Solutions to the
Fall 2018
<u>CAS</u> Exam <u>Five</u>

(Only those questions on Basic Ratemaking)

There were 24 questions worth 55 points, of which 12.5 were on ratemaking worth 28.75 points.
(Question 7b covered reserving, while the rest of Question 7 covered ratemaking.)

This exam used the paper-and-pencil format of exam administration.

(Incorporating what I found useful from the CAS Examiner's Report)

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1. (2.5 points) An insurer is considering changing the exposure base for boat owners line of business from boat-years to the insured value of the boat.

The insurer offers the following coverages for boat owners:

i. Liability coverage pays for damages to another boat or injuries of people not on
      the insured's boat.

ii. Physical damage coverage pays for damages to the insured's boat caused by common risks,
      such as sinking, fire, storms, theft, and collision.

Using three criteria for a good exposure base, evaluate the effectiveness of the proposed change in exposure base for both liability and physical damage coverages and provide a recommendation for the preferred exposure base.

1. 1. Proportional to Expected Loss:

For physical damage, the more expensive the boat the more monetary damage to the boat. While expected loss from physical damage coverage may not go up exactly proportional to the insured value of the boat, it is approximately the case, and thus this criteria is satisfied.

(Based on homeowners insurance and the existence of partial losses, I would expect that expected losses to go up somewhat less than linearly with insured value.)

The higher the insured value of the boat, the larger the boat is likely to be. Larger boats are able to do more damage to other boats and more likely to seriously injure passengers of those other boats. Thus expected liability losses should increase with insured value, however, it might not be close to proportional.

Insured value sort of satisfies this criterion; more so for physical damage than liability.

While two boats have twice the expected loss of one boat, all else being equal, the expected loss is very different for boats of very different value. Thus, boat-years does <u>not</u> satisfy this criterion.

2. Practical:

For physical damage coverages, the insurer already collects and uses the insured value of the boat. Thus using insured value would be neither difficult nor costly.

It would not be particularly subject to manipulation by the insured.

Insured value satisfies this criterion.

3. Historical Precedence:

Since it is not currently used as the exposure base, insured value violates this criterion.

There would be a lack of industry benchmarks. There would also be some difficulties in ratemaking for several years while moving to this new exposure base; this would be made easier since insured value is already collected for boats that buy physical damage coverage.

I would recommend staying with boat-years as the exposure base. This will avoid the disruption of changing the exposure base, for very little if any benefit. Insured value should continue to be used to rate physical damage coverages. If insured value is not being used to help rate liability coverages, nor something related like size of boat, then it would be worthwhile to start using insured value to help rate liability coverages.

<u>Comment</u>: There are many possible full credit answers.

2. (2 points) Given the following premium and loss information:

Premium Transactions					
Policy	Original Effective Date	Original Expiration Date	Transaction Effective Date	Full-Term Premium ($)	Notes
A	July 1, 2016	June 30, 2017	July 1, 2016	800	Start of New Policy
A	July 1, 2016	June 30, 2017	April 1, 2017	400	Additional Premium for Endorsement
B	April 1, 2017	March 31, 2018	April 1, 2017	1000	Start of New Policy
C	October 1, 2017	September 30, 2018	October 1, 2017	500	Start of New Policy
C	October 1, 2017	September 30, 2018	April 1, 2018	N/A	Policy Canceled

Loss Transactions			
Policy	Accident Date	Payment Date	Loss Payment ($)
A	October 1, 2016	October 15, 2016	500
A	January 1, 2017	January 15, 2017	200
B	October 1, 2017	January 15, 2018	500
C	January 1, 2018	January 15, 2018	750

• Each claim is closed on the payment date.

a. (0.75 point) Calculate the 2017 calendar year earned premium.

b. (1.25 points) Calculate the 2017 policy year loss ratio evaluated at December 31, 2018.

2. (a) Half of the original premium for policy A is earned during 2017.

All of the additional premium for policy A is earned during 2017, which is 400/4 = 100.

(400 is the <u>full</u> term premium.)

Three quarters of the premium for Policy B is earned during 2017.

One quarter of the original premium for Policy C is earned during 2017.

(The fact that Policy C is later cancelled in 2018, has no effect on this.)

800/2 + 100 + (3/4)(1000) + 500/4 = **1375**.

(b) Policy A does not contribute to Policy Year 2017.

As of December 31, 2018, the PY17 premium is: 1000 + 500/2 = 1250.

As of December 31, 2018, the PY17 losses are: 500 + 750 = 1250.

The loss ratio is: 1250/1250 = **100%**.

<u>Comment</u>: In part (b) we would usually take the ratio of reported losses to earned premium.

The question does not specify whether to use paid or reported losses and whether to use written or earned premium; fortunately in this case it makes no numerical difference.

3. (1.75 points) Given the following:

Effective Date	Overall Average Rate Change	Rate Per Exposure ($)	Class Factor X	Class Factor Y	Class Factor Z	Expense Fee ($)
January 1, 2016	0.0%	1,000	1.20	0.85	1.00	120
July 1, 2017	10.0%	1,112	1.20	0.85	1.00	120
October 1, 2017	0.0%	1,175	1.10	0.75	1.00	120
April 1 , 2018	1.0%	1,175	1.10	0.75	1.00	132

- All policies are semi-annual.
- Exposures are written uniformly throughout the year.
- Expense fee is a per exposure fee that is added in the last step of the rate calculation.

a. (0.75 point) Calculate the on-level factor for calendar year 2017 earned premium using the parallelogram method.

b. (0.5 point) Calculate the on-level factor for a policy effective on April 1, 2017 within Class Y using the extension of exposure method.

c. (0.5 point) Assess the appropriateness of using the parallelogram method to calculate indicated class factors using the loss ratio method.

3. (a)

Date	Rate Change	Rate Level Index
		1
July 1, 2017	10%	1.1
April 1, 2018	1%	1.111

Since policies are semi-annual, the line has a slope of 2:

12/31/17

    1/1/17   7/1/17
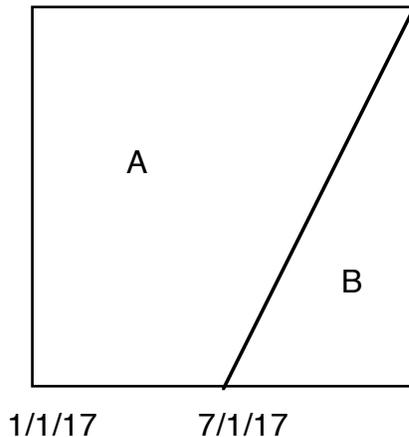
Area B = (1/2)(1/2)(1) = 1/4. Area A = 1 - 1/4 = 3/4.

Average rate level for CY17 earned premium is: (3/4)(1) + (1/4)(1.1) = 1.025.

OLF factor for CY17 earned premium is: 1.111/1.025 = **1.0839**.

(b) The current premium per exposure for Class Y is: (1175)(0.75) + 132 = 1013.25.

The April 1, 2017 premium per exposure for Class Y is: (1000)(0.85) + 120 = 970.

The ratio is: 1013.25/970 = **1.0446**.

(c) In general, one does <u>not</u> have enough detail to use the parallelogram method to put premiums by class on level in order to calculate indicated class factors using the loss ratio method.

Therefore, it is <u>not</u> appropriate to use the parallelogram method to calculate indicated class factors using the loss ratio method.

In this particular case, since the class factors change, unless one did a separate calculation for each class, the result of the parallelogram method would <u>not</u> be appropriate.

(An extra complication is that the class factors are not applied to the expense fee.)

<u>Comment</u>: One does not usually compute an "on-level factor" when using the extension of exposure method.

4. (1.5 points) Given the following information:

Report Year	Loss Costs by Report Year Lag ($)		
	0	1	2
2015	500	300	200
2016	525	330	210
2017	550	365	220

a. (0.25 point) Calculate the report year 2015 loss costs for a claims-made policy.
b. (0.25 point) Calculate the accident year 2015 loss costs for an occurrence policy.
c. (0.5 point) Compare the effect of an unexpected increase in underlying trend on the accuracy of
　　the pricing for a claims-made policy and an occurrence policy.
　　Briefly explain why one of the policies is impacted more than the other.
d. (0.25 point) Briefly explain why the risk of reserve inadequacy is reduced for
　　a claims-made policy relative to an occurrence policy.
e. (0.25 point) Briefly describe why the investment income earned from
　　claims-made policies is less than under occurrence policies.

4. (a) Assume this is a mature claims-made policy: 500 + 300 + 200 = **1000**.
(b) 500 + 330 + 220 = **1050**.
(c) An occurrence policy includes claims that will be reported many years in the future, while claims-made does not. For occurrence there is more time for inflation to increase these costs compared to known amounts. Thus, occurrence polices are more affected by an unexpected increase in underlying trend.
(d) Under claims-made, upon policy expiration there are no unreported claims. Under occurrence, upon policy expiration there are many unreported claims. Thus while under claims-made there is no pure IBNR reserve, under occurrence there is. Thus under occurrence the actuary has to estimate the number of unreported claims and their size. Thus there is more risk of reserve inadequacy for an occurrence policy than under a claims-made policy.
(e) Occurrence polices include the lag from time of occurrence to time of reporting, while claims-made policies do not. Thus there is more average time from when an occurrence policy is written to when a claim is paid, than there is for claims-made. Therefore, for claims-made there is less time for invested funds to earn investment income. Thus the investment income earned from claims-made policies is less than under occurrence policies.

5. (2 points) Given the following:

Accident Year	Incurred Loss and ALAE as of December 31,2017 ($000s)
2015	$15,000
2016	$8,000
2017	$2,000

Year Ending Quarter	Frequency	Severity ($)	Pure Premium ($)
March 31, 2014	0.055	18,200	1,001
June 30, 2014	0.054	18.000	972
September 30, 2014	0.056	18,100	1,014
December 31, 2014	0.058	18,300	1,061
March 31, 2015	0.058	18,500	1,073
June 30, 2015	0.059	19,000	1,121
September 30, 2015	0.062	19,200	1,190
December 31, 2015	0.063	19,500	1,229
March 31, 2016	0.065	19,750	1,284
June 30, 2016	0.066	19,885	1,312
September 30, 2016	0.066	20,000	1,320
December 31, 2016	0.068	20,250	1,377
March 31, 2017	0.069	20,445	1,411
June 30, 2017	0.070	20,882	1,462
September 30, 2017	0.069	21,000	1,449
December 31, 2017	0.065	21,250	1,381

Annual Exponential Trends			
# of Points	Frequency	Severity	Pure Premium
16	7%	5%	12%
12	6%	5%	11%
8	2%	4%	7%
6	0%	5%	5%
4	-7%	5%	-3%

- All policies are semi-annual.
- Rates are to be in effect for 2 years.
- There is no development after 36 months.
- An underwriting change went into effect on July 1, 2017,
    materially changing the composition of the book of business.
- A planned rate change will go into effect on January 1, 2019.

Calculate the projected 2015 accident year loss and ALAE to be used in the rate change analysis.

Justify any trend selections.

5. The average date of accident for AY15 is July 1, 2015.

The average data of writing under the new rates is January 1, 2019 + 1 year = January 1, 2020.

The average date of accident under the new rates is 3 months later (6 month policies): April 1, 2020.

Thus the trend period is from July 1, 2015 to April 1, 2020.

The severity trend is stable, so I will select 5%. There is a major change that occurs on July 1, 2017, so a two-step trend for frequency is best. I will select a 6% trend up until July 1, 2017, and then I will choose the -7% trend from July 1, 2017 and onward.

(15 million) (1.06^2) $(0.93^{2.75})$ $(1.05^{4.75})$ = **$17.4 million**.

Alternately, due to the change at July 1, 2017, I will select a two-step pure premium trend.

For the first period from July 1, 2015 to July 1, 2017 (2 years), I will use the all points average of 12%.

For the second trend, from July 1, 2017 to April 1, 2020 (2.75 years), I will select the 4 point trend of -3% in order to better reflect the impact of the underwriting change.

(15 million) (1.12^2) $(0.97^{2.75})$ = **$17.3 million**.

Alternately, for the quarter ending September 30. 2017, about half of the accidents should be from (6-month) policies written before the underwriting change on July 1, 2017, while half are from policies written after the change. For the quarter ending December 31. 2017, all of the accidents should be from (6-month) policies written after the underwriting change on July 1, 2017.

Thus the next to last quarter of data partially reflects the underwriting change, while the last quarter of data reflects the change. We note that the pure premiums for both of those quarters are significantly lower than previous quarters.

Prior to that we have a significantly high pure premium trend of 12%.

Somehow, I want to reflect the positive trend prior to the underwriting change, reflect the one-time impact of the underwriting change, and project what will happen in the future on the new book of business.

For the first piece, I will include one year and 10.5 months of pure premium trend at 12%, corresponding to July 1, 2015 to May 15, 2017.

For the second piece, I will include: 1381/1462 = 0.944.

(I would prefer not to rely on comparing two quarters of data.)

For the last piece I will use 5%, based on the severity trend, for 2 years and 4.5 months, corresponding to October 15, 2015 to April 1, 2020.

Projected AY2015 loss and ALAE:

(15 million) $(1.12^{22.5/12})$ (0.944) $(1.05^{28.5/12})$ = **$19.7 million**.

Comment: There are many reasonable possible answers.

The first 2 solutions are from the sample solutions provided by the CAS, while the final one is not. As always it would be very useful to be given more information.

Hopefully the elements of each of the trend series are at the same maturity level or have been adjusted to a common level of maturity.

6. (1.5 points) Two methods of deriving expense provisions in ratemaking include the
premium-based projection method and the exposure-based projection method.
a. (1 point) For each method, briefly describe how both fixed and variable expenses are treated.
b. (0.5 point) Briefly describe one shortcoming (or distortion) of each method.

6. (a) In both methods, expenses by category are split between variable and fixed.

In the premium-based projection method, both type of expenses are divided by premiums.

In the exposure-based projection method, variable expenses are divided by premiums while fixed expenses are divided by exposures; we then apply expense trend to the fixed expense pure premium.

(One divides by written premiums/exposures if expenses are incurred at the beginning of the policy period, and divides by earned premiums/exposures if expenses are incurred at the throughout the policy period.)

In both methods, in order to determine an indicated rate change or rate, fixed expenses go in the numerator while variable expenses go in the denominator.

For the premium-based projection method:

$$\text{Indicated Rate Change} = \frac{\text{Projected Loss \& LAE Ratio} + \text{Fixed Expense Ratio}}{1 - \text{Variable Expense Ratio} - \text{Profit \& Contingencies Provision}} - 1.$$

For the exposure-based projection method:

$$\text{Indicated Rate} = \frac{\text{Projected Loss \& LAE} + \text{Projected Fixed Expense}}{1 - \text{Variable Expense Provision} - \text{Profit \& Contingencies Provision}}$$

(b) Premium-based Projection Method:

1. The effect of rate changes. If there have been large rate increases or decreases, the provision for fixed expenses which was based on a ratio to premiums at a prior rate level, will not be appropriate.

2. The effect of a change in the average premium, for example due to Premium Trend.

If the average premiums increase at a faster rate than fixed expenses, then the needed fixed expenses as a percent of premium will be lower than it was in the historical data. Thus this method will overstate the needed provision of fixed expenses.

3. Misallocation of fixed expenses to state. More of countrywide fixed expenses will be allocated to states with higher average rates and less to states with lower average rates. This is unlikely to match the fixed expense needs by state.

4. If some expenses are truly fixed, then high premium policyholders would be paying more than they should in order to cover expenses.

Exposure-based Projection Method:

1. One should consider economies of scale. When there is a changing size of the book of business, economies of scale may affect the required average fixed expenses.

2. Some expenses could vary based on other policy characteristics such as for example new versus renewal business.

3. The method allocates fixed expenses to state based on exposures. By collecting more data at a finer level, a better allocation of expenses to state might be possible.

Comment: Either method can be distorted based on the split between the fixed and variable expenses, if the split is not approximately correct.

7. (5.75 points) Given the following information as of December 31, 2017:

Cumulative Reported Claim Counts as of (months)				Cumulative Reported Loss + ALAE ($) as of (months)			
Accident Year	12	24	36	Accident Year	12	24	36
2015	480	456	447	2015	7,200,000	8,208,000	8,850,600
2016	560	532		2016	8,120,000	9,256,800	
2017	590			2017	9,145,000		

$98,000	Expected reinsurance recoveries
$318,000	Cost of reinsurance (expected ceded premium)
3%	Expected annual exposure increase

Calendar Year	Earned Exposures
2015	14,000
2016	15,000
2017	17,000

5%	ULAE provision as a percent of loss and ALAE
3%	Annual pure premium trend
$21	Projected fixed expenses per exposure
15%	Variable expense ratio
10%	Profit provision
2%	Contingency provision
$950	On-leveled and projected earned premium per exposure

- Exposures are written evenly throughout each year.
- All policies are annual.
- There is no loss development or claim count development beyond 36 months.
- The reinsurance contract has a 12 month term length and an effective date of January 1, 2019.
- Rates are to be in effect for one year.
- Rate revision is planned to be effective April 1, 2019.

a. (1 point) Calculate the projected net reinsurance cost per exposure using a 12-month term for the reinsurance contract.

b. (2 points) Calculate the ultimate losses and ALAE for each accident year using an appropriate frequency-severity technique.

c. (1.75 points) Calculate the projected pure premium per exposure using even weights across the three accident years.

d. (1 point) Calculate the indicated rate change.

7. (a) Net Cost Reinsurance: 318,000 - 98,000 = $220,000.

Period from midpoint of latest year 2017 to the midpoint of the reinsurance contract: 2 years.

Project Exposures: (17,000) (1.03^2) = 18,035.

Projected Net Reinsurance Cost per Exposure: $220,000 / 18,035 = $12.20.

(b) Claim Count first to second report: (532 + 456) / (560 + 480) = 0.950.

Claim Count second to third report: 447/456 = 0.980.

AY16 estimated ultimate claim count: (532)(0.980) = 521.

AY17 estimated ultimate claim count: (590)(0.950)(0.980) = 549.

Severity			
Accident Year	12	24	36
2015	15,000	18,000	19,800
2016	14,500	17,400	
2017	15,500		

Severity first to second report: (18,000 + 17,400) / (15,000 + 14,500) = 1.200.

Severity second to third report: 19,800/18,000 = 1.100.

AY16 estimated ultimate severity: (17,400)(1.100) = 19,140.

AY17 estimated ultimate severity: (15,500)(1.100)(1.200) = 20,460.

AY15 ultimate losses (given): 8,850,000.

AY16 ultimate losses: (521)(19,140) = **9,971,940**.

AY17 ultimate losses: (549)(20,460) = **11,232,540**.

(c) The average date of writing under the new rates is: April 1, 2019 + 6 months = October 1, 2019.

Since we have annual policies, the average date of earning and accident under the new rates is:

October 1, 2019 + 6 months = April 1, 2020.

Thus the trend period from AY17 is 2.75 years.

AY	Exposures	Ultimate Loss & ALAE	Pure Premium	Pure Premium Trend	Trended P.P.
2015	14,000	$8,850,000	$632.14	1.1507	$727.43
2016	15,000	$9,971,940	$664.80	1.1172	$742.72
2017	17,000	$11,232,540	$660.74	1.0847	$716.69
Average					**$728.95**

(d) Indicated rate is: $\dfrac{(728.95)(1.05) + 21 + 12.20}{1 - 15\% - 10\% - 2\%}$ = $1093.97.

Indicated Rate Change is: $1093.97 / $950 - 1 = **15.15%**.

Comment: For part (a), see page B-8 of Basic Ratemaking.

In part (b), I used the first and also simplest method in Chapter 11 of

Estimating Unpaid Claims Using Basic Techniques.

8. (2.25 points) Given the following information:

$500	Current average premium
$600	Indicated average premium
$510	All competitors' average premium

a. (0.5 point) List two likely consequences of the company implementing the indicated rate.

b. (0.5 point) Briefly describe two factors that affect an insured's propensity to renew.

c. (0.75 point) The company has decided to not implement the indicated rate. List three non-pricing solutions the company could implement to ensure profitability does not deteriorate.

d. (0.5 point) Identify an issue with comparing one company's premium to another and briefly propose a solution to this issue.

8. (a) 1. The large 12% increase would cause many of its customers to shop around.
Since the average rate of its competitors is significantly lower, many of these customers will not
renew. (If better customers choose not to renew, this could lead to an increase in pure premiums.)
It will also be difficult for this insurer to attract new customers.
Thus this insurer's market share will decline.
2. The increase in rates should improve this insurer's loss ratio.
3. Some competitors might also raise their rates.
(b) 1. The prices charged by competitors. If insureds know another insurance company offers
the same product at a substantially lower price, they are likely to purchase the competing product.
2. Characteristics of the insured. For example, a young policyholder is more likely to shop and
subsequently change insurers than an older policyholder.
3. Time with the insurer. Insureds who have been the insurer longer are more likely to renew.
4. Overall cost of the product. If the insurance product is relatively cheap, then insureds are less likely
to spend time shopping for a cheaper product. On the other hand, if the cost of the product
represents a significant portion of disposable income, then insureds are more likely to compare
prices to determine any potential savings.
5. Rate increases. If the insured's rates have recently been increased, they are more likely to shop
around and thus are less likely to renew.
6. Type of insurer. Insurers who use independent agents have a lower renewal probability than
direct writers. Independent agents can switch the insured to another insurer in order to get a better
rate and/or a higher commission.
7. If the insured has had a major life event, for example: bought a new house, gotten married,
moved to a different state, retired, etc., then they are less likely to renew.
8. The place in underwriting cycle. During the hard market, commercial insureds are more likely to try
to renew, than they are during the soft market.
9. Customer satisfaction. Poor claims handling may cause existing insureds to be dissatisfied and
explore other options. Good loss control services may induce a Workers Compensation insured to
renew.
10. If a chief competitor has started an extensive advertising campaign, an insurer is more likely to
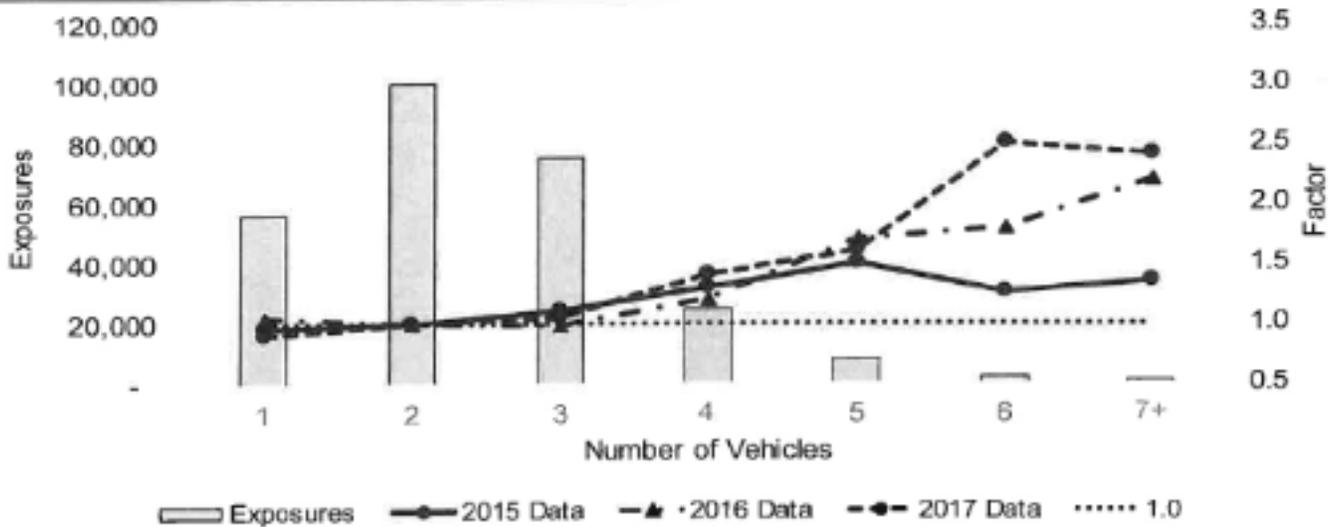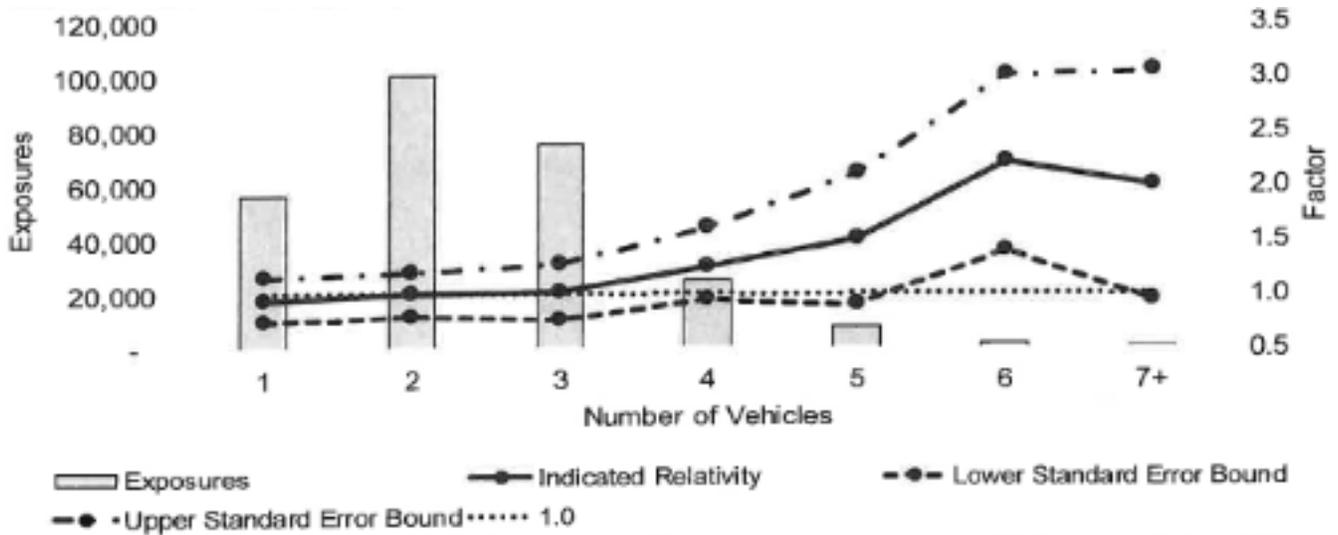see one of its insureds move to that competitor.

(c) 1. Expense reductions. For example, the company may try to reduce the marketing budget, reduce its staffing levels, or reduce commission rates.

2. Tighten the underwriting criteria or non-renew policies that have grossly inadequate premium relative to expected costs. The goal is to improve results by reducing losses more than the reduction in premiums.

3. Reduce the coverage provided by the policy. For example, a homeowners insurer may adjust the policy to exclude coverage for mold losses.

4. Institute better loss control procedures. For example, a workers compensation carrier may be able to reduce average severity by applying proactive medical management procedures and return-to-work programs for disability claims that are likely to escalate.

5. Order claims handlers to be less generous in settling claims.

6. Increase efforts to detect and fight fraudulent claims. (This might involve working with other insurers and/or the government.)

(d) The two insurance companies may have a significantly different mix of business by class and territory.

One could instead compare the rates charged by two (or several) insurer's for each of an appropriate set of specific insureds from carefully selected territories and classes.

Comment: In part (d), other difficulties include different underwriting standards or different distribution systems. Thus many insurers would first decide who they think their chief competitors are in a given line and in a given state, and then usually concentrate on comparing rates to those competitors. Take a homeowners example, where your insurer writes over the internet and concentrates on low value homes in the more rural part of the state. It would be meaningless to compare your average premiums to those of an insurer who instead uses independent agents and concentrates on high priced homes in more urban parts of the state.

9. (2 points) An auto insurer is evaluating the variable "number of vehicles" for inclusion in a rating plan. Given the following Generalized Linear Model (GLM) output:

Legend: Exposures | Indicated Relativity | Lower Standard Error Bound | Upper Standard Error Bound | 1.0

Legend: Exposures | 2015 Data | 2016 Data | 2017 Data | 1.0

- Number of vehicles chi-square percentage: 10%

a. (1 point) Fully justify whether number of vehicles should be included in the rating plan.

b. (0.75 point)
Briefly discuss three challenges associated with performing GLM analysis on loss ratio data.

c. (0.25 point) GLM analysis is widely accepted in classification ratemaking. Briefly discuss one reason that univariate analysis may be more appropriate than GLM analysis.

9. (a) The indicated relativities are not significantly different from one, with the exception of 6 vehicles where there is little data. This would argue against using the number of vehicles.

Consistency over time is poor outside of the first few buckets.

"A Chi-Square percentage of less than 5% generally suggests the actuary should reject the null hypothesis that the models are the same and should use the model with the greater number of parameters." Thus the p-value of 10% is not small enough to justify using the more complicated model including the number of vehicles.

For the above reasons, I would <u>not</u> use number of vehicles.

(b) 1. Modeling loss ratios requires premiums to be adjusted to current rate level at the granular level and that can be practically difficult.

2. Experienced actuaries have an a priori expectation of frequency and severity patterns (e.g., youthful drivers have higher frequencies). In contrast, the loss ratio patterns are dependent on the current rates and therefore actuaries do not have a priori expected patterns for loss ratios. Thus, the actuary can better distinguish the signal from the noise when building models, if they use frequency, severity, and pure premiums, rather than loss ratios.

3. Loss ratio models become obsolete when rates and rating structures are changed.

4. There is no commonly accepted distribution for modeling loss ratios.

(c) 1. Univariate analysis is simpler and thus easier to explain to nonactuaries, including others within the insurer as well as regulators.

2. There may not be a sufficient volume of data to justify the use of GLMs.

Other sample responses from the CAS Examiner's Report:

- Univariate analysis is easier to compute than GLM.
- The law/regulator in some states may require univariate analysis.
- Univariate analysis may be more appropriate if a company does not have the computing power to perform and set up a GLM.
- Univariate is more transparent than GLM.
- Simple rating algorithm, does not require higher complexity/cost GLM analysis.

<u>Comment</u>: For part (b), see page 177 of <u>Basic Ratemaking</u>.

In part (a), the quote is from pages F-2 to F-3 of <u>Basic Ratemaking</u>.

In part (a), I would have answered in part: "The relativities by year are similar, with the exception of 6 and 7+ vehicles where there is little data. This would argue for using the number of vehicles." However, the CAS did not accept this response.

In the graphs, the base level, whatever it is, should have a relativity of one with no error bars.

Note that <u>Basic Ratemaking</u> briefly discusses multivariate methods other than GLMs.

10. (2.5 points) A company writes homeowners insurance in a large state divided in half by a mountain range. The company currently uses two geographic rating territories, one on either side of the mountains, as the range has an effect on weather patterns. Each territory has sufficient exposures for its loss experience to be considered fully credible.

a. (0.5 point) Briefly discuss two disadvantages of the company's current territorial rating approach.

b. (2 points) Discuss the process by which an actuary would develop new rating territory definitions for this state. Briefly explain a consideration for each step in the process.

10. (a) 1. Weather only significantly affects some of the homeowners perils, but not others.
For example, expected theft losses and liability losses are not chiefly affected by weather.
2. The two territories are each too big; by splitting each of them up one should be able to obtain
more homogeneous territories that still contain sufficient data to determine relativities.
Other sample responses from the CAS Examiner's Report:
- The company's current approach doesn't take into account differences between rural and urban
    areas since there are only two relativities (not enough territories).
- The company's current approach doesn't have homogeneous risks in the territories since it's only
    based on one variable.
- Other companies may have more than two territories. You could gain a competitive advantage if
    you added more territories.

(b) The actuary has to decide what small geographical units to work with: zipcodes, census tracks,
etc. One must collect historical pure premiums for the chosen geographical units.
Then the actuary can use the fit of a GLM that models pure premium using a variety of
non-geographic and geographic explanatory variables. The actuary would have to test various
forms of GLM and sets of explanatory variables in order to select a GLM that works best. The
parameters from each of the various geographic predictors, including a predictor variable for the
geographic residual variation, can be combined to form one composite risk index that represents the
geographic signal for each geographic unit.
Then this index is spatially smoothed out somehow. The actuary must choose an amount of
smoothing that lessens the noise, but retains most of the signal. The actuary must decide which type
of smoothing method to use: distance-based or adjacency-based.
After smoothing the data, the geographical units are grouped together to form territories of units with
similar pure premiums, usually using a clustering algorithm. The actuary has to choose which
clustering method to use. There are choices to make with each method. (One would have to add a
constraint, if one wanted contiguous territories.)
When combining units to form territories, the actuary wants to minimize within group heterogeneity
and maximize between group heterogeneity. The smaller the within variance, and thus the larger
the between variance, the more homogeneous the territories.
However, a conflicting goal is to have territories with sufficient data in order to estimate relativities.
Thus the actuary will have to choose the number of territories so as to balance these two conflicting
goals.
As a final step, judgement will have to be used to try to avoid large rate differences across territory
boundaries, as well as in order to have the territory boundaries seem sensible to nonactuaries.
<u>Comment</u>: One could instead work directly with historical pure premiums rather than the output of a
GLM. We should now divide the pure premium by the average class relativity for each unit. (For
example, if a unit has a high average class relativity, then the pure premium will be reduced to what it
would have been with an average mix of classes.)
For states in which hurricanes are significant, for example Florida, separate sets of homeowners
territories will often be determined for wind and for all other perils.

11. (1.75 points) Given the following for a large deductible commercial general liability policy:

$500,000	Per occurrence deductible
90.0%	Loss elimination ratio for a $500,000 deductible
10.0%	ALAE/ground up loss ratio
$2,000,000	Ground up loss estimate
$100,000	Fixed expenses
12.0%	Variable expenses as % of premium
4.0%	Underwriting profit as % of premium
3.0%	Deductible processing cost as a % of losses below the deductible
1.0%	Credit risk as a % of losses below the deductible
7.0%	Additional risk margin as a % of excess losses

- The insurer will handle all claims, including those that fall below the deductible.
- The insurer will make the payments on all claims and will seek reimbursement for amounts below the deductible from the insured.
- The deductible is for loss only.
- All ALAE is paid by the insurer.

Calculate the premium for the large deductible policy.

11. Excess Losses = (1 - 90%)((2 million) = $200,000.
ALAE = (10%)(2 million) = $200,000.
Deductible Processing Cost = (3%)(1.8 million) = $54,000.
Credit Risk = (1%)(1.8 million) = $18,000.
Additional Risk Margin = (7%)(200,000) = $14,000.

$$\text{Premium} = \$1000 \ \frac{200 + 200 + 100 + 54 + 18 + 14}{1 - 12\% - 4\%} = \textbf{\$679,619}.$$

12. (2.5 points) Given the following for an insurance company:

Premium = (Base Rate) x (Rating Factor 1) x (Rating Factor 2)

Variable 1 Segment	Rating Factor 1
A	0.90
B	2.00
C	1.00

Variable 2 Segment	Rating Factor 2
X	0.75
Y	0.95
Z	1.00

Earned Exposures			
	Variable 1		
Variable 2	A	B	C
X	800	1,500	600
Y	300	750	500
Z	500	500	1,500

Reported Loss & ALAE ($)			
	Variable 1		
Variable 2	A	B	C
X	320,000	2,100,000	400,000
Y	170,000	1,535,000	500,000
Z	305,000	1,100,000	1,600,000

- The base rate is $1,000.
- The base classification for Variable 1 is C.
- The proposed overall rate level change is 0%.

a. (1.5 points)
Calculate the indicated relativities for Variable 1 using the adjusted pure premium method.

b. (0.5 point) Calculate the proposed base rate assuming the company selects half of the indicated relativity change for each segment of Variable 1.

c. (0.5 point) Briefly describe two social criteria for evaluating the appropriateness of rating variables.

12. (a) Adjusted exposures for Class A: (800)(0.75) + (300)(0.95) + (500)(1) = 1385.
Adjusted exposures for Class B: (1500)(0.75) + (750)(0.95) + (500)(1) = 2337.5.
Adjusted exposures for Class C: (600)(0.75) + (500)(0.95) + (1500)(1) = 2425.
Adjusted pure premium for Class A: ($1000) (320 + 170 + 305) / 1385 = $574.01.
Adjusted pure premium for Class B: ($1000) (2100 + 1535 + 1100) / 2337.5 = $2025.67.
Adjusted pure premium for Class C: ($1000) (400 + 500 + 1600) / 2425 = $1030.93.
Relativity for Class A: 574.01 / 1030.93 = **0.557**.
Relativity for Class B: 2025.67/ 1030.93 = **1.965**.
Relativity for Class C (base): **1.000**.
(b) Taking half of the change, the new relativity for Class A is: (0.90 + 0.557)/2 = 0.729.
The new relativity for Class B is: (2.00 + 1.965)/2 = 1.983.
The new relatively for Class C (base): 1.000.
Assume that the relativities for Variable 2 stay the same.
The current average relativity for Variable 1, using adjusted exposures is:
{(1385)(0.9) + (2337.5)(2) + (2425)(1)} / (1385 + 2337.5 + 2425) = 1.3577.
The new average relativity for Variable 1, using adjusted exposures is:
{(1385)(0.729) + (2337.5)(1.983) + (2425)(1)} / (1385 + 2337.5 + 2425) = 1.3127.
The off-balance to multiply by is: 1.3577/1.3127 = 1.0343.
Thus for no overall chance in rates, the new bas rate should be: ($1000)(1.0343) = **$1034**.
(c) 1. Affordability: high rates may cause problems in being able to afford insurance,

particularly if rates are negatively correlated with income. (low incomes ⟺high rates.)

Insurance should be affordable for everyone.
2. Causality: differences in the rating variable are responsible for differences in insurance costs.
There should be a causal relationship to loss that makes sense to regulators.
3. Controllability: a controllable variable is one which is under the control of the insured; the insured
can modify his behavior in order to reduce his insurance costs.
4. Privacy Concerns: reluctance to disclose personal information.
Should not infringe on someone's privacy.

13. (2.75 points) Given the following information about a home's propensity for loss:

Size of Loss	Loss Distribution
X ≤ $400,000	50.0%
$400,000 ≤ X ≤ $550,000	25.0%
$550,000 ≤ X ≤ $700,000	10.0%
$700,000 ≤ X ≤ $850,000	10.0%
$850,000 ≤ X ≤ $1,000,000	2.5%
$1,000,000 ≤ X ≤ $1,500,000	2.5%
Total:	100.0%

- Expected claim frequency is 2%.
- Expected losses are uniformly distributed within each layer of loss.
- The home is valued at $1,500,000.

a. (1.25 points)
Calculate the rate per $1,000 of coverage for the home at the following amounts of insurance:
     i. $1,500,000
     ii. $800,000

b. (0.5 point)
Briefly discuss a problem associated with underinsurance from the following perspectives:
     i. Insured
     ii. Insurer

c. (1 point) The home is insured for $1,000,000 with a coinsurance requirement of 80%.
Calculate the indemnity payments and coinsurance penalties for the following losses:
     i. $800,000
     ii. $1,200,000

13. (a) i. (2%)(1000)

$$\frac{(50\%)(200)+(25\%)(475)+(10\%)(625)+(10\%)(775)+(2.5\%)(925)+(25\%)(1250)}{1500} = \textbf{\$5.51}.$$

Alternately, the average severity is:

(0.5)(200K) + (0.25(475K) + (0.1)(625K) + (0.1)(775K) + (0.025)(925K) + (0.025(1250K) = $413,125.

The rate per $1,000 of coverage for the home: (2%)($413,125K) / (1500K / 1K) = **$5.51**.

ii. The payments for the fourth interval are uniform from 700K to 800K, while the remaining 1/3 are 800K. The average payment is: (750K)(2/3) + (800K)(1/3) = 767K.

$$(2\%)(1000)\ \frac{(50\%)(200)+(25\%)(475)+(10\%)(625)+(10\%)(767)+(2.5\%)(800)+(25\%)(800)}{800} = \textbf{\$9.95}.$$

(b) i. If there is a large or total loss to the property, the insured will not be provided enough money to replace their property.

ii. Due to the possibility of partial losses, as in part (a), the appropriate rate ot charge someone who buys less coverage is higher. Thus if rates are made assuming insurance to value, in the absence of a coinsurance clause, then the premiums for insureds who underinsure will be inadequate.

(c) The coinsurance requirement is: (80%)(1,500,0000) = $1,200,000.

The coinsurance apportionment ratio is: 1,000,000 / 1,200,000 = 0.8333.

i. The insured is paid: (0.8333)(800,000) = **$666,667**.

The coinsurance penalty is: 800,000 - 666,667 = **$133,333**.

ii. The insured is paid: (0.8333)(1,200,000) = **$1 million**.

The coinsurance penalty is: 1,000,000 - 1,000,000 = **0**.

<u>Comment</u>: The coinsurance penalty = reduction due to coinsurance of the amount an insured is paid = (Amount that would have been paid if there were no coinsurance clause) - (Amount actually paid).

$$\text{coinsurance penalty} = \begin{cases} L-I & \text{if } L \le F \\ F-I & \text{if } F < L < cV \\ 0 & \text{if } cV \le L \end{cases}.$$

In part (c), i. is in the first interval. L = 800,000 and I = 666,667.

ii. is in the last interval. L = 1,200,000 and cV = 1,200,000.