Solutions to the
Spring 2018
<u>CAS</u> Exam <u>Five</u>

(Only those questions on Basic Ratemaking)

There were 26 questions worth 55.5 points, of which 15.5 were on ratemaking worth 29.25 points.
(Question 8a covered reserving, while 8b covered ratemaking.)

This exam used the computer based testing with Excel.

(Incorporating what I found useful from the CAS Examiner's Report and sample solutions.)

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1. (1.50 points) Given the following for an insurance company:

- The exposure base is number of occupants in the home per year.
- The company writes a one-year homeowners policy effective April 1, 2017 for a home with four occupants.
- On October 1, 2017, two occupants leave the home, and the home has only two occupants for the remainder of the policy term.

a. (0.25 points) Calculate the total written exposures for calendar year 2017.
b. (0.25 points) Calculate the total earned exposures for calendar year 2017.
c. (0.25 points)
Calculate the total policy year 2017 written exposures evaluated as of September 30, 2017.
d. (0.75 points)
Briefly evaluate the number of occupants based on the three criteria of an exposure base.

1. (a) CY17 written exposures: (4)(1/2) + (2)(1/2) = **3**.
(b) CY17 earned exposures: (4)(1/2) + (2)(1/4) = **2.5**.
(c) As of September 30, 2017 we do not know of the change,
so the PY17 written exposures are **4**.
(d) 1. Proportional to Expected Loss:
The expected losses varies only slightly with the number of occupants.
For most perils, the number of occupants does not affect the expected losses.
4 occupants probably increase the expected losses very slightly compared to 2 occupants due to increased number of guests and thus an increased chance of liability claims. However, the expected homeowners losses are nowhere close to twice as much for twice the number of occupants.
Thus number of occupants violates this criterion.
2. Practical:
It would be difficult and costly to keep track of and verify the number of occupants.
It would be subject to manipulation by the insured.
It would be ambiguous; what if a relative comes to stay for a visit, how long does the visit have to be before they count as an occupant.
Thus number of occupants violates this criterion.
3. Historical Precedence:
Number of occupants is not currently used for homeowners insurance; house years (or amount of insurance years) is currently used. There would be a lack of industry benchmarks. There would also be serious difficulties in ratemaking for several years while moving to this new exposure base; all of the historical data would be on the old basis. Thus number of occupants violates this criterion.
Comment: While the original policy has 4 exposures, since the change takes place during 2017 it affects the CY17 exposures, similarly to endorsing an auto policy to add or remove a vehicle.
Assuming there are no further changes, at ultimate PY17 written exposures will be 3.
Number of occupants could be used as either a rating variable or underwriting variable.

2. (1.25 points) Given the following information:

Current Rate Review	
Number of Observations	100
Indicated Average Premium before credibility	\$750
Expected Value of Process Variance	7.5
Variance of Hypothetical Means	0.45
Annual Loss Trend	3%
Target Effective Date	July 1, 2018
Current Average Premium at Present Rates	\$600

Prior Rate Review	
Indicated Rate Change	20%
Implemented Rate Change	9%
Effective Date of Indication	July 1, 2016
Actual Effective Date	September 1, 2016

- The complement of credibility is trended present rates.

Calculate the credibility-weighted indicated premium using Buhlmann credibility.

2. K = EPV / VHM = 7.5 / 0.45 = 16.67.

Z = N / (N + K) = 100 / (100 + 16.67) = 85.7%.

The prior indicated rate was: (1.20/1.09)(\$600) = \$660.55.

Trending from the prior effective date to the current effective, the complement of credibility:

(660.55) (1.03^2) = \$700.78.

Credibility-weighted indicated premium: (85.7%)(750) + (1 - 85.7%)(700.78) = **\$743**.

Comment: It would be unusual to use Buhlmann Credibility in this context.

When using “trended present rates” with the pure premium method of ratemaking:

$$\text{Complement} = (\text{present rate})\ (\text{loss trend factor})\ \frac{\text{Prior Indicated Loss Cost}}{\text{Loss Cost Implemented with Last Review}}.$$

See pages 226-227 of Basic Ratemaking.

3. (1.5 points) An insurance company expanded its product offering into a new state last year.
a. (1.0 points) Briefly discuss an advantage and disadvantage of using the following data to determine loss trend for this state:
i. Competitor filings
ii. Internal company data
b. (0.5 points) Explain the loss development and trending overlap fallacy.

3. (a) i. Assuming the source of the loss trends is explained in the competitors fillings, the actuary can evaluate the appropriateness for this company of the different values selected by its competitors, and then select a reasonable value to use. For example, a similar competitor might have a large volume of (internal) data on which it based its loss trend; this would be a good choice.
On the other hand, the differences in books of business and underwriting may make a competitors trend not reflective of this company's expected future trends.
ii. Losses are subject to lots of random fluctuation.
Also in this case we would need to use data from other states, since this state is new.
Also if one uses recent reported data, one must estimate the effects of loss development.
Therefore, there can be considerable uncertainty in the estimate of loss trends based on internal company data.
On the other hand, internal company data is the most relevant to predict the future loss trends of this company.
(b) The "overlap fallacy" incorrectly asserts that loss development and loss trend capture the same change in loss patterns, and therefore, using both would be "double counting".
It is appropriate to both trend and develop losses; there is no overlap.
Loss trend projects losses from the midpoint of experience period to the midpoint of exposure period, while loss development adjusts immature losses to an ultimate level.
Comment: Part (b) has been asked many times before and will be asked again in the future.
The "overlap fallacy" is discussed at pages 117-118 of Basic Ratemaking.
If a competitor based its loss trend on either industrywide data or a CPI index, then this actuary could just do the same.

4. (1.75 points) An actuary uses classical credibility to develop full credibility standards for a private passenger auto indication.

Given the following information for State A:

	Exposures			Claim Counts		
Coverage	2015	2016	2017	2015	2016	2017
Bodily Injury	20,000	24,000	22,000	1,020	1,100	950
Collision	18,000	21,000	19,000	1,700	2,100	1,975

Claim counts standard for full credibility: 1,082.

a. (0.5 points) Calculate the number of exposures needed for full credibility for each coverage.

b. (0.25 points) Briefly explain why an actuary may prefer using an exposure standard for full credibility over a claim standard.

c. (0.5 points) Briefly describe one advantage and one disadvantage of using classical credibility.

d. (0.5 points) Propose a complement of credibility for the indication analysis and briefly evaluate the proposed complement.

4. (a) Average frequency for BI: (1020 + 1100 + 950) / (20,000 + 24,000 + 22,000) = 4.6515%.
Standard for full Credibility for BI: 1082/4.6515% = **23,261 exposures**.
Average frequency for Collision:
(1700 + 2100 + 1975) / (18,000 + 21,000 + 19,000) = 9.9569%.
Standard for full Credibility for Collision: 1082/9.9569% = **10,867 exposures**.
(b) The number of claims is subject to random fluctuation, while exposures are not.
If one uses claims, then a year of data in which there are fewer claims than expected will get lower credibility, while a year of data in which there are more claims than expected will get higher credibility.
This can make a significant difference when when working with relatively small amounts of data.
Alternately, if the definition of a claim has recently changed it would be better to use an exposure standard. (For example if claims closed with no payment were previously included in the claim count and now they are not.)
(c) "This approach has three main advantages. First, it is the most commonly used and is therefore generally accepted. Second, the data required for this approach is readily available. Finally, the computations are very straightforward.
The main disadvantage of this approach is that the derivation involves making several simplifying assumptions that may not be true in practice (e.g., no variation in the size of losses).
Another disadvantage of the classical credibility approach is that while it uses a selected complement, it does not take into account the quality of the estimator compared to the latest observation and therefore judgment is required."

(d) 1. Use countrywide data as the complement of credibility.
Desirable Properties: Accurate assuming the process variance of the larger group is smaller, logical relationship if the groups are closely related, readily available, and the calculations are easy to make.
This complement will be independent, if we exclude the subject experience from the complement of credibility, otherwise it is not.
While this complement is biased, appropriate adjustments may be able to reduce the bias.
However, any such adjustment may be difficult to explain.
Also deciding on what if any adjustment to make can be difficult.
2. Use trended rates from the previous rate indication as the complement of credibility.
Desirable Properties: The data required is readily available, the calculations are very straightforward, and the approach is easily explainable.
Unbiased since pure trended loss costs (i.e., no updating for more recent experience) are unbiased.
It is accurate when there is a lot of data and thus the process variance is small.
It is not accurate when there is little data and thus the process variance is large.
(The estimation errors of) this complement are independent of (the estimation errors of) the rate change based on latest experience provided the data periods do not overlap.
(The estimation errors of) this complement are dependent on (the estimation errors of) the rate change based on latest experience when the data periods do overlap.
From the CAS sample solutions:
3. Use a competitor's rates as the complement of credibility.
It is intuitive. However, it may not be ideal if the competitor has different underwriting standards or a different distribution of business/exposures.
4. Use private passenger auto data for a neighboring state as the complement of credibility.
This complement of credibility would be accurate (otherwise choose another neighboring state), is unlikely to be unbiased, is unlikely to be statistically independent from the base statistic (state A) especially if they're neighboring states, is easy to compute, and has a logical relationship to state A since they're neighboring states.
5. Use regional data, but excluding the data from State A, as the complement of credibility.
This would be an independent complement of credibility. It has a logical relationship to state A.
There should be a sufficient amount of data that is easily available. Because the data is from the same region one would not expect it to be very biased. Because the data is from the same region one would expect it to be reasonably accurate.
<u>Comment</u>: Part (b) is not mentioned in <u>Basic Ratemaking</u>.
In part (c), in contrast Buhlmann credibility does take into account the quality of the estimator compared to the latest observation.
In part (d), there are other possibilities such as using the rate change from the larger group applied to present rates.

5. (2.0 points) Given the following loss data for a property insurer:

Loss Peril, Fire

Accident Year	Sizes of loss	Claim Counts	Ground-Up Losses ($)
2014	0-$500,000	800	80,000,000
	> $500,000	20	12,000,000
2015	0-$500,000	780	81,900,000
	> $500,000	10	6,300,000
2016	0-$500,000	750	82,687,500
	> $500,000	25	13,891,500

Loss Peril, Flood

Accident Year	Sizes of loss	Claim Counts	Ground-Up Losses ($)
2014	0-$500,000	50	2,500,000
	> $500,000	30	16,500,000

Loss Peril, Other

Accident Year	Sizes of loss	Claim Counts	Ground-Up Losses ($)
2014	0-$500,000	1,500	15,000,000
	> $500,000		
2015	0-$500,000	1,450	15,225,000
	> $500,000	5	4,020,000
2016	0-$500,000	1,550	17,088,750
	> $500,000		

Excess loss threshold used by the insurer for individual reported losses: $500,000.
The insurer's claims information relating to all major events that occurred in the past 3 years are:

- The definition of catastrophe losses is $25 million in losses across the industry.
- All flood claims were caused by a single flood event that occurred in 2014, causing $500 million direct insured losses in the industry.
- In 2016, one fire claim occurred for the insurer as the result of a forest fire, causing $25 million direct insured losses in the industry.
- Ground-up loss of this fire claim is $1,000,000.
- There is no further development on losses.

a. (1.5 points) Calculate the excess loss factor.
b. (0.5 points)
Describe one reason to use an excess loss factor when setting property insurance rates.

5. (a) Excess Ratio = $\frac{\text{Excess Losses}}{\text{Total Losses - Excess Losses}}$.

Excess Loss Factor = 1 + Excess Ratio = $\frac{\text{Total Losses}}{\text{Total Losses - Excess Losses}}$.

Assume that catastrophe losses will be removed from the data, and a provision for catastrophes will be loaded back in. Thus the flood losses will not be used, but some provision for catastrophic flood losses will be included in the ratemaking.
For fire losses, we remove the $1 million catastrophe loss from the ratemaking data, but some provision for catastrophic fire losses will be included in the ratemaking.
Assume we will apply a single excess loss factor to fire plus other losses.
Excluding catastrophes, total losses are ($ million):
80 + 12 + 81.9 + 6.3 + 82.6875 + 13.8915 - 1 + 15 + 15.225 + 4.02 + 17.08875 = 327.11275.
The one large fire claim has 0.5 million excess of 0.5 million.
Excluding catastrophes, excess losses are ($ million):
{12 - (20)(0.5)} + {6.3 - (10)(0.5)} + {13.8915 - (25)(0.5)} - 0.5 + {4.02 - (5)(0.5)} = 5.7115.
Excess Loss Factor = 327.11275 / (327.11275 - 5.7115) = **1.018**.
Alternately, assume one will calculate separate excess loss factors for fire and other losses.
Excluding catastrophes, total losses for fire are ($ million):
80 + 12 + 81.9 + 6.3 + 82.6875 + 13.8915 - 1 = 275.779.
The one large fire claim has 0.5 million excess of 0.5 million.
Excluding catastrophes, excess fire losses are ($ million):
{12 - (20)(0.5)} + {6.3 - (10)(0.5)} + {13.8915 - (25)(0.5)} - 0.5 = 4.1915.
Excess Loss Factor for fire = 275.779 / (275.779 - 4.1915) = 1.015.
Excluding catastrophes, total other losses are ($ million):
15 + 15.225 + 4.02 + 17.08875 = 51.33375.
Excluding catastrophes, excess other losses are ($ million):
4.02 - (5)(0.5) = 1.52.
Excess Loss Factor for other losses = 51.33375 / (51.33375 - 1.52) = 1.031.
(b) A “shock loss” is a single loss so large that it has a material effect on the underwriting results of the insurer. The randomness of the number of such losses, if any, as well as the randomness of their size, introduces volatility into the yearly losses. Thus such large individual losses in the data can distort the estimate of future losses.
Therefore, the actuary should remove the amount excess of some limit from past data, and add back in a longer term average provision for such excess losses, via an excess loss factor. This adds stability.

Comment: In part (a), according to the CAS Examiner's report:

"Three cat treatments were accepted:

1. Calculating a non-cat excess loss factor, i.e. excluding all catastrophes from the loss data
2. Calculating an excess loss factor for both non-cat and cat losses, i.e. including all data
3. Calculating a combined cat load and excess loss factor, i.e. ground up cat losses plus excess non-cat losses"

In my solution, I did the first of these three alternatives; they did not mention my alternative of calculating separate excess factors for fire and "other".

In order to estimate future non-catastrophes losses, one would apply the excess loss factor that I calculated to the non-excess losses excluding catastrophes.

See Table 6.3 of Basic Ratemaking; part (a) is more detailed than what is shown there.

An actuary would usually use more than three years of data, for example 15 years as in Table 6.3, in order to calculate an excess loss factor.

The excess loss procedure would ideally be performed on reported losses that have been trended to future levels.

Typically flood losses would be covered by a separate flood insurance policy.

Provisions for catastrophes are either based on a computer model or on a longer term average.

Unlike shock losses that are individual high severity claims, a catastrophe denotes a natural or manmade disaster that is unusually severe and results in a very significant number of claims. This can include hurricanes, tornadoes, earthquakes, etc.

6. (1.75 points) Given the following information for a medical association:

Accident	Cumulative Reported Claim Counts Evaluated as of Month:					
Year	12	24	36	48	60	72
2010	26	52	71	80	85	89
2011	37	54	70	82	84	85
2012	44	81	85	100	110	112
2013	19	44	59	67	70	
2014	15	44	72	83		
2015	38	59	77			
2016	33	65				
2017	30					

Claims-Made Year	Step Factor
First	30%
Second	60%
Third	80%
Fourth	90%
Fifth	95%
Six and more	100%

- The medical association had claims-made policy coverages up until December 31, 2014.
- The claims-made policies have a retroactive date of January 1, 2010.
- The medical association switches to occurrence policies on January 1, 2015.
- All policies are annual.
- All policies incept January 1.

a. (0.25 points) Calculate the total reported claim counts covered under claims-made policies.
b.(0.25 points) Calculate the total reported claim counts covered under occurrence policies.
c. (1 point) Briefly describe the coverage gap and calculate the estimated number of ultimate claims falling in the gap for the medical association.
d.(0.25 points)
Briefly describe a solution for the medical association to address the coverage gap in part c. above.

6. (a) The claims-made policies cover losses reported during 2010 through 2014.
(Any claims that occur prior to 2010 are excluded by the retroactive date.)
This is: AY10 through 60 months, AY11 through 48 months, AY12 through 36 months, AY13 through 24 months, and AY14 through 12 months.
85 + 82 + 85 + 44 + 15 = **311**.
(b) The occurrence policies cover: AY15, AY16, and AY17.
The claim counts reported so far are: 77 + 65 + 30 = **172**.
(c) The gap includes: AY10 beyond 60 months, AY11 beyond 48 months, AY12 beyond 36 months, AY13 beyond 24 months, and AY14 beyond 12 months.

Accident	Incremental Reported Claim Counts Evaluated as of Month:					
Year	12	24	36	48	60	72
2010						4
2011					2	1
2012				15	10	2
2013			15	8	3	?
2014		29	28	11	?	?

While some of the numbers of claims reported are known, a few are still unknown.
One can estimate the number of unreported claims either by using the given step factors or via loss development techniques. Using the step factors assumes that the step factors are only based on the number of claims reported by lag and not also on any differences in average severity by lag.
Using the step functions, one estimates that the ultimate claims for AY13 are: (70)(100/95) = 73.7, or 73.7 - 70 = 4 more claims.
Using the step functions, one estimates that the ultimate claims for AY14 are: (83)(100/90) = 92.2, or 92.2 - 83 = 9 more claims.
Estimated ultimate number of claims for the coverage gap:
4 + 2 + 1 + 15 + 10 + 2 + 15 + 8 + 3 + 4 + 29 + 28 + 11 + 9 = **141**.
Alternately, I estimate the 48 to 60 claim count development factor as:
(85 + 84 + 110 + 70) / (80 + 82 + 100 + 67) = 1.061.
I estimate the 60 to 72 claim count development factor as:
(89 + 85 + 112) / (85 + 84 + 110) = 1.025.
One estimates that the ultimate claims for AY13 are: (70)(1.025) = 71.8, or 71.8 - 70 = 2 more claims.
One estimates that the ultimate claims for AY14 are: (83)(1.061)(1.025) = 90.3, or 90.3 - 83 = 7 more claims.
Estimated ultimate number of claims for the coverage gap:
4 + 2 + 1 + 15 + 10 + 2 + 15 + 8 + 3 + 2 + 29 + 28 + 11 + 7 = **137**.
(d) It can buy a **tail policy (extended reporting endorsement)**. This covers the gap when a policyholder who has been on claims-made coverage switches to occurrence coverage.

7. (1.75 points) Given the following information:
Underwriting profit provision 5%

	Expenses ($000)	% Fixed
Countrywide General Expenses	3,648	75%
Countrywide Other Acquisition Expenses	4,368	75%
State Tax, Licenses & Fees	315	25%
State Commission & Brokerage	1,868	0%

	Countrywide	State
Earned Premium ($000)	80,948	18,036
Written Premium ($000)	82,583	18,498
Earned Exposures	84,115	19,712
Written Exposures	87,476	20,217

Calculate the fixed expense fee per exposure.

7. For Fixed General Expense we take the ratio to CW earned exposures:
(75%) (3,648,000 / 84,115) = $32.53.
For Fixed Other Acquisition Expenses we take the ratio to CW written exposures:
(75%) (4,368,000 / 87,476) = $37.45.
For Fixed Taxes, Licenses and Fees we take the ratio to State written exposures:
(25%) (315,000 / 20,217) = $3.90.
Fixed Expenses: 32.53 + 37.45 + 3.90 = $73.88.
For Variable General Expense we take the ratio to CW earned premiums:
(25%) (3,648,000 / 80,948,000) = 1.13%.
For Variable Other Acquisition Expenses we take the ratio to CW written premiums:
(25%) (4,368,000 / 82,583,000) = 1.32%.
For Variable Taxes, Licenses and Fees we take the ratio to State written premiums:
(75%) (315,000 / 18,498,000) = 1.28%.
For Variable Commissions we take the ratio to State written premiums:
1,868,000 / 18,498,000 = 10.10%.
Variable expenses: 1.13% + 1.32% + 1.28% + 10.10% = 13.83%.
Fixed expense fee per exposure: $73.88 / (1 - 13.83% - 5%) = **$91.02**.

8. (6.50 points) Given the following information as of December 31, 2017:

Cumulative Reported Loss + ALAE ($000s) as of (months)			
Accident Year	12	24	36
2015	1,000	1,168	1,250
2016	1,100	1,285	
2017	1,050		

Calendar Year	Earned Premium ($000s)
2015	1,900
2016	2,085
2017	2,100

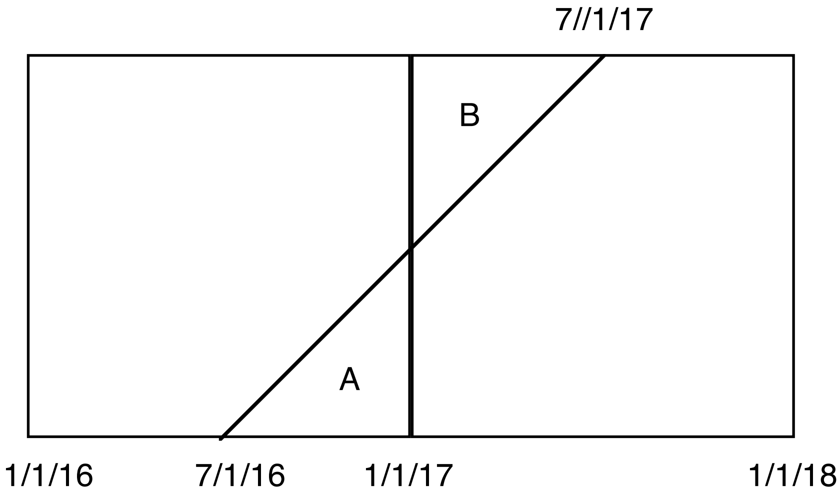
Annual loss and ALAE trend 2%
Annual premium trend 3%
Fixed Expense Ratio 10%
Variable Expense Ratio 30%
Profit and Contingencies Provision 5%
ULAE Provision (as % of Loss and ALAE) 7%
Rate change effective July 1, 2016 (only rate change in the past three years) +4%
36 to ultimate reported claim development factor 1.067

- All policies are annual.
- Exposures are written evenly throughout each calendar year.
- Rates will be in effect for one year.
- The historical experience is fully credible.

a. (4.25 points) Calculate the ultimate losses and ALAE for each accident year using the Cape Cod technique incorporating rate change and trend.

b. (2.25 points) Calculate the indicated rate change for policies effective January 1, 2019 using the latest three accident years of experience.

8. (a) Need to get on-level factors. AY15 is all written at the lower rate.
AY16 is written 7/8 at the lower rate and 1/8 at the higher rate.
AY17 is written 1/8 at the lower rate and 7/8 at the higher rate.



Area A = Area B = $(1/2)(1/2)^2 = 1/8$.
OLF for AY15 is: 1.04.
OLF for AY16 is: 1.04 / {(7/8)(1) + (1/8)(1.04)} = 1.035.
OLF for AY17 is: 1.04 / {(1/8)(1) + (7/8)(1.04)} = 1.005.
Put the premium on level and trend it to AY17.

AY	Earned Premium	OLF	Premium Trend	OL, Trended Premium
2015	1900	1.040	1.061	2096
2016	2085	1.035	1.030	2223
2017	2100	1.005	1.000	2110
Total				6430

Trend the latest reported Loss and LAE to AY17.

AY	Loss & ALAE	Loss Trend	Trended Loss & ALAE
2015	1250	1.040	1300
2016	1285	1.020	1311
2017	1050	1.000	1050
Total			3661

Compute link ratios from the reported Loss & LAE.
12-24: (1168+1285)/(1000+1100) = 1.168. 24-36: 1250/1168 = 1.070.

AY	Expected Percent Developed
2015	1/1.067 = 93.7%
2016	1/{(1.067)(1.070)} = 87.6%
2017	1/{(1.067)(1.070)(1.168)} = 75.0%

AY	Expected % Developed	OL, Trended Premium	Used Up Premium
2015	93.7%	2096	1964
2016	87.6%	2223	1947
2017	75.0%	2110	1582
Total			5494

Adjusted Expected Claim Ratio = $\frac{\text{Total Trended Loss \& ALAE}}{\text{Total Used Up Premium}}$ = 3661/5494 = 66.6%.

Then we "unadjust" this ratio to the earlier years by backing out trend and OLFs.
Unadjusted ratio = (Adjusted ratio) (On-level Factor) (Premium trend) / (Loss trend).
For example for AY15: (66.6%) (1.040)(1.03^2)/1.02^2 = 70.6%.

AY	Adjusted Expected Claim Ratio	OLF	Premium Trend	Loss Trend	Unadjusted Expected Claim Ratio
2015	66.6%	1.040	1.061	1.040	70.6%
2016	66.6%	1.035	1.030	1.020	69.6%
2017	66.6%	1.005	1.000	1.000	66.9%

Then for each AY, the estimated ultimate loss & ALAE is:
(premium) (unadjusted expected claim ratio).

AY	Earned Premium	Unadjusted Expected Claim Ratio	Ultimate Losses
2015	1900	70.6%	**1341**
2016	2085	69.6%	**1451**
2017	2100	66.9%	**1405**

(b) The average date of writing under the new rates is July 1, 2019.
The average date of accident under the new rates (annual policies) is January 1, 2010.
The trend period for AY17 is from July 1, 2017 to January 1, 2010, or 2.5 years.
Use the estimates of ultimate losses by AY from part (a).

AY	Earned Premium	OLF	Premium Trend	OL, Trended Premium
2015	1900	1.040	1.142	2257
2016	2085	1.035	1.109	2393
2017	2100	1.005	1.077	2272
Total				6923
AY	Ultimate Losses	Loss Trend	Trended Losses	Loss Ratio
2015	1341	1.093	1466	64.9%
2016	1451	1.072	1555	65.0%
2017	1405	1.051	1476	65.0%
Total			4497	65.0%

I will use the loss ratio based on the total losses divided by total premiums:
4497/6923 = 65.0%.

Indicated rate change is: $\frac{(65.0\%)(1.07) + 10\%}{1 - 30\% - 5\%} - 1 = \mathbf{22.4\%}$.

<u>Comment</u>: Part (a) is on reserving, while part (b) is on ratemaking.
The Cape Cod Method (Stanard-Buhlmann) is covered in Chapter 10 of Friedland.
The estimated IBNR by AY are in thousands :
1341 - 1250 = 91, 1451 - 1285 = 166, 1405 - 1050 = 355.
If you did the calculation in one big spreadsheet, your answer may differ somewhat from mine due to intermediate rounding.
In part (a), alternatively you could have brought all premiums to the average rate level for 2017 rather than current rate level, and should have gotten the same result; however, in part (b) we need to bring premiums to the current rate level.
As always, if one did not do part (a), one could just assume (reasonable) estimated ultimate losses to use in part (b); then if one does part (b) correctly, one should get credit for part (b).

9. (1.75 points) Given the following information for two personal automobile policies:

Three-Year Time Horizon for 21 Year-old Driver					
Year	Age	Premium ($)	Loss ($)	Expense ($)	Renewal Probability
1	21	1,215	1,200	50	
2	22	1,200	1,125	20	75%
3	23	1,185	1,050	20	75%
Total		3,600	3,375	90	

Three-Year Time Horizon for 65 Year-old Driver					
Year	Age	Premium ($)	Loss ($)	Expense ($)	Renewal Probability
1	65	900	795	50	
2	66	900	839	20	95%
3	67	900	859	20	95%
Total		2,700	2,493	90	

Annual discount rate 5%

- Policies are written on January 1.
- Premium is collected and the expense and losses are incurred on January 1.

Evaluate whether the 21 year-old or 65 year-old has a larger percentage return on premium over the three-year horizon.

9. For the first driver for example, for year 3, the profit is: 1185 - 1050 - 20 = 115.

We discount this profit and multiply by the cumulative persistency of 0.75^2:

$(115)(0.75^2) / 1.05^2 = 59$.

For the first driver for example: $(1185)(0.75^2) / 1.05^2 = 605$.

Year	Age	Premium	Losses	Expenses	Profit		
1	21	$1215	$1200	$50	-$35		
2	22	$1200	$1125	$20	$55		
3	23	$1185	$1050	$20	$115		
					PV of	PV of	
	Renewal	Cumulative	Adjusted	Discount	Adjusted	(Adjusted)	Profit
Year	Prob.	Persistency	Profit	Factor	Profit	Premium	Percent
1		100.0%	-$35.00	1.0000	-$35.00	$1215.00	-2.9%
2	75%	75.0%	$41.25	1.0500	$39.29	$857.14	4.6%
3	75%	56.2%	$64.69	1.1025	$58.67	$604.59	9.7%
Total					$62.96	$2676.73	**2.4%**

$62.96 / $2676.73 = 2.4%.

Year	Age	Premium	Losses	Expenses	Profit		
1	65	$900	$795	$50	$55		
2	66	$900	$839	$20	$41		
3	67	$900	$859	$20	$21		
					PV of	PV of	
	Renewal	Cumulative	Adjusted	Discount	Adjusted	(Adjusted)	Profit
Year	Prob.	Persistency	Profit	Factor	Profit	Premium	Percent
1		100.0%	$55.00	1.0000	$55.00	$900.00	6.1%
2	95%	95.0%	$38.95	1.0500	$37.10	$814.29	4.6%
3	95%	90.2%	$18.95	1.1025	$17.19	$736.73	2.3%
Total					$109.29	$2451.02	**4.5%**

65 year-old has a larger percentage return on premium over the three-year horizon at 4.5% than the 21 year-old at 2.4%.

Comment: “Loss” would have been better as instead “Expected Loss”.

Lifetime analysis as per Tables 13.16 and 13.17 in Basic Ratemaking.

10. (2 points) A company is considering whether to use vehicle color as a private passenger auto rating variable for bodily injury coverage.
a. (1 point) Evaluate the use of vehicle color as a rating variable using two operational criteria.
b. (1 point) Evaluate the use of vehicle color as a rating variable using two social criteria.

10. (a) 1. <u>Objectivity</u>:
It is not always clear what category in which to put a car. Some cars have two tones. Even one tone can be on the borderline between green and blue for example. There is a chance for overlap, ambiguity and administrative error. Color of car does not satisfy the operational criterion of objectivity.
2. Low Administrative <u>Expense</u>.
It would be inexpensive to get the color of the insured car from its official vehicle registration form. Color of car does satisfy the operational criterion of inexpensive to administer.
3. <u>Verifiability</u>: can be checked by insurer; not subject to manipulation or lying by the insured.
Color of car could be checked by the insurer; although, there is nothing to stop the insured from repainting the car once the insurer has inspected it.
Color of car does satisfy the operational criterion of verifiability.
(b) 1. Affordability: high rates may cause problems in being able to afford insurance, particularly if rates are negatively correlated with income. (low incomes $\Leftrightarrow$ high rates.)
It is unlikely that color of car is correlated with income.
Color of car does satisfy the social criterion of affordability.
2. Controllability: a controllable variable is one which is under the control of the insured; the insured can modify his behavior in order to reduce his insurance costs.
The insured can control the color of his car either at time of purchase, or by repainting it.
Color of car does satisfy the social criterion of controllability.
3. Causality: differences in the rating variable are responsible for differences in insurance costs.
Neither regulators nor insureds would believe that color of car causes a difference in bodily injury insurance costs.
Color of car does not satisfy the social criterion of causality.
4. Privacy: reluctance to disclose personal information.
The color of ones car is <u>not</u> personal information; rather it is easily observed as the car is driven.
Color of car does satisfy the social criterion of privacy.
<u>Comment</u>: Almost identical to 5, 5/08, Q.28.
These are just examples of possible answers. In each case, only list and evaluate two criteria.
From the CAS Examiner's Report: "Change in rating algorithm might be costly when involving change in the system, especially to the extent of a complicated algorithm."
I do not see this as one of the operational criteria in the textbook; it is related to the one-time cost to implement a new rating variable/algorithm, rather than the ongoing cost to administer the new rating variable/algorithm if it were put in place. Of course, both the initial and ongoing costs are important considerations.
Color of car does seem to have an effect on the theft rate, and thus on expected losses of Other Than Collision (Comprehensive) coverage.

11. (1.5 points) Given the following information:

		Company A		Company B	
Risk	True Expected Cost	Insured RIsks	Charged Rate	Insured RIsks	Charged Rate
High	170	5,000	160	5,000	150
Low	130	5,000	140	5,000	150
Total	150	10,000	150	10,000	150

- There are no underwriting expenses or profit provisions.
- Market consists of 10,000 high risk insureds and 10,000 low risk insureds.
- Both companies write only one line of business.
- 20% of all insureds shop at renewal and base their purchasing decision on price.

a. (1 point) Calculate the profitability for each company after one renewal cycle.
b. (0.5 points) Briefly describe two possible actions for the company experiencing adverse selection to reduce the risk of insolvency.

11. (a) 20% of high risk insured with A will go to B which charges a lower rate than A.
20% of low risk insured with B will go to A which charges a lower rate than B.
Thus 1000 high risk insureds switch from A to B, while 1000 low risk insureds switch from B to A.
Premium for A: (4000)(160) + (6000)(140) = 1.48 million.
Expected Losses for A: (4000)(170) + (6000)(130) = 1.46 million.
Profit for A: 1.48 million - 1.46 million = **20,000**.
Premium for B: (6000)(150) + (4000)(150) = 1.50 million.
Expected Losses for B: (6000)(170) + (4000)(130) = 1.54 million.
Profit for B: 1.50 million - 1.54 million = **-40,000**.
(b) 1. B can adjust its relativities so that it charges the high risk insureds more and the low risk insureds less. In other words, Insurer B can adopt the class plan used by A and use similar rates.
2. B can change its underwriting criteria so as to not write high risk insureds. Its rate charged to low risk insureds should then come down to more closely match the expected costs.
From the sample answers provided by the CAS:
3. Increase premiums and reduce costs with the expectation of becoming the insurer of choice for high-risk insureds. This can still be a profitable niche, if the policies are priced correctly.
4. Company B should alter their marketing strategy to focus more on low risk insureds to prevent losing so many to company A. This will boost their profit from low risk insureds if they can attract more.
<u>Comment</u>: Company B is experiencing adverse selection.
In part (b), insurer B may have to figure out how insurer A is distinguishing between the two types of insureds.

12. (2.5 points) Given the following information for an insurance company:

Class	Earned Exposures	Reported Loss and ALAE	Number of Claims	Current Relativity	True Relativity
A	15,271	\$864,000	924	1.00	1.00
B	7,250	\$732,000	623	1.10	1.73
C	10,532	\$505,000	185	1.80	0.72

Full credibility standard for number of earned exposures: 18,000

- Partial credibility is determined based on the square root rule.
- Complement of credibility is equal to normalized current class relativities.
- Class A remains the base class.

a. (2 points)
Calculate the indicated rate change for each class to achieve a revenue-neutral overall change.
b. (0.25 points)
Briefly describe one possible reason why the indicated relativities do not match the true relativities.
c. (0.25 points)
Briefly explain an adjustment to the univariate pure premium method to improve its result.

12. (a) For example, \$47.95/\$63.56 = 0.7543. 1.80/1.2768 = 1.4097. $\sqrt{10{,}532/18{,}000}$ = 76.5%.

(76.5%)(0.7543) + (1 - 76.5%)(1.4097) = 0.908. 0.9084/0.8816 = 1.0303.

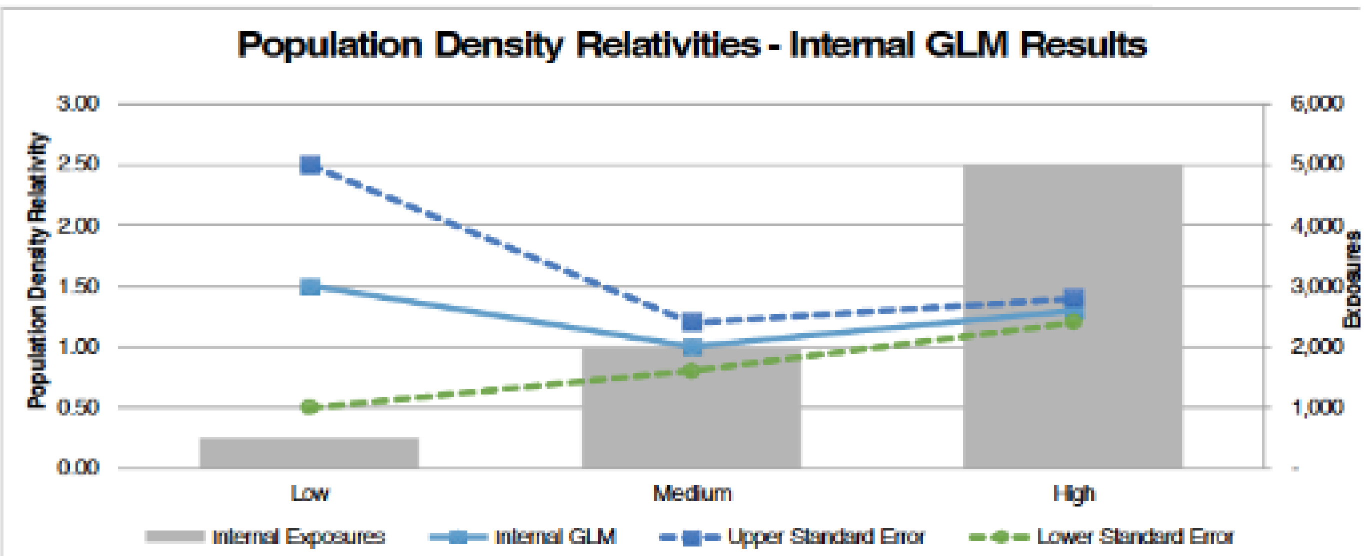
	[1]	[2]	[3]	[4]	[5]	[6]
Class	Earned	Reported	Pure	Indicated	Current	Normalized
	Exposures	Loss & ALAE	Premium	Relativity	Relativity	Current
						Relativity
			=[2]/[1]	= [3]/Total [3]		=[5]/Total [5]
A	15,271	\$864,000	\$56.58	0.8901	1.00	0.7832
B	7,250	\$732,000	\$100.97	1.5884	1.10	0.8615
C	10,532	\$505,000	\$47.95	0.7543	1.80	1.4097
Total	33,053	\$2,101,000	\$63.56	1.0000	1.2768	1.0000
	[7]	[8]	[9]			
Class	Credibility	Credibility	Cred-Weighted			
		Weighted	Relativity			
		Relativity	w.r.t. Base			
			=[8]/[8] for A			
A	92.1%	0.8816	1.0000			
B	63.5%	1.3228	1.5004			
C	76.5%	0.9084	1.0303			
Total		0.9869	1.1194			

The weighted average of the current relativities is:
{(15,271)(1.00) + (7250)(1.10) + (10,532)(1.80)} / (15,271 + 7250 + 10,532) = 1.2768.
The weighted average of the indicated relativities is:
{(15,271)(1.0000) + (7250)(1.5004) + (10,532)(1.0303)} / (15,271 + 7250 + 10,532) = 1.1194.
Therefore, the off-balance factor to multiply by is: 1.2768/1.1194.
Rate change for Class A (base): 1.2768/1.1194 - 1 = **14.1%**.
Rate change for Class B: (1.5004/1.10)(1.2768/1.1194) - 1 = **55.6%**.
Rate change for Class C: (1.0303/1.80)(1.2768/1.1194) - 1 = **-34.7%**.
(b) The classes may have different distributions of exposures by other rating variables.
For example, Class B could have a larger percentage of insureds from low rated territories, while Class C has a larger percentage of insureds from high rated territories.
Alternately, volatility in the loss experience by class, corresponding to the less than full credibility.
(c) One can use the adjusted pure premium method. One would instead use adjusted exposures, which are the product of the exposures for a Class and the average relativities over the other rating variables (including territory).
Comment: See page E-3 in Basic Ratemaking for the indicated relativities in part (a).
Assume for example that the current rates are: 100, 110, and 180.
Then the current premiums are: (15,271)(100) + (7250)(110) + (10,532)(180) = 4,220,360.
The indicated rates are: (100)(1.141) = 114.1, (110)(1.556) = 171.16, and (180)(0.653) = 117.54.
The corresponding premiums are: (15,271)(114.1) + (7250)(171.16) + (10,532)(117.54) = 4,221,262, the same subject to rounding.

13. (2 points) A company is implementing population density in its rating plan for a line of business. The company has three pieces of information.

- An analysis the company performed using a generalized linear model (GLM) on internal data only.
- The rating factors from a competitor's rate filing.
- The rating factors from an external industry benchmark.

Population Density	Internal GLM	Competitor	Industry
Low	1.5	0.9	0.4
Medium	1.0	1.0	1.0
High	1.3	1.1	1.6



Provide a recommendation for the low and high level rating factors given the chart above and the external rating factors.
Justify the recommendations considering:
i. GLM diagnostics
ii. Competitor
iii. Industry benchmark

13. The GLM relativity for Low is not significantly different from one; the confidence interval is from 0.5 to 2.5. (This is not surprising given the small number of exposures.) Also the pattern of indicated relativities is unusual; one would expect Medium to be between Low and High. Thus I will put no weight on the GLM result for Low.
Alternately, the Low Density confidence intervals contain the relatively for Medium Density of one.
The amount of data and thus the credibility for Low Density is very small.
Thus, it is not statistically proven that Low Density has a different relativity than Medium Density.
I would suggest grouping the Low and Medium Density levels together; they would both get a relativity of one.

In contrast, the GLM relativity for High is significantly different from zero; the confidence interval is from 1.2 to 1.4.
In the absence of information, I will assume that the industry benchmark is based on more data and analysis than my competitor's relativities. I will also assume that my competitor has similar underwriting guidelines to my company. For Low, I will select a relativity 0.6, between my competitor's and the industry, but closer to the industry.
For High I will use the GLM indication of 1.3, although anything between 1.2 and 1.4 makes sense.
This is between the competitor's and industry relativities.
Comment: One would usually take the class with the most exposures as the base class, which in this case is High. However, Medium may have the most exposures on an industrywide basis.
In any case, running the GLM with Medium as the base class results in a larger standard deviation of the class relativities, than if High were treated as the base class.
A final selection of relativities, within a range of reasonableness, would depend on marketing considerations. Since most of the exposures are in Medium and High, the relativity for High will be of the most practical importance to this insurer.

14. (1.5 points) Given the following information about a home:
Home value: $250,000
Insured value: $200,000
Coinsurance requirement: 90%
a. (1 point) Calculate the coinsurance penalty for the following loss amounts:
i. $50,000
ii. $220,000
iii. $250,000
b. (0.5 points) Briefly describe two problems with underinsurance.

14. (a) The coinsurance requirement is: (90%)($250,000) = $225,000, which is not met.
The coinsurance apportionment ratio is: 200/225 = 8/9.
i. The insured is paid: (8/9)(50,000) = 44,444.
The coinsurance penalty is: 50,000 - 44,444 = **$5556**.
ii. The insured is paid: (8/9)(220,000) = 195,556.
He would only be paid the insured amount of 200,000 in any case.
Thus the coinsurance penalty is: 200,000 - 195,556 = **$4,444**.
iii. The insured is paid: Min[200,000, (250,000)(8/9)] = 200,000.
There is no coinsurance penalty.
(b) 1. The insured is not fully covered in the event of a total or near-total loss. Thus, in these cases, the insurance payment will not return the insured to the condition prior to the loss.
2. If the insurer assumes all homes are insured to full value, then the premium will not be sufficient to cover the expected payments for a home that is underinsured; this is due to the existence of partial losses. Therefore, the rates will not be equitable.
<u>Comment</u>: The coinsurance penalty = reduction due to coinsurance of the amount an insured is paid = (Amount that would have been paid if there were no coinsurance clause) - (Amount actually paid).

$$\text{coinsurance penalty} = \begin{cases} L - I & \text{if } L \le F \\ F - I & \text{if } F < L < cV \\ 0 & \text{if } cV \le L \end{cases} .$$

i. is in the first interval. L = 50,000 and I = 44,444.
ii. is in the second interval. F = 200,000 and I = 195,556.
iii. is in the third interval.

15. (2.75 points) Given the following for a workers compensation policyholder:

Policy Year	Actual Primary Losses ($)	Actual Excess Losses ($)	Payroll ($)
2015	10,000	120,000	2,000,000
2016	10,000	80,000	2,100,000
2017	10,000	60,000	2,205,000
Total	30,000	260,000	6,305,000

Expected loss rate per $100 of payroll: 3.25
D-Ratio: 0.20
Ballast value: 40,000
Weighting value: 0.30
Minimum retrospective premium ratio: 0.70
Maximum retrospective premium ratio: 1.30
Loss conversion factor: 1.10
Per accident loss limitation: 100,000
Expense allowance (excludes tax multiplier): 0.25
Expected loss ratio: 0.70
Tax multiplier: 1.05
Standard premium: 800,000
Insurance charge for maximum premium: 0.40
Insurance savings for minimum premium: 0.05
Limited reported losses: 200,000
a. (1.25 points) Calculate the experience rating modification factor.
b. (1.5 points) Calculate the retrospective premium.

15. (a) E = (3.25)(6,305,000/100) = $204,912.

E_p = (0.2)(204,912) = $40,982. E_e = $204,912 - $40,982 = $163,930.

$$M = \frac{A_p + W\,A_e + (1 - W)E_e + B}{E + B} = \frac{30{,}000 + (0.3)(260{,}000) + (0.7)(163{,}930) + 40{,}000}{204{,}912 + 40{,}000}$$

= **1.073**. A 7.3% debit mod.

Alternately, Z_p = E / (E + B) = 204912 / (204,912 + 40,000) = 83.7%.

Z_e = W Z_p = (0.3)(83.7%) = 25.1%.

$$M = 1 + Z_p\,\frac{A_p - E_p}{E} + Z_e\,\frac{A_e - E_e}{E} = 1 + (83.7\%)\,\frac{30{,}000 - 40{,}982}{204{,}912} + (25.1\%)\,\frac{260{,}000 - 163{,}930}{204{,}912}$$

= **1.073**.

(b) Net Insurance Charge = (Insurance Charge - Insurance Savings) (Expected Loss Ratio) (LCF)
= (0.40 - 0.05) (0.70) (1.1) = 0.2695.

Basic Premium Factor =
Expense Allowance - (Expenses provided for through LCF) + (Net Insurance Charge)
= 0.25 - (1.1 - 1)(0.70) + 0.2695 = 0.4495.

Basic Premium = (0.4495)(800,000) = $359,600.

R = (b + cL)T = {$359,600 + (1.1)($200,000)} (1.05) = $608,580.

Minimum Premium = (0.7)($800,000) = $560,000.

Maximum Premium = (1.3)($800,000) = $1,040,000.

Thus the retro premium is **$608,580**.

Comment: The experience rating is as per the NCCI Plan.

The Standard Premium is the manual premium (not given) times the experience rating mod.
Presumably, the charge for the per accident loss limitation of 100,000 is included in the given insurance charge, as is the case in the example in Basic Ratemaking.

16. (1.5 points) Given the following transactional data:

Claim ID	Accident Date	Transaction Date	Amount Paid on Transaction Date	Ending Case Outstanding
A	October 31, 2014	February 1, 2015	200	300
		July 31, 2015	300	0
B	April 28, 2015	July 1, 2015	0	1000
		October 15, 2015	500	500
		August 28, 2016	500	200
C	June 1, 2015	July 7, 2015	100	650
		January 15, 2016	250	400
		May 15, 2017	300	0
D	September 24, 2015	March 1, 2016	0	325
		December 19, 2017	375	250

a. (0.75 points) Calculate accident year 2015 paid claims as of:
i. 12 months
ii. 24 months
iii. 36 months
b.(0.75 points) Calculate accident year 2015 reported claims as of:
i. 12 months
ii. 24 months
iii. 36 months

16. The first accident occurred in 2014, so contributes nothing to AY15.
(a) Paid as of 12/31/15: 500 + 100 = **600**.
Paid as of 12/31/16: 600 + 500 + 250 = **1350**.
Paid as of 12/31/17: 1350 + 300 + 375 = **2025**.
(b) Reported = Paid + Case Outstanding.
Reported as of 12/31/15: 600 + 500 + 650 = **1750**.
Reported as of 12/31/16: 1350 + 200 + 400 + 325 = **2275**.
Reported as of 12/31/17: 2025 + 200 + 250 = **2475**.
Alternately, Reported as of 12/31/16 =
Reported as of 12/31/15 + amounts paid during 2016 + changes in case reserves during 2016 =
1750 + 500 + (200 - 500) + 250 + (400 - 650) + 325 = **2275**.
Reported as of 12/31/17 = 2275 + 300 + (0 - 400) + 375 + (250 - 325) = **2475**.