Solutions to the
Fall 2015
CAS Exam 5

(Only those questions on Basic Ratemaking)

There were 25 questions worth 55.75 points, of which 12.5 were on ratemaking worth 28 points.

The exam and examiner's report are available from the CAS.
The solutions and comments are solely the responsibility of the author.

(Incorporating what I found useful from the CAS Examiner's Report)

prepared by
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1. (2.75 points) Given the following information:

Effective Date	Number of Autos Written on Effective Date
February 1, 2013	1,100
August 1, 2013	800
February 1, 2014	600
August 1, 2014	300

- All policies have six-month terms.
- The exposure base is earned car years.
- The premium per auto is $500 per six-month term for policies effective through August 31, 2014.
- A uniform rate change of -18% became effective September 1, 2014.

a. (0.75 point) Calculate the written and earned exposures for calendar year 2014.

b. (2 points) Calculate the earned premium at current rate level for calendar year 2014 using both the parallelogram method and extension of exposures method, and discuss which method is more appropriate for this situation.

1. (a) CY14 written exposures: (600 + 300) / 2 = **450 caryears**.

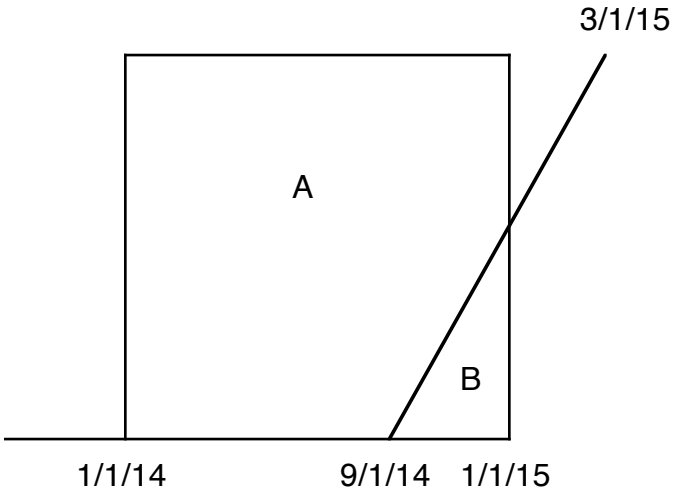
Policy Written 8/1/13 contributes 1/6 of its exposures to CY14 earned exposures.

CY14 earned exposures: {800/6 + 600 + (300) (5/6)} / 2 = **491.67 caryears**.

(b) The current rate is: (1 - 18%)($500) = $410. However, this is for one half caryear.

Using extension of exposures, CY14 Earned premium @ current rate level:

(491.67)($410)(2) = **$403,169**.



Area B is: (1/2)(4/12)(8/12) = 1/9.

CY14 earned average rate level index is: (8/9)(1) + (1/9)(0.82) = 0.980.

OLF is: 0.82 / 0.980 = 0.837.

The CY14 earned premium at actual rates is: (491.67)($500)(2) = $491,670.

Using parallelogram method, CY14 Earned premium @ current rate level:

(0.837)($491,670) = **$411,528**.

Whenever the appropriate data is available, extension of exposures is more accurate than the parallelogram method. In this case, the parallelogram method is undesirable since the exposures are neither written evenly throughout the year nor constant from year to year.

2. (1.5 points) a. (0.75 point) Based on two relevant criteria, propose and briefly justify an appropriate exposure base for a general liability policy for a restaurant.
b. (0.75 point) Based on two relevant criteria, propose and briefly justify an appropriate exposure base for a hospital professional liability policy.

2. (a) I propose **sales** as the exposure base.
Sales is proportional to expected losses. More sales corresponds to more customers, which corresponds to more expected losses. There is a uniform and continuous multiplicative relationship between sales and the expected losses. Sales is responsive to a change in the amount of business done by the restaurant and thus a change in exposure to risk. Sales also has the advantage of being inflation sensitive. During periods of high inflation, average severity will increase but so will sales.
Sales are practical. Sales are objective and well defined, relatively easy and inexpensive to obtain, and relatively easy and inexpensive to verify. Thus sales can be consistently measured, and are not subject to manipulation by insureds, agents, or underwriters.
(b) I propose a **combination of average number of occupied beds and annual number of outpatient visits**.
Average number of occupied beds is proportional to expected losses from liability related to inpatient events, while annual number of outpatient visits is proportional to expected losses from liability related to outpatient events.
These are practical. They are objective and well defined, relatively easy and inexpensive to obtain, and relatively easy and inexpensive to verify. They are each items hospitals have to keep track of anyway. They can be consistently measured, and are not subject to manipulation by insureds, agents, or underwriters.
Comment: In part (b) one could instead propose gross revenue from patients; this would have the advantage of being inflation sensitive. Other possibilities: number of patients, number of medical professionals (doctors and nurses) on staff, payroll for medical professionals,
number of physician-years (the number of physicians adjusted for those who are only on staff for part of a year in analogy to houseyears for homeowners), occupied beds.
In part (a) one could instead propose payroll.
In my solution I have not discussed the third desirable feature, Historical Precedence, since the exposure bases for these lines of insurance are not covered on the syllabus of this exam. You could do so if you are familiar with what is currently being used as an exposure base.
I believe sales is used for a general liability policy for a restaurant.
I believe a combination of average number of occupied beds and number of outpatient visits are used (by at least some insurers) for a hospital professional liability policy.

3. (1.5 points) A personal auto insurer has recently completed the acquisition of a smaller insurer in order to increase their market share in a state.
An actuary has calculated a rate level indication, using only the smaller insurer's historical data for that state.
a. (0.5 point) Explain the general role of credibility in ratemaking.
b. (1 point) Propose a complement of credibility for the analysis and evaluate it based on three desirable qualities.

3. (a) In overall ratemaking, classification ratemaking, or individual risk rating, an actuary will often weight together two estimates using Z and (1-Z). As here, when the subject experience has too small a volume of data to be statically reliable, the actuary supplements it with another estimate called the complement, in order to improve the statistical reliability.
Alternately, the general role of credibility in ratemaking is to assess how much weight should be reasonably given to the actual data and how to determine a reasonable complement. The purpose is to ensure that rate changes are credible and due to signals rather than noise created by random fluctuations in small amounts of data. If data is too sparse or erratic, it should not be used by itself when creating a rate indication.
(b) I propose a complement of credibility of the rate change based on the data of the larger insured excluding that of the smaller insured.
It is (largely) independent, since we have excluded the experience from the smaller insurer.
It should be accurate; since it is based on a larger volume of data it should have a smaller variance.
It should be available, since the insurer would normally produce such an indication as matter of course.
It is (relatively) easy to calculate; it would be a parallel calculation to that for the indication based on the small insurer's data.
It is unbiased. On average the indicated change in rates for the larger insurer should be equal to that for the smaller insured. (However, due to different mixes of business and underwriting guidelines, this complement may be biased.)
There is a logical relationship between the small insurer's experience in the state and the acquiring insurer's experience in the same state. (However, the rate level change for the small insurer depends on its current rate level as well as its needed rate level. Since this insurer has just been acquired, I do not see a close logical relationship between the needed rate changes for the two insurers.)
Comment: In part (b) discuss only three desirable qualities.
We would also like a logical relationship to the base statistic.
The two rate indications are not totally independent since they are probably based on similar trend and development selections made by the same actuary at the same point in time.
Due to different mixes of business there may not a logical relationship between the average rates for the two insurers
According to Basic Ratemaking, using trended present rates as a complement of credibility is not recommended when as here there is a larger group to use for the complement.

4. (2.25 points) An actuary is calculating a rate change to be effective July 1, 2016.
Given the following:

- Policies are written on a semi-annual basis.
- Rates are expected to be in effect for one year.
- The exposure base is non-inflationary.
- The annual frequency and severity exponential trend fits based on data for the 12 months ending each quarter evaluated through December 31, 2014 are as follows:

Number of Points	Frequency Exponential Fit	Severity Exponential Fit
20 point	-2.9%	3.4%
16 point	-3.2%	3.0%
12 point	-2.5%	2.8%
8 point	-0.5%	2.9%
6 point	3.0%	3.1%
4 point	2.8%	3.3%

Calculate a pure premium trend factor for accident year 2012, justifying the selected trends and methodology.

4. Average data of writing is 7/1/16 plus 6 months = 1/1/17.
Since there are 6 month policies, the trend to date is: 1/1/17 + 3 months = 4/1/17.
Trend from date is 7/1/2012. Trend period is 4.75 years for a one step method.
For the severity trend, I will select a 3% trend because all of the indicated trend values seem to hover around this value.
For the frequency trend, I will select a trend factor of 1%. This is because the trend value from year end 2012 (8 point trend) is negative, but there appears to be a positive trend going into the future. Therefore, I judgmentally selected a factor in between instead of doing a 2 step trend.
Thus, the one step pure premium trend factor is: $\{(1.01)(1.03)\}^{4.75}$ = **1.206**.
Alternately, the annual pure premium trends based on 12, 16, and 20 points are:
1.0040, 0.9970, 1.0023.
I will select the 16 point trend based on a compromise between stability and responsiveness.
Thus the one step pure premium trend factor is: $\{(0.968)(1.030)\}^{4.75}$ = **0.986**.
Alternately, one can use the two-step method.
Step one of the trend period is from the average date of accident for AY12 to the most recent data available in the trend series, which is the year 2014: 7/1/12 to 7/1/14.
Step two is from 7/1/14 to 4/1/17, which is the average date of accident under the new rates.
The frequency trend has changed significantly. I will therefore use a different trend for different periods. I will select -2.5% for the first step and 2.9% for the second step.
The severity trend is stable; I will select 3.1% for both steps.
Thus, the two step pure premium trend factor is: $\{(0.975)(1.031)\}^{2}$ $\{(1.029)(1.031)\}^{2.75}$ = **1.189**.
Comment: Many other selections would be reasonable.
Be sure to specify whether you are using the one-step or two-step method.

5. (3 points) An insurance company writes annual policies.

The history of law and coverage changes affecting benefit levels is as follows:

Effective Date	Direct Impact of Benefit Change
February 15, 2014	+6.5%
October 1, 2014	+4.3%

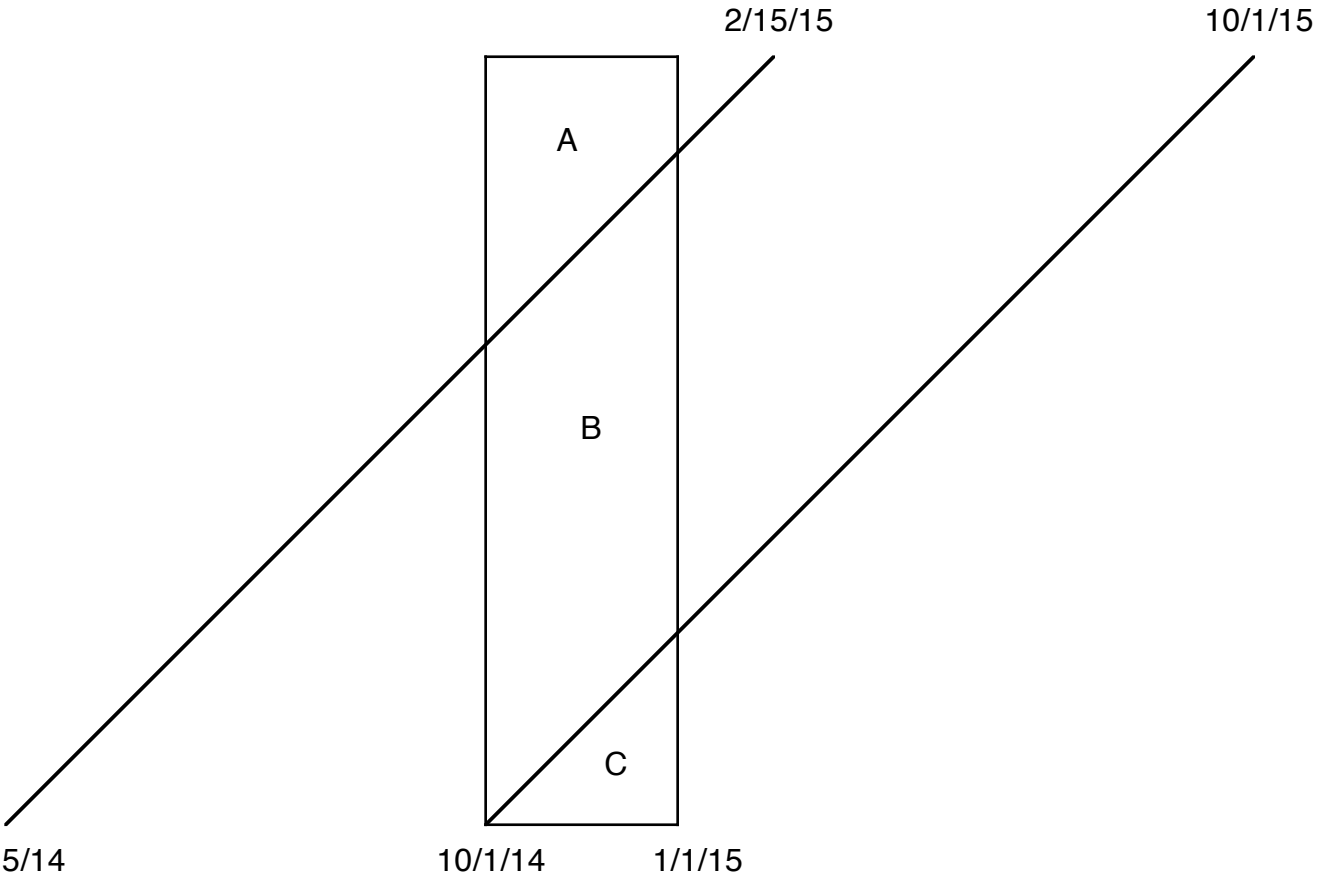
a. (1.25 points) Calculate the direct benefit change loss adjustment factor for fourth accident quarter 2014, assuming both changes only affect losses on policies written on or after the effective date of the change.

b. (1.25 points) Calculate the direct benefit change loss adjustment factor for first policy quarter 2014, assuming both changes affect all claims that occur on or after the effective date of the change.

c. (0.5 point) In doing a rate level calculation, the actuary for this insurance company has selected an annual loss trend based on unadjusted pure premium data from 2012 through 2014. Assess the appropriateness of this selection and suggest an adjustment, if necessary.

5. (a)

Effective Date	Benefit Level Index
Prior	1.0000
February 15, 2014	1.0650
October 1, 2014	1.1108



Area A = difference of two triangles = $(1/2)(4.5/12)^2 - (1/2)(1.5/12)^2 = 0.0625$.

Area B = parallelogram = $(7.5/12)(1/4) = 0.15625$.

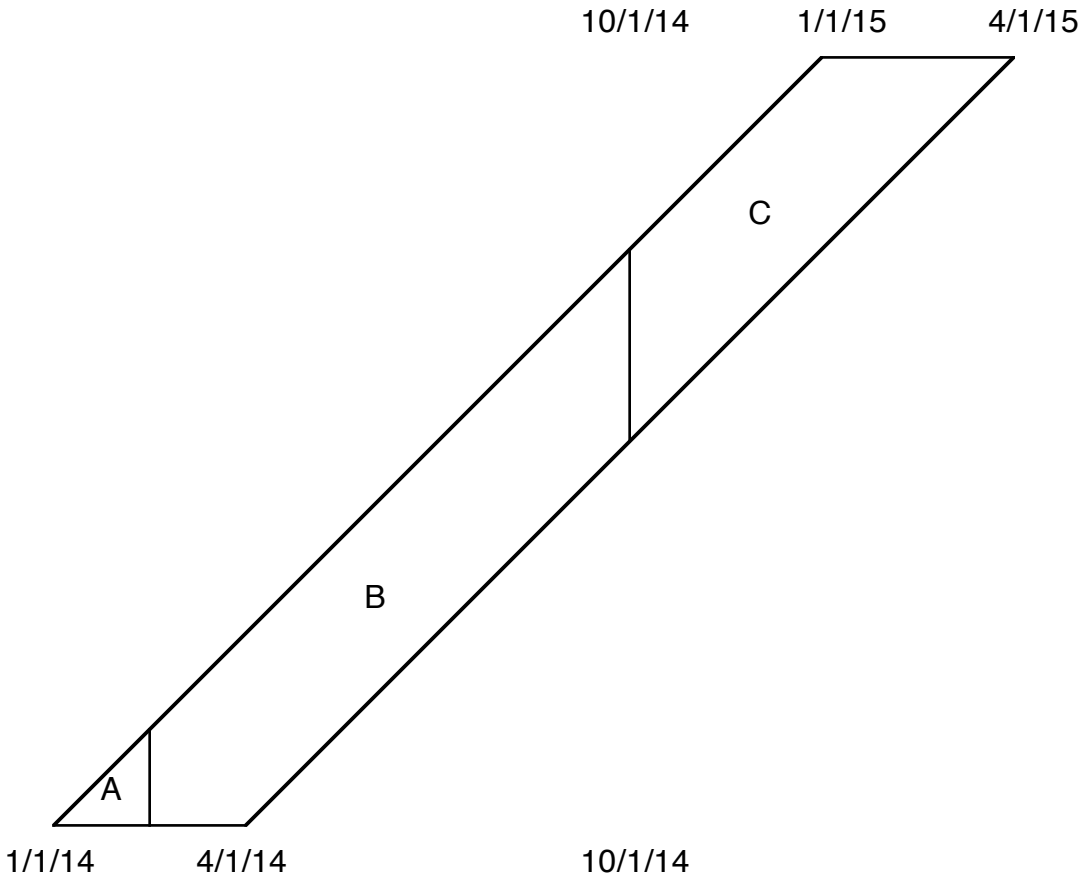
Area C = $(1/2)(1/4)(1/4) = 0.03125$.

Average benefit level for 4th accident quarter of 2014:

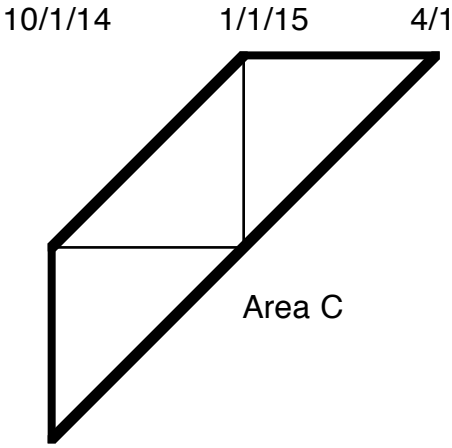
$$\frac{(0.0625)(1) + (0.15625)(1.065) + (0.03125)(1.1108)}{0.25} = 1.0545.$$

The direct benefit change loss adjustment factor for fourth accident quarter 2014 is:
1.1108/1.0545 = **1.053**.

(b) Area A = $(1/2)(1.5/12)^2 = 0.0078125$.



Area C = sum of three identical right triangles = $(3)(1/2)(1/4)^2 = 0.09375$.



Area B = 0.25 - 0.0078125 - 0.09375 = 0.1484375.

Average benefit level index for first policy quarter of 2014:

$$\frac{(0.0078125)(1) + (0.1484375)(1.065) + (0.09375)(1.1108)}{0.25} = 1.0801.$$

The direct benefit change loss adjustment factor for first policy quarter 2014 is:
1.1108 / 1.0801 = **1.028**.

(c) **This is inappropriate**, since the pure premiums have been affected by the one-time changes in benefit level. To use unadjusted data would implicitly assume that benefit changes just like these will also occur during the projection period. This would double-count the effect of the benefit change, resulting in an estimated trend that in this case would be too high.

The actuary should adjust the series of pure premium data to all be at the same benefit level; it is most common to adjust the series to the current benefit level. One should adjust for the direct impacts as well as your best estimate of any indirect impacts of the benefit changes.

Comment: Parts (a) and (b) are relatively difficult, and in my opinion worth at least twice the points that the exam committee assigned to them.

6. (1 point) Given the following information:

	Calendar Year 2014
Written premium	$560.00
Earned premium	$616.00
Commissions	$67.20
Taxes, licenses and fees	$19.60
General expenses	$73.92
LAE ratio (to loss)	8.2%
Combined ratio	100%

Calculate the 2014 operating expense ratio.

6. General Expense Ratio (to earned) = 73.92 / 616 = 12.0%.
Commissions Ratio (to written) = $67.20 / $560.00 = 12.0%.
Taxes, licenses and fees (to written) = $19.60 / $560.00 = 3.5%.
UW Expense Ratio = 12% + 12% + 3.5% = 27.5%.
Combined Ratio = Loss & LAE Ratio + UW Expense Ratio.
$\Rightarrow$ 100% = (1.082) (Loss Ratio) + 27.5%. $\Rightarrow$ Loss Ratio = 67.0%.
$\Rightarrow$ Ratio of LAE to Earned Premium: (8.2%)(67.0%) = 5.5%.
operating expense ratio = UW Expense Ratio + LAE / Earned Premium =
27.5% + 5.5% = **33.0%**.
Comment: Similar to 5, 5/11, Q.8.
The given figures for CY14 could be in millions of dollars.
I do not know why there is no other acquisition expanse.
In addition to the solution I gave, the CAS Examiner's Report included solutions where either implicitly one is relating General Expense to written premium, or Commissions and Taxes to earned premium. You should avoid doing so on your exam.

7. (4.5 points)

Given the following ratemaking information for a catastrophe-prone homeowners book of business:

		Non-Catastrophe Data		
Calendar/ Accident Year	Earned Exposures (EE)	Amount of Insurance Years (AIY) ($000)	Indicated Ultimate Frequency Trended to 2014	Indicated Ultimate Loss & ALAE Severity ($) Trended to 2014
2008	914,600	230,400	4.57%	14,638
2009	928,300	240,800	4.16%	12,624
2010	942,200	251,600	4.39%	13,445
2011	956,300	262,900	4.12%	12,306
2012	970,600	274,700	3.44%	14,564
2013	985,200	287,100	3.11%	11,634
2014	1,000,000	300,000	3.32%	13,726

- All policies are annual.
- The new rates will be in effect for one year, beginning April 1, 2016.
- Projected average rate = $1,070.
- Annual frequency trend = 3%.
- Annual loss and ALAE severity trend = 4%.
- Annual AIY/EE ratio trend = 3%.
- 20-year average historical ratio of non-modeled catastrophe losses and ALAE to AIY = 0.08.
- Projected modeled average catastrophe loss and LAE = $68.36.
- Variable expense ratio = 18%.
- Fixed expense provision = $54.36.
- ULAE provision = 4% of loss and ALAE.
- Target underwriting profit provision = 6%.

a. (1.75 points) Using a frequency-severity technique with trending, calculate the ultimate non-catastrophe loss and ALAE for accident years 2013 and 2014. Justify any selections.

b. (2.75 points) Using the results from part a. above, calculate the indicated rate change using the pure premium method.

7. (a) After being trended to 2014, frequency seems to be declining. Therefore, I will select 3.29% based on an average of the most recent three years.
This is for 2014; using the given 3% trend in frequency, for 2013 I will use: 3.29% / 1.03 = 3.19%.
After being trended to 2014, severity seems to be up and down, with no clear trend.
Therefore, I will select the average of all years: $13,277.
This is for 2014; using the given 4% trend in severity, for 2014 I will use: $13,277 / 1.04 = $12,766.

	AY2013	AY2014
Exposures	985,200	1,000,000
Selected Frequency	3.19%	3.29%
Projected Ultimate Claim Counts	31,428	32,900
Selected Severity	$12,766	$13,277
Projected Ultimate Loss & ALAE	**$401.210 million**	**$436.813 million**

(b) The average date of writing under the new rates is 4/1/16 plus 6 months = 10/1/16.
(Rates will be in effect for one year.)
The average date of accident under the new rates is 10/1/16 plus 6 months = 4/1/17.
(Policies are annual.)
Thus from the middle of 2014 the trend period is 2.75 years. It is 3.75 years for AY13.
Trended non-cat loss and ALAE Pure Premium based on AY13:
($401.210 million)$(1.03^{3.75})(1.04^{3.75})$ / 985,200 = $527.06.
Trended non-cat loss and ALAE Pure Premium based on AY14:
($436.813 million)$(1.03^{2.75})(1.04^{2.75})$ / 1,000,000 = $527.76
Select for trended non-cat loss and ALAE Pure Premium: ($527.06 + $527.76) / 2 = $527.41.
Loading for ULAE: (1.04)($527.41) = $548.51.
For CY14, AIY to Exposure Ratio is: 300 million / 1 million = 300.
Projecting to the effective period at the given 3% per year: $(300)(1.03^{2.75}) = $325.40.
Non-modeled cat loss and ALAE: (0.08)($325.40) = $26.03.
Loading for ULAE: (1.04)($26.03) = $27.07.
Indicated rate is: $\frac{548.51 + 26.03 + 68.36 + 54.36}{1 - 18\% - 6\%}$ = $917.45.
Indicated rate change is: $917.45 / $1070 - 1 = **-14.26%**.
Comment: In part (a), you are expected to use a "frequency-severity technique,"
as discussed in Chapter 11 of Estimating Unpaid Claims Using Basic Techniques by Friedland;
see in particular Sheet 10 of Exhibit III.
In part (a), there are other reasonable selections for frequency and severity.
In part (b) I have used AY13 and AY14 in order to follow the instructions in the question.
For homeowners I would normally use 5 years, as is done in Appendix B in Basic Ratemaking.

8. (1 point) A company has a combined ratio of 125% in the first year of writing policies.
Explain two reasons why the company could be profitable in the long run without increasing rates.

8. 1. The loss experience improves the longer insureds stays with an insurer. (This is one of the ideas behind asset share pricing.) Thus as this insurer writes renewals, its loss ratio should decline.
In the first year of an insurer, many of the underwriters do not have much experience selecting risks; this should improve with time, thus lowering the loss ratio.
2. Expense ratios are lower for renewal business than first year business. (This is one of the ideas behind asset share pricing.) First year business includes the cost of obtaining business. Also renewal commissions are lower for many insurers.
In the first year of an insurer, there are many one time startup costs.
Also in the first year, the volume of premium written is usually significantly smaller than it will be eventually, yet the insurer's expenses reflect to some extent the future level of premiums; this makes the first year expense ratio higher than it will be going forward.
3. The company may have a very profitable investment returns that could more than offset the underwriting loss. (However, the line of insurance would have to be very long-tailed and the investment rate of return high in order to overcome a combined ratio of 125%.)
4. In the first year of writing policies, there may not have been time for the reflection of eventual salvage and subrogation; if salvage and subrogation are sufficiently big they could offset the high combined ratio.
5. It is possible that the insurer has over-reserved; if there is sufficient future downward development on the initial loss reserves this could offset the reported high combined ratio for the first year.
6. In its first year, the insurer may have inexperienced claims adjusters, which may lead to claim settlements that are on average too generous. Over time the claims adjusters should get better at limiting claim settlements, which would improve the combined ratio.
7. In its first year, this insurer may have written a small book of business. In that case, the loss ratio would be subject to lots of random fluctuation, and the high combined ratio may just be due to bad luck, which is not reflective of future profitability.
Comment: Give only two reasons. I took some of the above from the CAS Examiner's Report.
The 125% combined ratio could be due to a bad year for catastrophes, which is not indicative of the average expected future loss ratio.
It is unclear to me whether the question writer intended to refer to new business written by an existing insurer or to a new insurer.

9. (2.75 points) Given the following information:

	Relativities
Territory A	0.60
Territory B	1.10
Smoke Detector	0.90
No Smoke Detector	1.00

	2014 Earned Exposures	
	Territory A	Territory B
Smoke Detector	750	600
No Smoke Detector	150	100

	Accident Year 2014 Incurred Loss and ALAE	
	Territory A	Territory B
Smoke Detector	$160,000	$260,000
No Smoke Detector	$40,000	$52,000

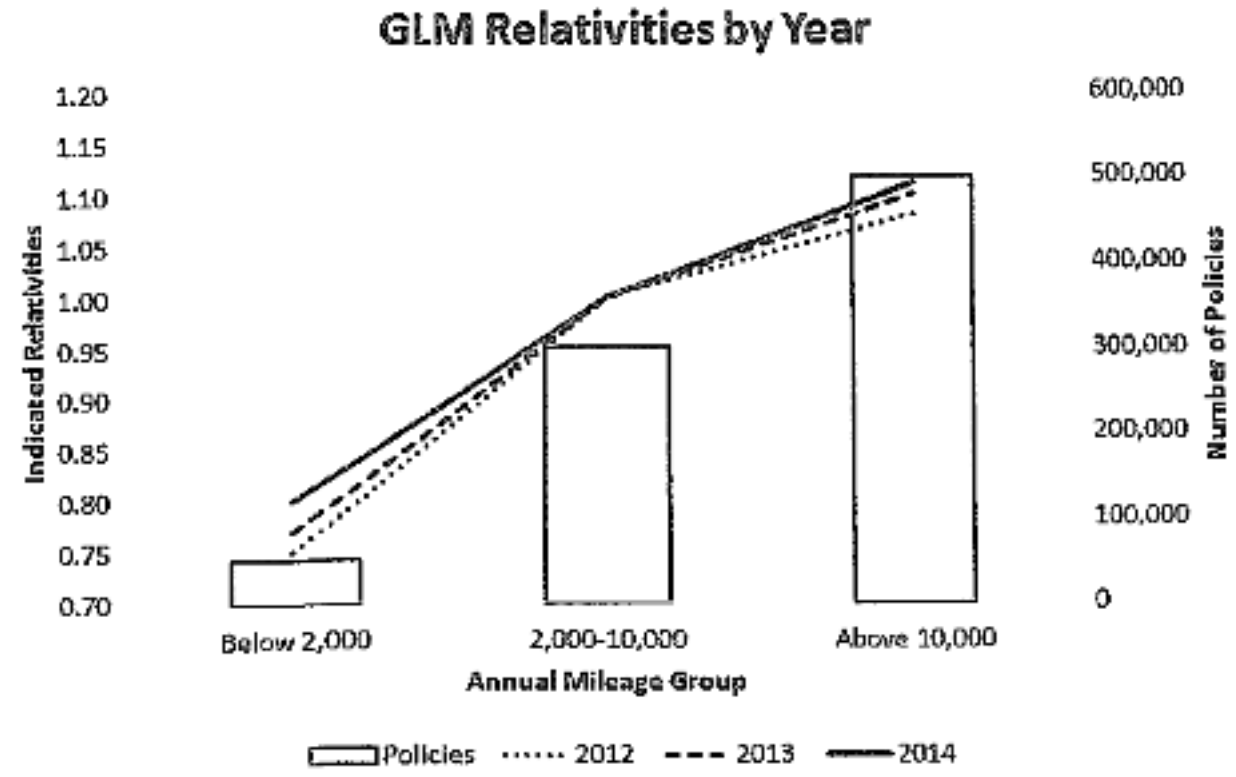
- Base rate = $550.
- All rates are effective January 1 of each year.
- Management has decided that the relativity of the highest-rated territory will not exceed 130% of the lowest-rated territory in any future rate level change.
- Assume data for 2014 is fully credible.

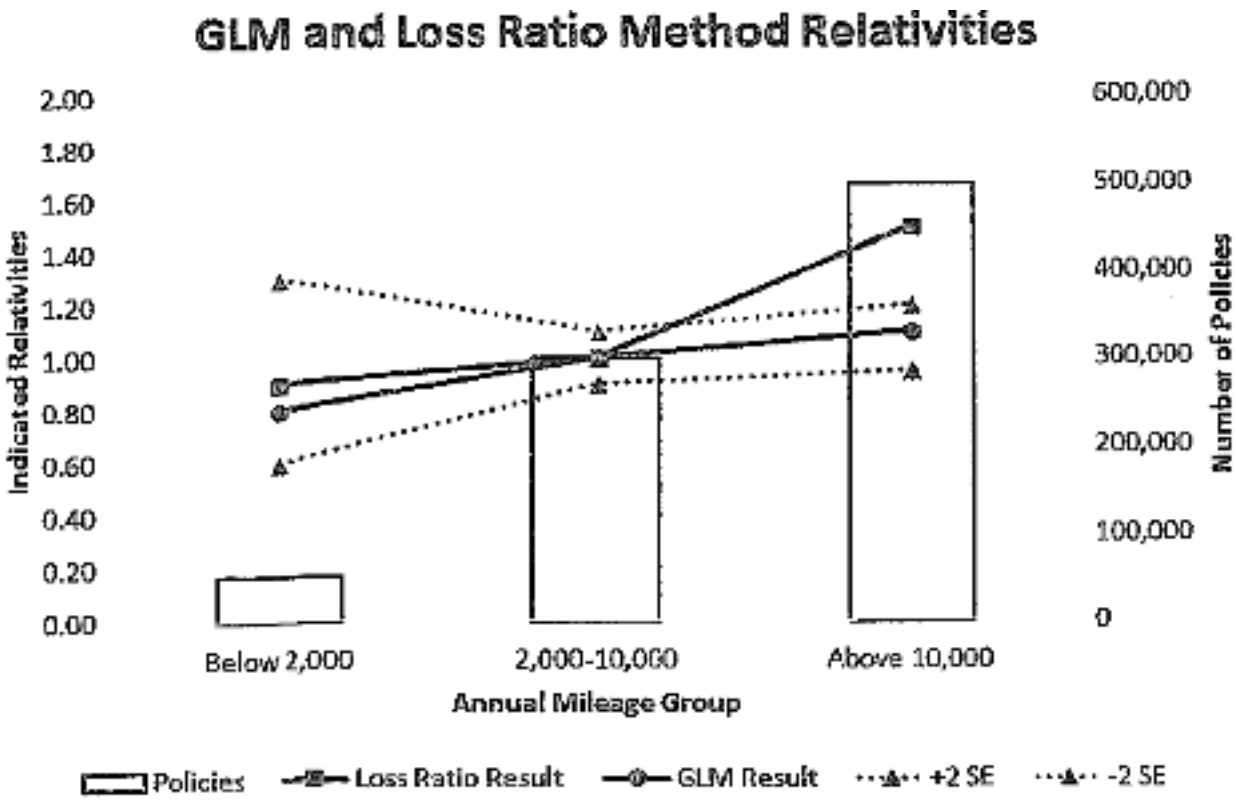
a. (2.25 points) Considering management constraints, use the loss ratio method to calculate the territorial relativity changes for a revenue-neutral overall change.

b. (0.5 point) Evaluate the impact that the relativity changes may have on this book of business in the short and long term.

9. (a) Territory A premiums: ($550)(0.6) {(750)(0.9) + (150)(1)} = $272,250.
Territory B premiums: ($550)(1.1) {(600)(0.9) + (100)(1)} = $387,200.
Territory A loss ratio: $200,000 / $272,250 = 73.46%.
Territory B loss ratio: $312,000 / $387,200 = 80.58%.
Overall loss ratio: (200,000 + 312,000) / (272,250 + 387,200) = 77.64%.
Indicated relativity Territory A: (73.46%/77.64%) (0.6) = 0.5677.
Indicated relativity Territory B: (80.58%/77.64%) (1.1) = 1.1417.
1.1417 / 0.5677 = 2.011 > 1.30.
Thus applying the management constraints, the relativity for Territory A is x and for B is 1.3x.
We can now pick different combinations of x and base rate in order to have a revenue-neutral overall change. Assume for example we want to continue to use a base rate of $550.
Then the new premiums are:
($550)(x) {(750)(0.9) + (150)(1)} + ($550)(1.3x) {(600)(0.9) + (100)(1)} = 911,350 x.
272,250 + 387,200 = 911,350 x. $\Rightarrow$ x = 0.7236.
Thus the relativity for Territory A is **0.73** and for Territory B is: (1.3)(0.7236) = **0.94**.
(b) The rates charged to Territory A will be excessive, while those charged to Territory B will be inadequate.
In the short term, since its rates for Territory A are too high and its rate for Territory B are too low, this insurer will lose insureds in Territory A and gain insureds in Territory B.
The insurer will suffer from adverse selection and lose money.
In the long term, assuming the insurer continues to use the management constraints, the insurer will have to raise its rates. It will lose even more business in Territory A. This insurer will write much less insurance in total. Eventually it will write most of its business in Territory B and very little if any in Territory A; it should now be able to charge an adequate rate in Territory B.
Comment: I do not know what the significance of "All rates are effective January 1 of each year."
We do not need the current base rate in order to apply the loss ratio method for relativities;
the $550 cancels out in the calculations of the changes in relativities.
There is no base territory; none of the current territory relativities is one. Thus in this case, many different combinations of new relativities for Territory A and new base rate will produce the same revenue level. Thus there is no unique set of "territorial relativity changes for a revenue-neutral overall change." The CAS Examiner's Report seemed to be oblivious to this inherent defect in their question as constructed.

10. (2 points) An automobile insurer has calculated indicated rating plan factors using both a loss ratio analysis and a generalized linear model (GLM). Data from years 2012-2014 was used in both analyses. Given the following output for the proposed Annual Mileage rating variable:





<QUESTION 10 CONTINUED FROM PREVIOUS PAGE>

a. (1 point) Using the data in each graph above, discuss whether annual mileage would be a good rating variable.
b. (0.5 point) Taking into account two other criteria of a good rating variable, discuss whether annual mileage would be a good rating variable.
c. (0.5 point) Recommend whether the insurer should add annual mileage to their rating plan.

10. (a) In the first graph, the relativities are fairly consistent from year to year. This is good.
Also, annual mileage is a good rating variable since there is a clear difference in the indicated relativity for each level.
In the second graph, the plus or minus two standard error intervals include a relativity of one.
Thus based on the GLM, the indicated relativities are not statistically significant.
The relativities from the loss ratio method and GLM differ, particularly for the over 10,000 mile group. This warrants further investigation; it is probably due to how the other rating variables currently being used interact with miles driven.
With respect to both graphs, since the less than 2K level contains far fewer policies than the other two levels, the actuary should consider combining it with the 2K to 10K level, and rerunning the models with only two groups.
(b) 1. Credibility: a rating group should be large enough to measure costs with sufficient accuracy. Using the groups in the graphs, below 2000 miles probably has too few exposures to be credible. However, assuming one grouped insureds into somewhat different mileage intervals, there should be enough data in each group and the criterion of credibility would be satisfied.
2. Homogeneity: all members of a group that receive the same rate or premium should have similar expected costs. The mileage intervals used in the graphs may be sufficient similarity within each group, although one may want to divide the large over 10,000 miles group into two in order to better satisfy criterion of homogeneity.
3. Statistical Significance: rating variables should be related to costs. Based on the second graph, the relativities are not statistically different than 1. Thus this criterion is not satisfied.
Alternately, mileage is intuitive and proportional to expected loss.
4. Verifiability: can be checked by the insurer; not subject to manipulation or lying by the insured.
Estimated annual mileage is subject to manipulation or lying by the insured.
It would be difficult for the insurer to check. This criterion is not satisfied.
Alternately, using telematics one can verify the mileage driven, so this criterion is satisfied.
5. Low Administrative Expense.
The administrative expense to verify the estimated mileage figures will be large compared to the differences in loss costs between the different mileage groups.
This criterion is not satisfied.
6. Objectivity: Since this is an objective definition, this criterion is satisfied.
7. Affordability. Miles driven is not negatively correlated with income. Thus using mileage driven will not create problems of affordability.
8. Causality. More miles driven generates more potential to be involved in an accident and thus generate a claim. Thus this criteria is satisfied, for other than Comprehensive.
9. Controllability. An insured can reduce his miles driven by using mass transit, car pooling, living closer to work, not driving on cross country vacations, etc. Thus this criterion is satisfied.
10. Privacy concerns. The number of miles driven does not raise privacy concerns.
Thus this criterion is satisfied.
11. Acceptable to regulators: This variable is widely used for personal auto and is largely considered to be acceptable to regulators, a good thing.

(c) I recommend that the insurer should not add annual mileage to their rating plan.
The relativities based on the GLM are not significantly different from one.
Also the "under 2000 miles" group is too small to be credible.
Alternately, I would include the rating variable given that mileage is proportional to loss, and does not violate any privacy concerns.
Comment: In the second graph, ± 2 SE refers to standard errors for the GLM.
(They should not show standard errors for the base group, whose relativity is one by definition.)
In part (b) only discuss two criteria. There are many full credit answers to parts (b) and (c).

11. (4 points) A homeowners insurance company uses only two rating variables, territory and amount of insurance.

The company wishes to accomplish the following as part of an upcoming rate filing:

- Achieve an indicated average rate increase of +15%.
- Update class plan relativities based on indicated results.
- Adopt a minimum premium requirement of $800.
- Keep the same base classes.

The following information applies to the company's current book of business:

- Current base rate per exposure is $1,250.

Amount of Insurance	Current Relativity	Indicated Relativity
Less than $100,000	0.750	0.600
Greater than or Equal to $100,000	1.000	1.200

Territory	Current Relativity	Indicated Relativity
Territory 1	0.800	0.850
Territory 2	1.000	1.000

In-force Exposure Distribution

Amount of Insurance	Territory 1	Territory 2
Less than $100,000	1,500	4,000
Greater than or Equal to $100,000	1,500	3,000

Using the extension of exposures method, calculate the base rate that satisfies all of the company's objectives.

11. The premiums using the current rates are:
(1250) {(1500)(0.75)(0.8) + (1500)(1)(0.8) + (4000)(0.75)(1) + (3000)(1)(1)} = $10,125,000.
We wish to have 15% more premium or: (1.15)($10,125,000) = $11,643,750.
In order to have AOI greater than $100,000 to continue to be the base class, we wish its new relativity to be one. This implies a new relativity for AOI less than $100,000 of: 0.6/1.2 = 0.5.
Ignoring the new minimum premium requirement, the premiums using the new rates are:
x {(1500)(0.5)(0.85) + (1500)(1)(0.85) + (4000)(0.5)(1) + (3000)(1)(1)} = 6912.5 x.
The new base rate is: x = $11,643,750 / 6912.5 = $1684.45.
However, the indicated rate for AOI less than $100,000 in Territory 1 is:
(0.5)(0.85)($1684.45) = $715.89 < $800.
The indicated rate for AOI less than $100,000 in Territory 2 is:
(0.5)(1)($1684.45) = $842.22. OK.
WIth $800 premium for AOI less than $100,000 in Territory 1, the premium is:
($800)(1500) + x {(4000)(0.5)(1) + (1500)(1)(0.85) + (3000)(1)(1)} = $1,200,000 + 6275 x.
$1,200,000 + 6275 x = $11,643,750. $\Rightarrow$ x = $**1664**.
Comment: I have assumed that the current rates do not include a minimum premium requirement.
For the current rates:

Amount of Insurance	Terr. 1	Terr. 2
Less than $100,000	750	937.5
Greater than or Equal to $100,000	1000	1250

(1500)($750) + (4000)($937.5) + (1500)($1000) + (3000)($1250) = $10,125,000.
For the proposed rates:

Amount of Insurance	Terr. 1	Terr. 2
Less than $100,000	800	832
Greater than or Equal to $100,000	1414	1664

(1500)($800) + (4000)($832) + (1500)($1414) + (3000)($1664) = $11,641,000.
$11,641,000 / $10,125,000 = 1.1497, which subject to rounding is a 15% average rate increase.

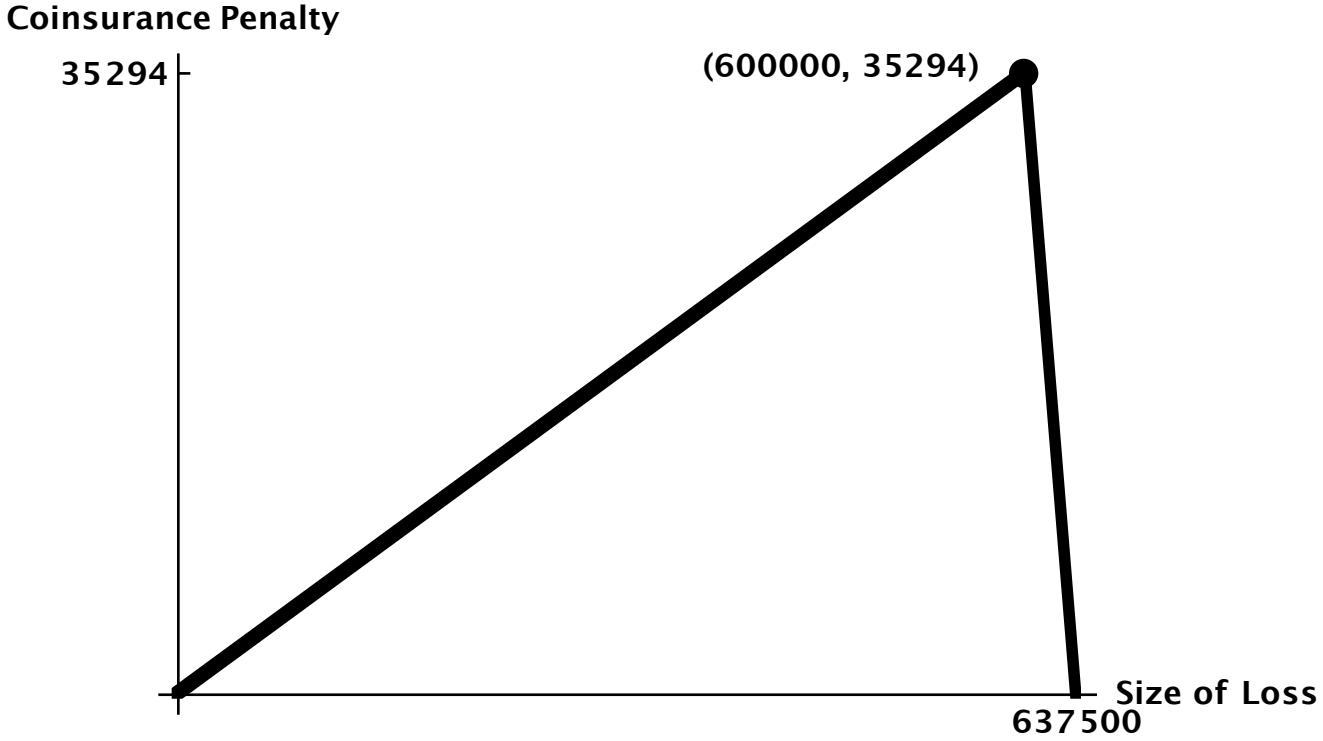
12. (2.5 points) Given the following information about a property:

- Value of property = $750,000
- Required Coinsurance = 85%
- Amount of Insurance purchased = $600,000
- There is a 30% chance of total toss, given a claim.
- All other losses are uniformly distributed between $0 and $750,000.
- Frequency of loss = 2%

a. (1.25 points) Draw a graph of the coinsurance penalty as a function of loss amount. Label and give values of all critical points.

b. (1.25 points) Calculate the rate per $1000 of insurance to be charged for this property, assuming no coinsurance penalty is used.

12. (a) The maximum penalty occurs for a loss of size equal to the policy face of $600,000.
For a loss of size $600,000, the policy pays: (80/85)(600,000) = $564,706.
The coinsurance penalty is: $600,000 - $564,706 = $35,294.
There is no coinsurance penalty for losses of size greater than the coinsurance requirement of:
(85%)($750,000) = $637,500.
A graph of the coinsurance penalty as a function of loss amount:



(b) I do not know what is meant by "assuming no coinsurance penalty is used."
I will calculate the appropriate rate for an insured who buys $600,000 of insurance in the absence of an coinsurance clause.
For losses of size greater than or equal to $600,000, the policy pays $600,000.
The probability of this is: 30% + (70%)(750 - 600)/750 = 44%.
The remaining 56% of the time, payments are uniform from 0 to $600,000.
Average severity is: (44%)($600,000) + (56%)($600,000/2) = $432,000.
Rate is: (2%)($432,000) / 600 = $**14.40 per thousand dollars**.
Alternately, I will calculate the appropriate rate for an insured who buys the coinsurance requirement of $637,500; this rate would also be appropriate for an insured who bought less than the coinsurance requirement for a policy with the coinsurance clause.
For losses of size greater than or equal to $637,500, the policy pays $637,500.
The probability of this is: 30% + (70%)(750 - 637.5)/750 = 40.5%.
The remaining 59.5% of the time, payments are uniform from 0 to $637,500.
Average severity is: (40.5%)($637,500) + (59.5%)($637,500/2) = $447,844.
Rate is: (2%)($447,844) / 637.5 = $**14.05 per thousand dollars**.
Comment: Based on the CAS Examiner's Report, my first solution was the intended one.

13. (1 point)
Below are the parameters for a retrospectively rated policy with an annual policy period:

Standard Premium	$813,546
Basic Premium	$343,137
Loss Conversion Factor	1.08
Tax Multiplier	1.03
Min Retro Premium Ratio	60%
Max Retro Premium Ratio	140%

Evaluation at Age	Limited Reported Losses
18 months	$115,000
30 months	$151,800

Calculate the retrospective premium at 18 months and 30 months.

13. The minimum premium is: (60%)($813,546) = $488,128.
The maximum premium is: (140%)($813,546) = $1,138,964.
At 18 months, preliminary premium is:
(1.03) {(1.08)($115,000) + $343,137} = $481,357.
Retro premium = minimum premium = **$488,128**.
At 30 months, preliminary premium is:
(1.03) {(1.08)($151,800) + $343,137} = **$522,293**,
which is in between the minimum and maximum premiums, so this is what will be charged.