Exercise: For a policy, the excess ratio is 20% and the expected loss ratio is 65%.
Determine the Excess Loss Factor.
[Solution: (20%)(65%) = 13%.]

The Excess Loss Factor times the Standard Premium gives the expected excess losses.
If the standard premium were $1 million, the expected excess losses would be $130,000.

Excess Loss Premium = c (Standard Premium) (Excess Loss Factor)
= c (Expected Excess Losses).

Thus if the loss conversion factor were 1.1, then the excess loss premium would be:
(1.1)(130,000) = $143,000.

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[22] The retro balance equations also change as does c. Thus so do somewhat r_G, r_H, and the net insurance charge.

[23] Quoted from Section 2.4 of "Individual Risk Rating".

[24] Excess Loss Factors are also known as Excess Loss Premium Factors (ELPFs).

Items Related to Excess Loss Factors:[25]

NCCI publishes ELFs in state where it publishes rates rather than loss costs.[26]
In states where instead NCCI publishes loss costs, NCCI instead publishes
Excess Loss Pure Premium Factors (ELPPFs).[27]

The ELPPF is applied to "loss cost premium", in order to get expected excess losses.[28] [29]

$$\text{ELPPF} = \frac{\text{Expected Excess Losses}}{\text{Loss Cost Premium}} = (\text{Excess Ratio}) / (1 + \text{LAE \%} + \text{Loss Assessment \%}).$$[30]

$$\text{ELPPF} = \frac{\text{ELF}}{(\text{Expected Loss Ratio})\ (1 + \text{LAE \%} + \text{Loss Assessment \%})}.$$

ELF = (ELPPF) (Expected Loss Ratio) (1 + LAE % + Loss Assessment %).

Exercise: For a policy the excess ratio is 20% and the expected loss ratio is 65%.
LAE is 10% of losses. There is a 0.5% loss assessment. Determine the ELPPF.
[Solution: ELF = (20%)(65%) = 13%. ELPPF = 13%/ {(65%)(1 + 10% + 0.5%)} = 0.181.
Alternately, 20% / (1 + 10% + 0.5%) = 18.1%.]

An insured that would have standard premium of $1 million would have loss cost premium of:
($1 million)(65%)(1.105) = $718,250.
Expected excess losses = (0.181)($718,250) = $130,003, matching before subject to rounding.

For retro plans in which LAE is included with losses, there are related factors:
Excess Loss and Allocated Expense Factor (ELAEF),
and Excess Loss and Allocated Pure Premium Expense Factor (ELAEPPF).[31]

Standard Premium times the ELAEF equals the expected excess loss and ALAE.
If the standard premium were $1 million, and the Excess Loss and Allocated Expense Factor were 0.15, then the expected excess loss and ALAE would be $150,000.[32]

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[25] See the third page of Rule 1 of the NCCI Retro Plan.
[26] Published ELFs vary by loss limit, state, and hazard group.
For policies with exposure in multiple states or hazard groups, the ELF is determined as the product of the policy excess ratio and the expected loss ratio.
[27] Published ELPPFs vary by loss limit, state, and hazard group.
[28] In contrast an Excess Loss Factor would be applied to standard premiums in order to get expected excess losses.
[29] In most states, loss cost premium is expected loss & ALAE plus any loss assessments.
[30] Workers Compensation insurers pay assessments on premium and/or losses to state governments.
These assessments pay for: the expenses of the state agency that administers the Workers Compensation law, the Second Injury Fund, and other special funds. States vary as to which if any of these expenses are recovered via assessments on losses. Any premium based assessments would be included in the Tax Multiplier.
[31] Published factors vary by loss limit, state, and hazard group.
[32] The expected loss and ALAE excess of for example a 500K limit that applies to loss and ALAE is not equal to c times the expected losses excess of a 500K limit that applies to losses.

The ELAEPPF is intended to be applied to loss cost premium, in order to get expected excess loss and ALAE.

$$\text{ELAEPPF} = \frac{\text{Expected Excess Losses \& ALAE}}{\text{Loss Cost Premium}}.$$

$$\text{ELAEPPF} = \frac{\text{ELAEF}}{(\text{Expected Loss Ratio})\ (1 + \text{LAE \%} + \text{Loss Assessment \%})}.$$

ELAEF = (ELAEPPF) (Expected Loss Ratio) (1 + LAE % + Loss Assessment %).

Exercise: For a policy the Excess Loss and Allocated Expense Factor is 0.15.
The expected loss ratio is 65%. LAE is 10% of losses. There is a 0.5% loss assessment.
Determine the ELAEPPF.
[Solution: ELAEPPF = 0.15 / {(65%)(1 + 10% + 0.5%)} = 0.209.]

An insured that would have standard premium of $1 million would have loss cost premium of:
($1 million)(65%)(1.105) = $718,250.
Expected excess loss & ALAE = (0.209)($718,250) = $150,114.[33]

Entry Ratios:

Entry Ratios have expected limited losses in their numerator and their denominator.[34]
Entry Ratio corresponding to the Maximum Premium is:

$$\frac{\text{Limited Losses Corresponding to the Maximum Premium}}{\text{Expected Limited Losses}} = \frac{\hat{L}_G}{\hat{E}}.$$

Entry Ratio corresponding to the Minimum Premium is:

$$\frac{\text{Limited Losses Corresponding to the Minimum Premium}}{\text{Expected Limited Losses}} = \frac{\hat{L}_H}{\hat{E}}.$$

[33] Matching the previous result, subject to rounding.
[34] With no loss limit, expected limited losses = expected total losses.

Terminology:

With their revision to their Retro Plan, NCCI has changed some of their terminology:

Current Term	Proposed Term	Explanation of Terms
Insurance charge	Aggregate Excess Loss Factors (AELFs)	Describes expected losses in excess of those provided by the maximum retrospective premium.
Insurance savings	Aggregate Minimum Loss Factors	Describes expected losses less than those that would produce the minimum retrospective premium.
Net insurance charge	Net aggregate loss factor	Determined by calculating the difference between the aggregate excess loss factor for possible losses that might produce more than the maximum retrospective premium and the aggregate minimum loss factor for losses that might produce less than the minimum retrospective premium, and then multiplying that difference by the product of the expected loss ratio and the loss conversion factor. The net aggregate loss factor may be less than zero, as long as the basic premium factor is not negative.
Table of Insurance Charges	Table of Aggregate Loss Factors	Precomputed table of values used in conjunction with a lookup procedure.

I would be prepared for your exam to use either the older or newer terminology in a question.

Insurance Charge ⇔ Aggregate Excess Loss Factor (AELF)

Insurance Savings ⇔ Aggregate Minimum Loss Factor

Aggregate Excess Loss Factor - Aggregate Minimum Loss Factor = 1 - Entry Ratio.[35]

Net Insurance Charge ⇔ Net Aggregate Loss Factor[36]

Table M ⇔ Table of Insurance Charges ⇔ Table of Aggregate Loss Factors[37]

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[35] The same relationship as between Insurance Charges and Insurance Savings.

[36] Net Aggregate Loss Factor =
cE(Aggregate Excess Loss Factor @ Max. - Aggregate Minimum Loss Factor @Min.).

[37] As will be discussed, NCCI now has a set of subtables, each of which is appropriate for a different range of excess ratios. Each of these subtables is similar to either a Table M_D or a Table L, although there are important differences, which will be discussed in subsequent sections.

NCCI's Example of Retrospective Rating:[38]

Note the very useful "Basic Premium Factor Calculation Example".

The following is assumed:
- Estimated Standard Premium = $500,000.[39]
- Maximum Retrospective Premium Factor = 130%
- Minimum Retrospective Premium Factor = 60%
- Loss Conversion Factor = 1.120
- Tax Multiplier = 1.070
- Loss Limit = $50,000
- Expenses (excluding Taxes) = 0.201.[40]

The Expected Loss Ratio (unlimited) is assumed to be 0.613.
Expected Losses = (0.613)($500,000) = $306,500.[41]

The Policy Excess Ratio is assumed to be 0.582.[42]
Excess Loss Factor = (0.613)(0.582) = 0.357.
Expected Limited Loss Ratio = 0.613 - 0.357 = 0.256.

The Expected Number of Claims is assumed to be 20.95.[43] [44]

Expense and Profit and Contingencies (Excluding Taxes) = (0.201)($500,000) = $100,500.
Expected Loss Plus Expense Ratio = ($306,500 + $100,500) / $500,000 = 0.814.
Loss and Expense in Converted Losses = (1.120)(0.613) = 0.687.
Expense and Profit and Contingencies (Excluding Expense in Converted Losses) =
0.814 - 0.687 = 0.127.[45]

Minimum Retrospective Premium Excluding Taxes = 60%/1.070 = 0.561.[46]
Maximum Retrospective Premium Excluding Taxes = 130%/1.070 = 1.215.[47]

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[38] See Appendix D of the NCCI Retro plan. This example applies to the new NCCI Retro plan.
[39] This is based on the initial estimate of payrolls, and would be finalized at the final payroll audit.
[40] Expenses and Profit and Contingency - ExcludingTaxes; includes LAE.
[41] For an interstate rated risk, the expected losses would be gotten by summing the expected losses for each state; for each state we would multiply standard premium by the expected loss ratio.
Then the overall loss ratio to use for retro rating would be the ratio of the expected losses over standard premium.
[42] The policy excess ratio would be a weighted average of the excess ratios by state and hazard group.
I subsequently discuss the NCCI example of this calculation, which is shown on page 2 of their Appendix D.
[43] The expected number of claims will be used to choose the column of the Table of Aggregate Loss Factors to use. One no longer uses ICRLL or State/Hazard Group Differentials in order to compute LUGs, nor does one use a Table of Expected Loss Groups to determine which column of the Table of Insurance Charges to use.
[44] The expected number of claims would be computed using the modified expected losses by state and hazard group. I subsequently discuss the NCCI example of this calculation, as shown on page 2e of their Appendix D.
[45] The expense in the basic is: e - (c-1)E = 0.201 - (1.120 - 1)(0.613) = 0.127.
[46] 60%/1.070 = H/T.
[47] 130%/1.070 = G/T.

Table of Aggregate Loss Factors Value Difference = $\frac{0.814 - 0.561}{(1.120)\ (0.256)} = 0.8824.$

This is one of the two balance equations with a loss limit: $X_H - X_G = \frac{E + e - H/T}{c\ \hat{E}}$.

Table of Aggregate Loss Factors Entry Difference = $\frac{1.215 - 0.561}{(1.120)\ (0.256)} = 2.28.$

This is the other balance equation with a loss limit: $r_G - r_H = \frac{G/T - H/T}{c\ \hat{E}}$.

Now one has to solve iteratively the two balance equations. One would look in the subtable of the Table of Aggregate Loss Factors that corresponds to the policy excess ratio of 0.582.[48]
Based on the Table of Policy Excess Ratio Ranges this would be Subtable 15.[49]

We would use the column based on the expected number of claims of 20.95.
Based on the Table of Expected Claim Count Groups this would be column 48.[50]
We are provided with an extract of column 48 of Subtable 15:[51]

Entry Ratio	Aggregate Excess Loss Factor		Entry Ratio	Aggregate Excess Loss Factor
0.04	0.9619		2.32	0.0736
0.05	0.9528		2.33	0.0727
0.06	0.9437		2.34	0.0718

For $r_H = 0.05$ and $r_G = 0.05 + 2.28 = 2.33$: $X_H - X_G = 0.9528 - 0.0727 = 0.8801$.
This as close as we can get to the desired value difference of 0.8824. ⇒

Ratio of Losses for Minimum Retrospective Premium to Expected Limited Losses = $0.05 = r_H$.
Ratio of Losses for Maximum Retrospective Premium to Expected Limited Losses = $2.33 = r_G$.

Aggregate Excess Loss Factor (for Maximum) = $0.0727 = X_G$.
Aggregate Minimum Loss Factor = $0.9528 + 0.05 - 1 = 0.0028 = S_H = X_H + r_H - 1$.
Net Aggregate Loss Factor = $(0.0727 - 0.0028)(0.256)(1.120) = 0.020 = (X_G - S_H)\hat{E}\,c$.

Basic Premium Factor = 0.020 + 0.127 = **0.147**.[52]
Thus, in dollars terms, the basic premium is: (0.147)($500,000) = $73,500.

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[48] While we have been provided with the Table of Policy Excess Ratios, we have not been provided with the Table of Aggregate Loss Factors.
[49] The Table of Policy Excess Ratio Ranges is shown at pages AA5 and AA6 of the NCCI Retro Plan.
[50] The Expected Claim Count Groups are shown at pages AA6 and AA7 of the NCCI Retro Plan.
[51] Aggregate Excess Loss Factor = Insurance Charge.
[52] The sum of the converted net insurance charge and the expenses in the basic.
For the NCCI Plan, the charge for the loss limit is not included in the basic premium.
In this example, the charge for the loss limit (prior to the application of the Tax Multiplier) would be:
(Loss Conversion Factor) (Standard Premium) (Excess Loss Factor) =
(1.120)($500,000)(0.357) = $199,920.

NCCI's Example of Calculation of Expected Number of Claims and Policy Excess Ratio:[53]

A workers compensation insured has exposures in two states.
The insured is buying a retrospective rating policy with a loss limit of $50,000.
In State X, there are exposures in two classes; these classes are in Hazard groups C and G.
In State Y, there are exposures in one class, which is in Hazard groups A.

State	Hazard Group of Class	Manual Premium	Excess Ratio at $50,000	Average Cost per Case
X	C	$217,170	0.5	$12,000
X	G	$305,873	0.7	$23,000
Y	A	$101,958	0.4	$9,000

The excess ratios depend on the loss limit, as well as the state and hazard group.[54] [55]
These excess ratios would be looked up in an NCCI publication, not on the syllabus.

The average costs per case depend on the state and hazard group.[56]
These average costs would be looked up in an NCCI publication, not on the syllabus.

The insured has an expected (unlimited) loss ratio of 61.3%.[57]
The insured has an experience modification of 0.80.
Modified Expected Loss is the manual premium times both the expected loss ratio of 61.3% and the experience modification of 0.80. For example: (61.3%)(0.80)($217,170) = $106,500.

In each case, we divide the modified expected loss by the average cost per case.

State	HG	Manual Premium	Modified Expected Loss	Average Cost per Case	Expected Number of Claims
X	C	$217,170	$106,500	$12,000	8.88
X	G	$305,873	$150,000	$23,000	6.52
Y	A	$101,958	$50,000	$9,000	5.56
Total			$306,500		**20.95**

The 20.95 is the expected number of claims used in the previous retro rating example.

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[53] See Pages AD2 and AD3 of the NCCI Retro Plan..
[54] The excess ratio is the expected percent of losses excess of the $50,000 loss limit.
[55] The excess ratios would differ based on whether the limit is per claim or per occurrence basis.
[56] I believe that these include medical only claims.
[57] As per the previous example of retro rating.

Using the modified expected losses, we take a weighted average of the excess ratios:[58]

State	HG	Manual Premium	Modified Expected Loss	Excess Ratio	Expected Excess Loss
X	C	$217,170	$106,500	0.5	$53,250
X	G	$305,873	$150,000	0.7	$105,000
Y	A	$101,958	$50,000	0.4	$20,000
Total			$306,500		$178,250

For example: ($106,500)(0.5) = $53,250.
The policy excess ratio is: 178,250/306,500 = **0.582**.
This is the policy excess ratio used in the previous retro rating example.

New versus Prior NCCI Retro Plan:

Prior NCCI Retro Plan	New NCCI Retro Plan
Loss Limits taken into account via ICRRL Procedure	Loss Limits taken into account via different subtables of Aggregate Loss Factors based on the policy excess ratio
State/Hazard Group Differentials used to help get LUGS	Average Cost per Case by State/Hazard Group used in the calculation of policy expected number of claims
Column of Table of Insurance Charges is determined via Losses Used for Group Selection (LUGS) to enter Table of Expected Loss Groups	Column of Table of Aggregate Loss Factors is determined via policy expected number of claims to enter Table of Expected Claim Count Groups
Table M insurance charges based on smoothing empirical results	Table of Aggregate Loss Factors are based on a use of the Panjer Algorithm to determine the aggregate distribution underlying each column in each subtable. (Empirical studies used to help determine the underlying frequency and severity distributions.)
	New Computer Product available: Aggregate Loss Factors on Demand. It is based on the same mathematics used to determine the published Table of Aggregate Loss Factors.

[58] Since the expected loss ratio and experience modification do not vary by state nor by hazard group, we would get the same answer using instead manual premium as the weights.

Table of Expected Claims Count Groups:

One determines which column of the Table of Aggregate Loss Factors to use based on the expected number of claims and the following Table:[59]

Table of Expected Claim Count Groups

Expected Claim Count Group	Expected Number of Claims (Rounded Values)	Expected Claim Count Group	Expected Number of Claims (Rounded Values)	Expected Claim Count Group	Expected Number of Claims (Rounded Values)
94	0.00 – 0.12	67	3.27 – 3.56	40	45.8 – 51.6
93	0.13 – 0.15	66	3.57 – 3.89	39	51.7 – 58.4
92	0.16 – 0.19	65	3.90 – 4.26	38	58.5 – 66.3
91	0.20 – 0.23	64	4.27 – 4.66	37	66.4 – 75.5
90	0.24 – 0.27	63	4.67 – 5.09	36	75.6 – 86.4
89	0.28 – 0.32	62	5.10 – 5.57	35	86.5 – 99.2
88	0.33 – 0.38	61	5.58 – 6.09	34	99.3 – 114
87	0.39 – 0.44	60	6.10 – 6.67	33	115 – 133
86	0.45 – 0.51	59	6.68 – 7.30	32	134 – 154
85	0.52 – 0.59	58	7.31 – 8.00	31	155 – 181
84	0.60 – 0.66	57	8.01 – 8.77	30	182 – 213
83	0.67 – 0.75	56	8.78 – 9.62	29	214 – 253
82	0.76 – 0.84	55	9.63 – 10.6	28	254 – 302
81	0.85 – 0.94	54	10.7 – 11.6	27	303 – 364
80	0.95 – 1.05	53	11.7 – 12.8	26	365 – 442
79	1.06 – 1.17	52	12.9 – 14.1	25	443 – 543
78	1.18 – 1.29	51	14.2 – 15.5	24	544 – 673
77	1.30 – 1.42	50	15.6 – 17.2	23	674 – 845
76	1.43 – 1.57	49	17.3 – 19.0	22	846 – 1,080
75	1.58 – 1.73	48	19.1 – 21.0	21	1,081 – 1,400
74	1.74 – 1.89	47	21.1 – 23.4	20	1,401 – 1,840
73	1.90 – 2.08	46	23.5 – 26.0	19	1,841 – 2,490
72	2.09 – 2.27	45	26.1 – 28.9	18	2,491 – 3,450
71	2.28 – 2.49	44	29.0 – 32.3	17	3,451 – 4,930
70	2.50 – 2.72	43	32.4 – 36.2	16	4,931 – 7,330
69	2.73 – 2.98	42	36.3 – 40.6	15	7,331 – and above
68	2.99 – 3.26	41	40.7 – 45.7		

For example, 20.95 expected claims corresponds to Expected Claim Count Group 48.
The larger the insured, the smaller the Expected Claim Count Group.
Insured with higher expected claim counts have a lower coefficient of variation of aggregate losses, and thus have smaller insurance changes (aggregate excess loss factors) at high entry ratios than smaller insureds.

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[59] See Pages AA6 and AA7 of the NCCI Retro Plan.
The number of claims corresponding to each claim count group was chosen so that for no loss limit (actually a loss limit of $50 million) the AELF at r = 1 is equal to: (claim count group) / 100.

In the past, which column to use was based on Expected Losses.
Expected Claim Counts have the advantage of not being affected by inflation.
Expected Claim Counts are a better measure of how the size of an insured affects the shape of its distribution of aggregate losses.

Table of Policy Excess Ranges:

One determines which subtable of the Table of Aggregate Loss Factors to use based on the policy excess ratio and the following Table:[60]

Table of Policy Excess Ratio Ranges

Sub-Table	Excess Ratio Range (Rounded Values)		
1	0.000	–	0.008
2	0.009	–	0.026
3	0.027	–	0.051
4	0.052	–	0.077
5	0.078	–	0.109
6	0.110	–	0.143
7	0.144	–	0.178
8	0.179	–	0.217
9	0.218	–	0.264
10	0.265	–	0.309
11	0.310	–	0.351
12	0.352	–	0.412
13	0.413	–	0.475
14	0.476	–	0.541
15	0.542	–	0.639
16	0.640	–	0.758
17	0.759	–	0.847
18	0.848	–	1.000

For example a policy excess ratio of 0.582 would correspond to Subtable 15.[61]

As the size of the loss limit increases, the excess ratio decreases. No loss limit would correspond to an excess ratio of 0, which is Subtable 1, similar to the Traditional Table M.

The distributions of aggregate losses in Subtable 6 (corresponding to an excess ratio of about 13%) have a smaller coefficient of variation than the corresponding aggregate distributions for no loss limit in Subtable 1. Thus the corresponding insurance charges for high entry ratios are smaller in Subtable 6 than Subtable 1.[62]
The higher the subtable number, the lower the loss limit and the smaller the insurance charges for high entry ratios.

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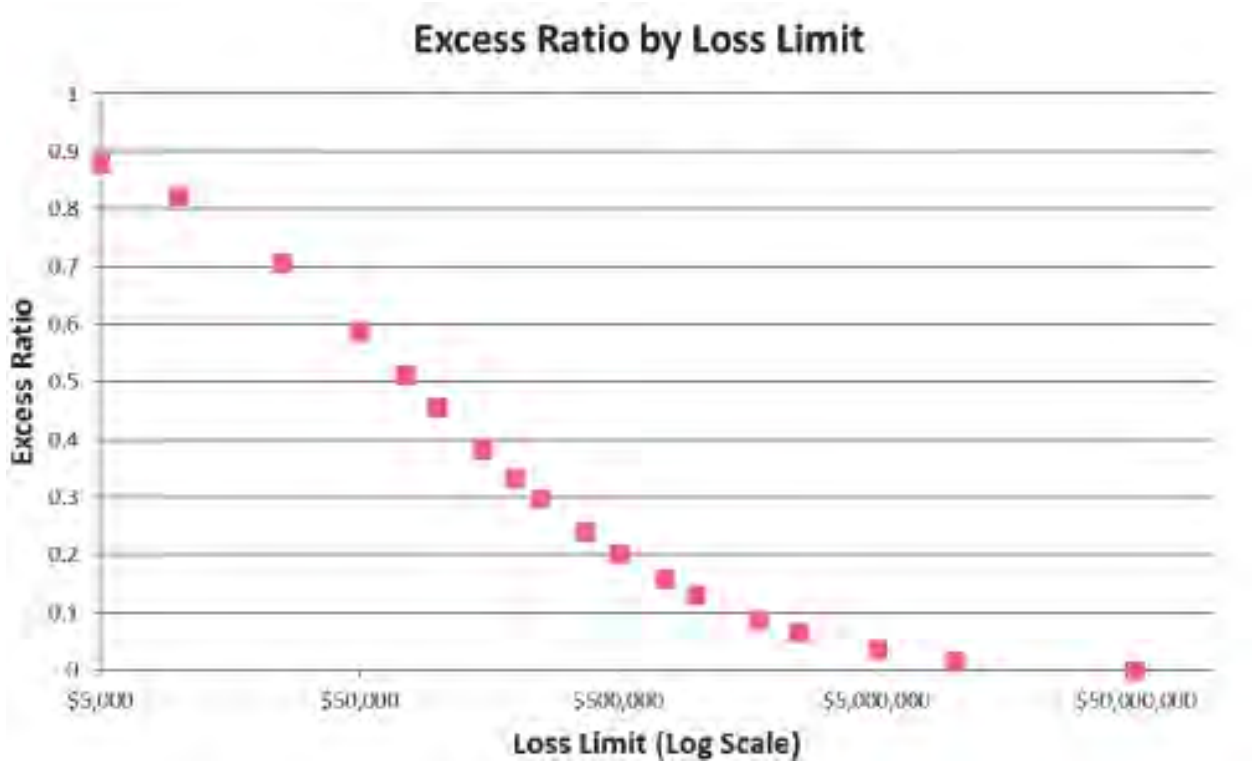
[60] See Pages AA5 and AA6 of the NCCI Retro Plan.
[61] I previously discussed an example of calculating the policy excess ratio.
[62] Unlike Table L, here the charge for the loss limit is not included in the tabulated charges.

More Detail on the Table of Policy Excess Ranges:[63]

"A total of 18 subtables were selected in order to keep the size of the published Table of Aggregate Loss Factors at a reasonable level, while also including enough subtables such that the Table of Aggregate Loss Factors is sufficiently accurate for any given policy excess ratio within the corresponding range for each subtable. The following chart illustrates the excess ratios which correspond to each Starting Point Loss Limit."



So for example, an excess ratio of 30% corresponds approximately to a loss limit of $250,000. Each of the 18 subtables of the Table of Aggregate Loss Factors corresponds to a policy excess ratio range. Each subtable was calculated using a loss limit which corresponds to an excess ratio (on a countrywide basis) that is the middle of that range. For example, Subtable 6 was calculated using a $1 million loss limit; when the Panjer algorithm was used to calculate an aggregate distribution, the severity distribution was censored from above at $1 million.[64]

The subtables are entered based on the policy excess ratio rather than the loss limit itself. If for example, an insured continues to buy retro policies with a $500,000 loss limit over many years, then due to inflation its excess ratio will increase.[65] [66] This will eventually lead to using a different subtable for this insured. However, neither the subtables themselves nor the published table of policy excess ranges need to be updated for inflation. If instead this retro insured's loss limit keeps up with inflation, then its excess ratio would stay approximately the same over time, and it would therefore use the same subtable.

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[63] Taken from NCCI Circular Letter CIF-2018-28, no longer on the syllabus.
[64] All of the probability for sizes greater than $1 million in the original unlimited severity distribution was placed instead at $1 million.
[65] With positive inflation, the percentage of losses excess of a fixed limit increases over time.
[66] NCCI updates its published excess ratios by loss limit (and state and hazard group) on a regular basis.

Details are shown in the following NCCI table:[67] [68] [69] [70]

Table of Policy Excess Ratio Ranges				
Subtable	Starting Point Loss Limit	Policy Excess Ratio Range		
Subtable 1	$50,000,000	0.000	-	0.008
Subtable 2	$10,000,000	0.009	-	0.025
Subtable 3	$5,000,000	0.026	-	0.051
Subtable 4	$2,500,000	0.052	-	0.077
Subtable 5	$1,750,000	0.078	-	0.110
Subtable 6	$1,000,000	0.111	-	0.145
Subtable 7	$750,000	0.146	-	0.181
Subtable 8	$500,000	0.182	-	0.221
Subtable 9	$375,000	0.222	-	0.269
Subtable 10	$250,000	0.270	-	0.316
Subtable 11	$200,000	0.317	-	0.358
Subtable 12	$150,000	0.359	-	0.420
Subtable 13	$100,000	0.421	-	0.484
Subtable 14	$75,000	0.485	-	0.550
Subtable 15	$50,000	0.551	-	0.648
Subtable 16	$25,000	0.649	-	0.765
Subtable 17	$10,000	0.766	-	0.852
Subtable 18	$5,000	0.853	-	1.000

Each of the 18 subtables is represented by a policy excess ratio range. A single countrywide loss limit (500K, 1 million, etc.) was used as the starting point. The policy excess ratio points corresponding to each of the countrywide loss limits were then expanded to ranges extending halfway to the excess ratio at adjacent loss limits.

In this way, the selection of policy excess ratio ranges began with single points of countrywide loss limits, and were then expanded to be able to handle various loss limits that correspond to different excess ratio values, depending on the state and hazard group.

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[67] Taken from Appendix A of NCCI Circular CIF-2018-28, not on the syllabus.
[68] The loss limits behind the creating of the subtables is background detailed information for actuaries, and is not needed to use the published countrywide Table of Aggregate Loss Factors.
[69] Actual retro insureds do not choose loss limits as small as $5000 or $10,000.
[70] No loss limit, equivalent to a loss limit of infinity, has an excess ratio of 0, and would correspond to Subtable 1.

Expense Ratio Tables:[71]

The NCCI Retro Plan contains Tables of Expense Ratios. They incorporate a set of premium discounts by size of insured.[72] They also assume a certain Expected Loss Ratio and Tax Multiplier.

One set is for expenses including profit and contingencies but excluding taxes. The other set is for expenses including profit and contingencies but excluding taxes and allocated loss adjustment expenses.

For example, for a standard premium of $1 million, the expense ratio excluding taxes is 0.264. Thus the expenses excluding taxes are $264,000. This would be an input used to determine the basic premium of a balanced retro plan.[73]

For a standard premium of $1 million, the expense ratio excluding taxes and ALAE is 0.204. Thus the expenses excluding taxes and ALAE are $204,000. This would be used with a retro plan that includes ALAE in with losses.

Table M:[74]

The NCCI Retro Plan contains a small extract of Table M as would be used in the prior NCCI Retro Plan.[75] We are given a Table of Insurance Charges for Expected Loss Groups 95 to 90, for Entry Ratios 0.01 to 0.37.

As shown in the Table of Expected Loss Groups in the NCCI Retro Plan, this applies to insureds with expected losses less than $6021, far too small to be retrospectively rated. In any case, one would need more entry ratios in order to solve the balance equations.[76]

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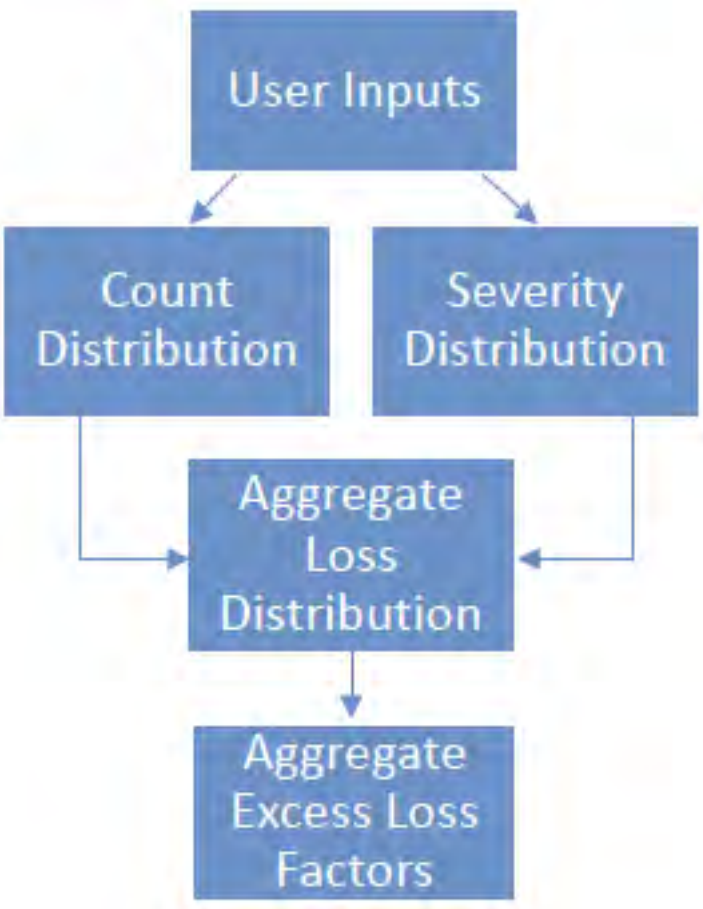
[71] See Pages AC7 and AC10 in the NCCI Retro Plan.
[72] Premium Discounts are discussed briefly in my section on Retrospective Rating.
The Expense Ratio Tables in the extract are for Type B Premium Discounts, which used to be called Mutual Company Premium Discounts, while Type A, so-called Stock Company Premium Discounts, are bigger.
[73] See the Example in Appendix D of the NCCI Retro Plan.
[74] See Pages AB1 and AB2 in the NCCI Retro Plan.
[75] As discussed in “Individual Risk Rating” and my section on Retrospective Rating.
[76] The traditional NCCI Table M went up to an entry ratio of 3.

Aggregate Loss Factors on Demand:[77]

In US Workers Compensation, the NCCI has recently replaced the "traditional" Table M. NCCI has created a computer program that performs real-time calculations and outputs the results to the user via an application known as Aggregate Loss Factors on Demand (ALFs on Demand).

The user inputs policy information: exposures by class and state, and the loss limit (if any). The computer program outputs for this policy: a frequency distribution, a severity distribution, an aggregate distribution, and aggregate excess loss factors by entry ratio.[78] The computer program can also output for the policy the projected ultimate distribution of claims by state, hazard group, and claim group.[79]



Using the same basic methodology as underlies this computer application, the NCCI has also published a countrywide Table of Aggregate Loss Factors, which replaces the traditional Table M. Since this table is on a countrywide basis, it is less accurate than ALFs on Demand, which is customized for the exposures by state and hazard group of each policy.

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[77] Information taken from NCCI Circular Letter CIF-2018-28, not on the syllabus.
[78] The aggregate excess loss factors are the insurance charges and savings used in determining retro rating plans.
[79] NCCI's claim groups differ somewhat from injury kinds.

Table of Aggregate Loss Factors:[80]

This is a new version of Table M. As in Table M, Aggregate Excess Loss Factors are given by:

$$\textbf{Entry Ratio} = \frac{\text{Limited Loss corrresponding to Maximum or Minimum Premium}}{\text{Expected Limited Loss}}.$$

Like Table M, and unlike Table L, these charges do not include the charge for the loss limit; there will be a separate charge added for any loss limit. The loss limit will be paid for via a separate Excess Loss Premium.[81]

The values contained in the Table of Aggregate Loss Factors are consistent with the general methodology underlying ALFs on Demand; however, due to its countrywide nature, the Table of Aggregate Loss Factors does not reflect the state and hazard group differences in severity distributions that are incorporated by ALFs on Demand.

The Table of ALFs has three dimensions:
- Rows corresponding to entry ratios.
- Columns corresponding to policy expected number of claims (size of insured).
- Subtables corresponding to the policy excess ratio (loss limit).[82]

Like Table M, each of these subtables has different columns which are to be used for different sized insureds; however, each column corresponds to a range of expected claim counts.[83] [84] Each column contains Aggregate Loss Factors (Insurance Charges) for entry ratios from 0 to 10 in increments of 0.01.[85]

The listed Aggregate Excess Loss Factors are based on the distribution of aggregate losses with the loss limit, and thus avoid any overlap for the charge for the effect of the maximum premium included in the basic premium and the charge for the loss limit contained in the excess loss premium.

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[80] Information taken from NCCI Circular Letter CIF-2018-28, not on the syllabus.
[81] Excess Loss Premium = c (Standard Premium) (Excess Loss Factor).
[82] The lower the loss limit, the higher the excess ratio.
[83] The larger the insured, the smaller the expected claim count group.
[84] In the traditional Table M, each column corresponded to an Expected Loss Group.
[85] The traditional NCCI Table M only went up to an entry ratio of 3.

Here is a small extract of Subtable 6, corresponding to a policy excess ratio of about 13%:[86]

Aggregate Excess Loss Factors (Sub-Table 6)			
Entry Ratio	Expected Claim Count Group		
	42	41	40
0.00	1.0000	1.0000	1.0000
0.25	0.7837	0.7810	0.7784
0.50	0.6160	0.6100	0.6040
0.75	0.4856	0.4768	0.4681
1.00	0.3836	0.3729	0.3622
1.25	0.3032	0.2912	0.2793
1.50	0.2392	0.2266	0.2140
1.75	0.1880	0.1751	0.1627
2.00	0.1465	0.1344	0.1232
2.25	0.1136	0.1028	0.0929
2.50	0.0879	0.0785	0.0699
2.75	0.0679	0.0598	0.0525
3.00	0.0524	0.0455	0.0393

ECG42 corresponds to about 38 expected claims, while ECG40 corresponds to about 49 expected claims. The larger insureds in ECG40 have smaller insurance charges than the smaller insureds in EGC42.

The values contained in the Table of Aggregate Loss Factors are based on a retro plan that uses pure losses (ALAE is not included with losses), and loss limits applied on a per occurrence basis. However, this table is to be used for all policies whether the applicable loss limit is on a per claim or per occurrence basis, and regardless of whether ALAE is included with ratable losses for purposes of computing the retrospective premium.

NCCI based this choice on an analysis observing that the relative change in the AELFs based on including versus excluding ALAE with losses, or for applying a loss limit on a per occurrence versus per claim basis was immaterial. In any case, the small relative differences would be further offset within the calculation of the net aggregate loss factors, applicable when there is both a minimum and maximum aggregate loss limitation.

[86] Taken from NCCI Circular Letter CIF-2018-28, not on the syllabus.

Here is an extract from NCCI's Table of Aggregate Loss Factors for different sub-tables:[87] [88]

Aggregate Excess Loss Factors			
Entry Ratio	Expected Claim Count Group 41		
	Sub-Table 5 (larger loss limit)	Sub-Table 6	Sub-Table 7 (smaller loss limit)
0.00	1.0000	1.0000	1.0000
0.25	0.7822	0.7810	0.7802
0.50	0.6143	0.6100	0.6073
0.75	0.4850	0.4768	0.4716
1.00	0.3853	0.3729	0.3649
1.25	0.3078	0.2912	0.2806
1.50	0.2470	0.2266	0.2139
1.75	0.1988	0.1751	0.1622
2.00	0.1604	0.1344	0.1226
2.25	0.1293	0.1028	0.0924
2.50	0.1039	0.0785	0.0694
2.75	0.0830	0.0598	0.0520
3.00	0.0657	0.0455	0.0389

Sub-Table 5 corresponds to a smaller excess ratio and thus a larger loss limit than Sub-Table 7. As we go from Sub-Table 7 to Table 5, in other words as we increase the loss limit, for a given Expected Claim Count Group and given entry ratio, the AELFs monotonically increase, as they should.

For larger entry ratios, $\frac{\text{AELF for higher loss limit}}{\text{AELF for lower loss limit}}$ is larger.

For example, 0.3853/0.3729 = 1.033, while 0.1604/0.1344 = 1.193.

[87] Taken from NCCI Circular Letter CIF-2018-28, not on the syllabus.
[88] Sub-Table 5 corresponds to a loss limit of approximately $1.75 million.
Sub-Table 6 corresponds to a loss limit of approximately $1 million.
Sub-Table 7 corresponds to a loss limit of approximately $0.75 million.

The Table of Aggregate Loss Factors is a countrywide table intended for use with policies having:[89]
o Exposure from any combination of state(s) and hazard group(s)
o Loss limits of any size
o Any number of expected claims

When publishing values in tabular form, a balance must be struck between:[90]
o Ensuring that the table contains enough values so that calculations performed using the table will be sufficiently accurate for any given policy
o Limiting the table to a practical size
o Limiting the number and complexity of calculations that the user is required to perform

Benefits of the Table of Aggregate Loss Factors:[91]

- Because the proposed table contains values that are based on a limited aggregate loss distribution, the table eliminates the need for an adjustment to account for overlap between the loss limit and aggregate loss limitation. AELFs obtained from the proposed table are more accurate for policies with a loss limit than is produced under the current methodology.[92]

- The proposed Table of Aggregate Loss Factors does not need periodic updates for claim inflation, as the introduction of policy excess ratio lookup ranges incorporates any and all loss limitations.[93]

- The parametric form used to produce the Table, provides users with a convenient method for calculating AELFs that are consistent with the values in the Table of Aggregate Loss Factors.

- The values contained in the Table of Aggregate Loss Factors are calculated in a manner that is consistent with the proposed methodology underlying ALFs on Demand, with certain exceptions due to the countrywide nature of the table.

- The Table of Aggregate Loss Factors leverages NCCI's 2014 Excess Loss Methodology, and replaces the current Table of Insurance Charges, which was created in the 1990's.[94]

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[89] Quoted from Informational Exhibit 3 of NCCI Circular Letter CIF-2018-28.
[90] Quoted from Informational Exhibit 3 of NCCI Circular Letter CIF-2018-28.
[91] Quoted from Informational Exhibit 3 of NCCI Circular Letter CIF-2018-28.
[92] The approximate ICRRL procedure has been replaced by using the appropriate subtable based on the policy excess ratio, which should be more accurate.
[93] Policy excess ratio rather than size of loss limit are used to determine which subtable to use.
If size of loss limit (50K, 100K, 250K, etc.) were used instead, this would require that the subtables be updated for inflation.
Policy expected claim counts are used to enter columns of the table.
If instead expected loss groups were used to enter columns of the table, then the expected loss groups would need to be updated for inflation.
[94] Updating the body of the old Table M based on new data after two decades would have been very worthwhile, even if there had been no change in methodology. Thus this is not a benefit of the new methodology.

Summary of NCCI's New Methodology to Compute Aggregate Excess Loss Factors:

The NCCI uses the Panjer algorithm to calculate an aggregate distribution.[95]
The Panjer algorithm has as inputs a frequency distribution and a discrete severity distribution; frequency and severity are assumed to be independent.
The severity distribution would be censored from above by any loss limit.[96]
Then this aggregate distribution is used to calculate Aggregate Excess Loss Factors (Insurance Charges).

The frequency distributions used are Negative Binomials, which vary by size of insured.

The discrete severity distributions were backed out of existing Excess Ratios underlying the Excess Loss Factors published by the NCCI.[97] You are not responsible for details of the NCCI methodology of determining Excess Loss Factors; for those who are interested, I do discuss a few details subsequently.[98]

However, the NCCI Filing Memorandum does mention that in determining Excess Loss Factors, the NCCI uses continuous severity distributions which are a splice of a mixture of two LogNormal Distributions with a (Generalized) Pareto Distribution. There is a different severity distribution for each claim group.[99] [100]

For a particular loss limit, the severity distribution is censored from above; any probability assigned to values above the loss limit is assigned to the loss limit.

For a particular size of insured, the appropriate frequency distribution is combined with the appropriate (discrete) severity distribution using the Panjer Algorithm, in order to determine the (discrete) aggregate distribution

This aggregate distribution is then used to compute Aggregate Excess Loss Factors.[101]

The methodology used in constructing the Table of Aggregate Loss Factors is that underlying the NCCI's ALFs on Demand with a few exceptions:
- The severity distributions, do not vary by state or hazard group.
  Rather, they are based on the countrywide parameters.
- The claim count weights used to combine the limited average severities for each claim group are based on expected claim counts across all hazard groups in states where NCCI performs ratemaking services.

Also AELFs are calculated assuming that the loss limit (if any) is on a per occurrence basis rather than a per claim basis, and that ALAE is not included with losses.

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[95] The Panjer algorithm is discussed in "Basics of Reinsurance Pricing," by David R. Clark, on Exam 9. See also Section 4.4 of Bahnemann, not on the syllabus.
[96] The NCCI used many discrete intervals, up to 15,000, to discretize a continuous severity distribution.
[97] The excess ratios differ by State and Hazard Group.
[98] For details see "NCCI's 2014 Excess Loss Factors," by Dan Corro and Yen-Chieh Tseng, Variance 2021, not on the syllabus.
[99] When applied to a particular State, the countrywide severity distributions are adjusted to be appropriate.
[100] NCCI claim groups are somewhat different than injury kinds.
[101] Subsequently I give a simple example of how to determine Aggregate Excess Loss Factors from a (discrete) aggregate distribution.

Benefits of the NCCI's New Methodology:[102]

- By utilizing severity distributions that vary to reflect the exposure characteristics of each risk(state, hazard group, ALAE handling, loss limit), the AELF values more directly reflect the exposure of the underlying policy than if a countrywide severity distribution is used.[103]

- By directly calculating the limited aggregate loss distribution, the proposed methodology eliminates the need for the adjustment to account for overlap between the loss limit and aggregate loss limitation. The result is that AELFs calculated using ALFs on Demand are more accurate for policies with a loss limit than is produced under the current methodology.[104]

- Because there is a vast array of possible exposure combinations (and resulting values) across all states, hazard groups and loss limits, it is not feasible for the AELFs to be published in a tabular form. Rather, the values for an individual policy will be available through an application, ALFs on Demand, that will be accessible on NCCI's website.[105] [106]

- The count and severity distributions underlying the proposed methodology are based on updated parameters that reflect more recent data than the distributions underlying the current Table of Insurance Charges, which have not changed since their creation in the late 1990s.[107]

- Future annual updates to the Excess Loss Factor parameters will automatically be incorporated into the Aggregate Loss Factors for each state.[108]

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[102] Quoted from the end of Informational Exhibit 1 of NCCI Circular Letter CIF-2018-28.

[103] This is only true of NCCI's computer product ALFs on Demand.
It is not true if one uses NCCI's countrywide published Table of Aggregate Loss Factors.

[104] This is also true if one uses NCCI's countrywide published Table of Aggregate Loss Factors, since the approximate ICRRL procedure has been replaced by using the appropriate subtable based on the policy excess ratio, which should be more accurate.

[105] The introduction of the new computer product is an improvement.
I am not sure that the elimination of published tables counts as an improvement.

[106] Nevertheless, NCCI did publish a countrywide Table of Aggregate Loss Factors.

[107] Updating the body of the old Table M based on new data after two decades would have been very worthwhile, even if there had been no change in methodology. Thus this is not a benefit of the new methodology.

[108] This is only true of NCCI's computer product ALFs on Demand.
NCCI's countrywide published Table of Aggregate Loss Factors will not automatically change.

Determining Aggregate Excess Loss Factors from a Discrete Distribution of Aggregate Losses:

The following discrete aggregate distribution has been calculated.[109] [110]

Aggregate Amount ($000)	Probability
0	8%
250	27%
500	19%
750	13%
1000	10%
1250	7%
1500	5%
1750	4%
2000	3%
2250	2%
2500	1%
2750	1%

The mean is: (0)(8%) + (250,000)(27%) + ... + (2,750,000)(1%) = 750,000.

An entry ratio of 1 corresponds to 750,000.
The expected aggregate excess of 750,000 is:
(250,000)(10%) + (500,000)(7%) + ... + (200,000)(1%) = 242,500.
Thus, $\phi(1)$ = 242,500 / 750,000 = 0.3233.

Exercise: Determine the Aggregate Excess Loss Factors for entry ratios of 2 and 3.
[Solution: An entry ratio of 2 corresponds to 1,500,000.
The expected aggregate excess of 1,500,000 is:
(250,000)(4%) + (500,000)(3%) + (750,000)(2%) + (1,000,000)(1%) + (1,250,000)(1%) = 62,500. $\phi(2)$ = 62,500 / 750,000 = 0.0833.
An entry ratio of 3 corresponds to 2,250,000.
The expected aggregate excess of 2,250,000 is:
(250,000)(1%) + (500,000)(1%) = 7,500. $\phi(3)$ = 7,500 / 750,000 = 0.0100.
Comment: $\psi(2) = \phi(2) + 2 - 1 = 1.0833$. $\psi(3) = \phi(3) + 3 - 1 = 2.0100$.]

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[109] I made up the given aggregate distribution. In practical applications, the Panjer algorithm would have as inputs a frequency distribution and a discrete severity distribution.
If there is a loss limit, then the severity distribution would be censored from above by any loss limit.
Also the NCCI used many more discrete intervals, up to as many as 15,000.

[110] The Panjer algorithm is discussed in "Basics of Reinsurance Pricing," by David R. Clark, on Exam 9.
See also Section 4.4 of Bahnemann, not on the syllabus.

NCCI uses a formula to compute Aggregate Excess Loss Factors at each of the evaluation points y_i from the discrete aggregate distribution:[111]

$$AELF_i = 1 - \frac{\sum_{j=0}^{j=i}(y_j\ PDF_j^{agg}) + \{y_i\ (1 - CDF_i^{agg})\}}{Agg_L},$$

where Agg_L is the average aggregate loss.[112]

For example let us assume we are given that the mean aggregate loss is 750,000, and only the first portion of the previous aggregate distribution:

Aggregate Amount ($000)	Probability
0	8%
250	27%
500	19%
750	13%
1000	10%
⋮	⋮

An entry ratio of 1 corresponds to 750,000.
The probability above 750,000 is: 1 - (8% + 27% + 19% + 13%) = 33%.

$$\phi(1) = 1 - \frac{\{(0)(8\%) + (250)(27\%) + (500)(19\%) + (750)(13\%)\} + (750)(33\%)}{750}$$

$= 1 - 507.5 / 750 = 0.3233$, matching the previous result.

$\psi(1) = \phi(1) + 1 - 1 = 0.3233$.

The NCCI calls the insurance savings the Aggregate Minimum Loss Factor (AMLF).
NCCI displays the following formula:

$$AMLF_i = \frac{y_i\ - [\sum_{j=0}^{j=i}(y_j\ PDF_j^{agg}) + \{y_i\ (1 - CDF_i^{agg})\}]}{Agg_L}.$$

For this example, to get the Aggregate Minimum Loss Factor at r = 1:

$$\psi(1) = \frac{750 - [\{(0)(8\%) + (250)(27\%) + (500)(19\%) + (750)(13\%)\} + (750)(33\%)]}{750}$$

$= (750 - 507.5) / 750 = 0.3233$, matching the previous result.

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[111] In order to get AELFs at the desired entry ratios, one has to interpolate between evaluation points.
[112] If the aggregate distribution was computed assuming a loss limit, then this average is after the impact of this per claim or occurrence limit.

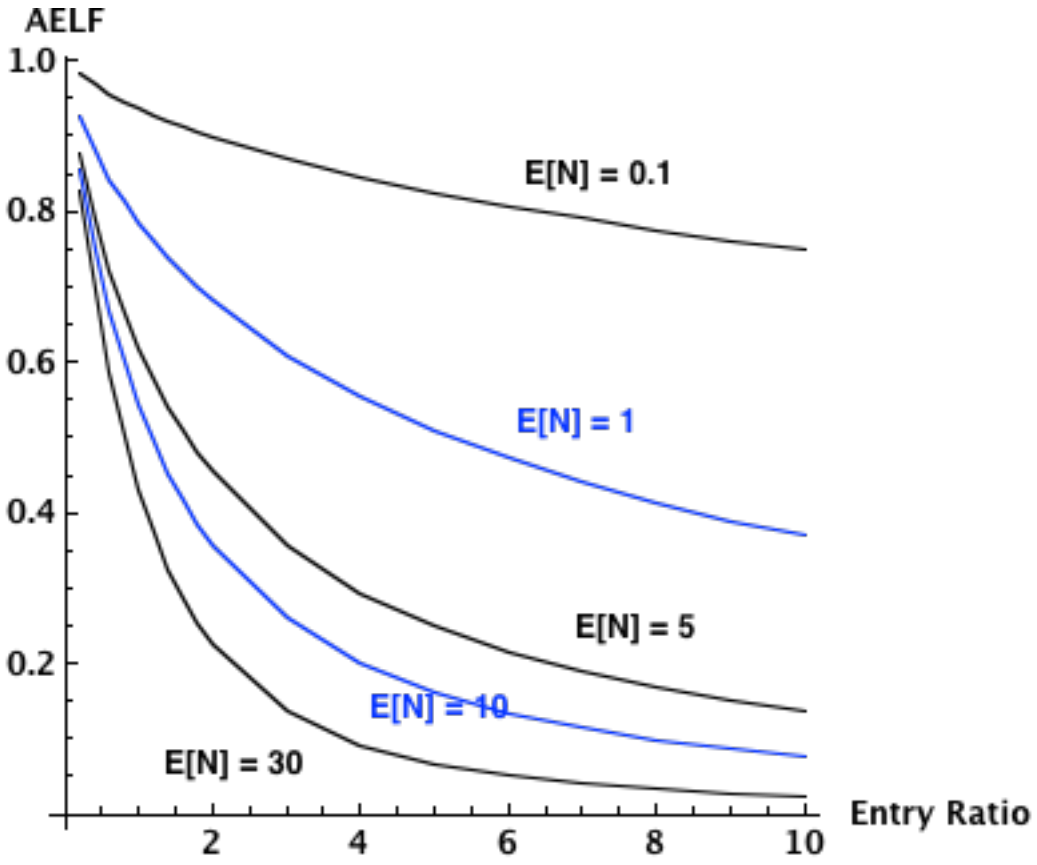
Examples of Output from NCCI's ALFs on Demand:[113]

For no loss limit, here is sample output from NCCI's computer product ALFs on Demand; these Aggregate Excess Loss Factors are for Hazard Group A in Alaska:[114] [115] [116] [117]

Entry Ratio	E[N] = 0.1	E[N] = 1	E[N] = 5	E[N] = 10	E[N] = 30	E[N] = 50	E[N] = 100	E[N] = 500	E[N] = 1000
0.2	0.9828	0.9253	0.8744	0.8543	0.8280	0.8191	0.8106	0.8019	0.8008
0.4	0.9683	0.8771	0.7873	0.7483	0.6931	0.6723	0.6495	0.6178	0.6108
0.6	0.9559	0.8393	0.7183	0.6641	0.5854	0.5542	0.5182	0.4604	0.4444
0.8	0.9449	0.8075	0.6611	0.5953	0.4987	0.4595	0.4133	0.3343	0.3104
1.0	0.9351	0.7797	0.6126	0.5378	0.4282	0.3836	0.3304	0.2382	0.2098
1.2	0.9261	0.7549	0.5708	0.4892	0.3705	0.3224	0.2653	0.1676	0.1381
1.4	0.9178	0.7324	0.5342	0.4475	0.3229	0.2729	0.2142	0.1169	0.0892
1.6	0.9100	0.7118	0.5020	0.4116	0.2834	0.2327	0.1741	0.0812	0.0568
1.8	0.9028	0.6928	0.4733	0.3802	0.2503	0.1999	0.1426	0.0565	0.0359
2.0	0.8960	0.6751	0.4476	0.3528	0.2225	0.1729	0.1178	0.0394	0.0226
3.0	0.8665	0.6019	0.3511	0.2552	0.1340	0.0926	0.0514	0.0078	0.0025
4.0	0.8422	0.5459	0.2879	0.1966	0.0900	0.0571	0.0274	0.0024	0.0004
5.0	0.8211	0.5010	0.2435	0.1583	0.0653	0.0389	0.0169	0.0011	0.0001
6.0	0.8023	0.4638	0.2106	0.1316	0.0501	0.0286	0.0115	0.0005	0.0000
7.0	0.7853	0.4325	0.1853	0.1120	0.0399	0.0220	0.0084	0.0003	0.0000
8.0	0.7696	0.4055	0.1652	0.0970	0.0328	0.0176	0.0064	0.0001	0.0000
9.0	0.7551	0.3821	0.1489	0.0853	0.0276	0.0144	0.0050	0.0000	0.0000
10.0	0.7415	0.3614	0.1354	0.0759	0.0236	0.0116	0.0041	0.0000	0.0000

[113] See Informational Exhibit 2 of NCCI Circular Letter CIF-2018-28, not on the syllabus.
[114] The differences between Hazard Groups is due to the different proportions of the different claim groups.
[115] With no loss limit these are similar to Subtable 1 of the NCCI's published Table of Aggregate Loss Factors; however, that table is based on a countrywide average and an average over hazard groups, and thus does not represent the exposure of an individual policy by state and hazard group.
[116] These AELFs would be appropriate for a policy with only exposure in a single state and the given Hazard Group. Even retro risks with exposure in only one state usually have exposure in more than one Hazard Group.
[117] These AELFs have been adjusted to be appropriate for this State and Hazard Group, and the columns are based on the expected number of claims rather than Expected Claim Count Groups.

For Hazard Group A, a graph of these AELFs for various Expected Numbers of Claims:[118]



[118] Each curve is decreasing and concave upwards. In every case, at r = 0 the AELF = 1.

For example, here are the Aggregate Excess Loss Factors for entry ratio 1 in Alaska:

HG	E[N] = 0.1	E[N] = 1	E[N] = 5	E[N] = 10	E[N] = 30	E{N] = 50	E[N] = 100	E[N] = 500	E[N] = 1000
A	0.9351	0.7797	0.6126	0.5378	0.4282	0.3836	0.3304	0.2382	0.2098
B	0.9361	0.7852	0.6202	0.5455	0.4351	0.3899	0.3359	0.2414	0.2121
C	0.9382	0.7895	0.6226	0.5475	0.4370	0.3918	0.3377	0.2427	0.2129
D	0.9391	0.7850	0.6136	0.5384	0.4293	0.3850	0.3321	0.2397	0.2108
E	0.9419	0.7899	0.6174	0.5420	0.4326	0.3881	0.3350	0.2414	0.2119
F	0.9443	0.7923	0.6173	0.5411	0.4310	0.3865	0.3335	0.2403	0.2110
G	0.9467	0.7958	0.6223	0.5468	0.4375	0.3931	0.3397	0.2439	0.2132

Here are the Aggregate Excess Loss Factors for an entry ratio of 2 in Alaska:

HG	E[N] = 0.1	E[N] = 1	E[N] = 5	E[N] = 10	E[N] = 30	E{N] = 50	E[N] = 100	E[N] = 500	E[N] = 1000
A	0.8960	0.6751	0.4476	0.3528	0.2225	0.1729	0.1178	0.0394	0.0226
B	0.8983	0.6837	0.4588	0.3638	0.2324	0.1819	0.1251	0.0425	0.0242
C	0.9020	0.6877	0.4615	0.3667	0.2355	0.1850	0.1279	0.0437	0.0247
D	0.9027	0.6781	0.4482	0.3542	0.2254	0.1763	0.1212	0.0407	0.0228
E	0.9072	0.6834	0.4533	0.3594	0.2306	0.1812	0.1253	0.0419	0.0231
F	0.9108	0.6846	0.4516	0.3568	0.2278	0.1786	0.1230	0.0404	0.0220
G	0.9141	0.6896	0.4602	0.3669	0.2389	0.1891	0.1317	0.0424	0.0226

Comparing a Retro with No Loss Limit to a Retro with a Loss Limit:[119] [120]

In its Table of Aggregate Loss Factors, the NCCI provides insurance charges on a countrywide basis. Sub-table 6 applies to policies with an excess ratio between 0.111 and 0.145, corresponding to a loss limit of about $1 million. With no loss limit, we would use Subtable 1.[121]

Let us compare these two situations for an example. One has to be careful in this comparison. The insurance charges shown in the Table of Aggregate Loss Factors are to be multiplied by the expected limited losses. Also, the insurance charges shown in the Table of Aggregate Loss Factors do not include the separate charge for the loss limit.

For the example I will assume in:[122]
- Standard Premium of $1.5 million.
- Expected Total Losses of $1 million.
- The expected number of claims is 100. ⇔ Expected Claim Count Group 34.
- When there is a loss limit it is $1 million and the policy excess ratio is 13%. ⇔ Sub-table 6.
- ALAE is treated separately from losses.

For example, let us assume that with no loss limit $3 million in losses correspond to the maximum premium. Then the entry ratio is: 3 million / 1 million = 3.
For no loss limit, the AELF in Sub-table 1 for r = 3 and Expected Claim Count Group 34 is 0.0714. Multiplying by the total expected losses of $1 million, this is equivalent to $71,400.

For the case with a $1 million loss limit, the expected limited losses are:
(1 - 13%)(1 million) = $870,000.
Let us assume the same maximum entry ratio of 3.
This corresponds to limited losses of: (3)($870,000) = $2,610,000.
In turn this corresponds to expected total losses of: $2,610,000/(1 - 13%) = $3 million.
$3 million in expected unlimited losses corresponds to the maximum premium.[123]

The AELF in Sub-table 6 for r = 3 and Expected Claim Count Group 34 is 0.0147.
Multiplying by the expected limited losses of $870,000, this is equivalent to $12,789.
Adding in the $130,000 in expected excess losses, we get a total of: $142,789.

This $142,789 due to the loss limit and the maximum premium, is more than the $71,400 for the maximum premium in the absence of the loss limit.[124] One should not be fooled by the fact that with a loss limit the AELF is lower than without a loss limit. Including the separate charge for the loss limit, in total the retro with a loss limit includes more fixed dollars in order to pay for the additional benefit to the insured of the loss limit.

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[119] See also my Section on Table L.
[120] Using information taken from NCCI Circular Letter CIF-2018-28, not on the syllabus.
[121] Subtable 1 applies to policies with an excess ratio of 0.000 to 0.008.
[122] All values are solely for illustrative purposes.
[123] This is the same $3 million as before because the entry ratios have limited losses in both their numerator and denominator.
[124] Both dollar figures are prior to being multiplied by the loss conversion factor.

For this example, I have varied the maximum premium and done a similar comparison:[125] [126] [127]

No Loss Limit				$1 million Loss Limit		
Losses ⇔ Maximum Premium	Entry Ratio	AELF	Dollar Charge	Entry Ratio	AELF	Dollar Charge
$200,000	0.20	0.8108	$810,800	0.20	0.8091	$833,917
$400,000	0.40	0.6510	$651,000	0.40	0.6431	$689,497
$600,000	0.60	0.5219	$521,900	0.60	0.5043	$568,741
$800,000	0.80	0.4199	$419,900	0.80	0.3906	$469,822
$1,000,000	1.00	0.3400	$340,000	1.00	0.2993	$390,391
$1,200,000	1.20	0.2777	$277,700	1.20	0.2273	$327,751
$1,400,000	1.40	0.2291	$229,100	1.40	0.1712	$278,944
$1,600,000	1.60	0.1910	$191,000	1.60	0.1281	$241,447
$1,800,000	1.80	0.1610	$161,000	1.80	0.0953	$212,911
$2,000,000	2.00	0.1372	$137,200	2.00	0.0705	$191,335
$3,000,000	3.00	0.0714	$71,400	3.00	0.0147	$142,789
$4,000,000	4.00	0.0446	$44,600	4.00	0.0029	$132,523
$5,000,000	5.00	0.0311	$31,100	5.00	0.0005	$130,435
$6,000,000	6.00	0.0231	$23,100	6.00	0.0001	$130,087
$7,000,000	7.00	0.0180	$18,000	7.00	0.0000	$130,000

As the maximum premium increases, the insured gets less benefit from the maximum, and the fixed dollars in the retro decrease. The insured benefits from the $1 million loss limit, and thus the fixed dollars in the retro are more with the loss limit than without it. With a $1 million loss limit, this insured would get very little additional benefit from a very large maximum premium such as an entry ratio of 7; the fixed dollars in the retro are equal to the $130,000 in expected excess losses.

As the maximum premium decreases, the additional benefit from the $1 million loss limit decreases. For example, at an entry ratio of 4, the difference is: 132,523 - 44,600 = $87,923. At instead an entry ratio of 2, the difference is only: 191,335 - 137,200 = $54,135.

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[125] In practical applications, the maximum premium would be more than the standard premium of $1.5 million. I have included smaller maximum premiums solely to illustrate the mathematics behind the AELFs.

[126] In the case of the loss limit, I have added in the $130,000 in expected excess losses.

[127] In the case of the loss limit, I have taken the same entry ratio as without a loss limit.

Problems:

14.1. (2 points)
The balanced retro plan provisions for a workers' compensation risk are given below:

Standard Premium	$300,000
Minimum Entry Ratio, r_H	0.40
Maximum Entry Ratio, r_G	3.00
Loss Conversion Factor	1.20
Provision for expenses and profit exclusive of taxes	$90,000
Expected Loss Ratio	66%
Expected Number of Claims	30
Aggregate Minimum Loss Factor at Minimum Entry Ratio	0.6965
Aggregate Maximum Loss Factor at Maximum Entry Ratio	0.1476

● There is no loss limit.
Calculate the basic premium ratio to standard premium.

14.2. (2 points) An insured that can either have 0, 1, or 2 claims, each of which can either be $1000 or $5000. The following tables summarize the probabilities of this occurring:

Claim Count Distribution		Severity Distribution	
Number of Claims	Probability	Loss Amount	Probability
0	50%	1000	80%
1	40%	5000	20%
2	10%		

Frequency and severity are independent.
Determine the aggregate distribution.

14.3. (0.5 point) An insured has Standard Premium of $700,000.
Using the Tables of Expense Ratios in the extract from the NCCI Retro Plan determine:
i. Expenses excluding taxes.
ii. Expenses excluding taxes and allocated loss adjustment expenses.

14.4. (1 point) For a policy the excess ratio is 70% and the expected loss ratio is 60%.
LAE is 12% of losses. There is a 0.8% loss assessment.
(a) Determine the Excess Loss Factor (ELF).
(b) Determine the Excess Loss Pure Premium Factor (ELPPF).

14.5. *(1.5 points) The NCCI lists five benefits of its Table of Aggregate Loss Factors Give three of these benefits.*

14.6. (3.25 points) A workers compensation insured has exposures in two classes in each of two states. The insured is buying a retrospective rating policy with an loss limit of $100,000.
The insured has an expected (unlimited) loss ratio of 63%.
The insured has an experience modification of 0.90.

State	Hazard Group of Class	Manual Premium	Excess Ratio at $100,000	Average Cost per Case
1	C	$50,000	0.363	$15,000
1	F	$250,000	0.491	$25,000
2	C	$30,000	0.264	$9,000
2	F	$200,000	0.383	$17,000

(a) (1.5 points) Determine the excess ratio for this policy.
(b) (1.5 points) Determine the expected number of claims for this policy.
(c) (0.25 points) Determine the Sub-Table and the Expected Claim Count Group.

14.7. (0.5 point)
For expected loses of $4000, using the extract of Table M in the NCCI's Retrospective Rating Plan, determine the Insurance Charge and Insurance Savings at an entry ratio of 0.30.

14.8. (2 points) Using the following Aggregate Excess Loss Factors (Sub-Table 6), for Expected Claim Count Group 42, estimate the distribution function and the density of the aggregate distribution at an entry ratio of 2.

Aggregate Excess Loss Factors (Sub-Table 6)	
Entry Ratio	Expected Claim Count Group 42
1.80	0.1790
1.90	0.1620
2.00	0.1465
2.10	0.1324
2.20	0.1196

14.9. (3 points) An insured is to be written under the NCCI Retrospective Rating Plan.
- Estimated Standard Premium = $1,000,000
- Maximum Retrospective Premium Factor = 140%
- Minimum Retrospective Premium Factor = 50%
- Loss Conversion Factor = 1.110
- Tax Multiplier = 1.060
- Loss Limit = $500,000
- Expenses (not covered in the Tax Multiplier) = 0.188
- Expected Loss Ratio = 0.640
- Policy Excess Ratio = 0.131
- Expected Number of Claims = 60

Use the following extract of NCCI's Table of Aggregate Loss Factors.

r	ALF		r	ALF
0.25	0.7735		1.65	0.1584
0.26	0.7654		1.66	0.1565
0.27	0.7574		1.67	0.1546
0.28	0.7494		1.68	0.1527
0.29	0.7415		1.69	0.1509
0.30	0.7337		1.70	0.1491
0.31	0.7260		1.71	0.1473
0.32	0.7183		1.72	0.1455
0.33	0.7107		1.73	0.1427
0.34	0.7032		1.74	0.1420
0.35	0.6958		1.75	0.1402

(a) (2.5 points) Determine the Basic Premium.
(b) (0.5 point) Determine the Excess Loss Premium.

14.10. (1 point) A retrospectively rated insured has an expected loss ratio of 65%.
It has an average cost per claim of $20,000.
If it is in Expected Claim Count Group 40, what is its guaranteed cost premium?

14.11. (4 points) You are given the following discrete aggregate distribution.

Aggregate Amount ($000)	Probability
0	7%
250	25%
500	18%
750	13%
1000	9%
1250	6%
1500	4%
1750	3%
2000	2%
2250	2%
2500	2%
2750	1%
3000	1%
3250	1%
3500	1%
3750	1%
4000	1%
4250	1%
4500	1%
4750	1%

(a) (1.5 points) Determine the Aggregate Excess Loss Factor at an entry ratio of 1, $\phi(1)$.
(b) (1.5 points) Determine the Aggregate Excess Loss Factor at an entry ratio of 2, $\phi(2)$.
(c) (1 point) Determine the Aggregate Excess Loss Factor at an entry ratio of 3, $\phi(3)$.

14.12. *(1.5 points) The NCCI lists five benefits of its New Methodology. Give three of them.*

14.13. (2 points) Using the following values of Aggregate Excess Loss Factors, estimate the distribution function and the density of the aggregate distribution at an entry ratio of 0.6.

Entry Ratio	AELF
0.2	0.8575
0.4	0.7529
0.6	0.6700
0.8	0.6022
1.0	0.5458

14.14. (2 points) Prior to 2019, the NCCI used a Table of Expected Loss Groups in order to determine which column to use in Table M. The expected losses corresponding to the different groups were updated annually for the effect of inflation.
Fully discuss how the NCCI now deals with the effect of inflation on using its Table of Aggregate Loss Factors.

14.15. You are provided the following information about a Workers Compensation account:
- Loss limit selected = $500,000
- Exposure only in one state.

Class	Payroll	Rate	Hazard Group	Excess Ratio at 500,000
1	1,400,000	5.85	G	0.025
2	600,000	4.65	E	0.017
3	1,000,000	2.75	D	0.012
4	200,000	1.50	B	0.008

Determine the excess ratio for this policy.

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14.16. (2 points) Given the following information about a retrospectively rated policy:

Standard Premium	$1,400,000
Maximum retro premium factor	160%
Minimum retro premium factor	50%
Loss Conversion Factor	1.175
Provision for expenses and profit exclusive of taxes (as percent of Standard Premium)	13.5%
Tax multiplier	1.061
Expected Loss Ratio	63.0%
Expected Number of Claims	100

- There is no loss limit.
- The retro rating plan is balanced.

Use the following Aggregate Loss Factors:

r	0.0	0.2	0.4	0.6	0.8	1.0
ALF	1.0000	0.8110	0.6508	0.5205	0.4166	0.3346
r	1.2	1.4	1.6	1.8	2.0	3.0
ALF	0.2702	0.2196	0.1798	0.1485	0.1236	0.0558

Calculate the basic premium.

14.17. (2 points) A retro rated insured has an expected loss ratio of 64%, a policy excess ratio of 12%, expected claims of 43, and a loss conversion factor of 1.1. The insured is such that sub-table 6 and expected claim count group 41 are appropriate to determine its Aggregate Excess Loss Factors.

Aggregate Excess Loss Factors (Sub-Table 6)	
Entry Ratio	Expected Claim Count Group 41
0.00	1.0000
0.25	0.7810
0.50	0.6100
0.75	0.4768
1.00	0.3729
1.25	0.2912
1.50	0.2266
1.75	0.1751
2.00	0.1344
2.25	0.1028
2.50	0.0785
2.75	0.0598
3.00	0.0455

The balance equations are:
Table of Aggregate Loss Factors Value Difference = 0.4758.
Table of Aggregate Loss Factors Entry Difference = 1.50.
Determine the Net Aggregate Loss Factor.

14.18. *(3 points) A Workers Compensation policy has exposure in only one Hazard Group in one state. The policy is retro rated with a $100,000 per claim loss limit.*
Use the following information.

Claim Group	*Average Unlimited Severity*	*Excess Ratio for $100K per claim limit*	*Expected Portion of Claims*
Fatal	*$200,000*	*0.597*	*0.05%*
PT	*$1,500,000*	*0.921*	*0.15%*
Likely PP/TT	*$150,000*	*0.564*	*5.00%*
Not Likely PP/TT	*$30,000*	*0.291*	*25.00%*
Med Only	*$1,000*	*0.044*	*69.80%*

(a) Determine for this policy the limited average severity per claim.
(b) Determine the excess ratio for this policy.

14.19. (0.5 point) An insured has Standard Premium of $4,000,000.
Using the Tables of Expense Ratios in the extract from the NCCI Retro Plan determine:
i. Expenses excluding taxes.
ii. Expenses excluding taxes and allocated loss adjustment expenses.

14.20. (2 points) Given the following information about a retrospectively rated policy

Standard Premium	$750,000
Maximum retro premium factor	240%
Minimum retro premium factor	40%
Loss Conversion Factor	1.120
Provision for expenses and profit exclusive of taxes (as percent of Standard Premium)	14.8%
Tax multiplier	1.041
Expected Loss Ratio	66.0%
Expected Number of Claims	50

- There is no loss limit.
- The retro rating plan is balanced.

Use the following Aggregate Loss Factors:

r	0.0	0.2	0.4	0.6	0.8	1.0
ALF	1.0000	0.8204	0.6755	0.5594	0.4664	0.3916
r	1.2	1.4	1.6	1.8	2.0	3.0
ALF	0.3314	0.2825	0.2427	0.2100	0.1831	0.1016
r	4.0	5.0	6.0	7.0	8.0	9.0
ALF	0.0645	0.0450	0.0335	0.0261	0.0210	0.0172

Calculate the basic premium.

14.21. (2 points) The mean aggregate loss is $50,000.
The following discrete aggregate distribution has been calculated.

Aggregate Amount ($000)	Probability
0	2%
10	6%
20	11%
30	14%
40	18%
50	15%
60	11%
70	8%
80	5%
90	3%
100	2%
⋮	⋮

Determine the Aggregate Excess Loss Factor at an entry ratio of 1.6, $\phi(1.6)$.

14.22. (3 points) An insured is to be written under the NCCI Retrospective Rating Plan.
- Estimated Standard Premium = $2,000,000
- Maximum Retrospective Premium Factor = 160%
- Minimum Retrospective Premium Factor = 40%
- Loss Conversion Factor = 1.113
- Tax Multiplier = 1.052
- Loss Limit = $1,000,000
- Expenses (not covered in the Tax Multiplier) = 0.179
- Expected Loss Ratio = 0.620
- Policy Excess Ratio = 0.116
- Expected Number of Claims = 121

Use the following extract of NCCI's Table of Aggregate Loss Factors.

r	ALF		r	ALF
0.25	0.7633		2.10	0.0543
0.26	0.7545		2.11	0.0535
0.27	0.7459		2.12	0.0526
0.28	0.7373		2.13	0.0518
0.29	0.7287		2.14	0.0510
0.30	0.7202		2.15	0.0501
0.31	0.7118		2.16	0.0493
0.32	0.7035		2.17	0.0485
0.33	0.6952		2.18	0.0478
0.34	0.6870		2.19	0.0470
0.35	0.6789		2.20	0.0462

(a) (2.5 points) Determine the Basic Premium.
(b) (0.5 point) Determine the Excess Loss Premium.

14.23. (3.25 points) A workers compensation insured has exposures in two classes in each of two states. The insured is buying a retrospective rating policy with an loss limit of $500,000.
The insured has an expected (unlimited) loss ratio of 66%.
The insured has an experience modification of 1.10.

State	Hazard Group of Class	Manual Premium	Excess Ratio at $500,000	Average Cost per Case
1	B	$150,000	0.131	$12,000
1	E	$500,000	0.182	$19,000
2	B	$200,000	0.145	$15,000
2	E	$900,000	0.204	$21,000

(a) (1.5 points) Determine the excess ratio for this policy.
(b) (1.5 points) Determine the expected number of claims for this policy.
(c) (0.25 points) Determine the Sub-Table and the Expected Claim Count Group.

14.24. (2 points) The mean aggregate loss is $100,000.
The following discrete aggregate distribution has been calculated.

Aggregate Amount ($000)	Probability
0	3%
20	7%
40	12%
60	16%
80	19%
100	15%
120	10%
⋮	⋮

Determine the Aggregate Excess Loss Factor at an entry ratio of 1.2, $\phi(1.2)$.

14.25. (9, 11/96, Q.9) (1 point) According to the National Council on Compensation Insurance's Retrospective Rating Plan Manual for Workers' Compensation and Employers Liability Insurance, which of the following are true?
1. Retrospective development premium may be included in the first three adjustments.
2. Risks are eligible for a one-year plan if the estimated standard premium is at least $25,000.
3. Retrospective rating may be applied to any of the following types of insurance alone or any combination of such insurance:
   - Workers compensation and employers liability insurance
   - Any other commercial casualty lines of insurance

Note: This past exam question has been rewritten to match the current syllabus.

14.26. (6, 5/97, Q.13) (1 point) The basic premium in the NCCI retrospective rating plan provides for which of the following costs?
1. Risk control services
2. Premium taxes
3. An allowance for profit and contingencies

14.27. (9, 11/97, Q.5) (1 point) According to the National Council on Compensation Insurance's Retrospective Rating Plan, which of the following statements are true?
1. The excess loss premium is an elective element of the retrospective premium formula.
2. The basic premium includes a provision for subsidy of the assigned risk market.
3. The standard premium used in the retrospective premium formula is prior to the effect of the premium discount.

Note: I have slightly reworded this past exam question.

14.28. (9, 11/98, Q.2) (1 point) According to the National Council on Compensation Insurance's Retrospective Rating Plan Manual for Workers Compensation and Employers Liability Insurance, which of the following are true?
1. The loss conversion factor is established by negotiations between the carrier and the insured.
2. Premium developed by the occupational disease rates for risks subject to the Federal Mine Health and Safety Act is included in the determination of standard premium.
3. Standard premium includes the expense constant.

Comment: This question has been rewritten to match the current syllabus.

14.29. (9, 11/98, Q.46) (3 points) As the actuary for Kryptonite Insurance Company, you are working with an insured, Kent & Lane Industries, to develop a retrospectively rated plan. You have developed the following initial proposal using the National Council on Compensation Insurance's Retrospective Rating Plan for Workers Compensation and Employers Liability Insurance.

Maximum Premium Factor	1.50
Minimum Premium Factor	0.50
Tax Multiplier	1.03
Basic Premium Factor	0.40
Loss Conversion Factor	1.09
Loss Limit	$250,000

Kent & Lane would like to change some of the parameters of your proposal.

a. (0.5 point) List which of the items above that the NCCI plan allows you to change if the risk qualifies for the "Large Risk Alternative Rating Option."

b. (0.5 point) List which of the items above that the NCCI plan allows you to change if the risk does not qualify for the "Large Risk Alternative Rating Option."

c. (1 point) Assuming that the risk does not qualify for the "Large Risk Alternative Rating Option," what will happen to the basic premium factor (will it increase, decrease, or stay the same) if the loss conversion factor is increased? Briefly explain your answer.

d. (1 point) The basic premium does not include the charge for the loss limit, which is separate. Assuming that the risk does not qualify for the "Large Risk Alternative Rating Option," what will happen to the basic premium factor (will it increase, decrease, or stay the same) if the loss limit is eliminated? Briefly explain your answer.

***14.30.* (9, 11/99, Q.8)** (1 point)

Which of the following are true in the NCCI Retrospective Rating Plan?

1. The maximum retrospective premium factor is established by agreement between the risk and the insurance carrier.
2. The excess loss premium is computed as the standard premium multiplied by the excess loss factor.
3. If the ALAE option is elected as part of incurred losses, the loss conversion factor must be adjusted to exclude ALAE.

Note: I have rewritten this past exam question to match the current syllabus.

Solutions:

14.1. Charge at r = 3.00 is 0.1476.
Charge at r = 0.40 is 0.6965. Savings are: 0.6965 - 1 + 0.4 = 0.0965.
Converted Net Insurance Charge is: (1.2)(0.66)(0.1476 - 0.0965) = 0.0405.
Basic Premium is: (0.0405)(300,000) + 90,000 - (1.2 - 1)(66%)(300,000) = 62,550.
Basic Premium Factor is: 62,550/300,000 = **0.209**.

14.2. There is 50% chance of zero claims and thus zero aggregate.
The probability of one claim for 1000 is: (80%)(40%) = 32%.
The probability of one claim for 5000 is: (20%)(40%) = 8%.
The probability of two claims each for 1000 is: $(80\%)^2(10\%) = 6.4\%$.
The probability of two claims, one for 1000 and one for 5000 is: (2)(80%)(20%)(10%) = 3.2%.
The probability of two claims each for 5000 is: $(20\%)^2(10\%) = 0.4\%$.

Aggregate Losses	Probability
0	50%
1000	32%
2000	6.4%
5000	8%
6000	3.2%
10,000	0.4%

14.3. i. (0.266)(700,000) = **$186,200**.
ii. (0.206)(700,000) = **$144,200**.
Comment: Using the 2024 Study Kit.

14.4. (a) ELF = (70%)(60%) = **42%**.
(b) ELPPF = 42% / {(60%)(1 + 12% + 0.8%)} = **0.621**.
Alternately, 70% / (1 + 12% + 0.8%) = 0.621.
Comment: See the third page of Rule 1 in the NCCI Retro Manual.
The ELPPF is applied to "loss cost premium", in order to get expected excess losses.

$$\text{ELPPF} = \frac{\text{Expected Excess Losses}}{\text{Loss Cost Premium}} = (\text{Excess Ratio}) / (1 + \text{LAE \%} + \text{Loss Assessment \%}).$$

$$\text{ELPPF} = \frac{\text{ELF}}{(\text{Expected Loss Ratio})\,(1 + \text{LAE \%} + \text{Loss Assessment \%})}.$$

14.5. Any three of the following five benefits from NCCI's Informational Exhibit 3 in NCCI Circular CIF-2023-28, formerly on the syllabus.

● Because the proposed table contains values that are based on a limited aggregate loss distribution, the table eliminates the need for an adjustment to account for overlap between the loss limit and aggregate loss limitation. AELFs obtained from the proposed table are more accurate for policies with a loss limit than is produced under the current methodology (using ICRLL).

● The proposed Table of Aggregate Loss Factors does not need periodic updates for claim inflation, as the introduction of policy excess ratio lookup ranges incorporates any and all loss limitations. (Policy excess ratio rather than size of loss limit are used to determine which subtable to use. If size of loss limit were used instead, this would require that the subtables be updated for inflation. Policy expected claim counts are used to enter columns of the table.)

● The parametric form used to produce the Table, provides users with a convenient method for calculating AELFs that are consistent with the values in the Table of Aggregate Loss Factors.

● The values contained in the Table of Aggregate Loss Factors are calculated in a manner that is consistent with the proposed methodology underlying ALFs on Demand, with certain exceptions due to the countrywide nature of the table.

● The Table of Aggregate Loss Factors leverages NCCI's 2014 Excess Loss Methodology, and replaces the current Table of Insurance Charges, which was created in the 1990's.

Comment: Updating the body of the old Table M based on new data after 2 decades would have been very worthwhile, even if there had been no change in methodology.

14.6. (a) Modified Expected Loss is the manual premium times both the expected loss ratio of 63% and the experience modification of 0.90. For example: (63%)(0.90)($50,000) = $28,350.

State	HG	Manual Premium	Modified Expected Loss	Excess Ratio	Expected Excess Loss
1	C	$50,000	$28,350	0.363	$10,291
1	F	$250,000	$141,750	0.491	$69,599
2	C	$30,000	$17,010	0.264	$4,491
2	F	$200,000	$113,400	0.383	$43,432
Total			$300,510		$127,813

For example: ($28,350)(0.363) = $10,291.
The policy excess ratio is: 127,813/300,510 = **0.425**.
(b) In each case, we divide the modified expected loss by the average cost per case.

State	HG	Manual Premium	Modified Expected Loss	Average Cost per Case	Expected Number of Claims
1	C	$50,000	$28,350	$15,000	1.89
1	F	$250,000	$141,750	$25,000	5.67
2	C	$30,000	$17,010	$9,000	1.89
2	F	$200,000	$113,400	$17,000	6.67
Total			$300,510		**16.12**

(c) The policy excess ratio of 0.425 corresponds to Sub-Table **13**.
Expected number of claims of 16.12 corresponds to Expected Claim Count Group **50**.
Comment: Similar to two examples shown in the NCCI Retro Manual.
In each case, the excess ratio is the expected percent of losses excess of $100,000, which differs by state and hazard group. The excess ratios and average costs per case would be looked up in an NCCI publication that is not on the syllabus.
In part (a), we are taking a weighted average of excess ratios. Therefore, since the expected loss ratio and experience modification do not vary by state nor by hazard group, we would get the same answer using instead manual premium as the weights.

14.7. Using the NCCI's Table of Expected Loss Ranges, expected losses of $4000 corresponds to Expected Loss Group 91. Using the NCCI's Table of Insurance Charges, at an entry ratio of 0.30, the Insurance Charge is **0.9527** and the Insurance Savings is **0.2527**.
Comment: 0.9527 - 0.2527 = 1 - 0.3. Charge - Saving = 1 - Entry Ratio.

14.8. Use the notation ϕ^* for the insurance charges in the Table, even though unlike Table L these charges do not include the charge for the loss limit.

$$\phi^*(r) = \int_r^\infty (y-r)\ dF^*(y) = \int_r^\infty S^*(y)\ dy\ .$$

Therefore, $\frac{d\ \phi^*(r)}{dr} = -S^*(r) = F^*(r) - 1.$ $\qquad \frac{d^2\ \phi^*(r)}{dr^2} = f^*(r).$

The derivative at 2 is approximately: (0.1324 - 0.1620) / (2.1 - 1.9) = -0.148.
Thus the distribution at 2 is approximately: 1 - 0.148 = **0.852**.
The derivative at 1.9 is approximately: (0.1465 - 0.1790) / (2.0 - 1.8) = -0.1625.
The derivative at 2.1 is approximately: (0.1196 - 0.1465) / (2.2 - 2.0) = -0.1345.
Thus the second derivative at 2 is approximately: {-0.1345 - (-0.1625)} / (2.1 - 1.9) = 0.140.
Thus the density at 2 is approximately **0.140**.

14.9. (a) Expected Losses = (0.64)($1 million) = $640,000.
Excess Loss Factor = (0.64)(0.131) = 0.084.
Expected Limited Loss Ratio = 0.640 - 0.084 = 0.556.
The two balance equations:

$$X_H - X_G = \frac{E + e - H/T}{c\ \hat{E}} = \frac{0.640 + 0.188 - 0.50/1.06}{(1.110)\ (0.556)} = 0.5773.$$

$$r_G - r_H = \frac{G/T - H/T}{c\ \hat{E}} = \frac{1.40/1.06 - 0.50/1.06}{(1.110)\ (0.556)} = 1.38.$$

Now one has to solve iteratively the two balance equations.

Try $r_H = 0.30$. $\Rightarrow r_G = 0.30 + 1.38 = 1.68$. $\phi(0.30) = 0.7337$. $\phi(1.68) = 0.1527$.

0.7337 - 0.1527 = 0.5810. A little more than the desired 0.5773.
In general, as r_H increases, the charge differences get smaller.

Thus we need to choose a bigger r_H.

Try $r_H = 0.31$. $\Rightarrow r_G = 0.31 + 1.38 = 1.69$. $\phi(0.31) = 0.7260$. $\phi(1.69) = 0.1509$.

0.7260 - 0.1509 = 0.5751. As close as we can get to the desired 0.5773.
Aggregate Minimum Loss Factor = $S_H = X_H + r_H - 1 = 0.7260 + 0.31 - 1 = 0.0360$.

Net Aggregate Loss Factor = $(X_G - S_H)\hat{E}\ c = (0.1509 - 0.0360)(0.556)(1.110) = 0.071$.

The expense in the basic is: e - (c-1)E = 0.188 - (1.110 - 1)(0.640) = 0.118.
Basic Premium Factor = 0.071 + 0.118 = 0.189.
Basic Premium = (0.189)(1 million) = **$189,000**.
(b) (Loss Conversion Factor) (Standard Premium) (Excess Loss Factor) =
(1.110)($1,000,000)(0.084) = **$93,240**.
Comment: The NCCI's Table of Aggregate Loss Factors is not included in the 2024 Study Kit.
Based on the Table of Policy Excess Ratio Ranges, subtable 6 corresponds to the policy excess ratio of 0.131. Based on the Table of Expected Claim Count Groups, column 38 corresponds to the expected number of claims of 60.
The given extract of ALFs is from column 38 of subtable 6 in NCCI Circular CIF-2023-28, not on the syllabus.
The retro premium would be: (1.06) {(1.11)(Limited Losses) + 189,000 + 93,240},
subject to a maximum premium of 1.4 million and a minimum premium of 0.5 million.

14.10. Based on the Table of Expected Claim Count Groups, the expected number of claims is 45.8 to 51.6 or about 49. Thus the premium is about: (49)($20,000) / 0.65 = **$1.5 million**.

14.11. The mean is: (0)(7%) + (250,000)(25%) + ... + (4,750,000)(1%) = 1,000,000.
(a) An entry ratio of 1 corresponds to 1,000,000.
The expected aggregate excess of 1,000,000 is:
(250,000)(6%) + (500,000)(4%) + ... + (3,750,000)(1%) = 380,000.
$\phi(1)$ = 380,000 / 1,000,000 = **0.3800**.
Alternately, $\psi(1) = \frac{(1 \text{ million})(7\%) + (750K)(25\%) + (500K)(18\%) + (250K)(13\%)}{1 \text{ million}} = 0.3800.$
$\phi(1) = \psi(1) + 1 - 1$ = **0.3800**.
(b) An entry ratio of 2 corresponds to 2,000,000.
The expected aggregate excess of 2,000,000 is:
(250,000)(2%) + (500,000)(2%) + ... + (2,750,000)(1%) = 172,500.
$\phi(2)$ = 172,500 / 1,000,000 = **0.1725**.
(c) An entry ratio of 3 corresponds to 3,000,000.
The expected aggregate excess of 3,000,000 is:
(250,000)(1%) + (500,000)(1%) + ... + (1,750,000)(1%) = 70,000.
$\phi(3)$ = 70,000 / 1,000,000 = **0.0700**.
Comment: I made up the given aggregate distribution.
The Panjer algorithm is not on the syllabus of this exam. In practical applications, the Panjer algorithm would have as inputs a certain frequency distribution and a discrete severity distribution. The severity distribution would be censored from above by any loss limit.

14.12. Any three of the following five benefits from the end of Informational Exhibit 1 in NCCI Circular CIF-2023-28, not on the syllabus.
● By utilizing severity distributions that vary to reflect the exposure characteristics of each risk(state, hazard group, ALAE handling, loss limit), the AELF values more directly reflect the exposure of the underlying policy than if a countrywide severity distribution is used.
(This refers to NCCI's new computer product ALFs on Demand.)
● By directly calculating the limited aggregate loss distribution, the proposed methodology eliminates the need for the adjustment to account for overlap between the loss limit and aggregate loss limitation. The result is that AELFs calculated using ALFs on Demand are more accurate for policies with a loss limit than is produced under the current methodology.
● Because there is a vast array of possible exposure combinations (and resulting values) across all states, hazard groups and loss limits, it is not feasible for the AELFs to be published in a tabular form. Rather, the values for an individual policy will be available through an application, ALFs on Demand, that will be accessible on NCCI's website.
● The count and severity distributions underlying the proposed methodology are based on updated parameters that reflect more recent data than the distributions underlying the current Table of Insurance Charges, which have not changed since their creation in the late 1990s.
● Future annual updates to the Excess Loss Factor parameters will automatically be incorporated into the Aggregate Loss Factors for each state.
Comment: The NCCI did publish a countrywide Table of Aggregate Loss Factors.
Updating the body of the old Table M based on new data after 2 decades would have been very worthwhile, even if there had been no change in methodology.

14.13. $\phi(r) = \int_r^\infty (y-r)\ dF(y) = \int_r^\infty S(y)\ dy$.

Therefore, $\phi'(r) = -S(r) = F(r) - 1$. $\phi''(r) = f(r)$.

The derivative at 0.6 is approximately: (0.6022 - 0.7529) / (0.8 - 0.4) = -0.377.
Thus the distribution at 0.6 is approximately: 1 - 0.377 = **0.623**.
The derivative at 0.4 is approximately: (0.6700 - 0.8575) / (0.6 - 0.2) = -0.4688.
The derivative at 0.8 is approximately: (0.5458 - 0.6700) / (1.0 - 0.6) = -0.3105.
Thus the second derivative at 0.6 is approximately: {-0.3105 - (-0.4688)}/(0.8 - 0.4) = 0.396.
Thus the density at 0.6 is approximately **0.396**.

14.14. In order to determine which column of the Table of Aggregate Loss Factors, one consults a Table of Expected Claim Count Groups. While the expected number of claims should not be affected by inflation, the calculation of the expected number of claims has to take into account inflation. This calculation uses the modified expected losses and average costs per case.
The modified expected losses typically increase due to inflation (as the payrolls and thus the manual premiums increase due to inflation.) The NCCI has to regularly update its average costs per case; the updated data used in this calculation should reflect the impact of inflation.
Comment: The 2019 update introduced a new methodology of determining the insurance charges in the body of the Table. It is assumed that over a decade or two the body of the Table of Aggregate Loss Factors will remain valid. However, every decade or two, the body of the Table of Aggregate Loss Factors should be updated, even if there is no change in methodology.

14.15. Manual premium = (Rate) (Payroll / 100).

Class	Payroll	Rate	Manual Premium	Excess Ratio	Manual Premium times Excess Ratio
1	1,400,000	5.85	$81,900	0.026	$2,129
2	600,000	4.65	$27,900	0.017	$474
3	1,000,000	2.75	$27,500	0.012	$330
4	200,000	1.50	$3,000	0.008	$24
Total			$140,300		$2,958

The policy excess ratio is: 2958/140,300 = **0.021**.
Comment: We are taking a weighted average of excess ratios. Usually modified expected losses would be used as the weights. Since the expected loss ratio and experience modification do not vary by hazard group, we get the same answer using instead manual premium as the weights.

14.16. The balance equations are:
$r_G - r_H = (G - H) / (c\ E\ T) = (160\% - 50\%) / \{(1.175)(63\%)(1.061)\} = 1.40$.
$X_H - X_G = (e + E - H/T) / (c\ E) = (0.135 + 0.63 - 0.50/1.061) / \{(1.175)(63\%)\} = 0.3968$.
Trying values:
$\phi(0.6) - \phi(2.0) = 0.5197 - 0.1230 = 0.3967$. OK.
$\psi(0.6) = 0.5197 - 1 + 0.6 = 0.1197$.
Net Converted Insurance Charge: (1.175)(63.0%)(0.1230 - 0.1197)(1.4 million) = $3420.
Basic Premium is: 3420 + (13.5%)(1.4 million) - (0.175)(63.0%)(1.4 million) = **$38,070**.
Comment: The NCCI's Table of Aggregate Loss Factors is not included in the 2024 Study Kit.
The given extract of ALFs is from the Sample Values of Aggregate Excess Loss Factors for the state of Alaska with only exposure in Hazard Group F in Informational Exhibit 2 of NCCI Circular CIF-2023-28, not on the syllabus.
The basic premium is a small percent of the standard premium because:
the savings from the minimum is almost as large as to the charge for the maximum, and
the loss conversion factor and the tax multiplier are each somewhat large.
(Some of the underwriting expenses and/or commissions may have been included in the loss conversion factor and/or the tax multiplier.)

14.17. $\phi(0.50) - \phi(2.00) = 0.6100 - 0.1344 = 0.4758$.
Thus the balance equation are solved for $r_H = 0.50$ and $r_G = 2.00$.
$\psi(0.50) = \phi(0.50) + 0.50 - 1 = 0.6100 - 0.50 = 0.1100$.
Net Aggregate Loss Factor = $c\ \hat{E}\ \{\phi(2.00) - \psi(0.50)\}$ = (1.1) (64%)(1 - 12%) (0.1344 - 0.1100)
= **0.015**.
Comment: The NCCI's Table of Aggregate Loss Factors is not included in the 2024 Study Kit.
The given extract of ALFs is from NCCI Circular CIF-2023-28, not on the syllabus.

14.18. (a) For example, (200,000)(1 - 0.597) = $80,600.

Claim Group	Average Unlimited Severity	Excess Ratio for $100K per claim limit	Average Limited Severity	Expected Portion of Claims	Product
Fatal	$200,000	0.597	$80,600	0.05%	$40
PT	$1,500,000	0.921	$118,500	0.15%	$178
Likely PP/TT	$150,000	0.564	$65,400	5.00%	$3,270
Not Likely PP/TT	$30,000	0.291	$21,270	25.00%	$5,318
Med Only	$1,000	0.044	$956	69.80%	$667
Total				100.00%	$9,473

The average limited severity per claim is **$9473**.
(b) The average unlimited severity per claim is $18,048.

Claim Group	Average Unlimited Severity	Expected Portion of Claims	Product
Fatal	$200,000	0.05%	$100
PT	$1,500,000	0.15%	$2,250
Likely PP/TT	$150,000	5.00%	$7,500
Not Likely PP/TT	$30,000	25.00%	$7,500
Med Only	$1,000	69.80%	$698
Total		100.00%	$18,048

The excess ratio is: 1 - 9473/18,048 = **47.5%**.
Comment: This type of calculation is not shown in the 2024 Study Kit.

14.19. i. (0.256)(4,000,000) = **$1,024,000**.
ii. (0.197)(4,000,000) = **$788,000**.
Comment: Using the 2024 Study Kit.

14.20. The balance equations are:

$$r_G - r_H = \frac{G - H}{c\,E\,T} = \frac{240\% - 40\%}{(1.120)\,(66\%)\,(1.041)} = 2.60.$$

$$X_H - X_G = \frac{e + E - H/T}{c\,E} = \frac{0.148 + 0.660 - 0.40 / 1.041}{(1.120)\,(66\%)} = 0.5733.$$

Trying values: $\phi(0.4) - \phi(3.0) = 0.6755 - 0.1016 = 0.5739$. As close as one is going to get.
$\psi(0.4) = 0.6755 - 1 + 0.4 = 0.0755$.
Net Converted Insurance Charge: (1.120)(66.0%)(0.1016 - 0.0755)(750,000) = $14,470.
Basic Premium is: 14,470 + (14.8%)(750,000) - (0.120)(66.0%)(750,000) = **$66,070**.
Comment: The NCCI's Table of Aggregate Loss Factors is not included in the 2024 Study Kit. The given extract of ALFs is from the Sample Values of Aggregate Excess Loss Factors for the state of Alaska with only exposure in Hazard Group B in Informational Exhibit 2 of NCCI Circular CIF-2023-28, not on the syllabus.

14.21. An entry ratio of 1.6 corresponds to aggregate losses of: (1.6)($50,000) = $80,000.
The probability above $80,000 is:
1 - (2% + 6% + 11% + 14% + 18% + 15% + 11% + 8% + 5%) = 10%.

$$\phi(1.6) = 1 - \frac{(0)(2\%)+(10)(6\%)+(20)(11\%)+(30)(14\%)+(40)(18\%)+(50)(15\%)+(60)(11\%)+(70)(8\%)+(80)(15\%)}{50}$$

= 1 - 45.9/50 = **0.082**.
Comment: What was done was analogous to calculating the loss elimination ratio and subtracting it from one to get the excess ratio.
See formula 22 in NCCI's Informational Exhibit 1 of NCCI Circular CIF-2023-28, not on the syllabus.
$\psi(1.6) = \phi(1.6) + 1.6 - 1 = 0.682$. Alternately,

$$\psi(1.6) = \frac{(80)(2\%)+(70)(6\%)+(60)(11\%)+(50)(14\%)+(40)(18\%)+(30)(15\%)+(20)(11\%)+(10)(8\%)}{50}$$

= 34.1/50 = 0.682.

14.22. (a) Expected Losses = (0.62)($2 million) = $1,240,000.
Expected Excess Losses = (0.116)($1,240,000) = $143,840.
Expected Limited Losses = $1,240,000 - $143,840 = $1,096,160.
Expected Limited Loss Ratio = $1,096,160 / $2,000,000 = 0.548.
The two balance equations:

$$X_H - X_G = \frac{E + e - H/T}{c \hat{E}} = \frac{0.620 + 0.179 - 0.40/1.052}{(1.113)(0.548)} = 0.6866.$$

$$r_G - r_H = \frac{G/T - H/T}{c \hat{E}} = \frac{1.60/1.052 - 0.40/1.052}{(1.113)(0.548)} = 1.87.$$

Now one has to solve iteratively these two balance equations.
Try $r_H = 0.25$. $\Rightarrow r_G = 0.25 + 1.87 = 2.12$. $\phi(0.25) = 0.7633$. $\phi(2.12) = 0.0526$.

0.7633 - 0.0526 = 0.7105. Too large.
In general, as r_H increases, the charge differences get smaller.
Thus we need to choose a bigger r_H.
Try $r_H = 0.28$. $\Rightarrow r_G = 0.28 + 1.87 = 2.15$. $\phi(0.28) = 0.7373$. $\phi(2.15) = 0.0501$.

0.7373 - 0.0501 = 0.6872. As close as we can get to the desired 0.6866
Aggregate Minimum Loss Factor = $S_H = X_H + r_H - 1 = 0.7373 + 0.28 - 1 = 0.0173$.
Net Aggregate Loss Factor = $(X_G - S_H)\hat{E}\, c = (0.0501 - 0.0173)(0.548)(1.113) = 0.020$.
The expense in the basic is: e - (c-1)E = 0.179 - (1.113 - 1)(0.620) = 0.109.
Basic Premium Factor = 0.020 + 0.109 = 0.129.
Basic Premium = (0.129)(2 million) = **$258,000**.
(b) Excess Loss Factor = (0.116)(0.620) = 0.072.
(Loss Conversion Factor) (Standard Premium) (Excess Loss Factor) =
(1.113)($2,000,000)(0.072) = **$160,272**.
Comment: The NCCI's Table of Aggregate Loss Factors is not included in the 2024 Study Kit. Based on the Table of Policy Excess Ratio Ranges, Subtable 6 corresponds to the policy excess ratio of 0.116. Based on the Table of Expected Claim Count Groups, column 33 corresponds to the expected number of claims of 121. The given extract of AELFs is from column 33 of Subtable 6 in NCCI Circular CIF-2023-28, not on the syllabus.
The retro premium would be: (1.052) {(1.113)(Limited Losses) + 258,000 + 160,272},
subject to a maximum premium of 3.2 million and a minimum premium of 0.8 million.

14.23. (a) Modified Expected Loss is the manual premium times both the expected loss ratio of 66% and the experience modification of 1.10. For example: (66%)(1.1)($150,000) = $108,900.

State	HG	Manual Premium	Modified Expected Loss	Excess Ratio	Expected Excess Loss
1	B	$150,000	$108,900	0.131	$14,266
1	E	$500,000	$363,000	0.182	$66,066
2	B	$200,000	$145,200	0.145	$21,054
2	E	$900,000	$653,400	0.204	$133,294
Total			$1,270,500		$234,680

For example: ($108,900)(0.131) = $14,266.
The policy excess ratio is: 234,680/1,270,500 = **0.185**.
(b) In each case, we divide the modified expected loss by the average cost per case.

State	HG	Manual Premium	Modified Expected Loss	Average Cost per Case	Expected Number of Claims
1	B	$150,000	$108,900	$12,000	9.08
1	E	$500,000	$363,000	$19,000	19.11
2	B	$200,000	$145,200	$15,000	9.68
2	E	$900,000	$653,400	$21,000	31.11
Total			$1,270,500		**68.97**

(c) The policy excess ratio of 0.185 corresponds to Sub-Table **8**.
Expected number of claims of 68.97 corresponds to Expected Claim Count Group **37**.
Comment: Similar to two examples shown in the NCCI Retro Manual.
In each case, the excess ratio is the expected percent of losses excess of $500,000, which differs by state and hazard group. The excess ratios and average costs per case would be looked up in an NCCI publication that is not on the syllabus.
In part (a), we are taking a weighted average of excess ratios. Therefore, since the expected loss ratio and experience modification do not vary by state nor by hazard group, we would get the same answer using instead manual premium as the weights.

14.24. An entry ratio of 1.2 corresponds to aggregate losses of: (1.2)($100,000) = $120,000.

$$\psi(1.2) = \frac{(120)(3\%)+(100)(7\%)+(80)(12\%)+(60)(16\%)+(40)(19\%)+(20)(15\%)}{100} = 40.4/100 = 0.404.$$

$\phi(1.2) = \psi(1.2) + 1 - 1.2 = 0.404 - 0.2 =$ **0.204**.

Alternately, the probability above $120,000 is:

1 - (3% + 7% + 12% + 16% + 19% + 15% + 10%) = 18%.

$$\phi(1.2) = 1 - \frac{(0)(3\%)+(20)(7\%)+(40)(12\%)+(60)(16\%)+(80)(19\%)+(100)(15\%)+(120)(10\% + 18\%)}{100}$$

= 1 - 79.6/100 = **0.204**.

Comment: In the second solution, what was done was analogous to calculating the loss elimination ratio and subtracting it from one to get the excess ratio.

The Panjer algorithm is not on the syllabus of this exam. In practical applications, the Panjer algorithm would have as inputs a certain frequency distribution and a discrete severity distribution. The severity distribution would be censored from above by any loss limit.

14.25. All three statements are true.

Comment: See Page 4 of Rule 1 and page 1 of Rule 2 in the NCCI Retro Plan.

14.26. Only #1 and #3 are true. Premium taxes are provided for via the tax multiplier.

14.27. Only #1 and #3 are true

The basic premium does not includes a provision for subsidy of the assigned risk market.

14.28. Statement #1 is true, see Page 2 of Rule 1.

Statement #2 is false, see Page 2 of Rule 1.

Statement #3 is false, see Page 2 of Rule 1.

14.29. a) All factors may be changed.
b) Maximum premium factor, minimum premium factor, loss conversion factor, loss limit, may each be changed.
The basic premium factor will change if any of these other factors are changed.
c) Together, the loss conversion factor and the expense portion of the basic premium should pay for expenses including expected LAE. When c is larger, more of the expenses (including expected LAE) are being recouped via the Loss Conversion Factor. Therefore, when c is larger, the expense portion of the basic premium factor should be smaller.
Therefore, when c is larger, the basic premium factor will decrease.
d) There is a charge for the loss limit via the excess loss premium. The insurance charge would overlap with the excess loss premium if one did not use the ICRLL procedure or the NCCI Table of Aggregate Loss Factors. The ICRLL procedure results in entering Table M at an increased size of expected losses. Thus the ICRLL procedure results in a smaller insurance charge and thus a smaller basic premium.
If instead there is no loss limit, then the ICRLL procedure is not used. (Alternately, with no loss limit the ICRLL procedure would result in no adjustment.)
Thus elimination of the loss limit will increase the insurance charge and thus the basic premium, compared to what it was with a loss limit.

	Loss Limit	No Loss Limit
Insurance Charge	Smaller	Bigger
ELF	Positive	0
Total of Basic Plus Excess Loss Premium	Bigger	Smaller

The same relationships hold for the use of the NCCI Table of Aggregate Loss Factors.
Comment: See Rule 1.II.K.
The question makes a “humorous” reference to the Superman comics, films, and television shows.
In part c, $b = e - (c - 1) E + c I = e - E - c(E - I)$.
E could be for example about 65%, while I could be for example about 5%.
Thus $E - I < 0$, and as c increases b decreases.
Note also that when c changes the balance equations change, and therefore the net insurance charge I also changes slightly.

14.30. #1 is true. See Page 4 of Rule 1.
The excess loss premium is computed as the standard premium multiplied by the excess loss factor and the loss conversion factor. #2 is false. See Page 2 of Rule 1.
#3 is true. See Page 2 of Rule 1.

Seminar Slides for Exam 8
Revision of NCCI Retro Plan Section
For the CAS 2024 Study Kit

Section 14

NCCI Retrospective Rating

The NCCI made major changes to its Retrospective Rating Plan effective January 2019:

- Changes in the methodology NCCI uses to determine tabulated Insurance Charges.
- Several different (sub)Tables of Insurance Charges; which one you should use depends on the policy excess ratio.
- One now determines which column of a table to use based on the policy expected claim count.
- Changes in terminology.
- A new computer based product, Aggregate Loss Factors on Demand (ALFs on Demand), that insurers can use rather than consulting published Tables of Insurance Charges.

An extract from the Retrospective Rating Plan Manual will be attached to your exam.

The first part contains rules for the NCCI Retro Plan.

Note the useful Table of Contents in the front of the manual.

Appendix D contains a very useful Basic Premium Factor Calculation Example.

Subsequently, I will discuss this example in detail. You should study it carefully.

Unfortunately, other than rules the rest of the 2024 study kit contains a mixture of elements that apply to the old and new NCCI Retro Plans:

- Table of Expected Loss Ranges,
  used in the old plan.
- Table of Policy Excess Ratios Ranges,
  used in the new plan.
- Table of Expected Claim Count Groups,
  used in the new plan.
- Table of Insurance Charges,
  used in the old plan.
- Table of Expense Ratios, used in both plans.
- Basic Premium Factor Calculation Example,
  for the new plan.

Despite what the CAS Content Outline says, the 2024 study kit does not contain Tables of Aggregate Loss Factors.

Taking into Account Loss Limits in Retro Rating:

In retrospective rating, when there is an loss limit, there is an overlap between the premium charge for a loss limit per loss and the insurance charge for the maximum premium.

In other words, if one calculate the effects of the loss limit and the maximum premium separately, then the average premium for a retrospectively rated policy with an loss limit would be too high.

NCCI used to deal with this issue via the ICRLL procedure, in which in the presence of a loss limit one shifts which column one uses in Table M. Instead, the new NCCI methodology is based on computing an aggregate distribution that reflects the effect of any loss limit.

The Interaction of Maximums and Loss Limits:

Let us assume a retrospectively rated insured had a basic premium of $30,000, an excess loss premium of $10,000, a loss conversion factor of 1.1, a tax multiplier of 1.05, an loss limit of $100,000, and a maximum premium of $250,000.

Exercise: If the insured has one large loss of $150,000 in year plus $100,000 in small losses, what is the retro premium?

{40,000 + (1.1)(200,000)}(1.05) = 273,000.
Limited to the maximum of $250,000.
Comment: The loss limit provided no benefit in this case.

This is an example of the “overlap” between the effects of the maximum premium and the loss limit.

In some years, even though there are large losses, the loss limit will not provide any additional benefit to the insured beyond that provided by the maximum premium.

Therefore, calculating independently additional amounts to charge an insured for the maximum premium and for the loss limit would overcharge the insured.

Retrospective Premium:

R = (b + cL)T,
subject to a minimum of H and a maximum of G.
H ≤ R ≤ G.

R = Retrospective Premium

b = basic premium
= (basic premium factor) (standard premium).

c = the loss conversion factor

L = reported losses
subject to any applicable limitation

T = Tax Multiplier

H = minimum premium
= (minimum premium factor) (standard premium).

G = maximum premium
= (maximum premium factor) (standard premium).

Basic Premium:

e = expenses plus profit but excluding taxes
(includes loss adjustment expense).

c = Loss Conversion Factor
(to include loss adjustment expense).

E = expected unlimited loss ratio.

Net Aggregate Loss Factor =
cE(Aggregate Excess Loss Factor @ Max.
- Aggregate Minimum Loss Factor @Min.).

e - (c-1)E is called the expenses in the basic premium. It does not cover the LAE included in c, nor does it cover taxes and assessments included in the tax multiplier T.

basic premium factor = b
= e - (c-1)E + Net Aggregate Loss Factor.

The basic premium =
(basic premium factor) (standard premium).

Optional Features of the NCCI Retro. Rating Plan:

The insured and the insurer can agree to have a loss limit.

The insured and insurer can agree to use retrospective development factors.

R =
(b + cL + Excess Loss Prem.
+ Retro. Develop. Prem.) T,
subject to a minimum of H and a maximum of G.

Excess Loss Premium =
c (Standard Premium) (Excess Loss Factor).

Allocated Loss Adjustment Expense Option:

Unless stated otherwise, a retro plan applies to only losses, in which case the provision for ALAE is included in the loss conversion factor, c.

However, if agreed upon by the insured and insurer, ALAE may be included with losses for purposes of the retro plan.

For the ALAE Option, E would be replaced by the expected loss and ALAE ratio, a different set of expense ratios is used, and the loss conversion factor would be smaller.

Large Risk Alternative Rating Option:

Provides that a large risk may be retrospectively rated as mutually agreed upon by carrier and insured.

All factors may be changed.

14.29. (9, 11/98, Q.46) (3 points) As the actuary for Kryptonite Insurance Company, you are working with an insured, Kent & Lane Industries, to develop a retrospectively rated plan. You have developed the following initial proposal using the National Council on Compensation Insurance's Retrospective Rating Plan Manual for Workers Compensation and Employers Liability Insurance.

Maximum Premium Factor	1.50
Minimum Premium Factor	0.50
Tax Multiplier	1.03
Basic Premium Factor	0.40
Loss Conversion Factor	1.09
Loss Limit	$250,000

Kent & Lane would like to change some of the parameters of your proposal.

a. (0.5 point) List which of the items above that the NCCI plan allows you to change if the risk qualifies for the "Large Risk Alternative Rating Option."

b. (0.5 point) List which of the items above that the NCCI plan allows you to change if the risk does not qualify for the "Large Risk Alternative Rating Option."

c. (1 point) Assuming that the risk does not qualify for the "Large Risk Alternative Rating Option," what will happen to the basic premium factor (will it increase, decrease, or stay the same) if the loss conversion factor is increased? Briefly explain your answer.

d. (1 point) Assuming that the risk does not qualify for the "Large Risk Alternative Rating Option," what will happen to the basic premium factor (will it increase, decrease, or stay the same) if the loss limit is eliminated? Briefly explain your answer.

9, 11/98, Q.46

a. (0.5 point) List which of the items above that the NCCI plan allows you to change if the risk qualifies for the "Large Risk Alternative Rating Option."

a) All factors may be changed.

b. (0.5 point) List which of the items above that the NCCI plan allows you to change if the risk does not qualify for the "Large Risk Alternative Rating Option."

b) Maximum premium factor, minimum premium factor, loss conversion factor, loss limit, may each be changed.
The basic premium factor will change if any of these other factors are changed.

c. (1 point) Assuming that the risk does not qualify for the "Large Risk Alternative Rating Option," what will happen to the basic premium factor (will it increase, decrease, or stay the same) if the loss conversion factor is increased?
Briefly explain your answer.

c) Together, the loss conversion factor and the expense portion of the basic premium should pay for expenses including expected LAE.
When c is larger, more of the expenses (including expected LAE) are being recouped via the Loss Conversion Factor.
Therefore, when c is larger, the expense portion of the basic premium factor should be smaller.
Therefore, when c is larger, the basic premium factor will decrease.

$b = e - (c - 1) E + c I = e - E - c(E - I)$.
E could be for example about 65%, while I could be for example about 5%. Thus $E - I < 0$, and as c increases b decreases.
Note also that when c changes the balance equations change, and therefore the net insurance charge I also changes slightly.

d. (1 point) Assuming that the risk does not qualify for the "Large Risk Alternative Rating Option," what will happen to the basic premium factor (will it increase, decrease, or stay the same) if the loss limit is eliminated? Briefly explain your answer.

d) There is a charge for the loss limit via the excess loss premium.

The insurance charge would overlap with the excess loss premium if one did not use the ICRLL procedure.

The ICRLL procedure results in entering Table M at an increased size of expected losses.

Thus the ICRLL procedure results in a smaller insurance charge and thus a smaller basic premium.

If instead there is no loss limit, then the ICRLL procedure is not used.

Thus elimination of the loss limit will increase the insurance charge and thus the basic premium, compared to what it was with a loss limit.

	Loss Limit	No Loss Limit
Insurance Charge	Smaller	Bigger
ELF	Positive	0
Total of Basic Plus Excess Loss Premium	Bigger	Smaller

Page 6 Excess Loss Factors:

Excess Loss Factor =
(Excess Ratio) (Expected Loss Ratio).

Exercise: For a policy, the excess ratio is 20% and the expected loss ratio is 65%.
Determine the Excess Loss Factor.
[Solution: (20%)(65%) = 13%.]

The Excess Loss Factor times the Standard Premium gives the expected excess losses.
If the standard premium were $1 million, the expected excess losses would be $130,000.

Excess Loss Premium =
c (Standard Premium) (Excess Loss Factor)

If the loss conversion factor were 1.1,
then the excess loss premium would be:
(1.1) (130,000) = $143,000.

Entry Ratios:

Entry Ratios have expected limited losses in their numerator and their denominator.

Entry Ratio corresponding to the Maximum Premium is:

$$\frac{\text{Limited Losses Corresponding to the Max. Premium}}{\text{Expected Limited Losses}} = \frac{\hat{L}_G}{\hat{E}}.$$

Entry Ratio corresponding to the Minimum Premium is:

$$\frac{\text{Limited Losses Corresponding to the Min. Premium}}{\text{Expected Limited Losses}} = \frac{\hat{L}_H}{\hat{E}}.$$

Terminology:

NCCI has changed some of their terminology.

I would be prepared for your exam to use either the older or newer terminology in a question.

Insurance Charge ⇔

Aggregate Excess Loss Factor (AELF)

Insurance Savings ⇔

Aggregate Minimum Loss Factor

Aggregate Excess Loss Factor
- Aggregate Minimum Loss Factor
= 1 - Entry Ratio.

Net Insurance Charge
⇔ Net Aggregate Loss Factor

Table M ⇔ Table of Insurance Charges

⇔ Table of Aggregate Loss Factors

NCCI's Example of Retrospective Rating:

The following is assumed:

- Estimated Standard Premium = $500,000.
- Maximum Retro Premium Factor = 130%
- Minimum Retro Premium Factor = 60%
- Loss Conversion Factor = 1.120
- Tax Multiplier = 1.070
- Loss Limit = $50,000
- Expenses (not covered in Loss Conversion Factor or Tax Multiplier) = 0.201

The Expected Loss Ratio is assumed to be 0.613.
Expected Losses = (0.613) ($500,000) = $306,500.

The Policy Excess Ratio is assumed to be 0.582.
Excess Loss Factor = (0.613) (0.582) = 0.357.
Expected Limited Loss Ratio
= 0.613 - 0.357 = 0.256.

Expected Number of Claims is 20.95.

Expense & Profit & Contingen. (Excluding Taxes)
= (0.201) ($500,000) = $100,500.

Expected Loss Plus Expense Ratio
= ($306,500 + $100,500) / $500,000 = 0.814.

Loss and Expense in Converted Losses
= (1.120) (0.613) = 0.687.

Expense and Profit and Contingencies
(Excluding Expense in Converted Losses)
= 0.814 - 0.687 = 0.127.

Minimum Retrospective Premium Excluding Taxes
= 60% / 1.070 = 0.561.

Maximum Retrospective Premium Excluding Taxes
= 130% / 1.070 = 1.215.

Table of Aggregate Loss Factors Value Difference

$$= \frac{0.814 - 0.561}{(1.120)\ (0.256)} = 0.8824.$$

This is one of the two balance equations with a loss limit: $X_H - X_G = \frac{E + e - H/T}{c\ \hat{E}}$.

Table of Aggregate Loss Factors Entry Difference

$$= \frac{1.215 - 0.561}{(1.120)\ (0.256)} = 2.28.$$

This is the other balance equation with a loss limit:

$$r_G - r_H = \frac{G/T - H/T}{c\ \hat{E}}.$$

Now one has to solve iteratively
the two balance equations.

One would look in the subtable of the
Table of Aggregate Loss Factors that
corresponds to the policy excess ratio of 0.582.

Based on the Table of Policy Excess Ratio Ranges
this would be Subtable 15.

We would use the column based on
the expected number of claims of 20.95.

Based on the
Table of Expected Claim Count Groups
this would be column 48.

We are provided with an extract of column 48 of Subtable 15:

Entry Ratio	Aggregate Excess Loss Factor		Entry Ratio	Aggregate Excess Loss Factor
0.04	0.9619		2.32	0.0736
0.05	0.9528		2.33	0.0727
0.06	0.9437		2.34	0.0718

For $r_H = 0.05$ and $r_G = 0.05 + 2.28 = 2.33$:

$X_H - X_G = 0.9528 - 0.0723 = 0.8801$.

This as close as we can get to the desired value difference of 0.8824. ⇒

Ratio of Losses for Minimum Retrospective Premium to Expected Limited Losses = 0.05 = r_H.

Ratio of Losses for Maximum Retrospective Premium to Expected Limited Losses = 2.33 = r_G.

Aggregate Excess Loss Factor (for Maximum)
= 0.0727 = X_G.

Aggregate Minimum Loss Factor
= 0.9528 + 0.05 - 1 = 0.0028 = $S_H = X_H + r_H - 1$.

Net Aggregate Loss Factor
= (0.0727 - 0.0028) (0.256) (1.120) = 0.020
= (X_G - S_H) Ê c.

Basic Premium Factor = 0.020 + 0.127 = **0.147**.

Thus, in dollars terms, the basic premium is:
(0.147) ($500,000) = $73,500.

14.22. (3 points) An insured is to be written under the NCCI Retrospective Rating Plan.

- Estimated Standard Premium = $2,000,000
- Maximum Retro. Premium Factor = 160%
- Minimum Retrospective Premium Factor = 40%
- Loss Conversion Factor = 1.113
- Tax Multiplier = 1.052
- Loss Limit = $1,000,000
- Expenses (not covered in Tax Multiplier) = 0.179
- Expected Loss Ratio = 0.620
- Policy Excess Ratio = 0.116
- Expected Number of Claims = 121.03

Use the following extract of NCCI's Table of Aggregate Loss Factors.

r	ALF		r	ALF
0.25	0.7633		2.10	0.0543
0.26	0.7545		2.11	0.0535
0.27	0.7459		2.12	0.0526
0.28	0.7373		2.13	0.0518
0.29	0.7287		2.14	0.0510
0.30	0.7202		2.15	0.0501
0.31	0.7118		2.16	0.0493
0.32	0.7035		2.17	0.0485
0.33	0.6952		2.18	0.0478
0.34	0.6870		2.19	0.0470
0.35	0.6789		2.20	0.0462

(a) (2.5 points) Determine the Basic Premium.

(b) (0.5 point) Determine the Excess Loss Premium.

14.22. (a) Expected Losses
= (0.62) ($2 million) = $1,240,000.
Expected Excess Losses
= (0.116) ($1,240,000) = $143,840.
Expected Limited Losses
= $1,240,000 - $143,840 = $1,096,160.
Expected Limited Loss Ratio
= $1,096,160 / $2,000,000 = 0.548.

The two balance equations:

$$X_H - X_G = \frac{E + e - H/T}{c \hat{E}} =$$

$$\frac{0.620 + 0.177 - 0.40/1.052}{(1.113)(0.548)} = 0.6833.$$

$$r_G - r_H = \frac{G/T - H/T}{c \hat{E}} =$$

$$\frac{1.60/1.052 - 0.40/1.052}{(1.113)(0.548)} = 1.87.$$

Now one has to solve iteratively
these two balance equations.

Try $r_H = 0.25. \Rightarrow r_G = 0.25 + 1.87 = 2.12.$

$\phi(0.25) = 0.7633.$ $\phi(2.12) = 0.0526.$
$0.7633 - 0.0526 = 0.7105.$ Too large.

In general, as r_H increases,
the charge differences get smaller.
Thus we need to choose a bigger r_H.

Try $r_H = 0.28. \Rightarrow r_G = 0.28 + 1.87 = 2.15.$

$\phi(0.28) = 0.7373.$ $\phi(2.15) = 0.0501.$
$0.7373 - 0.0501 = 0.6872.$
As close as we can get to the desired 0.6833.

Aggregate Minimum Loss Factor = S_H
= $X_H + r_H - 1 = 0.7373 + 0.28 - 1 = 0.0173$.

Net Aggregate Loss Factor = $(X_G - S_H)$ Ê c
= $(0.0501 - 0.0173)(0.548)(1.113) = 0.020$.

The expense in the basic is: e - (c - 1) E
= $0.179 - (1.113 - 1)(0.620) = 0.109$.

Basic Premium Factor = 0.020 + 0.109 = 0.129.

Basic Premium = (0.129) (2 million) = **$258,000**.

(b) Excess Loss Factor = (0.116) (0.620) = 0.072.

c (Standard Premium) (Excess Loss Factor)
= (1.113) ($2,000,000) (0.072) = **$160,272**.

Comment: The retro premium would be:
(1.052) {(1.113) (Limited Losses) + 258,000 + 160,272},
subject to a maximum premium of 3.2 million
and a minimum premium of 0.8 million.

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NCCI's Example of Calculation of Expected Number of Claims and Policy Excess Ratio:

A workers compensation insured has exposures in two states.

The insured is buying a retrospective rating policy with a loss limit of $50,000.

In State X, there are exposures in two classes; these classes are in Hazard groups C and G.

In State Y, there are exposures in one class, which is in Hazard groups A.

State	Hazard Group of Class	Manual Premium	Excess Ratio at $50,000	Average Cost per Case
X	C	$217,170	0.5	$12,000
X	G	$305,873	0.7	$23,000
Y	A	$101,958	0.4	$9,000

The excess ratios depend on the loss limit, as well as the state and hazard group.
These excess ratios would be looked up in an NCCI publication, not on the syllabus.

The average costs per case depend on the state and hazard group.
These average costs would be looked up in an NCCI publication, not on the syllabus.

The insured has an expected (unlimited) loss ratio of 61.3%.

The insured has an experience modification of 0.80.

Modified Expected Loss is the manual premium times both the expected loss ratio of 61.3% and the experience modification of 0.80. For example: (61.3%) (0.80) ($217,170) = $106,500.

In each case, we divide the modified expected loss by the average cost per case.

State	HG	Manual Premium	Modified Expected Loss	Average Cost per Case	Expected Number of Claims
X	C	$217,170	$106,500	$12,000	8.88
X	G	$305,873	$150,000	$23,000	6.52
Y	A	$101,958	$50,000	$9,000	5.56
Total			$306,500		**20.95**

The 20.95 is the expected number of claims used in the previous retro rating example.

Using the modified expected losses,
we take a weighted average of the excess ratios:

State	HG	Manual Premium	Modified Expected Loss	Excess Ratio	Expected Excess Loss
X	C	$217,170	$106,500	0.5	$53,250
X	G	$305,873	$150,000	0.7	$105,000
Y	A	$101,958	$50,000	0.4	$20,000
Total			$306,500		$178,250

For example: ($106,500) (0.5) = $53,250.

The policy excess ratio is:

178,250 / 306,500 = **0.582**.

This is the policy excess ratio used in
the previous retro rating example.

New versus Prior NCCI Retro Plan:

Prior NCCI Retro Plan	New NCCI Retro Plan
Loss Limits taken into account via ICRRL Procedure	Loss Limits taken into account via different subtables of Aggregate Loss Factors based on the policy excess ratio
State/Hazard Group Differentials used to help get LUGS	Average Cost per Case by State/Hazard Group used in the calculation of policy expected number of claims
Column of Table of Insurance Charges is determined via Losses Used for Group Selection (LUGS) to enter Table of Expected Loss Groups	Column of Table of Aggregate Loss Factors is determined via policy expected number of claims to enter Table of Expected Claim Count Groups
Table M insurance charges based on smoothing empirical results	Table of Aggregate Loss Factors are based on a use of the Panjer Algorithm to determine the aggregate distribution underlying each column in each subtable.
	New Computer Product available: ALFs on Demand. Based on the same mathematics used to determine the published Table of Aggregate Loss Factors.

Table of Expected Claims Count Groups:

One determines which column of the Table of Aggregate Loss Factors to use based on the expected number of claims. Here is a potion of the Table of Expected Claim Count Groups:

Expected Claim Count Group	Expected Number of Claims (Rounded Values)
40	45.8 – 51.6
39	51.7 – 58.4
38	58.5 – 66.3
37	66.4 – 75.5
36	75.6 – 86.4
35	86.5 – 99.2

**The larger the insured,
the smaller the Expected Claim Count Group**.

Insured with higher expected claim counts have a lower coefficient of variation of aggregate losses, and thus have smaller insurance changes at high entry ratios than smaller insureds.

In the past, which column to use was based on Expected Losses.

Expected Claim Counts have the advantage of not being affected by inflation.

Expected Claim Counts are a better measure of how the size of an insured affects the shape of its distribution of aggregate losses.

14.23. (3.25 points) A workers compensation insured has exposures in two classes in each of two states. The insured is buying a retrospective rating policy with an loss limit of $500,000.
The insured has an expected (unlimited) loss ratio of 66%.
Insured has an experience modification of 1.10.

State	Hazard Group of Class	Manual Premium	Excess Ratio at $500,000	Average Cost per Case
1	B	$150,000	0.131	$12,000
1	E	$500,000	0.182	$19,000
2	B	$200,000	0.145	$15,000
2	E	$900,000	0.204	$21,000

(a) Determine the excess ratio for this policy.
(b) Determine the expected number of claims for this policy.
(c) Determine the Sub-Table and the Expected Claim Count Group.

14.23. (a) Modified Expected Loss is the manual premium times both the expected loss ratio of 66% and the experience modification of 1.10.
For example: (66%) (1.1) ($150,000) = $108,900.

State	HG	Manual Premium	Modified Expected Loss	Excess Ratio	Expected Excess Loss
1	B	$150,000	$108,900	0.131	$14,266
1	E	$500,000	$363,000	0.182	$66,066
2	B	$200,000	$145,200	0.145	$21,054
2	E	$900,000	$653,400	0.204	$133,294
Total			$1,270,500		$234,680

For example: ($108,900) (0.131) = $14,266.

The policy excess ratio is:
234,680 / 1,270,500 = **0.185**.

(b) In each case, we divide the modified expected loss by the average cost per case.

State	HG	Manual Premium	Modified Expected Loss	Average Cost per Case	Expected Number of Claims
1	B	$150,000	$108,900	$12,000	9.08
1	E	$500,000	$363,000	$19,000	19.11
2	B	$200,000	$145,200	$15,000	9.68
2	E	$900,000	$653,400	$21,000	31.11
Total			$1,270,500		**68.97**

(c) The policy excess ratio of 0.185 corresponds to Sub-Table **8**.
Expected number of claims of 68.97 corresponds to Expected Claim Count Group **37**.

Comment: Similar to two examples shown in the NCCI Retro Plan.
In each case, the excess ratio is the expected percent of losses excess of $500,000,
which differs by state and hazard group.
The excess ratios and average costs per case would be looked up in an NCCI publication that is not on the syllabus.

Page 15 Table of Policy Excess Ranges:

One determines which subtable of
the Table of Aggregate Loss Factors to use
based on the policy excess ratio and the
Table of Policy Excess Ratio Ranges.
An extract of this table:

Sub-Table	Excess Ratio Range (Rounded Values)		
1	0.000	–	0.008
2	0.009	–	0.026
3	0.027	–	0.051
4	0.052	–	0.077
5	0.078	–	0.109
6	0.110	–	0.143
7	0.144	–	0.178
8	0.179	–	0.217

As the size of the loss limit increases,
the excess ratio decreases.

No loss limit would correspond to an excess ratio of 0, which is Subtable 1,
similar to the Traditional Table M.

The distributions of aggregate losses in Subtable 6 (corresponding to an excess ratio of about 13%) have a smaller coefficient of variation than the corresponding aggregate distributions for no loss limit in Subtable 1.

Thus the corresponding insurance charges for high entry ratios are smaller in Subtable 6 than Subtable 1.

**The higher the subtable number,
the lower the loss limit and the smaller
the insurance charges for high entry ratios**.

The subtables are entered based on the policy excess ratio rather than the loss limit itself.

If for example,
an insured continues to buy retro policies with a $500,000 loss limit over many years,
then due to inflation its excess ratio will increase.

This will eventually lead to using a different subtable for this insured.

However, neither the subtables themselves nor the published table of policy excess ranges need to be updated for inflation.

If instead this retro insured's loss limit keeps up with inflation, then its excess ratio would stay approximately the same over time, and it would therefore use the same subtable.

Expense Ratio Tables:

The NCCI Retro Plan contains
Tables of Expense Ratios.

They incorporate a set of premium discounts by size of insured.

They also assume a certain Expected Loss Ratio and Tax Multiplier.

One set is for expenses including
profit and contingencies but excluding taxes.

The other set is for expenses including
profit and contingencies but excluding taxes
and allocated loss adjustment expenses.
This would be used with a retro plan that includes ALAE in with losses.

Table M:

The NCCI Retro Plan contains
a small extract of Table M
as would be used in
the prior NCCI Retro Plan.

Table of Aggregate Loss Factors:

Aggregate Excess Loss Factors are given by:

$$\textbf{Entry Ratio} = \frac{\textbf{Limited Loss} \Leftrightarrow \textbf{Max. or Min. Premium}}{\textbf{Expected Limited Loss}}.$$

Like Table M, these charges do not include the charge for the loss limit; there will be a separate charge added for any loss limit.

The loss limit will be paid for via a separate Excess Loss Premium.

The values contained in
the Table of Aggregate Loss Factors
are consistent with the general
methodology underlying ALFs on Demand;
however, due to its countrywide nature,
the Table of Aggregate Loss Factors does not
reflect the state and hazard group differences in
severity distributions that are incorporated by
ALFs on Demand.

The Table of ALFs has three dimensions:

- Rows corresponding to entry ratios.
- Columns corresponding to policy
  expected number of claims (size of insured).
- Subtables corresponding to
  the policy excess ratio (loss limit).

Like Table M, each of these subtables has different columns which are to be used for different sized insureds; however, each column corresponds to a range of expected claim counts.

Each column contains Aggregate Loss Factors (Insurance Charges) for entry ratios from 0 to 10 in increments of 0.01.

The listed Aggregate Excess Loss Factors are based on the distribution of aggregate losses with the loss limit, and thus avoid any overlap for the charge for the effect of the maximum premium included in the basic premium and the charge for the loss limit contained in the excess loss premium.

A small extract of Subtable 6, corresponding to a policy excess ratio of about 13%:

	Aggregate Excess Loss Factors (Sub-Table 6)		
	Expected Claim Count Group		
Entry Ratio	42	41	40
0.00	1.0000	1.0000	1.0000
0.25	0.7837	0.7810	0.7784
0.50	0.6160	0.6100	0.6040
0.75	0.4856	0.4768	0.4681
1.00	0.3836	0.3729	0.3622
1.25	0.3032	0.2912	0.2793
1.50	0.2392	0.2266	0.2140
1.75	0.1880	0.1751	0.1627
2.00	0.1465	0.1344	0.1232

ECG42 corresponds to about 38 expected claims, while ECG40 corresponds to about 49 expected claims. The larger insureds in ECG40 have smaller insurance charges than the smaller insureds in EGC42.

The values contained in the Table of Aggregate Loss Factors are based on a retro plan that uses pure losses (ALAE is not included with losses), and loss limits applied on a per-occurrence basis.

However, this table is to be used for all policies whether the applicable loss limit is on a per-claim or per-occurrence basis, and regardless of whether ALAE is included with ratable losses for purposes of computing the retrospective premium.

14.8. (2 points) Using the following Aggregate Excess Loss Factors (Sub-Table 6), for Expected Claim Count Group 42, estimate the distribution function and the density of the aggregate distribution at an entry ratio of 2.

	Aggregate Excess Loss Factors (Sub-Table 6)
Entry Ratio	Expected Claim Count Group 42
1.80	0.1790
1.90	0.1620
2.00	0.1465
2.10	0.1324
2.20	0.1196

14.8. $\phi^*(r) = \int_r^\infty (y-r)\ dF^*(y) = \int_r^\infty S^*(y)\ dy$.

$$\Rightarrow \frac{d\ \phi^*(r)}{dr} = -S^*(r) = F^*(r) - 1. \quad \frac{d^2\ \phi^*(r)}{dr^2} = f^*(r).$$

The derivative at 2 is approximately:
(0.1324 - 0.1620) / (2.1 - 1.9) = -0.148.
⇒ The distribution at 2 is approximately:

1 - 0.148 = **0.852**.

The derivative at 1.9 is approximately:
(0.1465 - 0.1790) / (2.0 - 1.8) = -0.1625.
The derivative at 2.1 is approximately:
(0.1196 - 0.1465) / (2.2 - 2.0) = -0.1345.
⇒ The second derivative at 2 is approximately:

{-0.1345 - (-0.1625)} / (2.1 - 1.9) = 0.140.
Thus the density at 2 is approximately **0.140**.

Comment: I used the notation ϕ^* for the insurance charges in the Table, even though unlike Table L these charges do not include the charge for the loss limit.

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Benefits of the Table of Aggregate Loss Factors:

- Because the proposed table contains values that are based on a limited aggregate loss distribution, the table eliminates the need for an adjustment to account for overlap between the loss limit and aggregate loss limitation.
AELFs obtained from the proposed table are more accurate for policies with a loss limit than is produced under the current methodology.

- The proposed Table of Aggregate Loss Factors does not need periodic updates
for claim inflation,
as the introduction of policy excess ratio lookup ranges incorporates any and all loss limitations.

- The parametric form used to produce the Table, provides users with a convenient method for calculating AELFs that are consistent with the values in the Table of Aggregate Loss Factors.

- The values contained in the Table of Aggregate Loss Factors are calculated in a manner that is consistent with the proposed methodology underlying ALFs on Demand, with certain exceptions due to the countrywide nature of the table.

- The Table of Aggregate Loss Factors leverages NCCI's 2014 Excess Loss Methodology, and replaces the current Table of Insurance Charges, which was created in the 1990's.

Summary of NCCI's New Methodology to Compute Aggregate Excess Loss Factors:

The NCCI uses the Panjer algorithm to calculate an aggregate distribution.

The Panjer algorithm has as inputs
a frequency distribution
and a discrete severity distribution;
frequency and severity are assume
to be independent.

The severity distribution would be censored from above by any loss limit.

Then this aggregate distribution is used to calculate Aggregate Excess Loss Factors.

The frequency distributions used are
Negative Binomials,
which vary by size of insured.

The discrete severity distributions were backed out of existing Excess Ratios underlying the Excess Loss Factors published by the NCCI.

In determining Excess Loss Factors,
the NCCI uses continuous severity distributions which are a splice of a mixture of
two LogNormal Distributions with
a (Generalized) Pareto Distribution.

There is a different severity distribution
for each claim group.

For a particular loss limit,
the severity distribution is censored from above;
any probability assigned to values above
the loss limit is assigned to the loss limit.

For a particular size of insured,
the appropriate frequency distribution is combined
with the appropriate (discrete) severity distribution
using the Panjer Algorithm,
in order to determine
the (discrete) aggregate distribution

This aggregate distribution is then used to
compute Aggregate Excess Loss Factors.

Benefits of the NCCI's New Methodology:

- By utilizing severity distributions that vary to reflect the exposure characteristics of each risk(state, hazard group, ALAE handling, loss limit), the AELF values more directly reflect the exposure of the underlying policy than if a countrywide severity distribution is used.

- By directly calculating the limited aggregate loss distribution, the proposed methodology eliminates the need for the adjustment to account for overlap between the loss limit and aggregate loss limitation.

The result is that AELFs calculated using
ALFs on Demand are more accurate
for policies with a loss limit
than is produced under the current methodology.

● Because there is a vast array of possible exposure combinations (and resulting values) across all states, hazard groups and loss limits, it is not feasible for the AELFs to be published in a tabular form. Rather, the values for an individual policy will be available through an application, ALFs on Demand,
that will be accessible on NCCI's website.

● The count and severity distributions underlying the proposed methodology are based on updated parameters that reflect more recent data than the distributions underlying the current Table of Insurance Charges, which have not changed since their creation in the late 1990s.

● Future annual updates to the
Excess Loss Factor parameters
will automatically be incorporated into
the Aggregate Loss Factors for each state.

Determining Aggregate Excess Loss Factors from a Discrete Distribution of Severity:

The following discrete aggregate distribution has been calculated.

Aggregate Amount ($000)	Probability
0	8%
250	27%
500	19%
750	13%
1000	10%
1250	7%
1500	5%
1750	4%
2000	3%
2250	2%
2500	1%
2750	1%

The mean is: (0)(8%) + (250,000)(27%) +
... + (2,750,000)(1%) = 750,000.

An entry ratio of 1 corresponds to 750,000.

The expected aggregate excess of 750,000 is:

(250,000)(10%) + (500,000)(7%) +
... + (200,000)(1%) = 242,500.

Thus, $\phi(1)$ = 242,500 / 750,000 = 0.3233.

Aggregate Amount ($000)	Probability
0	8%
250	27%
500	19%
750	13%
1000	10%
1250	7%
1500	5%
1750	4%
2000	3%
2250	2%
2500	1%
2750	1%

Exercise: Determine the Aggregate Excess Loss Factor for an entry ratio of 2.

[Solution:
An entry ratio of 2 corresponds to 1,500,000.
The expected aggregate excess of 1,500,000 is:
(250,000)(4%) + (500,000)(3%) + (750,000)(2%)
+ (1,000,000)(1%) + (1,250,000)(1%) = 62,500.
$\phi(2)$ = 62,500 / 750,000 = 0.0833.
Comment: $\psi(2) = \phi(2) + 2 - 1 = 1.0833$.]

NCCI uses a formula to compute Aggregate Excess Loss Factors at each of the evaluation points y_i from the discrete aggregate distribution:

$$AELF_i = 1 - \frac{\sum_{j=0}^{j=i} (y_j \, PDF_j^{agg}) + \{y_i \, (1 - CDF_i^{agg})\}}{Agg_L},$$

where Agg_L is the average aggregate loss.

For example let us assume we are given that the mean aggregate loss is 750,000, and only the first portion of the previous aggregate distribution:

Aggregate Amount ($000)	Probability
0	8%
250	27%
500	19%
750	13%
1000	10%

An entry ratio of 1 corresponds to 750,000.
The probability above 750,000 is:
1 - (8% + 27% + 19% + 13%) = 33%.

$$\phi(1) = 1 - \frac{\{(0)(8\%) + (250)(27\%) + (500)(19\%) + (750)(13\%)\} + (750)(33\%)}{750}$$

= 1 - 507.5 / 750 = 0.3233,
matching the previous result.
$\psi(1) = \phi(1) + 1 - 1 = 0.3233$.

The NCCI calls the insurance savings the Aggregate Minimum Loss Factor (AMLF).
NCCI displays the following formula:

$$AMLF_i = \frac{y_i - [\sum_{j=0}^{j=i} (y_j \, PDF_j^{agg}) + \{y_i \, (1 - CDF_i^{agg})\}]}{Agg_L}.$$

For this example, to get
the Aggregate Minimum Loss Factor at r = 1:
$\psi(1) =$

$$\frac{750 - [\{(0)(8\%) + (250)(27\%) + (500)(19\%) + (750)(13\%)\} + (750)(33\%)]}{750}$$

= (750 - 507.5) / 750 = 0.3233,
matching the previous result.

14.24. (2 points)

The mean aggregate loss is $100,000.

The following discrete aggregate distribution has been calculated.

Aggregate Amount ($000)	Probability
0	3%
20	7%
40	12%
60	16%
80	19%
100	15%
120	10%
⋮	⋮

Determine the Aggregate Excess Loss Factor at an entry ratio of 1.2, $\phi(1.2)$.

14.24. An entry ratio of 1.2 corresponds to aggregate losses of: (1.2)($100,000) = $120,000.

$$\psi(1.2) = \frac{(120)(3\%)+(100)(7\%)+(80)(12\%)+(60)(16\%)+(40)(19\%)+(20)(15\%)}{100}$$

$$= 40.4 / 100 = 0.404.$$

$$\phi(1.2) = \psi(1.2) + 1 - 1.2 = 0.404 - 0.2 = \mathbf{0.204}.$$

Alternately, the probability above $120,000 is:
1 - (3% + 7% + 12% + 16% + 19% + 15% + 10%) = 18%.

$$\phi(1.2) = 1 - \frac{(0)(3\%)+(20)(7\%)+(40)(12\%)+...+(100)(15\%)+(120)(10\%+18\%)}{100}$$

$$= 1 - 79.6 / 100 = \mathbf{0.204}.$$

Comment: In the second solution, what was done was analogous to calculating the loss elimination ratio and subtracting it from one to get the excess ratio.

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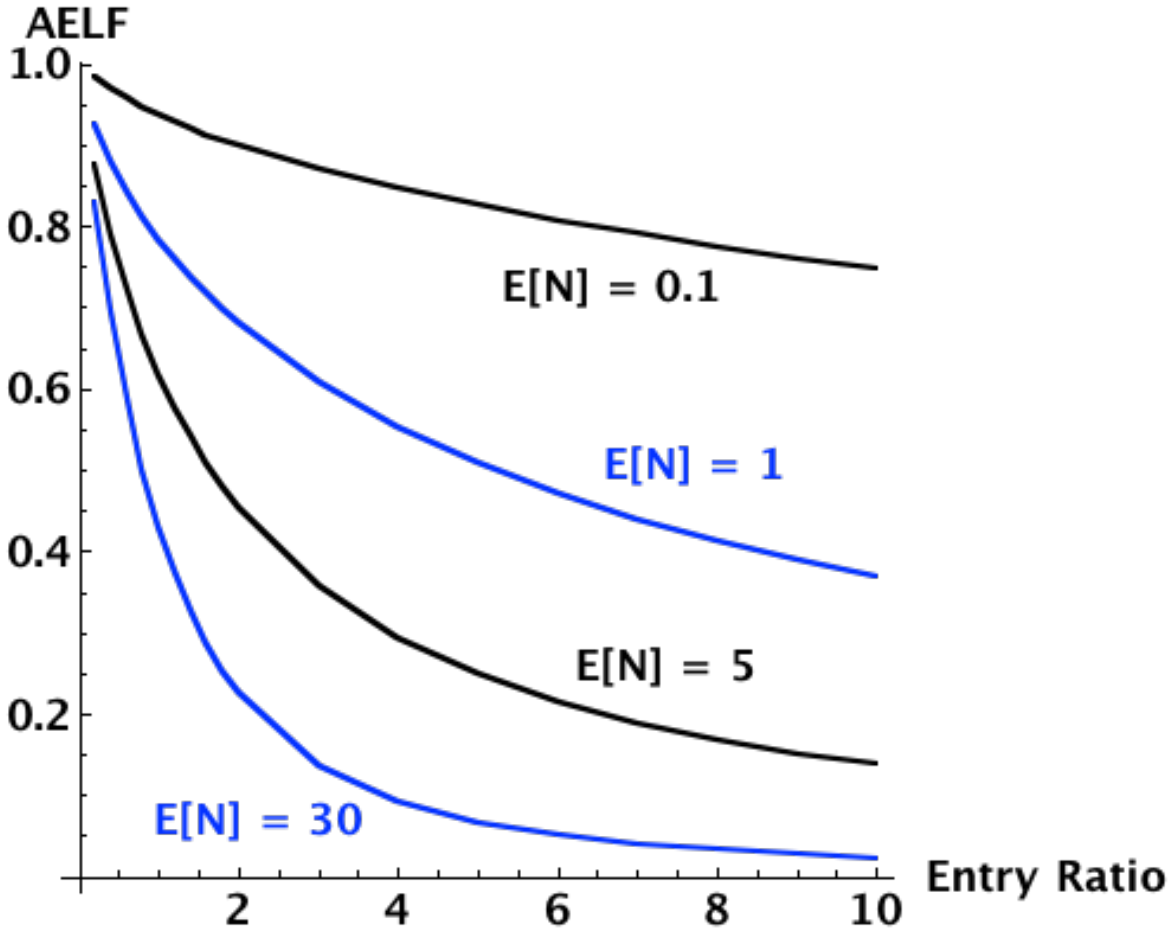
Examples of Output from ALFs on Demand:

By state, for no loss limit,
here is sample output from
NCCI's computer product ALFs on Demand.

These AELFs would be appropriate for
a policy with only exposure in a single state
and the given Hazard Group.

For Hazard Group A in Alaska, a graph of the AELFs for various Expected Numbers of Claims:



14.20. (2 points) Given the following information about a retrospectively rated policy:

Standard Premium	$750,000
Maximum retro premium factor	240%
Minimum retro premium factor	40%
Loss Conversion Factor	1.120
Provision for expenses and profit exclusive of taxes (as percent of Standard Premium)	14.8%
Tax multiplier	1.041
Expected Loss Ratio	66.0%
Expected Number of Claims	50

- There is no loss limit.
- The retro rating plan is balanced.

Use the following Aggregate Loss Factors:

r	0.0	0.2	0.4	0.6	0.8	1.0
ALF	1.0000	0.8204	0.6755	0.5594	0.4664	0.3916
r	1.2	1.4	1.6	1.8	2.0	3.0
ALF	0.3314	0.2825	0.2427	0.2100	0.1831	0.1016
r	4.0	5.0	6.0	7.0	8.0	9.0
ALF	0.0645	0.0450	0.0335	0.0261	0.0210	0.0172

Calculate the basic premium.

14.20. The balance equations are:

$$r_G - r_H = \frac{G - H}{c\,E\,T} = \frac{240\% - 40\%}{(1.120)\,(66\%)\,(1.041)} = 2.60.$$

$$X_H - X_G = \frac{e + E - H/T}{c\,E} =$$

$$\frac{0.148 + 0.660 - 0.40 / 1.041}{(1.120)\,(66\%)} = 0.5733.$$

Trying values:

$\phi(0.4) - \phi(3.0) = 0.6745 - 0.1012 = 0.5733.$ OK.

$\psi(0.4) = 0.6745 - 1 + 0.4 = 0.0745.$

Net Converted Insurance Charge:

$(1.120)\,(66.0\%)\,(0.1012 - 0.0745)\,(750{,}000)$
$= \$14{,}802.$

Basic Premium is:

$14{,}802 + (14.8\%)\,(750{,}000)$
$- (0.120)\,(66.0\%)\,(750{,}000) =$ **$66,402**.

Page 31 Comparing a Retro with No Loss Limit to a Retro with a Loss Limit:

One has to be careful in this comparison.

The insurance charges shown in the Table of Aggregate Loss Factors are to be multiplied by the expected limited losses.

Also, the insurance charges shown in the Table of Aggregate Loss Factors do not include the separate charge for the loss limit.

For the example I will assume:

- Standard Premium of $1.5 million.
- Expected Total Losses of $1 million.
- The expected number of claims is 100.
  ⇔ Expected Claim Count Group 34
- When there is a loss limit it is $1 million and the policy excess ratio is 13% ⇔ Sub-table 6.

Assume that with no loss limit $3 million in losses correspond to the maximum premium.

Then the entry ratio is: 3 million / 1 million = 3.

For no loss limit, the AELF in Sub-table 1 for r = 3 and Expected Claim Count Group 34 is 0.0714.

Multiplying by the total expected losses of $1 million, this is equivalent to $71,400.

For the case with a $1 million loss limit,
the expected limited losses are:
(1 - 13%)(1 million) = $870,000.

Let us assume the same maximum entry ratio of 3.
This corresponds to limited losses of:
(3)($870,000) = $2,610,000.
In turn this corresponds to expected total losses of: $2,610,000/(1 - 13%) = $3 million.
$3 million in expected unlimited losses corresponds to the maximum premium.

The AELF in Sub-table 6 for r = 3 and Expected Claim Count Group 34 is 0.0147.
Multiplying by the expected limited losses of $870,000, this is equivalent to $12,789.

Adding in the $130,000 in expected excess losses, we get a total of: $142,789.

This $142,789 due to the loss limit and the maximum premium, is more than the $71,400 for the maximum premium in the absence of the loss limit.

One should not be fooled by the fact that with a loss limit the AELF is lower than without a loss limit.

Including the separate charge for the loss limit, in total the retro with a loss limit includes more fixed dollars in order to pay for the additional benefit to the insured of the loss limit.