Solutions to the
Fall 2016
CAS Exam 5

(Only those questions on Basic Ratemaking)

There were 27 questions worth 56 points, of which 15 were on ratemaking worth 29 points.

(Incorporating what I found useful from the CAS Examiner's Report)

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1. (1.25 points)
Given the following automobile policies issued during calendar years 2013 through 2015:

Effective Date	Expiration Date	Number of Policies
April 1, 2013	September 30, 2013	100
October 1, 2013	March 31, 2014	110
April 1, 2014	September 30, 2014	105
October 1, 2014	March 31, 2015	100
April 1, 2015	September 30, 2015	110
October 1, 2015	March 31, 2016	105

• All policies have a 6-month term.
a. (0.5 point) Calculate the written car-years for calendar year 2014.
b. (0.25 point) Calculate the in-force car-years as of December 31, 2014.
c. (0.5 point) Calculate the earned car-years for calendar year 2015.

1. I will assume that each policy only covers one car.
(a) Only the six-month policies written in 2014 contribute: 105/2 + 100/2 = **102.5 car-years**.
(b) Only the six-month policies written October 1, 2014 are in force on December 31, 2014.
Since the question asks for in-force car-years one could answer: 100/2 = **50 car-years**.
(c) The six-month policies written October 1, 2014 contribute half their exposures.
The six-month policies written April 1, 2015 contribute all of their exposures.
The six-month policies written October 1, 2015 contribute half their exposures.
100/4 + 110/2 + 105/4 = **106.25** car-years.
Comment: In part (b), I think a better answer to a better written question would be 100 cars, since as of December 31, 2014 the insurer is covering 100 cars.

2. (1.5 points) Given the following:

Effective Date	Rate Change
September 1, 2012	-10%
September 1, 2013	-5%
September 1, 2014	-3%

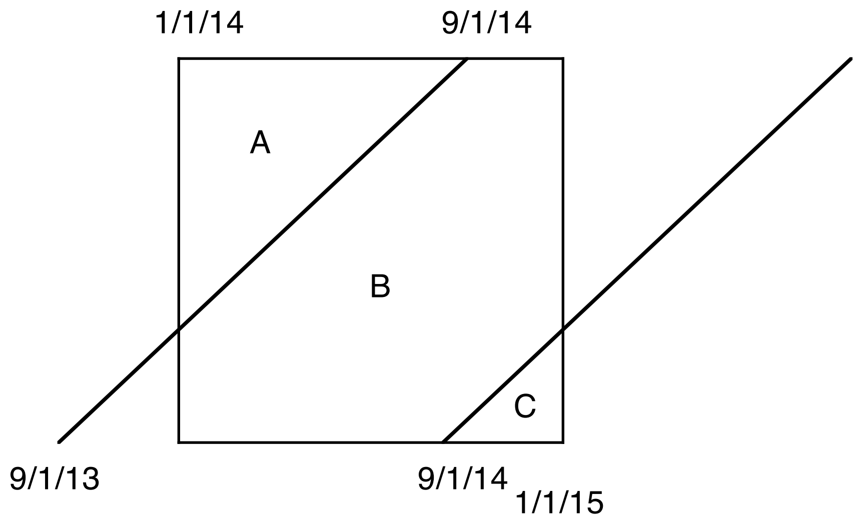
- A law change mandated a rate decrease of 15% effective February 1, 2015 applicable to all in-force policies.
- All policies are annual.

a. (1 point) Calculate the on-level factor to current rate level for calendar year 2014 earned premium.

b. (0.5 point) Identify a weakness with the parallelogram method and briefly describe a solution.

2. (a)

Effective Date	Rate Change	Rate Level Index
September 1, 2012	-10%	0.9
September 1, 2013	-5%	0.855
September 1, 2014	-3%	0.82935
February 1, 2015, outstanding	-15%	0.7049475



September 1 is 2/3 of the way through the year.

Area A = $(1/2)(2/3)^2$ = 4/18. Area C = $(1/2)(1/3)^2$ = 1/18. Area B = 1 - 4/18 - 1/18 = 13/18.

The February 1, 2015 law change has no effect on CY14 earned premiums.

The OLF is:

$$\frac{0.7049475}{(4/18)(0.9) + (13/18)(0.855) + (1/18)(0.82935)} = 0.7049475/0.863575 = \mathbf{0.8163}.$$

(b) 1. The parallelogram method assumes that policies are written evenly throughout the year.
In situations where this assumption fails to hold, one can apply the parallelogram method using more refined periods of time than a year, such as for example quarters or months.
Alternately, one can instead employ extension of exposures.
2. The parallelogram method is generally applied at the aggregate level using a series of overall average changes. So, while the overall premium may be adjusted to an approximated current rate level, the premium for certain classes will not be on-level if as is common the implemented rate changes varied by class. Consequently, the adjusted premium will likely be unacceptable for any classification ratemaking analysis.
The extension of exposures method does not share this shortcoming and can be used instead.
Comment: In part (b), only discuss one weakness.

3. (1.5 points) Given the following data for an insured:

Report Year	Reported Loss ($) by Report Year Lag 0	1	2	3	4
2011	75,300	84,000	62,400	59,000	39,800
2012	65,000	63,200	84,000	80,200	62,100
2013	82,100	49,900	55,000	60,600	72,300
2014	90,000	77,000	104,300	45,000	88,300
2015	71,800	89,000	62,000	91,500	46,600

- Policies run from January 1 through December 31.
- The insured's coverage changed from occurrence to claims-made on January 1, 2013 with a retroactive date of January 1, 2013.

a. (0.25 point)
Calculate the reported losses for the 2012 occurrence policy as of December 31, 2015.
b. (0.25 point)
Calculate the reported losses for the 2014 claims-made policy as of December 31, 2015.
c. (0.5 point) Describe how a switch from occurrence to claims-made coverage could affect an insurer's loss reserve risk.
d. (0.5 point) Describe how a switch from occurrence to claims-made coverage could affect the target underwriting profit provision.

3. Assume that all of the given values are as of December 31, 2015.
(Each of the boxes is subject to possible loss development as claims are settled and paid.)
(a) 65,000 + 49,900 + 104,300 + 91,500 = **$310,700**.
(b) 90,000 + 77,000 = **$167,000**.
(c) Eventually the loss reserve risk will decrease, because for claims-made there is no pure IBNR, while occurrence policies do have pure IBNR.
(d) Claims-made has less opportunity to earn investment income, which would increase the underwriting profit provision. (It would become more positive or less negative.)
On the other hand, the claims-made policy has less pricing and reserving risk; this would decrease the underwriting profit provision.
Which effect would predominate, depends on among other things the expected annual rate of investment income.
<u>Comment</u>: In part (a), the 2012 occurrence policy would eventually include RY16 at lag 4.
In part (b), a second year claims-made policies only includes the first two boxes in the row.

4. (3.75 points) Given the following information:

Accident Year	Frequency	Severity
2011	0.100	$25,000
2012	0.090	$27,250
2013	0.081	$30,248
2014	0.082	$33,423
2015	0.080	$36,599

Accident Year	Ultimate Losses ($000)
2013	48,000
2014	55,000
2015	60,000

- Exposures are constant.
- The company only writes semi-annual policies.
- The rate filing will be effective on January 1, 2017.
- Rates will be in effect for one year.

a. (2.5 points) Calculate the average annual trended ultimate losses that should be used to determine the indicated rate change. Briefly justify the frequency trend and severity trend selections.
b. (0.5 point) Discussions with the underwriting team reveal that changes in underwriting guidelines in the 2012 policy year resulted in lower claim counts. Describe how this information may change the estimate in part a. above without performing any additional calculations.
c. (0.75 point) Discussions with the underwriting team reveal that the company has been writing fewer high deductible policies, starting in policy year 2014. Fully describe how this information may change the estimate in part a. above without performing any additional calculations.

4. (a) The average date of writing under the new rates is July 1, 2017.
The average date of accident is 3 months later: October 1, 2017.
Thus the trend period from AY15 is from July 1, 2015 to October 1, 2017: 2.25 years.
Without fitting a linear or exponential regression, here is an analysis of the series:

AY	Frequency	Difference	Ratio	Severity	Difference	Ratio
2011	0.100			$25,000		
2012	0.090	-0.010	0.900	$27,250	2250	1.090
2013	0.081	-0.009	0.900	$30,248	2998	1.110
2014	0.082	0.001	1.012	$33,423	3175	1.105
2015	0.080	-0.002	0.976	$36,599	3176	1.095

I will apply a two piece trend method. (One could instead apply a one-piece method.)
There are many possible selections for the trend from AY15 to the policy effective period.
The most recent frequency changes have been very small, and thus I will select an annual frequency trend of 0%. The severity has increased by between 9% and 11% per year.
I will select an annual severity trend of 10%.

For example, (48,000) (1.012)(0.976) ($1^{2.25}$) (1.105)(1.095) ($1.1^{2.25}$) = 71,086.

AY	Ultimate Losses	1st piece frequency	2nd piece frequency	1st piece severity	2nd piece severity	Trended Losses
2013	48,000	(1.012)(0.976)	$1^{2.25}$	(1.105)(1.095)	$1.1^{2.25}$	71,086
2014	55,000	0.976	$1^{2.25}$	1.095	$1.1^{2.25}$	72,838
2015	60,000	1	$1^{2.25}$	1	$1.1^{2.25}$	74,351
Avg.						**72,758**

(b) Policy year 2012 contributes to accident years 2012 and 2013. Thus this change in underwriting guidelines would explain some or all of the decline in frequency seen from AY11 to AY13.
While I do not think it would cause any change in my particular selections in part (a), this should make one less likely to project a decrease in frequency into the future and more likely to select an approximately level frequency trend into the future.

(c) I assume this is a line of insurance such as collision where the insurer has no information on small losses, and that the data shown has had the deductible amounts subtracted from larger losses. Then the change in mix of deductible amounts sold will affect the data starting in policy year 2014. Policy year 2014 contributes to accident years 2014 and 2015.
Fewer policies with higher deductibles would increase the reported frequency compared to what it would otherwise have been. This may explain to some extent why the reported frequency leveled off. Thus, this information would make me more likely to project a decline in claim frequency going forward, for example an annual factor of 0.95.
The effect on the reported severity would depend on the ground up size of loss distribution; however, fewer policies with higher deductibles would usually decrease the average severity compared to what it would have otherwise been.
The pure premium would increase compared to what it would have otherwise been.
However, the change in mix of deductibles sold would also affect the historical average premiums. One would need to be consistent in how one took into account the effect of this change of mix of deductibles on loss and premium trends.

Comment: One can infer that there are about 1600 claims per year.
In part (c) I assumed that they were not referring to large deductible policies, under which the insurer settles all claims (and is later reimbursed by the insured for loss amounts below the deductible).
In such a case, the insurer would have data on ground-up losses. If the reported information were on ground-up losses, then the change in mix of large deductible amounts should have little if any effect.
In part (a), there are other selections that are reasonable.

5. (2.25 points) An insurance company purchases per risk excess-of-loss reinsurance each year that covers individual claims that exceed the retention.
Given the following information as of December 31, 2015:

Accident Year	Earned Exposures	Direct Ultimate Losses ($000)	Claim Counts
2013	1,850	185,000	185
2014	1,750	190,000	175
2015	1,650	199,500	165

Ultimate Value of Direct Claims Excess of $500,000

Accident Year	Claim	Direct Ultimate Loss of Individual Claims ($000)
2013	A	18,400
2013	B	3,200
2014	C	5,700
2014	D	5,200
2015	E	9,500
2015	F	6,200

Accident Year	Retention ($000)
2013	2,000
2014	5,000
2015	10,000

- Policies are annual.
- Policies are written uniformly throughout the year.
- Rates are expected to be in effect for one year.
- Planned rate revision to be effective January 1, 2017.

Calculate the average trended pure premium net of reinsurance at the current $10,000,000 retention.

5. The average date of writing under the new rates is July 1, 2017.
The average date of accident is 6 months later or January 1, 2018.
Thus the trend period from AY15 is from July 1, 2015 to January 1, 2018: 2.5 years.
I will use the direct data to estimate unlimited trend factors.

AY	Frequency	Severity	Pure Premium
2013	0.100	1,000,000	100,000
2014	0.100	1,085,714	108,571
2015	0.100	1,209,091	120,909

I will select an annual severity trend of: $\sqrt{1.209/1.000} = 1.10$.

Trending the individual large losses, claim A would be: $(18.4\text{M})(1.1^{4.5}) = 28.254\text{M}$.

Claim E would be: $(9.5\text{M})(1.1^{2.5}) = 12.056\text{M}$. Claim F would be: $(6.2\text{M})(1.1^{2.5}) = 7.868\text{M}$.

With a selected level frequency trend, the selected annual pure premium trend is 10%.
Thus the trended direct pure premiums are:

AY	Pure Premium	Trend	Trended P.P.
2013	100,000	$1.1^{4.5}$	153,556
2014	108,571	$1.1^{3.5}$	151,561
2015	120,909	$1.1^{2.5}$	153,441

However, the 10M retention reduces the trended AY13 net losses by:
28.254M - 10M = 18.254M. This reduces the trended pure premium by: 18.254M/1850 = 9867.
The 10M retention reduces the trended AY15 net losses by: 12.056M - 10M = 2.056M.
This reduces the trended pure premium by: 2.056M/1650 = 1246.

AY	Direct Trended P.P.	Net Trended Pure Premium
2013	153,556	153,556 - 9867 = 143,689
2014	151,961	151,561
2015	153,441	153,441 - 1246 = 152,195
AVG.		**149,148**

Alternately, apply the severity trend to the direct losses, and calculate the net losses by removing the trended excess of the $10,000,000 retention. And finally, divide by exposures to calculate the historical net pure premium and select a pure premium estimate.

AY	Direct Loss (million)	Trend Factor	Trended Direct	XS	Trended Net Loss	Exposure	Pure Premium
2013	185	$1.1^{4.5}$	284.079	18.254	265.824	1850	143,689
2014	190	$1.1^{3.5}$	265.233	0	265.233	1750	151,562
2015	199.5	$1.1^{2.5}$	253.177	2.056	251.121	1650	152,195
Total					782.178	5250	**148,986**

Alternately, limit the reported data to $500,000 per individual claim.

AY	Limited Losses	Number of Claims	Limited Severity	Ratio
2013	185M - 17.9 - 2.7 = 164.4M	185	888,648	
2014	190M - 5.2 - 4.7 = 180.1M	175	1,029,143	1.158
2015	199.5M - 9 - 5.7 = 184.8M	165	1,120,000	1.088

Thus a more stable estimate of an annual severity trend is about 12%.
(Unfortunately, mathematically the underlying severity trend of the unlimited losses has to be higher than that of the limited losses.)

Trending the individual large losses, claim A would be: $(18.4M)(1.12^{4.5}) = 30.641M$.

Claim E would be: $(9.5M)(1.12^{2.5}) = 12.612M$.

With a selected level frequency trend, the selected annual pure premium trend is 12%.
Thus the trend direct pure premiums are:

AY	Pure Premium	Trend	Trended P.P.
2013	100,000	$1.12^{4.5}$	166,526
2014	108,857	$1.12^{3.5}$	161,853
2015	120,909	$1.12^{2.5}$	160,511

However, the 10M retention reduces the trended AY13 net losses by:
30.641M - 10M = 20.641M.
This reduces the trended pure premium by: 20.641M/1850 = 11,157.
The 10M retention reduces the trended AY15 net losses by: 12.612M - 10M = 2.612M.
This reduces the trended pure premium by: 2.612M/1650 = 1583.

AY	Direct Trended P.P.	Net Trended Pure Premium
2013	166,526	166,526 - 11,157 = 155,369
2014	161,853	161,853
2015	160,511	160,511 - 1583 = 158,928
AVG.		**158,717**

Comment: Does not seem to closely parallel any example in the syllabus readings; thus it is not clear to me what you were expected to do.
The given data is inconsistent.
For example, for AY14 there are 175 - 2 = 173 smaller claims, totaling 190M - 5.7M - 5.2M = 179.1M. Thus the average size of these smaller claims is: 179.1M / 173 = 1,035,260.
However, these smaller claims are supposed to be of size less than or equal to $500,000.
The CAS Examiner's Report has my second solution, except it has the pure premiums all too small by a factor of 1000. The Direct Ultimate Losses are given in ($000).
"Per risk excess-of-loss reinsurance covers individual claims that exceed the retention."

6. (1.25 points) The following information is available for a single-state, mono-line insurer:

	Calendar Year ($000)		
	2013	2014	2015
General Expense	4,525	4,175	3,875
Other Acquisition	5,220	6,000	6,750
Commissions/Brokerage	8,700	8,000	7,500
Taxes, Licenses and Fees	3,480	3,200	3,000
Total Expenses	21,925	21,375	21,125

	Calendar Year ($000)		
	2013	2014	2015
Written Premium	87,000	80,000	75,000
Earned Premium	90,500	83,500	77,500

The company's pricing actuary is asked to calculate an expense provision for 2016, and does so using a ratio of three years' total expense to three years' earned premium as follows:

$$\text{Expense provision} = \frac{21{,}925{,}000 + 21{,}375{,}000 + 21{,}125{,}000}{90{,}500{,}000 + 83{,}500{,}000 + 77{,}500{,}000} = 25.6\%.$$

a. (0.75 point) Briefly discuss three reasons why the actuary's approach is not appropriate.

b. (0.5 point) Identify an alternative approach to calculate the expense provision and briefly explain its benefit relative to the actuary's approach without performing any additional calculations.

6. (a) 1. By dividing by earned premiums, the actuary has assumed that the expenses are incurred on average throughout the policy effective period. It is usual to instead divide Other Acquisition, Commissions/Brokerage, and Taxes, Licenses and Fees by written premiums. This assumes that these expenses are incurred on average at the time the policy is written.
2. The ratios of other acquisition expenses to written premium by year are 6%, 7.5%, and 9%. The actuary should investigate why these ratios are increasing and whether this pattern can be expected to continue into the future.
3. The actuary has treated all of the expenses as variable. The actuary should investigate whether it would be better to treat some of the General Expense and Other Acquisition Expense as fixed, in other words as varying with exposures rather than premiums.
4. The premiums are shrinking over time.
Thus using a ratio of total expenses to total premiums gives more weight to older years.
This is likely inappropriate, as recent years are likely more reflective of future expenses.
(b) I would for each year divide General Expenses by Earned Premium and each of the other expense items by written premium. This would better match when the latter types of expenses are incurred. Then I would select provisions for each. (For each year General Expense is 5%, Commissions/Brokerage is 10%, and Taxes, Licenses and Fees are 4%.)
The actuary's method ignores the increasing ratios of other acquisition expenses: 6%, 7.5%, and 9%. Instead I would select either the last year's value other acquisition expenses or something even higher, based on the upward trend in these ratios.
Alternately, instead of the all-variable approach we could use the Premium-Based Projection Method. By splitting into variable and fixed expenses It prevents us from misestimating fixed expenses in situations where the average premium is changing over time.
Alternately, use the Exposure-Based Projection Method, which divides the dollar amount of fixed expense by exposures, and takes a ratio of variable expenses to premium. One would then apply an expense tend to the fixed expense pure premiums. This would provide a better estimate of the expenses needed in 2016, since fixed expenses are a function of the exposures and not the level of premiums.
Comment: For a monoline insurer operating in one state, for purposes of determining an overall rate increase, the distinction between treating expenses as fixed or variable is unlikely to have much effect. For determining rates by class and territory this distinction would likely be important.

7. (1.75 points)
A regulator wants to benchmark the underwriting profit provisions between companies.

For Company A's rate filing, the following is assumed:

Projected total fixed costs	\$50,000
Projected total loss and LAE	\$600,000
Projected exposures	2,000
Indicated rate per exposure	\$500

For Company B's rate filing, the following is assumed:

Projected total fixed costs	\$50,000
Projected total loss and LAE	\$600,000
Projected premium at current rates	\$900,000
Indicated rate change	16.5%

• The variable expense ratio is the same for each company.
a. (1.25 points) Determine which company's filing includes the higher underwriting profit provision.
b. (0.5 point) List two reasons the underwriting profit provision might differ between companies with the same loss, LAE and expense experience.

7. (a) For Company A: (\$650,000 / 2000) / (1 - V - Q) = \$500. $\Rightarrow$ V+Q = 35%.
For Company B: (\$650,000 / \$900,000) / (1 - V - Q) = 1.165. $\Rightarrow$ V+Q = 38%.
Since V is the same for both companies, Company **B** has the higher underwriting profit provision.
b) 1. The length of the cashflow and thus the opportunity for investment income from funds held varies significantly from line of insurance to line of insurance.
2. The companies may have different desired rates of return on equity.
This may be due to differences in the (perceived) riskiness of the business being written.
3. One or both companies may have chosen their underwriting profit provision to some extent based on the competitive environment faced by that company.
4. The companies may have different estimates of the expected rate of return on investments.
5. The companies may be assuming different premium to surplus ratios.
6. The companies may face different regulatory restrictions on the profit provisions allowed to be used in rate filings.

<u>Comment</u>: Part (b) is barely discussed in <u>Basic Ratemaking</u>.
Differences in rates of return on equity may for example be based on differences on estimates of the riskiness of writing the line(s) of insurance. Alternately, differences in rates of return on equity can be based on differences in the structure of the insurer: mutual versus stock.
In part (b), besides using different inputs, the two companies may use different underwriting profit models or techniques for determining the appropriate underwriting provision.

8. (3.5 points) Given the following information about an insurance product:
- The product launched on January 1, 2012.
- All policies are annual.
- The rating algorithm is exposures multiplied by a fixed manual rate.
- The average written manual rate per exposure in 2013 = $5,000.
- Exposures are written uniformly throughout the year.
- A large loss of $2 million occurred and was paid in 2014.
  Underwriting guidelines have been revised such that further losses of this type are not expected.
- Losses do not develop after 36 months.
- The age-to-age factors in the latest diagonal are representative of future loss development.
- Rates will be in effect for two years.

Annual loss cost trend	5%
Annual premium trend	0%
Fixed expense ratio	0%
Variable expense ratio	22%
Profit and contingencies provision	6%
ALAE provision	12% of loss
ULAE provision	7% of loss

Rate Change History

Effective Date	Change
July 1, 2014	+7.5%
July 1, 2015	+3.0%

Calendar Year	2012	2013	2014	2015
Written Exposures	805	850	825	875

Cumulative Reported Loss ($000)

Accident Year	12 months	24 months	36 months
2013	--------------	1,100	1,150
2014	2,940	4,210	
2015	1,020		

Calculate the indicated rate change for policies effective between July 1, 2017 and July 1, 2019 based on the most recent three accident years of experience and assuming full credibility.

8. The current rate is: (5000)(1.075)(1.030) = $5536.
Since policies are annual and exposures are written uniformly throughout the year, I will estimate earned exposures by calendar year for 2013, 2014 and 2015, by averaging written exposures: (805 + 850)/2 = 827.5, (850 + 825)/2 = 837.5, and (825 + 875)/2 = 850.
"Underwriting guidelines have been revised such that further losses of this type are not expected."
Thus I will restate the AY14 losses to exclude the large loss paid in 2014:
2940 - 2000 = 940. 4210 - 2000 = 2210.
Link ratios: 1150/1100 = 1.045. 2210/940 = 2.351.
The average date of writing under the new rates is July 1, 2018.
The corresponding average date of accident is six months later: January 1, 2019.
The trend period from AY15 is 3.5 years.

AY	Losses	LDF	Trend	Trended Ultimate Losses (000)
13	1150	1	$1.05^{5.5}$	1504
14	2210	1.045	$1.05^{4.5}$	2876
15	1020	(1.045)(2.351)	$1.05^{3.5}$	2973

The loss pure premium for the 3 AYs combined is:
($1000) (1504 + 2876 + 2973) / (827.5 + 837.5 + 850) = $2924.
Indicated rate is: (2924)(1.19) / (1 - 22% - 6%) = $4833.
Indicated rate change is: 4833 / 5536 - 1 = **-12.7%**.
Alternately, the on-level premiums are: ($5536) (827.5 + 837.5 + 850) = $13,923,040.
Trended ultimate losses are: ($1000) (1504 + 2876 + 2973) = $7,353,000.
Loss ratio is: 7,353,000/13,923,040 = 52.81%.
Indicated rate change: (52.81%) (1 + 12% + 7%) / (1 - 22% - 6%) - 1 = **-12.7%**.
Comment: I used the latest three AYs, since this exam question said to so so.
The pure premiums by year are: (1000)(1504)/827.5 = 1818, (1000)(2876)/837.5 = 3434, and (1000)(2973)/850 = 3498. AY13 is very different from the other two, which might lead one to rely on only the two most recent accident years. On the other hand, we don't know what line of insurance this is, nor how much random fluctuation we should expect from year to year in the pure premiums when we have this volume of data.

9. (1 point) Given the following:

General Expenses	$225,000
Written Premium	$3,750,000
Earned Premium	$3,000,000
Other Acquisition Expense	8.0%
Commission	12.0%
Taxes, Licenses & Fees	3.0%
Projected Ultimate Loss and LAE Ratio	62.0%
Target Underwriting Profit Ratio	5.0%

- All expenses are paid at policy inception.
- Commission and Taxes, Licenses & Fees are 100% variable.
- All other expense categories are 50% variable.

Calculate the indicated rate change assuming the data is fully credible.

9. Due to the first bullet, I will divide the General Expenses by written premiums:
225,000 / 3,750,000 = 6%.

Category	%	% Fixed	Fixed	Variable
General	6%	50%	3%	3%
Other Acquisition	8%	50%	4%	4%
Commission	12%	0%	0	12%
Taxes, Licenses & Fees	3%	0%	0	3%
Total			7%	22%

Indicated rate change is: $\frac{62\% + 7\%}{1 - 22\% - 5\%} - 1 =$ **-5.48%**.

Comment: We usually assume that General Expenses are paid throughout the policy term, and thus divide the General Expenses by earned premiums.

10. (1 point) A homeowners insurance company is considering utilizing number of vehicles in the household as an additional risk characteristic within its risk classification system.
Briefly discuss the appropriateness of adding this risk characteristic to the company's risk classification system using four considerations from the Actuarial Standard of Practice No. 12: Risk Classification (for All Practice Areas).

10.
1. Relationship of Risk Characteristics and Expected Outcomes: The issue is whether the loss costs vary in a consistent way with the number of vehicles in the household, adjusting for those other risk characteristics already being used. Since we are given no data, further analysis of the appropriateness is not possible.
(I would expect the number of vehicles to be correlated with other possible variables which may be correlated with expected losses such as: value of home, territory, number of adults resident in the home, number of rooms, marital status, whether the home is owned or rented, etc.)
2. Credibility: Larger size categories (e.g., those with more than 5 vehicles) may lack enough volume to satisfy credibility concerns. Perhaps homes with more than 5 vehicles could be grouped together to determine loss costs for those homes associated with a large number of vehicles
3. Causality: There does not seem to be an obvious intuitive relationship between the number of vehicles and homeowners losses. However, it is not necessary for the actuary to establish a cause and effect relationship between the risk characteristic and expected outcome in order to use a specific risk characteristic. Thus this is not an issue.
4. Objectivity: The number of vehicles should be able to be objectively determined; however one would have to come up with a complete and careful definition. Presumably bicycles, riding mowers, etc., are not included. Also, does it matter whether or not a vehicle is principally garaged at the insured home?
5. Practicality: It should not be too expensive in time or money to establish the number of vehicles. (How much expense is reasonable would depend on the difference in expected loss costs for the different levels of the proposed risk characteristic.)
6. Applicable Law: As far as I am aware the use of this risk characteristic is not against the law.
7. Industry Practices: As far as I am aware, the proposed risk characteristic in not customarily used. This would make it less appropriate.
8. Business Practices: I am not aware of any business practices that would preclude the use of the proposed risk characteristic.
<u>Comment</u>: Given the lack of detail, I found it difficult to come up with things to say.
For only 1/4 point per item for four items, do not go into too much detail.
I did not see how adverse selection or homogeneity would be useful to answer this question.
“Adverse Selection - If the variation in expected outcomes within a risk class is too great, adverse selection is likely to occur. To the extent practical, the actuary should establish risk classes such that each has sufficient homogeneity with respect to expected outcomes to satisfy the purpose for which the risk classification system is intended.”

11. (3.25 points) Given the following ground-up uncapped loss profile for a book of business:

Claim Type	Number of Claims	Loss Amount of each Claim
A	200	\$5,000
B	100	\$20,000
C	10	\$100,000
D	10	\$400,000

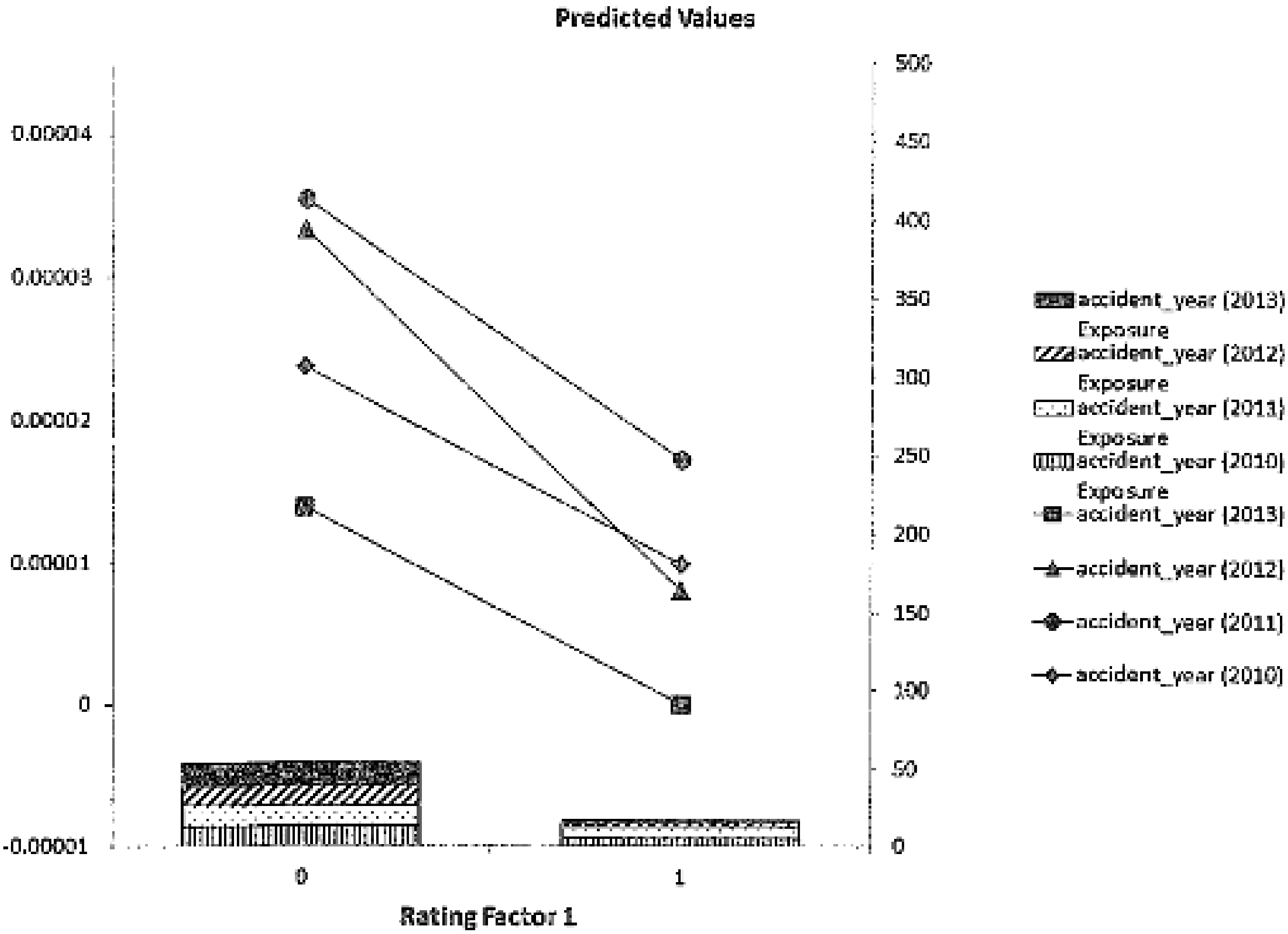
a. (1.25 points)
Calculate the increased limits factor for an increased limit of \$25,000 and a basic limit of \$10,000.
b. (1.5 points) Calculate the severity trend for the layer excess of \$50,000 assuming a ground-up severity trend of 10% over the next year.
c. (0.5 point) Provide one reason why the data above would not be appropriate to determine an increased limits factor for \$100,000 and suggest an alternative source that could be used.

11. (a) $E[X \wedge 10{,}000] = 1000 \dfrac{(200)(5) + (100)(10) + (10)(10) + (10)(10)}{200 + 100 + 10 + 10} = 6875.$

$E[X \wedge 25{,}000] = 1000 \dfrac{(200)(5) + (100)(20) + (10)(25) + (10)(25)}{200 + 100 + 10 + 10} = 10{,}937.5.$

ILF = $E[X \wedge 25{,}000] / E[X \wedge 10{,}000]$ = 10,937.5 / 6875 = **1.591**.
(b) Prior to trend, the layer excess of 50,000: (10)(50,000) + (10)(350,000) = 4 million.
After trend, the layer excess of 50,000:
(10)(110,000 - 50,000) + (10)(440,000 - 50,000) = 4.5 million.
Trend in the excess layer: 4.5/4 - 1 = **12.5%**.
(c) There are only 10 claims of size greater than 100,000, which creates credibility concerns.
One could instead use a larger set of similar data such as industrywide data.
Alternately, one could instead use increased limit factors from a rating bureau like ISO.
Alternately, one could instead base the ILFs on fitted distributions.
Alternately, one could instead base the ILFs on a competitor's rate pages or rate filing.
Comment: Since larger claims often develop differently than smaller claims, this can have an impact on the calculation of the increased limit factors. Ideally, all claims should be developed to ultimate before the application of these techniques. Also historical losses used in the analysis should be adjusted for any expected trend. If the given data has not been trended and developed to ultimate, it would not be appropriate to use it to determine an increased limits factor for \$100,000.

12. (1 point) The following graph provides the output from a generalized linear model (GLM):



a. (0.5 point) Briefly explain whether this variable should be included in the rating plan.
b. (0.5 point) Briefly discuss two reasons why GLM analysis is typically performed on loss cost data instead of loss ratios.

12. (a) Take level 0 as the base level. Then the effects of level 1 relative to level 0 by accident year are approximately: 0.000010 - 0.000024 = -0.000014, 0.000018 - 0.000035 = -0.000017, 0.000008 - 0.000033 = -0.000025, and 0.000000 - 0.000013 = -0.000013.
The relativities differ somewhat between the different accident years, but I believe are similar enough to include this variable in the rating plan if one uses an additive structure.
Alternately take instead multiplicative relativities: 0.000010 / 0.000024 = 0.42,
0.000018 / 0.000035 = 0.51, 0.000008 / 0.000033 = 0.24, and 0.000000 / 0.000013 = 0.
The relativities differ between the different accident years, and are dissimilar enough to not include this variable in the rating plan if one uses a multiplicative structure.
Alternately, we have only about 75 exposures combined for all accident years. Thus the results of the GLM are unlikely to be meaningful. Thus there is no evidence to include this variable in the rating plan.
(b) 1. Modeling loss ratios requires premiums to be adjusted to current rate level at the granular level and that can be practically difficult.
2. Experienced actuaries have an a priori expectation of frequency and severity patterns (e.g., youthful drivers have higher frequencies). In contrast, the loss ratio patterns are dependent on the current rates. Thus, the actuary can better distinguish the signal from the noise when building models based on frequency and severity.
3. Loss ratio models become obsolete when rates and rating structures are changed.
4. There is no commonly accepted distribution for modeling loss ratios.
Comment: In part (a), I believe that the CAS had been looking for my first response, but did accept others that were well reasoned.
Personally, I found it hard to figure out the meaning of the given graph; what are the “predicted values”? In my opinion, part (a) was a poorly constructed question.
I would want a lot more information, such as confidence intervals, in order to decide whether to include this variable in the rating plan. There is no way to determine whether the predicted values are statistically significant and/or significant from a business point of view.

13. (3 points)

An insurance company is considering updating its territorial relativities given the following information:

Territory	Number of Exposures	Trended and Ultimate Incurred Losses & ALAE	Current Territorial Relativity
1	30,000	$3,000,000	1.100
2	50,000	$4,000,000	1.000
3	25,000	$1,500,000	0.850

- The base territory remains the same.
- Exposures are homogeneous within each territory.
- The full credibility standard = 45,000 exposures.
- Partial credibility is determined by the square root rule.
- Complement of credibility is equal to normalized current territorial relativities.

a. (1.5 points)
Calculate the credibility weighted territorial relativities using the pure premium approach.

b. (0.75 point) Determine the percent change by territory, assuming the indicated relativities are to be adopted and no overall premium change is desired.

c. (0.75 point) Briefly discuss three reasons why proposed rate changes might deviate from indicated rate changes.

13. (a) The exposure weighted average current relativity is:

$$\frac{(30)(1.10) + (50)(1.00) + (25)(0.85)}{30 + 50 + 25} = 0.9929.$$ Then for example, 1.1/0.9929 = 1.1079.

For example, 100/80.95 = 1.2353. $\sqrt{30{,}000 \ / \ 45{,}000} = 81.65\%$.

(81.65%)(1.2353) + (1 - 81.65%)(1.1079) = 1.2119. 1.2119/0.9883 = 1.226.

Terr.	P.P.	Indic. Rel.	Normalized Curr. Rel.	Z	Cred. Rel.	Prop. Rel. To Base
1	100	1.2353	1.1079	81.65%	1.2119	**1.226**
2	80	0.9883	1.0072	100%	0.9883	**1.000**
3	60	0.7412	0.8561	74.54%	0.7705	**0.780**
Overall	80.95					

(b) Solely for convenience, assume for example a current base rate of $1000.

Then the current average rate is: $\frac{(30)(1100) + (50)(1000) + (25)(850)}{30 + 50 + 25} = \$992.86.$

If one kept the same base rate, the new average rate would be:

$$\frac{(30)(1226) + (50)(1000) + (25)(780)}{30 + 50 + 25} = \$1012.19.$$ Thus in order to keep the same average rate,

the new base rate has to be: (1000) (992.86/1012.19) = $980.90.

Terr.	Current Rate	New Rate	Change
1	1100	(1.226)(980.90) = 1202.58	**+9.3%** = 1202.58 / 1100 - 1
2	1000	980.90	**-1.9%**
3	850	(0.780)(980.90) = 765.10	**-10.0%**

(c) 1. Regulatory concerns. The insurance department may not have been willing to approve the (full amount) of an indicated increase. This could be the overall amount and/or the impact on individual insureds.

2. Competitive concerns. The insurance company may have worried about keeping its rates comparable to those of its chief competitors.

If insureds know another company offers the same product at a substantially lower price, they are likely to purchase the competing product.

3. Retention concerns. The insurance company may have worried that a large rate increase to individuals would cause many of its insureds to shop around for cheaper coverage.

Significant increases (or decreases) in premium for an existing policy can cause existing insureds to believe there may be better options available.

4. Lifetime analysis may lead the insurer to accept less premium now in order to retain more insureds and make more profit over the longterm on these policyholders.

5. For lines of business where very large magnitudes of rate indications are expected due to volatility and insufficient credibility of data, actuarial judgment may be used to propose a more reasonable rate change.

6. The insurer has decided to address the imbalance in rates by tightening underwriting guidelines in order to restrict business from being written at inadequate rates.

Comment: There are other possible reasonable answers to part (c).
In part (c) we could be considering: an insurer implementing less than the overall indicated increase, an insurer implementing less than the overall indicated decrease, an insurer capping the indicated changes in relativities, etc.
In part (b), the CAS Examiner's report has a sample solution that incorrectly computes the off-balance (to be multiplied by) as: 1/ (1 + exposure weighted change in relativities) = 1/1.013.
What I did instead results in no change in total premiums.
One can compare the exposure weighted relativities before and after; this is equivalent to what I did.

The current exposure weighted relativity is: $\frac{(30)(1.100) + (50)(1.000) + (25)(0.850)}{30 + 50 + 25} = 0.9929.$

Proposed exposure weighted relativity is: $\frac{(30)(1.226) + (50)(1.000) + (25)(0.780)}{30 + 50 + 25} = 1.0122.$

Thus the off-balance to multiply by is: 0.9929/1.0122 = 0.981.
Alternately, one can compute the (variable) premium weighted change in relativities:

$$\frac{(30)(1.100)(11.5\%) + (50)(1.000)(0) + (25)(0.850)(-8.3\%)}{(30)(1.100) + (50)(1.000 + (25)(0.850)} = 1.95\%.$$

Thus the off-balance to multiply by is: 1 / (1 + 1.95%) = 0.981.

14. (1.25 points) An insured purchases a $400,000 policy on a property valued at $500,000.

• The coinsurance requirement for the policy is 90% of property value.
• No deductible applies.

a. (0.25 point) Calculate the coinsurance penalty for a $300,000 loss.
b. (0.25 point) Calculate the maximum coinsurance penalty.
c. (0.25 point) Calculate the coinsurance apportionment ratio, assuming the property is valued at $425,000 instead of $500,000.
d. (0.5 point) Briefly describe two issues associated with underinsured properties.

14. (a) Coinsurance requirement is: (90%)(500,000) = 450,000.
Amount paid is: (300,000)(400/450) = 266,667.
Coinsurance penalty is: 300,000 - 266,667 = **$33,333**.
(b) The maximum coinsurance penalty occurs when the loss is equal to the face amount of $400K.
Amount paid is: (400,000)(400/450) = 355,555.
Coinsurance penalty is: 400,000 - 355,555 = **$44,445**.
(c) The coinsurance requirement is now: (90%)(425,000) = 382,500.
Coinsurance apportionment ratio is: Min[1, 400,000/382,500] = **1**.
(d) 1. Assuming partial losses are possible, a rate calculated based on assuming insurance to value will be <u>inadequate</u> for an underinsured property. Thus, premiums will not be equitable for underinsured versus fully insured policies.
2. If there is a total loss, the owner of the underinsured property will not be returned to the pre-loss condition; the insured will not be fully covered for a total loss. In other words, the property owner will not be able to replace the destroyed property without coming up with additional money of its own beyond the payment from the insurance company.
3. An insurance regulator might force an insurer to pay above the policy limit for underinsured policies in the event of a catastrophe.

15. (1.75 points) Given the following for a workers compensation policyholder:

Individual Claims Reported During the Experience Period

$19,000
$3,000
$102,500
$11,000

- Standard premium = $435,000.
- 3-year payroll = $14,590,000.
- Expected loss rate = 2.40 per $100 of payroll.
- D-ratio = 0.19.
- Primary loss cap = $5,000.
- Primary credibility = 0.75.
- Excess credibility = 0.15.

Calculate the policy's premium under an experience rating plan.

15. I will assume that the given premium is intended to be manual premium rather than standard premium, in other words prior to being multiplied by the experience modification. Also I will ignore expense constants and premium discounts since they are not mentioned in the question.
I assume the experience period is three years.
Actual Primary losses = 5000 + 3000 + 5000 + 5000 = 18,000.
Actual Excess losses = 14,000 + 0 + 97,500 + 6000 = 117,500.
Expected losses = (2.40) (14,590,000/100) = 350,160.
Expected Primary losses = (0.19)(350,160) = 66,530.
Expected Excess losses = 350,160 - 66,530 = 283,630.

$M = 1 + Z_p(A_p - E_p)/E + Z_e(A_e - E_e)/E$

$= 1 + (0.75)(18{,}000 - 66{,}530)/350{,}160 + (0.15)(117{,}500 - 283{,}630)/350{,}160 = 0.82.$

Standard Premium = (mod)(manual premium) = (0.82)(435,000) = **$356,700**.

Alternately, $M = \dfrac{Z_p A_p + (1 - Z_p) E_p + Z_e A_e + (1 - Z_e) E_e}{E}$

$$= \frac{(0.75)(18{,}000) + (0.25)(66{,}530) + (0.15)(117{,}500) + (0.85)(283{,}630)}{350{,}160} = 0.82.$$

Standard Premium = (mod)(manual premium) = (0.82)(435,000) = **$356,700**.

Comment: Standard premium is manual premium times the experience modification.
The policy's premium would then include any expenses constants and premium discounts.
The 3 year payroll should be for the 3-year past experience period used in the rating;
for simplicity we have assumed only one class with the given expected loss rate.