Solutions to the Spring 2017 CAS Exam Five

(Only those questions on Basic Ratemaking)

There were 26 questions worth 57.5 points, of which 13 were on ratemaking worth 30.25 points.

The exam and examiner's report are available from the CAS.
The solutions and comments are solely the responsibility of the author.

(Incorporating what I found useful from the CAS Examiner's Report)

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1. (1.25 points) An insurance company portfolio consists of the following:
- 1,000 two-year policies with an effective date of April 1, 2015.
- 1,000 one-year policies with an effective date of July 1, 2015.

a. (0.75 point) Calculate the following for calendar year 2015:
   i. Written exposures
   ii. Earned exposures

b. (0.5 point) Calculate the earned exposures for calendar year 2016.

1. I will assume that each policy insures one exposure per year.
a. Then the written exposures for CY15 are: (2)(1000) + (1)(1000) = **3000**.
Each of the policies written 4/1/15 contributes 3/4 of an earned exposure to CY15.
Each of the policies written 7/1/15 contributes 1/2 of an earned exposure to CY15.
The earned exposures for CY15 are: (3/4)(1000) + (1/2)(1000) = **1250**.
b. Each of the policies written 4/1/15 contributes 1 earned exposure to CY16.
(Each of these policies expires at the end of June 2017.)
Each of the policies written 7/1/15 contributes 1/2 of an earned exposure to CY16.
The earned exposures for CY16 are: (1)(1000) + (1/2)(1000) = **1500**.
Comment: The question should have said for example, that each policy covers one home.

2. (2 points) Given the following policy year information:

Effective Date	Overall Average Rate Change
October 1, 2015	5%
April 1, 2016	10%
October 1, 2016	5%

- All policies are annual.
- Policy year 2016 written premium = $100,000.
- Policy year 2016 earned premium = $100,000.
- Policy year 2016 ultimate losses including LAE = $80,000.
- Loss trend = 0%.
- Premium trend = 0%.
- There are no fixed expenses.

a. (1 point)
Calculate the policy year 2016 earned premium at current rate level using the parallelogram method.

b. (0.25 point) Calculate the variable expense ratio that would earn an underwriting profit of 5% at the current rate level.

c. (0.5 point) Assume the company rapidly increased exposures throughout 2016. Explain whether the parallelogram method would overstate or understate a rate level indication.

d. (0.25 point) Briefly describe a scenario in which policy year premium is not fixed at the completion of the policy year.

2. a.

Date	Rate Level Index
October 1, 2015	1.00
April 1, 2016	1.10
October 1, 2016	(1.1)(1.05) = 1.155

For PY16, assuming constant rate of writing, 1/4 is written at rate level 1, 1/2 is written at rate level 1.1, and 1/4 is written at rate level 1.115.

The OLF factor is: $\frac{1.155}{(1/4)(1) + (1/2)(1.1) + (1/4)(1.155)} = 1.061.$

On Level PY16 earned premium is: (1.061)(100,000) = **106,100**.

b. Assume a variable expense load of v.

We want: 106,100 = 80,000 / (1 - 0.05 - v). ⇒ v = **19.6**%

c. More of the exposures would be written at the more recent higher rate levels.

Thus the OLF factor calculated in part (a) would be too large.

Thus the estimated premium would be too large, and the loss ratio would be too low.

Thus the parallelogram method would understate a rate level indication.

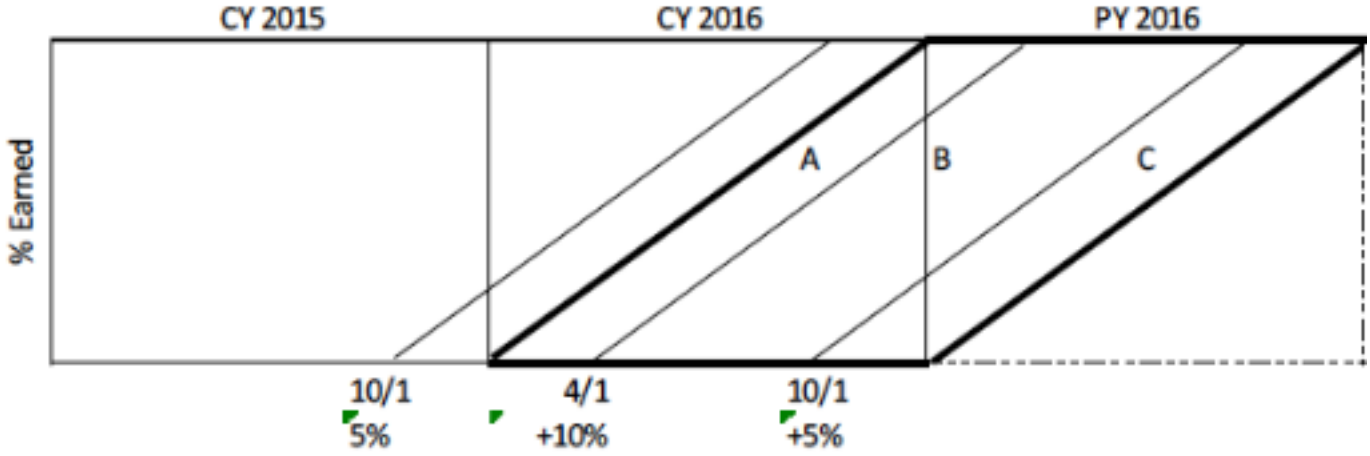
d. For a line of insurance where the exposure base is sales or payroll, for example commercial general liability insurance, the final premium would depend on the final audited values.

Retro policies would continue to have adjustments for many years after the end of the policy year.

Comment: The "end" of PY 2016 is usually considered December 31, 2017.

As of December 31, 2016, premiums would not be final since there are policies still in effect that may be either canceled or endorsed.

In part (a), one could draw a diagram, although many people would not have to:



Area A = 1/4. Area B = 1/2. Area C = 1/4.

Also in part (a), one could instead define October 1, 2015 as rate level 1.05; as long as one is consistent one would get the same on level factor.

3. (2 points) Given the following information about two claims:

Claim Number	Accident Date	Transaction Date	Incremental Payment	Ending Case Reserves
1	January 1, 2015	January 1, 2015	$0	$20,000
1	January 1, 2015	January 1, 2016	$25,000	$0
2	April 1, 2015	July 1,2015	$0	$50,000
2	April 1, 2015	October 1, 2015	$25,000	$75,000
2	April 1, 2015	April 1, 2016	$100,000	$20,000

a. (0.5 point) Calculate the incurred losses for accident year 2015 as of May 1, 2016.
b. (1 point) Calculate the incurred losses for calendar year 2015 and calendar year 2016.
c. (0.5 point) Briefly describe one advantage and one disadvantage of calendar year aggregation.

3. (a) Both claims contribute to AY15, since they both occurred during 2015.
All of the listed transactions are on or before May 1, 2016.
In each case, we take paid losses plus the change in case reserves:
20,000 + (25,000 - 20,000) + 50,000 + (25,000 + 25,000) + (100,000 - 55,000) = $**170,000**.
Alternately sum the total amount paid and add the final case reserves:
25,000 + 25,00 + 100,000 + 0 + 20,000 = $**170,000.**
(b) CY15: 20,000 + 50,000 + (25,000 + 25,000) = $**120,000**.
CY16: (25,000 - 20,000) + (100,000 - 55,000) = **$50,000**.
(c) An advantage is that calendar year data is available quickly and is not subject to development.
A disadvantage is that there is a poor match between calendar year losses and premiums. Calendar year losses include the effect of reserve changes and payments on older claims.

4. (2.25 points) Given the following information for an insurance company:

	($000)
Written Premium	15,000
Earned Premium	12,000
Ultimate Losses and LAE	10,000
Commissions and Brokerage	2,250
Other Acquisition Costs	750
Taxes, Licenses, and Fees	300
General Expenses	360

- All expenses are variable.
- Underwriting profit provision = -5%.

a. (1 point) Calculate the following expense ratios to premium and briefly justify the selection of the premium basis used in each calculation:
   i. Commissions and brokerage
   ii. General expenses

b. (0.5 point) Calculate the permissible loss and LAE ratio.

c. (0.25 point) Briefly explain how the company may return a profit with an underwriting profit provision less than 0%.

d. (0.5 point) Demonstrate whether or not the company met underwriting profit expectations.

4. (a) For commissions and brokerage take a ratio to written premium, since these expenses are paid when the policy is written. 2,250 / 15,000 = **15.0%**.
For General Expense take a ratio to earned premium, based on the assumption that these expenses are incurred approximately as coverage is provided and premiums are earned.
360 / 12,000 = **3%**.
(b) Other Acquisition: 750/15,000 = 5%. Taxes, Licenses, and Fees: 300/15,000 = 2%.
Permissible Loss and LAE: 1 - 15% - 3% - 5% - 2% - (-5) = **80%**.
(c) Between the time that premiums are collected and losses are paid, the insurer has an opportunity to earn investment income. If for the given line of insurance that average length of time is long enough and the return on investment income is large enough, then the insurer can earn a profit while running an underwriting loss.
Alternately, using lifetime analysis (the asset share pricing model), the company may gain a positive return in the long run with a negative profit provision in the one-year time horizon. The company may benefit from having a low profit provision now in order to gain market share, while planning to increase profits later.
(d) Ratio of Ultimate Loss and LAE to Earned Premium: 10,000 / 12,000 = 83.3% > 80%,
thus the insurer did <u>not</u> meet its underwriting profit expectations.
Alternately, with a loss ratio of 83.3%, the underwriting profit is: 100% - 83.3% - 25% = -8.3%.
Since -8.3% < -5%, the insurer did <u>not</u> meet its underwriting profit expectations.
Alternately, the indicated premium is: 10,000 / 0.80 = $12,500.
Since $12,000 < $12,500, the insurer did <u>not</u> meet its underwriting profit expectations.

5. (2 points) Given the following information:
- Experience period on-level trended earned premium = $250,000.
- Experience period trended and developed losses and LAE = $200,000.
- Experience period earned exposure = 8,000.
- Variable expense provision = 19%.
- Fixed expenses for the experience period = $16,000.
- Profit and contingency factor = 4%.

a. (0.75 point) Calculate the indicated average rate level change using the loss ratio method.
b. (0.75 point) Calculate the indicated average rate using the pure premium method.
c. (0.5 point) Briefly describe one situation where the loss ratio method cannot be used and one situation where the pure premium method cannot be used.

5. (a) Loss, LAE and fixed expense ratio: (200,000 + 16,000) / 250,000 = 86.4%.
Permissible Loss, LAE and fixed expense ratio: 1 - 19% - 4% = 77.0%.
Indicated rate change: 86.4% / 77.0% - 1 = **12.2% increase**.
(b) Loss, LAE and fixed expense pure premium: (200,000 + 16,000) / 8000 = $27.
The indicated average rate is: $27 / (1 - 19% - 4%) = **$35.06**.
(c) The loss ratio method cannot be used for a new line of business or new insurer, since this method requires existing rates and premiums.
Alternately, where historical overall rate change information is not available to compute on-level factors, and the detail is not available to apply extension of exposures; historical premiums cannot be brought to the current rate level and thus the loss ratio method cannot be used.
Where the exposure unit is not available or is not reasonably consistent between risks, as in the case of commercial fire insurance, the pure premium method cannot be used.
Alternately, use the loss ratio method rather than the pure premium method when there has been a recent change in the definition of the exposure base.
Alternately, Commercial General Liability (CGL) policies have multiple sublines intended to protect policyholders against a broad range of risks; as such, CGL policies can have different exposure bases for the various sublines included. Consequently, when pricing CGL, it may be easier to obtain and use premium at current rate level rather than trying to define a consistent exposure.
Thus in this case, the loss ratio method rather than the pure premium method should be used.
Comment: In making classification rates, the pure premium method should not be used if exposures are correlated between variables. One should instead use the Adjusted Pure Premium method.

6. (1.5 points) Given the following information:

Class	Exposures	Current Rate	True Expected Cost	Proposed Rate
A	3,500	$500	$550	$540
B	8,000	$400	$350	$370

- Scenario 1: If the proposed rates are implemented, the projected number of class A exposures will decrease to 3,150; the projected number of class B exposures will remain unchanged.
- Scenario 2: If the proposed rates are not implemented, the projected number of class A exposures will increase to 4,500; the projected number of class B exposures will decrease to 7,000.
- No other expenses are changed in either scenario.
- Profit provision is 0% in the indicated rate.

a. (1 point) Calculate the profit in each of the two scenarios.

b. (0.5 point)

Explain whether the proposed rates should be implemented given a $10,000 implementation cost.

6. (a) Scenario 1: Premiums: (3150)(540) + (8000)(370) = 4,661,000.
Costs: (3150)(550) + (8000)(350) = 4,532,500.
(Underwriting) Profit = 4,661,000 - 4,532,500 = **$128,500**.
Scenario 2: Premiums: (4500)(500) + (7000)(400) = 5,050,000.
Costs: (4500)(550) + (7000)(350) = 4,925,000.
(Underwriting) Profit = 5,050,000 - 4,925,000 = **$125,000**.
(b) The one time implementation cost of $10,000 is more than the increase in profit of $3,500.
However, this cost can be spread over several years.
The insurer is currently charging too little for Class A and too much for Class B.
If the rates remain misaligned with the true expected costs, this insurer will be subject to further anti-selection over coming years, gaining business in Class A and losing business in Class B, which will significantly reduce future profits. Thus I would implement the proposed rates, which bring the rates much closer to the true expected costs.
Alternately, the expected benefit to implement is: 128,500 - 125,000 = $3,500.
However the implementation cost is $10,000 > $3,500. The proposed rates should not be implemented because the overall benefit does not outweigh the costs.
Comment: The proposed rates are more equitable than the current rates.

7. (2 points) Given the following:

Class	Earned Exposures	Reported Loss and ALAE	Current Relativity
A	10,500	$512,000	1.00
B	5,200	$740,000	1.50
C	13,100	$632,000	1.30

- Full credibility standard is 13,260 exposures.
- Partial credibility is determined based on the square root rule.
- The complement of credibility is no change.

Calculate the indicated rate change for each class that results in a revenue-neutral overall change.

7. I will use the pure premium method, as per Appendix E of Basic Ratemaking:

	[1]	[2]	[3]	[4]	[5]	[6]
Class	Earned	Reported	Pure	Indicated	Current	Normalized
	Exposures	Loss & ALAE	Premium	Relativity	Relativity	Current
						Relativity
			=[2]/[1]	= [3]/Total [3]		=[5]/Total [5]
A	10,500	$512,000	$48.76	0.7454	1.00	0.8152
B	5,200	$740,000	$142.31	2.1754	1.50	1.2228
C	13,100	$632,000	$48.24	0.7375	1.30	1.0597
Total	28,800	1,884,000	$65.42	1.0000	1.2267	1.0000

	[7]	[8]	[9]
Class	Credibility	Credibility	Cred-Weighted
		Weighted	Relativity
		Relativity	w.r.t. Base
			=[8]/[8] for J
A	88.99%	0.7531	1.0000
B	62.62%	1.8193	2.4158
C	99.39%	0.7394	0.9819
Total		0.9394	1.2474

For example, for Class B: 142.31/65.42 = 2.1754. 1.50/1.2267 = 1.2228.

$\sqrt{5200 / 13260}$ = 62.62%. (0.6262)(2.1754) + (1 - 0.6262)(1.2228) = 1.8193.

1.8193/0.7531 = 2.4158.

The weighted average current relativity is:

{(10,500)(1) + (5200)(1.5) + (13,100)(1.3)} / 28,800 = 1.2267.

Assume for simplicity a base rate of $100.

Then the current premium is: (10,500)($100) + (5200)($150) + (13,100)($130) = $3,533,000

Then the premium if we keep the same base rate and the credibility weighted relativities were put in effect is: (10,500)($100) + (5200)($241.58) + (13,100)($98.19) = $3,592,505.

Thus in order to have no overall rate change, we need to multiply the current base rate by:

3,533,000 / 3,592,505 = 0.9834.

Rate change for class A: (1)(0.9834) = 0.9834. ⇔ **-1.66%**.

Rate change for class B: (2.4158/1.50) (0.9834) = 1.5838. ⇔ **+58.38%**.

Rate change for class C: (0.9819/1.30) (0.9834) = 0.7428. ⇔ **-25.72%**.

Alternately, the ratio of the weighted average current relativity to the weighted average indicated relativity: 1.2267/1.2474 = 0.9834. This is the off-balance to multiply by. Proceed as before.

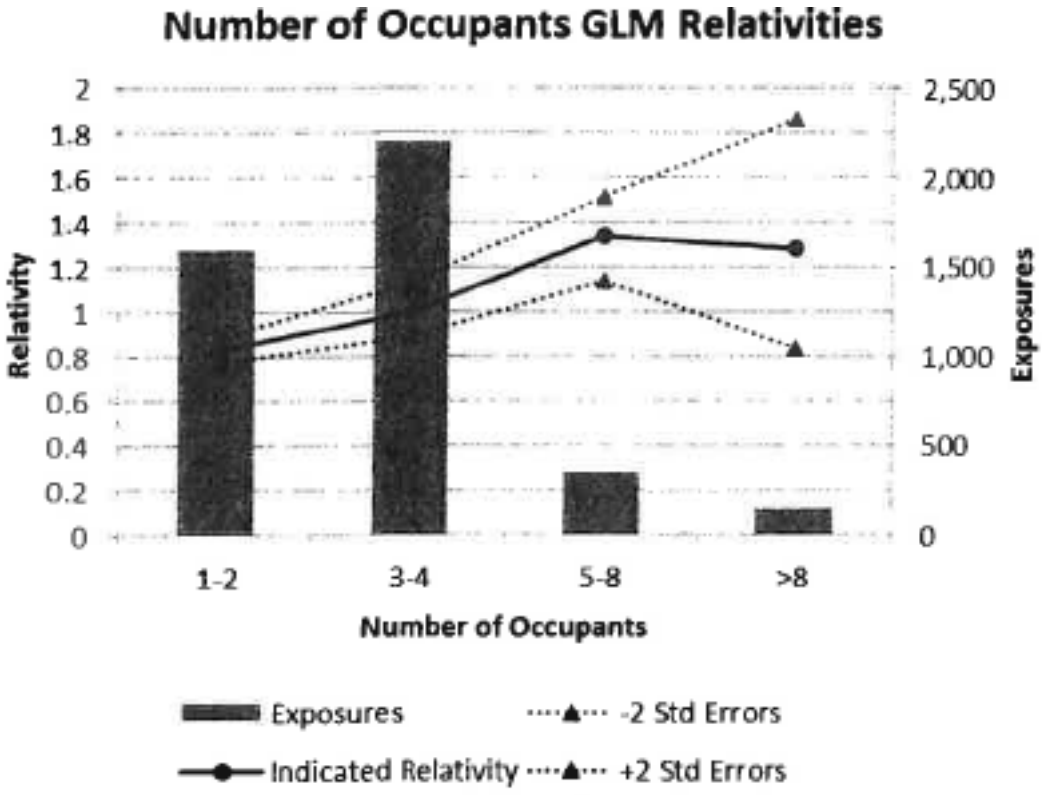
Comment: It would have been better for the question to say "the complement of credibility is the normalized current relativity." This is what is usually done in the pure premium method.
Alternately, based on the wording in the question, one could instead use no rate change as the complement of credibility. However, I doubt that this would have been given full credit.

	[1]	[2]	[3]	[4]	[5]	[6]
Class	Earned	Reported	Pure	Indicated	Current	Indicated
	Exposures	Loss & ALAE	Premium	Relativity	Relativity	Rate Change
				to Base		
			=[2]/[1]			
A	10,500	$512,000	$48.76	1.0000	1.00	0.00%
B	5,200	$740,000	$142.31	2.9184	1.50	94.56%
C	13,100	$632,000	$48.24	0.9894	1.30	-23.89%
Total	28,800	1,884,000	$65.42			

	[7]	[8]
Class	Credibility	Credibility
		Weighted
		Rate
		Change
A	88.99%	0.0000
B	62.62%	0.5922
C	99.39%	-0.2375

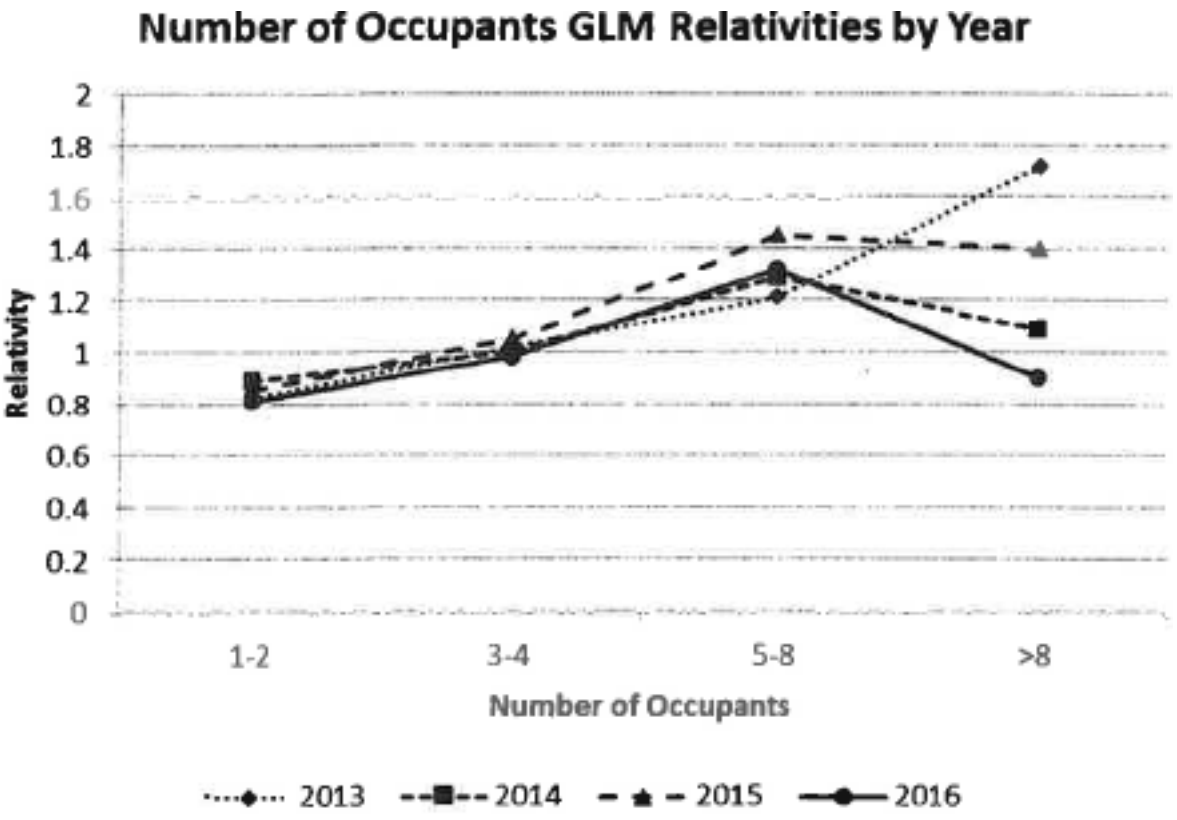
Assume for simplicity a base rate of $100.
Then the current premium is: (10,500)($100) + (5200)($150) + (13,100)($130) = $3,533,000
If we apply the above credibility weighted rate changes, then the premium would be:
(10,500)($100) + (1.5922)(5200)($241.58) + (1 - 0.2375)(13,100)($98.19) = $4,030,942.
Thus in order to have no overall rate change, we need to multiply the current base rate by:
3,533,000 / 4,030,942 = 0.8765.
Rate change for class A: (1)(0.8765) = 0.8765. ⇔ -12.35%.
Rate change for class B: (1.5922)(0.8765) = 1.3956. ⇔ +39.56%.
Rate change for class C: (1 - 0.2375)(0.87654) = 0.6683. ⇔ -33.17%.

8. (1.75 points) A company's current rating plan for fire coverage for personal property insurance only includes territory.
The following GLM outputs and experience are from a recent analysis of pure premium:



Number of Occupants	1~2	3-4	5-8	>8
Indicated Relativity	0.83	1.00	1.34	1.28

QUESTION 8 CONTINUED ON NEXT PAGE



• Number of occupants chi-squared percentage (entire variable) = 3.2%

a. (0.75 point) Fully justify whether number of occupants would be an appropriate addition to the rating classification plan.

b. (1 point) Identify and briefly describe two types of insurance environments which may discourage use of multivariate methods.

8. a) There are a number of reasons why I recommend that number of occupants be used:
1. The relativities for the number of occupants other than the base (3-4) are significantly different than one, with the exception of more than 8 which has few exposures.
2. With the exception of more than 8 which has few exposures, there is a pattern of increasing relativity with increasing number of occupants.
3. The relativities are similar for the different years of data, with the exception of more than 8 which has few exposures. This consistency is another reason in favor of using number of occupants.
4. The "chi-square percentage" of 3.5% is less than 5% indicates that I can reject the null hypothesis that number of occupants has no effect in favor of the alternate hypothesis that number of occupants has an effect on pure premium.
It would make sense to combine the 5 to 8 and over 8 categories, since the over 8 category has little data and its estimated relativity is not significantly different than that of 5 to 8.
b) 1. In a noncompetitive environment, such as for example social security or a monopolistic state Workers Compensation fund.
2. When each risk is sufficiently unique, such as when writing reinsurance or large commercial accounts.
3. Certain insurance regulators may not allow the use of multivariate methods for some lines of insurance.
4. When entering a brand new type of insurance market, often data is too limited to be able to accurately implement a multivariate method and other approaches are preferred.
5. If the insurer does not have the computing power and/or personnel to make use of multivariate methods, and the potential benefit does not outweigh the cost of upgrading computer systems and/or hiring the necessary people, then multivariate methods would be discouraged.
6. When there are too few insureds to make credible estimates; if a GLM is used, the standard errors would be very big and the results would be useless.
Comment: I do not believe that part (b) is covered in Basic Ratemaking.
Since only territory is currently used to rate insureds, there are other variables one should examine. When other variables are added to the model, it may turn out that number of occupants is no longer significant.
At page 170 of Basic Ratemaking: "Sequential analysis, a method related to minimum bias, may also be of interest to the ratemaking actuary. It is currently mandated as the only classification ratemaking method allowed for pricing voluntary private passenger automobile insurance in the state of California. In sequential analysis, the actuary performs a standard one-way analysis on the first variable selected to determine the indicated relativities. The exposures are adjusted for the results of the first variable's analysis (i.e., the adjusted one-way pure premium approach), and the indicated relativities are calculated for the second variable. This continues until the actuary has calculated the indicated relativities for every variable."

9. (2.75 points) Given the following information for an insurance company:

State	Class	Exposures	Losses	Current Pure Premium
A	1	200	$800	4.00
A	2	300	$2,100	7.00
A	Subtotal	500	$2,900	5.80
B	1	300	$600	2.00
B	2	300	$1,500	5.00
B	Subtotal	600	$2,100	3.50
C	1	500	$1,500	3.00
C	2	750	$4,500	6.00
C	Subtotal	1,250	$6,000	4.80
All	1	1,000	$2,900	2.90
All	2	1,350	$8,100	6.00
All	Total	2,350	$11,000	4.68

• Full credibility standard is 1,500 exposures.

a. (1.5 points)

Calculate the credibility-weighted pure premium for class 2, state B using Harwayne's method.

b. (0.5 point) Discuss the appropriateness of using Harwayne's method for this company.

c. (0.75 point)

Evaluate Harwayne's method using three desirable qualities for a complement of credibility.

9. (a) In each other state, weight the pure premiums by class by the distributions of exposures in the state for which we are making rates, B.

State A: $\frac{(300)(4.00) + (300)(7.00)}{300 + 300} = 5.50.$

State C: $\frac{(300)(3.00) + (300)(6.00)}{300 + 300} = 4.50.$

These are the pure premiums these states would have had if they had the same mix of exposures by class as in State B. Next using these adjusted state pure premiums, calculate relativities for Class 2 in these other states:

State A: 7.00/5.50 = 1.273.

State C: 6.00/4.50 = 1.333.

Weight these relativities by Class 2 exposures in these other states:

$\frac{(300)(1.273) + (750)(1.333)}{300 + 750} = 1.316.$

Convert this relativity to a pure premium by multiplying by the overall pure premium for State B: (1.316)($3.50) = $4.61. This is the complement of credibility.

The credibility is: $\sqrt{300 / 1500} = 44.7\%$.

The credibility weighted pure premium is: (44.7%)(5.00) + (1 - 44.7%)(4.61) = **$4.78**.

(b) This method is appropriate since it allows one to incorporate the data on class 2 form other states, while adjusting for the different distribution of exposures and level of losses in those other states.

Alternately, (see comment) it is mathematically appropriate to use the method here, and the result is likely to be better than relying on the data for a single state. However, Harwayne's method was designed for a situation with a greater volume of data, with more classes and more states. Here even supplementing with data from other states, we still have very little data on which to base pure premiums. There will a lot of random fluctuation, since there is only a total of $11,000 in losses. Thus if possible it would be better to supplement this company's data with data for the entire insurance industry.

(c) "The complement derived from this method is unbiased as it adjusts for the distributional differences. The use of multistate data generally implies the complement is reasonably accurate as long as there is sufficient countrywide data to minimize the process variance. Also, since the subject experience and related experience consider data from different states, the complement is considered mostly independent.
The data for the complement is usually available but the computations can be time-consuming and complicated. While the complement bears a logical relationship to the subject experience, the complement may be harder to explain because of the computational complexity."

Comment: The CAS Examiner's Report did not give credit for my alternative answer to part (b). "Common errors included stating Harwayne's method was not appropriate because of low volume in all 3 states. Harwayne's method addresses distributional bias in the overall experience and can be used in low volume situations." The last sentence is at best very misleading. Harwayne's method makes god use of the available information, but does not create information out of thin air. One can use Harwayne's method when there is low volume in some classes and/or states. However, having used Harwayne's method in practical applications many times, in my opinion the total volume of data in the three states is much too low to usefully apply the method. There is only a total of $11,000 in losses, so for someone with practical experience this is not even a close call.
The Exam Committee could have avoided penalizing people with practical experience in using Harwayne's method, by in their question just multiplying all of the given exposures, losses, and the standard for full credibility by for example 10,000.
In part (c), I have quoted from page 226 of Basic Ratemaking.
You of course do not have to quote the syllabus readings.
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10. (3.5 points) Given the following information about an insurance product:

Territory Factors

Territory	Factor
A	0.85
B	1.00
C	1.35

- Fixed expense per exposure = $50.
- Variable expense ratio = 17%.
- Underwriting profit provision = 3%.
- LAE provision = 16% of loss cost.
- Base rate = $435.
- Policy fee = $55.
- Policy fee is an additive fee added to each exposure in the last step of the rate calculation.

Based on a separate analysis, an actuary projects the following for calendar-accident year 2018:

Territory	Earned Exposures	Ultimate Loss Cost
A	150	$300
B	200	$350
C	100	$500

a. (1.5 points) Calculate the projected total underwriting profit for calendar-accident year 2018.

b. (1.5 points) Calculate the indicated policy fee, indicated territory factors, and indicated base rate.

c. (0.5 point) Management suggests reaching the targeted profit by only increasing the base rate. Discuss this approach.

10. (a) Projected total loss costs: (150)(300) + (200)(350) + (100)(500) = 165,000.
Project fixed expenses: (150 + 200 + 100) (50) = 22,500.
Projected premiums, excluding policy fees:
(435) {(150)(0.85) + (200)(1) + (100)(1.35)} = 201,187.5.
Projected policy fees: (150 + 200 + 100) (55) = 24,750.
Projected underwriting profit:
(1 - 17%)(201,187.5 + 24,750) - (1.16)(165,000) - 22,500 = **-$26.372**.
In terms of percentages, the projected underwriting profit is:
-26,372 / (201,187.5 + 24,750) = -11.7%.

(b) Indicated policy fee: 50 / (1 - 0.17 - 0.03) = **$62.5**.
Territory relativity to base:
A: 300/350 = 0.857. B: 1 C: 500/350 = 1.429.
Loss and lae pure premium: (1.16)(165,000) / 450 = 425.33.
Indicated average variable rate: 425.33 / (1 - 0.17 - 0.03) = 531.67.
Average indicated territory relativity: $\frac{(150)(0.857) + (200)(1) + (100)(1.429)}{150 + 200 + 100} = 1.048$.

Indicated base rate: 531.67 / 1.048 = **$507**.
Alternately, the indicated base rate is the projected loss and LAE for Territory B, loaded for variable expenses and profit: (350)(1.16) / (1 - 0.17 - 0.03) = $**507.5**.

(c) If the territory relativities and policy fee are not changed, then there will be a misalignment between costs and premiums. This could lead to antiselection: losing business in the overpriced territories and gaining business in the underpriced territory C. However, since the current and indicated relativities are not that different, the insurer should not be affected very much. Thus while the proposal is less than ideal, it should not create major problems if not also continued in future years. Thus I would reluctantly go along with what management suggests.

Alternately, by changing the policy fee and relativities prices will be more equitable and better aligned with expected loss per policy. If instead the territory relativities and policy fee are not changed, then there will be a misalignment between costs and premiums. This would lead to antiselection: losing business in the overpriced territories and gaining business in the underpriced territory C. This would lead to the insurer becoming unprofitable.

Therefore, I am against management's proposal.

Comment: If we do not change the current policy fee of 55, then the variable portion of the rate has to increase on average by: 62.5 - 55 = $7.5.

$\Rightarrow$ The average variable rate would need to be: 531.67 + 7.5 = $539.17.

The current average territory relativity is:

$$\frac{(150)(0.85) + (200)(1) + (100)(1.35)}{150 + 200 + 100} = 1.028.$$

Thus if we also do not change relativities, then the indicated base rate would be:

539.17 / 1.028 = $524.5.

A comparison of the premiums and costs if relativities and the policy fee are not changed:

Territory	Costs (Including Profit Provision)	Premiums
A	(300)(1.16)/0.8 + 50/0.8 = 497.5	(0.85)(524.5) + 55 = 500.8
B	(350)(1.16)/0.8 + 50/0.8 = 570	(1)(524.5) + 55 = 579.5
C	(500)(1.16)/0.8 + 50/0.8 = 787.5	(1.35)(524.5) + 55 = 763.1

Assuming the mix of exposures by territory does not change, the overall rate level would be achieved, but there is a misalignment between costs and premiums by territory. Of course due to antiselection as well as random fluctuation, the mix of exposures by territory is likely to change.

11. (2.5 points) Given the following information:

Size of Loss ($000)	Loss Distribution	Average Reported Loss ($000)
X <= 200	20%	100
200 < X <= 400	20%	300
400 < X <= 600	20%	500
600 < X <= 800	20%	700
800 < X <= 1,000	20%	900
Total	100%	500

- Expected claim frequency = 1%.
- Expected losses are uniformly distributed.
- A home is valued at $1,000,000.

a. (1 point)

Calculate the rate per $1,000 of coverage for the home at the following amounts of insurance:

i. $1,000,000

ii. $600,000

b. (0.5 point)

Briefly discuss a problem associated with underinsurance from the following perspectives:

i. Insured

ii. Insurer

c. (1 point)

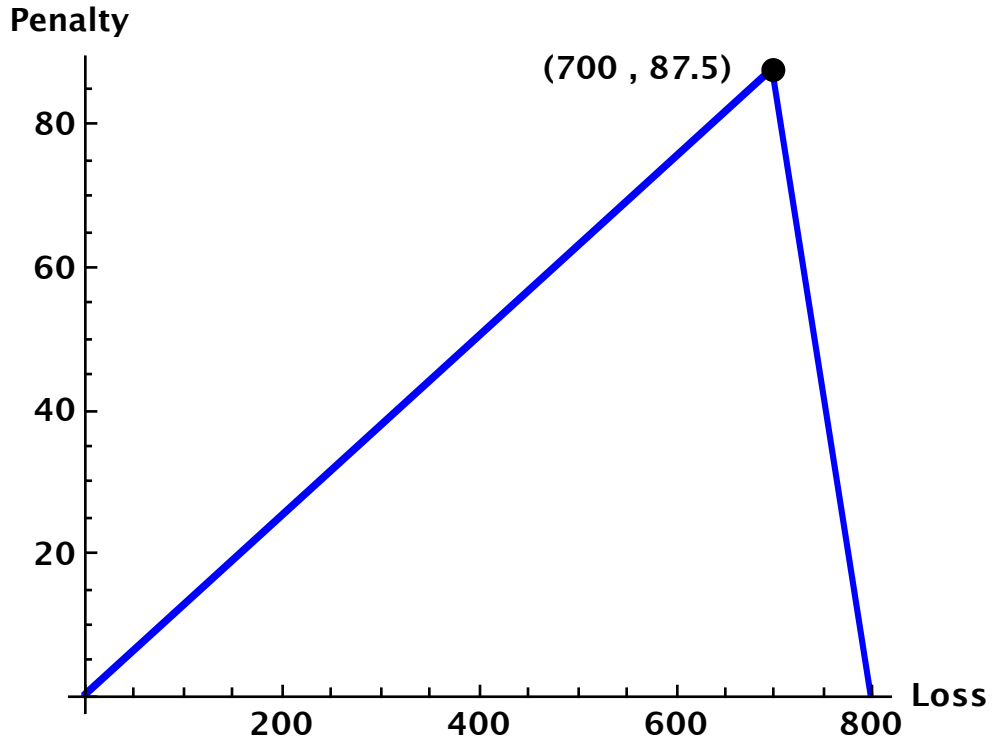
The home is insured for $700,000 with no deductible and a coinsurance requirement of 80%.

Calculate the indemnity payments and coinsurance penalties for the following losses:

i. $600,000

ii. $850,000

11. (a) If buy $1 million of insurance: (1%)($500,000) / (1,000,000 / 1000) = **$5** per $1000.
If buy $600,000 of insurance, average payment per loss is:
(60%)(300,000) + (40%)(600,000) = $420,000.
Rate: (1%)($420,000) / (600,000 / 1000) = **$7** per $1000.
(b) i. The insured will not receive the money to replace their home if there is a very large or total loss. Alternately, the insured will suffer coinsurance penalties for any losses below the coinsurance requirement, and thus not be fully reimbursed for their loss.
ii. In the absence of a coinsurance clause, if there is underinsurance, and the insurer based its rate on assuming insurance to value, then the insurer will be charging an inadequate rate.
(c) The coinsurance requirement is: (80%)(1 million) = $800,000.
i. The insured will be paid: (7/8)(600,000) = **$525,000**.
The coinsurance penalty is: 600,000 - 525,000 = **$75,000**.
i. The insured will be paid: Min[700,000 , (7/8)(850,000)] = **$700,000**.
Without a coinsurance clause, it would have been paid 700,000.
The coinsurance penalty is: 700,000 - 700,000 = **$0**.
Comment: The maximum penalty occurs for a loss of size equal to the policy face. There is no coinsurance penalty for losses of size greater than the coinsurance requirement. The coinsurance penalty in part (c) as a function of the size of loss, where everything is in thousands of dollars:



12. (1.25 points) Given the following information regarding an experience rating plan:

- Reported losses and ALAE limited by basic limits and maximum single limit per occurrence (MSL) for the policy being rated as of March 31, 2016 = $175,000.
- Company subject basic limit loss and ALAE for experience period = $225,000.
- Expected experience ratio = 0.875.
- Expected percentage basic limit loss and ALAE for experience period unreported at March 31, 2016 = 0.425.
- Credibility = 0.35.

a. (1 point) Calculate the experience modification factor.

b. (0.25 point) Briefly describe a scenario in which it would be appropriate for schedule rating to be used in addition to experience rating.

12. (a) Expected unreported losses: (0.425)(0.875)(225,000) = $83,672.

Actual experience ratio is: {175,000 + 83,672} / 225,000 = 1.150.

Mod = (1.150/0.875) (0.35) + (1 - 0.35) = **1.110**.

Alternately, mod = (0.35)(1.150 - 0.875) / 0.875 + 1 = **1.110**.

(b) Assume that we are rating a Workers Compensation policy to be effective January 1, 2018.

To experience rate this policy we would use experience from the 2014, 2015, and 2016 policies.

In July 2017, the employer instituted a new drug testing program of its employees which the underwriter believes will reduce expected losses.

Since this new program could not affected the experience used in the experience rating, it would be appropriate to apply a schedule rating credit in addition to the experience mod.

Alternately, in May 2017, the insured ends a safety program that had been in effect and which the underwriter believes reduced expected losses. Since this new program affected the experience used in the experience rating, it would be appropriate to apply a schedule rating debit in addition to the experience mod.

Comment: This is the ISO CGL Experience Rating Plan.

There are lots of possible full credit answers to part b.

13. (5.5 points) Given the following information for a book of business as of December 31, 2016:

Calendar Year	Earned Premium ($000)
2015	3,910
2016	4,410

Rate Change History

Effective Date	Average Rate Change
July 1, 2014	-2.0%
July 1, 2015	4.2%
July 1, 2016	3.6%

Accident	Reported Loss and ALAE ($000) Capped at $100,000 as of (months)		
Year	12	24	36
2014	1,116	1,448	1,610
2015	1,975	2,572	
2016	2,145		

Excess Loss and ALAE ($000) History

Accident	Trended Reported Loss and ALAE	
Year	Unlimited	Excess of $100,000
2009	3,538	718
2010	3,193	130
2011	1,990	234
2012	4,580	1,949
2013	2,369	120

- All policies are annual.
- Exposures are written evenly throughout each calendar year.
- Annual premium trend = 2.8%.
- Annual frequency trend = -2%.
- Annual severity trend capped at $100,000 = 4%.
- Fixed expense ratio = 4%.
- Variable expense ratio = 22%.
- Profit and contingencies provision = 6%.
- ULAE provision = 6% of loss and ALAE.
- Rates are to be in effect for one year.
- There is no loss development beyond 36 months.
- Assume full credibility.

a. (0.75 point)
Calculate the ultimate loss and ALAE capped at $100,000 for accident years 2015 and 2016.
b. (4.5 points)
Determine the indicated rate change effective July 1, 2017 using the results from part a. above.
c. (0.25 point) Briefly describe one reason the insurer might not take the full rate change determined in part b. above.

13. (a) 12-24 development factor: (2572 + 1448) / (1975 + 1116) = 1.3005.
24-36 development factor: 1610/1448 = 1.1119.
Estimated ultimate loss and ALAE capped at $100,000 for AY2015:
(2,572,000)(1.1119) = **$2.860 million**.
Estimated ultimate loss and ALAE capped at $100,000 for AY2016:
(2,145,000)(1.3005)(1.1119) = **$3.102 million**.
(b) The average date of writing under the new rates is January 1, 2018.
Since policies are annual, the average date of accident is six month later, July 1, 2018.
Thus the trend period from AY2015 is 3 years.

Trended ultimate limited loss and alae for AY15: (2.860 million) (0.98^3) (1.04^3) = $3.028 million.

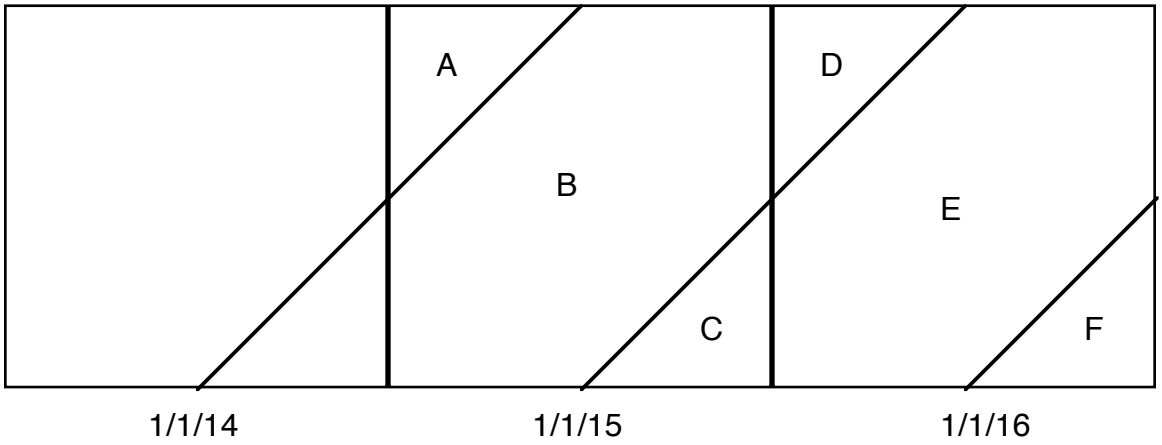
Trended ultimate limited loss and alae for AY16: (3.102 million) (0.98^2) (1.04^2) = $3.222 million.

Looking at history of excess losses: Excess / Limited =

$$\frac{718 + 130 + 234 + 1949 + 120}{(3568 + 3193 + 1990 + 4580 + 2369) - (718 + 130 + 234 + 1949 + 120)}$$

= 3151/ (15670 - 3151) = 0.252.
Thus to load the AY15 and AY16 limited loss for excess multiply by 1.252.
Since policies are annual, the average date of accident is July 1, 2018.
The trend period for AY15 earned premiums is 3 years.

Date	Rate Level Index
Prior	1.000
July 1, 2014	0.980
July 1, 2015	(0.980)(1.042) = 1.02116
July 1, 2016	(0.980)(1.042)(1.036) = 1.05792



Area A = $(1/2)(1/2)^2$ = 1/8 = Area C = Area D = Area F. Area B = Area E = 3/4.
Average rate level for CY15: (1/8)(1) + (3/4)(0.98) + (1/8)(1.02116) = 0.98765.
Average rate level for CY16: (1/8)(0.98) + (3/4)(1.02116) + (1/8)(1.05792) = 1.02061.
OLF for CY15: 1.05792 / 0.98765 = 1.07115.
OLF for CY16: 1.05792 / 1.02061 = 1.03656.

Trended on-level premiums for CY15: $(3910K)(1.028^3)(1.07115) = 4.550$ million.

Trended on-level premiums for CY16: $(4410K)(1.028^2)(1.03656) = 4.831$ million.

Combined loss and ALAE ratio, loaded for excess, for AY15 and AY16:
(1.252) (3.028 + 3.222) / (4.550 + 4.831) = 0.834.
Indicated rate change: {(1.06)(0.834) + 4%) / (1 - 22% - 6%) - 1 = **+28.3%**.
(c) 1. There may be competitive market reasons not to take the full indicated increase; the insurer is worried about losing market share to its chief competitors. If it raises rates on average by about 28%, with some individuals getting a larger increase, many of its insureds will shop around.
2. The insurer may have done a "lifetime analysis", and concluded that taking into account expected profits from renewals, it is better not to take the full indicated increase.
3. The insurance regulator may not be willing to approve taking the full increase.
Comment: In part (a), one could instead average the two 12-24 month development factors.
1448/1116 = 1.2975. 2572/1975 = 1.3023. (1.2975 + 1.3023)/2 = 1.2999.
In part (b), one could instead average the ratios of excess to limited for individual years.
(718/2820 + 130/3063 + 234/1756 + 1949/2631 + 120/2249) / 5 = 0.245.