Solutions to the Fall 2013 <u>CAS</u> Exam <u>5</u>

(Only those questions on Basic Ratemaking)

Revised January 10, 2014 to correct an error in solution 11.a.
Revised January 20, 2014 to correct an error in solution 9.
Revised February 9, 2014 to incorporate the CAS sample solutions.

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1. (1.5 points) An insurance company is considering changing its exposure base for workers compensation from payroll to hours worked.
Evaluate the merits of this change based on three different criteria of a good exposure base.

1. The three criteria of a good exposure base:
1. Proportional to Expected Loss
2. Practical
3. Historical Precedence
An increase in hours worked would increase the frequency of losses; but the payroll would also increase proportionally if more hours are worked. Payroll is also proportional to the Average Weekly Wage, which affects the severity of indemnity benefits, while hours worked is not. Since indemnity benefits are the majority of dollars paid, I think that payroll is better based on the first criterion.
On the other hand, the severity of medical benefits do not obviously vary with payroll, so there is some merit to hours worked.
Payroll is collected for other purposes, such as taxes.
Payroll is well-defined and verifiable.
Thus payroll is practical.
Number of hours worked is harder to verify than payroll.
Number of hours worked is easier for the employer to manipulate than payroll.
Number of hours worked may require additional expense to collect.
Thus, number of hours worked is less practical than payroll.
Payroll has been used for many years and thus is preferred based on historical precedent.
There would be significant difficulties transitioning from payroll to hours worked.
Possible difficulties include: large premium swings for individual insureds, reprogramming of the computers and other system changes, changes required in the rating algorithm, and for several years a lack of ratemaking data on the new exposure basis.
Based on all of the above, I would <u>not</u> recommended switching to hours worked.
<u>Comment</u>: Washington state uses hours worked (by class) as the exposure base.

2. (2 points) Given the following information:

- All policies have six-month terms.
- Policies are written uniformly during each six-month period and cannot be cancelled.
- The rating algorithm is base rate x class factor + expense fee.
- The proposed effective date of the next rate change is July 1, 2013.
- A rate review is performed every six months.

Effective Date of Rates	Base Rate Per Exposure	Class Factor A	Class Factor B	Expense Fee
January 1, 2011	$480	1.00	0.70	$45
July 1, 2011	$488	1.00	0.70	$45
January 1, 2012	$504	1.00	0.70	$50
July 1, 2012	$500	1.00	0.75	$50
January 1, 2013	$500	1.00	0.80	$55

	Written Exposures (000)	
Policy Effective Dates	Class A	Class B
January 1, 2011 - June 30, 2011	125	50
July 1, 2011 - December 31, 2011	150	100
January 1, 2012 - June 30, 2012	175	150
July 1, 2012 - December 31, 2012	200	200

Using the extension of exposures method, calculate the calendar year 2012 earned premium at current rate level.

2. All policies are 6-month, so policies effective January 1, 2011 - June 30, 2011 contribute nothing to the calendar year 2012 earned exposures,
policies effective July 1, 2011 - December 31, 2011 contribute on average half their exposures,
policies effective January 1, 2012 - June 30, 2012 contribute all of their exposures, and policies effective July 1, 2012 - December 31, 2012 contribute on average half their exposures.
At current rate level, class A pays: ($500)(1.00) + $55 = $555,
while class B pays: ($500)(0.80) + $55 = $455.
Calendar year 2012 earned premium at current rate level is:
($555)(150/2 + 175 + 200/2)(1000) + ($455)(100/2 + 150 + 200/2)(1000) = **$330.75 million**.
Comment: The expense constant is earned over time, just as with any other premium.
I have assumed that the $55 expense constant applies per exposure; this makes sense based on the given rating algorithm and the fact that the given base rates are per exposure, but this could have been made clearer.

3. (1.5 points)
When aggregating data for ratemaking purposes, two of the three general objectives are:

- To accurately match losses and premiums for the policy.

- To use the most recent data available.

Briefly discuss how well the following methods of data aggregation achieve these two general objectives.
a. (0.5 point) Calendar year
b. (0.5 point) Calendar/accident year
c. (0.5 point) Policy year

3. (a) Calendar year data is available the soonest; it is final at the end of the calendar year.
However, calendar year data has the poorest match between loss and premiums.
The calendar year earned premiums are portion of premiums from several years of policies.
Calendar year losses are amounts paid during the year plus changes in loss reserves during the year. Depending on how long-tailed the line of insurance is, this can reflect losses from many different policy years.
(b) Calendar/accident year data is not as recent as calendar year data. While calendar/accident year data is also available at the end of the year, the losses are subject to development. For lines of insurance with a lot of development, effectively calendar/accident year data is not useful as soon as calendar year data.
Calendar/accident year data is a better match between premiums and losses than calendar year data, but not as good as policy year. Accident year losses are determined by the accidents that occur during the year, which is an approximate match to the calendar year earned premiums.
(c) Policy year data is not available as soon as the others. For example, for 2014 policy year data, assuming annual policies, it takes until the end of 2015 for all of the policies to expire. Then as with accident year loses, policy year losses are subject to development. In addition policy year premiums are also subject to development.
Policy year data accurately matches losses and premiums for a given set of policies.
<u>Comment</u>: Calendar year is the most recent while policy year is the least recent.
Policy year has the best match, while calendar year has the worst match.

4. (7.5 points) Given the following information:

- The insurance company entered the market in State X at the beginning of 2008.
- All policies are annual.
- Rates will be in effect for 12 months beginning on July 1, 2014.
- Rate change history:
  - +5% effective July 1, 2010.
  - +7% effective April 1, 2012.
- Premiums are expected to increase at an inflationary rate of 2% annually.
- Annual loss cost trend = +4%.
- ULAE provision = 12% of loss and ALAE.
- Fixed expense ratio = 7%.
- Variable expense ratio = 21 %.
- Underwriting profit and contingencies provision = 8%.
- To simplify calculations, assume premium is earned evenly throughout the year.
- Assume no loss development after 48 months.

Calendar Year	State X Earned Premium	State X Earned Exposures	State X Written Premium	State X Written Exposures
2010	$400,000	400	$630,000	600
2011	$2,200,000	2,000	$3,105,000	2,700
2012	$16,800,000	14,000	$18,750,000	15,000

State X Incurred Losses and ALAE

Accident Year	12 Months	24 Months	36 Months	48 Months
2008	$0	$200	$800	$1,000
2009	$50,000	$61,300	$78,200	$80,000
2010	$380,500	$587,000	$624,486	
2011	$671,600	$1,316,239		
2012	$9,706,667			

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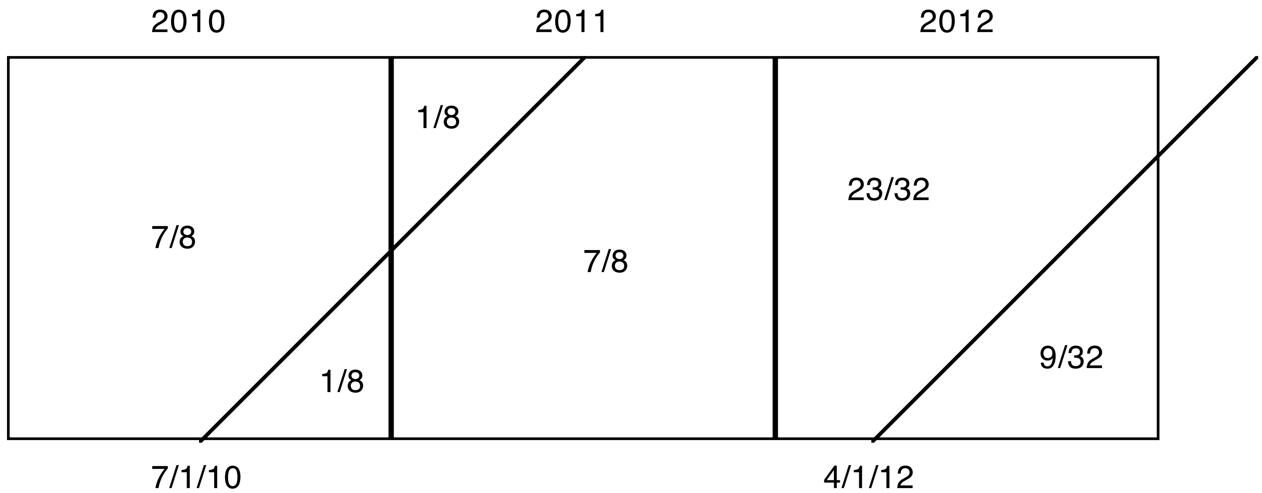
	Countrywide Incurred Losses and ALAE			
Accident Year	12 Months	24 Months	36 Months	48 Months
2007			$123,600,000	$125,000,000
2008		$62,700,000	$68,600,000	$70,000,000
2009	$75,000,000	$83,300,000	$88,200,000	$90,000,000
2010	$80,500,000	$87,000,000	$93,000,000	
2011	$71,600,000	$78,800,000		
2012	$86,900,000			

a. (6.25 points) Calculate the indicated rate level change for State X using the loss ratio method. Use two-step trending to project premiums.
b. (0.5 point) The assumption that the premium is earned evenly should not hold for State X given that it is a new book of business. Briefly describe two alternatives to the traditional parallelogram method that would improve the accuracy of the estimated projected premiums.
c. (0.75 point) Fully discuss the impact to the rate level indication for State X by assuming the premium is earned evenly. Include the directional change to the rate level indication that would result if adjusting for the actual earning of the premium.

4. (a)

Effective Date	Rate Level Change	Rate Level Index
		1.0000
7/1/2010	+5%	1.0500
4/1/2012	+7%	(1.05)(1.07) = 1.1235



On Level Factor for 2010: $\frac{1.1235}{(7/8)(1) + (1/8)(1.05)} = 1.1165.$

On Level Factor for 2011: $\frac{1.1235}{(1/8)(1) + (7/8)(1.05)} = 1.0764.$

On Level Factor for 2012: $\frac{1.1235}{(23/32)(1.05) + (9/32)(1.1235)} = 1.0493.$

Thus the on-level 2010 earned premium is: (1.1165)(400,000) = $446,600.
On-level 2011 earned premium is: (1.0764)(2,200,000) = $2,368,080.
On-level 2012 earned premium is: (1.0493)(16,800,000) = $17,628,240.
The average written premium in 2012 is: 18,750,000 / 15,000 = $1250.
One fourth of this was written at the 7/1/2010 rate and three quarters at the current 4/1/2012 rate.

Thus put on level, this average WP is: $\frac{1.07}{(1/4)(1) + (3/4)(1.07)}$ 1250 = $1270.78.

The on-level average earned premium in 2010 is: (1.1165)(400,000) / 400 = $1116.50.
The on-level average earned premium in 2011 is: (1.0764)(2,200,000) / 2000 = $1184.04.
The on-level average earned premium in 2012 is: (1.0493)(16,800,000) / 14,000 = $1259.16.
Thus the first step premium trend factors are:
1270.78/1116.50 = 1.1382, 1270.78/1184.04 = 1.0733, and 1270.78/1259.16 = 1.0092.
With annual policies, the average data of writing under the new rates is January 1, 2015, and in the second step we trend 2.5 years from an average writing date of July 1, 2012 for CY2012 written premium. Thus the trended on-level premium for the three years combined is:
{(446,600)(1.1382) + (2,368,080)(1.0733) + (17,628,240)(1.0092)} ($1.02^{2.5}$) = $21,898,102.

The state loss and ALAE data is too sparse to use for a loss development analysis.
(In addition, it may take some time for the claims handling and reserving pattern to stabilize in this new state.)
So I will instead rely on the countrywide data. (I am concerned that what little state data we have seems to show very high loss development but nevertheless will use the CW data.)
I will rely on volume weighted link ratios of all the available countrywide data.

$\frac{83.3 + 87.0 + 78.8}{75.0 + 80.5 + 71.6} = 1.0969.$ $\frac{68.6 + 88.2 + 93.0}{62.7 + 83.3 + 87.0} = 1.0721.$

$\frac{125.0 + 70.0 + 90.0}{123.6 + 68.6 + 88.2} = 1.0164.$

State AY2010 developed to ultimate: (624,486)(1.0164) = 634,728.
State AY2011 developed to ultimate: (1,316,239)(1.0721)(1.0164) = 1,434,283.
State AY2012 developed to ultimate: (9,706,667)(1.0969)(1.0721)(1.0164) = 11,602,114.
With annual policies, the average data of accident under the new rates is July 1, 2015.
Thus the trended loss and ALAE for the three years combined is:
$(634,728)(1.04^5) + (1,434,283)(1.04^4) + (11,602,114)(1.04^3) =$ \$15,500,952.
Including ULAE, we get loss and LAE of: (1.12)(\$15,500,952) = \$17,361,066.
The loss and ALAE ratio is: 17,361,066 / 21,898,102 = 79.28%.

Thus the rate indication is: $\frac{79.28\% + 7\%}{1 - 21\% - 8\%} - 1 =$ **21.5%**.

(b) One could work with smaller units of premium than a year, such as months or quarters. Applying separate on-level factors to each of these smaller time periods would better reflect the changing volume than would working with years.
Alternately, one could extend the past exposures by the current rates.
(c) There are past rate increases and the volume of business written is expanding over time.
Therefore, the past premiums were actually written at a higher rate level than if one assumed an even level of writing. Therefore, the on-level factors in part (a) are too high.
Therefore, the on-level premium in part (a) is too high.
Thus, the on-level ultimate projected loss ratio in part (a) is too low.
Therefore, the indicated rate change in part (a) is too small.
Comment: If the volume of business written is expanding rapidly, then in theory one should also adjust the trend from dates; they would be somewhat later in time than in part (a). In practice such an adjustment would have a smaller impact than the adjustments to on-level premium in part (b), and is rarely made.
There are other reasonable choices one can make on loss development; as always do something sensible and briefly explain what you are doing and perhaps why.
One need only show details for one of a series of similar calculations; show just enough detail so that the grader can follow what you did.

5. (2.75 points) Given the following information:

- All policies are annual, and rate level is reviewed annually.
- Rate change takes effect on January 1, 2013.
- Unlimited annual loss frequency trend = -1%.
- Unlimited annual loss severity trend = +5%.
- Annual average written premium trend per exposure = 0%.
- Assume the exposures are inflation sensitive.
- Annual exposure trend = +1%.

Calendar/ Accident Year	On-Level Earned Premium ($000s)	Reported Losses as of December 31, 2012 ($000s)	Reported Losses Excess of $1 Million as of December 31, 2012 ($000s)	Unlimited Loss Development Factor
2003	$60,612	$34,054	$456	1.00
2004	$61,941	$44,617	$4,888	1.00
2005	$66,893	$41,086	$5,348	1.00
2006	$67,092	$39,025	$8,774	1.00
2007	$65,960	$45,646	$8,134	1.00
2008	$65,037	$36,383	$0	1.00
2009	$65,242	$38,487	$1,398	1.00
2010	$67,732	$36,799	$0	1.03
2011	$69,450	$38,608	$2,002	1.08
2012	$67,213	$45,295	$9,000	1.20
Total	**$657,172**	**$400,000**	**$40,000**	**N/A**

a. (1.5 points) For accident year 2012, determine the trended ultimate loss ratio to use in the January 1, 2013 rate level analysis incorporating a large loss adjustment for claims above $1 million.
b. (0.5 point) Discuss the appropriateness of using a large loss adjustment in part a. above.
c. (0.75 point) Assume the rate associated with the first $1 million of coverage is analyzed using only the data above. Briefly discuss three modifications to loss and premium elements that would produce a more accurate rate analysis.

5. (a) With annual policies, the average date of accident under the new rates is January 1, 2014.
Using the totals, the large loss adjustment factor is: 1 + 40,000 / (400,000 - 40,000) = 1.111.
Subtract the excess losses for 2012 from the reported losses, then multiply by the adjustment factor: (1.111)(45,295,000 - 9,000,000) = $40,323,745.
Multiply by the loss development and trend factors:
(1.20)($40,323,745)($1.05^{1.5}$)($0.99^{1.5}$) = $51,283,640.
(It would be better to have loss development factors for the limited losses rather than the unlimited losses. It would be better to have a limited loss severity trend.)
Trend the 2012 on-level premiums: (67,213,000)($1.01^{1.5}$) = $68,223,711.
The trended ultimate loss ratio is: $51,283,640/$68,223,711 = **75.2%**.
(b) There is a lot of fluctuation year to year in the percentage of loss dollars that are excess of $1 million. Therefore, some sort of adjustment to smooth things out is appropriate. (There is not enough information to determine whether some other limit than $1 million would be more appropriate.)
(While using 10 years seems reasonable, there is not enough information to determine whether some other number of years would be preferable.)
There are a number of potential problems with the adjustment in part (a):
(i) Severity is increasing at 5% per year, which would change the meaning of $1 million over time.
For example $1 million in 2003 is equivalent to (1.05^9)($1 million) = $1.55 million in 2012.
Thus the expected percentage of losses over $1 million in the years would differ.
It would be better to trend each year of historical losses and see the effect of the $1 million limit on these trended losses, or to detrend the $1 million limit and see its effect on each year of losses.
(ii) Not all policies in any given year will have the same policy limits (unless this is Workers Compensation Insurance, where there are no policy limits.) As the mix of policy limits changes over time, the expected percentage of losses excess of $1 million will also change.
(iii) If this is property insurance, it may be more appropriate to use a threshold that is a percentage of the amount of insurance than to use a fixed threshold such as $1 million.
(c) (i) The limited loss severity trend should be less than the given unlimited loss severity trend.
(ii) Large claims tend to be reported later and have more development. Also capping the losses at $1 million would reduce the possible upward development of large losses.
So the given unlimited loss development factors are likely to be too high.
(iiii) The earned premiums are presumably for whatever mix of policy limits were purchased.
We would want to limit the historical premiums to $1 million coverage and then put them on-level.
(iv) Unless $1 million is the basic limit, which is unlikely, the data is for various limits, some lower than $1 million. So we either need to exclude polices with less than a $1 million limits from the analysis, or do separate analyses of basic limits rates and increased limit factors.
Comment: See Table 6.3 in Basic Ratemaking.
We would usually perform a rate indication for January 1, 2013 in late 2012, when AY2012 would not yet be available.

6. (3.25 points) Given the following information:

- All policies have six-month terms.
- New rates will take effect on January 1, 2014.
- Rates will be in effect for one year.
- Selected frequency trend = 0%.
- Selected severity trend = +5%.
- Selected ULAE provision = 10% of loss and ALAE.
- Two accident years with equal weights are used to calculate the pure premium.
- Assume no further development after 36 months.

	Cumulative Paid Loss and ALAE ($000s)		
Accident Year	12 Months	24 Months	36 Months
2009	$6,000	$17,200	$25,800
2010	$4,500	$12,200	
2011	$7,900		

	Open Claim Counts		
Accident Year	12 Months	24 Months	36 Months
2009	190	160	75
2010	120	110	
2011	165		

	Outstanding Case Loss and ALAE Reserves ($000)		
Accident Year	12 Months	24 Months	36 Months
2009	$16,500	$13,000	$5,500
2010	$10,000	$6,500	
2011	$9,000		

Calendar Year	Earned Exposures (000s)
2010	200
2011	300

Use the Berquist-Sherman case outstanding adjustment technique to calculate the projected ultimate loss and LAE pure premium of the rate level indication.

6. First use the Berquist-Sherman case outstanding adjustment technique.
The average costs on open claims along the latest diagonal are:
9,000,000/165 = 54,545, 6,500,000/110 = 59,091, and 5,500,000/75 = 73,333.
We use the assumed 5% severity trend to fill in the triangle:

$54,545/1.05^2 = 49,474$	59,091/1.05 = 56,277	73,333
54,545/1.05 = 51,948	59,091	
54,545		

Now we add the paid losses to these adjusted severities times the number of open claims.
(49,474)(190) + 6,000,000 = 15,400,060. (56,277)(160) + 17,200,000 = 26,204,320.
(51,948)(120) + 4,500,000 = 10,733,760.
Thus the restated triangle of reported loss and ALAE is:

15,400,060*	26,204,320*	31,300,000
10,733,760*	18,700,000	
16,900,000		

Now we can compute link ratios based on this restated Berquist-Sherman triangle.
First to second report: (26,204,320 + 18,700,000) / (15,400,060 + 10,733,760) = 1.7182.
Second to third report: 31,300,000/26,204,320 = 1.1945.
Thus the ultimate loss and ALAE for AY2010 is: (18,700,000)(1.1945) = 22,337,150.
Ultimate loss and ALAE for AY2011 is: (16,900,000)(1.7182)(1.1945) = 34,685,389.
With six-month policies, the average date of accident under the new rates is October 1, 2014.
Thus projected ultimate loss and LAE for AY2010 is: $(1.1)(22,337,150)(1.05^{4.25}) = 30,232,563$.
Projected ultimate loss and LAE for AY2011 is: $(1.1)(34,685,389)(1.05^{3.25}) = 44,709,981$.
AY2010 projected ultimate pure premium is: $30,232,563/200,000 = $151.16.
AY2010 projected ultimate pure premium is: $44,709,981/300,000 = $149.03.
Averaging the two pure premiums: (151.16 + 149.03)/2 = **$150.10**.
Comment: The Berquist-Sherman reserving technique is discussed in Chapter 13 of
Estimating Claims Using Basic Techniques, by Jacqueline Friedland.

7. (1.5 points) Two methods of deriving expense provisions in ratemaking include the Premium-Based Projection Method and the Exposure/Policy-Based Projection Method.
a. (1 point) For each method, briefly describe how both fixed and variable expenses are treated.
b. (0.5 point) Briefly describe one shortcoming (or distortion) of each method.

7. (a) In both methods, one divides historical expenses for each category into variable and fixed. (For example, Commissions are usually all variable, while some portion of Other Acquisition Expenses is assumed to be fixed.)
In both methods, one takes a ratio of variable expenses to premiums.
In the Premium-Based Projection Method, we divide historical fixed expenses by premiums.
Then the indicated rate change is:

$$\frac{\text{Projected Loss \& LAE Ratio} + \text{Fixed Expense Ratio}}{1 - \text{Variable Expense Ratio} - \text{Profit \& Contingencies Provision}} - 1.$$

$$\text{Expense Fee Ratio} = \frac{\text{Fixed Expense Ratio}}{1 - \text{Variable Expense Ratio} - \text{Profit \& Contingencies Provision}}.$$

Expense Fee = (Expense Fee Ratio) (Statewide Average Rate).
Variable Base Rate = (Base Rate) (1 - Expense Fee Ratio).
In the Exposure/Policy-Based Projection Method, we divide historical fixed expenses by exposures. Then we apply an expense trend to this fixed expense pure premium.
Then the indicated required premium per exposure is:

$$\frac{\text{Projected Loss \& LAE} + \text{Projected Fixed Expense}}{1 - \text{Variable Expense Provision} - \text{Profit \& Contingencies Provision}}.$$

The indicated expense fee (per exposure) is:

$$\frac{\text{Projected Average Fixed Expenses per Exposure}}{1 - \text{Variable Expense Provision} - \text{Profit \& Contingencies Provision}}.$$

(b) The “Premium Based Projection Method” can be inaccurate due to:
1. The effect of rate changes. Large changes in rates since the historical expense data period can change the ratio of needed fixed expenses to premium.
2. The effect of a change in the average premium, for example due to Premium Trend.
Different premium trends than trends in fixed expenses can change the ratio of needed fixed expenses to premium from that in the historical data period.
3. Misallocation of fixed expenses to state.
If this method is used to allocate countrywide fixed expenses to states, then states with higher average premiums per exposure will be allocated more fixed expenses than are needed, and vice-versa for states with lower than average premiums per exposure.
Potential shortcomings or enhancements to the “Exposure/Policy Based Projection Method”:
1. Expenses are split judgmentally between fixed and variable. Perhaps future activity based studies could make this split more accurate.
2. This method allocates fixed expenses to state based on exposures. By collecting more data at a finer level, a better allocation of expenses to state might be possible.
3. Some expenses could vary based on other policy characteristics such as for example new versus renewal business. This method does not take this into account
4. When there is a changing size of the book of business, economies of scale may affect the required average fixed expenses. This method does not take economies of scale into account.

Comment: It was unclear to me whether this exam question was asking about statewide rate indications and/or expense fees. I suspect the former. In part (b) only discuss one item per method.

8. (2 points) Given the following information:

Calendar/ Accident Year	Earned Exposures	Non-Catastrophe Number of Claims	Non-Catastrophe Reported Losses and ALAE ($000s)	Reported Loss and ALAE Development Factor	Loss Trend Factor
2010	20,725	350	$11,446	1.000	1.145
2011	21,220	310	$12,757	1.006	1.121
2012	23,015	320	$11,295	1.068	1.080

- ULAE = 2% of loss and ALAE.
- Regional non-catastrophe pure premium (including LAE) = $602.
- Non-modeled catastrophe pure premium (including LAE) = $30.
- Modeled catastrophe pure premium (including LAE) = $75.
- Projected net reinsurance cost per exposure = $22.
- Projected fixed expense per exposure = $35.
- Profit and contingency provision = 5.0%.
- Variable expense provision = 16.0%.
- Projected on-level average premium = $945.
- Claims required for full credibility for all three years combined = 1,082.
- The insurer uses the square root rule to determine partial credibility.

Calculate the credibility-weighted indicated rate change.

8. First for each year's loss and alae, develop, trend, and load for ULAE.
AY2010: (11,446,000)(1)(1.145)(1.02) = $13,367,783.
AY2011: (12,757,000)(1.006)(1.121(1.02) = $14,674,129.
AY2012: (11,295,000)(1.068)(1.080)(1.02) = $13,288,667.
Total Loss and LAE divided by total exposures:
($13,367,783 + $14,674,129 + $13,288,667) / (20,725 + 21,220 + 23,015) = $636.25.
We will credibility weight this with the regional non-cat pure premium.

Number of claim is: 350 + 310 + 320 = 980. Z = $\sqrt{980 / 1082}$ = 95.17%.

Credibility-Weighted Non-Cat Pure premium is: (95.17%)(636.25) + (4.83%)(602) = $634.60.

Thus the indicated rate is: $\frac{634.60 + 30 + 75 + 22 + 35}{1 - 16\% - 5\%}$ = $1008.35.

The indicated rate change is: $1008.35 / $945 - 1 = **6.70%**.

Comment: See Page 5 of Appendix B in Basic Ratemaking.
One could instead take the average of the pure premiums for the three years, getting $637.92 rather than $636.25 as I did.
Note that the given regional non-catastrophe pure premium, non-modeled catastrophe pure premium, and modeled catastrophe pure premium all include LAE; thus there is no reason to multiply them by the ULAE factor.

9. (2.5 points) Given the following information:

Territory	Premium	Current Relativity	Indicated Relativity
1	$195,000	0.85	0.75
2	$475,000	1.00	1.00
3	$330,000	1.30	1.20
Total	$1,000,000		

Management is requiring that both of the following objectives are achieved with the upcoming rate change:

- Target an overall rate level increase of 20%.
- Revise territorial relativities to the indicated relativity, while capping any territory rate impact at 25% overall.

Calculate the territorial relativities that will be implemented with the rate change.

9. The change in relativity for Territory 1 is: 0.75/0.85 - 1 = -11.76%

The overall average change in relativity is: $\frac{(195)(-11.76\%) + (475)(0) + (330)(-7.69\%)}{1000}$ = -4.83%.

The off-balance factor (to be multiplied by) is: 1/(1 - 4.83%) = 1.0508.
The total change for territory 1 is: (1 - 11.76%)(1.0508)(1.2) - 1 = 11.26%.
Without capping, the increase in Territory 2, the base territory, would be greater than 25%.

Terr.	Premium	Current Relativity	Indicated Relativity	Relativity Change	Off-Balance Factor	Overall Change	Total Change	Capping Shortfall
1	$195,000	0.85	0.75	-11.76%	1.0508	20.00%	11.26%	
2	$475,000	1.00	1.00	0.00%	1.0508	20.00%	26.09%	$5194
3	$330,000	1.30	1.20	-7.69%	1.0508	20.00%	16.39%	
Total	$1,000,000			-4.83%	1.0508	20.00%	20.00%	$5194

If we cap the change in Territory 2 at 25%, then the resulting shortfall in premium would be:
(26.09% - 25%)($475,000) = $5194, subject to rounding.
To make up for this, we will increase the rates charged to the other territories.
Premiums from noncapped territories: (1.1126)(195,000) + (1.1639)(330,000) = $601,044.
Therefore, in order to make up for the shortfall we need to increase the rates in these territories by an additional: $5194/$601,044 = 0.86%.
In order to cap the change in Territory 2, the base territory, we will need to reduce the otherwise indicated base rate by: 1.25/1.2609 - 1 = -0.86%.
There are two capping effects, the base rate goes down and we want the rates in Territories 1 and 3 to go up; they each affect the relativities.
Thus in order to get 0.86% more premium in Territories 1 and 3, the new relativities are:
Territory 1: (0.75)(1.0086)(1.2609/1.25) = **0.76**,
and Territory 3: (1.20)(1.0086)(1.2609/1.25) = **1.22**.
Comment: See Table 14.11 in Basic Ratemaking.
However, note that here the base territory is capped, while in Table 14.11 it is another territory other than the base territory that is capped.

10. (2 points) An insurance company develops territorial indications using a univariate pure premium analysis and has the following experience:

Territory	Earned Exposures	Reported Loss & ALAE ($000s)	Current Relativity
A	100,000	$60,000	1.00
B	250,000	$300,000	1.40
Total	350,000	$360,000	

		Exposures	
Amount of Insurance Group	Charged Factor.	Territory A	Territory B
Low	0.75	50,000	25,000
Medium	1.00	30,000	75,000
High	1.50	20,000	150,000
Total		100,000	250,000

a. (0.5 point) Describe how distortion can occur using a univariate approach.
b. (1.5 points) Calculate the indicated pure premium relativities, while accounting for distortion that may be occurring due to amount of insurance differences by territory.

10. (a) One basic problem is that there are usually unequal proportions of the levels of the other variables across the one dimension being looked at; there can be correlation of exposures.
The adjusted pure premium approach and loss ratio approach adjust to some extent for this problem. However, they rely on the current relativities or current class rates, and are therefore just an approximation to a proper reflection of all exposure correlations.
In addition, there may be interaction of effects. For example, for private passenger auto insurance the male/female relativity should depend on age of driver. Univariate methods can not reflect interaction of effects, while generalized linear models can.
(b) I will use the adjusted pure premium method.
The average AOI relativity in Territory A is: {(50)(0.75) + (30)(1) + (20)(1.5)} / 100 = 0.975
The average AOI relativity in Territory B is: {(25)(0.75) + (75)(1) + (150)(1.5)} / 250 = 1.275.
The adjusted pure premium for Territory A is: 60,000,000 / {(100,000)(0.975)} = $615.38.
The adjusted pure premium for Territory B is: 300,000,000 / {(250,000)(1.275)} = $941.18.
The indicated relativity for B compared to the base territory A is: 941.18 / 615.38 = **1.53**.
Comment: See Table 9.14 in Basic Ratemaking.

11. (2.25 points) Given the following information:

	Policies with a $100,000 Limit		Policies with a $250,000 Limit		Policies with a $500,000 Limit	
Size of Loss	Claims	Losses	Claims	Losses	Claims	Losses
X ≤ $100,000	100	$8,000,000	35	$1,800,000	35	$1,800,000
$100,000 < X ≤ $250,000			40	$7,400,000	25	$3,900,000
$250,000 < X ≤ $500,000					15	$5,200,000

Limit	Indicated factor (pure premium generalized linear model analysis)
$100,000	1.00
$250,000	0.95
$500,000	1.15

For the $250,000 policy limit:

a. (1.25 points) Calculate the indicated increased limits factor, assuming a basic limit of $100,000.

b. (0.5 point) Explain the difference between the indicated increased limits factor calculated in part a. above and the generalized linear model results.

c. (0.5 point) Select an increased limit factor and briefly explain the rationale for the selection.

11. (a) Limited expected value at \$100,000 is:

$$\frac{8M + 1.8M + (100K)(40) + 1.8M + (100K)(40)}{100 + 35 + 40 + 35 + 25 + 15} = \$78,400.$$

The 40 large claims on policies with \$250,000 limit contribute to the layer from \$100,000 to \$250,000: 7,400,000 - (40)(100,000).
The 15 largest claims on policies with \$500,000 limit contribute to the layer from \$100,000 to \$250,000: (15)(250,000 - 100,000).
Estimate of the difference between the limited expected values at \$100,000 and \$250,000 is:

$$\frac{7.4M - (40)(100K) + 3.9M - (25)(100K) + (150K)(15)}{35 + 40 + 35 + 25 + 15} = \$47,000.$$

Indicated \$250,000 ILF: 1 + \$47,000 / \$78,400 = **1.599**.
(b) The GLM considers the interaction of limit purchased with other characteristics such as class and territory. It is estimating the ratio of expected pure premiums of otherwise similar insureds (same class and territory) who buy different limits of coverage. Based on the model output, it seems as if those who choose to buy \$250,000 limits are better risks than those who choose to buy basic limits of \$100,000; there is favorable selection. Presumably the expected frequency of those who buy \$250,000 limits is significantly lower than that of similar insureds who buy basic limits. Thus even though those who buy \$250,000 limits are getting more coverage, the GLM estimates that their expected pure premium is lower than that of those who choose to buy basic limits. A GLM can sometimes produce counter intuitive results, such as lower increased limits factors for higher limits.
It should be noted that many class/territory/limit purchased cells will have little data.
Therefore, some of the GLM results may be due to random fluctuation (noise rather than signal.)
(It would have been useful to have the standard errors associated with the GLM output.)
In contrast, the calculation in part (a) implicitly assumes that the expected frequency does <u>not</u> vary by limit purchased. Also it does not consider the mix of classes and territories by limit purchased.

(c) Using an ILF of 0.95 for $250,000 would result in charging less for more coverage.
Soon anybody who would otherwise have bought basic limits would instead buy $250,000 limits; in addition we would attract insureds who currently buy basic limits from other insurers. These insureds would get more coverage for less premium, and therefore our premiums for $250,000 limits will be inadequate. (If we used the 0.95 ILF for $250,000, the favorable selection that the GLM said is in the data with respect to purchasers of $250,000 limits would vanish.)
Therefore, I will not use the ILF indicated by the GLM.
On the other hand, the GLM output leads me to believe that the indicated ILF from part (a) is too high. One could select something in between such as 1.30.
(It would be very helpful to have more information on the GLM output including but not limited to standard errors. It would be helpful to have more information such as the current ILFs, competitors ILFs, etc.)
Alternately, one can base ones selection on the 1.15 GLM indicated for $500,000.
The $250,000 ILF should be greater than one and less than the $500,000 ILF.
Linearly interpolating one would get: 1 + (0.15)(250 - 100) / (500 - 100) = 1.056.
However, ILFs should decrease at a decreasing rate, so I will select 1.10.
<u>Comment</u>: See Table 11.6 in <u>Basic Ratemaking</u>.
There are other reasonable selections you could make in part (c).
If one does not use the output of the GLM to set ILFs, one needs to go back and make sure the class and territory relativities from a GLM will work well with the ILFs actually selected.
The estimate of the difference between the limited expected values at $250,000 and $500,000 is:

$$\frac{5.2\text{M} - (15)(250\text{K})}{35 + 25 + 15} = \$19,333.$$

Therefore, in part (a) the Indicated $500,000 ILF would be:
1 + ($47,000 + $19,333) / $78,400 = 1.846.

12. (2 points) An insurance company has recently entered a new state and plans to invest heavily on marketing in an attempt to aggressively grow its homeowners book of business.
Relative to the low initial premium in the state, these marketing expenses will be significant.
The company's senior management proposes that the actuary develop a rate level where the expense provisions reflect only the typical variable costs.
Construct a thorough argument in support of this proposal.

12. Unusual and/or one-time expenses need to handled separately in pricing; some judgement may be required. This is particularly important when as is the case here the initial premium volume is small compared to the initial marketing expenses. If we were to load the initial marketing expenses into the rate, then we would overestimate the future expense needs as a percent of future premium.
If we were to include the initial heavy investment in marketing in the rate, then our rates would be noncompetitive, and we would not sell much business. If we consistently did this, we could never enter a new line and/or state and expand to a reasonable sized book of business.
We can treat the initial marketing expenses as a necessary investment to allow us to enter this new market. (There are probably other one-time expenses involved in entering this state.)
If one adopted lifetime value analysis as in the paper by Feldblum, then one could come up with a rate that reflected both this initial large marketing expenditure, the expected future level of expense, the new business that is expected to be written, and the anticipated results on renewals. This would probably be better than just ignoring the initial heavy marketing expenses.

13. (1.5 points) The chief underwriter of a company offering homeowners insurance informs the chief actuary that an analysis has been performed on the replacement costs of all properties in the book of business. The book of business contains only two territories, A and B.
The analysis indicates that for the past 10 years, all properties in territory A have been uniformly underinsured by 20% while all properties in territory B have been adequately insured. The current insurance contract at the insurance company does not include any guaranteed cost replacement endorsement or coinsurance clause. The chief actuary is aware that the rating structure has been fully reviewed in the past year.
Discuss the overall premium adequacy, territorial premium adequacy, and the premium equity among the insureds.

13. Assume no significant changes over the historical period or the future rate period in the relative amount of business written in the two territories. Assume no changes in the future rate period in the historical levels of insurance to value.
Then the reported amounts of insurance over the last 10 years reflect the same average level of insurance to value. Thus this should have no effect on the overall premium adequacy.
The reported amounts of insurance over the last 10 years in Territory A reflect the same level of insurance to value. Thus this should have no effect on the premium adequacy in Territory A.
Similarly, the reported amounts of insurance over the last 10 years in Territory B reflect the same level of insurance to value. Thus this should have no effect on the premium adequacy in Territory B.
(The underinsuring in Territory A has resulted in a higher indicated base rate in that territory than if there had been insurance to value. If there had been insurance to value, then the average amount of insurance relativity in Territory A would have been higher, and its indicated base rate would have been lower.)
Since everyone in each territory has the same level of insurance to value, there is no first order inequity in premiums between insureds within each territory. However, since the Amount of Insurance Relativities are probably based on statewide data, they are not quite appropriate for either territory due to their different levels of insurance to value. (For example, a home in territory A with a $100,000 amount of insurance is worth $125,000, while a home in territory B with a $100,000 amount of insurance is worth $100,000.) This probably introduces some inequity between insureds in the same territory with different value homes.
<u>Comment</u>: It would be very unusual to have this simplified situation.
I have assumed that each territory has a reasonable volume of data, enough to have significant credibility for making rates.
If instead we had insureds in the same territory buying different levels of insurance to value, then there would be inequity within that territory. In the absence of a coinsurance clause, the appropriate rate per $100 of insured value increases as the level of insurance to value decreases.