

# **Solutions to the Spring 2014 CAS Exam 5**

**(Only those questions on Basic Ratemaking)**

There were 23 questions worth 59.75 points, of which 11 were on ratemaking worth 30.75 points.

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1. (3.5 points) An insurance company writes annual policies.

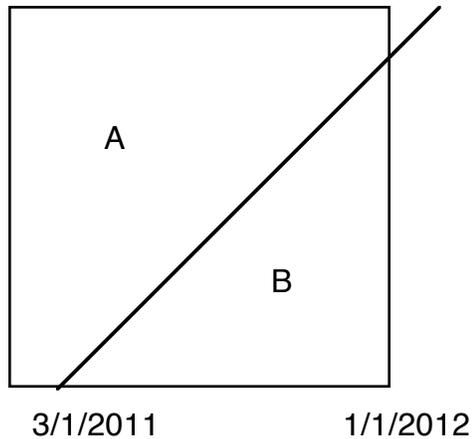
The history of rate changes is as follows:

| <u>Effective Date</u> | <u>Overall Rate Change</u> |
|-----------------------|----------------------------|
| January 1, 2010       | +4.2%                      |
| March 1, 2011         | +0.3%                      |
| January 1, 2012       | -1.7%                      |
| June 1, 2013          | +1.0%                      |

- a. (1 point) Calculate the on-level factor to current rate level for calendar year 2011 earned premium, assuming all policies are written uniformly throughout the year.
- b. (2 points) Assume that 25% of policies are written on the first day of the year and the remaining policies are written evenly throughout the year. Calculate the on-level premium factor to current rate level for policies in-force on February 1, 2012.
- c. (0.5 point) Assuming all policies are written uniformly throughout the year, and without performing additional calculations, discuss the effect on the on-level premium factor for calendar year 2011 if the policy term was 2 years instead of annual.

| 1. | <u>Effective Date</u> | <u>Overall Rate Change</u> | <u>Rate Level Index</u> |
|----|-----------------------|----------------------------|-------------------------|
|    |                       |                            | 1.0000                  |
|    | January 1, 2010       | +4.2%                      | 1.0420                  |
|    | March 1, 2011         | +0.3%                      | 1.0451                  |
|    | January 1, 2012       | -1.7%                      | 1.0274                  |
|    | June 1, 2013          | +1.0%                      | 1.0376                  |

(a) Area B =  $(10/12)^2 / 2 = 0.3472$ . Area A =  $1 - 0.3472 = 0.6528$ .



On level factor is:  $\frac{1.0376}{(0.6528)(1.042) + (0.3472)(1.0451)} = \mathbf{0.995}$ .

(b) The 25% of policies written on January 1, 2012, are all at rate level 1.0274.

For the remaining policies in-force on February 1, 2012, 1/12 are written prior to March 1, 2011 at rate level 1.0420, 1/12 are written during January 2012 at rate level 1.0274, while the remaining 10/12 are written at rate level 1.0451.

Thus the average rate level for the policies in-force on February 1, 2012 is:

$$(25\%)(1.0274) + (75\%) \left\{ (1/12)(1.0420) + (10/12)(1.0451) + (1/12)(1.0274) \right\} = 1.0394.$$

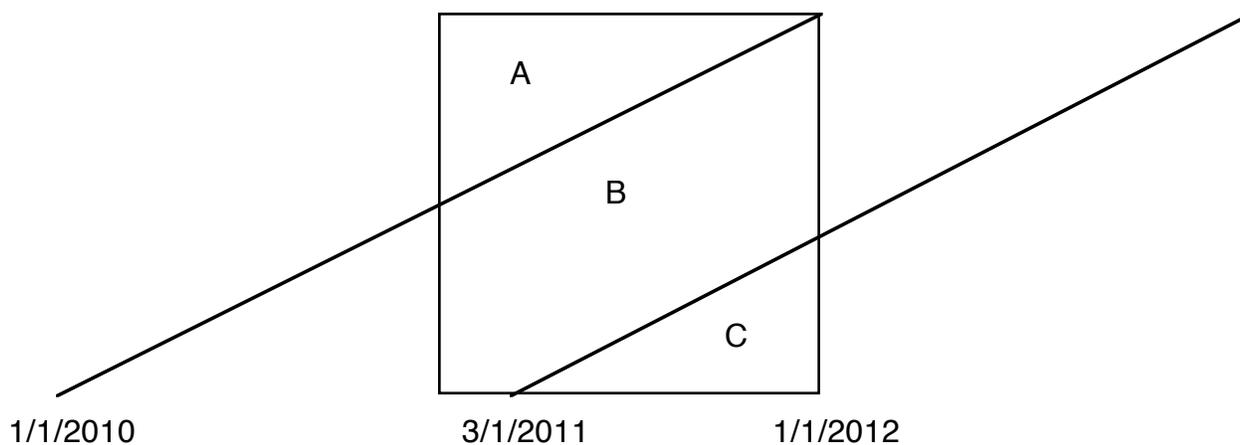
Thus the on level factor is:  $1.0376 / 1.0394 = \mathbf{0.998}$ .

(c) If we have two year policies, then more of the CY2011 earned premium was written at a lower rate level in effect prior to the 1/1/2010 rate increase, than if we have one year policies. Also less of the premium would be written at the higher rate level in effect after the 3/1/2011 rate increase.

Thus the average rate level during CY 2011 would be less with two year policies than it was with one year policies.

Therefore, the on-level factor is **greater** when we have two year policies.

Comment: Here is the calculation when there are two year policies.



Area A =  $(1/2)(1)/2 = 1/4$ . Area C =  $(5/12)(10/12)/2 = 25/144$ .  $\Rightarrow$  Area B =  $83/144$ .

On level factor is:  $\frac{1.0376}{(1/4)(1.0) + (83/144)(1.042) + (25/144)(1.0451)} = 1.005$ .

This on level factor of 1.005 is indeed greater than that with one year policies of 0.995.

2. (3.25 points)

A workers compensation insurance company uses the following data for ratemaking:

| <u>Year</u> | <u>Industry Loss Cost Premium (\$000)</u> | <u>Annual Payroll Level Change</u> | <u>Historical Average Experience Modification Factor</u> |
|-------------|---|------------------------------------|--|
| 2011        | 2,100                                     | 2.5%                               | 0.99   |
| 2012        | 2,500                                     | 2.0%                               | 0.98   |
| 2013        | 2,600                                     | 1.0%                               | 0.97   |

| <u>Year</u> | <u>Reported Indemnity Claims (\$000)</u> | <u>Annual Impact on Indemnity Claims Due to Benefit Level Changes</u> | <u>Indemnity Development Factor to Ultimate</u> | <u>Projected Ultimate Medical-Only Claims (\$000)</u> |
|-------------|--|---|---|---|
| 2011        | 850                                      | 2.0%  | 1.20  | 735   |
| 2012        | 670                                      | 1.5%  | 1.80  | 834   |
| 2013        | 460                                      | 0.5%  | 2.70  | 900   |

- Expected future wage level change = 1.5% per year.
- Expected effect on indemnity claims due to future benefit level changes = 1.0% per year.
- Projected average experience modification factor = 0.98.
- Projected LAE percentage (as a percent of losses) = 15.0%.
- Assume no other loss cost inflation other than indemnity benefit level changes.

Calculate the projected ultimate loss & LAE ratio for year 2015.

2. I will assume we are given Calendar/Accident year data. I will assume that “year 2015” is Policy Year 2015, with a trend period from AY13 to PY15 of 2.5 year.

The effect of benefit changes to go from 2011 to 2012 is: 1.015.

The effect of benefit changes to go from 2012 to 2013 is: 1.005.

The effect of benefit changes to go from 2013 to 2015 is:  $1.01^{2.5}$ .

Thus to adjust 2012 indemnity losses to the current level:  $(670) (1.005) (1.01^{2.5})$ .

Then multiply by 1.80 to develop these losses to ultimate:  $(670) (1.005) (1.01^{2.5}) (1.80) = 1243$ .

| <u>Year</u> | <u>Indem</u> | <u>LDF</u> | <u>OLF</u>                   | <u>Product</u> |
|-------------|--------------|------------|------------------------------|----------------|
| 2011        | 850          | 1.2        | $(1.015)(1.005)(1.01^{2.5})$ | 1067           |
| 2012        | 670          | 1.8        | $(1.005)(1.01^{2.5})$        | 1243           |
| 2013        | 460          | 2.7        | $1.01^{2.5}$                 | 1273           |

(In Exhibit D-8 in Basic Ratemaking there is an annual impact on benefits due to wage inflation applied to the indemnity losses, which differs from payroll trend; we are not given that in this question. One could treat the “Expected effect on indemnity claims due to future benefit level changes” of 1.0% per year as a trend for 2.5 years.)

Adding in the projected ultimate medical losses, the projected ultimate losses are:

$$1067 + 1243 + 1273 + 735 + 834 + 900 = 6052.$$

Assume that the given industry loss cost premium is on the current rate level.

Then we still want to trend and adjust for the changes in experience mods from the historical periods to the projected period. The effect of payroll trend to go from 2011 to 2012 is: 1.020.

The effect of payroll trend to go from 2012 to 2013 is: 1.010.

The effect of payroll trend to go from 2013 to 2015 is:  $1.015^{2.5}$ .

| <u>Year</u> | <u>Prem</u> | <u>Trend</u>                | <u>Experience Mod.</u> | <u>Adjust.</u> | <u>Product</u> |
|-------------|-------------|-----------------------------|------------------------|----------------|----------------|
| 2011        | 2100        | $(1.02)(1.01)(1.015^{2.5})$ | 98/99                  |                | 2223           |
| 2012        | 2500        | $(1.01)(1.015^{2.5})$       | 98/98                  |                | 2621           |
| 2013        | 2600        | $1.015^{2.5}$               | 98/97                  |                | 2726           |
| Total       |             |                             |                        |                | 7570           |

The projected ultimate loss & LAE ratio for year 2015 is:  $(1.15) (6052) / 7570 = 91.9\%$ .

Comment: See the Exhibits D-6 and D-8 in Basic Ratemaking.

A trending period of either 2.0 or 2.5 years was considered acceptable by the CAS, as long as the same trending period was used for both calculations.

“Medical-Only Claims” should have said “Medical Losses.”

“Indemnity Claims” should have said “Indemnity Losses.”

Most dollars of Workers Compensation losses are from claims that pay both medical and indemnity (lost wages). Medical-Only claims are the majority of claims but only constitute about 5% of losses.

Note that the loss cost premium does not include a loading for expenses (other than LAE), taxes, and profit. Thus prior to such considerations, a projected ultimate loss & LAE ratio of 100% would indicate no change in rates; see pages D-16 and D-17 in Basic Ratemaking.

3. (2.25 points) For a single personal auto policy with an annual policy term:

- $A$  = Calendar year 2013 written exposures as of December 31, 2013.
- $B$  = Calendar year 2012 earned exposures  
+ calendar year 2013 earned exposures as of February 1, 2013.
- $C$  = Calendar year 2013 unearned exposures as of February 1, 2013.
- $D$  = In-force exposures as of February 1, 2013.
- $A < 0 < B < C < D$ .
- Exposure is earned uniformly throughout the policy term.
- This policy cancels mid-term.

a. (1 point) Provide the range of valid effective dates for this policy.

b. (0.5 point) Provide the range of valid dates of the mid-term cancellation for this policy.

c. (0.75 point) Demonstrate that it would never be possible to have  $A < 0 < B < C < D$  if  $B$ ,  $C$ , and  $D$  were as of July 1, 2013 instead of February 1, 2013.

3. (a) Since  $D > 0$ , the policy was in force on February 1, 2013; thus the policy could not have been written after February 1, 2013, and also must have been canceled after February 1, 2013.

$A < 0$ , therefore, the policy was written in 2012 and cancelled in 2013.

Since  $D > 0$  and it was an annual policy, it must have been written after February 1, 2012.

If for example, it were written on March 1, 2012, then as February 1, 2013,  $B$  would be 11 months and  $C$  would be 1 month. However, we want  $B < C$ , so this is no good.

If instead it were written on August 2, 2012, then as February 1, 2013,  $B$  would be just less than 6 months and  $C$  would be just more than 6 months;  $B < C$ .

Put another way, in order for  $C > B$  since these are annual policies, we must have more than 6 months left of the policy term on February 1, 2013.

So the range of effective dates is August 2, 2012 to December 31, 2012.

(b)  $A < 0$ , therefore, the policy was written in 2012 and cancelled in 2013.

If it had been cancelled 2/1/2013, then  $D = 0$ , but we want  $D > 0$ .

If it had been cancelled 12/31/2013, then since it was written in 2012, the policy could not have been cancelled midterm.

Thus it must have been cancelled after February 1, 2013 and before December 31, 2013.

(c)  $A < 0$ , therefore, the policy was written in 2012 and cancelled in 2013.

$D > 0$  implies the policy is in-force on 7/1/2013.

(Assume for simplicity one car is insured.)

Thus since the annual policy was written in 2012, as of July 1, 2013,  $B \geq 1/2$  and  $C \leq 1/2$ .

Thus it cannot be that  $B < C$ .

Comment: A very unusual question; challenging under exam conditions.

4. (2 points) A product manager is proposing to revise rates in the scenarios described below. As an actuary, briefly assess the approach taken in each scenario and, if necessary, recommend an adjustment.
- a. (0.5 point) The loss provision in the indicated rate for next year is calculated as historical reported loss divided by exposure.
  - b. (0.5 point) The indicated rate for next year is calculated using projected losses and loss adjustment expenses based on historical experience. In the next month, the company will be revising its underwriting guidelines, increasing the minimum deductible from \$500 to \$1,000.
  - c. (0.5 point) The indicated rate for a classification is calculated based on one year of historical data that includes 25 earned car years.
  - d. (0.5 point) The indicated rate change is calculated using the ratio of developed, trended historical losses capped at \$100,000 to on-level total earned premium. Loss trend factors and loss development factors are determined using data limited to \$100,000. The company has a significant number of claims in excess of \$100,000.

4. (a) No good. We need to adjust the losses for trend, loss development, and if relevant law changes. (If relevant, we also need to remove any catastrophes in the loss data and load for an average catastrophe provision.) If relevant, we need to adjust the exposures for trend.

(b) If loss development is included this may be okay. If the relativities for the different deductibles are okay, then the rates charged should be okay. This could be thrown off if the frequency of loss were dependent on the deductible selected, since now all those who bought \$500 deductibles will buy \$1000 (or higher) deductibles.

Alternately, the provision for loss will be overstated, since the increase in deductible will lead to fewer covered losses as well as less paid out on losses that are covered. Restate the historical losses on the basis of the \$1000 deductible. (If we are using historical premiums, then they must also be restated.)

(c) No Good. There is not enough data to be fully credible. Using more than one year of data would help. The classification relativity should be based on a credibility weighting of the observed relativity with something else, such as the current relativity.

Alternately, this class could be grouped with other similar classes for ratemaking purposes, in order to create a more statistically significant group.

Alternately, one can use as the complement of credibility the class relativity from either a rating bureau such as ISO or a large competitor of this insurer.

(d) If \$100,000 is the basic limit, then this may be okay for making basic limits rates provided the premiums are also for basic limits. There would have to be a separate analysis to determine increased limits factors.

Alternately, the losses above \$100,000 still need to be added back in as a large loss load, otherwise if the analysis excludes these large losses altogether, then losses will be underestimated and rates will be inadequate.

Alternately, since the company has a significant number of claims in excess of \$100,000, we should use uncapped historical losses data to calculate the indication rate. The capped data does not reflect the true experience, and the rate might be too low by using the capped data.

Alternately, since the company has a significant number of claims in excess of \$100,000, I would use a higher cap than \$100,000. The goal of ratemaking is to include as many losses as possible into the projection process as long it doesn't introduce too much volatility. I would increase the cap and for the expected portion of losses above this cap I would include a large loss load.

Comment: When given limited information with a lack of detail, there are many possible reasonable answers.

5. (6.5 points) A countrywide insurer's rate filing for a state contains the following:

- All policies are annual.
- The filed rates are planned to be in effect for policy year 2015.
- There was a rate change of +7.5%, effective 7/1/2013.  
The prior rate change before that was in 2009.
- Loss trend is 3% annually.
- ULAE as a ratio of loss and ALAE = 10%.
- Profit and contingencies provision = 5%.
- Variable expense ratio = 20%.
- The company purchased new software in 2010 to assist with the processing of claims.
- Use an average of 2012 and 2013 for the rate level indication.

| <u>Calendar Year</u> | <u>Earned Premium (\$000)</u> | <u>Accident Year</u> | <u>Reported Loss and ALAE (\$000)</u> |
|----------------------|-------------------------------|----------------------|---------------------------------------|
| 2012                 | 1,250                         | 2012                 | 750                                   |
| 2013                 | 1,400                         | 2013                 | 500                                   |

| <u>Calendar Year</u> | <u>Current Level Average Policy Premium</u> | <u>Fixed Expense Ratio</u> |
|----------------------|---|----------------------------|
| 2009                 | \$500                                       | 10%                        |
| 2010                 | \$520                                       | 23%                        |
| 2011                 | \$540                                       | 15%                        |
| 2012                 | \$560                                       | 12%                        |
| 2013                 | \$583                                       | 10%                        |

| <u>Accident Year</u>       | <u>Reported Loss and ALAE Development Factors</u> |                     |                     |                     |                   |
|----------------------------|---|---------------------|---------------------|---------------------|-------------------|
|                            | <u>12-24 Months</u>                               | <u>24-36 Months</u> | <u>36-48 Months</u> | <u>48-60 Months</u> | <u>60+ Months</u> |
| 2006                       | 1.45  | 1.35                | 1.10                | 1.02                | 1.00              |
| 2007                       | 1.50  | 1.30                | 1.15                | 1.08                | 1.00              |
| 2008                       | 1.40  | 1.35                | 1.10                | 1.03                | 1.00              |
| 2009                       | 1.50  | 1.30                | 1.08                | 1.02                |                   |
| 2010                       | 1.85  | 1.15                | 1.10                |                     |                   |
| 2011                       | 1.75  | 1.15                |                     |                     |                   |
| 2012                       | 1.80  |                     |                     |                     |                   |
| All Year Average           | 1.61  | 1.25                | 1.11                | 1.04                | 1.00              |
| 5 Year Average             | 1.66  | 1.23                | 1.11                | N/A                 | N/A               |
| 3 Year Average             | 1.80  | 1.20                | 1.09                | 1.04                | 1.00              |
| Average Excluding High/Low | 1.60  | 1.25                | 1.10                | 1.03                | 1.00              |

a. (5.5 points) Calculate the indicated rate change.

Justify the selections of premium trend, all development factors, and the fixed expense ratio.

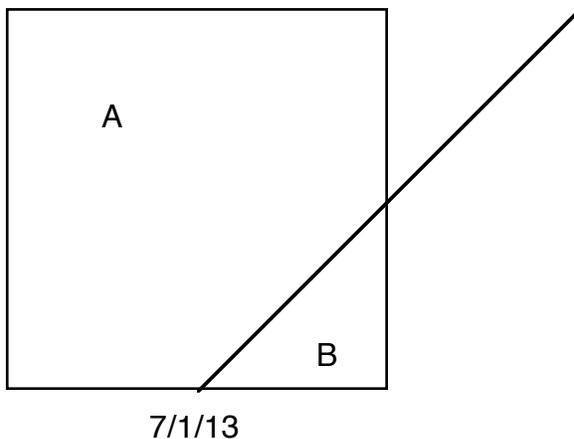
b. (1 point) The chief actuary is concerned about the credibility of company data in this state and would like to begin using credibility weighting with the company's countrywide loss costs. Assess this approach, considering two desirable qualities of a credibility complement.

5. (a) The 2010 fixed expense ratio is much higher than the other years, probably due to the company purchasing new software in 2010 to assist with the processing of claims. On the one hand, this is a one-time unusual expense that can not be expected to recur. On the other hand, the company will in the future undoubtedly face different unusual one-time expenses. Therefore, I do not want to exclude this unusual expense, but neither do I want to give it too much weight. Thus I will take a weighted average of the given five years, giving 2010 half weight. Selected fixed expense ratio is:  $(10\% + 23\%/2 + 15\% + 12\% + 10\%) / 4.5 = 13\%$ .

The new software was starting to be used to help process claims in 2010. Therefore, the diagonal starting with the loss development of 2011 from first to second report are probably more reflective of the pattern going forward, than are previous diagonals. Therefore, I will use the average of the latest two link ratios.  
 $(1.75 + 1.80)/2 = 1.775$ .  $(1.15 + 1.15)/2 = 1.15$ .  $(1.10 + 1.08)/2 = 1.09$ .  
 $(1.03 + 1.02)/2 = 1.025$ .

The calendar year average premiums are increasing at about \$20 per year; the last increase is \$23. There is 2.5 years from CY2013 to PY2015, so I will project a PY2015 average premium of:  $\$583 + (2.5)(\$20) = \$633$ .

OLF for 2012 is 1.075.



Area B is:  $(1/2)(1/2)(1/2) = 1/8$ . Area A is  $7/8$ .

OLF for CY2013 is:  $1.075 / \{(7/8)(1) + (1/8)(1.075)\} = 1.0650$ .

Projected On-level 2012 Premium:  $(1250)(1.075)(633/560) = 1518.9$ .

Projected On-level 2013 Premium:  $(1400)(1.065)(633/583) = 1618.9$ .

Trended & Developed AY2012 Losses & ALAE:  $(750)(1.15)(1.09)(1.025)(1.03^{3.5}) = 1068.7$ .

Trended & Dev. AY2013 Losses & ALAE:  $(500)(1.775)(1.15)(1.09)(1.025)(1.03^{2.5}) = 1227.7$ .

⇒ Loss & LAE Ratio:  $(1.1)(1068.7 + 1227.7) / (1518.9 + 1618.9) = 80.5\%$ .

⇒ Rate Indication is:  $(80.5\% + 13\%) / (1 - 20\% - 5\%) - 1 = 24.7\%$ .

Alternately, the 2010 fixed expenses are likely high due to the purchase of the new software; take a simple average of the other years: 11.75%.

Use an annual premium trend of 4%.

Projected On-level 2012 Premium:  $(1250)(1.075)(1.04^{3.5}) = 1541$ .

Projected On-level 2013 Premium:  $(1400)(1.065)(1.04^{2.5}) = 1645$ .

For loss development, select the following age to age factors:

12:24: 1.8 – used 3 year average because of impact of the new software

24:36: 1.15 – 2009 and prior is different due to new software

36:48: 1.10 – used average excluding High/Low since the experience seems to be similar even with the software change

48:60: 1.030 – used average excluding High/Low

60:Ultimate: 1.00 – no tail

Trended & Developed AY2012 Losses & ALAE:  $(750)(1.15)(1.10)(1.03)(1.03^{3.5}) = 1084$ .

Trended & Dev. AY2013 Losses & ALAE:  $(500)(1.8)(1.15)(1.10)(1.03)(1.03^{2.5}) = 1263$ .

Loss & LAE Ratio 2012:  $(1.1)(1084) / 1541 = 77.38\%$ .

Loss & LAE Ratio 2013:  $(1.1)(1263) / 1645 = 84.46\%$ .

⇒ Loss & LAE Ratio:  $(77.38\% + 84.46\%)/2 = 80.93\%$ .

⇒ Rate Indication is:  $(80.93\% + 11.75\%) / (1 - 20\% - 5\%) - 1 = 23.6\%$ .

(b) 1. The countrywide loss costs should have a smaller process variance than this state's loss costs. This is desirable.

2. The countrywide loss costs are a biased estimator of this state's loss costs. This is undesirable. (For most lines of insurance, the loss costs vary significantly between different states.

In addition, the mix of business written can differ significantly between the different states.)

3. Assuming this state is only a small portion of countrywide, the process estimation errors in the countrywide loss costs are largely independent of those for this state's loss costs. This is desirable. (It would be better to use as the complement countrywide excluding this state.

If the same loss development factors and/or trend factors are used for countrywide and this state, that would lessen the independence.)

4. The countrywide loss costs should be readily available. This is desirable. (Presumably, the company does rate indications for the other states.)

5. The countrywide loss costs should be relatively easy to compute. This is desirable. (It involves aggregating estimates from each state.)

6. While the countrywide loss costs bear some logical relationship to those of this state, they are for the same line of business and insurer, the relationship is not as close as one would desire.

Comment: Other reasonable choices can be made for the various rate components in part (a).

In part (b) discuss only two desirable qualities; there are other reasonable answers.

Fitting a linear regression to the average premiums, with 2009 corresponding to  $t = 1$ :

$P = 478.8 + 20.6t$ . Thus the fitted value for PY2015 is:  $478.8 + (20.6)(7.5) = 633.3$ .

**6.** (2.5 points) An auto insurance company is designing a risk classification system. The actuary has determined that the number of hours a driver sleeps each night is a predictive rating variable. Recommend whether the company should include this variable in their risk classification system. Justify this recommendation with respect to four relevant considerations.

6. Reasons not to use this variable:

1. it is not objective. If someone has a restless night of sleep, has trouble going to sleep, or is woken up in the middle of night, exactly how does that affect the number of hours of sleep?

The variables used for classification should be susceptible to convenient and reliable measurement; the average number of hours a driver sleeps each night is not.

2. It would be expensive to administer. There would be a lot of time and effort required to collect this information each year from each driver. Most drivers do not currently systematically record this information.

3. It is not verifiable. As a practical matter, there is no way to verify whatever information an insured gives the insurer. The information given would likely to be incorrect once insureds realized that it will affect their insurance rates. (The insurer could station cameras to record what goes on in the bedrooms of its insured drivers, or hook up its drivers each night to a sleep monitoring machine, but either of these is impractical and would also create huge privacy concerns.)

The system should minimize the ability to manipulate or misrepresent a risk's characteristics so as to affect the class to which it is assigned; this is not the case here.

4. It raises privacy concerns. How much someone sleeps each night on average is a personal matter which is none of the business of an insurance company. This is unnecessarily intrusive.

5. Affordability. Parents with very young children will on average get less sleep per night. Many such parents are already facing financial pressures, and this may make automobile insurance unaffordable. Also many poorer people have to work two jobs to make ends meet and may also work irregular hours with a shifting schedule. Thus this variable may disproportionately raise rates for poorer insureds, raising affordability concerns.

Reasons to use this variable:

1. It is statistically significant. I am assuming that the actuary determined that the expected cost estimates vary for the different levels of the rating variable, the estimated differences are within an acceptable level of statistical confidence, and the estimated differences are relatively stable from one year to the next.

2. Homogeneity. I assume that the insureds can be divided into relatively homogeneous subgroups based on the average number of hours slept.

3. Credibility. I assume that the insureds can be divided into large enough subgroups so that each one has sufficient data to be credible.

4. Causality. It makes sense that people who get too little sleep would have worse reflexes and decreased attention to what is happening around them. Therefore, in my opinion, less sleep is casually related to worse driving experience on average, all else being equal.

5. Controllability. Most insureds can control how much sleep they get per night.

Comment: One can either be in favor or opposed to the use of the variable, provided one gives reasonable justifications based on Chapter 9 of Basic Ratingmaking and/or the American Academy of Actuaries "Risk Classification Statement of Principles."

Be sure to make a final recommendation as to whether or not to use this variable.

There are many possible full credit answers other than those I gave.

One would have to check whether this variable is impermissible due to state law and/or regulation.

7. (2.25 points) Given the following information:

| <u>Limit</u> | <u>Premium</u> | <u>Increased Limits Factor</u> |
|--------------|----------------|--------------------------------|
| \$100,000    | \$1,000,000    | 1.00                           |
| \$250,000    | \$500,000      | 2.00                           |
| \$500,000    | \$400,000      | 2.75                           |
| \$750,000    | \$300,000      | 3.25                           |
| \$1,000,000  | \$200,000      | 3.50                           |
| TOTAL        | \$1,900,000    |                                |

a. (0.5 point) Given that the losses capped at \$250,000 are \$1,500,000, calculate a complement of credibility for the losses in the layer between \$500,000 and \$750,000.

b. (1.25 points) Assume that the expected total limits loss ratio is 65%. Using the limits analysis approach, calculate the complement of credibility for the layer between \$500,000 and \$750,000.

c. (0.5 point)

Provide two criticisms of using the limits analysis approach to develop a complement of credibility.

7. (a)  $(\$1,500,000) (3.25 - 2.75) / 2.00 = \mathbf{\$375,000}$ .

| (b) <u>Limit</u> | <u>Premium</u> | <u>Increased Limits Factor</u> | <u>% of losses in layer</u> |
|------------------|----------------|--------------------------------|-----------------------------|
| \$100,000        | \$1,000,000    | 1.00                           | 0                           |
| \$250,000        | \$500,000      | 2.00                           | 0                           |
| \$500,000        | \$400,000      | 2.75                           | 0                           |
| \$750,000        | \$300,000      | 3.25                           | $(3.25 - 2.75) / 3.25$      |
| \$1,000,000      | \$200,000      | 3.50                           | $(3.25 - 2.75) / 3.50$      |

Complement of credibility is:

$(65\%)(\$100,000) \{ (3)(3.25 - 2.75) / 3.25 + (2)(3.25 - 2.75) / 3.50 \} = \mathbf{\$48,571}$ .

(c) 1. It assumes these increased limits factors are appropriate for this book of business.

(The ILFs may be derived from industrywide data rather than the experience of this individual insurer. Since the size of loss distribution for this insurer may differ from industrywide, we have potential parameter type error.)

2. It assumes the same expected loss ratio applies regardless of limit.

3. It is not based on any loss data from this book of business.

Comment: Part (a) uses the “lower limits analysis” at page 233 of Basic Ratemaking.

The losses capped at \$250,000 are 2 times the expected basic limit losses. The losses in the layer between \$500,000 and \$750,000 are  $3.25 - 2.75 = 0.5$  times the basic limit losses.

In part (a), we are not making use of the premiums by limit; rather we are assuming that we start with unlimited losses which are then capped by \$250,000 or confined to a layer from 500K to 750K.

The “limits analysis” in part (b) is called “exposure rating” when pricing reinsurance.

8. (2.25 points)

A private passenger auto insurance company uses only two rating variables: territory and gender. The distribution of earned exposures is:

| <u>Territory</u> | <u>Earned Exposures</u> |               |
|------------------|-------------------------|---------------|
|                  | <u>Male</u>             | <u>Female</u> |
| 1                | 180                     | 120           |
| 2                | 500                     | 500           |
| 3                | 350                     | 150           |

| <u>Territory</u> | <u>Loss and LAE</u> |
|------------------|---------------------|
| 1                | \$11,127            |
| 2                | \$51,335            |
| 3                | \$32,983            |

| <u>Territory</u> | <u>Current Relativity</u> |
|------------------|---------------------------|
| 1                | 0.750                     |
| 2                | 1.000                     |
| 3                | 1.125                     |

| <u>Gender</u> | <u>Current Relativity</u> |
|---------------|---------------------------|
| Male          | 1.000                     |
| Female        | 0.800                     |

- Assume no adjustments are made to the relativities for expense considerations.
- Assume territory 2 remains the base territory.

For a revenue neutral overall change, calculate an indicated relativity change for policyholders in territory 3, accounting for any distortion that gender rating may cause.

8. Use the adjusted pure premium approach. The adjusted exposures are:

$$(180)(1) + (120)(0.8) = 276,$$

$$(500)(1) + (500)(0.8) = 900,$$

$$\text{and } (350)(1) + (150)(0.8) = 470.$$

| <u>Terr.</u> | <u>Loss &amp; ALAE</u> | <u>Adjusted Expos.</u> | <u>Pure Premium</u>    | <u>Indic. Rel. to Terr. 2</u> |
|--------------|------------------------|------------------------|------------------------|-------------------------------|
| 1            | 11,127                 | 276                    | $11,127 / 276 = 40.32$ | $40.32/57.04 = 0.707$         |
| 2            | 51,335                 | 900                    | 57.04                  | 1                             |
| 3            | 32,983                 | 470                    | 70.18                  | $70.18/57.04 = 1.230$         |

The indicated change in the relativity for territory 3 is:  $1.230 / 1.125 - 1 = 9.33\%$ .

The average current premium is proportional to:

$$(276)(0.75) + (900)(1) + (470)(1.125) = 1635.75.$$

The average premium with the proposed territory relativities is proportional to:

$$(276)(0.707) + (900)(1) + (470)(1.230) = 1673.23.$$

Thus, to have a revenue neutral change, we would have to multiply the base rate for territory 2 by:  $1635.75 / 1673.23 = 0.9776$ .

Thus the revenue neutral rate change for territory 3 is:  $(1.0933)(0.9776) - 1 = 6.88\%$ .

Comment: The revenue neutral rate change for territory 1 is:  $(0.707/0.750)(0.9776) - 1 = -7.84\%$

The revenue neutral rate change for territory 2 is:  $0.9776 - 1 = -2.24\%$ .

Assume for simplicity that the current base rate is \$100, then the current rates are:

| <u>Territory</u> | <u>Male</u> | <u>Female</u>            |
|------------------|-------------|--------------------------|
| 1                | 75          | $60 = (100)(0.8)(0.750)$ |
| 2                | 100         | 80                       |
| 3                | 112.5       | 90                       |

Then the current premiums are:

| <u>Territory</u> | <u>Male</u> | <u>Female</u>         | <u>Total</u> |
|------------------|-------------|-----------------------|--------------|
| 1                | 13,500      | $7,200 = (\$60)(120)$ | 20,700       |
| 2                | 50,000      | 40,000                | 90,000       |
| 3                | 39,375      | 13,500                | 52,875       |
| Total            | 102,875     | 60,700                | 163,575      |

The proposed base rate is  $(0.9776)(\$100) = \$97.76$ , and the proposed rates are:

| <u>Territory</u> | <u>Male</u> | <u>Female</u>                 |
|------------------|-------------|-------------------------------|
| 1                | 69.12       | $55.29 = (97.76)(0.8)(0.707)$ |
| 2                | 97.76       | 78.21                         |
| 3                | 120.24      | 96.20                         |

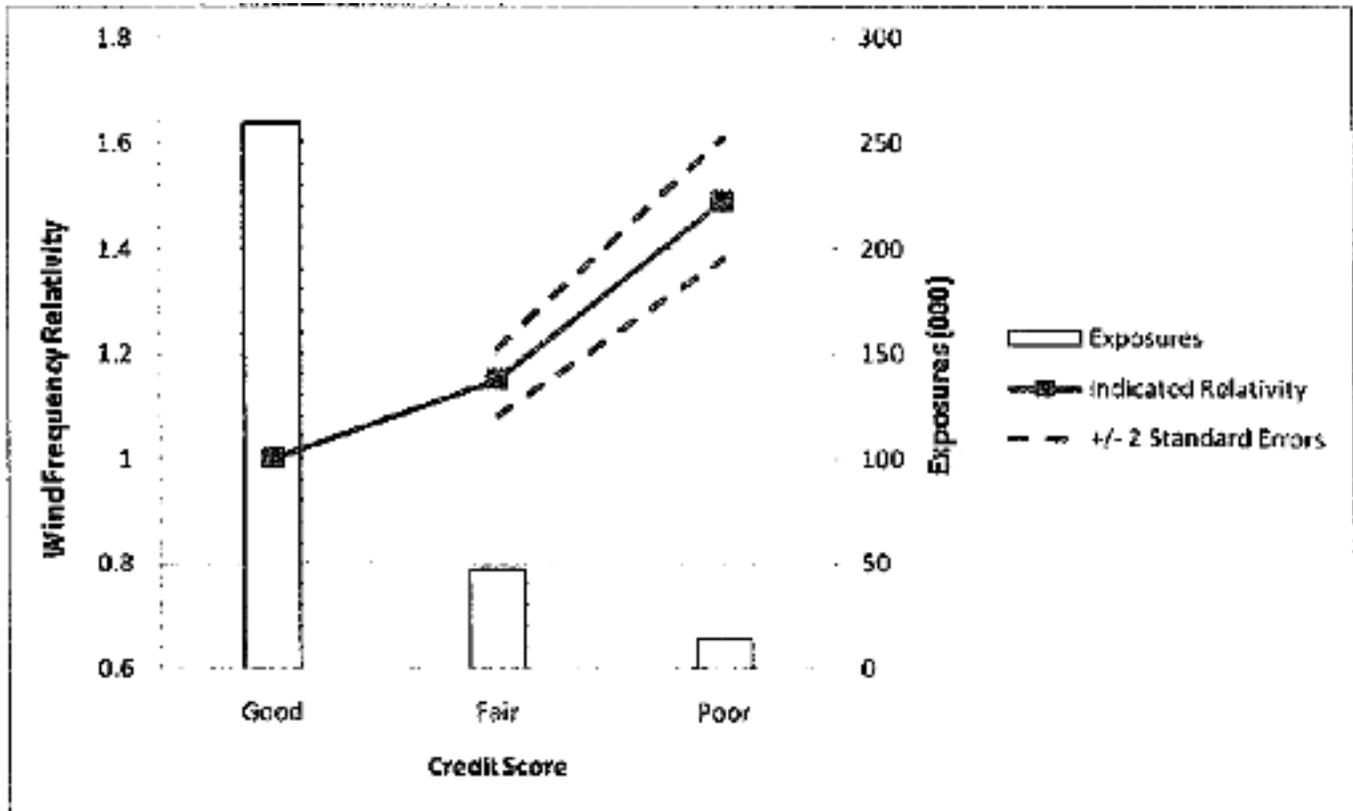
The proposed premiums are:

| <u>Territory</u> | <u>Male</u> | <u>Female</u>            | <u>Total</u> | <u>Change</u>                 |
|------------------|-------------|--------------------------|--------------|-------------------------------|
| 1                | 12,442      | $6,635 = (\$55.29)(120)$ | 19,077       | $-7.84\% = 19,077/20,700 - 1$ |
| 2                | 48,880      | 39,105                   | 87,985       | -2.24%                        |
| 3                | 42,084      | 14,430                   | 56,514       | +6.88%                        |
| Total            | 103,406     | 60,170                   | 163,576      | 0.00%                         |

9. (2 points)

An insurer is considering using credit score to further segment its homeowners book of business. The insurer has developed a generalized linear model to evaluate different variables' contribution to expected frequency of wind claims.

The following diagnostic chart displays the results of a countrywide analysis performed on one year of data from a generalized linear model:



(The solid line is the indicated relativity, while the dashed lines are  $\pm 2$  standard errors.)

Using the generalized linear model output, as well as other considerations, justify whether the insurer should add credit score to the homeowners rating plan for the wind peril.

9. Based solely on the given output of the GLM, it makes sense to add credit score to the homeowners rating plan for the wind peril. The  $\pm 2$  standard deviation bands around the indicated relativities for fair and poor each do not contain one; the indicated frequency relativities are statistically significantly different than 1. However, I would also want to see an analysis of pure premiums. Countrywide there are only about 45,000 exposures in the fair category and 15,000 exposures for the poor category. This raises concerns about the credibility of the data from those classes. (The vast majority of the exposures are in the good category. Perhaps some other breakdown of scores into categories would be better.)

I question whether there is casual relationship between credit scores and claim frequency from wind. This is a countrywide study. Could it be that the average credit scores may vary by state, with those states with higher average wind losses also having lower average credit scores?

(On the other hand, perhaps those with poorer credit scores are less likely to properly maintain the roof of their house, leading to some wind claims that would not have been otherwise made. It is important during hurricanes that the roof remain intact and attached to the home.)

Overall, I would recommend that the variable not be added (at this time) based on the lack of causality and the lack of reliable relativities due to the small volume of data for only one year. More data as well as more analysis is needed.

Comment: One should not spend much time commenting on the general issue of using credit scores in rating insurance. I think it should be sufficient to discuss the issue of causality, whether or not there is a logical connection between credit scores and wind losses.

On the general issue of using credit scores in rating insurance:

1. Assuming the insurer writes a reasonable amount of business and credit scores are grouped into intervals that are not tiny, there should be enough data in each rating group to measure costs with sufficient accuracy. The criterion of credibility is fulfilled.
2. Insureds with similar premiums after the use of credit scores have a range of expected costs, just as with any other rating variable. However, the use of rating scores decreases this variation and thus improves the homogeneity.
3. Studies have shown that credit scores are correlated with insurance costs. Credit scores have been used for several years and the relationship to costs has been reasonably stable over time. Thus the criterion of statistical significance is fulfilled.
4. There are errors in credit reports. Individuals can get copies of their credit reports and try to get the credit bureau to correct any errors. However, the information in the credit report are not subject to manipulation or lying by the insured. The criterion of verifiability is fulfilled.
5. There is considerable expense in obtaining credit reports and turning them into credit scores to use for rating insurance. Either the insurer will incur that cost or pay someone else to do this work. In either case the criterion of low administrative costs is not fulfilled.
6. One can construct credit scores for use in rating insurance using objective definitions, with little ambiguity. Class definitions based on ranges of credit scores can be mutually exclusive and exhaustive. There should not be much administrative error, as the credit scores can be calculated by computer. The criterion of objectivity can be fulfilled.

7. Since when they apply for a home mortgage or a car loan, their credit reports are examined, it is not an issue when these same reports are used for insurance. The criterion of privacy is fulfilled.
8. Both high and low income insureds have good and bad credit reports. The effect of using credit scores should not be correlated with income. The criterion of affordability is fulfilled.
9. The items recorded in a credit report, such as a late payment of a bill, are not responsible for differences in insurance costs. The criterion of causality is not fulfilled.
10. An insured can modify his behavior in order to improve his credit report in the future. The criterion of controllability is fulfilled.

10. (2.25 points) A company is implementing a recently approved private passenger automobile rate revision. The indicated rate change was 20% while the requested rate change was 8%.

- (0.75 point) Briefly describe three reasons the company may have decided to propose a rate increase substantially lower than the indicated rate change.
- (0.5 point) Briefly discuss two actions the company could take to offset the pricing shortfall.
- (1 point) Calculate the proposed base rate to achieve the overall 8% rate increase assuming no change in any rate differentials. Use the following information from the company's rate filing:

| Multiplicative Rating Factor 1<br>(R1) | Exposures<br>(000) | Rate<br>Differential |
|--|--------------------|----------------------|
| 1                                      | 150                | 1.350                |
| 2                                      | 500                | 1.000                |
| 3                                      | 100                | 0.990                |
| Overall                                | 750                | 1.069                |

| Multiplicative Rating Factor 2<br>(R2) | Exposures<br>(000) | Rate<br>Differential |
|--|--------------------|----------------------|
| A                                      | 250                | 0.870                |
| B                                      | 300                | 1.250                |
| C                                      | 200                | 1.000                |
| Overall                                | 750                | 1.057                |

| Additive<br>Discount | Exposures<br>(000) | Rate<br>Differential |
|----------------------|--------------------|----------------------|
| Yes                  | 450                | 0.050                |
| No                   | 300                | 0.000                |
| Overall              | 750                | 0.030                |

- Current average premium per vehicle = \$450.
- Indicated rate change = 20%.
- Requested rate change = 8%.
- Fixed expense per vehicle = \$35.

Vehicles are rated as follows:

$$P_p = [B \cdot R_1 \cdot R_2 \cdot (1-D) + A_p]$$

Where  $P_p$  = Proposed policy premium

B = Base rate

$R_1$  = Multiplicative Rating Factor 1

$R_2$  = Multiplicative Rating Factor 2

D = Additive Discount

$A_p$  = Additive Per Exposure Expense Fee

10. (a) 1. Regulatory concerns. The insurance department may not have been willing to approve more than an overall 8% increase (particularly if it were combined with relativity changes which would increase the effect on some insureds above the average overall effect.)

2. Competitive concerns. The insurance company may have worried about keeping its rates comparable to those of its chief competitors.

If insureds know another company offers the same product at a substantially lower price, they are likely to purchase the competing product.

3. Retention concerns. The insurance company may have worried that a 20% overall increase (at once) would cause many of its insureds to shop around for cheaper coverage (particularly if it were combined with relativity changes.)

Significant increases (or decreases) in premium for an existing policy can cause existing insureds to believe there may be better options available.

4. Lifetime analysis may lead the insurer to accept less premium now in order to retain more insureds and make more profit over the longterm on these policyholders.

(b) 1. The insurer can try to cut its underwriting and/or loss adjustment expenses. For example, the company may try to reduce the marketing budget or reduce the staffing levels.

2. The insurer can try to reduce its expected losses. One way to do this is to change the make-up of the portfolio of insureds. For example, a company may tighten the underwriting criteria or non-renew policies that have grossly inadequate premium relative to expected costs. Another way to reduce average expected loss is to reduce the coverage provided by the policy. For example, a homeowners insurer may adjust the policy to exclude coverage for mold losses.

3. Increase the expected rate of return on investments by investing more aggressively.

4. Market more aggressively to those groups that are adequately priced and avoid or nonrenew groups that are underpriced.

5. Educate policyholders on safe driving or provide increased safety incentives to try to reduce the number of claims and thus loss costs.

6. Be more aggressive in claims handling and less generous in claims settlements, particularly for third party claims, in order to reduce loss costs.

(c) We desire an average premium per vehicle of:  $(\$450)(1.08) = \$486$ .

Let X be the new base rate.

$$\$486 = \$35 + X (1.069)(1.057)(1 - 0.03). \Rightarrow X = \mathbf{\$411.48}.$$

Comment: See pages 239, 246, 263-264, and 274 of Basic Ratemaking.

Note that one can calculate the given overall proposed differentials yourself.

$$\{(150)(1.35) + (500)(1.00) + (100)(0.99)\} / 750 = 1.069.$$

$$\{(250)(0.87) + (300)(1.25) + (200)(1.00)\} / 750 = 1.057.$$

$$\{(450)(0.050) + (300)(0)\} / 750 = 0.030.$$

11. (2 points) A large commercial insured purchased a retrospectively rated annual policy to cover its workers compensation exposure in 2014. The first computation of the retrospective premium will occur on April 1, 2015, based on the following information and provisions:

|   |           |
|---|-----------|
| Limited Reported Losses valued as of April 1, 2015: | \$200,000 |
| Standard Premium:                                   | \$695,000 |
| Net Insurance Charge:                               | 0.181     |
| Minimum retrospective premium ratio:                | 75%       |
| Maximum retrospective premium ratio:                | 125%      |
| Loss Conversion Factor:                             | 1.08      |
| Expense Allowance (excludes tax multiplier):        | 15%       |
| Tax Multiplier:                                     | 1.04      |
| Expected Loss Ratio:                                | 62%       |

- (1.5 points) Calculate the retrospective premium for this policy as of April 1, 2015, including any necessary adjustments for minimum and maximum premium provisions.
- (0.5 point) Briefly describe two elements that the basic premium is intended to cover for a retrospectively rated policy.

11. (a) Minimum premium is:  $(75\%)(695,000) = \$521,250$ .

Maximum premium is:  $(125\%)(695,000) = \$868,750$ .

Basic Premium Factor = Expense Allowance

- (Expenses provided for through loss conversion factor) + (Net Insurance Charge).

Basic Premium is:  $\{15\% - (0.08)(62\%) + 0.181\} (\$695,000) = \$195,573$ .

Preliminary retro premium is:  $(1.04) \{(1.08)(\$200,000) + \$195,573\} = \$428,036$ .

This is less than the minimum premium, so the insured is charged **\$521,250**.

(b) The basic premium covers:

- Expenses: including profit but excluding taxes which are paid for via the tax multiplier, and excluding LAE which is paid for via the loss conversion factor.
- Net Insurance Charge: the net cost of limiting the retrospective premium to be between the minimum and maximum premium.

Comment: In Basic Ratemaking:

Net Insurance Charge = (Insurance Charge - Insurance Savings) (Expected Loss Ratio) (LCF).

In the example in Basic Ratemaking, the cost of the average effect of any per occurrence loss limitation is also included in the net insurance charge rather than as an additional charge.