Solutions to the Spring 2019 CAS Exam Five

(Only those questions on Basic Ratemaking)

There were 24 questions worth 52 points, of which 11.5 were on ratemaking worth 29 points.
(Question 7a covered reserving, while the rest of Question 7 covered ratemaking.)

This exam used the paper-and-pencil format of exam administration.

The exam and examiner's report are available from the CAS.
The solutions and comments are solely the responsibility of the author.

(Incorporating what I found useful from the CAS Examiner?s Report)

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Where each ratemaking question would go in my study guide:[1]

1. Section 4
2. Section 5

3. Section 16
4. Section 6

5. Section 6
6. Section 7

7. Section 8
8. Section 13

9. Section 10
10. Section 9

11. Section 15
12. Section 14

[1] Some questions rely on ideas in more than one section of my study guide. I have chosen the best place to put a question in my opinion.

1. (2.5 points) Given the following:

Policy	Number of Vehicles	Effective Date	Expiration Date
A	2	January 1, 2018	June 30, 2018
B	3	March 1, 2018	August 31, 2018
C	1	July 1, 2018	December 31, 2018
D	2	October 1, 2018	March 31, 2019
E	1	November 1, 2018	April 30, 2019

- All policies remain in-force until their expiration date.
- An exposure is defined as one vehicle insured for one year.

a. (0.25 point) Calculate the calendar year 2018 written exposures.

b. (0.5 point) Calculate the calendar year 2018 earned exposures.

c. (0,5 point) Calculate the policy year 2018 earned exposures as of February 28, 2019.

d. (0.25 point) Calculate the in-force exposures as of October 15, 2018.

e. (1 point) Identify two criteria of an exposure base and briefly evaluate miles driven as an exposure base for personal auto insurance using those criteria.

1. All polices are 6-month and written in 2018.
(a) CY 2018 written exposures: 2/2 + 3/2 + 1/2 + 2/2 + 1/2 = **4.5 caryears**.
(b) Policy D is 1/2 earned in 2018. Policy E is 1/3 earned in 2018.
CY 2018 earned exposures: 2/2 + 3/2 + 1/2 + (2/2)(1/2) + (1/2)(1/3) = **3.67 caryears**.
(c) As of February 28, 2019, only 5/6 of Policy D is earned and only 4/6 of E is earned.
PY 2018 earned exposures as of February 28, 2019:
2/2 + 3/2 + 1/2 + (2/2)(5/6) + (1/2)(4/6) = **4.167 caryears**.
(d) As of October 15, 2018, policies C and D are in force.
(Policy E has yet to become effective.)
Thus the in-force exposures as of October 15, 2018 are: 1 + 2 = **3 cars**.
Alternately, since the polices are semi-annual: (1+2)/2 = **1.5 caryears**.
(e) 1. Proportional to expected losses: Exposure base implies a uniform and continuous multiplicative relationship between the variable and the expected losses. The exposure base should be responsive to any change in exposure to risk.
As the number of miles driven increases, the expected number of accidents increases, and thus the expected losses increase. While it would matter whether these are miles driven on a highway or city streets, on balance this criterion is satisfied.
Miles driven does not satisfy this criteria for Other Than Collision Coverage (Comprehensive).
2. Practical. Objective and relatively easy and inexpensive to obtain and verify:
This will allow the exposure base to be consistently measured and not subject to manipulation by insureds, agents, or underwriters.
One could fit the car with a telemetric device that would allow the insurer to record the miles driven. This would be relatively easy and inexpensive. It is verifiable and not subject to manipulation.
Thus this criterion is satisfied.
3. Historical precedence: is this exposure base currently being used by this insurer.
If miles driven is currently being used then this criteria is satisfied.
Since it satisfies all three criteria, I conclude that miles driven is a good exposure base to use.
Comment: There are other reasonable answers to part (e).
If caryears are currently being used, then switching to miles driven would incur some expense.
For several years one would not have historical exposure data useful for ratemaking. Also switching exposure bases could produce large swings in premiums for individual insureds.
The CAS accepted either of the two answers to part (d).
In part (d), I would have followed Table 4.15 in Basic Ratemaking and answered 3 cars, which makes sense to me.
However, according to Basic Ratemaking:
"Most companies define insured units to be the count of items exposed to loss at a given point in time. For example, if an automobile policy insures three cars, that one policy could contribute three in-force exposures at a given point in time. Alternatively, some companies may define insured unit in terms of the number of policies (the auto example above would have one in-force exposure under this definition) or the written exposures (in the auto example, there could be three in-force exposures if the term is annual, or 1.5 in-force exposures if the term is semiannual)."

2. (2.25 points) Given the following:

Calendar Year	Earned Premium ($)
2017	3,850,000
2018	4,200,000

Rate Change Effective Date	Overall Rate Change
January 1, 2017	10%
July 1, 2017	5%

Quarter and Year	Average Written Premium at Current Rate Level
2Q 2016	$1,771
4Q 2016	$1,806
2Q 2017	$1,840
4Q 2017	$1,877
2Q 2018	$1,914
4Q 2018	$1,953

- No rate changes occurred in 2016 or 2018.
- Rates will be in effect for one year.
- All policies are semi-annual.
- All policies are written uniformly throughout the year.

a. (1.75 points) Calculate the trended on-level earned premium for 2017 to be used in a rate change effective July 1, 2019.

b. (0.5 point) Briefly describe two reasons why the 2017 trended on-level earned premium would be higher if all policies were annual rather than semi-annual.

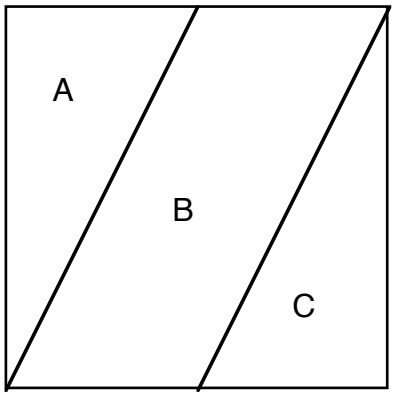
2. (a) I took the ratios of successive average written premiums:

Quarter and Year	Average Written Premium at Current Rate Level	Ratio
2Q 2016	$1,771	
4Q 2016	$1,806	1.0198
2Q 2017	$1,840	1.0188
4Q 2017	$1,877	1.0201
2Q 2018	$1,914	1.0197
4Q 2018	$1,953	1.0204

I select a trend of 2% per six months.
Since policies are semi-annual, the average data of writing of CY2017 Earned Premium is
July 1, 2017 minus 3 months = April 1, 2017.
Since new rates will be in effect for one year, the average date of writing under the new rates is
July 1, 2019 plus 6 months = January 1, 2020.
Thus the trend period for CY17 is 2 years and 9 months. (This is 5.5 half-years.)

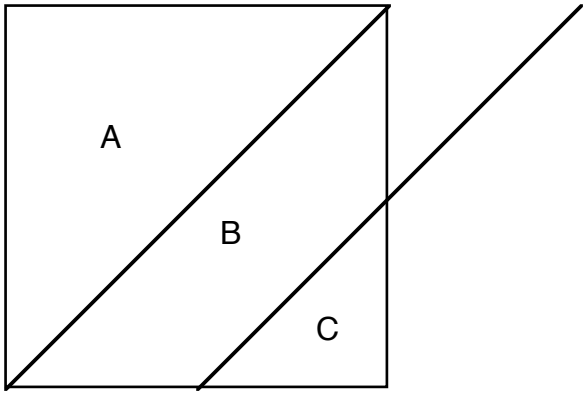
	Rate Level Index
Prior	1.00
1/1/17	1.10
7/1/17	(1.10)(1.05) = 1.155

Draw a diagram for CY17 earned premium; since policies are semi-annual, the lines have slope 2.



Area A = (1/2)(1/2)(1) = 1/4. Area C = (1/2)(1/2)(1) = 1/4. Area B = 1 - 1/4 - 1/4 = 1/2.
Average rate level for CY17: (1/4)(1) + (1/2)(1.1) + (1/4)(1.155) = 1.08875.
OLF = 1.155/1.08875 = 1.0608.
Trended on-level earned premium for CY17: $(1.0608)(1.02^{5.5})(3{,}850{,}000)$ = **4.554 million**.

(b) 1. The trend period would be three months longer.
Specifically the average date of writing of CY2017 Earned Premium would be:
July 1, 2017 minus 6 months = January 1, 2017.
The trend to date would remain the same.
2. The on-level factor would be bigger, since more of the CY2017 Earned Premium would have been written at lower rates.
Specifically, the diagram would now have lines with slope of 1 rather than 2:



Area A = (1/2)(1)(1) = 1/2. Area C = (1/2)(1/2)(1/2) = 1/8. Area B = 1 - 1/2 - 1/8 = 3/8.
Average rate level for CY17: (1/2)(1) + (3/8)(1.1) + (1/8)(1.155) = 1.0569.
OLF = 1.155/1.0569 = 1.0928.
Comment: In this case, in my opinion, there is no reason to bother to use a two-piece method of premium trend.
In part (b), I believe one could give somewhat less detail than I did.
If all policies were annual rather than semi-annual, then the trended on-level EP for CY17 would be:
$(1.0928)(1.02^6)(3{,}850{,}000) = 4.738$ million.

3. (1.5 points)
a. (0 .5 point)
Briefly describe one advantage and one disadvantage of calendar year data aggregation.
b. (0.5 point) Justify an appropriate data aggregation approach for a risk classification plan for a long-tailed line of business.
c. (0.5 point) Justify using report year data aggregation for a claims-made line of business.

3. (a) Advantages: Calendar Year Premium and losses are fixed at the end of the calendar year.
Once the Calendar Year is over, data is ready to be used and is thus available quickly.
Calendar Year data is easy to obtain since it is needed for accounting purposes.
Disadvantage: Calendar Year data has a poor match between premiums and losses.
(A given calendar year of losses includes the effect of reserve changes on older accidents.)

(b) I would recommend using policy year data. Policy year has a better match between exposures/premiums and losses than accident year. This is particularly important when it can take a long time for losses to be reported and settled. This is also particularly important when the data will be divided into many classification cells.
Alternately, I would recommend accident year data. It would provide a better match between premium/exposure and losses than would calendar year data, while it develops more quickly than policy year data.
Alternately, if the long-tailed line sells claims-made policies, then I would recommend using report year data because it will provide the best match of premium/exposures to losses.

(c) For example, report year 2018 would include data on all claims reported during 2018, regardless of when the accident/incident occurred. We can divide report year 2018 by reporting lag, so for example lag 0 would be those accidents that occurred during 2018, lag 1 would be those accidents that occurred during 2017, etc.
Under claims-made, at the expiration of the policy all claims are known. Therefore, the claims-made losses are a sum of the losses at certain lags for a given report year; which lags to include depends on the maturity of the policy. A mature claims-made policy written January 1 would be equivariant to a report year of losses. Put another way, since the coverage trigger for the claims-made policy is the report date, a claims-made policy written January 1 is represented by the entries in a row of the report year / lag diagram.
(Things get a little more complicated for policies written other than January 1.)
Alternately, at the expiration of a claims-made policy there is no pure IBNR; at the end of the policy period almost all losses are known (except for IBNER). This aligns with report year data aggregation, so it is a good match
Alternately, the coverage trigger in the case of a claims-made policy is the report of the claim.
That makes the match between premium and losses to be ideal using report year data.

Comment: I could not tell whether the class ratemaking examples in Appendix E of Basic Ratemaking use Accident Year or Policy Year data.

4. (2.25 points) Given the following information:

Claim Number	Total Limits Loss ($000s)
1	15
2	21
3	24
4	55

- Each total limits loss is subject to an 8% annual severity trend.
- Basic limit = $25,000.

a. (1 point) Calculate the basic limits loss trend over a one-year timeframe.
b. (0 .75 point) Calculate the excess loss trend over a one-year timeframe.
c. (0.5 point) Discuss the appropriateness of applying total limits trend to losses at basic limits for purposes of ratemaking.

4. The effect of trend on the total limit losses:

Claim Number	Total Limits Loss ($000s)	Losses Trended
1	15	(15)(1.08) = 16.2
2	21	22.68
3	24	25.92
4	55	59.40

(a) Prior to trend, the basic limit losses are: (1000)(15 + 21 + 24 + 25) = $85,000.
After trend the basic limit losses are: (1000)(16.2 + 22.68 + 25 + 25) = $88,880.
Basic limit trend = $88,880 / $85,000 - 1 = **4.6%**.
(b) Prior to trend, the excess losses are: (1000)(0 + 0 + 0 + 30) = $30,000.
After trend the excess losses are: (1000)(0 + 0 + 0.92 + 34.40) = $35,320.
Excess trend = $35,320 / $30,000 - 1 = **17.7%**.
(c) It is <u>not</u> appropriate to apply total limits trend to losses at basic limits. Due to the limit, basic limits losses increase more slowly than total limits losses (assuming a positive trend.)
Alternately, it is not appropriate since:

- For losses above basic limit, trend is entirely on the excess layer and none applied to basic layer.
- For losses close to basic limit, trend will push the loss into excess layer and thus not all of the trend will apply to the basic layer.

Alternately, it is not appropriate. Trend impacts basic limit losses and excess losses differently. The two layers should be trended separately.

5. (4.25 points) Given the following loss data as of December 31, 2018 for an insurer:

Cumulative Reported Losses ($000s)				
Accident Year	12	24	36	48
2012				169,000
2013				181,000
2014				180,000
2015				169,000
2016			161,000	
2017		150,000		
2018	121,000			

Shock Losses		
Accident Year	Claim Count	Reported Ground-Up Losses ($000s)
2012	3	3,000
2013	4	5,000
2014	3	3,000
2015	1	600
2016	1	900
2017	0	0
2018	1	900

Selected Age-to-Age Development Factors:		
12 - 24	24 - 36	36 -48
1.2	1.1	1.05

$500,000 = Excess loss threshold used by the insurer for individual reported losses

Question Continued on Next Page

The annual frequency and severity exponential trend fits based on data for the 12 months ending each quarter evaluated through December 31, 2018 are as follows:

Exponential Trend		
	Frequency	Total Severity
20 point	1.7%	5.0%
16 point	1.4%	5.4%
12 point	1.8%	5.1%
8 point	1.5%	2.0%
6 point	1.3%	0.1%
4 point	1.6%	0.0%

- All policies are annual.
- Rates will be in effect for one year.
- There is no development after 48 months.
- Loss development is the same for basic and excess losses.
- Accident years 2012 through 2015 are used to estimate the shock loss adjustment.

a. (3.75 points) Calculate the projected trended ultimate losses for accident year 2018 to be used to determine a rate change effective January 1, 2020. Briefly justify the trend selections.

b. (0.5 point) Briefly describe two approaches for selecting an excess loss threshold used for capping shock losses.

5. (a) The frequency trend appears to be stable over time.
Thus I will use the 16-point frequency trend of 1.4%.
The severity trend appears to be decreasing over time.
Thus I will use the 5.4% 16-point severity trend for the trend period and the 0.1%.6-point severity trend for the projection period.
The average date of writing under the new rates is July 1, 2020.
Since policies are annual, the average date of accident is January 1, 2021.
The average date of accident for AY18 is July 1, 2018.
The projection period from AY18 is 2.5 years.

For example, for 2102: $(3000)(1.0546)(1.001^{2.5}) = 4123.3$.

Accident Year	Shock Loss Claim Count	Reported Ground-Up Shock Losses ($000s)	Severity Trend	Trended Losses ($000s)	Excess Losses ($000s)
2012	3	3,000	$(1.054^6)(1.001^{2.5})$	4123.3	2623.3
2013	4	5,000	$(1.054^5)(1.001^{2.5})$	6520.2	4520.2
2014	3	3,000	$((1.054^4)(1.001^{2.5})$	3711.7	2211.7
2015	1	600	$(1.054^3)(1.001^{2.5})$	704.3	204.3

For 2012, the excess losses are: 4123.3 - (3) (500) = 2,623.3.
The total excess losses are 9559.5 thousand.

The total losses for these AYs trended for severity are: $(1.054^6)(1.001^{2.5})(169{,}000) + (1.054^5)(1.001^{2.5})(181{,}000) + (1.054^4)(1.001^{2.5})(180{,}000) + (1.054^3)(1.001^{2.5})(169{,}000) = 889{,}390$ thousand.
Thus the excess loss factor is: 889,390/(889,390 - 9559.5) = 1.0109.
AY18 excess losses are: (1000) (900 - 500) = 400,000.
Thus the projected trended ultimate losses for accident year 2018 are:
$(1.2)(1.1)(1.05)\ (1.014^{2.5})\ (1.001^{2.5})\ (1.0109)\ (121{,}000 - 400)\ (1000) =$ **175.388 million**.

(b) 1. One could select a certain high percentile (quantile) of the size of loss distribution.
2. One could choose the threshold so that approximately a certain small percent of total losses are expected to be excess of the threshold.
3. For property Insurance one could select a certain percent of the insured value of each property.
Some other acceptable responses from the CAS Examiner's Report, that are not mentioned in the syllabus reading (see my comment):

- Capping at a percentage of coverage,
  for example homeowners could cap at 70% of coverage A
- Look at losses and select a claim value where large losses start to get volatile and thin,
  in order to select a threshold
- Basic Limit
- Reinsurance Limit
- Policy Limit
- Industry Benchmark
- Balance stability and responsiveness of indication
- Actuarial Judgement

Comment: There are other reasonable trend selections. The CAS Examiner's report states that one should use a two-step severity trend in this case, and that you needed to say why.
However, one should be careful when selecting separate frequency and severity trends, that in combination they are consistent with observed or expected pure premium trends.
In their example calculation of an excess loss factor in Table 6.3 of Basic Ratemaking, Werner and Modlin do not use trend. However, "The simple excess loss procedure outlined above is ideally performed on reported losses that have been trended to future levels."
In calculating the excess loss factor one should use only severity trend and not frequency trend.
Only mature years are being used to estimate the excess loss factor; using immature data can bias this factor downwards as it take longer on average for large claims to be reported and settled.
"When the losses are not capped at the basic limit, the actuary must determine the large loss threshold that best balances the following goals: including as many losses as possible and minimizing the volatility in the ratemaking analysis."
For example, one could choose the 99th percentile of the size of loss distribution. Alternately, one could choose the threshold so that approximately 2% of total losses are expected to be excess.
In either case, some round number would be selected. Once a threshold is selected, it would usually be used for many years, being updated when appropriate for inflation.
In part (b), I would not have accepted the answers: Basic Limit or Policy Limit. I am not sure what was meant by the answer Reinsurance Limit. I think the answers Actuarial Judgement and Industry Benchmark are each too vague and general; they demonstrate no specific knowledge of the excess loss procedure being tested. Balancing stability and responsiveness are the goal; I do not think this describes an approach.

6. (2.5 points) Given the following countrywide information:

	2016 Expense Ratio	2017 Expense Ratio	2018 ($000s)
Direct Premium Written			6,100
Direct Premium Earned			5,920
Commission and Brokerage Expenses Incurred	12.0%	13.0%	945
Other Acquisition Expense Incurred	12.8%	12.7%	760
General Expenses	15.0%	5.5%	325
Taxes, Licenses, & Fees Incurred	2.1%	2.2%	130

- Profit provision = 7%.

a. (1.25 points)
Select and justify a total expense ratio for use in ratemaking assuming all expenses are variable.
b. (0 .25 point)
Calculate the variable permissible loss ratio using the expense ratio from part a. above.
c. (0.5 point) Briefly discuss two potential distortions from using the results in part b. above to determine rates in a new state.
d. (0.5 point) Calculate the variable permissible loss ratio if 100% of the taxes, license & fees and 75% of the general expenses do not vary by premium.

6. Calculate the 2018 expense ratios, with General Expense being divided by earned premium: 945/6100 = 15.5%, 760/6100 = 12.5%, 325/5920 = 5.5%, 130/6100 = 2.1%.
(a) The 2016 General Expense percentage seems very unusual. Therefore, in the absence of additional information, I will base my selection for general expenses on 2017 and 2018.
"If productivity gains led to a significant reduction in staffing levels during the latest historical experience period, then the selected ratios should be based on the expected expenses after the reduction rather than the all-year average."
The 2018 Commissions are higher than the other two years; however, in the absence of additional information I will take an average of the three years.

	2016	2017	2018	Selected
Commission and Brokerage Expenses Incurred	12.0%	13.0%	15.5%	13.5%
Other Acquisition Expense Incurred	12.8%	12.7%	12.5%	12.7%
General Expenses	15.0%	5.5%	5.5%	5.5%
Taxes, Licenses, & Fees Incurred	2.1%	2.2%	2.1%	2.1%
Total				**33.8%**

(b) 1 - 33.8% - 7% = **59.2%**.
(c) 1. If this state has average rates higher than the countrywide average rates, and some of the expense needs do not depend on rate level differences between states, we would allocating more expenses to this new state than may be needed. (The reverse would be true for lower than average rates.)
2. There may be some startup costs of entering the new state. The insurer may wish to load some portion of these startup costs into the rates.
3. Until there is significant premium volume in the new state, which may take several years, the infrastructure and staffing needs may be larger as a percent of premium; thus expense needs as a percent of premium in this new state may be higher than indicated by the countrywide data.
4. Taxes, Licenses and Fees vary by state (as sometimes do Commissions), and thus it may not be appropriate to use countrywide data for this state.
Some other acceptable responses from the CAS Examiner's Report (see my comment):

- If there are some expenses that are truly fixed this will understate premiums for small premium policies and overstate them for large policies.
- Historical expense ratios may be very different than future ratios.

(d) The variable expenses are: 13.5% + 12.7% + (25%)(5.5%) = 27.6%.
1 - 27.6% - 7% = **65.4%**.

Comment: The actuary should dig deeper to find out what is behind any unusual values.
The unusually high General Expenses in 2016 may represent one-time costs that it would be appropriate to spread out over a longer period of time; thus one might select for example a General Expense provision of 7%.
The higher commissions in 2018 may reflect a change in the schedule or manner of paying commissions; if this change will continue going forward, then 15.5% would be a better provision.
In part (c), often an insurer will not load into the rates any temporary extra expenses associated with expanding into a new territory.
In part (c), the two additional responses from the CAS Examiner's Report have nothing specific to do with determining rates in a new state. I would not have given such general responses credit.
In part (d), we are using the premium-based projection method. The fixed expense ratio is:

(75%)(5.5%) + 2.1% = 6.2%. The rate indication would be: $\frac{\text{(Loss \& LAE Ratio) + 6.2\%}}{65.4\%} - 1.$

7. (4.75 points) Given the following data as of December 31, 2018:

Accident Year	Reported Loss and ALAE ($000s)		Calendar Year	Earned Premium ($000s)
2016	2,000		2016	4,600
2017	1,750		2017	5,100
2018	800		2018	5,800

Development Age to Ultimate	Selected Cumulative Development Factors
36	1.46
24	2.08
12	4.90

65%	Expected loss and ALAE ratio
4%	Annual loss and ALAE trend
3%	Annual premium trend
6%	Fixed expense ratio
26%	Variable expense ratio
5%	Profit and contingencies provision
8%	ULAE provision as a % of loss and ALAE
0.7	Credibility of historical experience
+8%	Complement of credibility taken from a competitor rate change filing

Rate change history:

-2%	Rate change effective January 1, 2016
+3%	Rate change effective July 1, 2018

- All policies are annual.
- Exposures are written evenly throughout each calendar year.
- New rates will be in effect for one year starting January 1, 2020.

a. (0.5 point) Calculate the ultimate loss and ALAE for each accident year using the reported Bornhuetter-Ferguson technique.

b. (3.5 points)

Calculate the credibility-weighted indicated rate change using the latest three accident years.

c. (0.75 point) Briefly describe a disadvantage of using a competitor rate change filing as the complement of credibility and justify a more suitable alternative.

7. (a) First use the loss development factors to calculate the expected percent unreported:

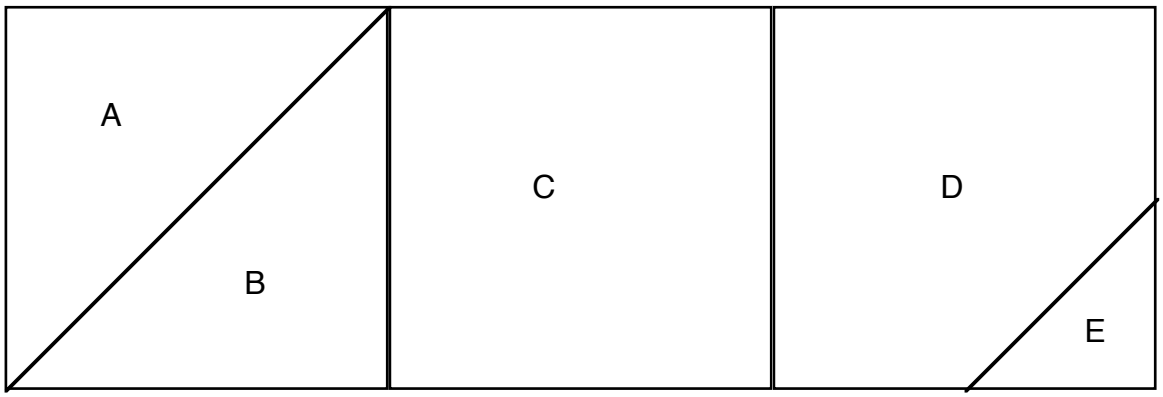
Age to Ultimate	Cumulative Development Factors	Expected % Unreported
36	1.46	1 - 1/1.46 = 0.3151
24	2.08	1 - 1/2.08 = 0.5192
12	4.90	1 - 1/4.90 = 0.7959

Multiply premiums by the expected loss & ALAE ratio and the expected percent unreported:

A Y	Loss and ALAE ($000s)	Premium ($000s)	Percent Unreported	Unreported ($000s)	Ultimate ($000s)
2016	2,000	4,600	0.3151	942	**2942**
2017	1,750	5,100	0.5192	1721	**3471**
2018	800	5,800	0.7959	3001	**3801**

For example: (4600)(0.65)(0.3151) = 942. 2000 + 942 = 2942.

(b) Draw a diagram for earned premiums; since policies are annual, the lines have slope 1.



Area A = 1/2 = Area B. Area C = 1. Area E = (1/2)(1/2)(1/2) = 1/8. Area D = 1 - 1/8 = 7/8.

	Rate Level Index
Prior	1.00
January 1, 2016	0.98
July 1, 2018	(0.98)(1.03) = 1.0094

Average rate level for CY16: (1/2)(1) + (1/2)(0.98) = 0.99. OLF = 1.0094/0.99 = 1.0196.

Average rate level for CY17: 0.98. OLF = 1.0094/0.98 = 1.0300.

Average rate level for CY18: (7/8)(0.98) + (1/8)(1.0094) = 0.98368.

OLF = 1.0094/0.98368= 1.0261.

Average date of writing under the new rates is July 1, 2020.

With annual policies, the average date of earning is January 1, 2021.

The average date of earning for CY16 is July 1, 2016. Trend period is 4.5 years.

Calendar Year	Earned Premium ($000s)	On Level Factor	Premium Trend	Trended On Level
2016	4,600	1.0196	$1.03^{4.5}$	5357
2017	5,100	1.0300	$1.03^{3.5}$	5826
2018	5,800	1.0261	$1.03^{2.5}$	6408
Total				17,591

I will use the ultimate loss and ALAE for each accident year from using Bornhuetter-Ferguson.

Accident Year	Ultimate Loss and ALAE ($000s)	Loss Trend	Trended Ultimate
2016	2942	$1.04^{4.5}$	3510
2017	3471	$1.04^{3.5}$	3982
2018	3801	$1.04^{2.5}$	4193
Total			11,685

The loss and ALAE ratio is: 11,685/17,591 = 66.43%.

Indicated rate change (prior to credibility) is: $\frac{(1.08)\ (66.43\%) + 6\%}{1 - 26\% - 5\%} - 1 = 12.67\%$.

Credibility-weighted indicated rate change: (70%)(12.67%) + (30%)(8%) = **11.27%**.

(c) The competitor's rate change depends on its current level of rates, which may be either higher or lower than the current average rates of this insurer. (For example, the competitor's 8% increase could result in average rates for the competitor that are lower than this insurer's current rates.) The 8% increase is not appropriate to apply to this insurer's current average rates.

One suitable complement of credibility would be to apply the net trend from the effective date of the last rate change to the proposed effective date, also taking into account any difference between the last implemented rate change and the corresponding last indicated rate change. This would be the current rate indication absent the new information available since the last rate review.

Comment: As I discussed, it makes little sense to rely on the competitor's rate change; if done properly it may make sense to rely on the competitor's average rate level.

Reasons not to rely on the competitor's average rate level:

- The competitor's mix of business by class and territory may differ significantly from this insurer.
- The competitor may have different underwriting guidelines.
- The competitor may have different expense needs and/or desired profit provision.

Hopefully, the competitor's rate filing was recent, otherwise one may be relying on what the competitor did for example almost a year ago rather than what the competitor is planning do now.

It would also be questionable to rely on this insurer's regional or countrywide rate change. For example, the current rates in other states could be on average much higher than in this state; then an indicated rate decrease in those other states is probably not relevant to this state.

8. (1.75 points) Given the following pricing strategies for an insurer with two classes of business:

- Strategy 1: Make no change to current rates.
- Strategy 2: Implement a new rating variable to charge different rates for Class A and Class B.
- Strategy 3: Do not implement a new rating variable, but increase rates for all risks.

Strategy 1 - No Change			
Class	Number of Risks	Losses & Expenses per Risk	Rate per Risk
A	5,000	900	1,050
B	10,000	1,000	1,050
One-time expense to implement Strategy 1:			0

Strategy 2 - New Rating Variable			
Class	Number of Risks	Losses & Expenses per Risk	Rate per Risk
A	5,000	900	1,000
B	10,000	1,000	1,100
One-time expense to implement Strategy 2:			500,000

Strategy 3 - Rate Increase for All Risks			
Class	Number of Risks	Losses & Expenses per Risk	Rate per Risk
A	5,000	900	1,100
B	10,000	1,000	1,100
One-time expense to implement Strategy 3:			500,000

a. (0.75 point) Calculate the expected total profit for the insurer under each strategy.

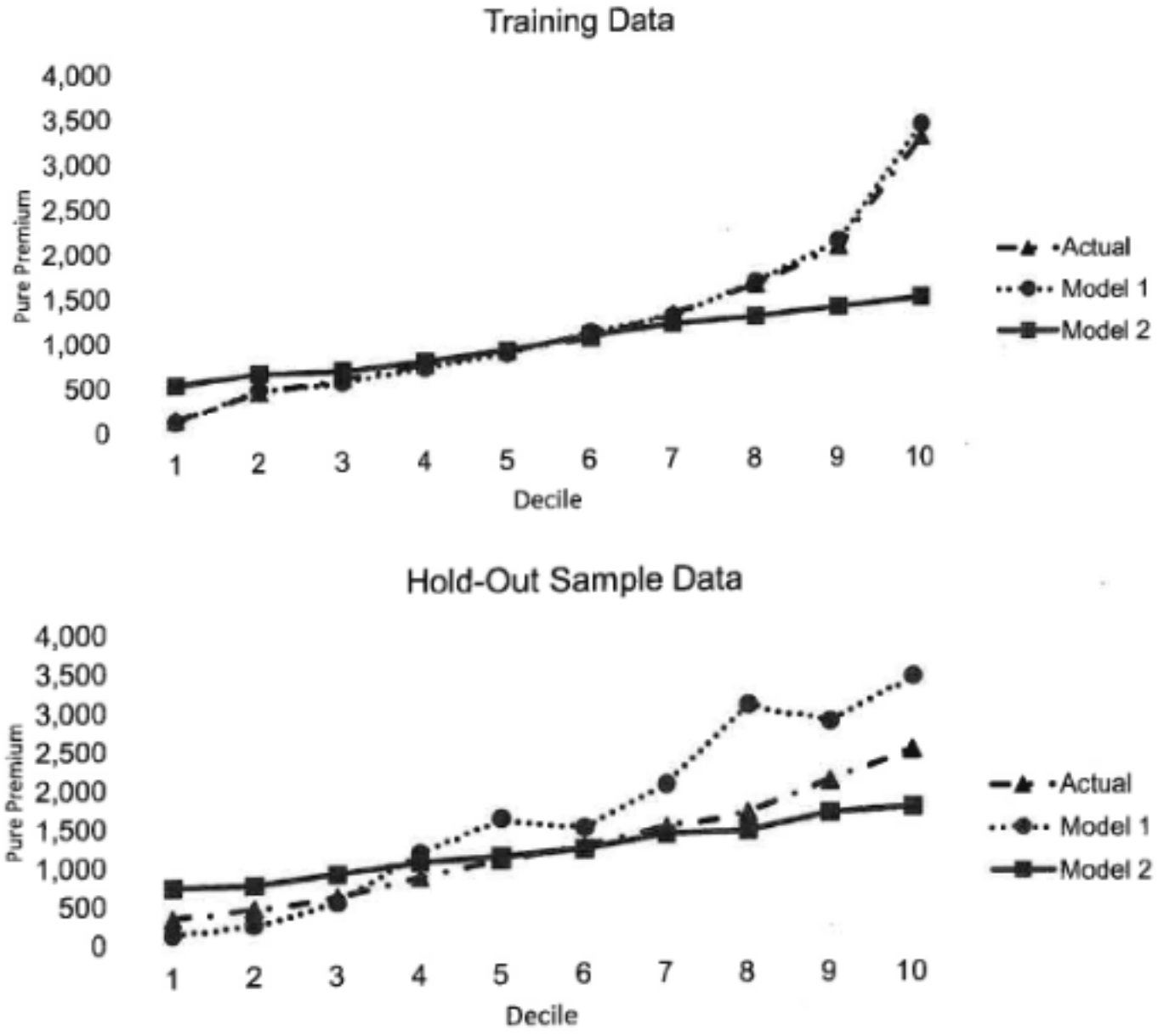
b. (1 point) Briefly evaluate the following for strategy 2 and strategy 3:

i. Assumption that the number of risks by class will be the same as strategy 1.

ii. Impact this assumption has on each strategy's expected total profit.

8. (a) Strategy 1: (5000)(1050 - 900) + (10,000)(1050 - 1000) = **1,250,000**.
Assume Y years over which to spread the implementation costs.
Strategy 2: (5000)(1000 - 900) + (10,000)(1100 - 1000) - 500,000/Y = 1,500,000 - 500,000/Y.
If Y = 1, then expected profit is: **1,000,000**.
Strategy 3: (5000)(1100 - 900) + (10,000)(1100 - 1000) - 500,000/Y = 2,000,000 - 500,000/Y.
If Y = 1, then expected profit is: **1,500,000**.
(b) i. In the case of Strategy 2, we are lowering the rate for Class A and raising the rates for Class B.
Thus all else being equal, we expect to write more Class A risks and fewer Class B risks.
In the case of Strategy 3, we are raising the rates for both classes.
Thus all else being equal, we expect to write fewer risks for both classes.
ii. In the case of Strategy 2, both classes generate 100 per risk. Thus if the decrease in business in Class B is balanced by an increase in business in Class A, then there should be no net effect on the expected total profit calculated in part (a). If on the other hand, there is more of a decrease in Class B than an increase in business in Class A, there would be a reduction in the expected total profit calculated in part (a).
In the case of Strategy 3, we are expecting to lose business in both classes, which would reduce the expected total profit calculated in part (a).
It should be noted that unlike Strategy 2, both Strategy 1 and Strategy 3 are subject to anti-selection. A competitor can charge less than this insurer for Class A, and attract many of the Class A risks currently written by this insurer. Strategy 3 is worse in this regard than Strategy 1.
Thus Strategy 2 is better in the long run.

9. (1 point) The following graphs show two competing generalized linear models' (GLMs) predictions versus the data used in modeling ("training") and a hold-out sample. Data in each graph has been sorted into equal volume deciles, ranked from low to high actual loss.



Assess each of the models.

9. Based on the first graph, Model 1 does a very good job of matching the training data.
However, based on the second graph, Model 1 does a very poor job of matching the test data, particularly for the high deciles. Model 1 is overfit; the model picks up too much of the random fluctuation (noise) in the training data.
Based on the first graph, Model 2 does a poor job of matching the training data.
Based on the second graph, Model 2 also does a poor job of matching the test data, although not as poor of a job as Model 1.
Model 2 is probably underfit; the model does not pick up enough of the signal in the training data.
Model 2 is monotone increasing, which is good. In the second graph, Model 1 is not monotone increasing; there are reversals, which is bad.
Model 1 has a larger vertical distance between the first and last deciles than does Model 2;
Model 1 has more "lift" than Model 2. All else being equal, larger lift is better, indicating that the model is able to maximally distinguish the best and worst risks.
Comment: According to the CAS Examiner's "recommendation of one model over the other was not required." I would not recommend using either model.
See Figure F.6 in Basic Ratemaking. However, there the deciles were based on sorting the modeled rather than actual losses.
My answer goes somewhat beyond what is discussed in Basic Ratemaking.
For more detail, see Section 7.2.1 of Generalized Linear Models for Insurance Rating on Exam 8.

10. (2.75 points)
Given the following information for the rating variables vehicle class, driver type, and territory:

Exposures	Territory				Exposures	Territory		
Vehicle Class	1	2	3		Driver Type	1	2	3
A	30	15	200		X	50	45	140
B	80	22	104		Y	80	72	224
C	20	80	60					

Loss ($000s)	Territory				Loss ($000s)	Territory		
Vehicle Class	1	2	3		Driver Type	1	2	3
A	30	15	200		X	40	65	115
B	100	33	135		Y	120	180	325
C	30	200	105					

- All segments are fully credible.
- Vehicle class A, Driver type X, and Territory 1 are used as the bases.

Propose rating factors for all three variables using a univariate method, considering any exposure correlation.

10. In order to consider any exposure correlation, in the absence of being given the current relativities, I will use a modified version of the adjusted pure premium method.
Using the grid by Vehicle Class (see my Comment), Territory pure premiums are:
160,000/130 = 1230.77, 248,000/117 = 2119.66, 440,000/364 = 1208.79.
Thus with Territory 1 as the base, the preliminary Territory relativities are:
1, 2119.66/1230.77 = 1.7222, 1208.79/1230.779 = 0.9821.
For Vehicle Class the adjusted exposures are:
(30)(1) + (15)(1.7222) + (200)(0.9821) = 252.3.
(80)(1) + (22)(1.7222) + (104)(0.9821) = 222.0.
(20)(1) + (80)(1.7222) + (60)(0.9821) = 216.7.
The adjusted pure premiums by Vehicle Class:
245,000/252.3 = 971.07, 268,000/222.0 = 1207.21, 335,000/216.7 = 1545.92.
Therefore with Vehicle Type A as the base, the estimated Vehicle Class relativities are:
1, 1207.21/971.07 = **1.243**, 1545.92/971.07 = **1.592**.
For Driver Type the adjusted exposures are:
(50)(1) + (45)(1.7222) + (140)(0.9821) = 265.0.
(80)(1) + (72)(1.7222) + (224)(0.9821) = 424.0.
The adjusted pure premiums by Driver Type:
220,000/265.0 = 830.19, 625,000/424.0 = 1474.06.
Therefore with Driver Type X as the base, the estimated Driver Type relativities are:
1, 1474.06/830.19 = **1.776**.
The average Vehicle Class Relativities by Territory are:

$$\frac{(1)(30) + (1.243)(80) + (1.592)(20)}{30 + 80 + 20} = 1.241, \quad \frac{(1)(15) + (1.243)(22) + (1.592)(80)}{15 + 22 + 80} = 1.450,$$

$$\frac{(1)(200) + (1.243)(104) + (1.592)(60)}{200 + 104 + 60} = 1.167.$$

The average Driver Type Relativities by Territory are:

$$\frac{(1)(50) + (1.776)(80)}{50 + 80} = 1.478, \quad \frac{(1)(45) + (1.776)(72)}{45 + 72} = 1.478, \quad \frac{(1)(140) + (1.776)(224)}{140 + 224} = 1.478.$$

(Since each territory has the same mixture by Driver Class, the average Driver Class Relativities by Territory will have no effect on the final result.)
The adjusted pure premiums by Territory:

$$\frac{160{,}000}{(130)(1.241)(1.478)} = 671.01, \quad \frac{248{,}000}{(117)(1.450)(1.478)} = 989.06, \quad \frac{440{,}000}{(364)(1.167)(1.478)} = 700.82.$$

Therefore with Territory A as the base, the estimated Territory relativities are:
1, 989.06/671.01 = **1.474**, 700.82/671.01 = **1.044**.
Comment: Beyond what is shown in Basic Ratemaking.
Way too long for the assigned points!

One could instead start your sequential analysis with for example vehicle class rather than territory as I did.
In order for “each segment to be fully credible” all of the given values should have been multiplied by for example 1000.
The total losses in the grid by Vehicle Class is 848,000, while the total losses in the grid by Driver Type is 845,000; the losses for Territory 2 differ between the two grids. This was unintended.
It would have been helpful to be given the full three dimensional grid of exposures.
One could have iterated the procedure, going back and using the estimated territory relativities to re-estimate the Vehicle Class and Driver Type relativities.

11. (1.5 points) A risk manager has the following concerns about a quote for a large deductible policy compared to the company's current small deductible policy.

i. The expense provision in the rates includes expenses associated with claims below the deductible.

ii. The rates include a provision associated with the chance of the insured becoming bankrupt. This does not appear to be relevant to the exposure being underwritten.

iii. The profit margin is higher than the one used for the small deductible option.

Explain why each of the concerns may be invalid.

11. i. The insurer is responsible for settling all claims, including small claims below the deductible. Therefore, the rates should include a provision for this loss adjustment expense.

ii. Some insureds are unable to reimburse the insurer for claim payments below the deductible. It can take many years for some reimbursements to be due. It is difficult to predict which businesses will get into financial difficulty in the future.

Therefore, the rates need to include a provision for such unrecoverable reimbursements.

iii. Net of the reimbursements, the insurer is responsible for the excess losses. With a large deductible the insurer is responsible for a higher layer of loses than with a small deductible; this higher layer is more volatile and thus more risky than the losses excess of a small deductible. Thus the insurer requires a larger profit provision as a percent of premium.

Comment: Rather than the rates including a provision associated with the chance of the insured becoming bankrupt, the insured could be required to post collateral.

12. (2.5 points) Given the following outputs from a retention model:

Number of Policies Renewed		
Rate Change	Territory A	Territory B
-20%	11,610	6,825
-10%	11,475	6,750
0%	11,408	6,600
+10%	10,800	6,300
+20%	9,450	6,000

	Territory A	Territory B
Current Policy Count	13,500	7,500
Current Premium ($)	2,700,000	1,350,000
Ultimate Loss Ratio	80%	70%

- All expenses are variable.
- Policy count retention level does not affect premium projections.
- Rates are set to meet management targets:
    i. 70% permissible loss ratio for total book.
    ii. 80% minimum retention ratio in each territory.

a. (1.5 points) Recommend rate changes for territories A and B using the loss ratio method, taking management targets into account.

b. (0.5 point)

Describe how the rate changes recommended in part a. above could drive adverse selection.

c. (0.5 point)

Identify two non-pricing solutions that could return the fundamental insurance equation to balance.

12. (a) In order to retain at least 80% of polices, in Territory A we need: (80%)(13,500) = 10,800, while in Territory B we need: (80%)(7500) = 6000.
Thus the maximum allowable rate increase in Territory A is 10%, while the maximum rate increase in Territory B is 20%.
The indicated rate increase in Territory A is: 80%/70% - 1 = 14.3% > 10%.
Thus we need to cap the increase for Territory A at **10%**.
The estimated ultimate losses are: (2.70)(80%) + (1.35)(70%) = 3.105 million.
To have a 70% loss ratio we need premiums of: 3.105/0.7 = 4.4357 million.
"Policy count retention level does not affect premium projections."
Thus the required premiums from Territory B are: 4.4357 million - (2.7 million)(1.1) = 1.4657 million.
The required rate increase for Territory B is: 1.4657 / 1.350 - 1 = **8.57%**.
(b) Territory A is underpriced and Territory B is overpriced. Therefore, assuming competitors have correctly priced these territories, this insurer can expect to gain customers in Territory B, while losing customers in Territory A. This will lead to the overall rate level being inadequate. This adverse selection will continue until this insurer correctly prices these territories.
(c) 1. Expense reductions. For example, the company may try to reduce the marketing budget, reduce its staffing levels, or reduce commission rates.
2. Tighten the underwriting criteria or non-renew policies that have grossly inadequate premium relative to expected costs. The goal is to improve results by reducing losses more than the reduction in premiums.
3. Reduce the coverage provided by the policy. For example, a homeowners insurer may adjust the policy to exclude coverage for mold losses.
4. Institute better loss control procedures. For example, a workers compensation carrier may be able to reduce average severity by applying proactive medical management procedures and return-to-work programs for disability claims that are likely to escalate.
5. Order claims handlers to be less generous in settling claims.
6. Increase efforts to detect and fight fraudulent claims. (This might involve working with the government.)
7. Reduce the target profit.
Comment: Presumably the insurer can write new business to make up for the nonrenewals.
In part (b) one could instead apply a 9.5% increase to both territories.
Part (c) has been asked before, and will be asked again.