Solutions to the Spring 2015 CAS Exam 5

(Only those questions on Basic Ratemaking)

There were 25 questions worth 57.25 points, of which 14 were on ratemaking worth 32.25 points.[1]

The exam and examiner's report are available from the CAS.
The solutions and comments are solely the responsibility of the author.

(Incorporating what I found useful in the CAS Examiner's Report)

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[1] Question 11 worth 5.75 points relies somewhat on material from the reserving textbook.
Question 15 worth 1.5 points while intended to be answered from the reserving textbook can also be answered from the ratemaking textbook.

1. (1.5 points) Given the following information for an ISO Personal Automobile Policy:

Primary Classification	Factor
Pleasure use	1.00
Work less than 15 miles	1.07
Work 15 or more miles	1.18
Farm use	0.82
Business use	1.25

Secondary Classification	Factor
0	0.30
1A	0.70
1B	0.75
2	1.20
3	1.80
4	2.50

- The policy has one 30-year-old driver who uses the vehicle to commute 10 miles to work one day per week.
- The driver has been licensed for more than two years and has no driving record points.
- The policy term is one year.
- The total premium is $1,480.
- The expense fee is $80 per policy.
- The policy coverages are Bodily Injury Liability and Property Damage Liability only.
- Only auto usage and safe driver insurance plan (SDIP) are used to determine primary and secondary classification.
- The policy is a single-car policy.

At six months into the policy term, a second driver is added as an operator with the following characteristics:

- 17 years old.
- Has been licensed for less than two years.
- Has 2 driving record points,
- Uses the vehicle 3 days per week to commute 10 miles to school.

Calculate the premium change in dollars for adding the new driver to the policy for the remainder of the policy term.

1. The expense fee of $80 does not depend on the classification and thus does not change.
Prior to the change, the Primary Class is Pleasure Use.
(Driving to work only one day less than 15 miles does not count.)
Prior to the change we have zero driver points for Secondary Class 0.
Thus the rating factor is: 1.00 + 0.30 = 1.30.
After the change, the Primary Class is Work less than 15 miles.
After the change we have two driver points for Secondary Class 2.
Thus the rating factor is: 1.07 + 1.20 = 2.27.
The change in classification is applied pro-rata, in this case to half of the $1400 annual variable premium, or $700.
Thus the change in premium is: (2.27/1.30 - 1) ($700) = $522.
The insured pays **$522 more in premium** for adding the new driver to the policy for the remainder of the policy term.
Alternately, if the new classification had been in effect for the whole year, the premium would have been: $80 + (1400)(2.27/1.30) = $2524.6.
Thus the change in premium is: ($2524.6 - $1480)/2 = **$522**.
Comment: See Rules 4.C.1, 5.C, and 9.A in the ISO P.P. Auto Manual.
In this exam question, age of driver was not used for classification solely for simplicity.
Since the second driver had been licensed less than 2 years, if he had been the principal operator and had had zero SDIP points, then the secondary classification would have been 1B.

2. (2 points) An insurer is considering changing the exposure base for a commercial auto line of business to one of the following:
i. Annual fuel expense
ii. Number of miles driven
Using three relevant actuarial criteria, evaluate the effectiveness of each of these potential exposure bases and provide a recommendation for the preferred exposure base.

2. i. While annual fuel expense would vary somewhat with the potential for loss, it would also be affected by the price of gasoline (or diesel fuel) and the fuel efficiency of vehicles neither of which varies with the expected losses. Thus this exposure is not proportional to loss costs.
For example, let us assume you make rates with cost of fuel as the exposure base, based on an average price of gas of $3 a gallon. However, during the period of time the rates are in effect the average price of gas is $2.50, then the projected premiums will be inadequate when the rates are applied to the annual fuel expense. If instead the price of average price of gas during the period of time the rates are in effect is $3.50, then the premiums will be excessive.
Annual fuel expense is practical, since the insured business already keeps track of this for other purposes. (Alternately, they may not keep careful track for each vehicle separately, which would be important if there are vehicles of different types in the insured fleet.)
Annual fuel expense lacks historical precedence. Changing to a new exposure base would cause large premium swings, it would be costly to implement, and would cause extra work for IT and actuarial staff.
I would not change to annual fuel expense as the exposure base since it is less proportional to losses than caryears.
ii. As the number of miles driven increases, the expected number of accidents increases, and thus the expected losses increase. While it would matter whether these are miles driven on a highway or city streets, this exposure base is more closely proportional to expected losses than is caryears.
Miles driven may be practical, depending on the technology used to determine the miles driven. (Odometer readings can be manipulated. On the other hand, a fleet of interstate trucks may already for other purposes keep careful track of the miles driven by each vehicle.)
Number of miles driven lacks historical precedence. Changing to a new exposure base would cause large premium swings, it would be costly to implement, and would cause extra work for IT and actuarial staff.
Assuming that the appropriate affordable technology (for example telematics) is available to verify the miles driven by each vehicle, I would change to number of miles driven as the exposure base, since it is more proportional to expected losses than caryears.
Comment: One could instead decide not to change the exposure base to either of the two choices.

3. (3 points) Given the following information for an insurance product:

Time Period	Written Car Years
January 1 through June 30, 2013	106.0
July 1 through December 31, 2013	107.5
January 1 through June 30, 2014	210.0
July 1 through December 31, 2014	45.0

- All policies written before January 1, 2014 had a six-month policy term.
- All policies written from January 1, 2014 onwards had an annual policy term.

a. (1.25 points) Calculate the earned car years for calendar year 2014, assuming that policies were written uniformly throughout each period.

b. (1.25 points) Assume new policies in 2014 were written uniformly over the year and the retention ratio prior to the policy term change was 77%.
Assess the effect of the policy term change on the retention ratio.

c. (0.5 point) Assess the appropriateness of the assumption of uniform writings in the calculation of calendar year 2014 earned car years.

3. (a) 6-month policies written during January 1 through June 30, 2013 contribute none of their written exposures to CY14 earned exposures.
6-month policies written during July 1 through December 31, 2013 contribute on average half of their written exposures to CY14 earned exposures.
12-month policies written during January 1 through June 30, 2014 contribute on average (1 + 1/2)/2 = 3/4 of their written exposures to CY14 earned exposures.
12-month policies written during July 1 through December 31, 2014 contribute on average (1/2 + 0)/2 = 1/4 of their written exposures to CY14 earned exposures.
(0)(106) + (1/2)(107.5) + (3/4)(210) + (1/4)(45) = **222.5**.

Time Period	Written caryears	Avg. Written Date	% earned in 2014	Earned caryears in 2014
1/1/13-6/30/13	106	4/1/13	0%	(0%)(106) = 0
7/1/13-12/31/13	107.5	10/1/13	50%	(50%)(107.5) = 53.75
1/1/14-6/30/14	210	4/1/14	75%	(75%)(210) = 157.5
7/1/14-12/31/14	45	10/1/14	25%	(25%)(45) = 11.25
Total				**222.5**

(b) Of the 6-month policies written during the second half of 2013, if 77% renew, then since they would be annual policies, they would produce (2)(77%)(107.5) = 165.55 caryears. We wrote 45 exposures during the second half of 2014, which are new policies.
If we had a similar 45 new exposures in the first half of 2014, then we would expect 166.55 + 45 = 210.55 caryears written during the first half of 2014. Since there were 210 caryears written during the first half of 2014, the retention ratio seems about the same as it was prior to the change in policy term.
We can estimate the retention ratio as: (210 - 45)/ {(2)(107.5)} = 76.7%, very close to 77%.
(c) Clearly an assumption of uniform writings each year would be inappropriate for the calculation of calendar year 2014 earned car years, since we observe that there are many more exposures written in the first half of 2014 than in the second half. In the absence of any additional information, an assumption of uniform writings each half year seems appropriate for the calculation of calendar year 2014 earned car years, as was done in part (a).
Alternately, the policies up for renewal from prior to PY2014 are 6 month which means they will be up for renewal in the first half of 2014. Since all policies are shifted from six months to annual, they will be up for renewal in the first half of 2015; the next wave of written car years will not occur until the first half of CY 2015 on these annual renewals. Thus the assumption of uniform writing during 2014 is inappropriate.
Comment: We have to assume no cancellations or endorsements; otherwise an endorsement can add cars to a policy, which may be some of the written exposures during the second half of CY14. Given enough time the number of exposures written would eventually even out between the two halves of a year, although it might take decades.

Retention Ratio = $\frac{\text{Number of Policies Renewed}}{\text{Number of Potential Renewal Policies}}$. Here there is no way to determine the number of policies invited to renew or that are able to be renewed.

4. (2.25 points) Given the following information:

- Policies are written on an annual basis.
- Proposed rates will be in effect from January 1, 2016 to January 1, 2017.
- Calendar year 2014 earned premium = $100,000.
- Beginning with July 1, 2012 renewals, the minimum deductible was increased from $500 to $1,000.
- The premium impact of any law change is applicable to all policies, including those in-force.
- The rate change history is as follows:

Effective Date	Overall Change	Type of Change
April 1, 2014	+5%	Law
July 1, 2014	+3%	Rate
July 1, 2015	-7%	Rate

- The annual premium exponential trend fit based on data for the 12 months ending each quarter evaluated through December 31, 2014 is as follows:

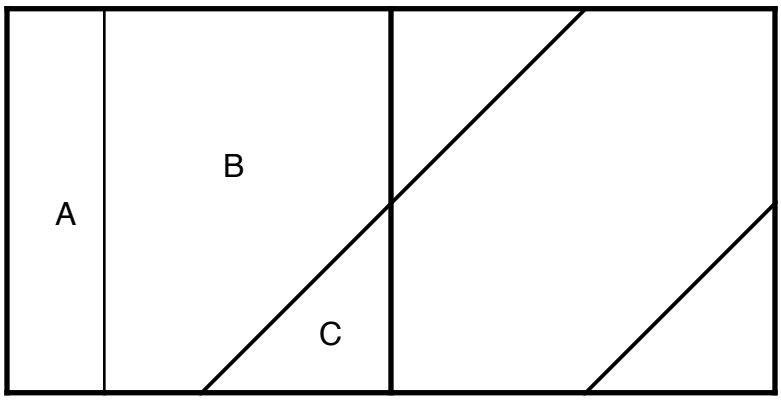
Calendar Year Ending	Average Earned Premium at Current Rate and Law Level
March 2012	$510.00
June 2012	$512.50
September 2012	$499.50
December 2012	$489.00
March 2013	$481.00
June 2013	$473.00
September 2013	$477.50
December 2013	$481.50
March 2014	$487.00
June 2014	$492.50
September 2014	$496.00
December 2014	$502.00

Number of Points	Annual Exponential Trend Fit
12 point	-0.5%
8 point	2.2%
6 point	3.4%
4 point	3.1%

Calculate the trended calendar year 2014 earned premium at current rate level.
Include justification of the premium trend selection.

4.

Effective Date	Overall Change	Type of Change	Rate Level Index
Prior			1.0000
April 1, 2014	+5%	Law	1.0500
July 1, 2014	+3%	Rate	1.0815
July 1, 2015	-7%	Rate	1.0058



Area A = 1/4. Area C = (1/2)(1/2(1/2) = 1/8. Area B = 1 - 1/4 - 1/8 = 5/8.
Thus the on-level factor for 2014 earned premium is:

$$\frac{1.0058}{(1)(1/4) + (1.050)(5/8) + (1.0815)(1/8)} = 0.9658.$$

Increasing the minimum deductible sold will decrease the average premium.
The series of average earned premiums does not seem to have been adjusted for the change in the minimum deductible sold.
Since annual policies are being sold, the year ending June 2013 is the first value all on the basis of the new minimum deductible. Thus we can use the last 7 points.
I will select a premium trend of 3.4% based on the last 6 points.
(The trend based on the last 4 points of 3.1% would be similar.)
The average data of writing under the new rates is 7/1/2016.
Since annual policies are being sold, the average date of earning is 6 months later or 1/1/2107.
Thus the premium trend is from 7/1/2014 to 1/1/2017 or 2.5 years.
The trended calendar year 2014 earned premium at current rate level is:
$(100{,}000)(0.9658)(1.034^{2.5}) =$ **$105,000**.
Comment: Slightly different premium trend selections would also be reasonable.

5. (2.5 points) Given the following information for a boat owners insurer:

- On July 1, 2014, a rate change of +10% went into effect.
- 2014 earned premium = $1,000.
- Policies are written on an annual basis.

a. (0.75 point) Assuming uniform writings, calculate the calendar year 2014 on-level earned premium using the parallelogram method.

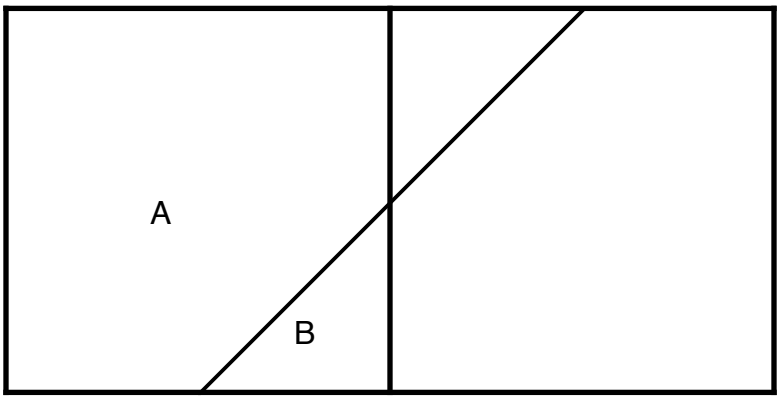
b. (1.25 points) Due to the seasonality of boat owners coverage, assume that the policy distribution of the book of business was as follows (with uniform distribution within each quarter):

Quarter 1	10%
Quarter 2	50%
Quarter 3	30%
Quarter 4	10%

Calculate the calendar year 2014 on-level earned premium, accounting for this assumed policy distribution.

c. (0.5 point) For the scenario in part b. above, describe another approach the insurer could take to calculate the on-level earned premium.

5. (a) Area B = (1/2)(1/2)(1/2) = 1/8. Area A = 1 - 1/8 = 7/8.



On-level factor is: $\frac{1.1}{(7/8)(1) + (1/8)(1.1)} = 1.08642$.

On-level earned premium is: (1.08642)(1000) = **$1086.42**.

(b) 1/8 of 2013 quarter 1 is earned during 2014.
3/8 of 2013 quarter 2 is earned during 2014.
5/8 of 2013 quarter 3 is earned during 2014.
7/8 of 2013 quarter 3 is earned during 2014.
(1+3/4)/2 = 7/8 of 2014 quarter 1 is earned during 2014.
5/8 of 2014 quarter 2 is earned during 2014.
3/8 of 2014 quarter 3 is earned during 2014.
1/8 of 2014 quarter 3 is earned during 2014.
Average rate level for 2014 earned premium is:
(1) {(1/8)(10%) + (3/8)(50%) + (5/8)(30%) + (7/8)(10%) + (7/8)(10%) + (5/8)(50%)}
+ (1.1) {(3/8)(30%) + (1/8)(10%)} = 0.875 + (1.1)(0.125) = 1.0125.
On-level factor is: 1.1 / 1.0125 = 1.08642.
On-level earned premium is: (1.08642)(1000) = **$1086.42**.
Alternately, assume that the given pattern is for the volume of earned exposures.
1/8 of the exposures earned during the 3rd quarter of 2014 is on the new rate level.
3/8 of the exposures earned during the 4th quarter of 2014 is on the new rate level.
Then the average rate level for 2014 earned premium is:
10% + 50% + (30%){7/8 + (1/8)(1.1)} + (10%){5/8 + (3/8)(1.1)} = 1.0075.
On-level factor is: 1.1 / 1.0075 = 1.09181.
On-level earned premium is: (1.09181)(1000) = **$1091.81**.
(c) The insurer could use extension of exposures, where each policy is rerated using the current rate manual, and then the amounts earned from each policy during CY2014 are aggregated.
Alternately, the insurer could use the parallelogram method on smaller time periods, such as monthly, and then aggregate the on-level premium from the smaller time periods to determine the total on-level earned amount.
Comment: The fact that part (a) and (b) can result in the same answer is a coincidence.

6. (1.5 points) Discuss the appropriateness of applying each of the following data aggregation methods to the given line of business:

a. (0.5 point) Calendar Year Aggregation for Auto Physical Damage

b. (0.5 point) Policy Year Aggregation for Homeowners

c. (0.5 point) Report Year Aggregation for Medical Professional Liability

6. (a) Since most Auto Physical Damage claims are reported and settled very quickly, Calendar Year data is appropriate. Calendar Year data is available more quickly than Accident Year Data. Alternately, not appropriate as calendar year is fixed at year-end (transaction based), but Auto Physical Damage losses will still develop.
(b) The property piece of Homeowners is reported and settled quickly, while the liability piece takes somewhat longer. The majority of the expected losses are from property rather than liability. Policy Year data would take longer to be available, and therefore Accident Year data is more appropriate for Homeowners.
Alternately, Policy Year aggregation provides for a good match between premium/exposures and losses. This would be a good choice if we are trying to isolate the effects of underwriting changes such as changes to policy limits or deductibles which could be appropriate for homeowners policies.
(c) Since Medical Professional Liability claims can take a very long time to be reported and then settled, Report Year is an appropriate method of aggregation.
Comment: Werner and Modlin use Accident Years for Homeowners in Appendix B and Accident Years for Medical Malpractice in Appendix C.
In its Examiner's Report, the exam committee stated that one had to mention claims-made policies in part (c)! This seems to have been based on page 42 of Estimating Unpaid Claims Using Basic Techniques: "For some lines of insurance, such as medical malpractice, products liability, errors and omission, and directors' and officers' liability, coverage may be dependent on the date on which the claim is reported to the insurer (i.e., claims-made coverage). For these lines of business, actuaries often prefer to use *report year data* for developing estimates of unpaid claims."
In fact, report year is also useful for Medical Malpractice occurrence policies, as shown in Chapter 16 of Basic Ratemaking. Occurrence policies have a longer average time from writing to claim settlement than do similar claims-made policies; see principle 5 of claims-made ratemaking at page 314 of Basic Ratemaking. Thus, report year can also be useful for analyzing occurrence policies.
For part (c), in its Examiner's Report the exam committee listed as a sample answer:
"RY aggregation is not appropriate because it cannot be used to estimate IBNR which is very important for a long-tailed line such as this."
This answer is based on page 43 of Estimating Unpaid Claims Using Basic Techniques:
"Estimation techniques based on claims aggregated by report year only measure development on known claims and not pure IBNR; and pure IBNR is frequently the more difficult part of the total unpaid claims estimate to determine. Other methods for developing unpaid claim estimates are required to derive the pure IBNR when using report year data."
First, for claims-made policies there is no (pure) IBNR, so that would not apply if we followed the CAS Examiner's Report and focused on claims-made policies. Second, for occurrence policies, one can estimate IBNR by analyzing a sufficient number of lags for old report years. Also one can estimate IBNER by analyzing a single RY/lag combination evaluated as of different points in time.

7. (2.5 points) The following information applies to a company's book of annual policies:

- All policies are written on January 1 each year.
- All losses occur on July 1 each year.
- Loss costs increase by 5% each report year.
- All losses are reported within 4 years of the occurrence date.
- Loss notification is always made on December 31 of a given year.
- Exposure levels are constant and each policy has one exposure.
- There are no expense or profit provisions.
- The ultimate loss costs for report year 2014 are as follows:

Report Year Lag	Loss Cost
0	$250
1	$350
2	$300
3	$100

a. (1 point) Calculate the premium of a policy issued in 2015 on:

i. a mature claims-made policy

ii. an occurrence policy

b. (0.5 point) Describe a scenario in which a mature claims-made policy would cost more than an occurrence policy.

c. (1 point) Beginning with losses occurring on or after January 1, 2015, a new law requires all losses to be reported within 2 years of the occurrence date. Explain the necessary adjustments, if any, to the premiums in part a. above without further calculations.

7. (a) For example, (1.05)(350) = 367.5. (300)(1.05^2) = 330.75.

	Lag 0	Lag 1	Lag 2	Lag 3
RY14	250	350	300	100
RY15	262.5	367.5	315	105
RY16		385.88	330.75	110.25
RY17			347.29	115.76
RY18				121.55

Premium for a mature claims-made policy issued in 2015 is the sum of a row:
262.5 + 367.5 + 315 + 105 = **$1050** = (1.05)($1000).
Premium for an occurrence policy issued in 2015 is the sum of a diagonal:
262.5 + 385.88 + 347.29 + 121.55 = **$1117.22**.
(b) More investment income is earned on an occurrence policy than a mature claims-made policy. Therefore, if interest rates are much bigger than the inflation rate of the relevant insurance loss costs, then a mature claims-made policy would cost more than an occurrence policy.
Alternately, if loss costs are decreasing, in other words if loss cost trends are negative, then the mature claims-made policy will cost more than an occurrence policy. This is because the claims-made policy will pay claims at today's higher costs while the occurrence policy will not have to pay some of the losses until later, with lower costs, due to the report lag.
Alternately, if there is huge abrupt change in the reporting pattern of claims, so that one report year has a big spike in claims reported during that year, then that particular mature claims-made policy would cost more than the corresponding occurrence policy.
(c) Assume that the same total number of claims will be reported to the insurer, with some of them reported sooner than they otherwise would. (It could instead be the case that some of the claims that currently take longer to be reported would not be reported sooner and under the new law the insurer would not be responsible for paying them.)
A loss that occurred on July 1, 2015 that is reported during 2017 is in RY17 lag 2.
However, it is assumed all losses are reported on some December 31.
A loss that occurred on July 1, 2015 that was reported on Dec. 31, 2017, would not be covered under the new law. Thus, losses reported in lag 2 and also lag 3 would not be covered under the new law!
I will assume that all of those claims that would have been reported in lags 2 and 3 will now be reported in lag 1; the new reporting pattern by lag is proportional to: 250, 350 + 300 + 100 = 750.
The first group affected is for claims occurring in 2015, that would have been reported at lag 2, or during 2017, which will instead be reported in 2016 at lag 1.
Claims occurring in 2015, that would have been reported at lag 3, or during 2018, I will assume will instead be reported in 2016 at lag 1.

	Lag 0	Lag 1	Lag 2	Lag 3
RY14	250	350	300	100
RY15	262.5	367.5	315	105
RY16		$(750)(1.05^2) = 826.88$	330.75	110.25
RY17			0	115.76
RY18				0

Premium for a mature claims-made policy issued in 2015 is the sum of a row:
262.5 + 367.5 + 315 + 105 = **$1050** = (1.05)($1000), **same as in part (a)**.
Premium for an occurrence policy issued in 2015 is the sum of a diagonal:
262.5 + 826.88 = **$1089.38**, **slightly less than in part (a)**.
Comment: In part (c) we are told we need not do calculations. Presumably if one makes the same assumptions as the examiner, one need only "explain the necessary adjustments if any".
A more complete set of loss costs including the effects of the law change:

	Lag 0	Lag 1	Lag 2	Lag 3	Total
RY14	250	350	300	100	1000
RY15	262.5	367.5	315	105	1050
RY16	275.63	826.88	330.75	110.25	1102.51
RY17	289.41	868.22	0	115.76	1273.39
RY18	303.88	911.63	0	0	1215.51
RY19	319.07	957.21	0	0	1276.28

Note how the abrupt change in reporting pattern causes a mature claims-made policy issued in 2017 to have an unusually large loss cost of $1273.39. Without the law change, this loss cost would have been instead: $(1000)(1.05^3) = \$1157.76$.
An occurrence policy issued in 2017 would have a premium of:
289.41 + 911.63 = $1201.04 < $1273.39.
If instead we assumed that the claims in lags 2 and 3 would not be reported sooner and thus would be not be covered under the new law, then the premium for an occurrence policy issued in 2015 is the sum of a diagonal: 262.5 + 385.88 = $648.38, much less than in part (a)
Note that this exam question, as well as the example in Basic Ratemaking, assumes that the inflation level depends on when the claim is reported. In practical applications, the inflation level depends on one or more of the following: when the event occurred, when the claim is reported, and when the claim is paid.

8. (2.25 points) An insurance company is using the following data to determine an appropriate large loss threshold and excess loss factor for use in its overall ratemaking calculation:

Accident Year	Reported Loss Including Severity Trend ($000)	Number of Claims Greater than $500,000	Excess Ratio at $500,000
1999	56,261	8	5.5%
...	...	...	...
2011	56,793	5	3.6%
2012	57,049	11	6.3%
2013	50,038	7	3.6%
Total	813,339	121	4.8%

- Excess ratio is the ratio of excess loss dollars to non-excess loss dollars.
- Total number of claims in accident years 1999 to 2013 = 12,435.

The following information is given for accident year 2014:

- Total reported losses including severity trend = $46,902,000.
- Total reported ALAE = $2,345,000.
- Loss and ALAE development factor = 1.08.
- Unlimited severity trend factor = 1.05.
- Unlimited frequency trend factor = 0.98.
- $500,000 limit applies to loss only. Individual claims, including severity trend, greater than $500,000 are:

Claim	Reported Loss Including Severity Trend
1	$504,000
2	$644,000
3	$817,000
4	$975,000

a. (1 point) Calculate the 2014 excess ratio at $500,000 and justify a recommended excess ratio to be used in the company's overall rate indication.

b. (1.25 points)

Calculate the 2014 projected ultimate losses and ALAE using an excess loss procedure.

8. (a) Excess losses for 2014: 4K + 144K + 317K + 475K = $940,000.
Excess Ratio for 2014 = Excess / Non-excess = 940 / (46,902 - 940) = **2.0%**.
Since there is a lot of random fluctuation in the excess ratios, and there are few claims of size greater than $500,000 per year, I recommend the use of the total excess ratio for 1999 to 2013 of **4.8%**.
Alternately, one can include 2014 with the other years.
One can back out that the excess losses for 1999 to 2013 are about 37,252,000 since:
37,252 / (813,339 - 37,252) = 4.8%.

Thus the total excess ratio for 1999 to 2014 is: $\frac{37{,}252 + 940}{(813{,}339 - 37{,}252) + (46{,}902 - 940)} = 4.6\%$.

(b) Using the 4.8% excess ratio, the estimated losses for 2014 (including severity trend) are:
(1.048) (46,902K - 940K) = 48,168K.
Including frequency trend, the 2014 losses are: (0.98)(48,168K) = 47,205K.
Adding trended ALAE: 47,205K + (2345K)(1.05)(0.98) = 49,618K.
Developing to ultimate: (1.08)(49,618K) = $**53,587K.**
Comment: See Table 6.3 in Basic Ratemaking.

9. (1.25 points) Given the following information:

Expense Type	2014 Expense Amount
Other Acquisition	$4,000
General	$17,000
Commission and Brokerage	$23,000
Taxes, License, and Fees	$5,000

- 2014 written premium = $200,000.
- 2014 earned premium = $170,000.
- Statewide average loss cost per exposure = $250.
- Profit and contingencies provision = 3%.
- General expenses are incurred throughout the policy term.
- All other expenses are incurred at the beginning of the policy.

Calculate the indicated average rate.

9. I assume that all expenses are variable.
General Expense = 17,000 / 170,000 = 10%.
Other Expense = (4000 + 23,000 + 5000) / 200,000 = 16%.
Indicated average rate = $250 / (1 - 10% - 16% - 3%) = **$352.11**.

10. (1.75 points) Given the following information for an insurance company:

Territory	Premium	Ultimate Loss Ratio
A	$45,000,000	80%
B	$55,000,000	60%

- The company sets its rates to achieve a permissible loss ratio of 65%.
- All expenses are variable.
- Due to market constraints, the company has decided to cap rate increases in any single territory to a maximum of +20%.

a) (1.25 points) Taking market constraints into account, use the loss ratio method to determine the rate changes for territories A and B.

b) (0.5 point) If the company decides to use a different permissible loss ratio due to the territorial cap, the fundamental insurance equation will not be in balance. Describe an alternative to pricing changes the company can take to achieve balance in the fundamental insurance equation.

10. a) The indicated rate changes by territory are: 80/65 - 1 = 23.1%, and 60/65 - 1 = -7.7%.
We would cap the rate change in territory A at +20%.
This would result in premiums for territory A of: (1.2)(45M) = $54M.
The total expected losses are: (80%)(45M) + (60%)(55M) = $69 million.
Thus the desired overall premium is: 69M / 0.65 = $106.15 million.
Thus the desired premium for territory B is: 106.15M - 54M = 52.15M.
Making up for the capping of the rate change in Territory A, the rate change in Territory B to achieve the desired overall loss ratio is: 52.15 / 55 - 1 = **-5.2%**.
Alternately, let x be the desired rate change factor for territory B after capping territory A.
The ultimate losses in millions are: (0.8)(45) + (0.6)(55) = 69.
The premiums in millions will be: (1.2)(45) + x(55) = 54 + 55x.
In order to have the desired 65% loss ratio:
0.65 = 69/(54 + 55x). $\Rightarrow$ x = 0.948. $\Rightarrow$ **-5.2%** rate change for Territory B.
(b) The insurance company can cut expenses, by for example cutting its number of employees or its marketing budget.
Alternately, the insurance company can try to cut expected loss ratios by for example having the underwriters be more strict in which insureds they choose to write, by nonrenewing blocks of policies that have high expected loss ratios, or by reducing coverage (without changing rates).
Comment: In part (b), the insurance company may want to use lifetime analysis which takes into account not just the current policy but also the profits from expected renewals. In light of this strategy, the insurance company may want to reduce the current underwriting profit provision in order to get the fundamental insurance equation in balance.

11. (5.75 points) An insurer filed the following information in support of a rate change to be effective October 1, 2015:

- All policies are semi-annual.
- The new rates will be in effect for one year.
- The indicated rate change from the prior rate review was 7%, with a target effective date of July 1, 2014.
- The implemented rate change from the prior rate review was 2.5%, with an effective date of October 1, 2014.
- The full credibility standard is 60,000 exposures over two years.
- The insurer uses the square root rule for partial credibility.
- The complement of credibility is selected using the trended present rates method.
- There have been no significant changes to paid loss patterns over the past four years.
- Variable expense ratio = 20%.
- Fixed expense ratio = 7%.
- LAE provision = 3% of loss.
- Profit and contingencies provision = 6%.
- Annual premium trend = 3%.
- Annual loss trend = 5%.
- All losses and subrogation/salvage recoveries are fully developed at 48 months.

Calendar/ Accident Year	Earned Exposures	On-Level Earned Premium ($000)	Gross Reported Loss Evaluated at December 31, 2014 ($000)	Ratio of Subrogation/Salvage Received to Paid Loss Evaluated at December 31, 2014
2013	20,700	15,900	17,500	0.471
2014	23,200	17,500	18,800	0.314

Gross Reported Loss Development Factors

Accident Year	12-24	24-36	36-48	48-Ult
2010	1.055	1.047	1.018	1.000
2011	1.052	1.043	1.020	
2012	1.057	1.048		
2013	1.061			

Ratio of Subrogation/Salvage Received to Paid Loss Development Factors

Accident Year	12-24	24-36	36-48	48-Ult
2010	1.012	1.006	1.006	1.000
2011	1.010	1.006	1.004	
2012	1.536	1.004		
2013	1.500			

Calculate the indicated rate change and justify any assumptions made.

11. For the complement of credibility, trend from the July 1, 2014 prior target date to the proposed effective date of October 1, 2015, or 1.25 years:

$$\frac{1.070}{1.025}\left(\frac{1.05}{1.03}\right)^{1.25} = 1.069. \Leftrightarrow 6.9\% \text{ increase.}$$

For loss development, in order to be responsive to any changes I will take an average of the last two link ratios: 1.059, 1.046, 1.019, 1.
(Other choices would be reasonable, such as an average of all available link ratios.)
Estimated ultimate gross losses for AY2013: (17,500)(1.046)(1.019) = 18,653.
Estimated ultimate gross losses for AY2014: (18,800)(1.059)(1.046)(1.019) = 21,220.
For subrogation and salvage ratios, something seems to have changed very significantly with the development factors for 12 to 24 months.
I will take an average of the last two development factors: 1.518, 1.005, 1.005, 1.
Estimated ultimate S&S Ratio for AY2013: (0.471)(1.005)(1.005) = 0.476.
Estimated ultimate S&S Ratio for AY2014: (0.314)(1.518)(1.005)(1.005) = 0.481.
I note that the AY2014 estimate of 0.481 is similar to the AY2013 estimate of 0.476.
Estimated ultimate net losses for AY2013: (18,653) (1 - 0.476) = 9774.
Estimated ultimate net losses for AY2014: (21,220) (1 - 0.481) = 11,013.
The average date of writing under the new rates is April 1, 2016.
Since policies are semi-annual, the average date of accident is July 1, 2016.
Thus the ultimate trended losses are: $(1.05^3)(9774) + (1.05^2)(11,013) = 23,456$.
The trended premiums are: $(1.03^3)(15,900) + (1.03^2)(17,500) = 35,940$.
Thus the estimated loss, LAE, and fixed expense ratio is:
(1.03)(23,456) / 35,940 + 7% = 74.2%.
The permissible loss, LAE, and Fixed expense ratios is: 1 - 20% - 6% = 74%.
Thus prior to credibility, the indicated rate change is: 74.2% / 74% - 1 = 0.3%.
Total exposures for the two years: 20,7000 + 23,200 = 43,900.

$$Z = \sqrt{\frac{43,900}{60,000}} = 85.5\%.$$

Thus the indicated rate change is: (85.5%)(0.3%) + (14.5%)(6.9%) = **1.3%**.
Comment: For Salvage and Subrogation Ratios see Chapter 14 of
Estimating Unpaid Claims Using Basic Techniques, by Jacqueline Friedland.
In my calculation, I have assumed that LAE is 3% of losses net of salvage and subrogation.
One could instead assume that LAE is 3% of gross losses.

12. (1 point)
Describe two primary purposes of risk classification.

12. Three primary purposes:
1. protect the insurance system's financial soundness.
An insurer who does not properly classify risks or whose classification system is not sufficiently refined, may be subject to adverse selection. Due to adverse selection, better risks will go to an insurer that prices appropriately, while poor risks who are underpriced by this insurer will want to get insurance from this insurer. This insurer attracts worse insureds whose premium is inadequate. Adverse selection can lead to decreased market share and to significant losses for the insurer, which can seriously impact the financial soundness of an insurer, and even lead to insolvency.

2. be fair.
"Differences in prices among classes should reflect differences in expected costs with no intended redistribution or subsidy among the classes.
Ideally, prices and expected costs should also match within each class. That is, each individual risk placed in a class should have an expected cost which is substantially the same as that for any other member of that class. Any individual risk with a substantially higher or lower than average expected cost should be placed in a different class."

3. permit economic incentives to operate and thus encourage widespread availability of coverage.
"Insurers need a risk classification system that will permit them to offer insurance to as many of their potential customers as possible, while at the same time assuring themselves that their prices will be adequate to cover the customers' financial uncertainty that they assume.
Therefore, prices for these better risks must be different from the prices charged the higher cost risks within that market."
Comment: Only describe two purposes from the AAA "Risk Classification Statement of Principles".

13. (2.5 points) An automobile insurance company uses the following information for ratemaking:

Earned Exposures

	Driver Age	
Vehicle Symbol	Under 25	25 and Older
A	250	100
B	75	200
C	75	500

Driver Age	Incurred Loss and ALAE
Under 25	$235,800
25 and Older	$482,560

Current Rating Factors	
Symbol A	0.80
Symbol B	1.00
Symbol C	1.30
Under 25	1.00
25 and Older	0.75

a. (0.5 point) Briefly discuss the use of driver age as a rating variable in the insurer's risk classification system with respect to two relevant considerations.

b. (2 points) Calculate the indicated driver age relativities and the corresponding base rate that would achieve a revenue neutral change. Justify the selection of a methodology.

13. (a) 1. Neither of the two age groups is homogeneous.
One could break teenagers out of the first group and senior citizens out of the second group.
2. Driver age is objective and measurable.
3. Given the small amount of data, each of the groups is not large enough to be credible.
4. Driver age is inexpensive to administer.
5. Driver age does not raise privacy concerns.
6. Driver age is verifiable from official documents. It can not be easily manipulated by the insured.
7. There is not as strong causal relationship between driver age and loss costs as between years of driving experience and loss costs.
Alternately, driver age meets the causality consideration, since young and very old drivers are thought to be less safe, due to inexperience or failing faculties.
8. It lacks controllability. People can't control their age, which means the rating variable doesn't provide incentive for the insured to take action which will lower its losses and thus improve its rate.
(b) One can not use the loss ratio method, since we are not given premiums.
Since the mix of symbols differs between the two age groups, I will use the adjusted pure premium method.
For the first age group, the average symbol relativity is:
{(250)(0.8) + (75)(1) + (75)(1.3)} / 400 = 0.93125.
For the second age group, the average symbol relativity is:
{(100)(0.8) + (200)(1) + (500)(1.3)} / 800 = 1.1625.
For the first age group, the adjusted pure premium is:
$235,800 / {(0.93125)(400)} = $633.02 = $235,800 / {(250)(0.8) + (75)(1) + (75)(1.3)} = $235,800 / 372.5.
For the second age group, the adjusted pure premium is:
$482,560 / {(1.1625)(800) = $518.88 = $482,560 / {(100)(0.8) + (200)(1) + (500)(1.3)} = $482,560 / 930.
Keeping the first group as the base class, the relativity for the second age group is:
$518.88 / $633.02 = **0.82**.
Taking into account the average symbol relativities by age group, the off-balance factor to multiply by is: $\frac{(400)(0.93125)(1) + (800)(1.1625)(0.75)}{(400)(0.93125)(1) + (800)(1.1625)(0.82)} = 0.943$.

Thus the new base rate for the first age group should be **0.943 times the current base rate** for the first age group (which is not given).

Alternately, the off-balance factor to multiply by is: $\frac{(400)(1) + (800)(0.75)}{(400)(1) + (800)(0.82)} = 0.947$.

Thus the new base rate for the first age group should be **0.947 times the current base rate** for the first age group (which is not given).
Comment: In part (a) give only two considerations.
In part (b), I prefer the first method of calculating an off-balance factor, since it is in essence using the full grid of exposures by cell rather than just the totals by age group.

Assume for example a current base rate of $1000.
Then the current rates are:

	Driver Age	
Vehicle Symbol	Under 25	25 and Older
A	800	600
B	1000	750
C	1300	975

Current total premiums are:
(250)(800) + (75)(1000) + (75)(1300) + (100)(600) + (200)(750) + (500)(975) = $1,070,000.
With a new driver age relativity of 0.82 and base rate of (0.943)(1000) = $943, the new rates are:

	Driver Age	
Vehicle Symbol	Under 25	25 and Older
A	754.40	618.61
B	943.00	773.26
C	1225.90	1005.24

Proposed total premiums are: (250)(754.40) + (75)(943.00) + (75)(1225.90) + (100)(618.61) + (200)(773.26) + (500)(1005.24) = $1,070,401, the same as currently subject to rounding.

14. (2.5 points) Given the following reported claims data:

Size of Loss	Reported Claim Counts	Reported Ground-Up Losses
$X \le \$1{,}000$	120	\$75,000
$\$1{,}000 < X \le \$4{,}000$	117	\$209,000
$\$4{,}000 < X$	3	\$22,000

Individual claims above \$4,000 are: \$5,500, \$6,500, and \$10,000.

a. (0.75 point)

Calculate the increased limits factor for an increased limit of \$6,000 and a basic limit of \$4,000.

b. (0.75 point) Calculate the combined loss elimination ratio for a policy with a \$1,000 deductible and a policy limit of \$3,000.

c. (1 point) Briefly discuss two potential issues when using historical data from policies with deductibles or limits to price deductible factors. Propose a solution for each issue.

14. (a) The losses paid with a $4000 limit are: 75K + 209K + (3)(4K) = 296K.
The losses paid with a $6000 limit are: 75K + 209K + 5.5K + 6K + 6K = 301.5K.
Increased limits factor is: 301.5 / 296 = **1.019**.
(b) I am assuming that the policy covers the layer from 1000 to 4000. (If one instead assumes that the policy covers the layer from 1000 to 3000, then one would have to interpolate.)
The total losses are: 75K + 209K + 22K = 306K.
With a $1,000 deductible and a maximum payment of $3,000, the losses paid are:
209,000 - (117)(1000) + (3)(3000) = 101,000.
The combined loss elimination ratio is: 1 - 101,000 / 306,000 = **67.0%**.
(c) i. With deductibles, some of the data will be truncated from below.
If the base deductible is no deductible, then to price the higher deductibles one can use the data from those policies with no deductible. If the base deductible were for example, $500, then to price an optional $1000 deductible one can use the data from policies with deductibles of $500 or less.
Alternately, one can estimate loss elimination ratios by fitting a distribution to the available data (taking into account the truncating from below and if applicable censoring from above.)
Alternately, one can treat deductible (and policy limit) purchased as a rating variable and fit GLMs to estimate the relativities.
ii. With policy limits, some of the data will be censored from above.
One can compute the loss elimination ratios for a basic limit policy. Then one can price the increased limits via the techniques for ILFs, being careful to compare to a basic limit policy that has the base deductible.
Alternately, one can estimate loss elimination ratios by fitting a distribution to the available data (taking into account the truncating from below and censoring from above.)
Alternately, one can try to estimate the losses in higher layers from empirical data in a manner similar to what is done with increased limits pricing. To compare a layer to the basic limit losses one would use data from policies such that the data in that layer would not be censored from above by their policy limits.
Alternately, one can treat policy limit and deductible purchased as a rating variable and fit GLMs to estimate the relativities.
iii. There could be favorable or adverse selection, with the ground-up unlimited severity distribution differing between purchasers of different deductibles and/or different policy limits.
(For example, this could be due to self-selection or differences in claim reporting behavior.)
One can treat policy limit and deductible purchased as a rating variable and apply GLMs to estimate the relativities.
iv. Higher limits will suffer higher severity trends.
Trend losses before calculating deductible factors.
v. Higher limits experience greater loss development.
Develop losses before calculating deductible factors.
vi. Data thin at higher layers.
Use smoothing techniques or fit severity data to a theoretical distribution.
Alternately, one can use industry data.

Comment: In part (b), apparently the examiner's are using the definition of the policy limit as the maximum amount paid for a claim; this is the definition in Loss Models. Basic Ratemaking is not clear on what is meant by the policy limit when there is also a deductible. In practical applications, the actuary should refer to the language in the policy.
The CAS Examiner's report mentions using the loss ratio method for deductible pricing, which is not discussed in Basic Ratemaking.

15. (1.5 points) The following table shows all claims transactions for an insurance company through December 31, 2014.

Claim ID	Accident Date	Report Date	Transaction Date	Incremental Payment	Ending Case Reserves
1	January 1, 2011	January 1, 2011	January 31, 2011	100	100
1			March 2, 2011	100	50
1			April 1, 2011	25	0
2	June 30, 2011	March 15, 2012	June 30, 2012	0	10,000
2			May 25, 2013	250	10,000
2			June 30, 2014	15,000	1,000
2			Sept. 12, 2014	200	0
3	February 25, 2012	March 15, 2012	March 15, 2012	0	0
3			January 15, 2013	0	1,000
3			July 15, 2013	2,500	0

• All three claims are closed as of December 31, 2014.

a. (0.75 point) Construct the cumulative reported claims triangle on an accident year basis.

b. (0.75 point) Construct the cumulative reported claims triangle on a report year basis.

15. (a) AY2011 includes the first two claims, while the last claim is in AY2012.
As of end of 2011, reported AY11 is: 100 + 100 + 25 = 225.
As of end of 2012, reported AY11 is: 225 + 10,000 = 10,225.
As of end of 2013, reported AY11 is: 225 + 250 + 10,000 = 10,475.
As of end of 2014, reported AY11 is: 225 + 250 + 15,000 + 200 = 15,675.

	12 months	24 months	36 months	48 months
AY2011	225	10,255	10,475	15,675
AY2012	0	2500	2500	

(b) RY2011 includes the first claim, while the last two claims are in RY2012.
As of end of 2012, reported RY12 is: 10,000.
As of end of 2013, reported RY12 is: 250 + 10,000 + 2500 = 12,750.
As of end of 2014, reported RY12 is: 250 + 15,000 + 200 + 2500 = 17,950.

	12 months	24 months	36 months	48 months
RY2011	225	225	225	225
RY2012	10,000	12,750	17,950	

Comment: See Table 5 in Chapter 5 of Estimating Unpaid Claims Using Basic Techniques, by Jacqueline Friedland.
Claim 1 is in lag 0 of RY2011. Claim 2 is in lag 1 of RY2012. Claim 3 is in lag 0 of RY2012.
For each Report Year, one could have separate development triangles for each lag.