Solutions to the
Spring 2018 Makeup Exam
CAS Exam Five

(Only those questions on Basic Ratemaking)

There were 26 questions worth 56.25 points, of which 12.5 were on ratemaking worth 28.25 points.
(Question 21 worth 2.75 points which covered reserving, was defective.
Question 6a covered reserving, while question 6b covers ratemaking.)

This exam used the computer based testing with Excel.

"Question 21 was deemed defective, as it was missing a key piece of information to solve the problem. The exam was graded excluding this question. However, any candidates that did answer the question appropriately, and who would have passed if the exam included the question but would fail with this question excluded, are included as part of the list of passing candidates."

The exam and examiner's report are available from the CAS.
The solutions and comments are solely the responsibility of the author.

(Incorporating what I found useful from the CAS Examiner's Report and sample solutions.)

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1. (1.25 points) Given the following information on policy transactions:

Policy	Transaction Date	Action	Full-Term Written Premium
A	January 1, 2015	Policy Inception	100
B	February 1, 2015	Policy Inception	250
B	August 1, 2015	Endorsement	60
B	June 5, 2017	Audit Premium	50
C	May 1, 2015	Policy Inception	150
C	October 1, 2015	Cancellation	-150
D	September 1, 2014	Policy Inception	175
D	September 1, 2015	Policy Renewal	200
D	March 1, 2016	Endorsement	80
E	April 1, 2016	Policy Inception	75

- Full-term written premium represents the policy premium if policy characteristics shown were in place from original effective date date until original expiration date.
- All policies are annual.

a. (0.5 points) Calculate the in-force premium as of September 15, 2015.
b. (0.75 points) Calculate the 2015 policy year written premium as of December 31, 2017.

1. (a) Policies A, B, C and D are in effect.
On September 15, 2015 policy B (endorses August 1, 2015) has full term premium of:
250 + 60 = 310.
On September 15, 2015 policy C has full term premium of 150.
On September 15, 2015 policy D (renewed September 15, 2015) has full term premium of 200.
In-force premium as of September 15, 2015: 100 + 310 + 150 + 200 = **760**
(b) Policy E is not part of Policy Year 2015, while the other policies are.
Policy B was endorsed half way through.
Policy C was cancelled after 5 months.
PY15 written premium as of December 31, 2017:
100 + (250 + 60/2 + 50) + {150 - (150)(7/12)} + (200 + 80/2) = **732.5**.
Comment: All of the given premiums are full-term.

2. (1.75 points) Given the following payment and reserve information about two different claims on two different policies:

Policy Number	Policy Effective Date	Accident Date	Transaction Date	Incremental Loss Payment ($)	Ending Case Reserve ($)
A	January 1, 2016	July 3, 2016	July 5, 2016	0	2,000
			September 15, 2016	1,000	3,000
			January 3, 2017	3,500	0
B	October 1, 2016	May 1, 2017	May 1, 2017	500	5,000
			November 2, 2017	2,500	3,000
			January 2, 2018	5,000	0

a. (0.5 points) Calculate the incurred losses for each of the following calendar years:
   i. 2016
   ii. 2017

b. (0.5 points) Calculate the incurred losses for each of the following accident years evaluated as of December 31, 2017:
   i. 2016
   ii. 2017

c. (0.25 points)
   Calculate the incurred losses for policy year 2016 evaluated as of December 31, 2017.

d. (0.5 points)
Briefly describe one disadvantage and one advantage of using calendar year losses for ratemaking.

2. (a) i. 1000 + 3000 = **$4000**.
ii. (3500 - 3000) + 500 + 2500 + 3000 = **$6500**.
(b) Claim from policy A contributes to AY16, while the claim from policy B contributes to AY17.
i. 1000 + 3500 = **$4500**.
ii. 500 + 2500 + 3000 = **$6000**.
(c) (1000 + 3500) + (500 + 2500 + 3000) = **$10,500**.
(d) Advantage: Calendar Year data is available quickly and is final at the end of the Calendar Year.
Disadvantage: Calendar Year data has a poor match between premiums and losses.
The premiums and losses are each from different combinations of policies.
The losses are the results of payments and reserve changes on accidents from many years.

3. (2.25 points) Given the following information evaluated as of December 31, 2017:

Calendar / Accident Year	Earned Premium ($000)	On-Level Factor	Ultimate Ground-up Losses ($000s)	Ultimate Losses ($000s) Excess of $750,000
2015	94,824	0.980	81,518	26,000
2016	97,230	1.010	54,051	0
2017	94,098	1.010	63,413	6,393

6%	Unlimited loss trend
4%	Limited loss trend
559,996	Ultimate trended ground-up losses for accident years 2008 - 2017 ($000s)
45,221	Ultimate trended losses excess of $750,000 for accident years 2008 - 2017 ($000s)

- There is no premium trend.
- Policies are written on an annual basis.
- Rates are in effect for one year.

a. (2 points) Calculate the trended ultimate loss ratio for accident years 2015 to 2017 to be used to determine a rate change effective July 1, 2018.

b. (0.25 points)
Briefly explain whether or not it is appropriate to use a large loss adjustment in this situation.

3. (a) The average data of writing under the new rates is January 1, 2019.
With annual policies, the average data of accident under the new rates is July 1, 2019.
Thus the trend period from AY17 is 2 years.
Using a large loss adjustment, we have to determine an Excess Loss Factor.

$$\text{Excess Loss Factor} = 1 + \frac{\text{Excess Losses}}{\text{Total Losses - Excess Losses}} = 1 + \frac{45,221}{559,996 - 45,221} = 1.088$$

$$= \frac{559,996}{559,996 - 45,221} = \frac{\text{Total Losses}}{\text{Total Losses - Excess Losses}}.$$

Using a large loss adjustment with premiums and losses in thousands:

AY	Premium	OLF	On-Level Premium	Limited Losses	Trend	Trended Limited Loss	ELF	Trended Losses
2015	94,824	0.98	92,928	55,518	1.1699	64,948	1.088	70,664
2016	97,230	1.01	98,202	54,051	1.1249	60,800	1.088	66,150
2017	94,098	1.01	95,039	57,020	1.0816	61,673	1.088	67,100
Total			286,169					203,914

For example, (94,824)(0.98) = 92,928. 81,518 - 26,000 = 55,518.

$(55,518)(1.04^4) = 64,948$. (64,948)(1.088) = 70,664.

Loss Ratio is: 203,914/286,169 = **71.2%**.

(b) The excess losses vary a lot from year to year. For example, AY15 has a significant amount of excess losses as a percent of the total, while AY16 has no excess losses.
Therefore, in order to smooth out the effects and thereby stabilize the estimated loss ratio, one should use a large loss adjustment; one removes excess losses and then loads them back in based on a longterm average.

Comment: Not using a large loss adjustment with premiums and losses in thousands:

AY	Premium	OLF	On-Level Premium	Losses	Trend	Trended Loss
2015	94,824	0.98	92,928	81,518	1.2625	102,915
2016	97,230	1.01	98,202	54,051	1.1910	64,376
2017	94,098	1.01	95,039	63,413	1.1236	71,251
Total			286,169			238,541

Loss Ratio is: 238,541/286,169 = 83.4%.

4. (2.75 points) Given the following exposure and loss data relevant to a line of property insurance:

Accident	Earned	Cumulative Reported Claims ($) evaluated as of (months):			
Year	Exposures	12	24	36	48
2014	898	363,572	490,822	564,445	620,890
2015	938	387,362	522,939	601,380	
2016	980	412,801	557,281		
2017	1,024	439,961			

- Losses are fully developed at 48 months.
- All policies are annual.

a. (1.25 points) Calculate the annual loss cost trend.

b. (1 points) Calculate the projected loss cost per exposure for policies written between January 1, 2018 and December 31, 2018 given a new legislative reform that will increase loss costs for claims reported on or after January 1, 2018 by 20%.

c. (0.5 points) Briefly describe two factors that drive loss cost trends.

4. (a) The 36 to 48 development factor: 620,880/564,445 = 1.100.
The 24 to 36 development factor: (564,445 + 601380) / (490,882 + 522,939) = 1.150.
The 12 to 24 development factor:
(490,882 + 522,939 + 557,281) / (363,572 + 387,362 + 412,801) = 1.350.
AY14 pure premium: 620,890 / 898 = $691.41.
AY15 pure premium: (1.1)(601,380) / 938 = $705.24.
AY16 pure premium: (1.1)(1.15)(557,281) / 980 = $719.35.
AY17 pure premium: (1.1)(1.15)(1.35)(439,961) / 1024 = $733.73.
Ratios of successive years are:
705.24/691.41 = 1.020. 719.35/705.24 = 1.020. 733.73/ 719.35 = 1.020.
Thus the annual loss cost trend is **2.0%**.
(b) The average data of writing is July 1, 2018.
With annual policies, the average data of accident is January 1, 2019.
Thus the trend period from AY17 is 1.5 years.
In this case, the trended pure premiums for each accident year are each the same:
$(733.73)(1.02^{1.5}) = \$755.85$.
Taking into account the law change: (1.20)($755.85) = **$907.02**.
(c) 1. Inflation increases the average severity over time.
2. Changes in mix of business can change the average pure premiums over time.
Alternately, from the CAS sample solutions:
Loss cost trends can be broken down into two factors: frequency and severity.
These two factors could change for different reasons, which is why trends are applied separately when possible; together they make the loss cost trend.
An example of how these two factors change for different reasons: suppose an insurance company decides to raise their minimum deductible from $250 to $500. Then frequency will decrease because the insurer will handle fewer small claims and severity will increase because the insurer will no longer have the data for the small claims below the deductible.
Comment: One would get the same result in part (a) if one fit an exponential regression.

5. (2.5 points) Given the following:

Countrywide							
Calendar Year	Policy Count	Written Premium ($000)	Earned Premium ($000)	Commission & Brokerage ($000)	Other Acquisition ($000)	Taxes, Licenses, & Fees ($000)	General Expense ($000)
2015	135,000	98,000	97,000	6,860	5,880	3,920	7,840
2016	138,000	100,000	99,000	7,000	6,300	3,900	8,007
2017	141,000	102,500	101,250	7,175	6,253	4,203	8,212

State A	
$725	Average Premium
6.2%	Average Taxes, Licenses & Fees
7.0%	Average Commission & Brokerage
67.2%	Selected Projected Loss + ALAE Ratio

3.5%	Profit Provision
6.0%	ULAE Provision (of Loss + ALAE)

- All policies are annual.
- Proposed effective date of rates is January 1, 2019.
- Rates are in effect for one year.

a. (1 point)
Calculate the underwriting expense provision for State A using the all variable expense method.

b. (0.5 point) Calculate the fixed and variable underwriting expense provisions for State A using the premium-based projection method, given the following:

- 75% of Other Acquisition expenses are fixed.
- 75% of General Expenses are fixed.

c. (1 point) Calculate the indicated rate change using the loss ratio method.
Include a brief justification for the selected underwriting expenses.

5. (a) CW General Expenses: (7840 + 8007 + 8212) / (97,000 + 99,000 + 101,250) = 8.09%.
CW Commissions: (6860 + 7000 + 7175) / (98,000 + 100,000 + 102,500) = 7.00%.
CW Other Acquisition: (5880 + 6300 + 6253) / (98,000 + 100,000 + 102,500) = 6.13%.
CW Taxes, Licenses & Fees: (3920 + 3900 + 4203) / (98,000 + 100,000 + 102,500) = 4.00%.
I will use the State Taxes, Licenses & Fees of 6.2% rather than the countrywide, as these items usually vary by state.
The countrywide and state commissions are both 7.0%.
Expense Provision: 8.09% + 7.00% + 6.13% + 6.2% = **27.4%**.
(b) Variable Expense Provision: (25%)(8.09%) + 7.00% + (25%)(6.13%) + 6.2% = **16.8%**.
Fixed Expense Provision: (75%)(8.09%) + (75%)(6.13%) = **10.7%**.
(c) Split fixed and variable expenses. If some expenses truly are fixed and you treat them as variable, if rates are indicated to increase, then you will include in the rate more than is needed to pay for fixed expenses.

Premium-based Projection Method: $\frac{(67.2\%)(1.06) + 10.7\%}{1 - 16.8\% - 3.5\%} - 1 =$ **2.8%**.

Comment: The only sample solution provided by the CAS used the premium-based projection method.
There is insufficient information to determine whether any expenses are actually fixed rather than varying with premiums. If one assumes that are expenses vary with premiums, then

All Variable Method: $\frac{(67.2\%)(1.06)}{1 - 27.4\% - 3.5\%} - 1 = 3.1\%$.

I prefer the exposure/policy-based projection method to the premium based projection method; the latter makes no sense to me; if some expenses do not vary with premiums, why take a ratio to premiums. Using the policy-based projection method:
Fixed General Expenses per policy:
(75%)($1000)(7840 + 8007 + 8212) / (135,000 + 138,000 + 141,000) = $43.59.
Fixed Other Acquisition per policy:
(75%)($1000)(5880 + 6300 + 6253) / (135,000 + 138,000 + 141,000) = $33.39.
The fixed expense ratio is: ($43.59 + $33.39) / $725 = 10.6%.

Rate Indication: $\frac{(67.2\%)(1.06) + 10.6\%}{1 - 16.8\% - 3.5\%} = 2.7\%$.

6. (4.5 points) Given the following as of December 31, 2017:

Rate Change History	
Effective Date	Change
July 1, 2016	8%
July 1, 2017	5%

Calendar Year	2015	2016	2017
Earned Premium	$2,500,000	$3,100,000	$2,100,000

Cumulative Paid Loss + ALAE ($) as of (months)			
Accident Year	12	24	36
2015	1,000,000	1,500,000	1,725,000
2016	1,100,000	1,650,000	
2017	900,000		

5%	Annual loss and ALAE trend
2%	Annual premium trend
6%	Fixed expense ratio
20%	Variable expense ratio
65%	Expected Loss & ALAE Ratio
4%	ULAE provision (% of loss and ALAE)
7%	Indicated rate change for policies effective July 1, 2018 using the last three accident years of experience

- All policies are annual.
- Exposures are written evenly throughout each calendar year.
- There is no development beyond 36 months.
- Rates are to be in effect for one year.
- The historical experience is fully credible.

a. (1 point) Calculate the ultimate loss & ALAE for each accident year using the paid Bornhuetter-Ferguson technique.

b. (3.5 points) Determine the profit and contingencies provision used to calculate the indicated rate change for policies effective July 1, 2018 using the ultimate loss & ALAE calculated in part a. above.

6. (a) The 12-24 loss development factor is: (1500 + 1650) / (1000 + 1100) = 1.500.
The 24-36 month loss development factor is: 1725/1500 = 1.15.

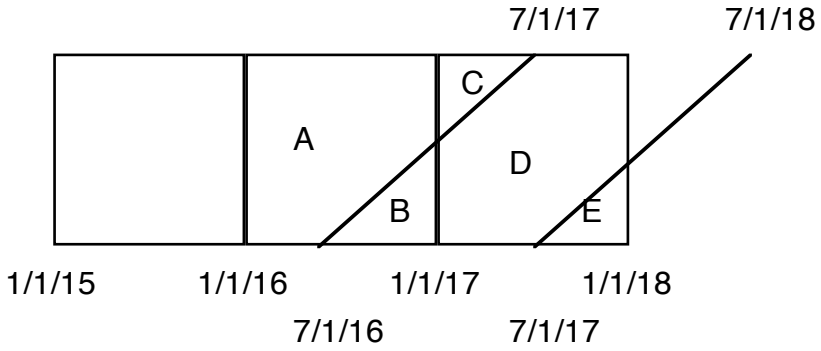
Estimated unpaid for AY17: $(65\%)(2{,}100{,}000)\left(1 - \frac{1}{(1.50)(1.15)}\right) = 573{,}676.$

Estimated ultimate loss and ALAE for AY17: 900,000 + 573,676 = **$1,473,696**.
Estimated unpaid for AY16: (65%)(3,100,000)(1 - 1/1.15) = 262,826.
Estimated ultimate loss and ALAE for AY16: 1,650,000 + 262,826 = **$1,912,826**.
Estimated ultimate loss and ALAE for AY15: **$1,725,000**.

(b)

Date	Rate Level Change	Rate Level Index
		1.00
7/1/16	8%	1.08
7/1/17	5%	1.134

On Level Factor for CY15 is 1.134.



Area B $= (1/2)(1/2)^2 = 1/8$. Area A $= 1 - 1/8 = 7/8$.

On Level Factor for CY16 is $1.134 / \{(7/8)(1) + (1/8)(1.08)\} = 1.1228$.

Area C $= (1/2)(1/2)^2 = 1/8 =$ Area E. Area D $= 1 - 1/8 - 1/8 = 6/8$.

On Level Factor for CY17 is $1.134 / \{(1/8)(1) + (6/8)(1.08) + (1/8)(1.134)\} = 1.0532$.

The average date of writing under the new rates is January 1, 2019.

The average date of accident under the new rates is 6 months later, July 1, 2019.

Therefore, the trend period from AY17 is 2 years.

AY	Premium	OLF	Premium Trend	On-Level Trended Prem.
2015	$2,500,000	1.1340	1.0824	$3,068,695
2016	$3,100,000	1.1228	1.0612	$3,693,725
2017	$2,100,000	1.0532	1.0404	$2,301,073
Total				$9,063,494
AY	Ultimate Loss	Loss Trend	Trended Ultimate Loss	Loss Ratio
2015	$1,725,000	1.2155	$2,096,748	68.33%
2016	$1,912,826	1.1576	$2,214,335	59.95%
2017	$1,473,696	1.1025	$1,624,750	70.61%
Total			$5,935,833	65.49%

Using the totals for the three AYs, the loss & ALAE ratio is: 5,935,833/9,063,494 = 65.49%.

$$1.07 = \frac{(65.49\%)(1.04) + 6\%}{1 - 20\% - Q}. \Rightarrow Q = \mathbf{10.7\%}.$$

Comment: There are other reasonable selections of a final loss & ALAE ratio in part (b); for example, one could take an average of the loss ratios for the three individual years.

7. (2 points) An insurance company has determined that average commute time to work is a predictive rating variable in their risk classification system for personal automobile coverage, based on statistical criteria.
Evaluate whether this variable should be included in rating based on four other criteria for evaluating rating variables.

7. I would recommend using this rating variable because:

- It satisfies objectivity. The different levels of the rating variable have objective definitions.
- Longer commute times (via the insured vehicle) are probably not positively correlated with income. Thus affordability is satisfied.
- The more time on the road, the more chance for an accident, and thus the higher the expected losses. One could convince (most) regulators that longer commute times (via the insured vehicle) are responsible for higher expected insurance costs. Causality is satisfied.
- Ones average commute time to work is not particularly personal information that someone would one reluctant to disclose.

Thus the criterion of not raising privacy concerns is satisfied.

Alternately I would recommend not using this rating variable because:

- This would be difficult to verify; it would be subject to manipulation or lying by the insured.

It does not satisfy verifiability.

- It would be expensive and time consuming to collect this information (unless it was self-reported by the insured.) It does not satisfy low administrative expense.
- Some insureds could reduce their commute time (by automobile) by some combination of: moving, changing jobs, or using public transit. However, these options are not available to many insureds. Controllability is not satisfied.
- As far I know using average commute times would not violate any laws or regulations.

Thus it satisfies the legal criterion. However, it still violates three other criteria.

From the samples solutions supplied by the CAS:

Controllable: This variable does not fit this criterion as drivers do not have control over traffic or road work which would affect commute time. Simply moving to an job with a shorter commute might not be feasible for most customers.

Comment: Other full credit answers are possible.

If a telematics device was installed in the vehicle's onboard diagnostic system port, this might allow the insurer to verify average commuting times. Also there would be no extra expense if telematics device was already installed for other purposes. However, in that case the insurer can probably find better ways to use all of the information from the device to rate insureds.

If the rating variable takes into account carpooling, not counting times when the insured is not using his own car, that would be another opportunity to reduce average commute times. In any case, one could argue that controllability is satisfied rather than not satisfied.

It would have been good to have a clear definition of commute time. Some people commute to work at least partially via public transit. Other people commute to work a majority of the time by walking or biking. I assume they meant time spent commuting to work in the insured vehicle.

8. (2 points)
An insurer is revising its class factors as part of their annual commercial general liability filing in a state. The actuary has calculated the indicated rate change for each class using the loss ratio method and the following information as of December 31, 2017:

Class	CY 2015-2017 Collected Premium	On-Level Factors	Premium at Current Rate Level	AY 2015-2017 Reported Loss and ALAE	AY 2015-2017 Reported Loss and ALAE Ratio
A	$1,212,729	1.037	$1,257,600	$505,300	40.2%
B	$995,661	1.037	$1,032,500	$1,134,500	109.9%
C	$622,179	1.037	$645,200	$201,400	31.2%
Total	$2,830,569	1.037	$2,935,300	$1,841,200	62.7%

Class	Indicated Change	Current Relativity	Indicated Relativity	Indicated Rel @ Base Class	Selected Rel @ Base Class	Relativity Change	Change with Off Balance
A	-35.9%	1.50	0.96	1.93	1.93	28.7%	-3.5%
B	75.2%	1.25	2.19	4.40	2.00	60.0%	19.9%
C	-50.2%	1.00	0.50	1.00	1.00	0.0%	-25.0%
Total	0.0%					33.4%	0.0%

- The company rates on class and territory only.
- Class and territory factors have been revised every year.
- The overall rate level is adequate.
- Premium trend is 0%.
- On-level factors reflect the overall historical rate changes for the state.

Discuss four potential shortcomings in this analysis.

8. 1. No use of credibility. The data by individual class may not be fully credible.
(In this case Class C has the least data, and its loss ratio is subject to the most random fluctuation.)
It would be a good idea to use no change from current as the complement of credibility and come up with a credibility weighted change in relativity.
2. Should do the analysis of class relativities together with the analysis of territory relativities.
Otherwise, the new class and territory relativities together may not accurately predict future losses.
The actuary is using univariate methods to separately select relativities for its two rating variables, but there may be significant distributional bias and/or correlation between these variables, so the actuary may be double counting the effect of these rating variables.
3. Premiums should be earned premiums rather than "collected premiums" so as to get a good match to reported losses.
4. Applying the same on level factor to each class has no effect on the indication. Rather one should put each class premium on the current rate level taking into account the changes in class and territory relativities that have occurred.
5. Balancing on the three years of premium does not reflect the latest mix of business by class.
Using the latest year for balancing would be better.
6. The current relativities should be first normalized to be with respect to average before working with them. With the use of credibility, normalizing would make a difference.
7. Losses are uncapped, so a single large loss may disrupt the entire analysis. Instead unusual large losses should be capped. (If possible a longer term average factor to load back in the effect should be estimated by class.)
8. It would have been better to develop the losses to ultimate or a common maturity level.
This would have an effect if the classes have different mixes of losses by AY.
9. It would have been better to trend the losses to a common level.
This would have an effect if the classes have different mixes of losses by AY.
10. The actuary selected a relativity of 2 for Class B, but the indicated relativity was much higher, at 4.4. Therefore, the rates for class B may be inadequate.
Comment: In their examples of classification ratemaking, Werner and Modlin neither trend nor develop the losses. "In this example, loss development and trend are assumed to have a negligible effect on the relativities and therefore have been ignored." However, that does not imply that in general they should be ignored.

9. (1.5 points) Given the following exposures and losses for states A, B and C:

State	Class	Exposure	Losses	Pure Premium
A	1	200	7,000	35.00
	2	225	12,000	53.33
	Subtotal	425	19,000	44.71
B	1	250	10,000	40.00
	2	300	17,000	56.67
	Subtotal	550	27,000	49.09
C	1	300	14,000	46.67
	2	350	20,000	57.14
	Subtotal	650	34,000	52.31
All	1	750	31,000	41.33
	2	875	49,000	56.00
	Total	1,625	80,000	49.23

a. (1 point) Calculate the complement of credibility for state A, class 1 using Harwayne's method.
b. (0.5 points) Evaluate an alternative complement of credibility for this company.

9. (a) The pure premium for State B if it had the same mix of exposures as State A:
{(40.00)(200) + (56.67)(225)} / (200 + 225) = 48.83.
Adjusted relativity for Class 1 in State B: 40.00/48.83 = 0.8192.
The pure premium for State B if it had the same mix of exposures as State A:
{(46.67)(200) + (57.14)(225)} / (200 + 225) = 52.21.
Adjusted relativity for Class 1 in State B: 46.67/52.21 = 0.8939.
Weighted average of State B and State C adjusted relativities for Class 1:
{(250)(0.8192) + (300)(0.8939)} / (250 + 300) = 0.8599.
To get the complement of credibility, now multiply by the overall pure premium for State A:
(0.8599)(44.71) = **38.45**.
(b) 1. The overall data of Class 1 pure premium, which is equal to 41.33
Advantages:

- Relatively accurate because there are more data than just state A.
- Has a logical relationship with Class 1 in state A.
- Data is readily available
- Obviously easy to compute.

Disadvantages:

- Not independent with the subject experience (state A) because it includes the information from state A.
- Biased, since unlike Harwayne's method it does not adjust for the different average cost levels by state.

2. One could use the pure premium for Class 1 in State A for all other insurers.
This complement is accurate since it is based on a larger volume of data.
This complement is independent, since we have excluded the data from this insurer.
It is very likely to be biased, since different insurers have different underwriting standards and different reserving procedures. (The data use to make class rates is usually not mature.)
It is logically related to the data for this insurer.
Data by class for all insurers combined may be available from a rating bureau.
If data for all insurers is available, one can easily subtract out the data for this insurer and then compute the pure premiums for all other insurers.
3. One could use the current relativity compared to average for Class 1 in State A; one could convert to a pure premium by multiplying by the total pure premium for State A.
This complement is accurate, since via credibility the previous relativity is based to some extent on the pure premiums compared to average for Class A over many years.
This complement is independent to the extent there is no overlap between the years of data used in the previous and current classification rate reviews.
It is unbiased. It has a logical relationship.
Data is readily available. Easy to compute.
Comment: The first solution to part (b) is from the sample solutions provided by the CAS.
Frank Harwayne developed his method for Workers' Compensation insurance, where there are many classes and states provide different Workers' Compensation benefits.

10. (2 points) Given the following censored loss data:

	Policy Limit $100,000		Policy Limit $250,000	
Size of Loss	Claims	Losses ($000s)	Claims	Losses ($000s)
X <= $50,000	600	21,522	700	27,468
$50,000 < X <= $100,000	550	46,200	500	39,900
$100,000 < X <= $250,000			200	35,620

Basic Limit is $50,000.

a. (0.75 points) Calculate the increased limits factor for a limit of $100,000.
b. (1.25 points) Calculate the increased limits factor for a limit of $250,000.

10. (a) The basic limit losses are:
21,522,000 + (550)(50,000) + 27,468,000 + (500 + 200)(50,000) = 111,490,000.
The losses limited to 100,000 are:
21,522,000 + 46,200,000 + 27,468,000 + 39,900,000 + (200)(100,000) = 155,090,000.
$E[X \wedge 50,000] = 111,490,000 / (600 + 550 + 700 + 500 + 200) = 43,722.$
$E[X \wedge 100,000] = 155,090,000 / (600 + 550 + 700 + 500 + 200) = 60,820.$
ILF for a limit of $100,000 is: 60,820 / 43,616 = **1.391**.
Alternately, ILF for a limit of $100,000 is: 155,090,000 / 111,490,000 = **1.391**.
(b) We can not use the data from the policies with a $100,000 limit in order to get the limited expected value at $250,000, since there is no way to know what the losses would have been with instead a $250,000 limit.
For the layer from 100,000 to 250,000 we only consider policies with limits of $250,000.
Using the data from the policies with $250,000 limit:
$E[X \wedge 100,000] = (1000)\{27,468 + 39,900 + (200)(100)\} / (700 + 500 + 200) = 62,406.$
$E[X \wedge 250,000] = (1000)\{27,468 + 39,900 + 35,620\} / (700 + 500 + 200) = 73,563.$
Thus I estimate: $E[X \wedge 250,000] - E[X \wedge 100,000] = 73,563 - 62,406 = 11,157.$
Using the results of part (a), $E[X \wedge 250,000] = 60,820 + 11,157 = 71,977.$
ILF for a limit of $250,000 is: $E[X \wedge 250,000] / E[X \wedge 50,000] = 71,977 / 43,722 =$ **1.646**.
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11. (3.75 points) Given the following information:

Accident Year / Calendar Year	Written Premiums ($000s)	Earned Premiums ($000s)	Projected Ultimate Claims ($000s)	General Expenses ($000s)	Commission, Brokerage, and Other Acquisition Expenses ($000s)	Taxes, Licenses, and Fees ($000s)
2014	49,000	43,000	39,000	3,250	4,200	350
2015	48,000	50,000	43,000	2,750	2,000	290
2016	51,500	49,000	47,000	1,900	1,750	200

Territory	In-force Premium ($000s)	Current Relativity	Loss Ratio	Claim Count
A	32,000	1.00	87%	930
B	14,000	1.22	105%	450
C	8,000	1.35	68%	78

24,000	Exposures required for full credibility standard
3%	Expected frequency per exposure
3.5%	Target underwriting profit provision
1,000	One time commission included in 2014 expense data ($000)

- All expenses are variable.
- There have been no rate changes in the past 5 years.
- There are no premium or expense trends.
- Territory A remains the base territory.

a. (1.5 points) Calculate the permissible loss ratio. Briefly justify the selected expense ratio.

b. (0.5 points) Calculate the indicated overall rate change.

c. (0.5 points) Calculate the credibility factor for each territory using the classical credibility approach.

d. (1.25 points) Calculate the indicated change to the base territory after revising the territory relativities and overall rate level.

11. (a) The General Expense Ratios have been declining.

CY	Earned Prem.	General Exp.	Ratio
2014	43,000	3,250	7.56%
2015	50,000	2,750	5.50%
2016	49,000	1,900	3.88%
Total	142,000	7,900	5.56%

I am hesitant to rely solely on the latest calendar year of expense data.
Therefore, I will select 4.5% for General Expenses, between the latest year and the three year total.
Exclude the one time commission from 2014.
The Acquisition plus Tax ratios have been declining.

CY	Written Prem.	Acquisition	Taxes	Ratio
2014	49000	3200	350	7.24%
2015	48000	2000	290	4.77%
2016	51500	1750	200	3.79%
Total	148,500	6,950	840	5.25%

I am hesitant to rely solely on the latest calendar year of expense data.
Therefore, I will select 4.5%, between the latest year and the three year total.
Permissible loss and LAE ratio is: 1 - 4.5% - 4.5% - 3.5% = **87.5%**.
(b) The historical loss ratio (to earned premium) is: (39 + 43 + 47) / (43 + 50 + 49) = 90.85%.
Indicated overall rate change is: 90.85%/87.5% - 1 = **3.8%**.
(c) The standard for full credibility in terms of claims: (24,000)(3%) = 720.
Credibility for Territory A: **100%**.
Credibility for Territory B: $\sqrt{450/720}$ = **79.1%**.
Credibility for Territory C: $\sqrt{78/720}$ = **32.9%**.

(d) The weighted average loss ratio is:

$$\frac{(87\%)(32{,}000) + (105\%)(14{,}000) + (88\%)(8000)}{32{,}000 + 14{,}000 + 8000} = 88.85\%.$$

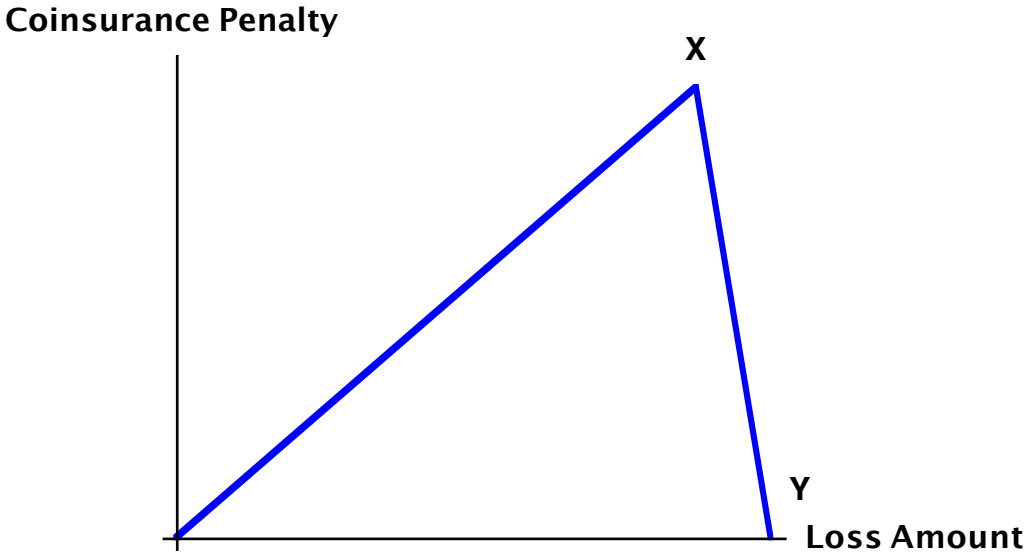
For example, 68%/88.85% - 1 = -23.47%. (32.9%)(23.47%) = -7.72%.
(1.35)(1 - 7.72%) = 1.246. 1.246/0.979 = 1.272.

Terr.	Premium	Loss Ratio	Indicated Change	Cred.	Cred. Wght. Change	Current Relat.	Cred. Wght. Ind. Rel.	Rel. wrt Base
A	32,000	87%	-2.08%	100.0%	-2.08%	1.00	0.979	1.000
B	14,000	105%	18.17%	79.1%	14.38%	1.22	1.395	1.425
C	8,000	68%	-23.47%	32.9%	-7.72%	1.35	1.246	1.272
Overall	54,000	88.85%						

The premium using the proposed relativities with no change in base rate would be:
32,000 + (14,000)(1.425/1.22) + (8000)(1.272/1.35) = 55,890.
Thus for no overall change we would need to multiply the base rate by: 54,000/55,890.
The indicated change to the base territory including a 3.8% overall rate change is:
(1 + 3.8%)(54,000/55,890) - 1 = **0.3%**.
Comment: In part (a), there are other reasonable selections, for example using the latest CY.
Your answers to parts (b) and (d) depend on your selections in part (a).
Part (d) is similar to the loss ratio example on Page E-4 in Basic Ratemaking.

12. (1.5 points) Given the following:

$500,000	Property value
$350,000	Insured value
80%	Coinsurance requirement



a. (1 point)
Calculate the loss amount and coinsurance penalty at point X on the coinsurance penalty chart.
b. (0.25 point) Calculate the loss amount at point Y on the coinsurance penalty chart.
c. (0.25 points) Describe the consequence for an insured at loss amounts greater than Y.

12. (a) The maximum coinsurance penalty is for a loss equal to the insured value of $350,000.
The coinsurance requirement is: (80%)(500,000) = 400,000.
For a loss of 350,000, the payment is: (350,000) (350,000 / 400,000) = 306,250.
The coinsurance penalty is: 350,000 - 306,250 = $43,750.
Thus X = (**$350,000, $43,750**).
(b) The coinsurance penalty declines to zero at the coinsurance requirement of $400,000.
Thus the loss amount for Y is $400,000.
(c) For loss amounts greater than the coinsurance requirement of $400,000, the coinsurance penalty remains zero. The payment is capped at the insured value of $350,000; the insured will have to pay the amount of the loss above $350,000 themselves.

13. (1.5 points) Given the following information:

1,000	Projected Fixed Expenses ($000)
7,000	Projected Losses ($000)
2,000	Projected LAE ($000)
40	Projected Exposures (000)
1.5	Expected Exposures per Policy
15%	Variable Expense Ratio
5%	Target Underwriting Profit %

- The company charges a fixed expense fee per policy written.

a. (1 point) Calculate the indicated premium for a policy with four exposures.

b. (0.5 point) Describe a scenario where the company would prefer the pure premium method to the loss ratio method.

13. (a) The average policy has 1.5 exposures.
Thus projected policies are: 40,000/1.5 = 26,667.
The indicated expense fee per policy is: (1,000,000/26,667) / (1 - 15% - 5%) = $46.87.
Indicated variable rate per exposure is: {(7000 + 2000) / 40} / (1 - 15% - 5%) = $281.25
Since the expense fee is per policy, the indicated premium for 4 exposures is:
(4)($281.25) + 46.87 = **$1172**.
(b) 1. If the company has no rate change data available and thus cannot put premiums on-level, then the loss ratio method cannot be used, and thus the pure premium method is preferred.
2. Pure premium method is preferable where on-level premium is difficult to calculate.
Therefore, for commercial lines where individual risk rating adjustments are made to individual policies, it is more appropriate to use the pure premium method if possible.
For private passenger automobile insurance, it may be difficult to quantify the effects of many changes to rating variables from the experience period to the current rate manual, making it difficult to estimate on-level premiums and thus to use the loss ratio method.
3. The loss ratio method cannot be used for a new line or new insurer, thus the pure premium method is preferred. One can estimate a pure premium from that for a similar line of business or from data for insurers already writing that line of business.