Solutions to the Fall 2019 CAS Exam Five

(Only those questions on Basic Ratemaking)

There were 25 questions worth 53.5 points, of which 15 were on ratemaking worth 31.25 points. (Question 7 included the use of Bornhuetter-Ferguson method, also covered under reserving, thus the ratemaking/reserving split was about 30.75 points versus 22.75 points.)

This exam used the paper-and-pencil format of exam administration.

The exam and examiner's report are available from the CAS.
The solutions and comments are solely the responsibility of the author.

(Incorporating what I found useful from the CAS Examiner's Report)

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Where each ratemaking question would go in my study guide:[1]

1. Section 4
2. Section 5

3. Section 12
4. Section 16

5. Section 6
6. Section 7

7. Section 8
8. Section 17

9. Section 9
10. Section 10

11. Section 10
12. Section 12

13. Section 11
14. Section 11

15. Section 15

[1] Some questions rely on ideas in more than one section of my study guide. I have chosen the best place to put a question in my opinion.

1. (1.75 points) Given the following quarterly exposure information:

Calendar Year and Quarter	Written Exposures	Earned Exposures
2017 Q 1	100	5.00
2017 Q 2	450	247.50
2017 Q 3	400	427.50
2017 Q 4	100	52.50
2018 Q 1	125	53.75
2018 Q 2	550	528.75
2018 Q 3	475	562.50
2018 Q 4	30	59.00

- The company started writing business on January 1, 2017.
- The company stops writing business on December 31, 2018.
- The quarterly earnings pattern was set by analyzing historical experience across the industry and is not uniform.
- All policies are annual.
- All policies are written on the first day of the quarter.
- There are no policy cancellations and no mid-term adjustments.

a. (0.5 point) Calculate the 2017 policy year earned exposures as of March 31, 2018.

b. (0.25 point) Calculate the in-force exposures as of May 31, 2018.

c. (0.5 point) Calculate the calendar year 2018 unearned exposures.

d. (0.5 point) Calculate the calendar year 2019 quarter 1 earned exposures.

1. (a) We can cumulate the earned exposures through March 31, 2018:
5 + 247.5 + 427.5 + 52.5 + 53.75 = 786.25.
However, this includes a contribution from policies written in the first quarter of 2018.
Based on the contribution of policies written in the first quarter of 2017 to the earned exposures in the first quarter of 2017, I estimate the similar contribution of 2018 Q1 as: (125)(5/100) = 6.25.
Thus, the 2017 policy year earned exposures as of March 31, 2018 are: 786.25 - 6.25 = **780**.
(b) This includes 2017 Q3, 2017 Q4, 2018 Q1, 2018 Q2:
400 + 100 + 125 + 550 = **1175**.
(c) Assume that they meant to ask for the unearned exposures as of the end of 2018.
Given that the insurer started writing business on January 1, 2017, we can add up all of the written exposures by quarter and subtract the sum of all of the earned exposures by quarter, through the end of 2018:
2230 - 1936.5 = **293.5**.
(d) We need to figure out how the written exposures in a quarter are earned over time.
Each annual policy (written at the beginning of a quarter) is earned over the four quarters it is in effect.
It turns out the given table is based on the following.
For each policy, regardless of in which quarter it is written, it earns 5% in Q1 of a year, 45% in Q2 of a year, 45% in Q3 of a year, and the remaining 5% of its written exposures in Q4 of a year.
There would be contributions to Q1 of 2019 earned exposures from the written exposures in each of the last three quarters of 2018, since annual policies written in these three quarters are still in effect during the first quarter of 2019. (There is no new business written in 2019.)
These policies each contribute 5% of their written exposures to Q1 of 2019 earned exposures:
(5%)(550 + 475 + 30) = **52.75**.
Comment: A question that covers a situation not discussed in any detail in the syllabus reading. In my opinion, it was totally unfair to expect students to infer what the underlying earnings pattern was under exam conditions; personally I would not have been able to.
For example, 2018 Q1: (450 + 400 + 100 + 125)(5%) = 53.75.
For example, 2018 Q2: (400 + 100 + 125 + 550)(45%) = 528.75.
This earnings pattern is based on some kind of seasonality; for this line of insurance most of the incidents that result in claims must happen in the second and third quarters of a year.
Unearned exposures is not a useful concept for a calendar year as opposed to a policy year.
Nevertheless, I would have answered part (c) for CY18:
unearned exposures = written exposures - earned exposures =
(125 + 550 + 475 + 30) - (53.75 + 528.75 = 562.50 + 59) = 1180 - 1204 = -24.
This was not the solution given in the CAS Examiner's Report.

2. (1.75 points) Given the following policies for an insurance company:

Policy	Effective Date	Expiration Date	Written Premium
A	March 1, 2017	February 28, 2018	1,200
B	June 1, 2017	November 30, 2017	1,500
C	July 1, 2017	June 30, 2018	2,000
D	October 1, 2017	September 30, 2018	750
E	January 1, 2018	December 31, 2018	900
F	April 1, 2018	September 30, 2018	1,650
G	August 1, 2018	July 31, 2019	1,350

a. (0.25 point) Calculate the written premium for the fiscal year ending July 31, 2018.

b. (0.25 point) Calculate the in-force premium as of December 15, 2018.

c. (0.5 point)

Calculate the 2018 calendar year written premium if Policy C is cancelled on March 31, 2018.

d. (0.5 point) Discuss if it is appropriate for this insurer to estimate earned premium for the current year by averaging the in-force premium at the end of the current year and prior year.

e. (0.25 point) Identify one potential use of in-force premium other than estimating earned premium.

2. (a) We are looking for policies written between August 1, 2017 and July 31, 2018: D, E, F.
750 + 900 + 1650 = **3300**.
(b) Policies E and G are in force on December 15, 2018. 900 + 1350 = **2250**.
(c) Policies E, F, and G are written in 2018, with total premium of: 900 + 1650 + 1350 = 3900.
Canceling Policy C on March 31, 2018 results in negative written premium of: 2000/4 = 500.
3900 - 500 = **3400**.
(d) If one has only annual policies and a large book of business, then the earned premium would be approximately equal to the average of the in-force premium at the end of the current year and prior year. However, since some of the policies are six-month and we have a small book of business, this approximation does not hold.
(e) "As in-force premium is the best estimate of the company's mix of business as of a given date, the most recent in-force premium is often used to measure the impact of a rate change on an existing portfolio of customers."
Alternately, In-force premium can be used (with caution) to monitor the size of a book of business over time.
Also from the CAS Examiner's Report:

- Determine current potential for loss.
- In-force premium could be used to estimate how much reinsurance to purchase.

Comment: A fiscal year is a generalization of a calendar year, with a starting date other than January 1.
Basic Ratemaking does not discuss averaging in-force premiums in order to approximate the earned premium. Therefore, in my opinion, it was unfair to expect students to answer part d under exam conditions; this is the basic ratemaking exam.
Graph the in-force premium from time zero to one. If the insurer has only annual policies, then the earned premium is the area under this curve. With a large book of business, the area under this curve from zero to one is approximately equal to the average of the values at times zero and one. (This approximation is exact if the curve of the in-force premium is a straight line.)
Similarly if the insurer has only six-month policies, then the earned premium would be twice the area under this curve. Thus the earned premium would be approximately the sum of the in-force premiums at the end of the current year and prior year.

3. (3 points) Given the following information:

Current Rate Review	
Number of Exposures	20,000
Indicated Rate Change before credibility	7.9%
Projected Frequency	3.0%
Annual Loss Trend	-1.0%
Annual Premium Trend	1.5%
Target Effective Date	January 1, 2019

Prior Rate Review	
Indicated Rate Change	8.0%
Implemented Rate Change	3.5%
Effective Date	January 1, 2017

Normal Distribution Table	
p	z(p)
0.800	0.842
0.850	1.036
0.900	1.282
0.950	1.645
0.975	1.960
0.990	2.326

- The loss experience is considered fully credible if there is a 90% probability that the observed experience is within 2.5% of its expected value.

a. (2.25 points) Calculate the credibility-weighted indicated rate change using the classical credibility approach and trended present rates as the complement of credibility.

b. (0.75 point) Identify three other complements of credibility appropriate for first dollar ratemaking.

3. (a) P = 90%. $\Phi(1.645) = (1+P)/2. \Rightarrow y = 1.645$. k = 2.5%.

$n_0 = (y/k)^2 = (1.645/0.025)^2 = 4330$ claims.

To convert the standard for full credibility to number of exposures, divide by the mean frequency: 4330/0.03 = 144,333 exposures.

$Z = \sqrt{20,000/144,333} = 37.2\%$.

There are two years between the current and prior effective dates.

Thus the complement of credibility is: $\frac{1.080}{1.035}\left(\frac{0.99}{1.015}\right)^2 = 0.9927$.

Credibility weighted: (32.7%)(1.079) + (1 - 32.7%)(0.9927) = 1.0248. $\Leftrightarrow$ **2.5% rate increase**.

(b) Loss costs of a larger group that includes the group being rated
Loss costs of a larger related group
Rate change from the larger group applied to present rates
Harwayne's method
Competitors' rates

Comment: Assume one desires that the chance of being within ±k of the mean to be at least P, then $n_0 = \frac{y^2}{k^2}$, where y is such that $\Phi(y) = \frac{1+P}{2}$. n_0 is what Basic Ratemaking has for the standard for full credibility in terms of number of claims. In order to convert to a standard for full credibility in terms of exposures, divide n_0 by the mean frequency.

4. (2.5 points)

Given the following information for an insurance company that sells claims-made policies:

- Exposure levels are constant.
- Loss costs increase by 3% each report year.
- An equal number of claims are reported each year.
- All claims are reported within four years of occurrence.

	Loss Cost by Report Year Lag			
Report Year	0	1	2	3
2014	100	100	100	100
2015	103	103	103	103
2016	106	106	106	106
2017	109	109	109	109
2018	113	113	113	113

a. (0 .75 point) Demonstrate and briefly explain why a claims-made policy will cost less than an occurrence policy.

b. (1 point) Demonstrate and briefly explain whether a claims-made policy or an occurrence policy would be more underpriced if the actual loss cost trend by report year is 10%.

c. (0.75 point) Briefly describe one difference between occurrence policies and claims-made policies regarding each of the following:

i. Coverage trigger

ii. Loss development

iii. Investment income

4. (a) For example, a 2015 mature claims-made policy will have loss costs of:
103 + 103 + 103 + 103 = 412.
A 2015 occurrence policy will have loss costs of: 103 + 106 + 109 + 112 = 430 > 412.
This was the case, since the claims-made policy pays for older accidents on average than the similar occurrence policy, and here inflation is positive.
(b) If the actual loss cost trend by report year is instead 10%:

	Loss Cost by Report Year Lag			
Report Year	0	1	2	3
2014	100	100	100	100
2015	110	110	110	110
2016	121	121	121	121
2017	133.1	133.1	133.1	133.1
2018	146.41	146.41	146.41	146.41

A 2015 mature claims made policy will have loss costs of:
110 + 110 + 110 + 110 = 440. An increase of 28 or 6.8% from before.
A 2015 occurrence policy will have loss costs of: 110 + 121 + 133.1 + 146.41 = 510.51.
This is an increase of 80.51 or 18.7% from before.
The increase is more for occurrence than claims-made; an occurrence policy would be more underpriced than a claims-made policy.
This was the case, since the later lags paid by the occurrence policy have been more affected by inflation than was the case for the claims-made policy, and here inflation is positive.
(c) i. The major difference between claims-made and occurrence coverage is that the coverage trigger is the date the claim is reported rather than the date the event occurs.
ii. For a claims-made policy, all of the claims are known at policy expiration. In contrast, for an occurrence policy there are still unreported claims at policy expiration. Thus while for claims-made we have only development of known claims, for occurrence we also have the emergence of yet to be reported claims. Thus there is more loss development for an occurrence than a claims-made policy.
iii. For an occurrence policy there is a delay waiting for some claims to be reported after policy expiration; this is not true for claims-made. Thus there is more time on average from when premiums are collected to when losses are paid for an occurrence policy than a claims-made policy. Therefore, there is more opportunity to earn investment income for an occurrence policy than a claims-made policy.

5. (1.5 points)
Given the following to be used in developing a rate indication effective January 1, 2021:
- All policies are annual.
- Rates are expected to be in effect for one year.
- The selected annual loss trend is 2%.

a. (0.5 point) Calculate the loss trend factor applied to losses from accident year 2018.
b. (0.5 point) Calculate the loss trend factor applied to losses from policy year 2018.
c. (0.5 point) Explain why trending and developing losses do not result in overlapping adjustments.

5. (a) Since the new rates are expected to be in effect for one year, the average date of writing under the new rates is January 1, 2021 plus 6 months: July 1, 2021.
Since policies are annual, the average date of accident under the new rates is 6 months later or January 1, 2022. Thus the trend period is from July 1, 2018 to January 1, 2022, or 3.5 years.
$1.02^{3.5}$ = **1.072**.

(b) The average data of writing for PY18 is July 1, 2018. Since policies are annual, the average date of accident for PY18 is 6 months later or January 1, 2019.
As in part (a), the average date of accident under the new rates is January 1, 2022.
Thus the trend period is from January 1, 2019 to January 1, 2022, or 3 years. 1.02^3 = **1.061**.
Alternately, the average date of writing under the new rates is July 1, 2021.
The average data of writing for PY18 is July 1, 2018.
Thus the trend period is 3 years long. 1.02^3 = **1.061**.

(c) There is no overlap or double counting between the two adjustments when developing and trending losses. Trending losses will adjust the level of losses from the midpoint of the experience period to the midpoint of the exposure period. Developing losses will adjust them from an early report to the ultimate level.
Comment: Part (c) has been asked many times before, and will be asked again in the future.

6. (2.25 points)

a. (1 point)

Discuss whether there is a need to explicitly account for the following costs in primary ratemaking:

i. Proportional reinsurance

ii. Non-proportional reinsurance

b. (0.5 point) Identify two sources of investment income considered in the total profit provision.

c. (0.75 point) Briefly discuss whether trending is necessary for the following:

i. Variable expenses

ii. Fixed expenses when using the exposure-based projection method

iii. Fixed expenses when using the premium-based projection method

6. (a) i. No. Under proportional reinsurance, the same percent of premiums and losses are ceded; this does not affect the loss ratio.
Thus there is no need to consider proportional reinsurance in the ratemaking process.
ii. Yes. Under non-proportional reinsurance, the insurer pays the reinsurance premium and in return in specified situations can cede specified losses to the reinsurer. In ratemaking, one should include as an expense the net cost of such reinsurance: the reinsurance premiums paid minus expected reinsurance recoveries.
Also from the CAS Examiner's Report:
• Should be addressed explicitly because amounts are not proportional and will distort triangles and loss ratios.
(b) 1. On the cashflows between when premiums are collected and losses and expenses are paid. This is equivalent to the investment income earned on the assets held as unearned premium and loss reserves.
Another way to say the same thing is investment income earned on policyholder-supplied funds.
2. Investment income earned on the assets held as policyholder surplus (or capital).
Also from the CAS Examiner's Report:
• Stocks (including dividends and capital gains)
• Bonds (interest)
• Mutual funds
• Real estate
(c) i. One determines a variable expense percentage by comparing historical (variable) expenses to premiums. There is no need for an expense trend. (Implicitly the dollar provision for variable expenses increases with premiums.)
ii. One determines a dollar provision for fixed expenses (per exposure or per policy) based on historical (fixed) expense data. One needs to apply a (fixed) expense trend in order to bring this provision to the prospective level.
However, if fixed expenses are determined per exposure, and the exposure base in inflation sensitive, then if also the trends in exposures and fixed expenses are approximately the same, one need not trend the fixed expense provision.
iii. "In the Premium-based Projection Method, the fixed expense ratio is the fixed expenses divided by premium. Approaches for trending expenses vary by company. If the average expenses and average premium are changing at the same rate, then the fixed expense ratio will be consistent and no trending is necessary. However, some companies trend the fixed expense ratio, which implies that average fixed expenses are changing at a different rate than average premium. For the purpose of this text, the fixed expense provision calculated using that methodology is not trended."
In other words, if one assumes that on average fixed expenses increase at approximately the same rate as premiums, then there is no need to trend the fixed expense provision.

7. (4.5 points) Given the following data as of December 31, 2018:

Accident Year	Cumulative Reported Loss + ALAE ($000s) as of (months)		
	12	24	36
2016	3,440	4,107	4,522
2017	3,427	4,109	
2018	3,545		

Calendar Year	Earned Premium ($000s)	Fixed Expenses ($000s)
2016	10,500	1,155
2017	12,000	3,600
2018	12,500	1,500

Rate Change History	
Effective Date	Change
July 1,2017	5%
July 1,2018	2%

4%	Annual loss and ALAE trend
3%	Annual premium trend
60%	Expected Loss and ALAE Ratio
30%	Variable Expense Ratio
5%	Profit and Contingencies Provision
7%	ULAE Provision (as % of Loss and ALAE)
1.031	36-to-ultimate tail factor

- In 2017 the company implemented a new policy issuance system.
- Rates are in effect for one year.
- All policies are annual.
- Exposures are written evenly throughout each calendar year.

Calculate the indicated rate change for policies effective January 1, 2020 using the reported Bornhuetter-Ferguson technique for the last three accident years.

7. Compute link ratios and determine loss development factors to ultimate:

	12 to 24	24 to 36	36 to ultimate
	4107/ 3440 = 1.1939	4522/4107 = 1.1010	
	4109/ 3427 = 1.1990		
Selected	1.196	1.101	1.031
Factor to Ultimate	1.358	1.135	1.031

Apply the Bornhuetter-Ferguson technique:

Calendar Accident Year	Earned Premium ($000s)	Unreported Losses ($000s)	Ultimate Losses ($000s)
2016	10,500	(0.6)(10,500)(1 - 1/1.031) = 189	4522 + 189 = 4711
2017	12,000	(0.6)(12,000)(1 - 1/1.135) = 856	4109 + 856 = 4965
2018	12,500	(0.6)(12,500)(1 - 1/1.358) = 1977	3545 + 1977 = 5522

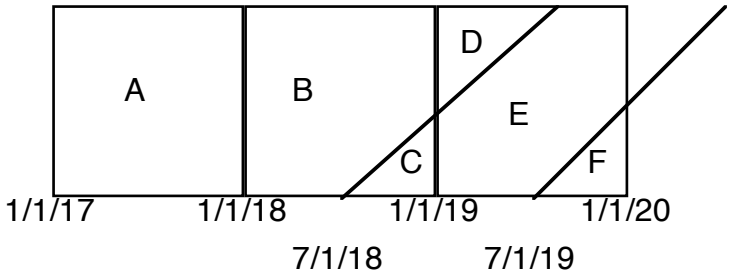
Compute Fixed Expense Ratios:

Calendar Year	Earned Premium ($000s)	Fixed Expenses ($000s)	Expense Ratio
2016	10,500	1,155	11%
2017	12,000	3,600	30%
2018	12,500	1,500	12%

In 2017 the company implemented a new policy issuance system, thus 2017 is not typical of what we expect in the future. On the other hand, there will always be such one time expenses which have to be paid for somehow.
Thus I will select a fixed expense ratio of 13%, somewhat higher than the other two years.
(There are other reasonable selections.)

The average date of writing under the new rates is July 1, 2020.
Since there are annual policies, the average date of accident is 6 months later or January 1, 2021.
Thus the trend period from AY18 is 2.5 years long.

Date	Rate Level Change	Rate Level Index
Prior		1.000
July 1, 2017	5%	1.050
July 1, 2018	2%	(1.05)(1.02) = 1.071



Area A = 1. Area C = Area D = Area F = $(1/2)(1/2)^2 = 1/8$. Area B = 7/8. Area E = 6/8.

Average Rate Level CY17 = 1.

Average Rate Level CY18 = (1)(7/8) + (1.05)(1/8) = 1.00625.

Average Rate Level CY19 = (1)(1/8) + (1.05)(6/8) + (1.071)(1/8) = 1.046375.

On Level Factors: 1.071/1 = 1.071, 1.071/1.00625 = 1.064, 1.071/1.06375 = 1.024.

Calendar Year	Earned Premium ($000s)	On-Level Factor	Premium Trend	On-Level Trended Premium ($000s)
2016	10,500	1.071	$1.03^{4.5}$	12,845
2017	12,000	1.064	$1.03^{3.5}$	14,160
2018	12,500	1.024	$1.03^{2.5}$	13,782

Calendar Accident Year	Ultimate Losses ($000s)	Loss Trend	Ultimate Trended Losses ($000s)	On-Level Trended Premium ($000s)	Loss Ratio
2016	4711	$1.04^{4.5}$	5620	12,845	43.8%
2017	4965	$1.04^{3.5}$	5696	14,160	40.2%
2018	5522	$1.04^{2.5}$	6091	13,782	44.2%
Total			17,407	40,787	42.7%

I will select the loss ratio of 42.7% based on all years combined.

$$\frac{(42.7\%)(1.07) + 13\%}{1 - 30\% - 5\%} = 0.903. \Leftrightarrow \textbf{9.7\% rate decrease}.$$

8. (1.75 points) An insurer's retention model predicts the following:

	Retention Ratio	
% Change in Premium	First Renewal	Second Renewal
0%	85%	0%
5%	75%	0%

The following information is known:

0.00%	Discount rate
$0	Fixed expenses
$0	Implementation costs
$1,000	Premium per policy
$800	Loss & LAE per policy

- Senior management will consider only the following at renewal:
  - i. No rate change
  - ii. +5% rate change

a. (1.25 points) Select the rate change that will maximize the insurer's profit.

b. (0.5 point) Briefly evaluate the selected rate change with respect to the *Statement of Principles Regarding Property Casualty Insurance Ratemaking*, citing one relevant principle.

8. (a) Assume for example 100 policies.
With no rate increase, 85 policies renew.
The insurer expects profits of: (85)(1000 - 800) = $17,000.
With an 5% rate increase, 75 policies renew.
The insurer expects profits of: (75)(1050 - 800) = $18,750.
Thus the 5% increase will maximize the insurer's profit.
Alternately, consider both the current policy and the first renewal.
Again assume 100 policies.
Then under the current policies the insurer profit is: (100)(1000 - 800) = 20,000.
With no rate increase, 85 policies renew.
The insurer expects profits of: 20,000 + (85)(1000 - 800) = $37,000.
With an 5% rate increase, 75 policies renew.
The insurer expects profits of: 20,000 + (75)(1050 - 800) = $38,750.
Thus the 5% increase will maximize the insurer's profit.
(b) “A rate is reasonable and not excessive, inadequate, or unfairly discriminatory if it is an actuarially sound estimate of the expected value of all future costs associated with an individual risk transfer.”
Here the loss and alae are $800. There is insufficient information given in order to determine whether $200 (20% of premium) is a reasonable provision for variable expenses plus profit and contingencies.
The market based manner in which management is deciding whether or not to increase the rate has nothing to do with the Principles Regarding Property/Casualty Insurance Ratemaking; it focuses totally on the maximizing the insurer’s profits, and not on the expected value of all future costs associated with the individual risk transfer.
Comment: An extremely simplified example of lifetime analysis.
In part (a), I have ignored any variable expenses, which are nowhere mentioned in the question.

9. (1.75 points)

a. (0.5 point) Briefly describe one similarity and one difference between the purposes of risk classification and individual risk rating.

b. (0.5 point) Briefly describe a situation for each of the following:

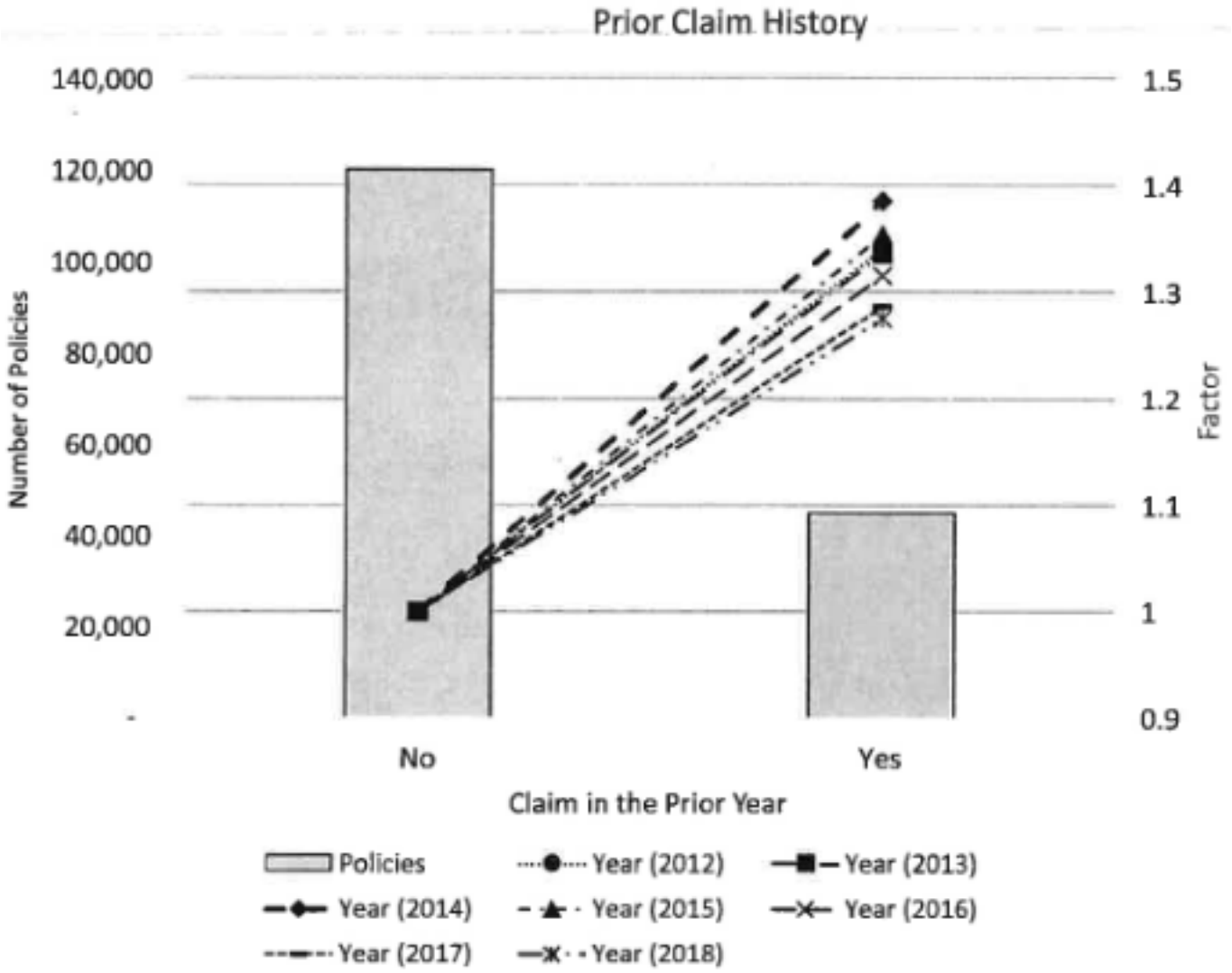
i. A classification rating plan is more appropriate than individual risk rating

ii. Neither a classification rating plan nor individual risk rating is necessary

c. (0.75 point) Briefly describe three reasons a rating characteristic might not be included in a classification rating plan.

9. (a) Both attempt to improve the estimate of the expected future loss costs of insureds using past experience.
Classification rating tries to identify a group of insureds with similar loss costs and then predict their future loss costs using the group's past experience, while individual risk rating tries to distinguish between the expected loss costs of otherwise similarly classified insureds using the past experience of each individual.
Classification rating tries to distinguish between different groups of insureds, while individual risk rating tries to distinguish between individual insureds within a group.
(b) i. Homeowners insurance, where each policy has limited data to which to apply individual risk rating and the data has a lot of randomness from year to year, but there are many good classification variables to use to distinguish the expected loss costs of insureds.
Alternately, private passenger automobile insurance, where each policy has limited data to which to apply individual risk rating, but there are many good classification variables to use to distinguish the expected loss costs of insureds.
Also from the CAS Examiner's Report:
• When the company is covering all small risks.
• Many risks with similar characteristics.
• When rating homogeneous groups of risks.
ii. Social Insurance programs, such as Social Security.
Alternately, when the insurance premiums are very small, for example when buying one trip travel insurance.
Also from the CAS Examiner's Report:
• Rate set by state regulators.
• If expected loss costs are the same for every exposure.
• There is no competition in the market and all insureds appear to be equally risky.
(c) 1. Too expensive to implement (compared to the identified difference in loss cost)
2. Illegal to use
3. The resulting relativities are not significantly different than unity
4. It is not verifiable; it can be easily manipulated by insureds or agents.
5. One can not create objective definitions of the different levels of the rating characteristic.
6. It is highly correlated with already existing rating characteristics
7. Privacy Concerns: it may require insureds to disclose very personal information.
Also from the CAS Examiner's Report:
• No historical precedent.
• No causality relationship with expected loss.
• It may not be controllable by the insured.
• If the use of this rating variable were to make insurance unaffordable.

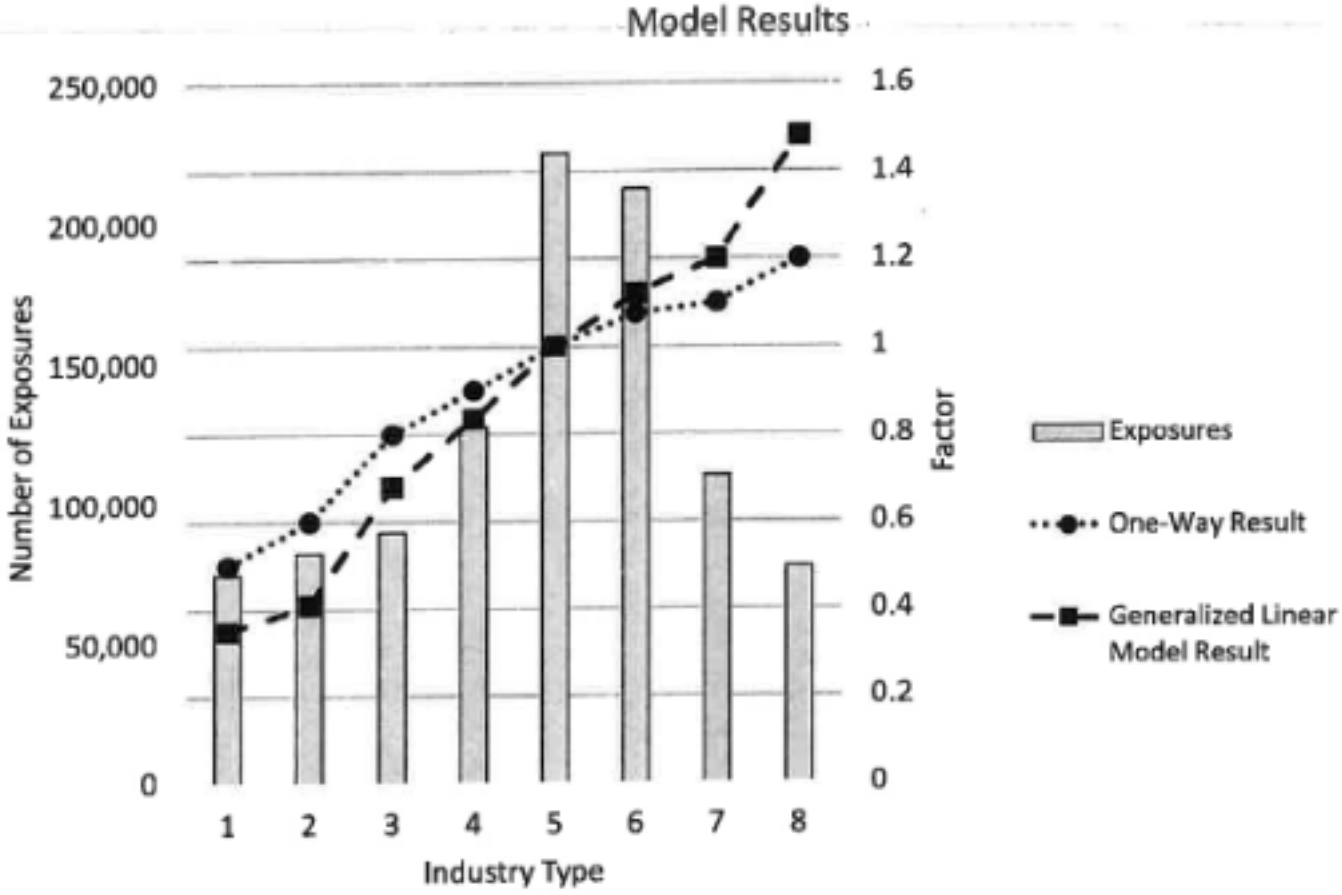
10. (1.75 points) The following graph shows pure-premium relativities produced by a generalized linear model (GLM).
The variable indicates whether the risk has had a claim in the most recent prior year or not.



a. (0.5 point) Describe the type of test for which the above graph is used.
b. (0.25 point) Briefly state the conclusion that can be drawn from the above graph, using the test described in part a.
c. (1 point) Describe two other tests to consider when evaluating the inclusion of this variable in the model.

10. (a) We are testing whether the results of the model are consistent over time.
The test is to verify that the relativities are stable when you examine them across many individual years.
(b) The results of the model appear to be relatively consistent by year. Thus it reasonable to use the model, at least based on this test.
(c) 1. A graph for all years combined with error bars, in order to see if the relativity for "yes" is significantly different than one.
2. Do the model results make sense? (In this case, it makes sense that those who had a claim previously have a higher expected future expected claim frequency.)
3. Use a Chi-Square test to compare the model with the variable to an otherwise similar model without the variable.
4. A comparison of the model predictions and the actual results on a hold-out set of data.
Comment: See Figure F.2 in Appendix F of Basic Ratemaking.

11. (1.25 points) The graph below shows frequency model results for industry types.
The-same data and assumptions are used in both models.



a. (0.25 point) Briefly discuss why the relativity for both line graphs match for industry 5.
b. (0.5 point) Explain why the models in part a. above produce different results.
c. (0.5 point) Describe how the use of one-way results might impact profitability for an insurance company.

11. (a) Industry Type 5 was chosen as the base level, which has a relativity of one by definition.
(b) The GLM takes into account both correlation of exposures and interaction of effects with other variables, while the one way model does not.
(c) Assume that the GLM is a more accurate estimate of future frequency than the one way model. Also assume that a competitor makes use of the GLM results to set its rates.
Then this insurer will be subject to adverse selection. This insurer will attract insureds in Industry Types 6 to 8, which it has underpriced. This insurer will lose insureds in Industry Types 1 to 4, which it has overpriced. Thus this insurer will lose profitable business and gain unprofitable business; its financial situation will continue to deteriorate unless it adopts relativities that better match the expected results.
Comment: Industry Type 5 has the most exposures, which is common for the chosen base level.

12. (1.75 points) Given the following information:

		State A		Countrywide	
Vehicle Type	Vehicle Use	Exposures	Losses ($)	Exposures	Losses ($)
Car	Pleasure	2,500	500,000	30,000	6,600,000
Car	Work	1,000	500,000	25,000	16,250,000
Truck	Pleasure	0	0	40,000	14,400,000
Truck	Work	3,000	300,000	50,000	8,500,000

• An actuary is developing a pure premium estimate for trucks used for work in state A.
• The actuary is using experience from all other states as a complement of credibility.
a. (1.25 points) Calculate the complement of credibility using Harwayne's Method.
b. (0.5 point) Briefly describe two advantages of using the current rate as a complement of credibility instead of calculating the complement with Harwayne's method.

12. Assume that the given countrywide data includes State A; we need to subtract out the State A data from the given Countrywide data.
Compute the pure premiums by class for the other states data:
6,100,000 / 27,500 = 221.82, 15,750,000 / 24,000 = 656.25,
14,400,000/40,000 = 360, 8,200,000 / 47,000 = 174.47.
Weight these pure premiums by the exposures in State A:

$$\frac{(2500)(221.82) + (1000)(656.25) + (0)(360) + (3000)(174.47)}{2500 + 1000 + 0 + 3000} = 266.80.$$

Compare the other states pure premium for trucks used for work to this weighted pure premium:
174.47/266.80 = 0.65394.
Apply this relativity to the State A overall pure premium of 1,300,000/6500 = 200:
(200)(0.65394) = **$130.79**.
This is the desired complement of credibility using Harwayne's Method.
(b) 1. The current rate is simpler to compute than Harwayne's method.
2. The current rate has a more logical relationship to the item of interest (the base statistic) than does Harwayne's method. It is easier to explain why one is using the current rate as the complement of credibility than it is to explain why one is using Harwayne's method.
3. Harwayne's method relies on an assumption that the relationship of the classes in other states is the same as it is in this state. The use of the current rate as the complement does not depend on this assumption.
Comment: Harwayne's method usually uses the individual data in the other states rather than the data for these other states all combined.

13. (1.75 points) Given the following information:

Policy Limit	Claims	% of Claims at Policy Limit
$50,000	145	100%
$100,000	550	60%
$200,000	875	40%

• All claim payments are either 50% of the policy limit or 100% of the policy limit.
• $50,000 is the basic limit.
Calculate the indicated increased limit factor for the $200,000 limit.

13. For the 50,000 limit policies, all 145 claim payments are of size $50,000.
For the 100,000 limit policies, 220 claim payments are of size $50,000, while the remaining 330 claim payments are of size $100,000.
For the 200,000 limit policies, 525 claim payments are of size $100,000, while the remaining 350 claim payments are of size $200,000.
The 100,000 limit policies have many losses of size 50,000, while the 200,000 limit policies have none. The 100,000 limit policies have 60% of losses of size 100,000 or more, while the 200,000 limit policies has all of its losses of size 100,000 or more.
Since the size of loss distributions are significantly different by limit, I will not try to combine their data in an attempt to estimate expected layers of loss.
The average severity for the $50,000 basic limit is: $50,000.
The average severity for the $200,000 limit policies is:
(60%)(100,000) + (40%)(200,000) = $140,000.
Making the usual assumption that the frequency does not vary by the limit, the indicated increased limit factor for the $200,000 limit is: 140,000 / 50,000 = **2.80**.
Here is a solution as per the CAS Examiner's Report (see my Comment):
We use data from all policies to estimate the first layer. Every claim is of size at least $50,000.
LAS($50K) = 50,000.
We use data from the policies with $100,000 and $200,000 limits to estimate the next layer.
The 220 claims of size $50,000 contribute nothing to this layer.
The 330 + 875 = 1205 claims of size at least $100,000 each contribute $50,000 to this layer.
LAS($50K xs $50K) = (1205)($50,000) / (550 + 875) = $42,281.
We only use data from the policies with a $200,000 limit to estimate the next layer.
The 525 claims of size $100,000 contribute nothing to this layer.
The 350 claims of size $200,000 each contribute $100,000 to this layer.
LAS($100K xs $100K) = (350)($100,0000 / 875 = $40,000.

ILF for the $200,000 limit is: $\frac{\$50,000 + \$42,281 + \$40,000}{\$50,000} = \mathbf{2.65}$.

Comment: A very unusual set of data!
The CAS Examiner's Report required one to use data from multiple policy limits in order to estimate Layer Average Severities. This is the technique shown in Basic Ratemaking, applied to a different type of data set. (See Exhibit 11.6 and following in Basic Ratemaking.) However, it relies on an assumption that the underlying unlimited size of loss distribution does not vary by policy limit purchased. As discussed in my solution, this is not appropriate in the case of this given data.

14. (2 points) Given the following:

Home Value	$300,000
Insured Value	$250,000
Amount of Loss	$260,000

a. (1 point) Calculate the amount paid by the policyholder at the time of loss under the following:
   i. Coinsurance percentage of 80%
   ii. Coinsurance percentage of 90%
b. (0.5 point) Describe how coinsurance provisions promote equitable rates.
c. (0.5 point) Describe how coinsurance provisions promote adequate rates.

14. (a) i. Coinsurance requirement is: (80%)(300,000) = $240,000.
Since 250,000 > 240,000, the coinsurance requirement has been met.
Thus there is no coinsurance penalty.
However, the payment is capped at the insured value of $250,000.
Thus the policyholder pays: $260,000 - $250,000 = **$10,000**.
ii. Coinsurance requirement is: (90%)(300,000) = $270,000.
Since 250,000 < 270,000, the coinsurance requirement has not been met.
The payment is: ($260,000) (250/270) = $240,741.
Thus the policyholder pays: $260,000 - $240,741 = **$19,259**.
(b) In the absence of a coinsurance requirement, a rate that is adequate for full coverage will be inadequate for those who buy less than full coverage. Thus without a coinsurance clause, those who underinsure will be subsidized by those who insure their properties to value.
Coinsurance reduces claim payments to those who underinsure.
With a coinsurance clause, the proper rate is the same for otherwise similar insureds who buy the coinsurance requirement or less, resulting in equitable rates.
(c) Without the coinsurance clause, the insurer would have great difficulty charging underinsured properties an adequate rate, since such a rate would be excessive for properties insured to value.
With the coinsurance clause, the adequate rate is the same for properties that buy the coinsurance requirement or less.
Alternately, from the CAS Examiner's Report:
"Coinsurance provisions promote adequate rates by penalizing and thus discouraging underinsurance. Underinsured exposures are usually underpriced because policies are priced assuming full coverage."
"Coinsurance promotes adequate rates by incentivizing risks to insure to the appropriate value. Thus, the insurer will not be exposed to unknown levels of risk and can rate similar risks accordingly."

15. (2 points) A workers compensation annual policy for a large insured is expiring on January 1, 2020. The following changes have taken place at the insured since the policy was last issued on January 1, 2019:

• The insured has implemented a new job-specific training program to reduce the expected number of claims.

• The insured has doubled the number of their employees.

a. (0.5 point) Briefly describe how each of these changes could be accounted for in the final premium for the January 1, 2020 renewal policy.

b. (1 point) Describe how each of these changes are accounted for in the final premium on January 1, 2024 if the insured has no additional changes over the next four years.

c. (0.5 point) Describe one additional rating mechanism that would benefit the company if the insured has grown substantially larger and more operationally complex between January 1, 2020 and January 1, 2024.

15. (a) The increase in employees will result in increased payroll, which will be picked by the payroll audit(s). These revised exposures (by class and state) will be multiplied by the rates (by class and state) in order to get the final manual premium. These increased exposures will result in an increase in the manual premium from before. (This will in turn result in a larger premium discount.)
Assuming the insurer believes that the new job-specific training program will reduce expected claims frequency, then it can give the insured a schedule credit (of appropriate size.)
(b) Again the audited payrolls (exposures by class and state) will be used to get the manual premium. These exposures (by class and state) will be multiplied by the rates (by class and state) in order to get the final manual premium.
By this time, the effect of the job-specific training program should be reflected in the experience used to calculate this insured's experience modification; thus the schedule credit should no longer be given.
(c) If it is not already on one, the insured can go onto a retrospective rating plan.
The insured's premium would vary based on its experience on this policy, subject to a maximum and a minimum. The insured would be taking on more of its own risk.
Alternately, if it is not already on one, the insured can go onto a large deductible plan.
For example with a $100,000 deductible, the insurer would pay all losses, but the insured would have to reimburse the insurer for the first $100,000 of each loss; in exchange the insured's premium is lower. The insured would be taking on more of its own risk.
Alternately, if it is not already doing so, the insured can switch to a self-insured retention together with appropriate excess insurance. The insured would be taking on more of its own risk.
Alternately, assuming that this insurer writes this insured for lines besides workers compensation, if it is not already doing so, the insured can go onto composite rating. A single exposure basis is chosen for several different lines of insurance. Then the composite rate is based on the insured's recent experience.
Alternately, from the CAS Examiner's report:
"Premium discount since insured should be large enough that the fixed expense portion of the premium is a lower percentage of premium."
(Note that premium discounts based on expenses by size of insured are not an optional rating mechanism; this large insured would be big enough to get a premium discount both before and after the increase in the number of employees.)
Comment: The manual premiums would only double if the new employees matched up one-to-one with the current employees as to class and their payroll. For example, the mix by class of the new employees may differ from current. For example, the hourly salary and/or number of hours worked by the new employees might be different than the current employees.
Even if the employees stayed the same, the payrolls would vary from year to year due to inflation and variation in the number of hours worked.