Solutions to the
Fall 2017
CAS Exam Five

(Only those questions on Basic Ratemaking)

There were 28 questions worth 55.75 points, of which 14 were on ratemaking worth 27.75 points.
(Questions 6 and 15 covered both reserving and ratemaking.)

The exam and examiner's report are available from the CAS.
The solutions and comments are solely the responsibility of the author.

(Incorporating what I found useful from the CAS Examiner's Report)

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1. (1.5 points) Given the following information:

Calendar Year	Average Earned Premium at Current Rate Level	Average Written Premium at Current Rate Level
2014	$210	$212
2015	$220	$224
2016	$235	$240

- The projected annual premium trend = -2%.
- Fourth quarter 2016 average earned premium at current rate level = $236.
- Fourth quarter 2016 average written premium at current rate level = $242.
- The company uses a calendar-accident year aggregation of data for indications.
- All policies are annual.
- Rates are in effect for one year.
- The rate revision is planned to be effective January 1, 2018.

a. (1 point) Calculate the premium trend factor for each year using two-step trending.

b. (0.5 point) Identify two scenarios that could lead to a negative premium trend when analyzing average premium at current rate level.

1. (a) Since it is more current than the earned premium, I will use the fourth quarter 2016 average written premium as the pivot point for the two-step trending method.
The average date of writing under the new rates is: January 1, 2018 + 6 months = July 1, 2018.
The projection period is from average date of writing November 15, 2016 to July 1, 2018 or 19.5 months.
I am assuming we are using CY earned premiums in the rate indication.
Thus the first step is the ratio of the fourth quarter 2016 average written premium of $242 over the CY earned premium.

C Y	First Piece of Trend	Second Piece of Trend	Trend Factor for Earned Premium
2014	242/210 = 1.152	$0.98^{19.5/12} = 0.968$	(1.152)(0.968) = **1.115**
2015	242/220 = 1.100	$0.98^{19.5/12} = 0.968$	(1.100)(0.968) = **1.065**
2016	242/235 = 1.030	$0.98^{19.5/12} = 0.968$	(1.030)(0.968) = **0.997**

Alternately, since we are working on earned premiums to be used in the rate level indication, I will use the fourth quarter 2016 average earned premium as the pivot point for the two-step trending method.
The average date of writing under the new rates is: January 1, 2018 + 6 months = July 1, 2018.
Since we have annual policies, the average date of earning under the new rates is:
July 1, 2018 + 6 months = January 1, 2019.
The projection period is from average date of earning November 15, 2016 to January 1, 2019 or 25.5 months.
The first step is the ratio of the fourth quarter 2016 average earned premium of $236 over the CY earned premium.

C Y	First Piece of Trend	Second Piece of Trend	Trend Factor for Earned Premium
2014	236/210 = 1.124	$0.98^{25.5/12} = 0.958$	(1.124)(0.958) = **1.077**
2015	236/220 = 1.073	$0.98^{25.5/12} = 0.958$	(1.073)(0.958) = **1.028**
2016	236/235 = 1.004	$0.98^{25.5/12} = 0.958$	(1.004)(0.958) = **0.962**

(b) 1. Over the recent past, the insurer has been writing more business in lower rated classes and territories and less business in higher rated classes and territories.
2. For Homeowners Insurance, if the average value of the homes insured has been decreasing.
(I am assuming that the average premiums are calculated per houseyear.)
3. Insureds are choosing significantly higher deductibles over time, so that the average premium is declining.
Comment: Since the observed average premiums have been increasing, while the projection is that they will decrease, in this case the two alternative methods in part (a) give significantly different results.

2. (2 points) Given the following information:

Rate Change Effective Date	Overall Rate Change
July 1, 2013	5%
October 1, 2015	2%
October 1, 2016	-4%

Calendar Year	Earned Premium ($000)	Earned Premium ($000) at Current Rate Level
2014	15,000	14,775
2015	18,000	17,622

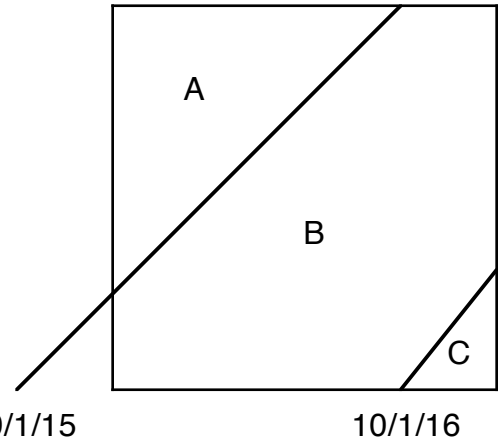
- 2016 Earned Premium = $22,000,000.
- 2014 through 2016 combined projected ultimate loss and LAE = $40,000,000.
- Selected annual premium trend = 2%.
- Fixed expense provision = 8%.
- Variable expense provision = 20%.
- Target underwriting profit provision = 5%.
- All policies are annual.
- Rates are to be in effect for one year.
- The rate revision is planned to be effective October 1, 2017.

a. (1.5 points) Calculate the projected earned premium at current rate level for 2014 through 2016.

b. (0.5 point) Calculate the indicated rate change.

2. (a) While we are given CY14 and CY15 on current rate level, we need to put CY16 on-level.

Rate Change Effective Date	Overall Rate Change	Rate Level Index
July 1, 2013	5%	1.05
October 1, 2015	2%	(1.05)(0.98) = 1.071
October 1, 2016	-4%	(1.071)(0.96) = 1.028



Area A = $(1/2)(3/4)^2 = 9/32$. Area C = $(1/2)(1/4)^2 = 1/32$. Area B = 1 - 9/32 - 1/32 = 22/32.

Average Rate Level CY16 = (9/32)(1.05) + (22/32)(1.071) + (1/32)(1.028) = 1.064.

CY16 OLF is: 1.028/1.064 = 0.966.

CY16 premium on current rate level = (22 million)(0.966) = 21.252 million.

The average date of writing under the new rates is: October 1, 2017 + 6 months = April 1, 2018.

With annual policies, the average date of earning under the new rates is:

April 1, 2018 + 6 months = October 1, 2018.

The trend period for CY16 Earned premium is from July 1, 2016 to October 1, 2018 or 2.25 years.

$(14.775)(1.02^{4.25}) + (17.22)(1.02^{3.25}) + (21.252)(1.02^{2.25})$ = **$57.086 million**.

(b) The loss and LAE ratio is: 40/57.086 = 70.07%.

(70.07% + 8%) / (1 - 20% - 5%) = 1.041. $\Leftrightarrow$ **4.1%** indicated rate increase.

3. (1.5 points) Given the following for an insurance company that writes only annual policies:

Policy	Effective Date	Annual Premium
A	July 1, 2014	$200
B	October 1, 2014	$240
C	January 1, 2015	$260
D	July 1, 2015	$280

• Policy D was cancelled March 31, 2016.

a. (0.5 point) Calculate the following for calendar year 2015:

i. Earned premium

ii. Written premium

b. (0.5 point) Calculate the following as of December 31, 2016:

i. Policy year 2015 earned premium

ii. Policy year 2015 written premium

c. (0.5 point)

Briefly describe one advantage and one disadvantage of calendar year data aggregation.

3. (a) i. CY15 Earned Premium: 200/2 + (240)(3/4) + 260 + 280/2 = **$680**.

ii. CY15 Written Premium: 260 + 280 = **$540**.

(b) i. PY15 earned premium as of the end of 2016: 260 + (3/4)(280) = **$470**.

ii. Since by the end of 2016 all of policies written during 2015 have expired, the PY15 written premium is the same as the earned: 260 + (3/4)(280) = **$470**.

(c) Advantages: Calendar Year Premium and losses are fixed at the end of the calendar year.
Once the Calendar Year is over, data is ready to be used and is thus available quickly.
Calendar Year data is easy to obtain since it is needed for accounting purposes.
Disadvantage: Calendar Year data has a poor match between premiums and losses.
Comment: Policy D written on July 1, 2015 was cancelled March 31, 2016, but it still contributes the full $280 to CY15 written premium; it would contribute -$70 to CY16 written premium.

4. (1.5 points) Given the following information for an insurance company:

- All policies are annual.
- For all claims reported in one year, 40% of the ultimate loss is from claims occurring in the same year, 35% from the prior year and 25% from the 2nd prior year.
- Annual report year loss cost trend = 3%.
- The company writes policies uniformly through the year.
- Exposure levels are constant.
- For report year 2013, the loss cost per exposure from claims occurring in 2013 = $200.

a. (1 point) Calculate the loss cost per exposure for the following:

i. Occurrence policy effective January 1, 2016

ii. Claims-made policy effective January 1, 2018 with retroactive date of January 1, 2017

b. (0.5 point) A customer is switching from a claims-made policy to an occurrence policy effective January 1, 2016. Calculate the total loss cost per exposure that would provide complete coverage without overlap for this customer.

4. Report Year 2013 losses are divided by lag into:
$200 given, ($200) (35%/40%) = $175, and ($200) (25%/40%) = $125.
Then we apply the 3% annual report year loss cost trend:

Report Year	Lag 0	Lag 1	Lag 2
2013	$200.00	$175.00	$125.00
2014	$206.00	$180.25	$128.75
2015	$212.18	$185.66	$132.61
2016	$218.55	$191.23	$136.59
2017	$225.10	$196.96	$140.69
2018	$231.85	$202.87	$144.91

For example, (200)(1.03) = 206. (206)(1.03) = 212.18.
(a) i. The northwest-southeast diagonal, starting with 2016 @ lag 0:
218.55 + 196.96 + 144.91 = **$560.42**.
ii. Usually this would be the all of RY18. However due to the retroactive date, accidents occurring prior to January 1, 2017 are not covered by this policy.
This excludes RY18 @ lag 2, which are accidents occurring in 2016 reported during 2018.
231.85 + 202.87 = **$434.72**.
(b) Prior to filling in the coverage gaps:

Report Year	Lag 0	Lag 1	Lag 2
2013	CM13	CM13	CM13
2014	CM14	CM14	CM14
2015	CM15	CM15	CM15
2016	Occ16		
2017	Occ17	Occ16	
2018	Occ18	Occ17	Occ16

Thus in order to provide complete coverage we need to fill in three boxes:
191.23 + 136.59 + 140.69 = $468.51.
Adding in the cost of a 2016 occurrence policy from part (a): 468.51 + 560.42 = **$1028.93**.

5. (2 points) Given the following information for an insurance company as of December 31, 2016:

Accident Year	Earned Premium ($000)	Reported Loss ($000)	Cumulative Loss Development Factors
2012	3,000	1,500	1.05
2013	3,500	1,925	1.10
2014	3,300	1,749	1.20
2015	3,200	1,984	1.35
2016	3,800	2,470	1.40

- All policies are annual.
- Annual loss cost trend = 3%.
- The company has increased rates by 5% every year on January 1.
- The company writes policies uniformly throughout the year.

Calculate accident year 2016 trended ultimate loss using the Bornhuetter-Ferguson method with the expected loss ratio based on accident years 2012 through 2014 experience.

5. We need to develop to ultimate and trend to a common date the AY12, AY13, and AY14 losses; I will choose the average AY14 level. Then we also need to put the AY12, AY13, and AY14 premiums on the corresponding same rate level, that for AY 2014.

AY	Earned Prem. On-Level	Ultimate Trended Losses	Loss Ratio
2012	$(3000)(1.05^2) = 3307.5$	$(1500)(1.05)(1.03^2) = 1670.9$	1670.9/3307.5 = 50.52%
2013	(3500)(1.05) = 3675	(1925)(1.10)(1.03) = 2181.0	2181.0/3675 = 59.35%
2014	3300	(1749)(1.2) = 2098.8	2098.8/3300 = 63.58%

I will select the average of the three loss ratios: (50.52% + 59.35% + 63.58%) / 3 = 57.82%.
(There are other reasonable ways to proceed, such as the ratio of the totals.)
We now need to adjust this loss ratio from the average 2014 level to the average 2016 level:
$(57.82\%)\ (1.03^2) / 1.05^2 = 55.64\%$.
Based on this loss ratio, the expected ultimate losses for AY16 would be:
(3,800,000)(55.64%) = $2,114,320.
From the given development factor to ultimate, the expected % unreported is: 1 - 1/1.40 = 28.57%.
Thus the expected unreported losses for AY16 are: (28.57%)(2,114,320) = $604,601.
Add this to the reported AY16 losses: 2,470,000 + 604,601 = **$3075 thousand**.
Comment: Can be answered either based on pages C-13 and C-14 of Appendix C of Basic Ratemaking or Chapter 9 of Estimating Unpaid Claims Using Basic Techniques.
At the first step, one could instead bring the AY12, AY13, and AY14 to the average AY16 level; which would have been quicker since we are not also applying Bornhuetter-Ferguson to AY15.
The expected unreported for AY15 is: (57.82%)(1.03/1.05) ($3.2 M) (1 - 1/1.35) = $470,555.
Thus the estimated ultimate losses for AY15 are: 1,984,000 + $470,555 = $2455 thousand.

6. (2 points) The current workers compensation indemnity benefit structure in a state is as follows:

- The compensation rate is 80% of the workers pre-injury wage.
- The state average weekly wage (SAWW) is currently $1,500.
- The minimum indemnity benefit is 50% of the SAWW.
- The maximum indemnity benefit is 125% of the SAWW.

The following changes have been proposed to the workers compensation indemnity benefit structure:

- The proposed minimum indemnity benefit is 75% of the SAWW.
- The proposed maximum indemnity benefit is 100% of the SAWW.

The distribution of injured workers for Company A is shown below:

Ratio to SAWW	# Workers	Total Weekly Wages
<50%	150	$108,750
50%-75%	100	$110,000
75%-100%	95	$137,750
100%-125%	50	$87,500
>125%	45	$216,000
Total	440	$660,000

a. (1.5 points) Calculate the impact to Company A of the state's proposed workers compensation indemnity benefit change.

b. (0.5 point) Briefly describe two potential indirect effects of the benefit change.

6. (a) Under the current law the minimum benefit is: (50%)(1500) = $750,
and the maximum benefit is: (125%)(1500) = $1875.
In order to receive the minimum benefit, need a wage of: 750/0.8 = $937.5.
937.5/1500 = 62.5% = 50%/0.8.
In order to receive the maximum benefit, need a wage of: 1875/0.8 = $2343.75.
$2343.75/1500 = 156.25% = 125%/0.8.
In the category 50%-75% of SAWW, the average weekly wage is $110,000/100 = $1100,
or 1100/1500 = 73.3% of the SAWW. Thus most of these workers must be at the high end of the range. For example, one might have 7 workers making $768 and 93 workers making $1125.
In the category >125%, the average weekly wage is $216,000/45 = $4800,
or 4800/1500 = 320% of the SAWW. For simplicity assume that all of these workers make more than 156.25% of the SAWW.
Under current law the expected total benefits (assuming each worker was receiving benefit) are:
(150 + 7) ($750) + (80%) {(93)($1125) + $137,750 + $87,500} + (45) ($1875) = $466,025.
Under the proposed law the minimum benefit is: (75%)(1500) = $1125,
and the maximum benefit is: (100%)(1500) = $1500.
In order to receive the minimum benefit, need a wage of: 1125/0.8 = $1406.25.
1406.25/1500 = 93.75% = 75%/0.8.
In order to receive the maximum benefit, need a wage of: 1500/0.8 = $1875.
$1875/1500 = 125% = 100%/0.8.
In the category 75%-100% of SAWW, the average weekly wage is $137,750/95 = $1450,
or 1450/1500 = 96.67% of the SAWW. Thus most of these workers must be at the high end of the range. For example, one might have 13 workers making $1135 and 82 workers making $1500.
Under the proposed law the expected total benefits are:
(150 + 100 + 13) ($1125) + (80%) {(82)($1500) + $87,500} + (45) ($1500) = $531,775.
Thus the estimated direct impact is: $531,775 / $466,025 - 1 = **14.1%** increase.
(b) Since low wage workers would have a higher minimum benefit, they would have an incentive to stay on benefits longer and would also have a somewhat greater probability to file for benefits.
Since high wage workers would have a lower maximum benefit, they would have an incentive to come back to work sooner and would also have a somewhat lower probability to file for benefits.
Comment: One usually evaluates a proposed law change using the wage data for the whole state.
A much finer division of wages into groups would have been very desirable;
depending on how one deals with this issue, one will get somewhat different answers in part (a).

7. (2 points) Given the following information for the past year for an insurance company:

	($000)	% Fixed
Written Premium	25,000	-
Earned Premium	20,000	-
Agent Commission	3,000	0%
Other Acquisition Cost	300	100%
Premium Tax & Licensing Fees	1,000	0%
General Expense	2,500	100%
Loss Adjustment Expenses	1,200	0%

• Underwriting profit provision = 5%.

a. (0.75 point)

Calculate the underwriting expense ratio using the premium-based projection method.

b. (0.25 point) Calculate the operating expense ratio using the premium-based projection method.

c. (0.25 point) Calculate the total permissible loss ratio.

d. (0.75 point) Calculate the indicated rate change using a projected loss ratio of 65% (excluding loss adjustment expenses).

7. (a) We take the ratio of General Expenses to Earned Premium.

We take the ratio of other expenses to Written Premium.

Underwriting expense ratio is:

(3000 + 300 + 1000)/25,000 + 2500/20,000 = 17.2% + 12.5% = **29.7%**.

(b) Operating Expense Ratio is:

29.7% + LAE/(Earned Prem.) = 29.7% + 1200/20,000 = 29.7% + 6% = **35.7%**.

(c) Permissible Loss Ratio = 1 - Total Expense Ratio - Target Profit = 1 - 35.7% - 5% = **59.3%**.

(d) Fixed expenses = 300/25,000 + 2500/20,000 = 13.7%.

Variable Expenses = (3000 + 1000)/ 25,000 = 16% = 29.7% - 13.7%.

LAE / Losses = 1200 / {(65%)(20,000)} = 9.23%.

$$\frac{(65\%)(1.0923) + 13.7\%}{1 - 16\% - 5\%} = 1.072. \Leftrightarrow \textbf{7.2\%} \text{ increase.}$$

Alternately, LAE/(Earned Prem.) = 1200/20,000 = 6%.

$$\frac{65\% + 6\% + 13.7\%}{1 - 16\% - 5\%} = 1.072. \Leftrightarrow \textbf{7.2\%} \text{ increase.}$$

8. (2.25 points) Given the following information about an insurance product filing with annual policies:

- 2018 projected pure premium = $350.
- Loss cost annual trend = 3%.
- Premium annual trend = 4%.
- Fixed expense per exposure, new business = $50.
- Fixed expense per exposure, renewals = $6.
- Variable expense ratio = 18%.
- Profit and contingencies provision = 6%.
- LAE provision = 10% of loss.
- Retention ratio = 80%.
- Discount rate = 5%.

a. (1.5 points) Calculate the new business premium per exposure for 2018 indicated by a lifetime value analysis using a two-year time horizon.

b. (0.75 point) Fully justify the use of lifetime value analysis in a rate indication using the Statement of Principles Regarding Property and Casualty Ratemaking.

8. (a) Assuming a new policy in 2018:

Year	P.P.	Fixed Expense P.P.	Loss + LAE + Fixed Expense P.P.
2018	350	50	(350)(1.1) + 50 = 435
2019	360.5	6	(360.5)(1.1) + 6 = 402.55

Assume the 2018 premium is x, then the underwriting profits by years are:
0.82 x - 435, and (0.82 x)(1.04) - 402.55.
Taking into account persistency, the discounted underwriting profit is:
{0.82 x - 435} + {(0.82 x)(1.04) - 402.55} (0.8)/1.05 = 1.46975 x - 741.70.
Taking into account persistency, the discounted premiums are:
x + x(1.04)(0.8)/1.05 = 1.7924 x.
Setting the underwriting profit provision equal to the desired 6%:
(0.06) (1.7924 x) = 1.46975 x - 741.70. $\Rightarrow$ x = **544.48**.
(b) "A rate is reasonable and not excessive, inadequate, or unfairly discriminatory if it is an actuarially sound estimate of the expected value of all future costs associated with an individual risk transfer."
In lifetime value analysis, one treats an individual policy plus potential renewals as one risk transfer; the rate is determined based on the expected (discounted) value of all future costs associated with the new policy plus set of renewals, taking into account assumptions of persistency.
Comment: See Tables 3.16 and 3.17 in Basic Ratemaking.
In lifetime analysis, one usually assumes that controlling for inflation the loss pure premium for renewal business is better than that for new business.
In part (b), the Statement of Principles Regarding Property and Casualty Ratemaking was written to apply to an individual policy rather than a string of policies in which there is not a guarantee of renewals; it was adopted prior to lifetime analysis being used extensively in Property and Casualty Insurance. Lifetime analysis is not consistent with the intent of the authors of the Statement of Principles Regarding Property and Casualty Ratemaking.

9. (1.75 points) An actuary is developing a rating algorithm for a new product covering professional liability for nurses working in hospitals.

Characteristics considered:

- Age of nurse
- Gender of nurse
- Hours worked per week by each nurse
- Number of nurses employed by the hospital
- Specialty of the nurse (Cardiac or General)

Given the following:

- 20% of customers will switch insurers based on price.
- The company's competitor uses specialty of nurse in their rating algorithms and charges the true expected cost.
- At the start of the program the company and the competitor each write 100 policies for Cardiac Nurses and 100 policies for General Nurses.
- There are no underwriting expenses or profit.

Specialty	True Expected Cost
Cardiac	$500
General	$200

a. (0.5 point) For one of the characteristics, briefly discuss two reasons why it would be an appropriate exposure base.

b. (0.5 point) Assess if age of nurse is an appropriate rating variable using two social criteria.

c. (0.75 point) The company decides not to use specialty of nurse in their rating algorithm. Quantify the effect on profitability for the company after one renewal cycle.

9. (a) I will assume that a policy is issued to each individual nurse. I will discuss why number hours worked per week by each nurse would be an appropriate exposure base.
1. It is approximately proportional to expected loss. All other things being equal, a nurse that works twice as many hours would have twice the expected number of incidents and thus twice the expected losses.
2. It is practical. The number of hours worked by an individual nurse is already recorded by the employer (the hospital) and therefore should be available and easy to verify. It is objective.
Alternately, I will assume that the policy is to be issued to the hospital. I will discuss why number of nurses employed by the hospital would be an appropriate exposure base.
1. It is approximately proportional to expected loss. All other things being equal, a hospital that employs twice as many nurses would have twice the expected losses.
2. It is practical. The number of nurses employed by the hospital should be available and easy to verify. It is objective.
(b) 1. Affordability: high rates may cause problems in being able to afford insurance, particularly if rates are negatively correlated with income.
I will assume that the individual nurses are paying the premium rather than the hospital.
If younger nurses are to be charged more and also earn somewhat lower salaries, this could be a concern; however, I do not think it is a major concern that would prevent the use of age of nurse.
2. Causality: differences in the rating variable are responsible for differences in insurance costs.
In my opinion, there is not a clear causal relationship between the age of a nurse and the probability of a negligent act leading to a professional liability claim; age of nurse does not satisfy this criterion.
(Years of experience as a nurse would probably satisfy this criterion.)
3. Controllability: a controllable variable is one which is under the control of the insured; the insured can modify his behavior in order to reduce his insurance costs.
Age is not under control of the nurse; age of nurse does not satisfy this criterion.
4. Privacy Concerns: While some nurses may be reluctant to disclose their ages, this is something that their employer (the hospital) would already know for other purposes. So there are not privacy concerns that would prevent the use of age of nurse.
(c) Assume that the company charges (500+ 200)/2 = $350 per nurse.
The competitor charges less for General nurses, so 20 of the company's General nurses will move to its competitor.
The company charges less for Cardiac nurses, so 20 of the competitor's Cardiac nurses will move to the company.
Thus after one renewal cycle we expect the company to write 120 Cardiac nurses and 80 General Nurses. The expected total cost is: (120)($500) + (80)($200) = $76,000.
However, the premiums are only: (120 + 80)($350) = $70,000.
There is an expected **underwriting loss of $6000**.
Comment: In part (a), the other desirable property of an exposure base is historical precedent. Since this is a new product, this characteristic does not really apply.
In part (b), discuss only two social criteria.
In part (c), the company has been subject to anti-selection.

10. (1.75 points) Given the following:

Territory	True Relativity	Univariate Indicated Relativity	Loss & ALAE ($000)
1	0.50	0.46	3,680
2	1.00	1.00	8,000
3	1.20	1.28	11,636

Earned Exposures (000)

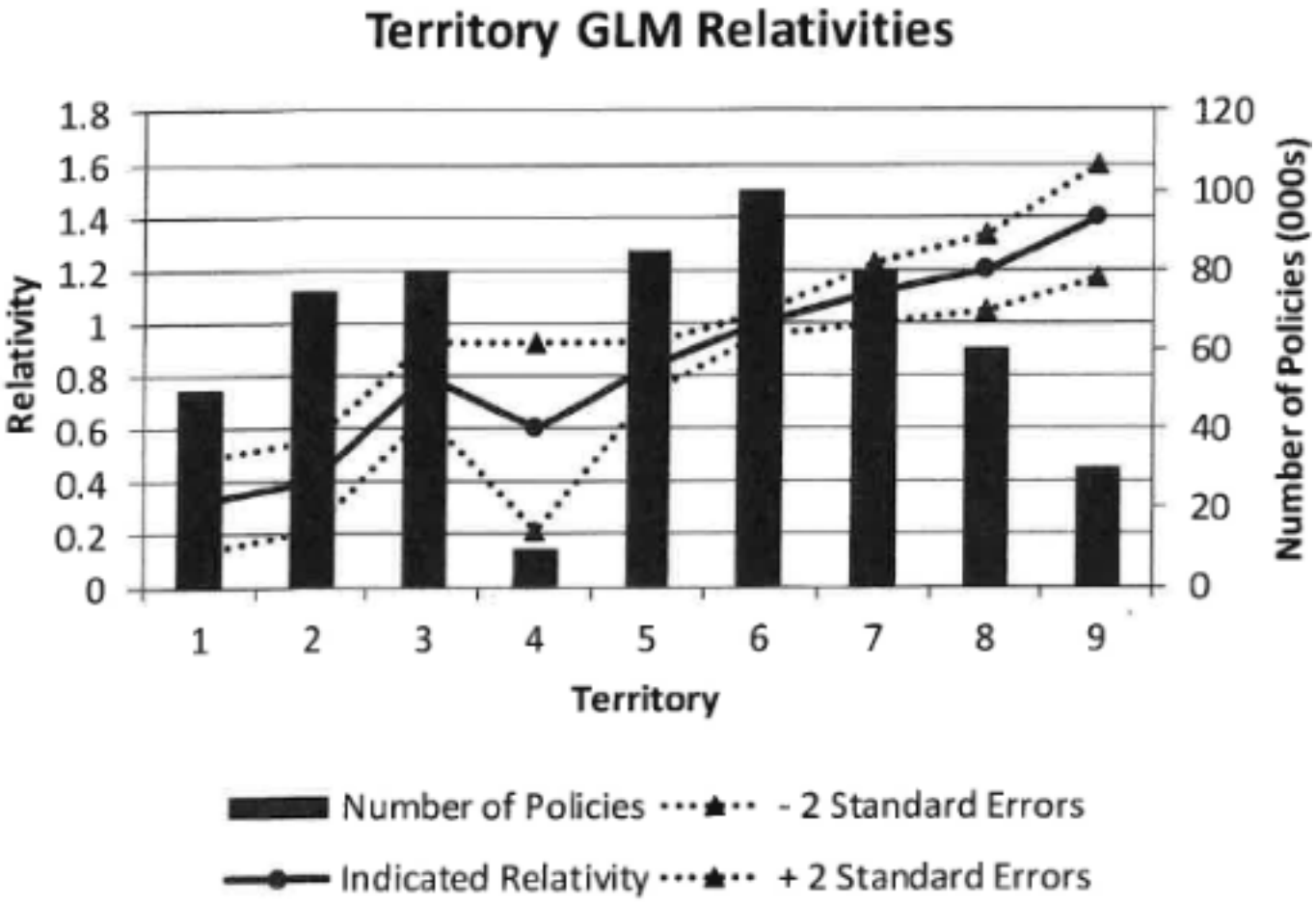
Territory	Class A	Class B	Class C
1	150	70	110
2	105	115	110
3	70	180	125

Class	A	B	C
Charged Factor	0.85	1.15	1.00

a. (0.5 point) Explain why the univariate indicated relativities are different from the true relativities.
b. (1.25 points) Calculate territory relativities using the adjusted pure premium method, keeping territory 2 as the base level.

10. (a) The univariate indicated relativities ignore the different mix of classes by territory that is present here, which creates a problem when as here the expected pure premiums differ by class. So for example, Territory 1 with a larger percentage of Class A exposures than average, will just for that reason have an expected lower pure premium than it would otherwise have.
(b) Using the class factors, the adjusted exposures by territory are:
(150)(0.85) + (70)(1.15) + (110)(1) = 318.
(105)(0.85) + (115)(1.15) + (110)(1) = 331.5.
(70)(0.85) + (180)(1.15) + (125)(1) = 391.5.
The adjusted pure premiums by territory are:
3680/318 = 11.57, 8000/331.5 = 24.13, 11,636/391.5 = 29.72.
Indicated territory relativities with respect to territory 2:
11.57/24.13 = **0.48** , 1, 29.72/24.13 = **1.23**.

11. (1.5 points) The output of a generalized linear model (GLM) analysis on relativities for pure premium by territory is shown below:



• Chi-Square Percentage (entire variable) = 0.1%.

a. (0.5 point) Explain whether the GLM output supports including territory as a rating variable.

b. (0.75 point) Briefly describe three benefits of using multivariate methods over univariate methods for classification ratemaking.

c. (0.25 point)

Briefly describe how spatial smoothing can be used to improve territory relativity estimates.

11. (a) If the "Chi-Square percentage" < 5%, then reject the null hypothesis, use the model with more variables. In this case, the Chi-Square Percentage is 0.1%. Thus based on this statistical test we would use the more complicated model; we use the model including territory.
Also there are number of territories for which the indicated relativity is significantly different than 1; In other words for many territories, 1 is outside the ±2 standard error confidence intervals.
(b) 1. Multivariate methods take into account correlation of exposures by considering all of the rating variables at once. "The main benefit of multivariate methods is that they consider all rating variables simultaneously and automatically adjust for exposure correlations."
2. Multivariate methods take into account correlation of effects.
"A benefit of multivariate methods is that they allow consideration of the interaction, or interdependency, between two or more rating variables."
3. For multivariate methods such as GLMs there are statistical tests that can be applied to test the adequacy of the model or to choose between two different models.
"Many multivariate methods produce model diagnostics, additional information about the certainty of results and the appropriateness of the model fitted."
(c) One has for example, estimated pure premiums by zipcode. Spatial smoothing will take a weighted average of the pure premium for a given zipcode with that for nearby zipcodes. This reduces the random fluctuation in the estimated pure premiums by zipcodes, prior to using them to group similar zipcodes together into territories. This will produce more homogenous territories and thus improve the estimates of territory relativities.
Sample answers from the CAS Examiner's Report (see my comment):
"Two methods of spatial smoothing include distance-based and adjacency-based. Often, defined territories are so granular that very little data exists. Spatial smoothing allows one to have more data, and thus more credibility, when analyzing these granular territories. Both methods stated above incorporate neighboring territory data (based on distance away or adjacency) which will most likely lead to more narrow confidence intervals and more refined relativities. I would recommend spatial smoothing to get a finer relativity for territory 4 in the GLM output."
"Spatial smoothing can credibility-weight the territory's experience with the experience of surrounding territories. The further away from the territory, the less weight is given."
Comment: In part (a), "a clear upward trend in relativity" is irrelevant in this case, since the numbering of the territories is somewhat arbitrary. Due to its small number of policies, it would make sense to group territory 4 with territory 3.
Basic Ratemaking states, "Multivariate methods attempt to remove unsystematic effects in the data (also known as noise) and capture only the systematic effects (also known as signal) as much as possible. This is not the case with univariate methods, which include both signal and noise in the results." However, assuming the appropriate use of credibility, univariate methods reduce the effect of noise (random fluctuation) on the result. So I do not agree with their statement. Nevertheless, this would be a full credit response to part (b) for one benefit of using multivariate methods over univariate methods for classification ratemaking.

Spatial smoothing helps to produce better territories, rather than directly improving the estimation of territory relativities. As discussed at pages 190-191 of Basic Ratemaking, spatial smoothing is performed prior to clustering units such as zipcode into territories There are two types of spatial smoothing: distance-based and adjacency-based. The former is based on miles, while the latter is based on being adjacent.

12. (1.75 points) Given the following information for an insurance company:

Limit of Liability	Current Increased Limits Factor	Indicated Increased Limits Factor
100,000	1.00	1.00
250,000	2.20	2.20
500,000	2.50	2.75
750,000	2.75	3.00
1,000,000	2.90	3.00

- The indicated increased limit factors are based on the company's own loss experience.
- Losses limited to $100,000 have been consistent over time.
- Expected losses limited to $100,000 = $500,000,000.

a. (0.75 point) Compare the expected losses for the excess layer between $500,000 and $1,000,000 based on the current increased limits factors and the indicated increased limits factors.
b. (0.5 point) Assess the appropriateness of implementing the indicated increased limits factors.
c. (0.5 point) The company wishes to offer policy limits exceeding $1,000,000 in the future. Propose an approach to calculating increased limits factors for the higher limits and briefly describe an implementation challenge the company may encounter.

12. (a) Using the current increased limits factors: (500 million) (2.9 - 2.5)/1 = $200 million.
Using the indicated increased limits factors: (500 million) (3.0 - 2.75)/1 = $125 million.
Using the current increased limits factors the estimate is $75 million more than using the proposed.
(b) The indicated increased limit factors should not be used, since there would be a free cover; since the ILFs for 750K and 1000K are the same, the insurer would be charging nothing for the additional coverage for the layer 250K excess of 750K.
(This is due to no losses for this company piercing the layer 250K excess of 750K.)
(c) The company could fit a size of loss distribution to the available data and use it to extrapolate beyond 1,000,000. For example, ILF(2 million) = LAS(2 million) / LAS(100,000), where the limited average severities are for the fitted size of loss distribution. One difficulty is that the extreme righthand tail of the fitted distribution is very sensitive to the choice of the form of he distribution and how many very large claims are in the data. Another difficulty would be to make sure the selected ILFs above $1 million are consistent with those at $1 million and lower limits.
Alternately, the company can use increased limits factors from an industry rating bureau (ISO) or major competitor. One difficulty would be to make sure the selected ILFs above $1 million are consistent with those at $1 million and lower limits.
Comment: In part (c), regardless of how the ILFs above $1 million are determined, the insurer will have to allocate additional capital and/or buy additional reinsurance due to the additional risk associated with writing higher limit policies.

13. (2.25 points) Given the following information:

Territory	Current Premium ($000)	Current Territory Factor	Indicated Territory Factor
1	90	0.80	0.70
2	300	1.00	1.00
3	260	1.15	1.10
Total	650		

Management requires achieving the following objectives with the upcoming rate change:

- Target an overall rate level increase of 10%.
- Revise territorial relativities to the indicated relativity, while capping the overall rate impact to any territory at 13%.
- Territory 2 remains the base territory.

Calculate the territorial relativities that will be implemented with the rate change.

13. The off-balance factor to divide by is:
{(90)(0.7/0.8) + (300)(1/1) + (260)(1.10/1.15)} / 650 = 0.9653.
Prior to capping:

Territory	Current Premium	Indicated Premium, 10% increase	Change
1	90	(1.1)(90)((0.7/0.8) / 0.9653 = 89.74	-0.29%
2	300	(1.1)(300)(1/1) / 0.9653 = 341.86	13.95%
3	260	(1.1)(260)(1.10/1.15) / 0.9653 = 283.40	9.00%
Total	650	715 = (1.1)(650)	

The (preliminary) increase for the base territory 2 is greater than 13%; we need to cap.
(300)(1.13) = 339. So the premium short fall would be: 341.86 - 339 = 2.86.
We would need to increase the other territories by: 2.86 / (89.94 + 283.40) = 0.766%.
(89.74)(1 + 0.766%) = 90.43. (283.40)(1 + 0.766%) = 285.57.
Then the increases for territories 1 and 3 would be: 90.43 / 90 - 1 = 0.48%, 285.57/260 = 9.83%.
(Territory 2 increases by 13%, due to the cap.)

Territory	Current Relativity	Capped Relativity
1	0.80	(0.80) (1.0048/1.13) = **0.711**
2 (base)	1.00	(1.00) (1.13/1.13) = **1.000**
3	1.15	(1.15) (1.0983/1.13) = **1.118**

Comment: There are many alternative ways to proceed to the correct answer.

14. (1.5 points) Given the following information about a homeowners insurance loss:

- Face amount of policy = $300,000.
- Value of property = $500,000.
- Coinsurance penalty = $22,000.
- Indemnity payment = $84,000.
- There is no deductible.

a. (0.5 point) Calculate the required coinsurance percentage.

b. (1 point) Identify two homeowners insurance to value initiatives insurers could implement and briefly describe how each initiative encourages insurance to full value.

14. (a) Since there is a coinsurance penalty, the coinsurance requirement must be more than 300K.

The loss was: 22,000 + 84,000 = 106,000.

The portion of the loss that was paid was: 84/106.

Thus the coinsurance requirement must be: 300,000 / (84/106) = 378,571.

Thus the required coinsurance percentage is: 378,571 / 500,000 = **75.7%**.

(b) 1. Policies that allow replacement cost to exceed the policy limit, subject to cap such as 125%, if the property is 100% insured to value. (Extended replacement cost.)
This gives insureds an incentive to insure to value in order to receive replacement cost.

2. The use of sophisticated property estimation tools, that estimate the insured value both when a home is first written and at renewal. (Software.) Then require insureds to insure to value.

3. Coinsurance clauses limit payments on partial losses which gives insureds an incentive to insure to value in order to avoid the coinsurance penalty.

4. Clauses in the insurance policy that automatically index the amount of insurance to inflation. (Inflation guard.)

5. Education of insureds of the importance to them of insuring their house to value, through for example mailing inserts or advertising.

6. Frequent property inspections. Then require insureds to insure to value.

Comment: In part (b), give only two reasons.

In part (a), 75.7% would be a very unusual coinsurance percentage as opposed to 75% or 80%.

I believe that the ISO HO2 and HO3 forms contain an 80% coinsurance clause.

15. (4.75 points) Given the following information for a book of business as of December 31, 2016:

Accident Year	Cumulative Reported Loss & ALAE ($000)
2014	5,615
2015	4,315
2016	2,745

Calendar Year	Earned Premium ($000)
2014	10,800
2015	11,250
2016	12,375

Selected Reported Loss & ALAE Age to Age Factors

12-24	24-36	36-48
2.089	1.368	1.070

- All policies are annual.
- Exposures are written evenly throughout each calendar year.
- Annual loss and ALAE trend = 5%.
- Annual premium trend = 4%.
- There has been one rate change in the past five years: +5%, effective July 1, 2015.
- Fixed expense ratio = 15%.
- Variable expense ratio = 25%.
- Profit and contingencies provision = 5%.
- ULAE provision = 6% of loss and ALAE.
- Rates are to be in effect for one year.
- There is no loss development beyond 48 months.

a. (0.5 point) Calculate the ultimate losses and ALAE for each accident year using the loss development technique.

b. (0.75 point) Calculate the ultimate losses and ALAE for each accident year using the Bornhuetter-Ferguson technique using an expected loss and ALAE ratio of 56%.

c. (0.5 point) Briefly justify an appropriate ultimate loss and ALAE selection from parts a. and b. above for accident years 2014 through 2016.

d. (3 points) Calculate the indicated rate change for policies effective July 1, 2017 using the ultimate loss and ALAE selections from part c. above, assuming full credibility.

15. (a)

AY	Reported ($000)	Estimated Ultimate ($000)
2014	5,615	(5615)(1.070) = **6008**.
2015	4,315	(4315)(1.368)(1.070) = **6316**.
2016	2,745	(2745)(2.089)(1.368)(1.070) = **8394**.

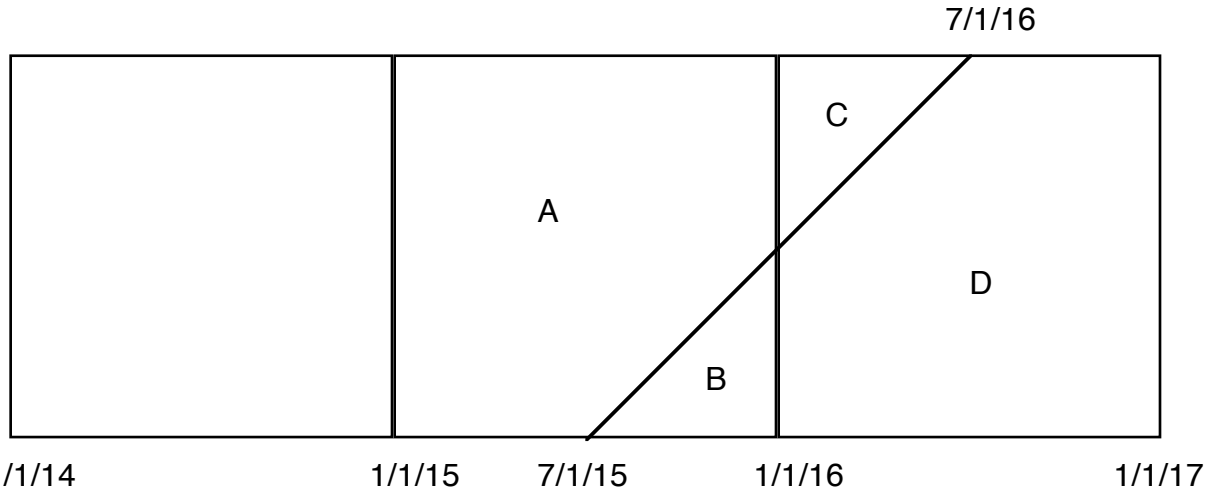
(b) Factor from 1st report to ultimate: (2.089)(1.368)(1.070) = 3.05779.
Expected percent unreported: 1 - 1/3.05779 = 67.297%.
Factor from 2nd report to ultimate: (1.368)(1.070) = 1.46376.
Expected percent unreported: 1 - 1/1.46376 = 31.683%.
Factor from 3rd report to ultimate: 1.070.
Expected percent unreported: 1 - 1/1.070 = 6.542%.

AY	Reported ($000)	Expected Unreported ($000)	Estimated Ultimate ($000)
2014	5,615	(56%)(10,800)(6.542%) = 396.	5615 + 396 = **6011**.
2015	4,315	(56%)(11,250)(31.683%) = 1996.	4315 + 1996 = **6311**.
2016	2,745	(56%)(12,375)(67.297%) = 4664.	2745 + 4664 = **7409**.

(c) Since a majority of the losses are not reported by first report, for AY16 I will select the estimate from Bornhuetter-Ferguson in part (b). The Bornhuetter-Ferguson method is more stable.
Relying on the loss development method would apply too much leverage to the losses at first report, which consist to a large extent of case reserves estimated by claims adjustors.
Since almost all of the loses have been reported by third report, for AY14 I will select the estimate from the loss development technique in part (a). In this case, there is no need to rely on the estimated loss ratio used in Bornhuetter-Ferguson method, which has the potential to introduce errors.
Since a majority but far from all of the losses have been reported by second report, for AY15 I will select an average of the estimates by the two techniques in parts (a) and (b).

AY	Estimate from Loss Dev.	Estimate from B-F	Selected ($000)
2014	6008	6011	**6011**
2015	6316	6311	(6316 + 6311)/2 = **6314**
2016	8394	7409	**7409**

(d) Area B = (1/2)(1/2)2 = 1/8 = Area C. Area A = 1 - 1/8 = 7/8 = Area D.



C Y	Average Rate Level	On-Level Earned Premium ($000)
2014	1.000	(10,800)(1.05/1) = 11,340.
2015	(7/8)(1.000) + (1/8)(1.050) = 1.00625	(11,250)(1.05/1.00625) = 11,739.
2016	(1/8)(1.000) + (7/8)(1.050) = 1.04375	(12,375)(1.05/1.04375) = 12,449.

The average date of writing under the new rates is: July 1, 2017 + 6 months = January 1, 2018.
Since policies are annual, the average date of earning under the new rates is:
January 1, 2018 + 6 months = July 1, 2018.
Thus we need to trend AY16 for 2 years.

A Y	Trended Ultimate Losses & ALAE	Trended On-Level Premiums	Loss & ALAE Ratio
2014	$(6011)(1.05^4) = 7306$	$(11,340)(1.04^4) = 13,266$	55.07%
2015	$(6314)(1.05^3) = 7309$	$(11,739)(1.04^3) = 13,205$	55.35%
2016	$(7409)(1.05^2) = 8168$	$(12,449)(1.04^2) = 13,465$	60.66%

I will use the average of the three loss ratios: (55.07% + 55.35% + 60.66%) / 3 = 57.03%.
(Other selections make sense, such as taking the ratio of the two totals.)

Indicated rate change: $\frac{(57.03\%)(1.06) + 15\%}{1 - 25\% - 5\%} = 1.078. \Leftrightarrow$ **7.8%** rate increase.

Comment: See page C-15 in Appendix C of Basic Ratemaking.
In part (c), the numerical effect is small of your selections for AY14 and AY15, in this example.
Therefore, somewhat different selections for AY14 and AY15 would also be okay.