Solutions to the Spring 2016 CAS Exam 5

(Only those questions on Basic Ratemaking)

There were 25 questions worth 57.75 points, of which 13 were on ratemaking worth 32 points.

(Incorporating what I found useful from the CAS Examiner's Report)

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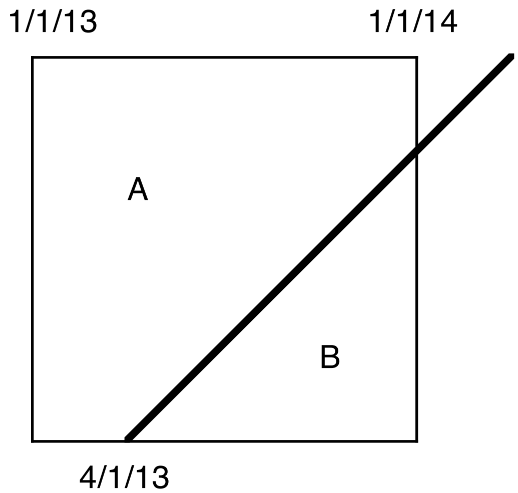
1. (2.5 points) Given the following information for an insurance company:

Accident Year	Earned Premium ($000)	Ultimate Losses ($000)
2013	1,500	800
2014	1,600	800
2015	1,800	1,200

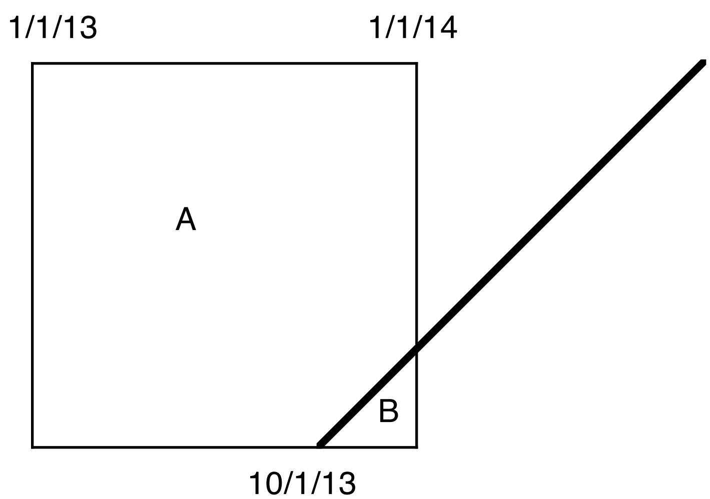
• A benefit level change increased losses by 10% for policies written after April 1, 2013.
• A second benefit level change decreased losses by 5% for accidents occurring after January 1, 2014.
• A rate change of +5% was effective October 1, 2013.
• Annual loss cost trend is +2%.
• All policies have a term of one year.
• The company writes policies uniformly throughout the year and files rates only one time per year.
• Planned rate revision to be effective January 1, 2017.

Calculate the on-level loss ratio for accident year 2013 for the planned rate revision.

1. The average date of writing under the new rates is July 1, 2017.
Since policies are annual, the average data of accident is January 1, 2018.
Thus the trend period from AY2013, average date July 1, 2013, is 4.5 years.
The current benefit level is: (1.1)(0.95) = 1.045.



Area B = (1/2)(3/4)(3/4) = 9/32. Area A = 1 - 9/32 = 23/32.
Average benefit level of AY13 is: (23/32)(1) + (9/32)(1.1) = 1.0281.
Factor to put AY13 on current benefit level: 1.045/1.0281 = 1.0164.
The current rate level is 1.05.



Area B = (1/2)(1/4)(1/4) = 1/32. Area A = 1 - 1/32 = 31/32.
Average rate level for AY2013 is: (31/32)(1) + (1/32)(1.05) = 1.0016.
Factor to bring 2013 premiums to the current rate level: 1.05/1.0016 = 1.0483.

Thus, the on-level loss ratio for accident year 2013 is: $\frac{(800)(1.0164)}{(1500)(1.0483)}$ $1.02^{4.5}$ = **56.53%**.

Comment: I assumed there is no premium trend, since none is mentioned in the question.

2. (2.5 points) An insurance company writes both 6-month and 12-month automobile policies. Given the following information:

Policy	Original Effective Date	Original Expiration Date	Transaction Effective Date	Territory	Full-Term Written Premium	Notes
A	January 1, 2015	December 31, 2015	January 1, 2015	1	$1,000	Start of New Policy
A	January 1, 2015	December 31, 2015	July 1, 2015	1	N/A	Policy Canceled
B	July 1, 2015	June 30, 2016	July 1, 2015	1	$500	Start of New Policy
B	July 1, 2015	June 30, 2016	September 30, 2015	2	$400	Relocated to Territory 2
C	October 1, 2015	March 31, 2016	October 1, 2015	2	$1,000	Start of New Policy

• Full-term written premium represents the policy premium if policy characteristics shown were in place from original effective date to original expiration date.

a. (0.75 point) Calculate the 2015 calendar year written premium as of December 31, 2015.

b. (0.75 point) Calculate the 2015 calendar year earned premium as of December 31, 2015.

c. (0.5 point) Calculate the in-force premium as of October 1, 2015.

d. (0.5 point) Calculate the 2015 calendar year earned exposures separately for Territory 1 and Territory 2 as of December 31, 2015.

2. (a) Policy A is canceled before the end of 2015, so it contributes 1000/2 = 500.
Policy B is rerated prior to the end of 2015, so it contributes: (1/4)(500) + (3/4)(400) = 425.
Total CY2015 written premium is: 500 + 425 + 1000 = **$1925**.
(b) Policy B has CY2015 earned premium of: (1/4)(500) + (1/4)(400) = 225.
Six-month Policy C has CY2015 earned premium of: (1/2)(1000) = 500.
Total CY2015 earned premium is: 500 + 225 + 500 = **$1225**.
(c) In-force premium as of October 1, 2015 is: 0 + 400 + 1000 = **$1400**.
(d) I will assume that each policy covers one car.
CY2015 earned exposures for Territory 1: 1/2 + 1/4 + 0 = **3/4 car year**.
CY2015 earned exposures for Territory 2: 0 + 1/4 + 1/4 = **1/2 car year**.
Comment: For Policy B, 3 months is at $500 (annual basis),
while the remaining 9 months is at $400 (annual basis).

3. (2.25 points) A personal automobile insurance company is considering changing its exposure base from car-years to hours driven.
a. (1.5 points) Evaluate hours driven using three criteria of a good exposure base.
b. (0.75 point) The company is also considering keeping its current exposure base as car-years but including hours driven in its risk classification system. Briefly discuss the appropriateness of adding this risk characteristic to the company's risk classification system using three considerations from the Actuarial Standard of Practice No. 12: Risk Classification (for All Practice Areas).

3. (a) 1. Proportional to Expected Loss:
The more hours driven, the larger the potential for an insured loss.
The expected loss is approximately proportional to the hours driven, so this criterion is met.
Alternately, some areas have more traffic than others; driving in the city is not the same as driving in the country. For that reason, more hours driven does not necessarily translate to higher expected loss. Thus hours driven is not proportional to expected loss, and this criteria is <u>not</u> met.
2. Practical: Hours driven is subject to manipulation by the insured and would be costly to verify, so this criterion is not met.
Alternately, hours driven is objective and well defined.
Using telematics, hours driven is relatively easy and inexpensive to obtain and verify.
Thus this criterion is satisfied.
3. Historical Precedence: We are told that car-years is currently used. Hours driven is not currently used as the exposure base, so this criterion is <u>not</u> met.
(While I think this is a full credit response, the CAS Examination Committee required one to say something more on historical precedence.)
If one changed to hours driven: this can be very expensive and time consuming to implement, it may cause large premium swings to insureds, would require one to try to restate historical data, and there may not be industry benchmarks to use.

(Since some criteria may be met and others may not be met, one can conclude either that the change to hours driven should be or should not be made.)

(b) Hours driven is correlated with expected costs, which is good. Charging an insured who drives more hours than an otherwise similar insured who drives fewer hours is fair, since hours driven is materially related to expected costs. Hours driven is causally related to expected costs.
However, if self-reported, hours driven is not practical since it is subject to manipulation by the insured and would be very hard for the insurer to verify. Thus hours driven is not appropriate.
Alternately, hours driven is objective and well defined. Using telematics, hours driven is relatively easy and inexpensive to obtain and verify. Thus hours driven is appropriate.
Additional possible answers:

- Hours driven does <u>not</u> have a strong relationship to expected loss because people driving on the highway will travel further than someone driving the same amount of time in the city.
- Causality: Hours driven has a cause and effect relationship with losses. It is intuitive that more hours driven would result in more accidents and more losses, thus public acceptance would occur.
- Legal: This is a legal variable
- Using hours driven may help avoid adverse selection. Risks that drive much more than average can be priced appropriately, as opposed to using car years which underprices such risks, thereby attracting such risks to the insurer while losing otherwise similar risks who drive fewer hours.
- Credibility: Classes based on hours driven can be made large enough to have enough drivers in them to provide reasonable credibility.
- Homogeneity: With different types of driving, urban, rural, and highway, the same hours driven would have different probabilities of an accident as well as different types of accidents. Thus there would be subgroups of each hours driven class that have significantly different expected losses. Thus this criteria is <u>not</u> met.
- Affordability: Using hours driven might make insurance unaffordable for low income insureds with longer commutes. This would argue against using hours driven as a classification variable.
- Hours driven is controllable by the insured. They can drive less hours in order to keep their premium down.
- Alternately, insureds likely have little control over how much they drive, as many drive primarily to and from work. Thus this criteria is <u>not</u> met.
- Privacy: If devices are installed in the insured car to track hours driven, this could be seen as an invasion of personal privacy. Thus this criteria is <u>not</u> met.
- Alternately, hours driven is <u>not</u> private information so it is likely to be accepted by the public.

<u>Comment</u>: One could drive 60 miles in one hour on an interstate, 30 miles in one hour on uncrowded rural roads, or 10 miles in one hour in crowded city traffic. Expected loss is not exactly proportional to either hours driven or miles driven. Which one is closer to proportional is an empirical question, which may vary between sublines of automobile insurance or even by region.
If one used hours driven (or miles driven) as a rating variable, one would group it into categories.
When there are two people who customarily drive a car, such as a teenager and their parent, it would be useful to know how many hours (or miles) are driven by each. However, this would be very hard information to collect and verify.
There could be some ambiguity in hours driven; for example one should probably exclude minutes spent warming up an engine on a very cold morning, prior to driving the vehicle.

4. (2.25 points) Given the following information:

Claim	Policy Effective Date	Accident Date	Report Date	Transaction Date	Claim Status	Loss Payment	Case Reserve Change
				January 7, 2014	Open		+$5,000
1	October 1, 2013	Decem. 15, 2013	January 5, 2014	March 1, 2014	Open	$4,000	-$4,000
				January 5, 2015	Closed	$500	-$1,000
				March 1, 2014	Open		+$6,000
2	Novem. 1, 2013	Feb. 1, 2014	Feb. 10, 2014	March 15, 2014	Closed	$6,000	-$6,000
				June 10, 2014	Open		+$10,000
3	January 1, 2014	June 1, 2014	June 5, 2014	Septem. 1, 2014	Open	$1,000	+$10,000
				January 3, 2015	Open	$4,000	-$5,000
				July 20, 2015	Open	$500	+$5,000
4	June 1, 2014	August 15, 2014	July 15, 2015	March 1, 2016	Open		+$5,000
				June 1, 2016	Open	$5,000	+$7,000

• Calendar year 2014 earned premium = $50,000.
• Calendar year 2015 earned premium = $60,000.

a. (0.5 point) Calculate the 2015 calendar year case incurred losses.

b. (0.5 point)
Calculate the 2014 policy year case incurred losses, evaluated at December 31, 2014.

c. (0.75 point)
Calculate the 2014 accident year case incurred loss ratio, evaluated at December 31, 2015.

d. (0.5 point)
Provide one advantage and one disadvantage of using policy year data in ratemaking analyses.

4. (a) Claim 1 contributes: 500 - 1000 = -500.
Claim 2 contributes 0.
Claim 3 contributes: 4000 - 5000 = -1000.
Claim 4 contributes: 500 + 5000 = 5500.
2015 calendar year case incurred losses are: -500 - 1000 + 5500 = **$4000**.
(b) Only claims 3 and 4 can contribute, since they are from policies written in 2014.
Claim 3 contributes: 10,000 + 1000 + 10,000 = 21,000.
Claim 4 contributes 0.
2014 policy year case incurred losses, evaluated at December 31, 2014 are: **$21,000**.
(c) Only claims 2, 3, and 4 can contribute, since they are from accidents that occurred in 2014.
Claim 2 contributes: 6000 + 6000 - 6000 = 6000.
Claim 3 contributes: 10,000 + 1000 + 10,000 + 4000 - 5000 = 20,000.
Claim 4 contributes: 500 + 5000 = 5500.
2014 accident year case incurred losses, evaluated at December 31, 2015 are:
6000 + 20,000 + 5500 = $31,500.
Loss ratio is: $31,5000 / $50,000 = **63.0%**.
(d) Policy year data compares premiums and losses on a given set of policies, producing a better match than either calendar year or accident year data. This is an advantage.
Another advantage is that Policy Year data can isolate changes in policy limits or underwriting guidelines.
Policy year data takes longer to be available than either calendar year or accident year data.
This is a disadvantage.
Another disadvantage is that policy year premiums and losses take longer to develop to ultimate.
<u>Comment</u>: In parts (b) and (c), do <u>not</u> include transactions after the given evaluation dates.

5. (1.5 points) An insured had a mature claims-made policy with Insurer A in 2011 and 2012 before switching to an occurrence policy with Insurer B in 2013 and 2014.
Below are the losses incurred over a 5-year period:

Accident Date	Report Date	Claim Amount
July 1, 2010	October 1, 2012	\$1,000
August 1, 2010	November 1, 2011	\$2,000
January 1, 2011	March 1, 2014	\$2,000
April 1, 2011	May 1, 2011	\$3,000
June 1, 2012	December 1, 2012	\$4,000
March 1, 2013	February 1, 2015	\$5,000
April 1, 2013	June 1, 2014	\$3,000
April 1, 2014	August 1, 2014	\$2,000

• Policies are effective on January 1 of each year.
• All policies are annual.

a. (0.5 point) Determine the loss amount each insurer pays.
b. (0.5 point) Briefly discuss two reasons why occurrence policy ultimate loss estimates are more volatile than claims-made policy ultimate loss estimates.
c. (0.5 point) Discuss whether an occurrence policy or a claims-made policy is likely to earn more investment income, assuming a stable interest rate environment.

5. (a) Insurer A pays for any claims reported during 2011 and 2012. (mature claims-made policies.)
Insurer B pays for accidents during 2013 and 2014.
There is thus a gap in coverage, that would normally be covered by a tail policy bought from Insurer A, but of which there is no mention.
Thus Insurer A pays for claims 1, 2, 4, and 5, totaling: (1000)(1 + 2 + 3 + 4) = $10,000.
Insurer B pays for claims 6, 7, and 8, totaling: (1000)(5 + 3 + 2) = $10,000.
Claim 3 would be paid by a tail policy, which this insured should have purchased from insurer A.
(b) Under claims-made, at the expiration of a policy there are no unreported claims covered by that policy. (Under special circumstances there may a very small number of such covered claims.)
Therefore, with claims-made one does not have to estimate pure IBNR.
For an occurrence policy, for long tailed lines of insurance such as malpractice, at the expiration of a policy there may be many unreported claims.
Therefore, if selling occurrence policies one has to estimate a provision for pure IBNR.
For long tailed lines of insurance such as malpractice, the provision for pure IBNR can be quite large and difficult to estimate. Thus occurrence policies ultimate loss estimates are more volatile than those of claims-made policies. This affects both reserving and ratemaking.
Specifically when there is a sudden shift in reporting pattern, claims-made will be affected very little while occurrence will be affected a lot. Second, when there is an unexpected loss trend change, claims-made will be impacted very little compared to occurrence policies.
(c) Claims from occurrence policies have a reporting and settlement lag, while those from claims-made policies only have a settlement lag. Therefore, an occurrence policy has a longer time on average from when premiums are received until losses are paid. Thus occurrence policies have more opportunity to earn investment income than otherwise similar claims-made policies.
<u>Comment</u>: For part (a), the CAS Examiner's Report also had a sample solution:
"For claims-made assume retroactive date of 1/1/2011. Insurer A = 3000 + 4000 = 7,000."
However, this is incorrect, since in that case the 2011 policy would have been a first year claims-made policy rather than a mature claims-made policy.
For part (a), the CAS Examiner's Report had another sample solution:
"Assuming at the end of 2012 claims-made policy, the insured bought tail coverage to cover reported losses after 12-31-12 for accidents which occurred during the mature claims-made period, insurer A = 12,000."
This is correct, given the additional assumption that is not stated in the exam question.

6. (2.25 points) Given the following for an individual state:

Selected Loss and ALAE Ratio	105.5%
Expense and ULAE Ratio	30.7%
Profit and Contingency Provision	-5.0%
Number of Reported Claims	109
Claims Required for Full Credibility Standard	683
Countrywide Indicated Rate Change	8.5%

• Partial credibility is determined using the square root rule.

a. (1 point) Calculate the credibility-weighted indicated rate change for this state.

b. (0.25 point)

Briefly describe a situation where the given profit and contingency provision may be appropriate.

c. (1 point) Discuss two situations where the pure premium method is preferable to the loss ratio method for calculating an indicated rate change.

6. (a) 105.5% / (1 - 0.307 + 0.05) = 142.0%. ⇔ 42.0% indicated rate increase.

$Z = \sqrt{\frac{109}{683}} = 39.9\%.$

Credibility-weighted indicated rate change is: (39.9%)(42.0%) + (1 - 39.9%)(8.5%) = **21.9%**.
Alternately, assume that the Expense and ULAE are to be treated as fixed rather than variable.
Then, (105.5% + 30.7%) / (1 + 0.05) = 129.7%. ⇔ 29.7% indicated rate increase.
Credibility-weighted indicated rate change is: (39.9%)(29.7%) + (1 - 39.9%)(8.5%) = 17.0%.
(b) If we have a long-tailed line of insurance, such as general liability, where the average time from receiving premiums to when losses are paid is long, and if the risk free rate of interest is large, then the expected investment income is large and the appropriate underwriting provision could be -5%.
Alternately, the insurer may have a chosen a -5% profit and contingency provision in order to be very competitive and gain new business; they are hoping that they will make a profit in later years.
(c) 1. For a new line of insurance for which there are no current rates, since there are no current premiums one can not use the loss ratio method. It may nevertheless be possible to estimate loss pure premiums by looking at similar lines of insurance plus the use of underwriting judgement.
2. Pure premium method is preferable where on-level premium is difficult to calculate.
Therefore, for commercial lines where individual risk rating adjustments (such as schedule rating) are made to individual policies, it is more appropriate to use the pure premium method if possible.
3. For private passenger automobile insurance, it may be difficult to quantify the effects of many changes to rating variables from the experience period to the current rate manual, making it difficult to estimate on-level premiums and thus to use the loss ratio method. (When new rating variables are introduced and they are not available in the historical dataset, it is impossible to put premiums on-level.) Thus in such situations the pure premium method is preferable.

7. (2.25 points) Given the following information:

	Calendar Year			
Expense Ratios	2013	2014	2015	% Fixed
General Expenses	4.2%	5.1%	5.9%	70%
Other Acquisition	9.9%	10.6%	11.5%	80%
Taxes, Licenses and Fees	1.5%	1.4%	1.5%	30%
Commission and Brokerage	11.1%	10.4%	11.0%	0%

• Projected ultimate pure premium, including LAE = $600.

• Underwriting profit provision = 12%.

• Projected average premium per exposure = $1,000.

a. (1.75 points) Calculate the indicated average rate using the premium-based projection method for determining expense provisions. Justify all selections.

b. (0.5 point) Management would like to achieve its targeted underwriting profit without changing rates. Discuss whether this is a reasonable expectation based on the information above.

7. (a) I am concerned about the increasing pattern of ratios for General Expenses and Other Acquisition; however, in the absence of additional information, I will take a 3 year average.

Expense Ratios	3-Year Average	Fixed	Variable
General Expenses	5.067%	3.55%	1.52%
Other Acquisition	10.667%	8.53%	2.13%
Taxes, Licenses and Fees	1.467%	0.44%	1.03%
Commission and Brokerage	10.833%	0	10.83%
Total		12.52%	15.51%

Indicated average rate: $\dfrac{(600) + (1000)(12.52\%)}{1 - 15.51\% - 12\%}$ = **$1000**.

Alternately, due to the increasing trend, one may use only the latest year of data for General Expenses and Other Acquisition Expenses.

Fixed Expenses: (70%)(5.9%) + (80%)(11.5%) + (30%)(1.467%) = 13.77%.

Variable Expenses: (30%)(5.9%) + (20%)(11.5%) + (70%)(1.467%) + 10.83% = 15.93%.

Indicated average rate: $\dfrac{(600) + (1000)(13.77\%)}{1 - 15.93\% - 12\%}$ = **$1024**.

(b) The projected average premium per exposure at $1000 is close to the indicated average rate, so it reasonable to expect that management would like to achieve its targeted underwriting profit without changing rates.

Alternately, if the upwards trend in expenses in the data is meaningful, one would expect expense needs to be bigger than in the rate indicated in part (a). In that case, it is <u>not</u> reasonable to expect that management would be able to achieve its targeted underwriting profit without changing rates.

Alternately, the premium-based projection method is inherently flawed and subject to distortions; it treats as a percent of premium expenses which are assumed to fixed. Therefore, it is <u>not</u> reasonable to expect that management would achieve its targeted underwriting profit without changing rates.

Alternately, management could achieve its targeted underwriting profit without changing rates by lowering expenses (for example by lower advertising or layoffs), lowering losses (for example by tighter claim settlement practices), or by marketing/underwriting to lower risk insureds.

<u>Comment</u>: Other reasonable choices can be made in part (a).

In an actual application, the actuary would try to investigate further whether the differences in expense percentages by year are meaningful or represent randomness.

Your answer to part (b) may depend on your answer to part (a).

8. (4.75 points) Given the following information for a book of business as of December 31, 2015:

Rate Change History

Effective Date	Change
October 1, 2013	+4.5%
April 1, 2015	+2.5%

Calendar Year	2013	2014	2015
Earned Premium	$1,870,000	$2,228,000	$2,404,000
Earned Exposures	1,420	1,530	1,610

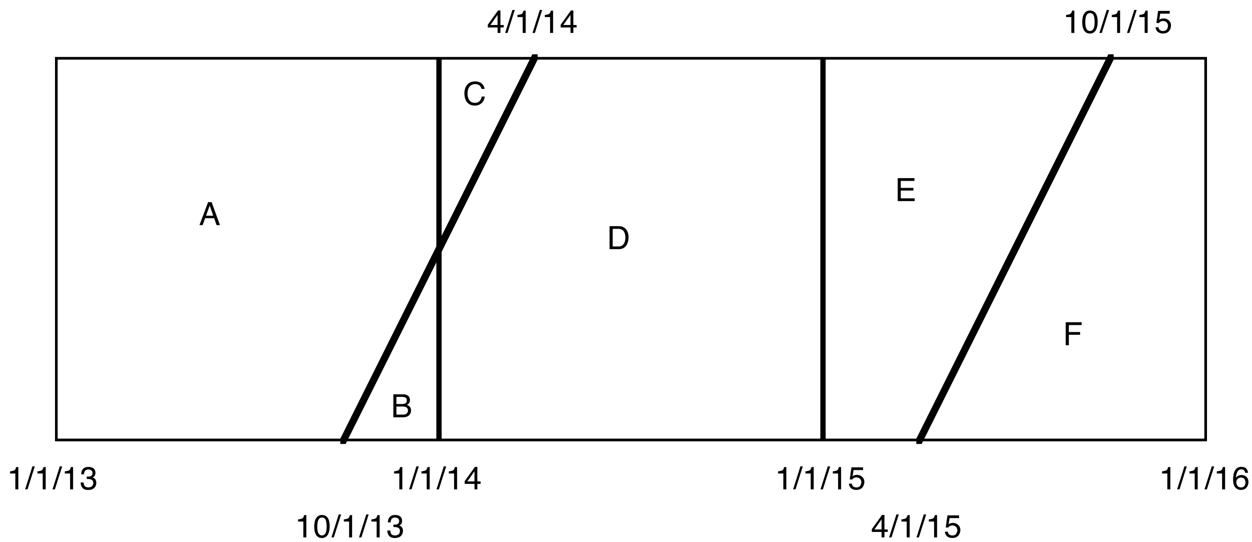
	Cumulative Reported Loss + ALAE ($) as of (months)		
Accident Year	12	24	36
2013	2,150,000	2,395,000	2,495,000
2014	925,000	1,085,000	
2015	1,250,000		

	Cumulative Reported Loss + ALAE excluding Catastrophes ($) as of (months)		
Accident Year	12	24	36
2013	750,000	895,000	975,000
2014	825,000	975,000	
2015	900,000		

- All policies are semi-annual.
- Exposures are written evenly throughout each calendar year.
- Annual severity trend = 5%.
- Annual frequency trend = -1%.
- Annual premium trend = 2%.
- Fixed expense ratio = 5%.
- Variable expense ratio = 22%.
- Profit and contingencies provision = 6%.
- ULAE provision = 7% of loss and ALAE.
- Projected catastrophe load including ALAE = $235 per exposure.
- There is no loss development beyond 36 months.
- Rates are to be in effect for one year.

Calculate the indicated rate change for policies effective January 1, 2017 using the latest three accident years of experience and assuming full credibility.

8. The average date of writing under the new rates is July 1, 2017.
Since policies are semi-annual, the average date of earning and accident is October 1, 2017.
Thus the trend period from AY13 is 4.25 years.



Area B = (1/2)(1/4)(1/2) = 1/16. Area A = 1 - 1/16 = 15/16.
Area C = (1/2)(1/4)(1/2) = 1/16. Area D = 1 - 1/16 = 15/16.
Area E = 1/2 = Area F.
Average rate level for AY13 = (15/16)(1) + (1/16)(1.045) = 1.0028.
Average rate level for AY14 = (1/16)(1) + (15/16)(1.045) = 1.0422.
Average rate level for AY15 = (1/2)(1.045) + (1/2)(1.045)(1.025) = 1.0581.
On level factor for AY13: (1.045)(1.025)/1.0028 = 1.0681.
On level factor for AY14: (1.045)(1.025)/1.0422 = 1.0278.
On level factor for AY15: (1.045)(1.025)/1.0581 = 1.0123.
AY13 on level and trended premium: (1.0681)(1,870,000)($1.02^{4.25}$) = 2,172,722.
AY14 on level and trended premium: (1.0278)(2,228,000)($1.02^{3.25}$) = 2,442,161.
AY15 on level and trended premium: (1.0123)(2,404,000)($1.02^{2.25}$) = 2,544,451.
Working with the losses and ALAE excluding catastrophe.
The estimated development from 1st to 2nd report: (895 + 975) / (750 + 825) = 1.1873.
The estimated development from 2nd to 3rd report: 975/895 = 1.0894.
AY13 ultimate and trended loss & ALAE: (975,000) $\{(1.05)(0.99)\}^{4.25}$ = 1,149,499.
AY14 ultimate and trended loss & ALAE: (1.0894)(975,000) $\{(1.05)(0.99)\}^{3.25}$ = 1,204,680.
AY15 ultimate and trended loss & ALAE:
(1.1873)(1.0894)(900,000) $\{(1.05)(0.99)\}^{2.25}$ = 1,270,122.
I will assume that the exposures are not inflation sensitive.
In order to load in the expected catastrophe losses, multiply by the exposures for the 3 AYs:
(1420 + 1530 + 1610)($235) = $1,071,600.

Thus the loss and ALAE ratio is: $\frac{1,149,499 + 1,204,680 + 1,270,122 + 1,071,600}{2,172,722 + 2,442,161 + 2,544,451}$ = 65.59%.

Indicated rate change factor: $\frac{(1.07)(65.59\%) + 5\%}{1 - 22\% - 6\%}$ = 1.044. ⇔ **4.4% rate increase**.

Comment: I have assumed that the ULAE is the same for catastrophes and non-catastrophes.
Alternately, one could assume that exposures are inflation sensitive and trend at 2% per year.
One could get a somewhat different answer by using a two-piece premium trend method rather than a one-piece premium trend as I have done.
One could use the pure premium method rather than the loss ratio method as I have done.

9. (2 points) The following are considerations for pricing a large deductible policy:

- Deductible = $750,000 per occurrence.
- Expected total ground-up losses = $1,500,000.
- ALAE = 12% of total ground-up losses.
- Fixed expenses = $75,000.
- Variable expenses = 15% of premium.
- Underwriting profit provision = 3%.
- Risk margin = 10% of excess losses.
- Cost of processing losses below the deductible = 5% of losses below the deductible.
- Credit risk = 1.5% of expected deductible payments.
- Deductible applies to losses only and does not reduce ALAE.
- Loss elimination ratios (LER) and excess ratios are:

Loss Limit ($000)	LER	Excess Ratio
$500	85%	15%
$750	90%	10%
$1,000	95%	5%

Calculate the large deductible premium.

9. Fixed expenses = $75,000.
Full ALAE: (12%)(1,500,000) = 180,000.
Excess Losses: (10%)(1,500,000) = 150,000.
Risk Margin: (10%)(150,000) = 15,000.
Expected losses below the deductible: (90%)(1,500,000) = 1,350,000.
(Additional) Cost of processing losses below the deductible: (5%)(1,350,000) = 67,500.
Credit Risk = (1.5%)(1,350,000) = 20,250.

Large deductible premium is: $1000 \dfrac{150 + 180 + 75 + 15 + 67.5 + 20.25}{1 - 15\% - 3\%}$ = **$619,207**.

10. (2.5 points) Given the following:

Class	Premium at Current Rate Level	Reported Loss and ALAE	Number of Claims	Current Relativity
A	$1,257,600	$964,200	924	1.00
B	$879,500	$632,800	623	1.10
C	$254,900	$201,400	185	1.80

• Full credibility standard is 800 claims.

• Partial credibility is determined based on the square root rule.

a. (2 points)

Calculate the indicated rate change for each class to achieve a revenue-neutral overall change.

b. (0.5 point) Briefly discuss two benefits of multivariate classification ratemaking.

10. (a) Compare the loss ratios by class to the overall loss ratio:

Class	Premium	Losses	Loss Ratio	Compared to Overall
A	\$1,257,600	\$964,200	76.67%	1.020
B	\$879,500	\$632,800	71.95%	0.957
C	\$254,900	\$201,400	79.01%	1.051
Total	\$2,392,000	\$1,798,400	75.18%	

The credibilities are: 100%, $\sqrt{\frac{623}{800}}$ = 88.2%, $\sqrt{\frac{185}{800}}$ = 48.1%.

Class	Indicated Change	Credibility	Credibility Weighted Change
A	2.0%	100%	2.0%
B	-4.3%	88.2%	-3.8%
C	5.1%	48.1%	2.5%

The premium weighted credibility weighted change is:

$$\frac{(1{,}257{,}600)(2.0\%) + (879{,}500)(-3.8\%) + (254{,}900)(2.5\%)}{1{,}257{,}600 + 879{,}500 + 254{,}900} = -0.08\%.$$

In order to be revenue neutral we need to divide by: 1 - 0.08% = 0.9992.

Class	Credibility Weighted Change	Revenue Neutral Rate Change
A	2.0%	1.020/0.9992 - 1 = **2.1%**.
B	-3.8%	0.962/0.9992 - 1 = **-3.7%**.
C	2.5%	1.025/0.9992 - 1 = **2.6%**.

(b) 1. Multivariate methods consider all rating variables simultaneously and automatically adjust for exposure correlations between rating variables

2. Multivariate methods attempt to remove unsystematic effects in the data (also known as noise) and capture only the systematic effects (also known as signal) as much as possible.

3. Many multivariate methods produce model diagnostics, additional information about the certainty of results and the appropriateness of the model fitted.

4. Multivariate methods allow consideration of the interaction, or interdependency, between two or more rating variables.

Comment: For part (b), see pages 174 to 175 of Basic Ratemaking. Discuss only 2 benefits.
For part (a), see Appendix E page 4 of Basic Ratemaking.

Class	Cred. Weighted Change	Current Rel.	Indicated Rel.	Rel. to Base
A	2.0%	1.00	1.020	1.000
B	-3.8%	1.10	1.058	1.037
C	2.5%	1.80	1.845	1.809

If we use these relativities with respect to base class A, then the premiums collected would be:
1,257,600 + (879,500)(1.037/1.1) + (254,900)(1.809/1.8) = 2,342,903.
Thus the off-balance factor to multiply by is: 2,392,000/2,342,903 = 1.021.
Note that: (1.021)(1.037/1.10) - 1 = -3.7%, and (1.021)(1.809/1.80) - 1 = 2.6%.

11. (1.75 points) Given the following loss distribution for accident year 2015 by policy limit:

	\$100,000 Limit		\$250,000 Limit		\$500,000 Limit	
Size of Loss	Claims	Losses (\$000)	Claims	Losses (\$000)	Claims	Losses (\$000)
X ≤ \$100,000	210	14,000	40	3,000	50	3,000
\$100,000 < X ≤ \$250,000			50	9,000	40	7,000
\$250,000 < X ≤ \$500,000					10	4,000
Total	210	14,000	90	12,000	100	14,000

a. (1 point) Calculate the increased limits factor for \$250,000 assuming a basic limit of \$100,000.

b. (0.25 point) Assume a ground-up annual severity trend of 10% applies to the data above. Briefly discuss how the increased limits factor estimate would change for future accident years without performing any additional calculations.

c. (0.5 point) Calculate the complement of credibility for the excess layer between \$250,000 and \$500,000 using the industry increased limits factors below.

Limit of Liability	Increased Limits Factor
\$100,000	1.00
\$250,000	1.50
\$500,000	1.90
\$750,000	2.25
\$1,000,000	2.50

11. (a) The limited expected value at \$100,000 is: E[X $\wedge$ 100,000] =

$$\frac{14{,}000{,}000 + 3{,}000{,}000 + (50)(100{,}000) + 3{,}000{,}000 + (50)(100{,}000)}{210 + 90 + 100} = 75{,}000.$$

E[X $\wedge$ 250,000] - E[X $\wedge$ 100,000] =

$$\frac{9{,}000{,}000 - (50)(100{,}000) + 7{,}000{,}000 - (40)(100{,}000) + (10)(150{,}000)}{90 + 100} = 44{,}737.$$

Estimated increased limits factor for \$250,000: 1 + 44,737/75,000 = **1.60**.

(b) The trend would have a greater impact in the excess layers because losses already contributing to the excess would get a leveraged effect, while those just under the limit get pushed into the excess layer. Since excess losses increase faster than basic limit losses, the ILFs will increase.

(c) I assume we are trying to estimate the losses in the layer for this set of policies. (Think from the point of view of an excess reinsurer.) Only the policies with a \$500,000 limit can contribute to this layer. We can start with their basic limit losses of: \$3,000,000 + (50)(100,000) = 8,000,000.

Then the complement of credibility is: $\frac{1.90 - 1.50}{1.00}$ = (\$8,000,000) = **\$3,200,000**.

Alternately, we can start with their losses limited to \$250,000 of:
\$3,000,000 + 7,000,000 + (10)(250,000) = 12,500,000.

Then the complement of credibility is: $\frac{1.90 - 1.50}{1.50}$ (\$12,500,000) = **\$3,333,333**.

Alternately, we can try to estimate what the losses in the layer between \$250,000 and \$500,000 for all of the policies would be if these policies had <u>no</u> limit. Basic limit losses of:

14,000,000 + 3,000,000 + (50)(100,000) + 3,000,000 + (50)(100,000) = \$30,000,000.

Then the complement of credibility is: $\frac{1.90 - 1.50}{1.00}$ (\$30,000,000) = **\$12,000,000**.

<u>Comment</u>: In part (b), since the excess losses are actually a layer with a finite upper endpoint, for unusual data sets the empirical excess losses can increase slower than the basic limits losses.
For example, let us assume only two losses: 50,000 and 500,000.
Then prior to inflation for a basic limit of \$100,000, the basic limit losses are \$150,000.
After 10% inflation, the basic limit losses are \$160,000.
Prior to inflation for an increased limit of \$250,000, the excess losses are \$150,000.
After 10% inflation, the excess losses are still \$150,000.
The increased limit factor goes from 2 to 310/160 = 1.94.
The CAS Examiner's report, did not include my first two solutions to part (c); I disagree!
Their report also included: Losses capped at 250,000 for policies with at least 250,000 limits:
(\$1 million) (3 + 9 + 3 + 7 + 2.5) = 24,500,000.

Then the complement of credibility is: $\frac{1.90 - 1.50}{1.50}$ (\$24,500,000) = \$6,533,333.

This complement would apply to estimating what the losses in the layer from 250,000 to 500,000 for those polices with at least 250,000 limits would be if these policies had <u>no</u> limit.

12. (2.5 points) An insurer is proposing the following changes in order to address inadequate rates:

	Rating Factor		
Building Type	Current	Proposed	Exposures
Commercial	1.00	1.00	300
Large Industrial	1.15	1.15	500
Small Industrial	1.20	1.40	100

	Discount		
Years Since Claim	Current	Proposed	Exposures
0	0%	0%	50
1	10%	5%	150
2+	15%	10%	700

- Additive expense factor (after applying rating factors and discounts) = $20.
- Base premium = $100.

a. (1.75 points) Estimate the change to average premiums.
b. (0.25 paint) Briefly explain a shortcoming with the calculation performed in part a. above.
c. (0.5 point) Briefly describe two non-pricing solutions that can address inadequate rates.

12. (a) Current average building type rating factor:

$$\frac{(1.00)(300) + (1.15)(500) + (1.20)(100)}{300 + 500 + 100} = 1.1056.$$

Current average discount: = $\frac{(0)(50) + (10\%)(150) + (15\%)(700)}{50 + 150 + 700} = 0.1333.$

Current average premiums: (\$100)(1.1056)(1 - 0.1333) + \$20 = \$115.82.
Proposed average building type rating factor:

$$\frac{(1.00)(300) + (1.15)(500) + (1.40)(100)}{300 + 500 + 100} = 1.1278.$$

Proposed average discount: $\frac{(0)(50) + (5\%)(150) + (10\%)(700)}{50 + 150 + 700} = 0.0861.$

Proposed average premiums: (\$100)(1.1278)(1 - 0.0861) + \$20 = \$123.07.
The change to average premiums is: 123.07/115.82 = 1.0626. ⇔ **6.26% increase**.
(b) In the calculation we have <u>no</u>t taken into account the different mix of discounts between the different building types and vice versa.
For example, 7/9 of the overall exposures get the biggest discount, however, each building type may have more or less than 7/9 of its exposures get this biggest discount.
(We would want a gird of exposures by building type and discount.)
Alternately, the rate change will effect those who will purchase insurance from this insurer. Therefore, the past distribution of exposures is likely not identical to that in the future. This calculation does not take this into account
(c) 1. Reduce expenses: reduce commissions, reduce marketing expenses, reduce staff, etc.
2. Reduce loss costs: be stricter in settling claims, reduce coverage provided in the policy language, tighten up underwriting standards, etc.
3. Require insureds to fulfill certain loss mitigating practices such as safety seminars.
4. Adopt a more aggressive investment strategy in order to increase expected investment income.
<u>Comment</u>: In part (c), discuss only 2 solutions.

13. (3 points) An insurance company sells workers compensation insurance, which includes both indemnity and medical loss types. In preparation for its next rate filing, effective January 1, 2017, the company uses the following information about its book of business for accident year 2015, evaluated as of December 31, 2015:

Territory	Exposures	Indemnity Loss & ALAE	Medical Loss & ALAE	Workers Compensation Total Current Relativity
A	2,500	\$2,000,000	\$2,000,000	1.20
B	3,500	\$3,000,000	\$500,000	0.90
C	4,500	\$4,000,000	\$1,000,000	1.00

• Indemnity development factor to ultimate = 2.50.
• Medical development factor to ultimate = 1.50.
• Indemnity annual loss and ALAE trend = 3%.
• Medical annual loss and ALAE trend = 6%.
• Accidents are evenly distributed throughout the experience period.
• All policies are annual.
• Rates are in effect for one year.
• The base territory remains the same.

a. (2.25 points) Calculate the indicated territorial relativities to the base territory.
b. (0.75 point) Determine the percent change by territory, assuming the indicated relativities are to be adopted and no overall premium change is desired.

13. (a) The new rates will be in effect from January 1, 2017 to December 31, 2017.
The average date of writing is July 1, 2017.
Since policies are annual, the average date of accident is January 1, 2018.
Thus the trend is from July 1, 2015 to January 1, 2018 or 2.5 years.
The trend factor for indemnity is: $1.03^{2.5} = 1.0767$.
The trend factor for medical is: $1.06^{2.5} = 1.1568$.
We trend and develop the losses and divide by the exposures:

Indemnity	Trend	Devel.	Product	Medical	Trend	Devel.	Product
2	1.0767	2.5	5.3835	2	1.1568	1.5	3.4705
3	1.0767	2.5	8.0752	0.5	1.1568	1.5	0.8676
4	1.0767	2.5	10.7670	1	1.1568	1.5	1.7352
Losses	Expos.	Pure Prem.	Relat.				
8.8539	2500	$3542	**1.275**				
8.9428	3500	$2555	**0.920**				
12.5022	4500	$2778	**1.000**				

Then we compare the pure premiums to those for the base territory C.
For example for Territory A: 3542/2778 = 1.275.
I have assumed full credibility.

(b) The current average relativity is: $\dfrac{(2500)(1.20) + (3500)(0.90) + (4500)(1.00)}{2500 + 3500 + 4500} = 1.0143.$

The current average relativity is: $\dfrac{(2500)(1.275) + (3500)(0.920) + (4500)(1.00)}{2500 + 3500 + 4500} = 1.0388.$

For no change in average premiums, the changes by territory are:
Territory A: (1.0143/1.0388) (1.275/1.20) = 1.037. ⇔ **3.7% increase**.
Territory B: (1.0143/1.0388) (0.920/0.90) = 0.998. ⇔ **0.2% decrease**.
Territory C: (1.0143/1.0388) (1/1) = 0.976. ⇔ **2.4% decrease**.
Comment: In part (b), the off-balance factor to multiply by is: 1.0143/1.0388 = 0.9764.
Here since the indemnity and medical losses have different loss development and trend factors, taking into account trend and loss development makes a difference in the indicted relativities.
The same would be true if instead we were using several different years of data.
In most states, workers compensation rates do not vary by territory.