Solutions to the Fall 2014 CAS Exam 5

(Only those questions on Basic Ratemaking)

There were 24 questions worth 58.25 points, of which 12 were on ratemaking worth 29.25 points.

The exam and examiner's report are available from the CAS.
The solutions and comments are solely the responsibility of the author.

(Updated January 17, 2015, after looking at the CAS Examiner's Report)

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1. (2 points)
In an attempt to improve poor workers compensation underwriting results, an insurance company is considering changing its exposure base from number of employees to number of hours worked.
a. (0.5 point) Identify two criteria of a good exposure base.
b. (0.5 point) Briefly discuss whether this change in exposure base is appropriate for each of the criteria from part a. above.
c. (0.25 point) Briefly describe the impact the exposure base change could have on frequency.
d. (0.25 point) Briefly describe the impact the exposure base change could have on severity.
e. (0.5 point) Discuss an impact the exposure base change could have on the company's loss ratio.

1. (a) 1. Proportional to Expected Loss
2. Practical: objective, easy to obtain, and inexpensive to verify.
3. Historical Precedence
(b) The number of hours worked should be closer to proportional to expected losses than would the number of employees; the more hours someone works the more chance for a work place accident and thus a workers compensation claim.
The number of hours should be recorded for other purposes and thus should be practical (with the possible exception of workers not paid by the number of hours worked. However, such workers usually have much smaller expected losses than those who are paid based on the number of hours worked.)
On the other hand, number of hours worked would be more subject to manipulation by the employer than would number of employees.
The number of hours worked has not been used prior, so this does not satisfy the criteria of historical precedence. There would be difficulties in transitioning to a new exposure base with:
difficulty estimating class rates, expensive changes to systems, and the likelihood of large premium swings for individual insureds.
(c) Frequency is number of claims per exposure. The number of hours worked is greater than the number of employees, and thus the frequency per exposure would decrease.
(d) Severity is dollars of loss per claim, and should be unaffected by a change in exposure base.
(e) The exposure base change should not directly affect the companies loss ratio, provided the insurer manages to create new rates by class based on the new exposure base which produce the same total premium as would the old rates based on the old exposure base.
(This is not an easy task.)
However, this insurer would now attract employers whose workers have fewer hours worked per week than average for a class, and would lose employers whose workers have more hours worked per week than average for a class. The former are expected to have fewer work place accidents on average and thus lower workers compensation costs. This favorable selection would lead to an improvement in the insurer's loss ratio, all else being equal.
Alternately, the exposure base change could lead to wide premium swings. Thus many good customers will go to other insurers, while bad risks who cannot get coverage elsewhere remain. Therefore this insurer's loss ratio is likely to get worse.
Comment: Switching to a new exposure base that is closer to proportional to expected losses has a similar affect to introducing a new useful classification variable. The insurer can attract better insureds which are currently being written by competitors who stay with the old scheme.

2. (1.5 points) Given the following policy data:

Policy	Effective Date	Expiration Date	Initial Policy Premium
1	June 1, 2012	May 31, 2013	480
2	July 1, 2012	December 31, 2012	125
3	March 1, 2013	February 28, 2014	225
4	August 1, 2013	March 31, 2014	300

- Six months after the policy expires, the initial policy premium on every policy increases by 8% due to the final audit.

a. (0.5 point) Calculate calendar year 2013 earned premium as of December 31, 2013.
b. (0.5 point) Calculate calendar year 2013 written premium as of December 31, 2013.
c. (0.25 point) Calculate policy year 2013 earned premium as of December 31, 2013.
d. (0.25 point) Calculate policy year 2013 written premium as of December 31, 2014.

2. Policy 1 is a 1-year policy that has 5 out of 12 months in 2013. Its audit occurs on 11/30/13.
Policy 2 is a 6-month policy that has 6 out of 6 months in 2012. Its audit occurs on 6/30/13.
Policy 3 is a 1-year policy that has 10 out of 12 months in 2013. Its audit occurs on 8/31/14.
Policy 2 is an 8-month policy that has 5 out of 8 months in 2013. Its audit occurs on 9/30/14.

(a)

Policy	Contribution from Initial Premium	Contribution from Audit	Total
1	(5/12)(480)	(8%)(480)	238.4
2	0	(8%)(125)	10
3	(10/12)(225)	0	187.5
4	(5/8)(300)	0	187.5
Total			**623.4**

(b)

Policy	Contribution from Initial Premium	Contribution from Audit	Total
1	0	(8%)(480)	38.4
2	0	(8%)(125)	10
3	225	0	225
4	300	0	300
Total			**573.4**

(c)

Policy	Contribution from Initial Premium	Contribution from Audit	Total
1	0	0	0
2	0	0	0
3	(10/12)(225)	0	187.5
4	(5/8)(300)	0	187.5
Total			**375**

(d)

Policy	Contribution from Initial Premium	Contribution from Audit	Total
1	0	0	0
2	0	0	0
3	225	(8%)(225)	243
4	300	(8%)(300)	324
Total			**567**

Comment: No contribution to PY2013 from the first two policies not written during 2013.
Audit premium is earned as soon as it is written.

3. (2 points) A personal auto insurer has a highly-refined classification rating plan. In the calculation of a rate level indication for this insurer, fully assess the use of the following methods to adjust premium to current rate level:

i. Parallelogram method
ii. Extension of Exposures method

3. The parallelogram method requires less detailed information. However, it is based on the estimated overall impacts of past rate changes, which are in turn based on the corresponding prior estimates of portion of exposures written in each classification cell. In addition, the parallelogram method assumes a constant level of exposures are written over time. Thus the result of using the parallelogram method will be an approximation, which depending on circumstances may not be very good. The very detailed class plan decreases the likely accuracy of using the parallelogram method.

In contrast, extension of exposures requires the historical exposures by classification cell. In other words for each past policy we need to know how it would have been classified using the current classification rating plan. (Unless there have been major changes to the plan since the historical premiums have been written, this should not be a problem.) Then we just extend the past exposures by the current rates to adjust the past premiums to the current level. Extension of exposures is generally more accurate than the parallelogram method.

In summary, the parallelogram method is easier while the extension of exposures is more accurate. Assuming the necessary detailed information is available, extension of exposures would be greatly preferred to the parallelogram method, particularly for a personal auto insurer with a highly-refined classification rating plan.

In addition, the parallelogram method uses overall rate changes and thus would not work for an analysis of classification rates (assuming past rate changes varied by class) while the extension of exposures would work.

4. (1.5 points) Given the following workers compensation information for an employer:

Ratio of Wage to the State Average Weekly Wage (SAWW)	Percentage of Workers
0.50	6%
0.85	18%
1.00	31%
1.45	26%
1.90	17%
2.20	2%

- Minimum benefit = 45% of State Average Weekly Wage (SAWW).
- Current Compensation Rate = 80% of Worker's Pre-Injury Wage.
- Proposed Compensation Rate = 85% of Worker's Pre-Injury Wage.
- Current Maximum Benefit = 130% of SAWW.
- Proposed Maximum Benefit = 115% of SAWW.

a. (1 point)
Assuming no changes to claim frequency, calculate the combined percent impact of both the compensation rate and maximum benefit changes to the average weekly expected claim benefit.
b. (0.5 point)
Briefly describe a potential indirect effect of the maximum benefit changes on:

i. Frequency
ii. Duration

4. (a) For example, (0.85)(80%) = 0.68. (1.90)(80%) = 1.52, capped at maximum of 1.30.

Ratio of Wage to (SAWW)	Percentage of Workers	Benefit Current	Benefit Proposed
0.50	6%	0.45*	0.45*
0.85	18%	0.68	0.7225
1.00	31%	0.80	0.85
1.45	26%	1.16	1.15**
1.90	17%	1.30**	1.15**
2.20	2%	1.30**	1.15**
Total		0.94600	0.93805

* minimum benefit ** maximum benefit

Impact of benefit changes: 0.93805 / 0.94600 - 1 = **-0.84%**.

(b) The maximum benefit is being decreased, which makes it slightly less likely that a high-wage worker will file a workers compensation claim, since the percent of their salary being replaced is lower. Thus the claim frequency may decline slightly due to this change.

The maximum benefit is being decreased, which makes it slightly less likely that a high-wage worker will remain on workers compensation, since the percent of their salary being replaced in lower. Thus some of these high-wage workers will return to work somewhat sooner, leading to a decrease in the average duration of benefits due to this change.

Comment: We are given a discrete distribution of weekly wages for an employer.

Some might find it helpful to make up numbers, for example 100 employees and an average weekly wage of $1000, and translated that into what was given.

The increase in the compensation rate will makes it more likely that a medium-wage worker will file a workers compensation claim. Thus the claim frequency will increase due to this change.

The increase in the compensation rate will makes it more likely that a medium-wage worker will remain on workers compensation. Thus the average duration of benefits will increase due to this change. The affects of increasing the compensation rate are in the opposite direction from those due to the decrease in the maximum benefit.

5. (3.25 points) The following information is available for a homeowners insurance company as of December 31, 2013:

Period (months)	Reported Loss and ALAE Age-to-Age Development Factors
12-24	1.10
24-36	1.05
36-48	1.01

Calendar/ Accident Year	Earned Exposures (000)	Amount of Insurance Years (AIY) ($000)	Reported Non-Catastrophe Loss and ALAE ($000)
2011	45	13,500	23,000
2012	50	15,300	25,000
2013	40	12,500	20,000

- Annual loss and ALAE trend = 4%.
- Historical non-catastrophe ULAE to loss and ALAE ratio = 1.05.
- Historical catastrophe ULAE to loss and ALAE ratio = 1.09.
- Long-term non-modeled catastrophe loss and ALAE-to-AIY ratio = 0.25.
- Modeled catastrophe loss and ALAE-to-AIY ratio = 0.07.
- Rates will take effect on January 1, 2015, and will be in effect for one year.
- All policies are annual.
- Assume no development after 48 months.

Using three years of historical data, determine the provision for loss and LAE to be used in the pure premium indication.

5. For the non-catastrophe losses and ALAE, develop to ultimate, trend to an average date of accident of January 1, 2016:

$(1.01)(1.04^{4.5})(23{,}000) + (1.01)(1.05)(1.04^{3.5})(25{,}000) + (1.01)(1.05)(1.10)(1.04^{2.5})(20{,}000) = 83{,}862.$

Load for ULAE and divide by exposures: $(1.05)(83{,}862)/ (45 + 50 + 40) = \$652.26.$

The average amount of insurance for the three historical years are:

13,500/45 = \$300, 15,300/50 = \$306, 12,500/40 = \$312.5.

$\sqrt{\$312.5 / \$300} = 1.0206.$

Assume an increase in the average amount of insurance of about 2.06% a year for 2.5 years, and thus the projected average amount of insurance is: $(312.5)(1.0206^{2.5}) = \$328.84.$

Thus the provision for catastrophe loss and ALAE is: $(\$328.84)(0.25 + 0.07) = \$105.23.$

Loading this for ULAE: $(1.09)(105.23) = \$114.70.$

Thus, total pure premium for loss and LAE is: \$652.26 + \$114.70 = **\$766.96**.

Comment: In Appendix B of Basic Ratemaking, the Homeowners Indication uses the pure premium method. However unlike here, the pure premium for modeled catastrophe losses is just given with no further details provided.

6. (2.25 points) Given the following information:

- Projected ultimate pure premium, including LAE = $450.
- Underwriting profit provision = 5%.
- Projected average premium per exposure = $750.

Expense Category	Selected Expense Ratio	% Fixed
General Expenses	6.0%	75%
Other Acquisition	9.5%	75%
Taxes, Licenses and Fees	2.8%	25%
Commission and Brokerage	12.0%	0%

a. (0.5 point) Calculate the indicated average rate using the all variable expense method for determining expense provisions.

b. (1 point) Calculate the indicated average rate using the premium-based projection method for determining expense provisions.

c. (0.75 point) Assume the historical average premium per exposure on which the selected expense provisions are based is $675. Discuss whether the result calculated in part b. above is excessive or inadequate.

6. (a) Treat all of the expenses as variable.
Indicated average rate is: \$450 / (1 - 6% - 9.5% - 2.8% - 12% - 5%) = **\$695.52**.
(b) The fixed expenses are: (6.0%)(75%) + (9.5%)(75%) + (2.8%)(25%) = 12.325%.
Variable expense plus profit is:
(6.0%)(25%) + (9.5%)(25%) + (2.8%)(75%) + 12% + 5% = 22.975%.

Rate change factor is: $\frac{\$450 / \$750 + 12.325\%}{1 - 22.975\%} = 0.93898$.

Thus the indicated average rate is: (0.93898)(\$750) = **\$704.24**.
Alternately, the projected fixed expenses are: (12.325%)(750) = \$92.44.

The indicated average rate is: $\frac{\$450 + \$92.44}{1 - 22.975\%} =$ **\$704.24**.

(c) The rate of \$704.24 includes a provision for fixed expenses of \$92.44.
This compares to fixed expenses based on the \$675 of: (12.325%)(\$675) = \$83.19.
However, one would want to apply fixed expense trend to this provision for some past period of time. We are given neither an annual fixed expense trend nor the trend period.
\$92.44/\$83.19 = 1.111.
Thus if the total fixed expense trend is 11.1%, then the indicated rate is adequate.
If the total fixed expense trend is larger than 11.1%, then the provision for fixed expenses in the indicated rate is too low, and the indicated rate is inadequate.
If the total fixed expense trend is smaller than 11.1%, then the provision for fixed expenses in the indicated rate is too high, and the indicated rate is excessive.
Alternately, the projected average premium of \$750 is higher than the historical one of \$675.
Using the premium based method to evaluate fixed expense assumes fixed expense scales with premium. This is not very accurate since some fixed expense does not depend on size of policy, and fixed expense may trend differently from premium. So, fixed expense may be overestimated, and result in part (b) being excessive.
Alternately, the result in part (b) is excessive. Since the expense ratios were calculated using an average premium of \$675, the true fixed expense amount is: (675)(0.12325) = \$83.19. However, in the rate calculation an average premium of \$750 was used, which means our fixed expense amount was estimated to be: (750)(0.12325) = \$92.44. Since this estimated fixed expense is greater than the true fixed expense of \$83.19, the indicated rate is excessive.
Comment: The final alternative solution to part (c) is from the CAS Examiner's Report; while the CAS apparently finds it acceptable I do not since it ignores the fact that the \$83.19 is for an earlier period of time than the policy effective period for which rates are being made.
I find the "premium based projection method" less intuitive than the exposure based method.
In answering part (c) one has to assume that the provisions for items other than fixed expenses are adequate.

7. (2.5 points) An insurance company began writing personal automobile policies in 2011. Given the following information for the insurance company:

Calendar/Accident Year	Written Policies	Ultimate Loss & LAE ($000)
2011	44,000	14,250
2012	48,400	19,500
2013	53,240	22,000

Variable expense ratio	20%
Profit and contingency provision	5%
Fixed expense per exposure	$50

- Expense and profit provisions are not expected to change.
- Policies have six-month terms, are written uniformly throughout the year, and include one automobile per policy.
- The company is currently charging an average premium per policy of $500.
- The annual loss trend factor = 3%.
- The data is fully credible.
- When calculating the indication, consider data from all three years.
- Rates are assumed to be effective July 1, 2014, and in effect for six months.

Calculate the overall indicated rate change, including justification for the selection of projected ultimate pure premium.

7. The average date of writing under the new rates is three months past July 1, 2014 or October 1, 2014, since the new rates will be in effect for six months.
The average date of accident under the new rates is three months beyond October 1, 2014, or January 1, 2015, since the policies are for six month terms.
Thus the trend period from AY 2013 is from July 1, 2013 to January 1, 2015 or 1.5 years.
Each policy insures one car for six months, and thus is 1/2 caryear of exposure.
Thus the written exposures by year are: 22,000, 24,200, and 26,620.
Since we have six month policies, only polices written in the second half of 2011 contribute any earned exposures to 2012; on average such policies contribute half of their earned exposures to 2012. Thus for each calendar year earned exposures are approximately:
(1/4)(prior year written exposures) + (3/4)(current year written exposures).
Thus CY2012 earned exposures are: (1/4)(22,000) + (3/4)(24,200) = 23,650.
CY2013 earned exposures are: (1/4)(24,200) + (3/4)(26,620) = 26,015.
Since the insurance company began writing policies in 2011,
CY2011 earned exposures are: (3/4)(22,000) = 16,500.
Trended pure premiums for each year separately:

AY2011: $(1.03^{3.5})(14.25 \text{ million}) / 16{,}500 = \$957.77.$

AY2012: $(1.03^{2.5})(19.5 \text{ million}) / 23{,}650 = \$887.76.$

AY2013: $(1.03^{1.5})(22 \text{ million}) / 26{,}015 = \$884.00.$

The pure premiums for the two most recent years are very similar, while 2011 seems like an outlier.
It would not be surprising that being the first year policies are written, 2011 would be unusual.
I will use the ratio of the totals for 2012 and 2013. (One could instead average the two pure premiums; and the result would be very similar.)
For 2012 and 2013, dividing the total trended loss and LAE by the total earned exposures, the projected pure premium is:

$$\frac{(1.03^{2.5})(\$19{,}500{,}000) + (1.03^{1.5})(\$22{,}000{,}000)}{23{,}650 + 26{,}015} = \$885.79.$$

Thus the indicated rate per exposure is: $\frac{\$885.79 + \$50}{1 - 25\%} = \$1247.72.$

The current rate per policy is \$500, which is \$1000 per exposure.
Indicated rate change is: \$1247.72 / \$1000 - 1 = **24.8%**.
Comment: One could instead put the indicated rate on a per policy basis and compare it to the current rate per policy. (Note that the fixed expense is \$50 per exposure or \$25 per policy.)
Earned exposures are a better comparison to AY ultimate losses than would be written exposures.
Note that since this insurer only began writing policies in 2011, the average accident date for AY2011 is slightly later than halfway through the year.
For a policy written at time t, $0 \le t \le 1/2$, all the premium is earned during 2011 and the average date of accident is $t + 1/4$. For a policy written at time t, $1/2 \le t \le 1$, the portion earned in 2011 is: $2(1- t)$, and the average corresponding date of the accident is: $(t + 1)/2$.

Thus the overall average date of accident in AY2011 is:

$$\frac{\int_0^{1/2} (t+1/4)\,dt + \int_{1/2}^{1} 2(1-t)(t+1)/2\,dt}{\int_0^{1/2} dt + \int_{1/2}^{1} 2(1-t)\,dt} = \frac{1/8 + 1/8 + \int_{1/2}^{1} 1 - t^2\,dt}{1/2 + 1/4} = \frac{1/4 + 1/2 - (1 - 1/8)/3}{3/4}$$

= 11/18.

Alternately, assume for simplicity that the insurer writes policies in the middle of each month.

Assume all of the premiums are 240.

Month	Premiums Earned in 2015	Average Date of These Earned Premiums
Jan.	240	1/24 + 1/4 = 14/48.
Feb.	240	3/24 + 1/4 = 18/48.
⋮		
June	240	11/24 + 1/4 = 34/48.
July	(11/12)(240) = 220	(13/24 + 24/24)/2 = 37/48.
August	(9/12)(240) = 180	(15/24 + 24/24)/2 = 39/48.
⋮		
Nov.	(3/12)(240) = 60	(21/24 + 24/24)/2 = 45/48.
Dec.	(1/12)(240) = 20	(23/24 + 24/24)/2 = 47/48.

Average date of earning is:

$$\frac{(240)(14/48) + (240)(18/48) + ... + (240)(34/48) + (220)(37/48) + ... + (60)(45/48) + (20)(47/48)}{(6)(240) + 220 + 180 + 140 + 100 + 60 + 20}$$

= (63,400 / 48) / 2160 = 0.611. In other words, the average of earning is about 11/18 of the way through the year, which is also the average date of accident.

The average date of accident is not the usual middle of the year, due to the fact that there are no policies written in 2010 that would normally contribute to the AY2011 reported losses.

Thus the trend from date differs, and the trend period for AY2011 should actually be:

3.5 - (11/18 - 1/2) = 3.39.

This would make the trended pure premium for AY2011:

$(1.03^{3.39})$(14.25 million) / 16,500 = \$954.66, rather than \$957.77.

Almost all actuaries would ignore this refinement which makes little practical difference.

8. (4.25 points) A company is reviewing the rate level adequacy in State X.
Given the following information for a book of business:

- All policies are annual.
- Rate change history:
  - 5% effective April 1, 2012, mandated by law to apply to all policies in force with no impact on losses.
  - 10% effective January 1, 2013.
- New rates will be in effect for 12 months beginning on April 1, 2015.
- Selected annual underlying loss trend = 2%.
- Selected annual premium trend = 0%.
- Loss adjustment expense provision = 4% of loss.
- Projected expense ratios:
  - Fixed = 5%.
  - Variable = 27%.
- Underwriting profit and contingencies provision = 8%.
- Ultimate losses are estimated using the reported development technique.
- Credibility of the indicated rate change = 0.6.
- State X's earned premium is 5% of the total earned premium countrywide.
- State X is part of Region A, and accounts for 50% of the total earned premium for that region.
- Potential complements of credibility include:
  - Countrywide rate indication = 10%.
  - Total Region A rate indication = 8%.
  - Major competitor rate indication for State X = 4%.
  - Annual inflation trend for State X = 3%.

Calendar Year Ending	Earned Premium ($000)
December 31, 2012	9,500
December 31, 2013	9,800

Accident Year as of December 31, 2013	Reported Losses ($000)
2012	4,800
2013	4,100

question continued on the next page

Age-to-Age Reported Loss Development Factors

Accident Year	12-24 months	24-36 months	36-48 months	48-60 months	60-72 months
2008	1.37	1.15	1.06	1.02	1.00
2009	1.35	1.15	1.05	1.02	
2010	1.32	1.12	1.07		
2011	1.28	1.09			
2012	1.25				

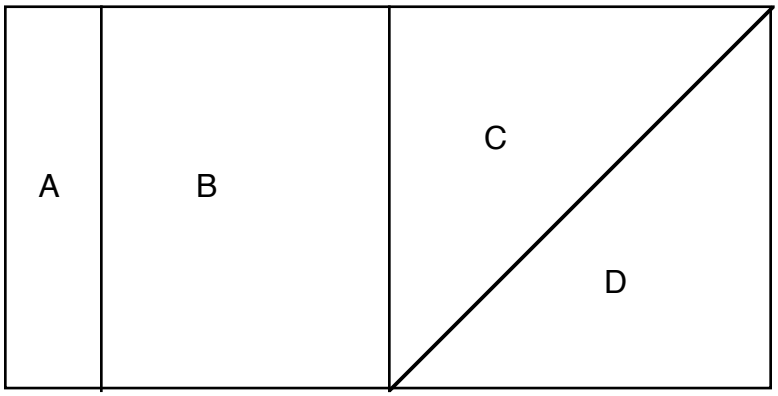
a. (1 point) Recommend a complement of credibility from the list above. Briefly explain the recommendation, including a brief discussion of each potential complement not selected.
b. (3.25 points) Calculate the indicated rate change using the complement of credibility recommended in part a. above.
Briefly justify your selection of age-to-age reported loss development factors.

8. (a) Using a major competitor's rate indication might make sense, provided you had some reason to believe that its current rate level adequacy in State X is similar to that of this company.
It would be better to use the competitor's rates themselves.
Assuming the January 1, 2013 change was thought at the time to make rates adequate, then using the ratio of the loss trend to the premium trend over the appropriate period of time would make sense. This would be an example of using "Trended present rates." The appropriate period to trend is from the last rate increase to the current rate increase or 2.25 years. The net annual trend is 1.02, not 1.03. Therefore, I would not use the annual inflation trend for State X of 1.03.
Using either the rate indication for Region A or countrywide are examples of "Rate change from a larger group applied to present rates."
Using the rate indication for Region A would make some sense provided you had some reason to believe that the current rate level adequacy in Region A is similar to that in State X. However, since State X is 50% of Region A, the two rate indications are dependent. Thus I would not use this complement. Region A excluding State X would have been much better.
Using the rate indication for countrywide would make some sense provided you had some reason to believe that the current rate level adequacy countrywide is similar to that in State X. Since State X is 5% of countrywide, the two rate indications are close to independent. Countrywide excluding State X would be better.
Given the limited information provided in the question, **I will use the major competitor's rate indication for State X of 4%** This complement is logical, available, easy to compute, and independent of our company's indicated rate change.
Alternately, I would not use the CW rate indication because there may be many differences between the "average CW company" and companies in State X (operational, territorial.)
I would not use State A indication because State X accounts for 50% of the region which would make the complement not independent from base. The major competitor may also have many operational differences with State X and the data is not available to make proper adjustments.
My selection would be trended present rates. The information is available. This complement will be unbiased and independent since projection and experience period to not overlap. It is easy to compute and has logical relationship to base.
Alternately, I would choose the CW indication as the complement of credibility because it is credible and accurate. State A only has 5% of CW data, so CW can also be seen as independent.
Total Region A is not appropriate because State X is 50% of the exposure, so it is not independent. Major competitor rate indication is not unbiased because those two companies are not targeting the exact same market.
Annual inflation is not appropriate because it will not accurately reflect the rate need in State X.

(b) The link ratios seem to be decreasing, thus I will select of the average of the last three available factors. (There are other reasonable selections.)
12-24: 1.28 24-36: 1.12 36-48: 1.06 48-60: 1.02 60-72: 1.00.
Average date of writing is 11/1/2015. Average date of accident is 4/1/2016.

AY	Reported Losses	Loss Development	Trend	Projected Losses at Ultimate
2012	4,800,000	(1.12)(1.06)(1.02)	$1.02^{3.75}$	6,260,600
2013	4,100,000	(1.28)(1.12)(1.06)(1.02)	$1.02^{2.75}$	6,710,708

Date of Rate Change	Rate Level Index
	1.00
4/1/12	0.95
1/1/13	(0.95)(1.1) = 1.045.



On-level Factor for CY12: 1.045 / {(1/4)(1) + (3/4)(0.95)} = 1.0857.
On-level Factor for CY13: 1.045 / {(1/2)(0.95) + (1/2)(1.045)} = 1.0476.
There is no premium trend, and the sum of the on-level premiums is:
(1.0857)(9,500,000) + (1.0476)(9,800,000) = 20,580,630.
The loss and LAE ratio is: (1.04)(6,260,600 + 6,710,708) / 20,580,630 = 0.6555.
Thus the rate change factor indicated by the data is: (0.6555 + 0.05) / (1 - 0.27 - 0.08) = 1.0854.
Using a complement of credibility of 4%, the indicated rate change is:
(0.6)(8.54%) + (1 - 0.6)(4%) = **6.724%**.
Comment: As is common in exam questions, one would be better able to answer part (a) with a lot more information about the situation.
There are many other reasonable choices in part (b), for example taking the average of the latest two link ratios. You need to explain the reason(s) for whatever your choice.
Also the rate indication in part (b) depends on your choice of complement of credibility in part (a).

9. (2.75 points) An insurance market with a fixed number of insureds consists of two insurers - Company A and Company B. Company A has identified a new potential rating variable to segment its risks, consisting of High Risk and Low Risk.

		Insured Risks	
Variable	True Expected Cost	Company A	Company B
High Risk	$200	10,000	90,000
Low Risk	$100	10,000	90,000

- All policies are annual.
- True expected cost is known only to Company A
- The probability each risk will switch insurers at renewal if they are offered a lower price by the new insurer is given by the following equation:
  Probability = 0.9 x (Difference in Offered Rates) / True Expected Cost
- The probability each risk will switch insurers at renewal if they are offered a higher or equal price by the new insurer is 0.

Company A intends to charge the true cost for High Risk insureds, and is evaluating two different prices for Low Risk insureds: $130 or $140. Company B charges $150 for all risks.

a. (1.75 points) Determine which of the two rates Company A should charge the Low Risk insureds to maximize profits, assuming Company B does not adjust its price.

b. (0.5 point) Describe the ultimate impact on the distribution of risks and each company's profitability if Company B does not adjust its strategy.

c. (0.5 point) Briefly describe two possible strategies Company B could utilize in response to Company A's new rate plan.

9. (a) For the Low Risks, B charges $150.
Thus if A charges $130 the probability of switching from A to B each year is: (0.9)(20)/100 = 18%.
If A charges $140 the probability of switching from A to B each year is: (0.9)(10)/100 = 9%.
(Since we are only evaluating the choice of how to price Low Risks, we can ignore High RIsks.)
With a price of $130, the number of Low Risks written by A grows more quickly.

The number of Low Risks written by B in year n is: $(90{,}000)(0.82^n)$.

Thus the number of Low Risks written by A in year n is: $100{,}000 - (90{,}000)(0.82^n)$.

The profit made by A is: $(30)\{100{,}000 - (90{,}000)(0.82^n)\}$.

With a price of $140, the number of Low Risks written by A grows less quickly.

The number of Low Risks written by B in year n is: $(90{,}000)(0.91^n)$.

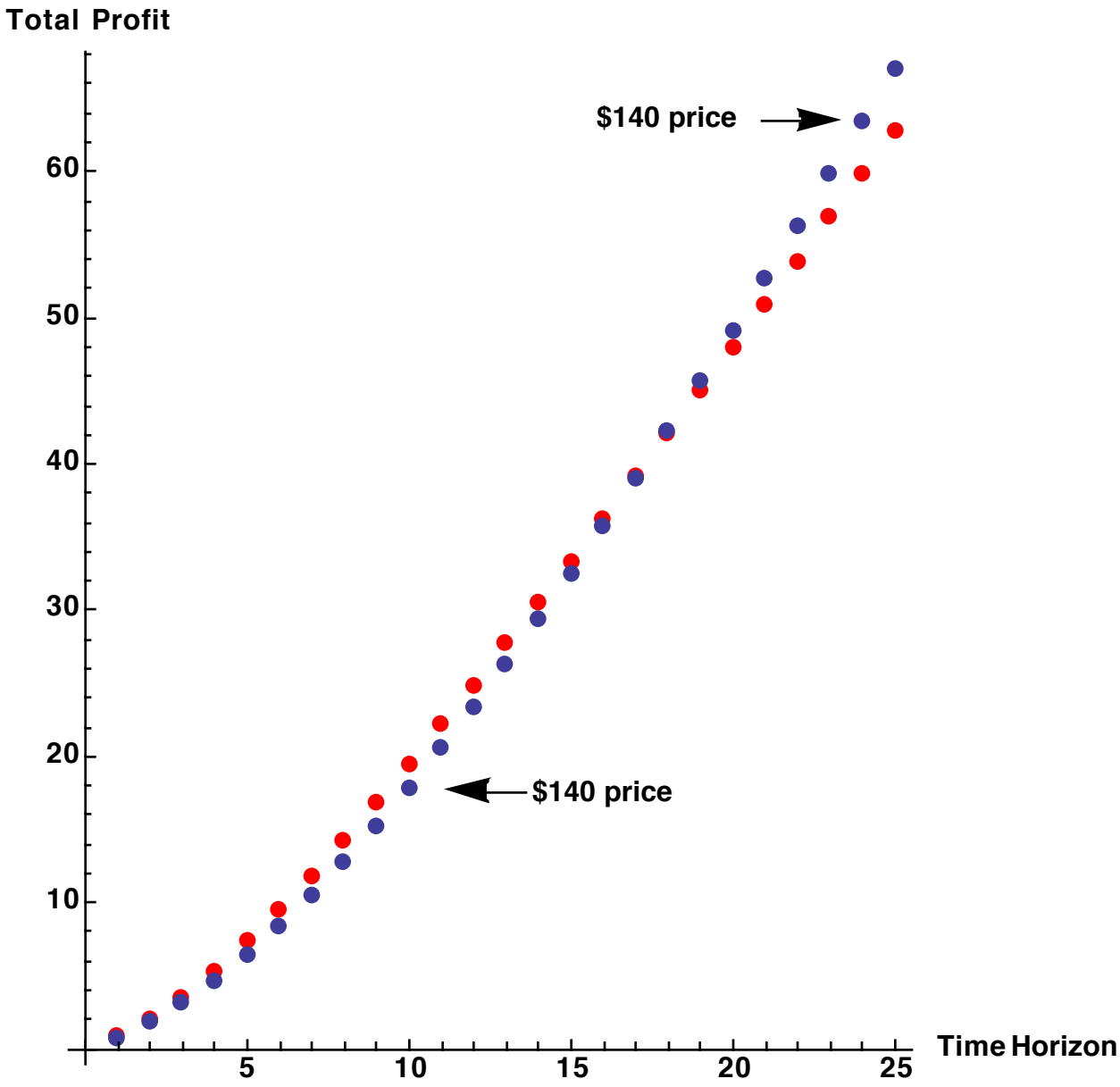
Thus the number of Low Risks written by A in year n is: $100{,}000 - (90{,}000)(0.91^n)$.

The profit made by A is: $(40)\{100{,}000 - (90{,}000)(0.91^n)\}$.

Over a one-year time horizon, with a rate of $140, A would write low risks:
10,000 + (9%)(90,000) = 18,100; profit is: ($140 - $100) (18,100) = $724,000.
Over a one-year time horizon, with a rate of $130, A would write low risks:
10,000 + (18%)(90,000) = 26,200; profit is: ($130 - $100) (22000) = $786,000.
Thus, over a one-year time horizon, A should charge $130 for the low risk insureds.
Eventually, A will write most of the Low Risks and would prefer to make a profit of $40 per Low Risk rather than $30 per Low Risk. Thus which strategy is more profitable depends on the time horizon and the discount factor used.
Trying values of n, the profit is more for the $140 price for n = 11.
Starting in year 11, charging the higher $140 all along for Low Risks will be better.
However, it will take even longer for the cumulative profit to be bigger with the $140 price.
Given a reasonable time horizon such as ten years, and/or a reasonable discount factor, the $130 price is better for A.
(b) For the High Risks, A charges $200 while B charges $150. Thus the probability of switching from A to B is: (0.9)(50)/200 = 22.5%.
Ultimately, A will write almost all of the Low Risks and B will write almost all of the High Risks.
B will charge $150 for risks that cost $200 and will lose a lot of money.
Company B has been hurt by adverse selection.
As the adverse selection continues, Company B will go through a cycle of increasing rates which leads to more adverse selection until it adopts a strategy as per part (c).
Company A will charge either $130 or $140 for risks that cost $100 and will be very profitable.
(c) 1. If B can figure out the rating variable being used by A, then B can adopt something similar to A's pricing strategy.
2. B can raise the price on all risks to $200, and thus eventually it will write almost exclusively High Risks. However, B will maintain a share of this market without losing money on each risk written.
3. B could exit this market.
4. B can find other rating characteristics to segment the market in a more refined manner that A has not discovered.

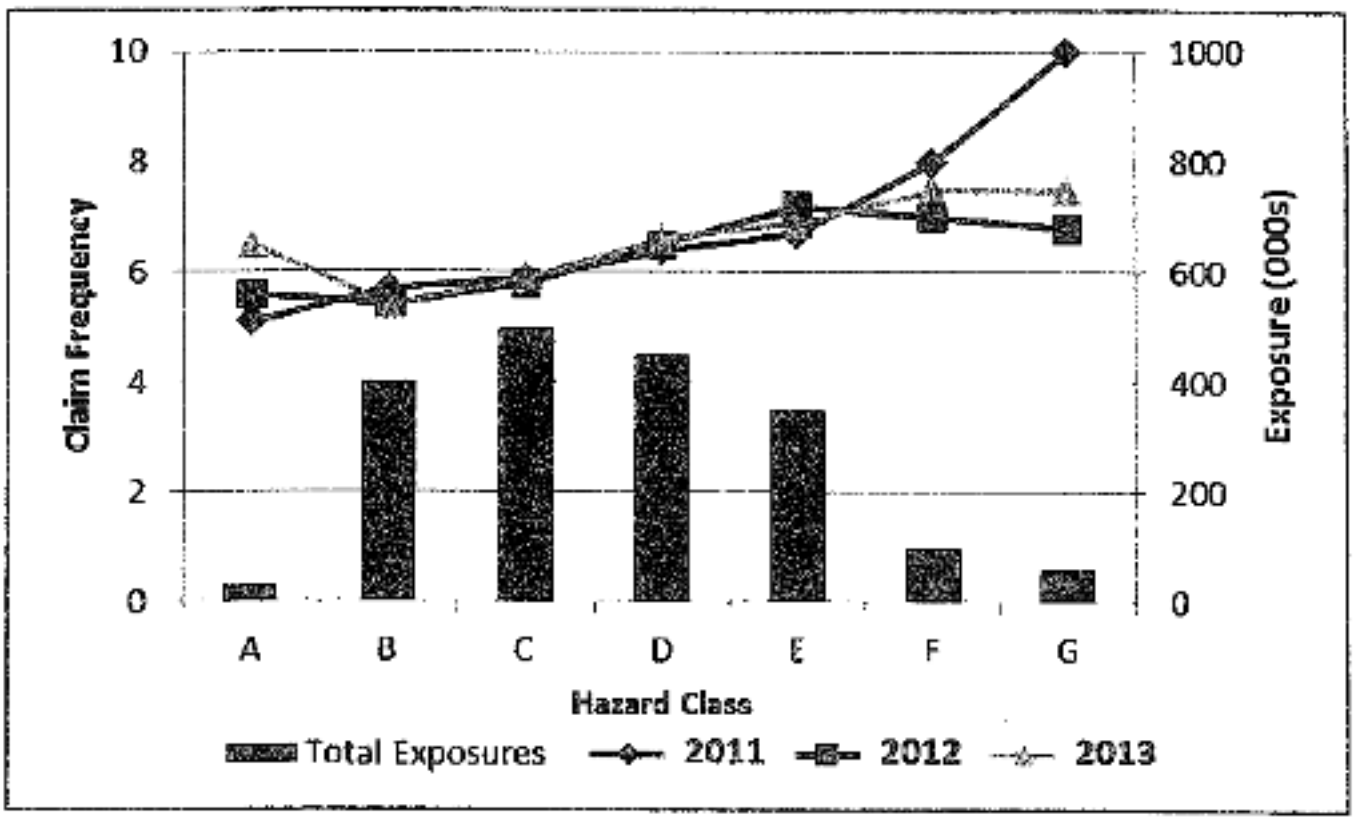
5. B can change its marketing strategy or provide better customer service to attract more low risk insureds.

Comment: Here is the cumulative profit ($ million) for Company A, with the result of using the $140 price shown in blue and the result of using the $130 price shown in red:



The two strategies are about equal for an 18 year time horizon. For shorter time horizons, the $130 price produces more total profit for A, while for longer time horizons the $140 price produces more total profit for A. I ignored the time value of money, since we are not given a discount factor to use.

10. (1.75 points)
An actuary performed an analysis of a products liability class plan using a Generalized Linear Model (GLM) for the first time on this book of business. The insureds are categorized by hazard classes A through G. The following graph shows claim frequency and exposure data by hazard class.



a. (0.75 point)
Fully evaluate the predictive value of hazard class based on the information provided above.
b. (1 point) Briefly describe two data mining techniques and how each might be used to enhance a GLM multivariate classification analysis.

10. (a) The indicated frequencies differ significantly by hazard group. (We are not given information in order to determine whether these differences are statistically significant.) Indicated relativities increase with Hazard Group, with the exceptions of Hazard Groups A and G which have much less data than the others. The separate indications for the three years are consistent, with the exceptions of Hazard Groups A and G which have much less data than the others. Therefore, I conclude that hazard group is useful for predicting expected frequency.

(b) 1. Factor analysis, of which principle components analysis may be the most commonly used, is a technique to reduce the number of parameter estimates in a classification analysis. This can imply a reduction in the number of variables or a reduction in the levels within a variable.

2. Cluster analysis is an exploratory data analysis tool that seeks to combine small groups of similar risks into larger homogeneous categories or "clusters." It generally aims to minimize the differences within a category and maximize the difference between categories. For example, cluster analysis could be used to group zipcodes into territories or to group classes into Hazard Groups.

3. The purpose of CART (Classification and Regression Trees) is to develop tree-building algorithms to determine a set of if-then logical conditions that help improve classification. Examination of the tree may help ratemaking actuaries whittle down a long list of potential variables to a more manageable yet meaningful list and determine how to categorize each variable. CART can also help detect interactions between variables.

4. The Multivariate Adaptive Regression Spline (MARS) algorithm operates as a multiple piecewise linear regression where each breakpoint defines a region for a particular linear regression equation. This technique is generally used to select breakpoints for categorizing continuous variables.

5. Neural networks are very sophisticated modeling techniques. The neural network user gathers test data and invokes training algorithms designed to automatically learn the structure of the data.

Comment: For part (b) describe only two techniques. See pages 183-185 of Basic Ratemaking.

11. (3 points) Given the following information for a homeowners book of business:

Territory	Earned Exposures	Earned Premium ($000)	Ultimate Losses Excluding Catastrophes ($000)	Current Relativity
1	2,500	3,375	3,200	1.150
2	7,000	11,200	6,200	1.000
3	500	700	1,000	0.900

- Ratio of ALAE to loss = 4%.
- Full credibility standard for exposures = 5,000.
- Use square root rule for credibility calculations.
- Territory 2 is the base class.
- The rating algorithm is Base Rate x Territory Factor x Amount of Insurance Factor.

a. (2 points) Calculate the credibility-weighted indicated non-catastrophe relativity to the base for each territory using the pure premium method.
b. (0.5 point) Territory 1 has a high percentage of low-value homes relative to territories 2 and 3. Describe a possible distortion to the indicated territory 1 relativity resulting from the distribution of home values.
c. (0.5 point) Assume that $1,000,000 of the loss in territory 2 came from a single loss. Discuss an appropriate adjustment to the analysis.

11. (a) The credibility for territory 3 is: $\sqrt{500 / 5000} = 31.62\%$.

Then as per Appendix E in Basic Ratemaking:

	[1]	[2]	[3]	[4]	[5]	[6]
Terr	Earned	Ultimate	Pure	Indicated	Current	Normalized
	Exposures	Losses	Premium	Relativity	Relativity	Current
		Excluding				Relativity
		Catastrophes	=[2]/[1]	= [3]/Total [3]		=[5]/Total [5]
1	2,500	$3,200,000	$1280.00	1.2308	1.150	1.1138
2	7,000	$6,200,000	$885.71	0.8516	1.000	0.9685
3	500	$1,000,000	$2000.00	1.9231	0.900	0.8717
Total	10,000	10,400,000	$1040.00	1.0000	1.0325	1.0000

	[7]	[8]	[9]
Class	Credibility	Credibility	Cred-Weighted
		Weighted	Relativity
		Relativity	w.r.t. Base
			=[8]/[8] for 2
1	70.71%	1.1965	**1.4049**
2	100.00%	0.8516	**1.0000**
3	31.62%	1.2042	**1.4139**
Total		0.9555	1.1219

Note that one can include the ALAE factor of 1.04 without changing the relativities, since the same factor would apply to every territory.

(b) Low-value homes have lower average severities and thus lower average pure premiums. Thus the observed pure premium for territory 1 is lower than it would otherwise be just because of this different mix of business. Assuming we already take into account the lower pure premium for low-value homes through appropriate relativities for amount of insurance, we would be double counting; the premiums charged to homes in territory 1 would be on average too low. Such differences in mix of business, in other words correlation of exposures, can cause distortions when each dimension is analyzed separately via the pure premium method as is done here.

(c) One could cap the impact of large losses at for example $250,000. (We are not given enough information to decide what would be the most appropriate cap to apply.)

Then any amount above $250,000 would not enter into the calculation of territory relativities. It would be a good idea to load back in an average provision by territory for the amounts excluded by capping via a longterm ratio of excess losses to non-excess losses; if practical this would be based on many more years of data. (The frequency of large property claims could vary by territory based on differing values of homes, protection classes, and construction types.)

Also one should use basic limits liability losses.

Alternately, this shock loss should be excluded and we should include an appropriate large loss load based on an analysis of a larger volume of data.

Comment: According to the CAS Examiner's Report, one was expected to include ALAE in part (a), unless one mentioned that ALAE had no effect on this particular problem's answer.

12. (2.5 points) Given the following information:

Policy Year	Premium	Present Value of Losses	Variable Expenses New	Variable Expenses Renewal	Fixed Expenses New	Fixed Expenses Renewal	Income
1	$800	$656	$242	-	$142	-	$(240)
2	$872	$701	-	$54	-	$32	$86
3	$950	$748	-	$59	-	$33	$110
4	$1,036	$799	-	$64	-	$34	$139
5	$1,129	$853	-	$70	-	$36	$170

Policy Year	Persistency	Cumulative Persistency	Profit	Discount Factor	Present Value of Profits	Present Value of Premiums
1	100%	100%	$(240)	1.00	$(240)	$800
2	85%	85%	$73	1.12	$65	$662
3	86%	73%	$81	1.25	$64	$554
4	87%	64%	$88	1.40	$63	$469
5	88%	56%	$95	1.57	$61	$402
Total					$13	$2,886

- Premium-to-surplus ratio is 2 to 1.
- Surplus equals GAAP equity.
- The company seeks growth in this market.
- Management requires the present value of profit of policy years 1 to 5 to be positive in total.

a. (0.5 point) Briefly describe two differences between asset share pricing and pure premium ratemaking when they are used to price property and casualty products.
b. (1 point) After preparing the asset share model shown above, the actuary evaluates an alternative set of persistency assumptions in which persistency in the third and fourth policy years are changed to 81% and 82%, respectively. Calculate the revised present value of premiums.
c. (1 point) Briefly discuss the results of the asset share model under each set of persistency assumptions with regard to Management's profitability requirement. Provide a recommendation to management on whether to make a change to the current rating structure.

12. (a) 1. The insurer can expect to renew a majority of its policies each year and lifetime analysis considers not just the current policy but also the profits from expected renewals, while pure premium ratemaking does not.
2. Lifetime analysis takes into account that the expected losses depend on how long an insured has been with the insurer, while pure premium ratemaking does not.
3. Lifetime analysis takes into account that the expenses depend on how long an insured has been with the insurer, expenses on renewal polices are lower than on new policies, while pure premium ratemaking does not.
4. Lifetime analysis takes into account renewal probabilities, while pure premium ratemaking does not.
5. Lifetime analysis uses a discount factor to take present values of profit and premium, while pure premium ratemaking does not.

(b) ($950)(69%)/1.25 = $524. ($1036)(56%)/1.40 = $414. ($1129)(50%)/1.57 = $360.

Policy		Present Value of	Variable Expenses		Fixed Expenses		
Year	Premium	Losses	New	Renewal	New	Renewal	Profit
1	$800	$656	$242	-	$142	-	$(240)
2	$872	$701	-	$54	-	$32	$86
3	$950	$748	-	$59	-	$33	$110
4	$1,036	$799	-	$64	-	$34	$139
5	$1,129	$853	-	$70	-	$36	$170

Policy Year	Renewal Probability	Cumulative Persistency	Adjusted Profit	Discount Factor	Present Value of Profits	Present Value of Premiums
1	100%	100%	$(240)	1.00	$(240)	$800
2	85%	85%	$73	1.12	$65	$662
3	81%	68.9%	$76	1.25	$61	$524
4	82%	56.5%	$79	1.40	$56	$418
5	88%	49.7%	$84	1.57	$54	$357
Total					$(4)	**$2,761**

(c) The return on surplus using the original persistency assumption is: 13 / (2,886/2) = 0.9%.
The return on premium (profit percentage) using the revised persistency assumption is:
-4 /2,761 = -0.1%
The return on surplus using the revised persistency assumption is: -4 / (2,761/2) = -0.3%.
Under the original persistency assumption the present value of profit of policy years 1 to 5 is positive in total. Under the revised persistency assumption the total profitability the present value of profit of policy years 1 to 5 is not positive in total. Whether the requirement of management is met depends on which of the two persistency assumptions is more realistic.
We are asked to "Provide a recommendation to management on whether to make a change to the current rating structure." However, we are not told whether the exhibit provided relates to the current rating structure or to some proposed new rating structure. Neither are we told which of the two persistency assumptions is more realistic. So there is insufficient information in order to provide a recommendation to management.
Alternately, management could offer a renewal discount to improve persistency; the discount could be calculated so that overall profits remain positive.
Alternately, I recommend that marketing is increased to boost persistency so that there can be growth and remain profitable. I am skeptical about increasing rates as this will reduce growth and persistency.
Comment: Given the wording of this exam question and limited information provided, I do not see how one could "Provide a recommendation to management on whether to make a change to the current rating structure." In my opinion, this portion of part (c) is extremely poorly written; perhaps you are expected to read the minds of the examiners.
Apparently, the two alternatives I have provided from the CAS Examiner's Report are the types of responses desired. A good example of how sometimes writing down something related to what is in the syllabus reading can get you some points.
In my opinion, there is no way to know that a renewal discount would raise persistency sufficiently (from whichever of the two assumptions used is more realistic) to increase the profit percentage over the given five year time-horizon. So I do not believe that the first of the two alternative recommendations is reasonable coming from an actuary.
In my opinion, there is no way to know whether additional marketing would raise persistency sufficiently (from whichever of the two assumptions used is more realistic) in order to make up for its cost. In addition there is no way to tell how much persistency would be decreased by raising rates and therefore whether raising rates somewhat might be a good idea. So I do not believe that the second of the two alternative recommendations is reasonable coming from an actuary.