Solutions to the Spring 2013 CAS Exam 5

(Only those questions on Basic Ratemaking)

The exam is available from the CAS.
The solutions and comments are solely the responsibility of the author.

prepared by
Howard C. Mahler, FCAS

Howard Mahler
hmahler@mac.com
www.howardmahler.com/Teaching/

1. (2 points)
Given the following information for an insurance company that writes 24-month term policies:

Policy Group	Effective Date	Expiration Date	Number of Vehicles
A	January 1, 2010	December 31, 2011	50
B	July 1, 2010	June 30, 2012	100

All policies within each group have the same effective date.
a. (0.5 point) Calculate the earned car-years for calendar year 2011.
b. (0.5 point) Calculate the earned car-years for policy year 2010 evaluated as of December 31, 2010 and as of December 31, 2011.
c. (0.5 point) Assume Policy Group B cancels on January 1, 2011. Calculate the 2010 policy year written car-years evaluated as of December 31, 2010 and as of December 31, 2011 for Policy Group B.
d. (0.5 point) Assume Policy Group B cancels on July 1, 2011. Calculate the 2010 and 2011 calendar year written car-years for Policy Group B.

1. (a) Half of the exposures for A are earned in CY2011: (50)(2)/2 = 50.
Alternately, one year of A is earned in CY2011: (50)(1) = 50.
Half of the exposures for B are earned in CY2011: (100)(2)/2 = 100.
Alternately, one year of B is earned in CY2011: (100)(1) = 100.
CY2011 Earned Exposures are: 50 + 100 = **150**.
(b) Evaluated as of 12/31/2010:
Only half of A has been earned: (50)(2)/2 = 50.
Only one quarter of B has been earned: (100)(2)/4 = 50.
Total earned exposures: 50 + 50 = **100**.
Evaluated as of 12/31/2011:
All of A has been earned: (50)(2) = 100.
Only three quarters of B has been earned: (100)(2)(3/4) = 150.
Total earned exposures: 100 + 150 = **250**.
(c) Evaluated as of 12/31/2010:
B PY2010 written exposures: (100)(2) = **200**.
Evaluated as of 12/31/2011:
B has been canceled and policies were in effect for only 1/2 year rather than 2 years.
B PY2010 written exposures: (100)(1/2) = **50**.
Alternately, B has been canceled and we need to subtract 3/4 of the exposures from the written.
B written exposures: (100)(2) - (100)(2)(3/4) = **50**.
(d) CY2010 B written exposures: (100)(2) = **200**.
Due to the cancelation, the written exposures decrease by 100.
However, since the cancelation took place after the end of 2010, the change goes into CY2011.
CY2011 B written exposures : (-100)(2)/2 = **-100**.
Comment: The calendar year exposures are not changed after the end of the year.
In part (d), the total written exposures for the two calendar years total to the correct 100.

2. (2 points) Given the following information for an insurance company:

- Proposed effective date of the next rate change is January 1, 2014.
- Rates will be in effect for 1 year.
- All policies have 12-month terms and are written uniformly throughout the year.
- Calendar year 2012 earned premium at current rate level is $114,208,050.

12 Month Period Ending	Written Premium at Current Rate Level	Written Exposures
December 31, 2011	$104,500,000	110,000
June 30, 2012	$113,800,500	121,000
December 31, 2012	$123,916,100	133,100

a. (1 point) Utilizing one-step trending, calculate the calendar year 2012 projected earned premium at current rate level for use in calculating the rate change.

b. (0.25 point) Briefly discuss why a premium trend should be utilized in a rate level indication.

c. (0.25 point) Briefly discuss why it is inappropriate to use written premium at historical rate levels to determine premium trends.

d. (0.5 point) The insurance company decides to move all existing business with a $100 deductible to a $500 deductible upon renewal during calendar year 2013.
Given this new information, discuss whether the true projected earned premium will be higher, lower, or unchanged from that in part a. above.

2. (a) The average premiums are: \$104,500,000 / 110,000 = \$950,
\$113,800,500 / 121,000 = \$940.50, and \$123,916,100 / 133,100 = \$931.00.
940.5/950 = 0.990. 931/945.5 = 0.990.
Thus select a semi-annual premium trend factor of 0.990.
CY2012 earned premium has an average date of earning of 7/1/2012.
Since there annual policies, the average date of writing is 6 months earlier 1/1/2012.
The average date of writing under the new rates is 7/1/2014.
Thus the trend period is 2.5 years or 5 half-years.
The calendar year 2012 projected earned premium at current rate level is:
(0.990^5) (\$114,208,050) = **\$108,610,719**.

(b) There are continuous gradual effects, for example shifts in the mix of business, that affect both premiums and losses. Assuming the loss trend is based on insurance data, then it will reflect these effects. Therefore, we need to apply premium trend reflecting these effects so that the losses and premiums are on a comparable basis.
Premium trend is particularly important for a line of insurance such as homeowners, where insured value (usually) increases over time, thus raising the average premiums in the absence of rate changes. For the same reason, premium trend is particularly important for a line of insurance that uses an inflation sensitive exposure base such as sales or payroll.

(c) For example, if there were a large rate increase during the historical period, then historical written premium levels would show a large jump, and the calculated premium trend would be large.
We would implicitly be assuming that a similar large rate increase would be occurring during the trend period; this is probably not the case. It is better to put the historical premiums on a common rate level, so that the premium trend is an estimate of what would occur in the future in the absence of rate changes. Then we can incorporate the one-time effects of actual rate changes by applying on-level factors to premiums.

(d) Less coverage. $\Leftrightarrow$ Less Premium.

$\Rightarrow$ The premium for a \$500 deductible is smaller than that for a \$100 deductible.

The projected premium in part (a) does <u>not</u> take this into account.
Therefore, the true projected premium will be **smaller** than in part (a).
<u>Comment</u>: It would have been helpful to me to know what line of insurance we are dealing with.
In part (d), the true trended losses will also be smaller than would be gotten by trending losses, without taking into account this planned change. Assuming the current deductible relativities are approximately correct, it would not have a significant effect on the overall rate indication, provided this proposed change were ignored with respect to <u>both</u> premiums and losses.

3. (2.5 points) An actuary has submitted the following analysis for a rate level indication:

Calendar/ Accident Year	Calendar Year Earned Premium	Accident Year Reported Losses and Paid ALAE	Accident Year Reported Loss and Paid ALAE Ratio
2010	$1,023,549	$703,902	68.8%
2011	$1,086,756	$773,430	71.2%
2012	$1,222,930	$749,249	61.3%

Three Year Average Reported Loss and Paid ALAE Ratio	67.1%
Fixed Expense Provision	11.0%
Variable Expense Provision	15.0%
Underwriting Profit Provision	8.0%
Variable Permissible Loss Ratio	77.0%
Indicated Rate Chance	1.4%

Recommend five improvements to the analysis and briefly explain the purpose of each.

3. 1. Put the premiums on the current rate level. We need to take into account any rate changes that have taken place, so that we can determine the indicated change from the current rate level.
2. Develop the loss and ALAE to ultimate. We need to estimate the total cost of providing coverage, which includes the loss and ALAE ultimately paid on a year of accidents.
Reported losses develop as they mature, due to unreported claims, reopened claims, and differences between the amount ultimately paid and the amount reported on open claims.
Paid ALAE develops with maturity, as additional amounts are paid to handle and settle claims.
3. Trend the losses and ALAE for each AY to the policy effective period. Losses and ALAE are affected by inflation; in addition there may be changes in frequency.
4. Include a provision for Unallocated Loss Adjustment Expense. We need to include all expenses associated with the risk transfer.
5. Trend the premiums for each CY to the the policy effective period. Premiums are affected by shifts in the mix of business and for some lines of insurance by inflation acting on the exposure base.
6. Take a weighted average of loss ratios rather than a straight average.
One could weight years by their premium volume, in order to take into account the different volumes of data by year. It may also make sense to weight more recent years more heavily, since they are more similar to the policy effective period.
7. Apply a trend to the fixed expenses. (It is not clear how the given "Fixed Expense Provision " was determined.) Inflation affects the amount paid for "fixed" expenses.
Comment: Give only 5 improvements.

4. (3 points) Given the following information:

- Annual loss trend rate = +4%.
- Rate change history:

  +3% effective April 1, 2009.
  +2% effective July 1, 2010.
- All policies have annual terms.
- Calendar year 2012 earned premium = $50,000.
- Accident year 2012 reported losses at December 31, 2012 = $4,200.

Percentage of Loss Reported at:

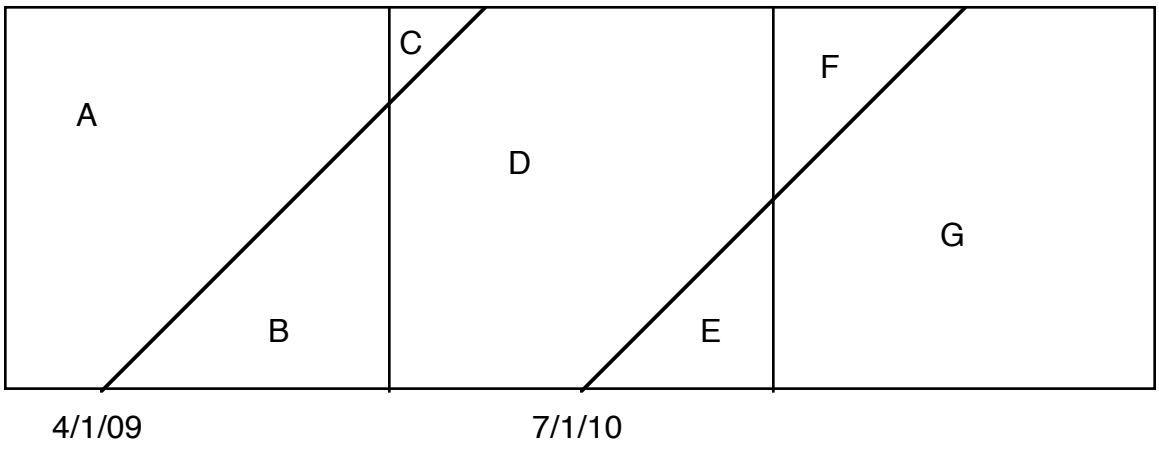
12 months	10%
24 months	35%
36 months	65%

Selected Ultimate Loss Ratio

Accident Year 2009	66%
Accident Year 2010	67%
Accident Year 2011	70%

Use the reported Bornhuetter-Ferguson technique to estimate ultimate losses for accident year 2012.

4. First we calculate on-level factors for each of the three accident years: 2009, 2010, 2011.



A = 1 - 9/32 = 23/32. B = (1/2)(3/4)^2 = 9/32. C = (1/2)(1/4)^2 = 1/32. D = 1 - 1/32 - 4/32 = 27/32.

E = (1/2)(1/2)^2 = 4/32. F = (1/2)(1/2)^2 = 4/32. G = 1 - 4/32 = 28/32.

2009 OLF is: $\frac{(1.03)(1.02)}{(1)(23/32) + (1.03)(9/32)} = 1.0418.$

2010 OLF is: $\frac{(1.03)(1.02)}{(1)(1/32) + (1.03)(27/32) + (1.03)(1.02)(4/32)} = 1.0184.$

2011 OLF is: $\frac{(1.03)(1.02)}{(1.03)(4/32) + (1.03)(1.02)(28/32)} = 1.0025.$

We put the selected ultimate loss ratios on the 2012 rate level and trend the losses to 2012.

2009 LR is: (66%) (1.04^3) / 1.0418 = 71.26%.

2010 LR is: (67%) (1.04^2) / 1.0184 = 71.16%.

2011 LR is: (70%) (1.04) / 1.0025 = 72.62%.

Select for 2012 the average of these three loss ratios: 71.68%.

AY2012 is at 12 months, so we assume 90% of ultimate losses are unreported.

Thus the estimated unreported losses for 2012 are: (90%)(71.68%)($50,000) = $32,256.

Adding the reported losses, the estimated ultimate loses for 2012 are: 4200 + 32,256 = **$36,456**.

Comment: We are implicitly assuming that there is no premium trend.

The Bornhuetter-Ferguson technique is discussed at pages C-10 to C-16 of Basic Ratemaking and in Chapter 9 of Estimating Unpaid Claims Using Basic Techniques by Jacqueline Friedland.

5. (4 points) A company is reviewing the rate level adequacy.
Given the following information for a book of business:

- All policies are annual.
- Current rates have been in effect for three years.
- New rates will be in effect for 18 months beginning on July 1, 2013.
- Annual premium trend = -1%.
- Annual loss trend = +3%.
- Loss adjustment expense provision = 2.5% of loss.
- Historical expense ratios:

  Fixed = 6%.

  Variable = 30%.

- Underwriting profit and contingencies provision = 5%.
- Ultimate losses are estimated using the reported development technique.
- On January 1, 2014, the company will reduce agency commissions by 3% of premium.

Calendar Year Ending	Earned Premium ($000s)
December 31, 2011	$2,163
December 31, 2012	$2,120

Reported Losses ($000s)

Accident Year	12 months	24 months	36 months	48 months	60 months
2008	$780	$928	$1,030	$1,083	$1,094
2009	$765	$921	$1,004	$1,053	
2010	$760	$920	$1,012		
2011	$805	$966			
2012	$890				

Calculate the indicated rate change.

5. Average date of writing under the new rates is April 1, 2014.
Thus with annual policies, average date of earning and accident is October 1, 2014.
The trend period is 3.25 years from 2011 and 2.25 years from 2012.
The premium on-level factors are one.
Thus the projected on-level earned premiums are:

2011: $(2163)(1)(0.99^{3.25}) = 2093.5$. 2012: $(2120)(1)(0.99^{2.25}) = 2072.6$.

The link ratios are:

	1st to 2nd	2nd to 3rd	3rd to 4th	4th to 5th	5th to ultimate
2008	1.190	1.110	1.051	1.01	
2009	1.204	1.090	1.049		
2010	1.211	1.100			
2011	1.200				
Selected	1.20	1.10	1.05	1.01	1.00

(Other similar selections are reasonable.)
Thus the projected ultimate losses are:

2011: $(966)(1.10)(1.05)(1.01)(1.03^{3.25}) = 1240.5$.

2012: $(890)(1.20)(1.10)(1.05)(1.01)(1.03^{2.25}) = 1331.6$.

We are given no instructions on how to combine 2011 and 2012; I will take the ratio of the combined losses to the combined premiums in order to get the projected loss ratio:
(1240.5 + 1331.6) / (2093.5 + 2072.6) = 61.74%.
The reduction in commission will take place 6 months through the 18 month period in which the new rates will be in effect, resulting in an average reduction in variable expenses of: (12/18)(3%) = 2%.
Thus the variable expenses will be: 30% - 2% = 28%.

The indicated rate change factor is: $\frac{(61.74\%)(1.025) + 6\%}{1 - 28\% - 5\%} = 1.034$. A **3.4% rate increase**.

Comment: Somewhat different choices would be reasonable.

6. (2.5 points)

a. (0.5 point) Contrast the components of IBNR for a claims-made policy and an occurrence policy.

b. (0.5 point) Explain why a claims-made policy should cost less than an occurrence policy, provided claim costs are increasing.

c. (0.5 point) Explain why a change in underlying trends will impact the estimated premium for an occurrence policy more than for a claims-made policy.

d. (0.5 point) Briefly describe the provision that exists to eliminate coverage overlap if an insured switches from an occurrence policy to a claims-made policy, and why an overlap would exist without it.

e. (0.5 point) Explain why there would be a coverage gap if an insured switches from a claims-made policy to an occurrence policy and what an insurer can do to provide coverage.

6. (a) For claims-made policy, once the coverage period ends there are no unreported claims.
Thus there is only the incurred but not enough reported (IBNER) reserve, the development on known claims. In contrast, when writing occurrence policies, there is also a reserve for unreported claims, pure IBNR.
(b) For example, let us assume we have policies effective January 1, 2013.
Then the claims-made policy would cover claims reported during 2013, while the occurrence policy would cover events that occurred during 2013, in other words AY2013.
Assume the retroactive date is so far in the past that the claims-made policy is mature; note that an immature claims-made policy costs less than a mature claims-made policy.
Both the claim-made and the occurrence policy would cover AY2013 at lag 0.
The claims-made policy would cover AY2012 at lag 1, while the occurrence policy covered AY2013 at lag 1.
The claims-made policy would cover AY2011 at lag 2, while the occurrence policy covered AY2013 at lag 2, etc.
The claims-made policy covers the same lags as the occurrence policy, but with the exception of lag 0, for an earlier accident year.
Thus if claims costs are increasing, the expected losses from the claims-made policy would be less.
Therefore, the claims-made policy would cost less than the occurrence policy.
Put more succinctly, claims-made policies have a much shorter average period of time between the coverage trigger and the settlement date and therefore are not impacted as much by loss cost increases. Occurrence policies incur liability for claims that occur now but are reported much later so loss trend affects these costs, whereas claims-made policies incur liability for claims reported at today's cost levels.
(c) As discussed in part (b), the claims-made policy covers the same lags as the occurrence policy, but with the exception of lag 0, for an earlier accident year. Thus the average trend period is smaller for the claims-made policy. Therefore, the claims-made policy is less affected by a change in the underlying trend in losses than the occurrence policy.
Alternately, with an occurrence policy, claims are covered that are reported much further out into the future. Thus changes in loss trends will have a greater impact on the losses covered by an occurrence policy than a claims-made policy.
(d) A claims-made policy has a retroactive date. Any event that occurred prior to the retroactive date is not covered by the claims-made policy.
For example, let us assume that an insured switches to claims-made on January 1, 2013.
(This would be called a first-year claims-made policy.)
We assume there were occurrence policies covering the Accident Years prior to 2013.
Then in the absence of a retroactive date, a claim reported during 2013 from any of these prior AYs would be covered by both an occurrence policy and the claims-made policy.
By using a January 1, 2013 retroactive date, we avoid this overlap in coverage.

This is illustrated in the following diagram, where for simplicity lags only out to 3 are shown:

		Lag		
Report Year	0	1	2	3
2010	Occ10	Occ09	Occ08	Occ07
2011	Occ11	Occ10	Occ09	Occ08
2012	Occ12	Occ11	Occ10	Occ09
2013		Occ12	Occ11	Occ10

Without a retroactive date, the claims-made policy would cover the whole row represented by Report Year 2013; this would overlap with occurrence policies for lags greater than 0.

With the January 1, 2013 retroactive date, the first-year claims-made policy only covers lag 0 of Report Year 2013, thus avoiding the overlap.

(e) Let us assume that an insured switched from claims-made to occurrence policy on January 1, 2013. Then an event that occurred prior to 2013 but was first reported during 2013 would not be covered by either policy. This gap in coverage would be closed by an extended reporting endorsement. An extended reporting endorsement (or tail coverage) covers claims that occurred prior to 2013, but were not reported before the expiration of the last claims-made policy.

This is illustrated in the following diagram, where for simplicity lags only out to 3 are shown:

		Lag		
Report Year	0	1	2	3
2012	CM12	CM12	CM12	CM12
2013	Occ13	X	X	X
2014	Occ14	Occ13	X	X
2015	Occ15	Occ14	Occ13	X
2016	Occ16	Occ15	Occ14	Occ13

Gaps in coverage that would be filled by an extended reporting endorsement are indicated by X.

Comment: Figures similar to the report year versus lag diagrams in Chapter 16 of Basic Ratemaking may help to understand and explain all of these concepts.

In part (b), the occurrence policy would have more associated investment income, since there is more time on average between when premiums are collected and losses are paid.

This would tend to make the occurrence policy cost less than the claims-made policy. On balance, in most situations, the claims-made policy would still cost less than the occurrence policy.

7. (3 points) An actuary is reviewing workers compensation indemnity loss experience for a rate level indication analysis. Given the following information:

- A benefit change having an impact of +5.0% applies to all indemnity losses for accidents occurring after July 1, 2011.
- A benefit change having an impact of +2.0% applies to indemnity losses on policies written after October 1, 2012.
- No other benefit changes are expected within the next few years.
- The annual impact on benefits due to wage inflation has been +2.0% and is expected to continue.
- The proposed effective date for revised loss costs is July 1, 2013.
- Policies are annual.
- Revised loss costs would be in effect for one year.
- Losses occur uniformly throughout the year.

Accident Year	Estimated Ultimate Losses at Pre-July 1, 2011 Benefit Levels ($000s)
2010	$1,875
2011	$1,875
2012	$2,000

Calculate the 2010, 2011, and 2012 accident year projected ultimate losses to be used in the rate level indication.

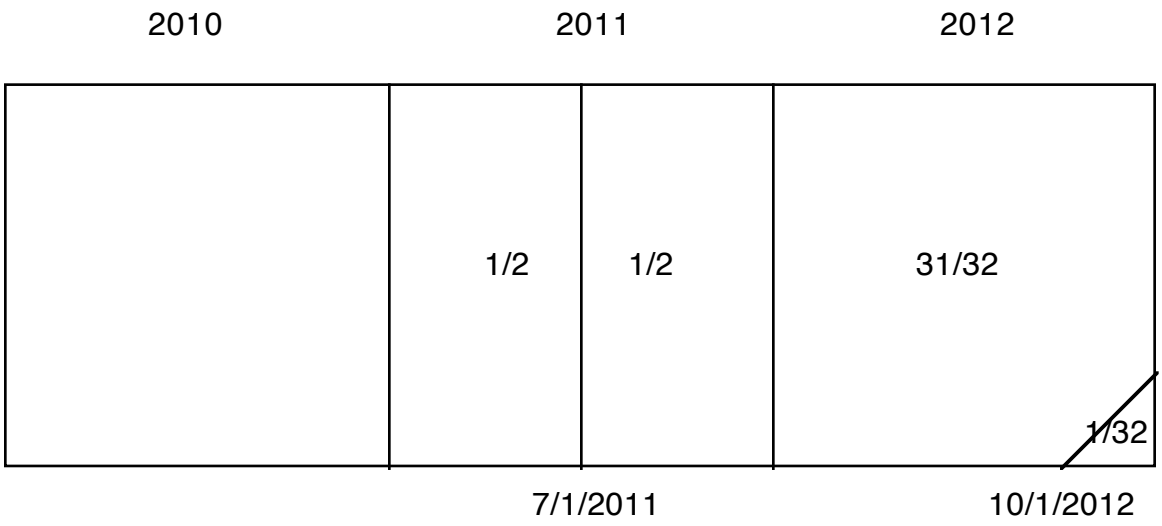
7. The average date of writing is January 1, 2014.
Since the policies are annual, the average date of accident is July 1, 2014.
Thus the trend periods are 4 years, 3 years, and 2 years.
Since all losses are reported at pre-July 2011 benefit levels, all years need both the 2% and 5% adjustment in order to be put them on the current benefit level.

2010: (1875) (1.02^4) (1.02)(1.05) = **2173.7**.

2011: (1875) (1.02^3) (1.02)(1.05) = **2131.0**.

2012: (2000) (1.02^2) (1.02)(1.05) = **2228.5**.

Comment: It is very unusual to start with losses on a different benefit level than at which they were paid. If instead the losses were reported at the benefit levels in effect at the various times, then the benefit level factors would be different. The 7/1/2011 change applies to all accidents after that date, so is represented by a vertical line. In contrast, the 10/1/2012 change applies to all policies written after that date, and thus is represented by a 45 degree line.



The factor to adjust 2010 losses to the current benefit level would is: (1.05)(1.02) = 1.071.
The factor to adjust 2011 losses to the current benefit level would be:

$$\frac{(1.05)(1.02)}{(1)(1/2) + (1.05)(1/2)} = 1.045.$$

The factor to adjust 2012 losses to the current benefit level would be:

$$\frac{(1.05)(1.02)}{(1.05)(31/32) + (1.05)(1.02)(1/32)} = 1.019.$$

8. (3 points) Given the following information:

- All policies are annual and written on January 1.
- Rate change effective date is January 1, 2013.
- Rate level is reviewed annually.
- Underwriting guidelines were revised on January 1, 2011, substantially changing the composition of the book of business.

Accident Year	Reported Loss & ALAE as of June 30, 2012
2010	$ 10,000,000
2011	$ 6,000,000
2012	$ 1,500,000

Selected Reported Loss & ALAE Age-to-Ultimate Factors

Month	6	12	18	24	30	36	42	48	54	60
Factor	6.50	2.00	1.55	1.20	1.12	1.08	1.05	1.02	1.01	1.00

Reported Loss & ALAE

Calendar Year Ending	Frequency	Severity	Pure Premium
Sep 2009	0.058	$20,355	$1,181
Dec 2009	0.059	$20,125	$1,187
Mar 2010	0.062	$20,500	$1,271
Jun 2010	0.063	$21,575	$1,359
Sep 2010	0.063	$21,388	$1,347
Dec 2010	0.065	$19,903	$1,294
Mar 2011	0.078	$19,567	$1,526
Jun 2011	0.078	$19,238	$1,501
Sep 2011	0.079	$19,538	$1,543
Dec 2011	0.082	$20,063	$1,645
Mar 2012	0.081	$20,050	$1,624
Jun 2012	0.082	$19,950	$1,636

# of Points	Annual Frequency Exponential Fit	Annual Severity Exponential Fit	Annual Pure Premium Exponential Fit
12	15.9%	-1.7%	13.9%
8	16.0%	-1.7%	14.0%
6	4.7%	2.9%	7.7%
4	4.1 %	2.5%	6.7%

Calculate the 2010 accident year trended ultimate loss & ALAE to be used in a rate change analysis. Justify any trend selections.

8. Accident Year 2010 is at 30 months.
So developed to ultimate the losses are: (10 million)(1.12) = 11.2 million.
The new guidelines affect all of the policies written January 1, 2011.
Thus the calendar year ending Dec 2011 will be largely under the new underwriting guidelines, while the calendar year ending Dec 2010 is unaffected by the new underwriting guidelines.
The ratio of pure premiums is: 1645/1294 = 1.271.
Select 25% to go from AY2010 before the guidelines to AY2011 after the guidelines.
Since all policies are written on January 1, the average date of writing under the new rates is January 1, 2013; average date of accident is July 1, 2013.
The trend period from AY2011 to the effective period is 2 years.
Based on the 4 and 6 point pure premium trends, which are largely based on data on the new underwriting guidelines, select 7% pure premium trend for these 2 years.
Trended ultimate losses AY2010: (11.2 million)(1.25)(1.07^2) = **$16.0 million**.
Comment: There are many reasonable choices one could make.
I found it difficult to see what they were trying to get at in this question.
The change in underwriting guidelines would have a one-time abrupt effect on losses, which is hard to quantify from the given information.
One would also have to determine what the impact of the new underwriting guidelines was on premiums due to change of mix of business by class and territory. If for example, the new guidelines resulted in a larger proportion of business in high rated classes and territories, then the average premium collected would increase even if rates remain the same.
I do not know what use one could make of the given AY2011 and AY2012 losses, since in order to somehow relate them to AY2010, we would need to know what the change if any has been in the volume of business being written.

9. (2 points)
An actuary develops an overall indicated rate increase of 4.5% using the following assumptions:

- All expenses are variable.
- Total permissible loss ratio = 65%.
- Profit and contingency provision = 5%.

The actuary's manager asks that the expenses be split into fixed and variable components as follows:

- Fixed = 75% of total expenses.
- Variable = 25% of total expenses.

a. (1.25 points) Calculate the revised overall rate indication with the new expense split suggested by the actuary's manager.
b. (0.25 point) Briefly explain why splitting the expenses as described above results in a different indication.
c. (0.5 point) Identify two reasons an actuary may want to split expenses into fixed and variable components.

9. (a) Expenses = 100% - 65% - 5% = 30%.
Splitting the expenses: 22.5% is fixed and 7.5% is variable.
We can back out the projected ultimate loss ratio from the 4.5% indication:

$1.045 = L / 0.65. \Rightarrow L = 0.67925.$

So the new rate indication is: $\frac{0.67925 + 0.225}{1 - 0.075 - 0.05} = 1.0334. \Leftrightarrow$ **3.3% rate increase**.

(b) Originally we were assuming that when premiums increase all expenses increase in proportion. In part (a) we were assuming that the fixed expenses do not increase when the premiums increase. Thus the rate indication was lower. In general, when we assume some expenses are fixed the rate indication would be closer to no change than it would be if one assumed all expenses are variable.
(c) 1. To more accurately reflect the costs by class and territory.
If some expenses do not vary with premium, then treating all expenses as variable would lead to excessive rates for insureds in high rated classes and territories, and inadequate rates in low rated classes and territories.
2. To get a more accurate overall rate indication.
If some expenses do not vary with premium, then treating all expenses as variable would lead to a rate indication that included an amount other than that needed to pay the expected expenses.
As an example compare the rate indication in part (a) to the original rate indication.
3. Allows us to apply trend factors to fixed expenses in order to get more accurate expense loads.
4. To more accurately allocate expenses to line and state.
Fixed expenses would usually be allocated based on exposures, while variable expenses would be allocated based on premiums.
Comment: In my opinion, part (a) reflects a basic misunderstanding of how to treat fixed expenses (in the Loss Ratio method). Originally when we assumed all expenses are variable, the 30% figure for expenses was probably based on looking at historical ratios of expenses to premium. When instead we assume some of the expenses are fixed, the provision for those fixed expenses would be gotten in a different manner. One would look at historical dollars for "fixed" expenses and compare them to exposures written. Then one get a dollar provision for fixed expenses for the future effective period; this would include expense trend and be consistent with the amount of business assumed to be written in the future effective period in the rest of the rate indication. Then one would take a ratio of these fixed expense dollars to the projected premiums at current rates. The result of all of this would be very unlikely to be the 22.5% used for fixed expenses in part (a). Fixed expenses fit more naturally in the pure premium method.

10. (2.25 points) Given the following information for a policy:

- Annual earned premium = $1,000.
- New business expected loss ratio = 60%.
- Losses expected to decrease $25 per year.
- New business expenses = $420.
- Renewal business expenses = $350.
- Probability of first renewal = 85%.
- Probability of second renewal = 90%.
- Probability of third renewal = 0%.
- Assume an annual discount rate of 3%.

a. (1.75 points) Calculate the lifetime value of the expected total profit as a percentage of premium.
b. (0.5 point) Identify two considerations used in the analysis in part a. above that differ from standard actuarial ratemaking techniques.

10. (a)

Policy	Premium	Losses	Expenses	Profit
New	1000	600	420	1000 - 600 - 420 = -20
1st renewal	1000	575	350	1000 - 575 - 350 = 75
2nd renewal	1000	550	350	1000 - 550 - 350 = 100

Present value of premium is: $1000 + (1000)(0.85)/1.03 + (1000)(0.85)(0.9)/1.03^2 = 2546.33$.

Present value of profit is: $-20 + (75)(0.85)/1.03 + (100)(0.85)(0.9)/1.03^2 = 114.00$.

Profit as a percent of premium is: 114.00 / 2546.33 = **4.48%**.

(b) 1. Standard (casualty) actuarial ratemaking techniques look at an individual policy and do not take into account potential profit from renewals.

2. Standard (casualty) actuarial ratemaking techniques do not distinguish between expenses for new business and renewals.

3. Standard (casualty) actuarial ratemaking techniques do not distinguish between expected losses for new business and renewals.

4. Standard (casualty) actuarial ratemaking techniques do not consider persistency, the likelihood of an insured renewing his policy.

Comment: They assumed no probability of a third renewal in order to reduce the amount of calculations required to solve the problem.

I have assumed that the given probabilities of renewal are what Feldblum calls:
Persistency Rate = (number of policies that renew) / (number of policies that could have renewed).

11. (3.5 points) An insurance company is researching three new rating variables to include in its homeowners risk classification system.

The insurer has determined the following information about the existing book of business:

Credit	Exposures	Pure Premium	Competitor's Rating Plan Factor	Base Class
Excellent	1,500	$116.67	0.85	No
Good	2,500	$128.00	1	Yes
Fair	1,000	$155.00	1.3	No
Total	5,000	$130.00		

Age of Homeowner	Exposures	Pure Premium	Competitor's Rating Plan Factor	Base Class
Under 30 years	800	$150.00	0.7	No
30 to 40 years	1,200	$116.67	1	Yes
Over 40 years	3,000	$130.00	1.2	No
Total	5,000	$130.00		

Loss Prevention	Exposures	Pure Premium	Competitor's Rating Plan Factor	Base Class
Fire extinguisher	100	$100.00	0.9	No
Smoke detector	4,700	$128.72	1	Yes
None	200	$175.00	1.5	No
Total	5,000	$130.00		

- Credit is determined using the credit score for the primary homeowner.
- Age of homeowner is determined using the age of the primary homeowner.
- A homeowner with both a fire extinguisher and smoke detector would be classified with a smoke detector.
- Full credibility claim standard = 400.
- The square root rule is used to determine partial credibility.
- A competitor's rating relativities are used as the credibility complement.
- Frequency for every risk classification = 10%.
- Assume that the insurer can implement only one new rating variable at this time.
- Assume that each variable is independent.

a. (1.5 points) For each potential rating variable, briefly describe two possible concerns of adding it to a risk classification system.

b. (0.75 point) Without performing any calculations, recommend and justify which rating variable the insurer should implement within a risk classification system.

c. (1.25 points) Develop the indicated credibility weighted rating factors for the variable recommended in part b. above.

11. (a) Credit:
Difficult to show causality; how would a low credit score cause a higher loss potential.
May be illegal to use for rating in a given state.
Raises privacy concerns; invades the privacy of insureds.
If credit scores are correlated with income, then its use in rating would raise affordability concerns.
Age of Homeowner:
Lacks controllability since a homeowner can't control their age.
The indicated relativities from the insurer's data differ significantly from competitor relativities; age of homeowner may not be a statistically significant risk differentiator.
The definition is not objective, as it does not tell what to do when a married couple jointly owns a home; should one use the age of the husband or of the wife.
Difficult to show causality; age of homeowner is probably correlated with other variables that affect the potential for loss such as: number of children living at home, age of home, territory, etc.
Loss Prevention:
There is very little data in the classes other than the base class, which raises credibility concerns.
Difficult and expensive to verify; subject to manipulation from the insureds.
The definition makes no sense; why would someone with both a fire extinguisher and a smoke detector be rated higher than someone with just a fire extinguisher:
(b) I would recommend credit score as a rating variable.
There is significant loss cost differentiation. There is an objective definition.
Credit scores can be purchased from a vendor for a reasonable amount of money.
Assuming it is legal, potential social concerns are not sufficient to prevent using credit scores.
(c) Assume that the standard for full credibility is 400 claims, and use the given 10% frequency.

Thus, for example, the credibility for excellent is: $\sqrt{\frac{(0.1)(1500)}{400}} = 0.6124$.

For example, 116.67 / 130.00 = 0.8975.
{(0.85)(1500) + (1.00)(2500) + (1.30)(1000)} / 5000 = 1.0150. 0.85 / 1.0150 = 0.8374.
(0.6124)(0.8975) + (1 - 0.6124)(0.8374) = 0.8742.
Good is given as the base class. 0.8742 / 0.9847 = 0.8877.

	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]
Class	Expos.	Pure Premium	Indicated Relativity	Competitor's Relativity	Normalized Competitor's Relativity	Cred.	Credibility Weighted Relativity	Cred-Weighted Relativity w.r.t. Base
			= [2]/Total [2]		=[4]/Total [4]			
Exc.	1500	$116.67	0.8975	0.85	0.8374	0.6124	0.8742	**0.8877**
Good	2500	$128.00	0.9846	1.00	0.9852	0.7906	0.9847	**1.0000**
Fair	1000	$155.00	1.1923	1.30	1.2808	0.5000	1.2365	**1.2557**
Total	5000	$130.00	1.0000	1.0150	1.0000			1.0175

In order to implement the new rating variable in a revenue neutral manner, the base rate would have to be divided by an off-balance factor of 1.0175.

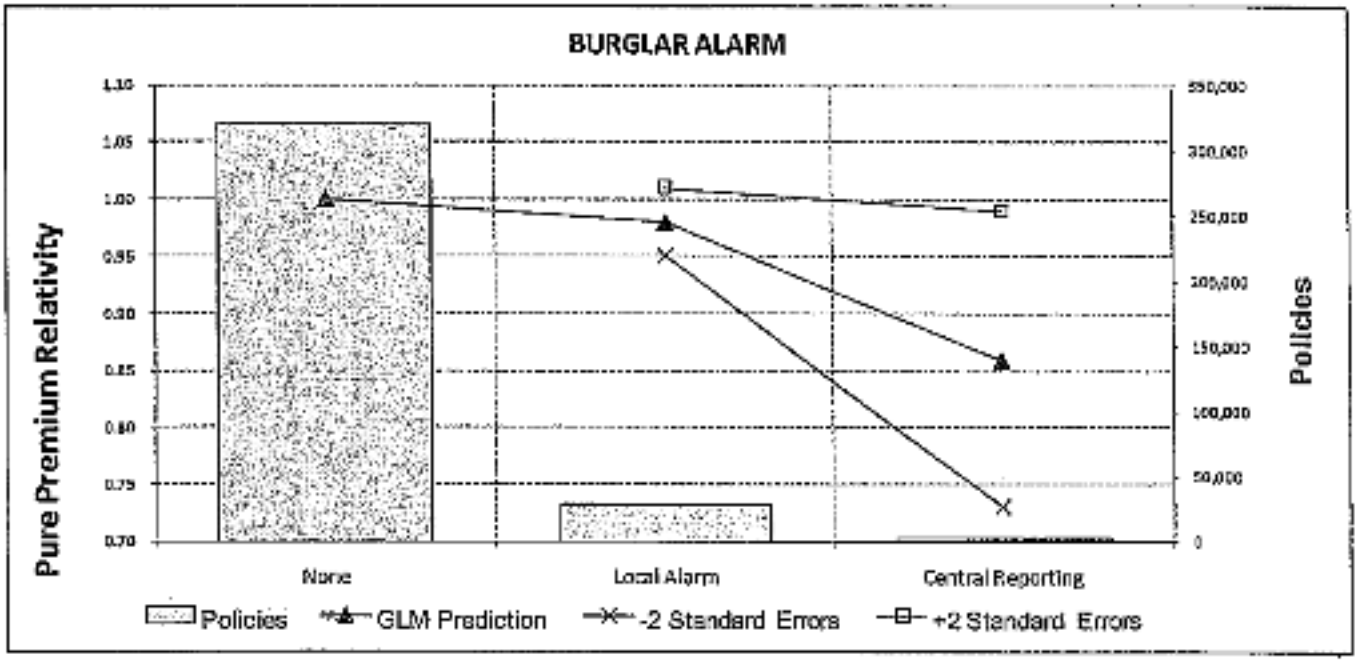
Comment: In parts (a) and (b) there are other possible answers, although in part (b) I would not recommend Loss Prevention.

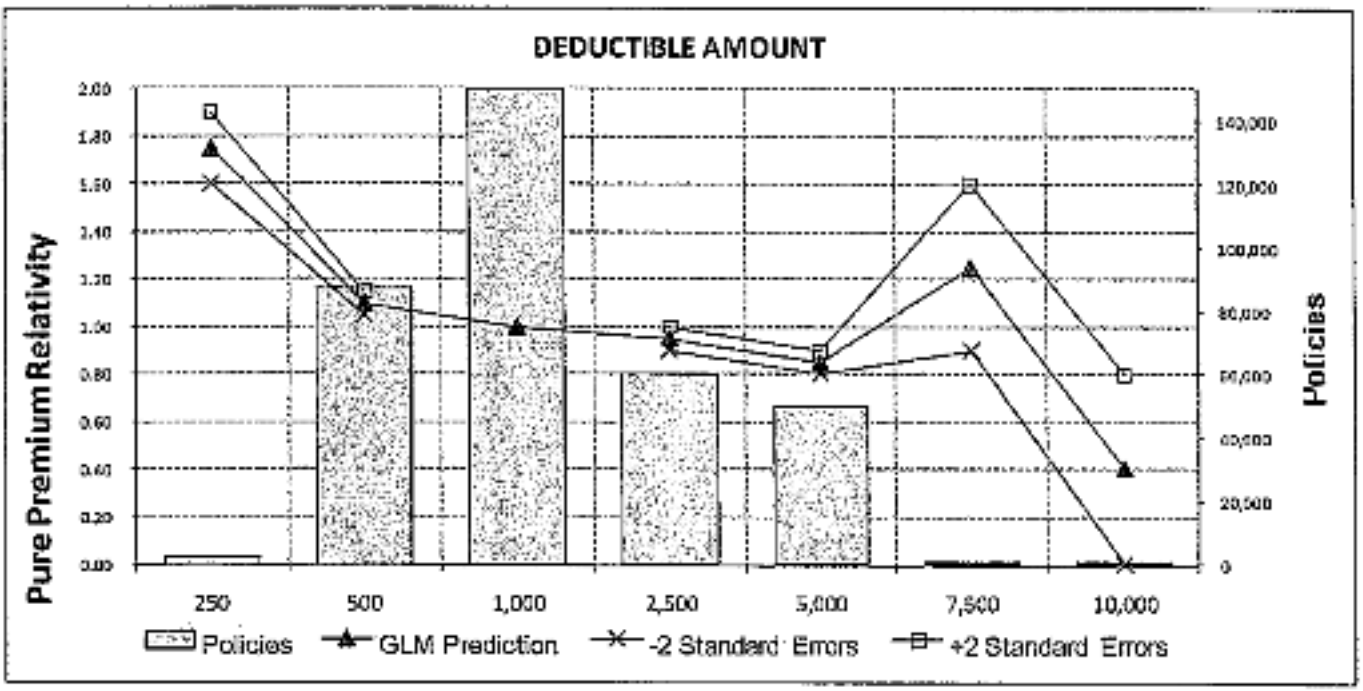
In part (a) give only two reasons for each variable. It may be hard to come up with reasons. Do not worry about whether you agree with a possible concern; make sure it illustrates one of the criteria in Basic Ratemaking. It is probably better not to use a criterion more than once in total.

In part (c), see Page E-3 in Basic Ratemaking.

One could instead assume that the given standard for full credibility is 400 exposures.

12. (3 points) An insurer is planning to revise burglar alarm and deductible rating plan factors for its Homeowners program. Given the following generalized linear model output:





Question 12 is continued on the next page.

12. (continued)

Burglar Alarm	GLM Prediction	-2 Standard Errors	+2 Standard Errors	Policies
None	1.00			320,000
Local Alarm	0.98	0.950	1.010	27,500
Central Reporting	0.86	0.730	0.990	2,500

Deductible	GLM Prediction	-2 Standard Errors	+2 Standard Errors	Policies
$250	1.75	1.60	1.90	2,700
$500	1.10	1.05	1.15	87,000
$1,000	1.00			150,000
$2,500	0.95	0.90	1.00	60,000
$5,000	0.85	0.80	0.90	50,100
$7,500	1.25	0.90	1.60	150
$10,000	0.40	0.00	0.80	50

Propose revised burglar alarm and deductible rating plan factors.
Document the relevant analysis and rationale to support the proposal.

12. Burglar Alarm:
Based solely on the GLM, there is little evidence to support a discount; there is relatively little data for the non-base classes, particularly for central reporting.
There are wide confidence intervals for both Local Alarm and Central Reporting groups. The Local Alarm standard errors suggest it is not significantly different than the None category; the confidence interval encompass a relativity of one. Central reporting has very for few exposures and large standard errors.
I would recommend this variable not be used; in other words, 1.00 factor for all groups.
Alternately, based solely on the GLM, there is little evidence to support a discount.
On the other hand, it is logical that a local burglar alarm will reduce theft losses.
It is logical that a central reporting burglar alarm would be more effective at reducing theft losses than a local alarm.
However, theft losses are only one of many perils covered by Homeowners.
We are given no information on what portion of the expected losses are due to theft; this varies by geographical location.
Based on the logic and the limited statistical support from the GLM, small discounts make sense.
I judgmentally select 0.98 for local alarm and 0.96 for central reporting.
Deductible:
Based solely on the GLM, due to the small amount of data, there is little evidence to support a discount for the $7500 and $10,000 deductibles. Also the discount for a $7500 should be smaller than for a $10,000 deductible; the GLM fitted relativities indicate the opposite.
There is somewhat more data for the $250 deductible, but the error bars are relatively wide.
On the other hand, we know that expected losses paid are more for a lower deductible and are lower for a higher deductible.
For the $7500 and $10,000 deductible, based on the difference between the indicated relativities for 2500 and 5000, I will judgmentally select relativities of 0.78 and 0.74.
(For evenly spaced deductibles, the difference in Loss Elimination Ratios gets smaller as the deductible increases, in the absence of either favorable or adverse selection.)
For the $250 deductible, based on the difference between the $500 and the $1000 relativities, a relatively of something like 1.20 might make sense. The 1.75 prediction from the GLM might be due to adverse selection. So I will judgmentally select a relativity of 1.30.
For the other deductibles, I will use the GLM output.
Thus my selected relativities are: 1.30, 1.10, 1.00, 0.95, 0.85, 0.78, 0.74.
Comment: There are many possible reasonable answers. Additional information besides the GLM output would be very helpful, for example competitor's rates.

13. (2 points) Given the following for a large deductible commercial general liability policy:

Per occurrence deductible	$250,000
Loss elimination ratio for a $250,000 deductible	80%
ALAE/ground up loss ratio	10%
Ground up loss estimate	$2,000,000
Fixed expenses	$100,000
Variable expenses as % of premium	12%
Underwriting profit as a % of premium	3%
Deductible processing cost as a % of losses below the deductible	5%
Credit risk as a % of losses below the deductible	2%
Additional risk margin as a % of excess losses	8%

- The insurer will handle all claims, including those that fall below the deductible.
- The insurer will make the payments on all claims and will seek reimbursement for amounts below the deductible from the insured.
- The deductible is for loss only.
- All ALAE is paid by the insurer.

Calculate the premium for the large deductible policy.

13. Losses excess of the deductible: ($2 million)(1 - 0.8) = $400,000.
Total ALAE: (0.1)($ 2 million) = $200,000
Fixed Expenses: $100,000.
Deductible processing cost: (5%)(0.8)$2 million) = $80,000.
Credit risk charge: (2%)(0.8)$2 million) = $32,000.
Additional risk margin: (8%)($400,000) = $32,000.
Premium for the large deductible policy:

$$1000 \frac{400 + 200 + 100 + 80 + 32 + 32}{1 - 12\% - 3\%} = \textbf{\$992,941}.$$

14. (1.25 points) An insurer proposes to increase rates by 6.0% where many individual policy impacts will be above 10%. The insurer proposes a capping rule that will restrict premium changes at the policy level to plus or minus 10.0%.
a. (0.5 point) Identify two problems that a capping rule may cause for an insurer.
b. (0.75 point) Explain why an insurer would propose a capping rule in light of the problems identified in part a. above.

14. (a) Rates for some individual insureds will be excessive (those whose decreases were capped) while others are inadequate (those whose rate increases were capped).
Besides violating the fundamental equation of insurance on an individual risk basis, this may lead to attracting more insureds whose rates are inadequate and losing insureds whose rates are excessive, which will hurt underwriting results.
The insurer will have to apply an off-balance to all rates for the net effect of capping, so as to achieve the desired overall rate increase. This will mean that no insured will get its indicated rate.
The insurer will need to program this capping rule into its computer systems.
(b) Increasing an insured's rate by a larger amount (all at once) will reduce retention; maintaining a high retention rate usually helps the long term profitability of insurers.
Allowing large decreases all at once for an individual may make it difficult to get back to adequate rates if in the following year a large increase would be indicated. In any case, by capping both decreases and increases, the off-balance from the net effect of capping will be closer to one.
The insurance regulator may not allow large rate increases for individual policies.
(Note that for capped policies, an additional change can be applied next year if it is still indicated.)
Finally, capping large changes makes sense from a statistical standpoint. Credibility is a linear estimator, and thus extreme data can result in extreme indicated changes for a given class/territory cell. The expected squared errors are reduced by capping large changes. (It is not clear that capping at ±10% rather than for example ±20% would achieve this goal in this case.)
Comment: Since the capping is applied at the policy level rather than to each classification dimension separately, this is what Basic Ratemaking calls a Premium Transition Rule.

15. (2.5 points) An employer negotiated a workers compensation retrospective policy with an insurer, effective from January 1, 2011 to December 31, 2011.
The first adjustment of the retrospective premium occurs six months after the end of the policy period and annually thereafter until the tenth adjustment.

The reported losses during the policy period evaluated as of June 30, 2012 are as follows:

Claim	Reported Losses
#1	$300,000
#2	$200,000
#3	$100,000

The provisions for this retrospective rating plan are as follows:

Minimum retrospective premium ratio	50%
Maximum retrospective premium ratio	150%
Loss Conversion Factor	1.2
Per Accident Loss Limitation	$150,000
Expense Allowance Excluding Tax Multiplier	25%
Expected Loss Ratio	60%
Tax Multiplier	1.05
Net Insurance Charge	44.6%
Standard Premium	$540,000

a. (2 points) Calculate the retrospective premium as of June 30, 2012.
b. (0.5 point) Discuss what could cause the retrospective premium in part a. above to change for the insured between June 30, 2012 and the tenth adjustment.

15. (a) The three claims are limited to $150,000 each.
Thus the losses entering the plan are: $150,000 + $150,000 + $100,000 = $400,000.
The expenses in the basic premium are: 25% - (1.2 - 1)(60%) = 13%.
Thus the basic premium factor is: 44.6% + 13% = 57.6%.
The basic premium is: (57.6%)(540,000) = $311,040.
Thus the preliminary retro premium is: {$311,040 + (1.2)($400,000)} (1.05) = $830,592.
However, the maximum premium is: (150%)(540,000) = $810,000.
Thus the retro premium is: **$810,000**.
(b) In general losses develop; there may be some claims that have yet to be reported, and the reported losses may settle for amounts different than the amounts reported as of a given date.
In this case, one or more of the three claims would have to settle for less than its value reported as of June 30, 2012; in order to change the retro premium, the first or second claim would have to settle for less than the accident limit of $150,000.
The net effect of all loss development would have to be such that the limited losses at tenth adjustment would be sufficiently lower than at first adjustment so that the insured pays less than the maximum premium; in this case the reported losses would have to be less than:
(810,000/1.05 - 311,040) / 1.2 = $383,657.
Also, although it is unlikely, it is possible for the Standard Premium to change due to a late revision in the premium audit.
Comment: See Table 15.7 in Basic Ratemaking and the associated example.
In Basic Ratemaking, the net insurance charge includes the loss conversion factor;
in contrast in the NCCI Retro Rating Plan, the net insurance charge is prior to multiplication by the loss conversion factor, while the converted insurance charge would be after multiplication by the loss conversion factor.
The minimum premium should be greater than the basic premium times the tax multiplier, which is not the case here; the retro premium when there are no losses (and no excess loss premium) is: (basic premium) (tax multiplier).
When there is an accident limit, then there should be a corresponding charge.
In Basic Ratemaking this charge for the loss limitation is included in the basic premium.
In contrast, in the NCCI Retro Rating Plan, the loss limitation is paid for through the separate Excess Loss Premium.