Mahler's Guide to

Basic Ratemaking

CAS Exam 5

prepared by
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Study Aid 2020-**5**

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This study guide covers the ratemaking topics on CAS Exam 5.

Not covered are the reserving topics on CAS Exam 5 which are covered in:
"Estimating Unpaid Claims Using Basic Techniques," by Jacqueline F. Friedland,
Statement of Principles Regarding Property and Casualty Unpaid Claim Estimates, Casualty Actuarial Society, November 2014,
and Actuarial Standard of Practice No. 43: Property/Casualty Unpaid Claim Estimates, American Academy of Actuaries, 2007 and Updated for Deviation Effective May 2011.

Concepts in Basic Ratemaking by Werner and Modlin are demonstrated in my first 16 sections, with each section corresponding to the chapter in Basic Ratemaking with the same number.[1]

Afterwards are covered the syllabus readings:
"CAS Statement of Principles Regarding Property and Casualty Insurance Ratemaking"
"ASOP No. 13, Trending Procedures in Property/Casualty Insurance Ratemaking"
Actuarial Standard of Practice No. 12: Risk Classification (for all Practice Areas), American Academy of Actuaries, 2005.

Finally I cover Lifetime Value Analysis.[2]

Information in bold or sections whose title is in bold are more important for passing the exam. Larger bold type indicates it is extremely important.

Information presented in italics (including subsections whose titles are in italics) should rarely be needed to directly answer exam questions and should be skipped on first reading. It is provided to aid the reader's overall understanding of the subject, and to be useful in practical applications.

Highly Recommended problems are double underlined.
Recommended problems are underlined.
Additional problems are starred.
Solutions to the problems in each section are at the end of that section.[3]

[1] I have incorporated all of the changes included in the May 2016 Fifth Edition of Basic Ratemaking.
The minor changes between the 4th and 5th edition are listed at the end of Basic Ratemaking.
[2] Discussed in Chapter 13 of Basic Ratemaking.
[3] Note that problems include both some written by me and some from past exams. The latter are copyright by the CAS or SOA. They are reproduced here solely to aid students in studying for exams. The solutions and comments are solely the responsibility of the author; the CAS bears no responsibility for their accuracy. While some of the comments may seem critical of certain questions, this is intended solely to aid you in studying and in no way is intended as a criticism of the many volunteers who work extremely long and hard to produce quality exams.

Section #	Pages	Section Name
1	10-26	Introduction
2	27-52	*Rating Manuals*
3	53-70	Ratemaking Data
4	71-150	Exposures
5	151-359	**Premium**
6	360-578	**Losses and LAE**
7	579-702	Other Expenses and Profit
8	703-944	**Overall Indication**
9	945-1104	Traditional Risk Classification
10	1105-1264	Multivariate Risk Classification
11	1265-1473	Special Classification
12	1474-1583	Credibility
13	1584-1618	Other Considerations
14	1619-1710	Implementation
15	1711-1846	Commercial Lines Rating Mechanisms
16	1847-1938	Claims-Made Ratemaking
17	1939-1962	CAS Principles of Ratemaking
18	1963-1973	Trending Procedures in Property/Casualty Insurance Ratemaking
19	1974-1992	ASOP 12: Risk Classification
20	1993-2070	*Lifetime Value Analysis*

Seminar Style Slides Sold Separately.

Past Exam Questions by Section of this Study Guide[4]

Section	1992	1993	1994	1995	1996
1					
2					22
3					
4	53	6, 36, 51	8, 11, 40, 49	3, 5, 36	27, 29
5	44		1, 48	30	31, 36
6	22, 37	1, 37, 46, 49	22, 36	4, 6, 7, 31, 37	
7	54	34	13	49	3, 50
8	15, 31	50	2, 32, 41, 51	38	30
9		31a, 42a	42	33, 40	32
10					
11	5, 6, 35	24, 27	17, 18, 43, 47	18, 23, 27a, 35, 45, 46	16, 21, 41a, 44, 46, 48
12	46	44			11, 42
13			44		
14	55	35			
15	33	7, 8	26, 27	32	5, 6
16	1, 29, 30, 60	47	28, 29	39	12
17		23	39	1, 28	1
18		21	19		28
19					
20					38

[4] Prior to 2000, Exam 6 was the basic ratemaking exam. Since 2000, Exam 5 has been the basic ratemaking exam.

Section	1997	1998	1999	2000
1				
2				
3				
4	25	3, 45	14, 43	43
5	4, 12, 19	25, 39, 41, 47	37a, 38, 44, 58	38
6			39, 47	
7	26		52	29
8	44	36, 42	30	39, 40
9	31, 43	26, 43	13	21, 54
10	45			
11	8, 17, 24, 42	5, 7, 27, 34, 37	12, 15, 16, 40, 56, 57	24, 25, 27, 31, 45, 48, 50, 52
12	7, 21	4, 53	18, 21, 50, 53	23
13				30, 53a
14				19
15	13, 35, 36	19, 32, 33	24, 35b, 36	9, 28, 51
16	5, 33	15, 35	31, 32	44
17	25	46	41	22, 42
18	2	1 , 40	3, 7	55
19				
20	30	52b	34	47

Section	2001	2002	2003	2004	2005
1					
2					
3					
4	5, 6	9, 38	26, 27	30	14, 36
5	15, 38		10, 11, 31, 33	7, 11, 31, 32, 35	12, 37, 38
6	2, 45, 47, 48	28, 31	12, 15	8, 37, 45	16, 40, 50
7	43	4, 18	35	13	42, 43
8	1, 37	17, 27, 44	28, 36	10, 33	44, 46
9	27	19		29, 36, 40	45, 49
10					
11	7	14, 29, 30, 32, 42	38, 40	15, 34, 39, 41	19, 20, 39, 48, 51
12	17	10	39	46	17, 18, 41
13		11	24	26a	
14	36, 40		37	14	
15	25, 26, 44	2, 3	17, 42	17, 43	22, 53, 54
16	46	43	22, 29	12	15
17	3, 4	1	30	9, 38	35
18	14	33		47	13
19					
20	11, 23		43	44	56

Questions on the P.P. Auto Manual, no longer on the syllabus: 2002 Q 8, 2004 Q.21, 2005 Q.6.

Section	2006	2007	2008	2009
1				
2				
3				
4	24	33	12	17
5	5, 26, 27, 28	34, 35, 36, 37	13, 14, 15	18, 19, 20, 21
6	7, 29a, 31, 32	22, 40, 46, 53	17, 18, 19	22, 24, 25, 26, 27
7	33, 34	7, 41	23, 24	29
8	36	8, 42, 43	25, 26, 27	23, 30, 31
9	8, 38, 39	44, 48	28, 29, 30, 33	33, 34, 37
10				
11	37, 41, 43, 44	9, 10, 47, 49, 50	16, 31, 32, 34, 35, 36, 45	36, 39b, 40, 41
12	35, 42	32, 38, 39	22	32
13	22a		42a	14, 15
14				38
15	10, 20, 40, 49	45, 51, 52	39, 40, 41	35, 43, 44
16		13	20, 21	28
17	25	11		39a
18		6		
19				
20	50	28	43, 44	45

Prior to 2010, the material currently covered in Basic Ratemaking by Werner and Modlin, was covered in many separate readings.

Questions on the ISO P.P. Auto Manual, no longer on the syllabus: 2006 Q 3, 2007 Q.5.

Section	2010	2011	2012	Spring 2013	Fall 2013
1	11, 12	8			
2					
3					
4	16, 17	2, 3	2, 3	1	1
5	18, 19	4, 5	4, 5, 6	2	2
6	20, 21, 23, 24	6, 7, 17	7	7, 8	3, 6
7	25				7
8	26, 27	9, 10	9	3, 4, 5, 9	4, 5, 8
9	29	11	13, 14	11	10
10	36	13	11	12	
11	30, 31, 32	14, 15, 16	12, 15		11, 13
12					
13					12
14	28	18		14	9
15	33, 34	19		13, 15	
16	22		8	6	
17			10		
18					
19					
20	35	20		10	

Questions on the ISO P.P. Auto Manual, no longer on the syllabus: 2011 Q 1, 2012 Q.1.
Question on AAA “Risk Classification” no longer on the syllabus: 2011 Q.12.

Starting in Spring 2011, Basic Ratemaking and Basic Reserving were put on the same exam. Prior to that, Basic Reserving was on Exam 6 rather than Exam 5.

Starting in 2013, Exam 5 was given in both the Spring and Fall.

Section	Spring 2014	Fall 2014	Spring 2015	Fall 2015	Spring 2016	Fall 2016
1				6		
2						
3			6			
4	3	1	2, 3	2		1
5	1	2, 3	4, 5	1, 4	2	2
6		4, 5	8, 15	5	1, 4	4, 5
7		6	9		7	6, 7
8	2, 4	7	11	7	6, 8	8, 9
9	6, 8	9, 11	13		13	
10	9	10		10	10	12
11			14	12		11, 14
12	5, 7	8		3	11	
13						
14	10		10	9, 11	12	13
15	11			13	9	15
16			7		5	3
17						
18						
19					3	10
20		12		8		

Question on the ISO P.P. Auto Manual, no longer on the syllabus: Spring 2015 Q.1.
Question on AAA “Risk Classification” no longer on the syllabus: 5/15 Q.12.

For the 2015 exams, “Personal Automobile Premiums: An Asset Share Pricing Approach for Property-Casualty Insurance,” by Sholom Feldblum, was dropped from the syllabus.

For the 2016 exams: the I.S.O. P.P. Auto Manual and Chapter 2 of Basic Ratemaking were dropped from the syllabus. AAA Committee on Risk Classification, “Risk Classification Statement of Principles,” June 1980, was replaced by Actuarial Standard of Practice No. 12: Risk Classification (for all Practice Areas), American Academy of Actuaries, 2005.

				Makeup Exam			
Section	Spring 2017	Fall 2017	Spring 2018	Spring 2018	Fall 2018	Spring 2019	Fall 2019
1							
2							
3							
4	1		1		1	1	1
5		1, 3		1	3	2	2
6	3	6	3, 5, 16	2, 3, 4	2, 5	4, 5	5
7	2, 4	7	7	5	6	6	6
8	5, 13	2, 5, 15	8	6, 13	7	7	7
9			10, 11 ,12	7, 8, 11	12	10	9
10	8	10	13		9	9	10, 11
11	11		14	10, 12	10, 13		13, 14
12	9		2, 4	9			3, 12
13						8	
14	6, 7, 10	13			8	12	
15	12		15		11	11	15
16		4	6		4	3	4
17							8
18							
19							
20		9	9				

Both the original and makeup exams for Spring 2018 were via computer based testing using Excel. For Fall 2018, the CAS went back to the traditional paper exam, using a calculator.

For Spring 2019 and Fall 2019, Exam 5 used the paper-and-pencil format of exam administration.

The CAS hopes to use an improved version of computer based testing at some point in the future. Be sure to check the CAS webpage for any developing news on this issue.

Section 1, Introduction

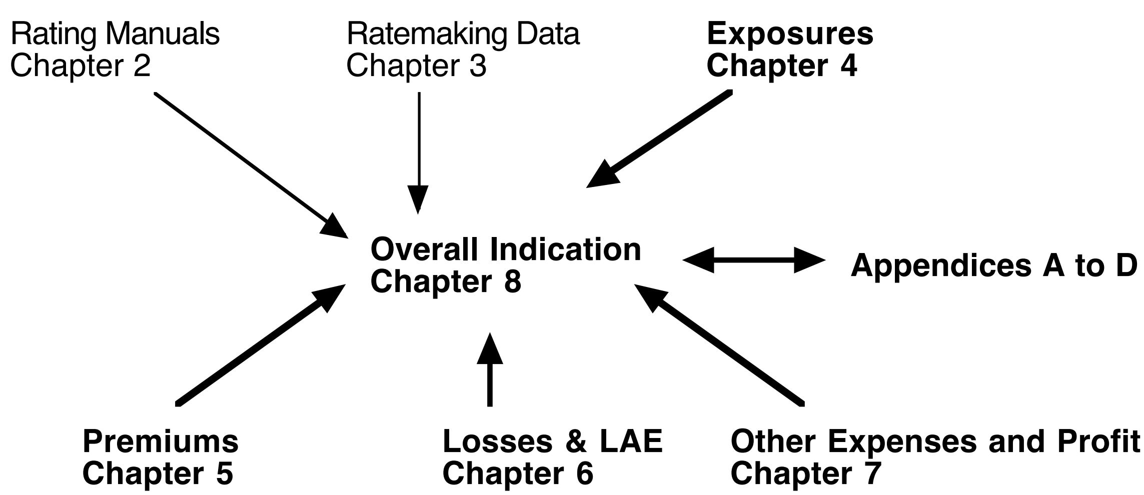
Basic Ratemaking replaced many separate papers that were formerly on the syllabus.[5] In some cases, the authors have taken or adapted material from these papers. Material has been updated and put on the level of comprehension and detail intended for your exam.[6] There is also much new material.

A single textbook has the advantage of a consistent notation, terminology, and approach. However, you should bear in mind that Basic Ratemaking was written by consulting actuaries who work for a single firm. Different actuaries would have presented some things from a somewhat different point of view, chosen different examples, and emphasized some different items.

Most of us learn by looking at specific, detailed examples. Be sure to carefully study those presented in the Appendices of Basic Ratemaking as well as the tables within the chapters. Learn these well for your exam; however, you should treat these as examples of the types of choices that could be made, rather than the one right way to do things.

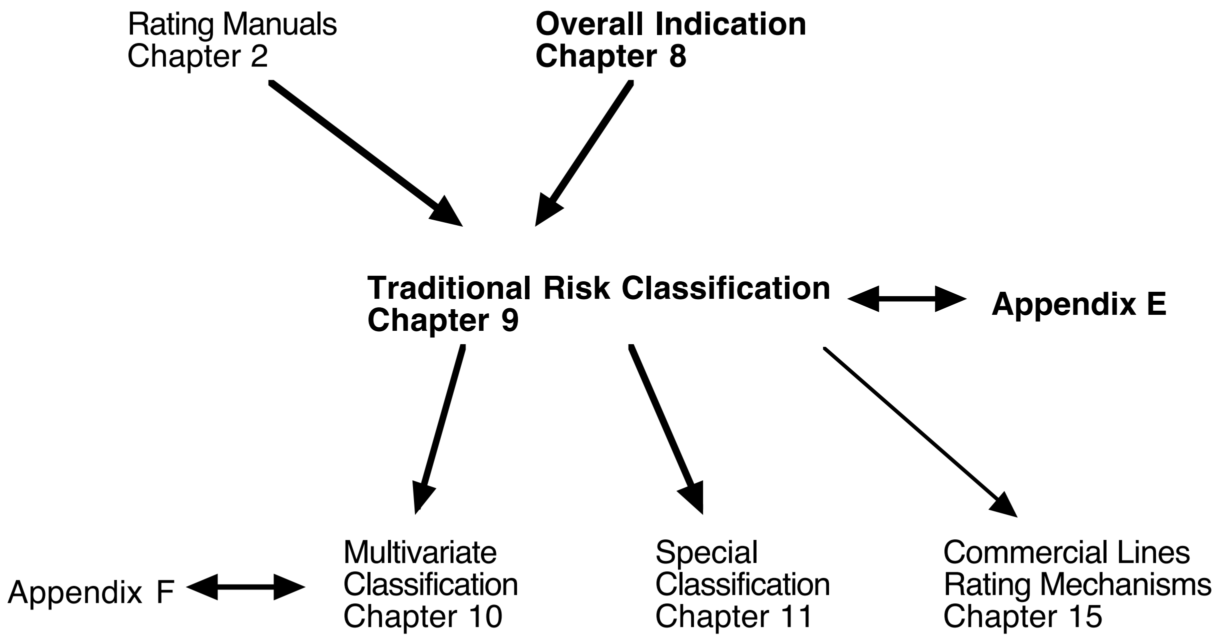
The purpose is to expose you to the general principles so that you will be able to do ratemaking yourself. In a practical application, you will have to make decisions as to what is appropriate for that particular situation, in light of any practical constraints.

In Chapter 1, the Introduction, basic ideas and terms are introduced, which are used in later chapters of Basic Ratemaking. The following diagrams attempt to describe the relationships of the chapters and possible orders in which you can read them.
Chapter 8 on Overall Rate Indications brings together material from earlier chapters. Before studying Chapter 8, you need to study Chapters 4 to 7 first, and glance at Chapters 2 and 3.[7]



[5] Basic Ratemaking was added to the syllabus for the 2010 exam.
[6] The authors were guided by a Committee of the CAS.
[7] While Chapter 2 is no longer on the syllabus, it has some useful background material.

Chapter 9 on Classification Ratemaking should be studied after Chapters 2 and 8, and before Chapters 10, 11, and 15.



Chapter 12 on Credibility should be studied after Chapter 8 on Overall Indications and Chapter 9 on Traditional Risk Classification.

Chapter 13 on Other Considerations should be studied after Chapter 9 on Traditional Risk Classification.

Chapter 14 on Implementation should be studied after Chapter 2 on Rating Manuals.

Chapter 15 on Commercial Lines Rating Mechanisms should be studied after Chapter 9 on Traditional Risk Classification and Chapter 12 on Credibility.

Chapter 16 on Claims-Made Ratemaking should be studied after Chapter 8 on Overall Indications.

Premiums and Exposures:

Price = Cost + Profit.

Price of Insurance = Premium.

Premium is the amount the insured pays for insurance coverage.

An **exposure** is the basic unit of risk that underlies the insurance premium.

Insurance premiums are calculated using a **rating manual** based on the exposures insured.[8]

The insured pays premiums to an insurer in exchange for a promise to pay claims.

Claims:

The person making the **claim** is called a **claimant**, and can be an insured or a third party.

There are many examples of first party claims. Your home burns down in a fire, and you make a claim with your Homeowners Insurer. Your car is stolen and you make a claim with your Automobile Insurer.

There are many examples of third party claims. Alice's dog bites Bob, and Bob makes a claim with Alice's Homeowners Insurer. Cal hits Debra's car with his car, and Debra makes a claim with Cal's Automobile Insurer. Eunice slips and falls in a store; she is injured and she makes a claim with the store's Liability Insurer.

The date of the event that caused the loss is called the date of loss, **accident date**, or occurrence date.

Usually, the accident is a sudden event. If instead the loss is the result of continuous or repeated exposure to substantially the same general hazardous conditions, the accident date is the date when the damage, or loss, is apparent.

For example, a coal miner is exposed to the silica and carbon in the coal dust in a mine over many years. If the miner is diagnosed with "black lung disease", the date of diagnosis is usually the accident date. Due to repetitive motions, a worker may develop tendonitis. Again the date of diagnosis is usually the accident date.

[8] The rating manual may be hardcopy and/or electronic.

Oil may seep from an old underground storage tank, over many years. The date that the damage caused by this pollution is recognized is usually the accident date.[9]

The date on which the insurer receives notice of a claim is the **report date**.

Claims not currently known by the insurer are referred to as **unreported claims** or incurred but not reported claims.

Until the claim is settled, the reported claim is considered an **open claim**. Once the claim is settled, it is categorized as a **closed claim**. In some instances, further activity may occur after the claim is closed, and the claim may be **reopened**.[10]

Losses:

Loss is the amount of compensation paid or payable to the claimant under the terms of the insurance policy. The terms associated with losses are: paid loss, case reserve, reported or case incurred loss, IBNR reserve, IBNER reserve, and ultimate loss.

Paid losses are those amounts that have been paid to claimants.

When a claim is reported, the insurer establishes a **case reserve**, which is an estimate of the amount of money required to ultimately settle that claim, minus anything that has already been paid. The amount of the case reserve is adjusted as payments are made and additional information is obtained about the claim.

Reported Losses = Case Incurred Losses = Paid Losses + Case Reserves.

Ultimate loss is the amount of money required to close and settle all claims for a defined group of policies.

The amount estimated to be needed to ultimately settle unreported claims is referred to as an **incurred but not reported (IBNR) reserve**.[11]

The **incurred but not enough reported (IBNER) reserve** is the difference between the aggregate amount estimated to ultimately be needed settle the reported claims and the aggregate reported losses (paid + case) at the time the losses are evaluated.[12]

[9] Courts ultimately decide which insurance policy or policies will provide coverage for pollution claims.
[10] For example, a worker who injured his back on the job, returns to work. If there are no more medical payments expected for treatments on his back, this claim is closed. However, he may re-injure his back, and not be able to work. This claim would then be reopened.
[11] Also called pure IBNR, in order to make it clear that it only relates to unknown claims.
[12] IBNER is called Bulk reserves at many insurers. It is the expected increase in incurred losses on known claims.

IBNER reserve = Estimated Ultimate on Reported Claims - Reported Losses.[13]

Current Estimate of Ultimate Losses = Reported Losses + IBNR Reserve + IBNER Reserve.

Neither the IBNR Reserve nor the IBNER Reserve are associated with individual claims. They are probably estimated separately by the reserving actuary.[14] They are probably estimated separately by line of insurance. They may be estimated separately by state or a countrywide reserve may be allocated to individual states.

Loss Adjustment Expenses:

Expenses that the insurer incurs in the process of settling claims are called loss adjustment expenses (**LAE**).

Allocated loss adjustment expenses (**ALAE**) are claim-related expenses that are directly attributable to a specific claim.

Unallocated loss adjustment expenses (**ULAE**) are claim-related expenses that cannot be directly assigned to a specific claim.

LAE = ALAE + ULAE.

For statutory financial reporting purposes, LAE is separated into defense and cost containment (DCC) and all other (AO) expenses.[15]

Underwriting Expenses:

Commissions and brokerage are amounts paid to insurance agents or brokers as compensation for generating business.

Other acquisition costs are expenses other than commissions and brokerage expenses paid to acquire business.

General expenses include the remaining expenses associated with the insurance operations and any other miscellaneous costs.

Taxes, licenses, and fees include all taxes and miscellaneous fees paid by the insurer excluding federal income taxes.

[13] Reported losses = paid losses + case reserves.
[14] See for example, Estimating Claims Using Basic Techniques, by Jacqueline Friedland, CAS Study Note.
[15] In the United States, the National Association of Insurance Commissioners (NAIC) determines how data will be reported by insurers in their Annual Statements and Insurance Expense Exhibits.

Also the insurer will load into the premiums an (expected) Underwriting Profit.

Since premiums are collected before losses and lae are paid, the insurer earns investment income in addition to any underwriting profit or loss. For lines of insurance where the average delay in paying loss and lae is long, such as liability insurance, investment income is extremely important part of determining an appropriate target underwriting profit.

<u>Fundamental Insurance Equation</u>:[16]

Premium = Losses + LAE + Underwriting Expenses + Underwriting Profit.

The goal of ratemaking is to determine rates such that the premium is expected to cover all costs and achieve the target underwriting profit.[17] This should be the case both for a book of business and to the extent possible for each individual insurance policy.[18]

It is common ratemaking practice to use relevant historical experience, suitably adjusted, to estimate the <u>future</u> expected costs that will be used in the fundamental insurance equation.

The following are some items that may necessitate a restatement of the historical experience:
• Rate changes
• Operational changes
• Inflationary pressures
• Changes in the mix of business written
• Law changes.

A rate is reasonable and not excessive, inadequate, or unfairly discriminatory if it is an actuarially sound estimate of the expected value of all future costs associated with an individual risk transfer.[19]

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[16] When determining premiums to be charged, everything on the righthand side the equation is on a predicted, estimated, or target basis.
Once the business is written and loss and lae are at ultimate, this equation can be used to calculate the achieved underwriting profit.
[17] See the CAS Statement of Principles Regarding Property and Casualty Insurance Ratemaking.
[18] Overall rate indications are discussed in Chapter 8 of <u>Basic Ratemaking</u>. How to determine the rates for classes, territories, etc. relative to average is discussed in Chapters 9 to 11 of <u>Basic Ratemaking</u>.
Individual risk rating is discussed in Chapter 15 of <u>Basic Ratemaking</u>.
[19] "Statement of Principles Regarding Property and Casualty Insurance Ratemaking."

Various Ratios:

Frequency = $\dfrac{\text{Number of Claims}}{\text{Number of Exposures}}$.

Average Claim Cost = **Severity** = $\dfrac{\text{Total Losses}}{\text{Number of Claims}}$.[20]

Loss Cost = **Pure Premium** = $\dfrac{\text{Total Losses}}{\text{Number of Exposures}}$ = (Frequency) (Severity).

Average Premium = $\dfrac{\text{Total Premium}}{\text{Number of Exposures}}$.

Loss Ratio = $\dfrac{\text{Total Losses}}{\text{Total Premium}} = \dfrac{\text{Pure Premium}}{\text{Average Premium}}$.[21]

LAE Ratio = $\dfrac{\text{Total Loss Adjustment Expenses}}{\text{Total Losses}}$.

Underwriting (UW) **Expense Ratio** = $\dfrac{\text{Total Underwriting Expenses}}{\text{Total Premium}}$.[22]

Operating Expense Ratio = OER = UW Expense Ratio + $\dfrac{\text{LAE}}{\text{Earned Premium}}$.[23]

Combined Ratio = (Pure) Loss Ratio + Operating Expense Ratio.[24]

[20] "Losses" in the numerator will in some cases be "Losses plus ALAE."
[21] "Losses" in the numerator will in some cases be "Losses plus ALAE," or "Losses plus LAE."
The denominator will usually be earned premium rather than written premiums.
[22] Underwriting Expenses include: commissions and brokerage, other acquisition costs, general expenses, and taxes, licenses, and fees. Usually general expenses are divided by earned premiums, while the other categories are divided by written premiums. This is discussed in my section on "Other Expenses and Profit."
[23] Insurers that use independent agents will have higher commissions. The independent agent may do some of the work in handling claims, which would otherwise result in the insurer incurring more LAE. The Operating Expense Ratio takes into account all expenses, regardless of whether they are called LAE or underwriting expenses.
[24] Each dollar of LAE should be one and only one of the two pieces

Retention is a measure of the rate at which existing insureds renew their policies upon expiration.

Retention Ratio = $\frac{\text{Number of Policies Renewed}}{\text{Number of Potential Renewal Policies}}$.[25] [26]

A policy may not be renewed for many reasons, not necessarily disjoint:
Insurer decides not continue to insure this insured.
Insured decides not to continue to buy from this insurer.
Insurer and insured cannot agree on a price acceptable to both of them.
Insurer decides to not renew a whole book of business.
The insurer ends its relationship with an agent or vice versa.
The insured decides to self-insure.
Insurance is no longer required.[27]

Particularly for large commercial insureds, an insurer will offer to write coverage and quote a price. Sometimes the insured will accept this offer of coverage and sometimes they will not.[28]

For personal lines, some of the potential new insureds who contact the insurer and who the insurer is willing to insure will become customers and some will not, possibly due to price.

The close ratio compares the rate quotes given to potential new business and the number of such insureds who end up being written by the insurer:

Close Ratio = $\frac{\text{Number of Accepted Quotes}}{\text{Number of Quotes}}$.

The close ratio is also known as hit ratio, quote-to-close ratio, or conversion rate. In general business jargon, the quote-to-close ratio (or “conversion rate”) is the measurement of actual customers or clients who buy, compared to the number of prospects you contacted or to the number of potential customers who visited your business.

[25] It can be for a month or a year. It can be for a state or countrywide.
[26] Some insurers exclude from the denominator policies that cancel due to death and policies that an underwriter non-renews, while others do not.
[27] For example, the insured died, the insured went out of business, or the insured home was sold.
[28] Negotiation may lead to an agreed upon price.

Notation:[29]

While it should not be vital to learn all of the notation used in Basic Ratemaking, it can not hurt to know some of it. You should certainly learn and use this notation to the extent that you find it helpful.

X = Exposures.

P = Premium.
$\overline{P}$ = Average premium = P / X.

P_C = Premium at current rates.
$\overline{P}_C$ = Average premium at current rates = P_C / X.

P_I = Premium indicated by rate review.
$\overline{P}_I$ = Average indicated premium = P_I / X.

P_P = Premium at proposed rates.[30]
$\overline{P}_P$ = Average premium at proposed rates = P_P / X.

L = Losses.
$\overline{L}$ = Pure Premium = Loss Pure Premium = L / X.

E_L = Loss Adjustment Expense (LAE).
$\overline{E}_L$ = Average LAE per exposure = Loss Adjustment Expense Pure Premium = E_L / X.

E_F = Fixed underwriting expenses.[31]
$\overline{E}_F$ = Average fixed underwriting expense per exposure = Fixed Expense Pure Premium = E_F/X.
F = Fixed expense ratio = E_F / P.

E_V = Variable underwriting expenses.
V = Variable expense provision = E_V / P.

[29] See page vi in Basic Ratemaking. There is no standard actuarial notation for ratemaking.
[30] An insurer will file or adapt rates based to some extent on an actuarial rate indication. However, the proposed rates may differ to some extent from those indicated.
[31] As discussed in my section on "Other Expenses and Profit," underwriting expenses are sometimes divided into fixed and variable.

Q_C = Profit percentage at current rates.

Q_T = Target profit percentage = Underwriting Profit Provision.

B_C = Current base rate.[32]

B_P = Proposed base rate.

$R1_{C,i}$ = Current relativity for the i^{th} level of rating variable R_1.[33]

$R1_{P,i}$ = Proposed relativity for the i^{th} level of rating variable R_1.

A_C = Current fixed additive fee.[34]

A_P = Proposed fixed additive fee.

[32] Base rates are discussed in Chapter 2 of Basic Ratemaking on Rating Manuals, as well as subsequent chapters on classification rating.

[33] A class relativity is the pure premium or variable premium of a class relative to average or relative to the base class. For example, the relativity for class 2 might be 1.5, one and half times that for the base class.
We might have several different rating variables; the resulting rates could be displayed in a multidimensional array. This is discussed in Chapter 2 of Basic Ratemaking on Rating Manuals, as well as subsequent chapters on classification rating.

[34] This refers to a fixed amount added to the premium of each policy.
See Chapter 14 of Basic Ratemaking on Implementation.

Problems:

1.1. (2 points) Define each of the following:
a. Frequency.
b. Severity.
c. Pure Premium.
d. Average Premium.

1.2. (1.5 points) Define each of the following:
a. LAE.
b. ALAE.
c. ULAE.

1.3. (2.5 points) Define each of the following ratios:
a. Loss Ratio.
b. LAE Ratio.
c. UW Expense Ratio.
d. Operating Expense Ratio (OER).
e. Combined Ratio.

1.4. (1/2 point) State the "Fundamental Insurance Equation."

1.5. (2 points) Define each of the following underwriting expenses:
a. Commissions and brokerage.
b. Other acquisition costs.
c. General expenses.
d. Taxes, licenses, and fees.

∗**1.6**∗**.** (3 points) Define each of the following:
a. Paid loss.
b. Case reserve.
c. Reported loss.
d. IBNR reserve.
e. IBNER reserve.
f. Ultimate loss.

1.7. (1 point) Define each of the following ratios:
a. Retention Ratio.
b. Close Ratio.

1.8. (1 point) Use the following information:
Written Premium = $85 million
Earned Premium = $90 million
Commissions and Brokerage = $8 million
Other Acquisition Expenses = $5 million
Taxes, Licenses and Fees = $1 million
General Expenses = $4 million
Loss Ratio (excluding LAE) = 65%
LAE ratio (to loss) = 7%
(a) (0.5 points) Calculate the Underwriting Expense Ratio
(b) (0.25 points) Calculate the Operating Expense Ratio
(c) (0.25 points) Calculate the Combined Ratio

1.9. (1 point) Given the following information:

	Calendar Year 2015
Written premium	$550 million
Earned premium	$580 million
Commissions	$44 million
Taxes, licenses and fees	$12 million
General expenses	$31 million
Other Acquisition Expenses	$18 million
LAE ratio (to loss)	12%
Combined ratio	106%

Calculate the 2015 operating expense ratio.

1.10. (5, 5/10, Q.11) (2 points)
a. (0.75 point) Explain how the standard economic formula, Price = Cost + Profit, relates to the fundamental insurance equation.
b. (1.25 points) Company ABC replaced inexperienced adjusters with experienced adjusters who have a greater knowledge of the product.
Explain the impact of this change on each component of the fundamental insurance equation.

1.11. (5, 5/10, Q.12) (1 point) Given the following information:
2008 earned premium = $200,000
2008 incurred losses = $125,000
Loss adjustment expense ratio = 0.14
Underwriting expense ratio = 0.25
Calculate the combined ratio.

1.12. (5, 5/11, Q.8) (1.25 points) Given the following information:

	Calendar Year 2010
Written premium	\$280.00
Earned premium	\$308.00
Commissions	\$33.60
Taxes, licenses and fees	\$9.80
General expenses	\$36.96
LAE ratio (to loss)	8.2%
Combined ratio	100%

Calculate the 2010 operating expense ratio.

1.13. (5, 11/15, Q.6) (1 point) Given the following information:

	Calendar Year 2014
Written premium	\$560.00
Earned premium	\$616.00
Commissions	\$67.20
Taxes, licenses and fees	\$19.60
General expenses	\$73.92
LAE ratio (to loss)	8.2%
Combined ratio	100%

Calculate the 2014 operating expense ratio.

Solutions to Problems:

1.1. Frequency = $\frac{\text{Number of Claims}}{\text{Number of Exposures}}$.

Severity = $\frac{\text{Total Losses}}{\text{Number of Claims}}$.

Pure Premium = $\frac{\text{Total Losses}}{\text{Number of Exposures}}$ = (Frequency) (Severity).

Average Premium = $\frac{\text{Total Premium}}{\text{Number of Exposures}}$.

1.2. a. Expenses that the insurer incurs in the process of settling claims are called loss adjustment expenses (LAE).
b. Allocated loss adjustment expenses (ALAE) are claim-related expenses that are directly attributable to a specific claim.
c. Unallocated loss adjustment expenses (ULAE) are claim-related expenses that cannot be directly assigned to a specific claim.

1.3. Loss Ratio = $\frac{\text{Total Losses}}{\text{Total Premium}} = \frac{\text{Pure Premium}}{\text{Average Premium}}$.

LAE Ratio = $\frac{\text{Total Loss Adjustment Expenses}}{\text{Total Losses}}$.

Underwriting (UW) Expense Ratio = $\frac{\text{Total Underwriting Expenses}}{\text{Total Premium}}$.

Operating Expense Ratio = OER = UW Expense Ratio + $\frac{\text{LAE}}{\text{Total Premium}}$.

Combined Ratio = (Pure) Loss Ratio + Operating Expense Ratio.

1.4. Premium = Losses + LAE + Underwriting Expenses + Underwriting Profit.

1.5. a. Commissions and brokerage are amounts paid to insurance agents or brokers as compensation for generating business.
b. Other acquisition costs are expenses other than commissions and brokerage expenses paid to acquire business.
c. General expenses include the remaining expenses associated with the insurance operations and any other miscellaneous costs.
d. Taxes, licenses, and fees include all taxes and miscellaneous fees paid by the insurer excluding federal income taxes.

1.6. a. Paid losses are those amounts that have been paid to claimants.
b. A case reserve is an estimate of the amount of money required to ultimately settle a particular claim, minus anything that has already been paid.
c. Reported Losses = Paid Losses + Case Reserves.
Note that for Calendar Year data, Reported Losses = Paid Losses + Change in Case Reserves.
d. Incurred but not reported (IBNR) reserve is the amount estimated to be needed to ultimately settle unreported claims.
e. The incurred but not enough reported (IBNER) reserve is the difference between the aggregate amount estimated to ultimately be needed settle the reported claims and the aggregate reported losses (paid + case) at the time the losses are evaluated.
f. Ultimate loss is the amount of money required to close and settle all claims for a defined group of policies.

1.7. a. Retention Ratio = $\dfrac{\text{Number of Policies Renewed}}{\text{Number of Potential Renewal Policies}}$.

b. Close Ratio = $\dfrac{\text{Number of Accepted Quotes}}{\text{Number of Quotes}}$.

1.8. (a) Take the ratio of General Expense to Earned premium: 4/90 = 4.44%.
Take the ratio of Commissions, Other Acquisition, plus Taxes, licenses and fees
to written premiums: (8 + 5 + 1)/85 = 16.47%.
Underwriting Expense Ratio = 4.44% + 16.47% = **20.91%**.
(b) Operating Expense Ratio = LAE Ratio + UW Expense Ratio = (7%)(65%) + 20.91% = **25.46%**.
(c) Combined Ratio = Loss Ratio + Operating Expense Ratio = 65% + 25.46% = **90.46%**.

1.9. Take the ratio of General Expense to Earned premium: 31 / 580 = 5.34%.
Take the ratio of Commissions, Other Acquisition plus Taxes, licenses and fees
to Written premiums: (44 + 12 + 18) / 550 = 13.45%.
Underwriting Expense Ratio: 5.34% + 13.45% = 18.79%.
Combined Ratio = Loss & LAE Ratio + UW Expense Ratio.
$\Rightarrow$ 106% = (1.12) (Loss Ratio) + 18.79%. $\Rightarrow$ Loss Ratio = 77.87%.
$\Rightarrow$ Ratio of LAE to Earned Premium: (12%)(77.87%) = 9.34%.
Operating expense ratio = LAE / Earned Premium + UW Expense Ratio
= 9.34% + 18.79% = **28.13%**.
Comment: Similar to 5, 5/11, Q.8.

1.10. a. The “Fundamental Insurance Equation” is:
Premium = Losses + LAE + UW Expenses + UW Profit.

Premium. ⇔ Price.

Losses + Loss adjustment expenses + UW Expenses. ⇔ Cost.

UW Profit. ⇔ Profit.

b. Loss will go down due to better adjusting.
Loss adjustment Expense will go up due to larger salaries or fees paid to experienced adjuster.
UW expense should have no significant change.
Premium would go down if one assumes the same UW Profit.
On the other hand, UW profit would increase if one instead assumes the same premium.

1.11. 0.14 = LAE Ratio = LAE / Losses = LAE / 125,000.

⇒ LAE = (0.14)(125,000) = 17,500.

Combined Ratio = 125/200 + 17.5/200 + 0.25 = **96.25%**.
<u>Comment</u>: In the combined ratio, the Loss and LAE have earned premium in the denominator, but the underwriting expense ratio would have written premium in the denominator.
The Operating Expense Ratio = 17.5/200 + 0.25 = 33.75%.
The (pure) Loss Ratio = 125/200 = 62.5%. 62.5% + 33.75% = 96.25%.

1.12. Take the ratio of General Expense to Earned premium: 36.96/308.00 = 12.00%.
Take the ratio of Commissions plus Taxes, licenses and fees to written premiums:
(33.60 + 9.80)/280.00 = 15.50%.
Underwriting Expense Ratio = 12.00% + 15.50% = 27.50%.
Combined Ratio = Loss & LAE Ratio + UW Expense Ratio.

100% = (1.082) (Loss Ratio) + 27.50%. ⇒ Loss Ratio = 67.00%.

⇒ Ratio of LAE to Earned Premium = (8.2%)(67.00%) = 5.5%.

Operating expense ratio = LAE / Earned Premium + UW Expense Ratio
= 5.5% + 27.5% = **33.0%**.
<u>Comment</u>: Usually there would Other Acquisition Expenses.

1.13. General Expense Ratio (to earned) = 73.92 / 616 = 12.0%.
Commissions Ratio (to written) = \$67.20 / \$560.00 = 12.0%.
Taxes, licenses and fees (to written) = \$19.60 / \$560.00 = 3.5%.
UW Expense Ratio = 12% + 12% + 3.5% = 27.5%.
Combined Ratio = Loss & LAE Ratio + UW Expense Ratio.
$\Rightarrow$ 100% = (1.082) (Loss Ratio) + 27.5%. $\Rightarrow$ Loss Ratio = 67.0%.

$\Rightarrow$ Ratio of LAE to Earned Premium: (8.2%)(67.0%) = 5.5%.

operating expense ratio = UW Expense Ratio + LAE / Earned Premium =
27.5% + 5.5% = **33.0%**.
Comment: Similar to 5, 5/11, Q.8.
The given figures for CY14 could be in millions of dollars.
I do not know why there is no other acquisition expanse.
In addition to the solution I gave, the CAS Examiner's Report included solutions where either implicitly one is relating General Expense to written premium, or Commissions and Taxes to earned premium. You should avoid doing so on your exam.

Section 2, Rating Manuals

While **no longer on the syllabus**, Section 2 of Basic Ratemaking contains some useful background material. I suggest that you look at one example of a rating algorithm.

Rating Manuals are used by insurers to determine the premium that will be charged a particular insured for a particular policy. The information in a rating manual can be divided into three pieces: Rules, Rate Pages, and the Rating Algorithm.[35] In addition, the insurer will have Underwriting Guidelines which help determine how the rating manual is used. The rules include: general definitions, rules for determining classifications, discussions of endorsements, etc.

For example, in the ISO Personal Automobile Manual formerly on the syllabus, they define such items as: private passenger auto, liability, single limit liability, comprehensive coverage, etc.[36] The different classification variables are defined and discussed. For example, primary classification depends on: age, sex, and marital status of the operators, the use of the auto, and the eligibility of youthful operators for the Driver Training and/or Good Student Classes. The Safe Driver Insurance Plan, an individual risk rating plan, is discussed. Model Year / Age groups for physical damage are discussed. Cancellation rules are discussed.

The rate pages are a list of rates for different classes and territories. There may be additional items that have to be applied to the listed numbers in order to get the final premium to be charged. For example, for the Michigan Automobile Insurance Placement Facility:[37]

Age of Any Resident Operator	Adult Driver	Not Owner or Principal Operator	Owner or Principal Operator
Under 19		4A	5A
19–20		4B	5B
21–22		4C	5C
23–24		4D	5D
Adult 25 and Older	1B		
Retired or Unemployed, 60–64	1A		
Senior Citizen, Unemployed 65–69	1AS		
Senior Citizen, Unemployed 70 and Older	1SS		
Business Use	3		

In addition, the state is divided into territories. For example, Territory 13 consists of the cities of: Dearborn, Allen Park, Lincoln Park, and Melvindale.[38]

[35] In some cases, the line between them can be blurry.
[36] The policy itself contains detail on the coverages provided, etc. The rating manual and the policy work together.
[37] This is for the residual market in one state. This is just an example. The classes used by ISO or an individual insurer would differ somewhat.
[38] Some territories may consist of several counties. Other territories may be parts of a large city.

Usually there is a base class or base risk.[39] For the above example, the base class would 1B: Adult Driver (25 years and older). The actuary would determine the rate to be charged the base class in each territory, and the relativities to go from the rate for the base class to the rates for the other classes.

For example, if the rate of Bodily Injury Liability (Basic Limits of 20/40) in Territory 13 for class 1B were $103, and the relativity for class 1A were 0.80, then the rate for class 1A would be: (0.8)($103) = $82. If the relativity for class 4D were 1.31, then the rate for class 4D would be: (1.31)($103) = $135. The base rate in Territory 19 would be different, for example $95.

For automobile liability there would be a list of increased limits factors, used to determine the premium if the insured buys more than the basic limits of liability.[40] For automobile physical damage, there would be a list of deductible credits for those who bought a larger deductible than the base deductible.[41]

How to get the final premium is explained via the Rating Algorithm. Basic Ratemaking gives three examples: Homeowners, Medical Malpractice, and Workers Compensation. While you should study these examples, remember that the details would differ by insurer and there are other features that would apply to other lines of insurance.

The manual rate is the rate calculated directly from the rate tables and factors in the manual.

Underwriting Guidelines:

Each insurer has its own guidelines for which insureds its underwriters should write.

So for example, a particular Workers Compensation insurer may not write nursing homes. Another Workers Compensation insurer might not write small risks. A private passenger automobile insurer may not insure sports cars. In each case, this may be due to perceived unprofitability of the risk or lack of expertise of the insurer.

Many times a group of insurers will have common ownership.[42] In that case, different insurers in the group can have different rates for the same line of insurance in the same state. In that case, the insurer needs underwriting guidelines to determine which insureds will be written in which company.

[39] This will be used in Classification Rating, as discussed in Chapter 9 of Basic Ratemaking.
[40] Increased Limits Factors are discussed in Chapter 11 of Basic Ratemaking.
[41] Pricing of deductibles is discussed in Chapter 11 of Basic Ratemaking.
[42] Where allowed by law, a single insurer may have more than one set of rates, in other words have different underwriting tiers. See the Homeowners example in Basic Ratemaking, to be discussed subsequently.

For example, the All Star Insurance Group writes private passenger automobile insurance in three companies. The preferred business is written in Vega Insurance Company at the lowest rate. The standard business is written in the Sirius Insurance Company, at a medium rate level. The substandard business is written in the Polaris Insurance Company at the highest rate.

Sometimes, an underwriting guideline may turn into a rating variable, or be used to provide a credit to insureds. For example, credit scores of insureds, or some of the items that were later used to create credit scores, were first used by a few insurers as underwriting guidelines for private passenger automobile insurance.[43] As discussed in Chapter 9 of Basic Ratemaking, now many insurers use credit scores as a rating variable for personal lines of insurance.

An underwriting guideline can be somewhat subjective; the underwriter may have to apply some judgement. As will be discussed, in contrast a classification variable must be objective in its application.[44]

Note that most states have restrictions on underwriting guidelines. For example, one could not use race or religion to determine whether to write an insured or which underwriting tier to place them in. A state might not allow credit scores to be used as a rating variable, but might allow its use in underwriting guidelines.[45]

Basic Ratemaking list some examples of typical characteristics used in underwriting:[46]

Personal Automobile Insurance: Credit Score, Homeownership, Prior Bodily Injury Limits.[47]
Homeowners Insurance: Credit Score, Prior Loss Information, Age of Home.[48]
Workers Compensation: Safety Programs, Number of Employees, Prior Loss Information.
Commercial General Liability Insurance: Credit Score, Years in Business, Number of Employees.
Medical Malpractice: Patient Complaint History, Years Since Residency,
Number of Weekly Patients.[49] [50]
Commercial Automobile: Driver Tenure, Average Driver Age, Earnings Stability.[51]

[43] *Among the many items from credit reports that may be used to calculate a credit score for an individual are: late payments, bad debts, and financial leverage. See "A View Inside the Black Box: A Review and Analysis of Personal Lines Insurance Credit Scoring Models Filed in the State of Virginia," by Cheng-sheng Peter Wu and John R. Lucker, Winter 2004 CAS Forum.*
[44] See my section on "Traditional Risk Classification."
[45] A state might allow credit scores to be used for both.
Another state might allow credit scores to be used for neither.
[46] See Exhibit 2.2 in Basic Ratemaking. As mentioned, sometimes some of these would be rating variables.
[47] They are referring to the limit for Bodily Injury Liability the insured has on their policy that is about to expire.
[48] Prior Loss Information refers to the claims history of the insured.
[49] Residency is a stage of graduate medical training. In the United States, it leads to eligibility for board certification, and the ability of the doctor to practice medicine on his own.
[50] Presumably, the insurer might be concerned if the doctor has too heavy a work load. In any case, the more patients the doctor sees, the more chance for a medical malpractice claim, all else being equal.
[51] They mean the stability of profitability of the business being insured. These criteria seem suited to a trucking firm.

Homeowners Example:[52] [53]

$500 would be charged to insure a “base rate” home for one year.[54] [55]
This base rate of $500 applies to a home with $200,000 of insurance (Coverage A amount), in Territory 3, with frame construction, in Protection Classes 1-4, written in underwriting Tier C, that received no miscellaneous credits and bought no addition coverages.[56]

This base rate will be adjusted for the following items: Amount of Insurance (AOI), Territory, Protection Class and Construction Type, Underwriting Tier, Deducible Amount, Miscellaneous Credits, any Additional Optional Coverages, and the Expense Fee.

Coverage A of the Homeowners Policy covers the value of the dwelling itself. The Coverage A amount is the Amount of Insurance (AOI). The larger the amount of insurance purchased the larger the premium, all else being equal. In this example, the premium for a $125,000 home would be 75% of that for a $200,000 home. The premium for a $500,000 home would be 169% of that for a $200,000 home.[57] Note that the AOI relativities are not linear![58]

A state is divided into geographical territories. In this example there are five territories.[59] Territory 3 is the base territory. A home in territory 5 would pay 1.15 times that of a similar home in Territory 3.[60]

Masonry construction is less susceptible to damage by fire than frame construction.
Therefore, a masonry home pays less than a frame home.[61]

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[52] See pages 17 to 23 in Basic Ratemaking.
[53] Later I show an example of rate pages from the Illinois Fair Plan.
[54] This rate covers all perils. Some insurer’s would charge a separate rate for hurricanes, with separate classes and territories. See for example, “Homeowners Ratemaking Revisited (Use of Computer Models to Estimate Catastrophe Loss Costs),” by Michael A. Walters and Francois Morin, PCAS 1997.
[55] They do not specify the level of coverage, which would differ between HO2 and HO3 for example.
[56] As will be discussed, the premium would be the base of rate of $500 plus a $50 policy fee, for a total premium of $550.
[57] See Exhibit 2.4 in Basic Ratemaking.
[58] *Helping to determine the curve of relatives by amount of insurance is an important task of homeowners actuaries. See for example, “Homeowners Insurance Pricing” by Mark Homan and “Homeowners Ratemaking” by Stacy Weinman, both in the 1990 CAS Discussion Paper Program. One could have different curves for different types of construction.*
[59] Typically, there would be several dozen territories. Typically, the relativities would vary much more.
The number assigned to each territory is arbitrary and may not have any relation to its relative rate.
[60] Maybe territory 5 is nearer the coast and thus more susceptible to hurricanes.
Constructing territories is an important task for actuaries, but is not discussed in detail in Basic Ratemaking.
See for example, “The Construction of Automobile Rating Territories in Massachusetts, “ by Robert F. Conger, PCAS 1987, and “Determination of Statistically Optimal Geographic Territory Boundaries,” by Klayton N. Southwood, CAS Special Interest Seminar on Predictive Modeling, October 2006.
[61] While fire (and lightning) is the single most important peril for Homeowners Insurance, constituting less than half of the loss dollars, there are other important perils such as: Wind & Hail, Water Damage & Freezing, Theft, Vandalism & Malicious Mischief, and Liability.

Towns are rated for their "Public Protection Class." Lower numbers indicate a quicker and/or better capability of fighting fires. Thus a home in a town with protection class 5 pays more than a similar home in a town with protection class 3.

There are combined relativities for Protection Class/Construction.[62] In this example, the base rate is for a Frame Home in Protection Classes 1-4. A Masonry Home in Protection Class 9 would pay 1.75 times the base rate.[63]

In this example, the insurer has four underwriting tier.[64] Those insured placed in Tier A pay less than the base rate. while those in tier D base the highest rate.[65]

Insureds may choose a deductible amount.[66] The larger the deductible amount the lower the premium, all else being equal.[67] $250 is the base deductible; they pay the base rate.[68] Those insured with a $5000 deductible get less coverage, and pay 70% of the base rate, all else being equal.[69]

This insurer offers Miscellaneous Credits: New Home Discount of 20%,
5-Year Claims-Free Discount of 10%, and Multi-Policy Discount of 7%.[70]

The basic limits for Homeowners Insurance are $100,000 of Liability Coverage and $500 of Medical Payments.[71] Insureds may purchase increased limits; they pay more premium. They pay $25 extra if they buy $300,000/$1000 limits, while they pay $45 extra if they instead buy $500,000/$2500 limits.[72] This rate for higher limits does not depend on the other rating factors.

Homeowners Insurance includes $2500 coverage for jewelry. Insureds may purchase increased limits. They pay $35 extra if they buy $5000 limits, while they pay $60 extra if they instead buy $10,000 limits.[73]

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[62] *Determining construction or protection/construction relativities is an important task of homeowners actuaries. See for example, "Fire Protection Classifications of Homeowners Insurance," by William Von Seggern, et al., Winter 1996 CAS Forum.*
[63] See Exhibit 2.6 in Basic Ratemaking.
[64] In this example, we assume the tiers are all written in the same company. They could instead each be written in a different insurer who is part of the same group, with common ownership and management.
[65] See Exhibit 2.7 in Basic Ratemaking.
[66] The homeowners deductible applies to property losses but not liability losses.
[67] Pricing of deductibles is discussed in Chapter 11 of Basic Ratemaking.
[68] They still pay less than if they had a $100 deductible, which is not offered by this insurer.
[69] See Exhibit 2.8 in Basic Ratemaking.
[70] See Exhibit 2.9 in Basic Ratemaking.
The Multi-Policy discount would apply to somebody who also bought auto insurance from this insurer.
[71] See Personal Insurance by C.M. Nyce, for a discussion of the coverages provided by Homeowners Insurance.
[72] See Exhibit 2.11 in Basic Ratemaking.
[73] See Exhibit 2.10 in Basic Ratemaking.

The insurer charges each policy a policy fee of $50.[74]

Exercise: Lincoln Penny is buying insurance from the Wicked Good Insurance Company for his home. He is written in Underwriting Tier B. He chooses a $1000 deductible.
Since he also has his automobile policy with Wicked Good, he receives the multi-policy discount.
He buys increased limits of liability of $500,000/$2500.
His house is insured for $440,000. It is in Territory 5.
It is masonry and in Protection Class 3.
Determine his premium.
[Solution: AOI Relativity is 1.57, Territory Relativity is 1.15, Protection/Construction relativity is 0.90.
His Underwriting Tier Relativity is 0.95. His Deductible Credit Factor is 0.85.
His Multi-Policy Discount results in a factor of: 1 - 7% = 0.93.
Increased Liability / Medical Coverage Rate = $45.
The policy fee is $50.
Premium = ($500)(1.57)(1.15)(0.90)(0.95)(0.85)(0.93) + $45 + $50 = $705.]

Rating Algorithm for this Homeowners Example:

R1 = Amount of Insurance Relativity
R2 = Territory Relativity
R3 = Protection Class / Construction Type Relativity
R4 = UnderWriting Tier Relativity
C1 = Deductible Credit Factor
C2 = 1 - (New Home Discount) - (Claims-Free Discount).[75]
C3 = 1 - Multi-Policy Discount

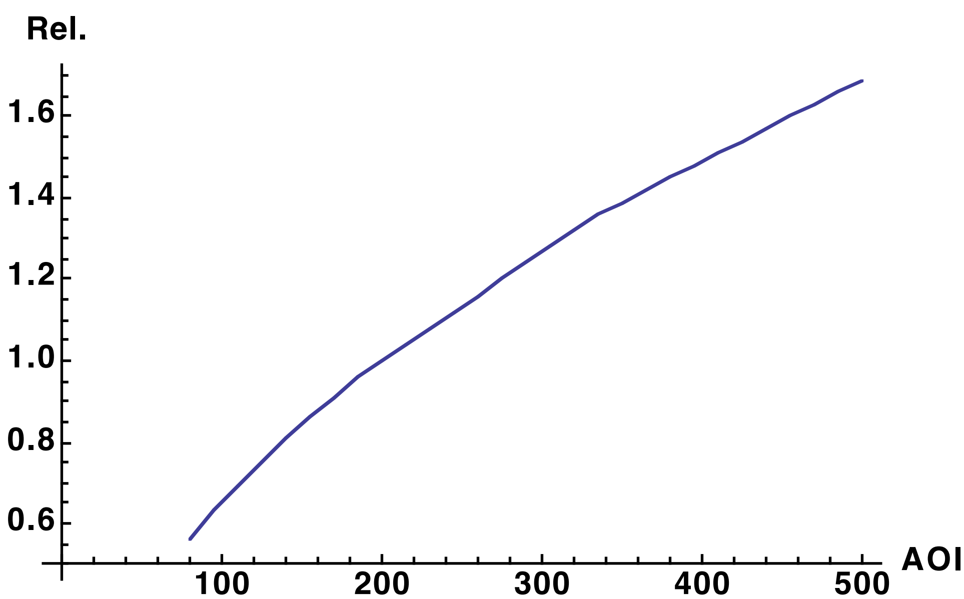
Premium = (Base Rate) R1 R2 R3 R4 C1 C2 C3 + Increased Jewelry Coverage Rate
+ Increased Liability / Medical Coverage Rate + Policy Fee.[76]

[74] This covers fixed expenses that do not vary with premium, as discussed in my section on "Other Expenses and Profit." In this case, this is a fee per policy rather than a fee per home.
Many insurers write each home on a separate policy.
[75] Thus in this example, the new home and claims-free discounts are added together.
[76] *Premium is rounded to the nearest penny after each step, and rounded to the nearest dollar at the end.*

Exercise: Barbara Seville is buying insurance from the Wicked Good Insurance Company for her new home.
She is written in Underwriting Tier A. She chooses a $5000 deductible.
She buys increased jewelry coverage with a limit of $5000.
Her house is in Territory 1. It is insured for $110,000.
It is frame and in Protection Class 6.
Determine her premium.
[Solution: AOI Relativity is 0.69, Territory Relativity is 0.80, Protection/Construction relativity is 1.10.
Her Underwriting Tier Relativity is 0.80. Her Deductible Credit Factor is 0.70.
Her new home discount results in a factor of: 1 - 20% = 0.80.
Her Increased Jewelry Coverage Rate = $35.
The policy fee is $50.
Premium = ($500)(0.69)(0.80)(1.10)(0.80)(0.70)(0.80) + $35 + $50 = $221.]

Table 2.4:

Basic Ratemaking gives an example of Rate Relativities by Amount of Insurance (AOI) for Homeowners. $200,000 is chosen as the base, with a relativity of 1.[77]



For example, $500,000 has a rate relativity 1.69.
The expected losses go up less than linearly with the amount of insurance.

[77] This is one of many reasonable choices and makes no difference in the rates charged.

<u>Medical Malpractice Example</u>:[78] [79]

This example involves nurses being covered for their profession liability.[80]

The base rate for an self-employed nurse is $3000.
The base rate for employed nurses, in other words other nurses, is $2500.[81]

There is a rate relativity based on the nurses specialty.[82]
The base is Family Practice.
For example, Psychiatric nurses are charged 80% of the base rate.

Part time nurses pay 50% of full-time nurses.[83]

Rates vary by territory.

There is a "claims-free" discount of 15%, that applies to each individual nurse.[84]

There is a schedule rating plan, that applies to each individual nurse,
with a maximum debit or credit of 25%:

Item	Debit/Credit
Continuing Education	up to 25% credit
Procedure[85]	up to 25% debit
Workplace Setting	up to 25% debit

The limits of liability are per claim with an annual aggregate limit.
For example, $500,000/$1 million, means the insurer will pay at most $500,000 for any one claim and will pay at most $1 million for losses covered by a single annual policy.[86] [87]
Buying higher limits of liability provides more coverage for a higher premium.[88]

[78] See pages 23 to 28 in <u>Basic Ratemaking</u>.
[79] You should probably just glance at this example, and then return while you have studying Chapter 16 of <u>Basic Ratemaking</u> on Claims-Made Ratemaking.
[80] Doctors and hospitals also buy Medical Malpractice Insurance.
[81] Nurses might be employed by a hospital, medical practice, nursing home, psychiatric institution, school, etc.
[82] See Exhibit 2.15 in <u>Basic Ratemaking</u>.
[83] *Part-time is defined as 20 hours or less per week, at being a nurse.*
[84] To get the credit the nurse can not have had more than $5000 in reported losses over the last 3 years.
Basing this on reported losses invites arguments about the insurer's case reserves.
[85] *Debit for nurses who are involved in higher risk procedures such as invasive surgery or pediatric care.*
[86] The aggregate applies per policy not per nurse. So a large group practice would want a higher aggregate limit.
In theory, the relativities should differ somewhat for a large group as opposed to a single nurse.
[87] The limit here applies to loss without ALAE.
Other insurers write policies where the limit applies to losses plus ALAE.
[88] See Exhibit 2.18 in <u>Basic Ratemaking</u>.

One can buy different deductibles. A larger deductible provides less coverage for less premium.[89]

These are claims-made rather than occurrence policies.[90] A "first-year" claims-made policy written 1/1/2010 would cover claims reported during 2010 from incidents that occurred during 2010.
A "second-year" claims-made policy written 1/1/2011 would cover claims reported during 2011 from incidents that occurred during either 2010 or 2011.

A second-year claims-made policy provides more coverage and therefore costs more than a first-year claims-made policy.[91] The base rate is for a "mature" policy.[92]

As discussed in my Chapter 16, an extended reporting endorsement (tail policy) should be purchased when a nurse who has been on claims-made coverage retires or leaves this practice.[93]
A 12-month extended reporting endorsement purchased 1/1/2010 would cover claims reported during 2010 from incidents that occurred during 2009. It would be purchased by a nurse who had been on claims-made coverage for 12 months. A 24-month extended reporting endorsement purchased 1/1/2010 would cover claims reported during 2010 from incidents that occurred during either 2008 or 2009. It would be purchased by a nurse who had been on claims-made coverage for 24 months.

A 24-month extended reporting endorsement provides more coverage and therefore costs more than a 12-month extended reporting endorsement.[94] [95]
One pays 94% of a mature claims-made policy for a 12 month extended reporting endorsement, and 170% for a 24 month extended reporting endorsement.

Groups with more than one nurse insured on the policy get a discount.[96]
For example, a group with 15 or more nurses gets a 10% credit.

Exercise: Sharona Fleming is a full-time self-employed nurse, who works in Territory 4.
Her specialty is pediatrics.
Determine the premium for her 3rd year claims-made policy with limits of $100K/$300K and no deductible.
[Solution: ($3000)(1.1)(1.50)(0.60)(0.800) = $2376.]

[89] See Exhibit 2.19 in Basic Ratemaking. The deductible applies per claim.
[90] As discussed in Chapter 16 of Basic Ratemaking.
[91] See Exhibit 2.20A in Basic Ratemaking. Claims-made Maturity Factors are commonly called step factors.
[92] Here mature means a 7-year policy or more.
[93] It could also be purchased by the estate of a nurse or doctor who died.
[94] See Exhibit 2.20B in Basic Ratemaking.
These factors are multiplied by the base rates just as are the claims-made maturity factors.
[95] Pricing extended reporting endorsements is not discussed in Basic Ratemaking.
See for example "Liabilities for Extended Reporting Endorsement Guarantees under Claims-Made Policies," by Charles L. McClenahan, 1988 CAS Discussion Paper Program.
[96] See Exhibit 2.21 in Basic Ratemaking.

Exercise: Mildred Ratched is retiring as a nurse.
She worked at the Salem Mental Hospital located in Territory 3.
Her specialty is psychiatric.
She worked full-time.
She gets a 15% schedule rating credit.
Determine the premium for her 60 month extended reporting endorsement with limits of $2M/$4M and a $5000 deductible.
[Solution: ($2500)(0.80)(1.25)(0.85)(1.15)(0.92)(2.400) = $5396.]

Rating Algorithm for this Medical Malpractice Example:

R1 = Specialty Relativity
R2 = Part-time/Full-time Relativity
R3 = Territory Relativity
R4 = Limit Relativity
C1 = 1 - Claims Free Discount
C2 = 1 - Schedule Rating Credit/Debit[97]
C3 = 1 - Deductible Credit
C3 = 1 - Group Credit.
F1 = Claims-Made Factor (Either Step Factor or Extended Reporting Factor)

Premium = $\sum$(Base Rate for Nurse) R1 R2 R3 C1 C2 R4 C3 F1 C4.[98]

There is a minimum premium of $100, per nurse.

The total premium for a policy with several nurses is the sum of the premium for the individual nurses on the policy.

[97] A credit of 10% corresponds to factor of 0.90. A debit of 15% corresponds to a factor of 1.15.
[98] *Premium is rounded to the nearest penny after each step, and rounded to the nearest dollar at the end.*

Workers Compensation Example:[99] [100]

There are hundreds of different classes. An employer will usually have different workers and their payroll assigned to more than one class.[101] Each class in each state has its own rate.
Examples of classes: Clerical, Trucking, Plumbing, Box Manufacturing, Biotechnology, etc.

Exposures are in units of $100 of payroll.[102]

Manual premiums are: $\sum$(class rate) (payroll / $100).[103]

Exercise: An employer has payroll in three classes with the following rates:

Class	Payroll	Rate
1	$100,000	$3
2	$500,000	$1
3	$200,000	$2

Determine this employers manual premium.
[Solution: (3)(1000) + (1)(5000) + (2)(2000) = $12,000.]

The manual premium would normally be multiplied by an experience modification factor.[104]
However, in the example in Basic Ratemaking, the insured is too small to be experience rated.[105]

In addition, one can Schedule Rate an insured.[106] For each of a set of characteristics, there will be a range of credits or debits.[107]

[99] See pages 29 to 34 in Basic Ratemaking.
[100] For a different example, see The Massachusetts Workers Compensation Rate Manual at:
https://www.wcribma.org/Mass/Products/Manuals/MA_Manual.aspx
[101] *While it is usually a relatively straightforward process, it can be difficult to classify some employers. There are foot thick manuals going into all of the details.*
[102] As discussed in my section on Exposures, while payroll is the most commonly used exposure base for Workers' Compensation, one might consider using manweeks.
[103] Basic Ratemaking here does not use the term "manual premiums."
[104] Experience Rating is discussed in Chapter 15 of Basic Ratemaking. An insured with more than the expected losses pays more, while one with less than the expected losses pays less.
[105] In most states, all insureds above a certain size are experience rated.
[106] Schedule Rating is discussed in Chapter 15 of Basic Ratemaking.
Schedule rating is used to alter manual rates to reflect characteristics of the specific insured that are expected to affect future losses compared to the average reflected in the manual rate. If Experience Rating applies, then only characteristics that are not yet reflected in the prior experience should be reflected in Schedule Rating.
[107] See Exhibit 15.6 in Basic Ratemaking for a detailed example of a Schedule Rating Plan.

In the example, they list 6 characteristics with the following ranges:[108]

Characteristic	Range of Credits and Debits
Premises	10% credit to 10% debit
Classification Peculiarities	10% credit to 10% debit
Medical Facilities	5% credit to 5% debit
Safety Devices	5% credit to no credit
Employees-Selection, Training, Supervision	10% credit to 10% debit
Supervision, Management-Safety Organization	5% credit to 5% debit

Usually there is a maximum total credit and debit. In this example, the maximum credit is 25%, while the maximum debit is also 25%.

There is some underwriting judgement involved in applying Schedule Rating.

In this example, a particular insured is given a 10% credit for Premises, a 2.5% credit for Safety Devices, and a 5% credit for Employees-Selection, Training, Supervision.
The total credit is: 10% + 2.5% + 5% = 17.5%.[109]

The manual premium is multiplied by the effect of Schedule Rating.
In this example we multiply by: 1 - 17.5% = 0.825.

This insurer gives addition credits for specific items: Pre-employment Drug Screening 5%, Employee Assistance Program 10%, Return-to-Work Program 5%.[110]

Any of these credits that are applicable to a given insured employer are applied separately and multiplicatively. In the example, a 5% credit is given for Pre-employment Drug Screening , so the manual premium is multiplied by: 1 - 5% = 95%.

In addition, each policy is charged an Expense Constant of $150.[111]

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[108] See Exhibit 2.24 in Basic Ratemaking.
[109] The maximum does not have an effect in this case. Schedule Rating credits and debits add.
[110] See Exhibit 2.25 in Basic Ratemaking.
[111] This covers fixed expenses that do not vary with premium, as discussed in my section on "Other Expenses and Profit."

Exercise: An employer has a payroll of $500,000, all in a class with a rate of $2.
They have an experience modification of 0.96, a 4% credit.
They have a schedule debit of 3%.
They are given a credit of 10% for an Employee Assistance Plan.
They are given a credit of 5% for a Return-to-Work Program.
The Expense Constant is $150.
Determine the premium.
[Solution: ($500,000/100) ($2) = $10,000.
(0.96) (1 + 3%) (1 - 10%) (1 - 5%) ($10,000) + $150 = $8604.
Comment: The example in Basic Ratemaking does not include Experience Rating.
Larger insureds would get Premium Discounts, as discussed in Chapter 11 of Basic Ratemaking.]

Rating Algorithm for this Workers Compensation Example:

Manual Premium = $\sum$(class rate) (payroll / $100).

Factor 1 = 1 + Schedule Rating Credit/Debit.[112]
Factor 2 = 1 - Pre-Employment Drug Screening Credit
Factor 3 = 1 - Employee Assistance Program Credit
Factor 4 = 1 - Return-to-Work Program Credit

Premium = (Manual Premium) (Factor 1) (Factor 2) (Factor 3) (Factor 4) + Expense Constant.[113]

The Minimum Premium is $1500.

[112] A 5% credit corresponds to a factor of 0.95, while a 5% debit corresponds to a factor of 1.05.
[113] *Premium is rounded to the nearest penny after each step, and rounded to the nearest dollar at the end.*

Sample of Illinois Fair Plan Homeowners Rates:[114]

This is a concrete example of rates for homeowners insurance, for those who have not worked in that line of insurance.

Lake County, Protection Classes 1-8[115]

	MASONRY		FRAME	
Amount of Insurance	HO 2	HO 3	HO 2	HO 3
$10,000	$135	$145	$150	$160
$20,000	$179	$191	$202	$216
$30,000	$224	$239	$251	$268
$40,000	$287	$307	$318	$340
$50,000	$352	$377	$395	$423
$75,000	$559	$599	$623	$666
$100,000	$799	$859	$883	$946
$150,000	$1,279	$1,379	$1,403	$1,506
$200,000	$1,759	$1,899	$1,923	$2,066
$250,000	$2,239	$2,419	$2,443	$2,626
$300,000	$2,719	$2,939	$2,963	$3,186
$350,000	$3,199	$3,459	$3,483	$3,746
$400,000	$3,679	$3,979	$4,003	$4,306
$450,000	$4,159	$4,499	$4,523	$4,866
$500,000	$4,639	$5,019	$5,043	$5,426
Premium per $10,000 additional	$96	$104	$104	$112

[114] www.illinoisfairplan.com Rates effective 1/1/02. The Fair Plan is the involuntary market for property insurance.
[115] Lake County in suburban Chicago, is north of Cook County, on Lake Michigan and the Wisconsin border.

The more insurance that is purchased, the more the premium. However, the relativities by amount of insurance are not linear. Here are selected relatives to $100,000:

	MASONRY		FRAME	
Amount of Insurance	HO 2	HO 3	HO 2	HO 3
$50,000	0.44	0.44	0.45	0.45
$100,000	1.00	1.00	1.00	1.00
$250,000	2.80	2.82	2.77	2.78
$500,000	5.81	5.84	5.71	5.74

Less is charged for Masonry than Frame Construction.
For example for $100,000, for HO3: $859/$946 = 0.908.

Less is charged for HO2 than HO3.[116]
For example for $100,000, for Masonry: $799/$859 = 0.930.

[116] HO3 as an "all risk" policy provides more coverage than the HO2 "named peril" policy.
Determining coverage relativities is an important task of homeowners actuaries.

Lake County, Protection Class 9

	MASONRY		FRAME	
Amount of Insurance	HO 2	HO 3	HO 2	HO 3
$10,000	$166	$178	$184	$197
$20,000	$220	$236	$245	$263
$30,000	$276	$295	$307	$328
$40,000	$350	$375	$390	$417
$50,000	$433	$463	$484	$518
$75,000	$688	$736	$765	$819
$100,000	$978	$1,047	$1,085	$1,164
$150,000	$1,558	$1,668	$1,725	$1,854
$200,000	$2,138	$2,290	$2,365	$2,544
$250,000	$2,718	$2,911	$3,005	$3,234
$300,000	$3,298	$3,533	$3,645	$3,924
$350,000	$3,878	$4,154	$4,285	$4,614
$400,000	$4,458	$4,776	$4,925	$5,304
$450,000	$5,038	$5,397	$5,565	$5,994
$500,000	$5,618	$6,019	$6,205	$6,684
Premium per $10,000 additional	$116	$124	$128	$138

More is charged for Protection Class 9 than Protection Classes 1-8.
For example, for $100,000, HO3, Masonry: $1,047/$859 = 1.22.
For example, for $100,000, HO3, Frame: $1,164/$946 = 1.23.

Cook County (except Chicago), Protection Class 9

	MASONRY		FRAME	
Amount of Insurance	HO 2	HO 3	HO 2	HO 3
$10,000	$198	$212	$220	$235
$20,000	$263	$282	$292	$312
$30,000	$329	$352	$366	$391
$40,000	$419	$448	$466	$499
$50,000	$520	$556	$578	$619
$75,000	$822	$880	$914	$978
$100,000	$1,167	$1,250	$1,299	$1,388
$150,000	$1,857	$1,990	$2,069	$2,208
$200,000	$2,547	$2,730	$2,839	$3,028
$250,000	$3,237	$3,470	$3,609	$3,848
$300,000	$3,927	$4,210	$4,379	$4,668
$350,000	$4,617	$4,950	$5,149	$5,488
$400,000	$5,307	$5,690	$5,919	$6,308
$450,000	$5,997	$6,430	$6,689	$7,128
$500,000	$6,687	$7,170	$7,459	$7,948
Premium per $10,000 additional	$138	$148	$154	$164

More is charged in more urban Cook County than in less urban Lake County.[117]
For example, for $100,000, HO3, Masonry: $1,250 / $1,047 = 1.19.
For example, for $100,000, HO3, Frame: $1,388 / $1,164 = 1.19.

[117] There are other territories. Helping to determine territories and territory relativities is an important task of actuaries.

All of the above rates are for a $250 deductible.[118] If the insured chooses a larger deductible, then he pays less. The above base rates are multiplied by the following factors:

Deductible:	$500	$1000	$2500	$5000	$10,000
Up to $200,000 of Insurance:	0.92	0.79	0.62	N/A	N/A
$200,001 and over:	0.96	0.89	0.75	0.67	0.62

Note that the 8% credit for a $500 deductible rather than a $250 deductible could be based on an Loss Elimination Ratio of 0.15 for $250 and a Loss Elimination Ratio of 0.22 for $500;
$(1 - 0.22) / (1 - 0.15) = 0.78 / 0.85 = 0.92$.[119]

All of the above rates are for one or two family homes. For 3 or 4 family homes, the insured has to pay more premium; a factor of 1.3 is applied to the base rate shown.

All of the above rates are for $100,000 per person limits of Liability (coverage E) and $1000 per person Medical Payments (Coverage F). The insured can choose optional higher limits of liability and pay additional premiums. The extra flat amounts do not vary by territory, form, construction, etc.[120]

Limit of Liability($000)	100	200	200	300	300
Medical Payments ($000)	2	1	2	1	2
Additional Premium	$24	$20	$25	$27	$30

[118] The deductible does not apply to Liability Coverage.
[119] These Loss Elimination Ratios were made up by me solely for illustrative purposes. Besides the ratio of loss elimination ratios, pricing of deductibles can include expense considerations and the issue of self-selection.
[120] I do not know the source of the particular values shown.

Problems:

2.1. (1.5 points) List the three items in a rating manual and briefly describe each one.

2.2. (2 points)
The Workers Compensation rates charged by Dependable Insurance in State X are:

Class	Description	Rate
3383	Jewelry Manufacturing	1.46
7219	Trucking	9.41
8810	Clerical Employees	0.22

The Jay Jewelry Company is located in State X and has the following payrolls:

Class	Description	Payroll
3383	Jewelry Manufacturing	$200,000
7219	Trucking	$30,000
8810	Clerical Employees	$50,000

Dependable Insurance gives Jay Jewelry Company a 10% schedule credit.
In addition. they are given a 7% credit for a drug free work place.
Dependable Insurance company charges an Expense Constant of $200.
Determine the premium charged to Jay Jewelry Company. Show all work

2.3. (1 point) List four possible uses of Underwriting Guidelines.

2.4. (1 point) An insurer writes homeowners insurance in a large diverse state.
A particular type of burglar alarm has been shown to lead to a significant reduction in losses due to theft. The insurer plans to introduce a percentage discount for a this type of burglar alarm.
Briefly discuss a potential problem with this idea and a possible solution.

2.5. (3 points) Use the following information for Medical Malpractice Insurance:

- The Base Rate is $10,000 per doctor.
- Full-Time doctors are the base; part-time doctors have a relativity of 0.60.
- $100K/$300K limits of liability is the base; $1M/$3M has a relativity of 1.7.
- No deductible is the base; the credit for a $5000 deductible is 10%.
- A mature claims-made policy is the base. The relativities are as follows:

1st year 0.20	2nd year 0.35	3rd year 0.75
4th year 0.85	5th year 0.95	mature 1.00

60 month extended reporting endorsement: 2.00

The Friendly Pediatrics Group Practice consists of four practicing doctors.
Dr. Burns just retired and Dr. Fever took his place.

	Type of Policy		Schedule Rating
Dr. George Burns	60 month extended reporting	Full-Time	5% debit
Dr. Flo Schotte	Mature	Part-Time	10% credit
Dr. Major Payne	Mature	Full-Time	20% credit
Dr. Vera Sharpe-Needles	4th year	Full-Time	10% debit
Dr. Hy Fever	1st year	Full-Time	no credit or debit

The practice is located in Territory 5 with a relativity of 1.15.
They are all pediatricians; pediatrics has a relativity of 0.95.
The Friendly Pediatrics Group Practice purchases a claims-made policy covering all five doctors.
They purchase limits of $1M/$3M with a deductible of $5000.
They get a group credit of 7%, which is applied separately from the deductible credit.
Determine the premium for the Friendly Group Practice claims-made policy. Show all work.

2.6. (1 point) Basic Ratemaking list some examples of typical rating variables.
For each of the following four lines of insurance, list one rating variable:
Personal Automobile, Homeowners, Workers Compensation, and Medical Malpractice.
Do not use a given rating variable more than once.

Use the following information for Homeowners Insurance for the next two questions:

- Base Rate = $600.

- Amount of Insurance ($000) / Relativity

Amount of Insurance ($000)	Relativity
50	0.6
100	1.0
200	1.3
300	1.5

- Territory / Rate Relativity

Territory	Rate Relativity
1	0.5
2	1.0
3	1.4

Underwriting Tier	Rate Relativity
Preferred	0.85
Standard	1.00
Substandard	1.25

- Protection Class / Construction Type Rate Relativities

Protection Class	Frame	Masonry
1-3	1.00	0.90
4-6	1.15	1.05
7-10	1.35	1.20

- Deductible / Rate Relativity

Deductible	Rate Relativity
$250	1.1
$500	1.0
$1000	0.9

Increased Liability and Medical Limits

Limit	Rate
$100,000/$500	Included
$500,000/$1000	$30

- Policy Fee = $70.

- Rating Algorithm:

R1 = Amount of Insurance Relativity
R2 = Territory Relativity
R3 = Protection Class / Construction Type Relativity
R4 = Underwriting Tier Relativity
C1 = Deductible Credit Factor
C2 = 1 - (New Home Discount) - (Claims-Free Discount).
Premium = (Base Rate) R1 R2 R3 R4 C1 C2
+ Increased Liability / Medical Coverage Rate + Policy Fee

2.7. (1 point) Frame House. Protection Class 8. Territory 1. Amount of Insurance = $50,000. Written in the Substandard Tier. $250 deductible. Determine the premium.

2.8. (1 point) Masonry House. Protection Class 4. Territory 3. Amount of Insurance = $200,000. Written in the Preferred Tier. $1000 deductible. Increased Limits of $500,000/$1000. Receives a 12% credit for being claims free for 5 years. Determine the premium.

2.9. (2 points) A rate manual for the Gecko Insurance Company in State X, shows the following information for private passenger automobile liability insurance:
Expense Fee (per automobile): $40.

Territory	Base Rate
1	$78
2	$141
3	$126

Class	Relativity
1	1.00
2	1.45
3	2.40

Combined Single Limit	Increased Limits Factor
$25,000	1.00
$100,000	1.30
$500,000	1.40

Determine the premium for the following policies.
(a) 1 automobile in Territory 1, Class 2, with $25,000 limit of liability.
(b) 2 automobiles in Territory 2, Class 3, with $500,000 limit of liability.
(c) 1 automobile in Territory 3, Class 1, with $100,000 limit of liability.

2.10. (1.5 points)
Basic Ratemaking list some examples of typical characteristics used in underwriting.
For each of the following six lines of insurance, list one characteristic:
Personal Automobile, Homeowners, Workers Compensation, Commercial General Liability, Medical Malpractice, and Commercial Automobile.
Do not use a given characteristic more than once.

2.11. (3 points) The Workers Compensation rates are:

Class	Description	Rate in State X	Rate in State Y
5183	Plumbing	4.07	6.80
5190	Electrical Wiring	3.17	4.92
5437	Carpentry	3.52	9.11
5474	Painting	8.50	11.97
5551	Roofing	42.57	54.04
8810	Clerical	0.18	0.25

The Cohen Construction Company operates in States X and Y. It has the following payrolls:

Class	Description	Payroll in State X	Payroll in State Y
5183	Plumbing	\$500,000	\$300,000
5190	Electrical Wiring	\$300,000	\$200,000
5437	Carpentry	\$900,000	\$400,000
5474	Painting	\$400,000	\$300,000
5551	Roofing	\$600,000	\$200,000
8810	Clerical	\$200,000	0

The Cohen Construction Company has an experience modification of 1.18, an 18% debit.
Their insurer gives Cohen Construction a 5% debit under schedule rating.
In addition, Cohen Construction is given an 8% credit for having recently put in place a back-to-work program.
An Expense Constant of \$150 is charged.
For simplicity ignore the effects of any Premium Discounts.
Determine the premium charged to Cohen Construction Company. Show all work

2.12. (6, 5/96, Q.22) (1 point) Which of the following describe characteristics of the ISO expense fee program for Private Passenger Automobile Insurance?
1. Expense fees are considered a component of premium.
2. Expense fees are subject to increased limits factors.
3. Expense fees are subject to deductible relativities.

A. 1 only B. 3 only C. 1, 2 only D. 2, 3 only E. 1, 2, 3

Solutions to Problems:

2.1. Rules: contain definitions, particularly how to classify a risk.
Rates pages: contain base rates, rating tables, etc.
Rating Algorithm: describes how to combine the components to determine the premium for a specific insured.

2.2. (1.46)(200,000/100) + (9.41)(30,000/100) + (0.22)(50,000/100) = $5853.
(1 - 10%)(1 - 7%)($5853) + $200 = **$5099**.
Comment: The payrolls would have been estimated at the time the policy was written.
While in theory the payrolls could be audited after policy inception, this insured is too small to make that worthwhile. Presumably, Jay Jewelry Company is to small to be Experience Rated.
There could have been a loss constant added to the premium of small insureds.

2.3. 1. Deciding whether to accept or reject a risk, or to refer it to a more senior underwriter.
2. Deciding in which of several companies in the group to place the insured.
3. Deciding in which of several underwriting tiers to place the insured.
4. How to apply Schedule Rating.

2.4. Assuming the alarm produces the same expected percentage reduction in theft losses in the different territories in the state, there is still the problem that the percent of loss dollars due to the theft almost certainly varies significantly across the state. (For example, theft may be 10% in a rural part of the state, but 30% in an urban area.) Thus as a percent of premiums, the appropriate discount would vary by territory.
The insurer could implement a rating plan that has separate base rates for each major peril covered and the individual rating variable relativities are applied to the applicable base rate; the burglar alarm discount applies to the theft base rate only.
Alternately, the insurer could have different percentage discounts by territory.

2.5. The factors that apply to the whole policy are:
Territory 1.15. Limit Factor 1.7. Deductible Factor 0.90. Group Credit 0.93.

	Base Rate	Specialty	Part-Time	Schedule Rating	Claims-Made	Product
Dr. Burns	$10,000	0.95	1.00	1.05	2.00	$19,950
Dr. Schotte	$10,000	0.95	0.60	0.90	1.00	$5,130
Dr. Payne	$10,000	0.95	1.00	0.80	1.00	$7,600
Dr. Sharpe-Needles	$10,000	0.95	1.00	1.10	0.85	$8,882
Dr. Fever	$10,000	0.95	1.00	1.00	0.20	$1,900
Total						$43,462

(1.15) (1.7) (0.9) (0.93) ($43,462) = **$71,118**.
Comment: Dr. Burns still needs coverage for incidents that occurred while he was practicing but have yet to be reported.

2.6. Personal Automobile: Driver Age and Gender, Model Year, Accident History.
Homeowners: Amount of Insurance, Age of Home, Construction Type, Protection Class.
Workers Compensation: Occupation Class Code.
Medical Malpractice: Specialty, Territory, Limit of Liability.
<u>Comment</u>: See Exhibit 2.1 in <u>Basic Ratemaking</u>.

2.7. (\$600)(0.6)(0.5)(1.35)(1.25)(1.1) + \$70 = **\$404**.

2.8. (\$600)(1.3)(1.4)(1.05)(0.85)(0.9)(0.88) + \$30 + \$70 = **\$872**.

2.9. (a) (1.45)(\$78) + \$40 = **\$153**.
(b) (1.40)(2.40)(\$141) + \$40 = \$514. (2)(\$514) = **\$1028**.
(c) (1.30)(\$126) + \$40 = **\$204**.
<u>Comment</u>: See page G-2, Rule 3E, of the ISO Personal Vehicle Manual.
One does <u>not</u> apply the class relativities or the increased limit factors to the Expense Fee.
An Expense Fee is charged per automobile.
In the ISO Manual, what is referred to as a “base rate” is what an actuary might refer to as a “variable base rate”.

2.10. Personal Automobile Insurance: Credit Score, Homeownership, Prior Bodily Injury Limits.
Homeowners Insurance: Credit Score, Prior Loss Information, Age of Home.
Workers Compensation: Safety Programs, Number of Employees, Prior Loss Information.
Commercial General Liability Insurance: Credit Score, Years in Business, Number of Employees.
Medical Malpractice: Patient Complaint History, Years Since Residency,
Number of Weekly Patients.
Commercial Automobile: Driver Tenure, Average Driver Age, Earnings Stability.
<u>Comment</u>: See Exhibit 2.2 in <u>Basic Ratemaking</u>.

2.11. Multiply the rates by the payrolls in hundreds of dollars:

Class	Rate in X	Payroll in X	Premium in X	Rate in Y	Payroll in Y	Premium in Y
5183	4.07	$500,000	$20,350	6.80	$300,000	$20,400
5190	3.17	$300,000	$9,510	4.92	$200,000	$9,840
5437	3.52	$900,000	$31,680	9.11	$400,000	$36,440
5474	8.50	$400,000	$34,000	11.97	$300,000	$35,910
5551	42.57	$600,000	$255,420	54.04	$200,000	$108,080
8810	0.18	$200,000	$360	0.25	$0	$0
Sum			$351,320			$210,670

The manual premium is: $351,320 + $210,670 = $561,990.
(1.18)(1 + 5%)(1 - 8%)($561,990) + $150 = **$640,751**.
Comment: The payrolls would have been estimated at the time the policy was written.
The payrolls would be audited after policy expiration.
Premium Discounts are discussed in Chapter 11 of Basic Ratemaking.

2.12. A. 1. True.
2. False. Increased Limit Factors are applied to the variable portion of the rate, not to the Expense Fee.
3. False. Deductible Relativities are applied to the variable portion of the rate, not to the Expense Fee.
Comment: See page G-2, Rule 3E, of the ISO Personal Vehicle Manual.

Section 3, Ratemaking Data

Data is used by actuaries for many purposes including ratemaking.

For indications of the overall rate level, summarized data on exposures, premiums, losses, and ALAE is used.

Data on ULAE and underwriting expenses is usually from the insurer's accounting system; an aggregate amount may be allocated to line of insurance and/or state.

For classification and territory ratemaking, more detailed data on exposures, premiums, losses, and ALAE is used, broken down by class and territory.
For individual risk rating, data from a particular insured is used.

Actuaries also use detailed information for special studies, for example: determining territory definitions, determining increased limits factors, marketing studies, calculating individual risk rating plan parameters, determining deductible credits, determining the usefulness of a possible new rating variable or underwriting criterion, etc.

Ratemaking data is usually aggregated into years, as discussed below.

An insurer's data is often contained in a policy data base with exposures and premiums, and a separate claims data base with losses and alae.
Actuaries and underwriters rely on information besides the exposure, premium, loss, and expense data from their own insurer.

Data Quality:

Any analysis performed by an actuary is no better than the quality of the data that goes into that analysis![121]

For example, assume that under the Workers Compensation Statistical Plan one insurer mistakenly reports $500 million of payroll in a class for an employer rather than $5 million. Assume that this results in about twice as much reported exposure in total for that class as was actually the case. Then the reported pure premium will be half of the actual pure premium. Unless this mistake is caught and corrected, the class relativity that results from any actuarial analysis will be erroneous!

Therefore, assuring the quality of data is important to actuaries. *Statistical Agents have many techniques to try to catch and correct any mistakes in the data reported to them. Insurers need to try to eliminate as many errors as possible in their data that is used for internal purposes.*

[121] Garbage in, garbage out.

<u>Years of Data</u>:[122]

Premiums and losses can be organized in different ways.

Calendar Year: All premiums and losses related to a given calendar year.

Calendar year data might be used for ratemaking on lines on insurance where claims are reported and settled quickly, such as Automobile Collision.

Fiscal Year: Similar to a Calendar Year; however a fiscal year starts on a date other than January 1.[123]

For example, a fiscal year might start on July 1 and end on the subsequent June 30.

Accident Year: All the losses with accident dates during a given year.[124]

For example, an accident occurs on March 15, 2003. All payments related to claims resulting from this accident, are part of Accident Year 2003, regardless of when those payments are made.

Calendar/Accident Year 2003 would consist of premiums for Calendar Year 2003 and losses for Accident Year 2003.

Policy Year: All premiums and losses related to policies with effective dates during a given year.

For example, an accident occurs on March 15, 2003. If the policy providing coverage was written effective November 1, 2002, then all payments related to claims resulting from this accident, are part of Policy Year 2002, regardless of when those payments are made.

Policy Year data has the best match of losses to premiums, with Accident Year next, and Calendar Year worst. Calendar Year data is available quickest, with Accident Year next, and Policy Year slowest.

Report Date: the date the insurer receives notice of the claim.

Report Year: All the losses on claims for which the insurer first receives notice during a given year.

For example, an accident occurs on March 15, 2003. If on June 5, 2004 the insurer first receives notice of a claim resulting from this accident, then all payments related to this claim, are part of Report Year 2004, regardless of when those payments are made.

[122] While a "year" could be any consecutive 12 months, unless stated otherwise a year starts on January 1 and ends on December 31.
[123] A calendar year is a special case of a fiscal year, that starts on January 1.
[124] Accident Year data is used in the examples of rate indications in Appendices A to D of <u>Basic Ratemaking</u>.

As will be discussed, Report Year data can be used for ratemaking for lines of insurance with a long reporting lag for claims, such as Medical Malpractice Insurance. Such lines of insurance are often written on a claims-made rather than occurrence basis.[125]

Policy Database:

The policy database contains detailed information on the exposures and premiums for policies written by an insurer.

Each data element is reported in its own “field”. A group of fields is called a “record”.
A policy may be reported as one record or may be broken down by coverage and/or class into several records, depending on the line of insurance. Cancelations and any midterm endorsements would each result in additional records being reported.

For a given line of insurance at a particular insurer, each record would contain the same fields in the same order, although occasionally a field is left blank in a particular record.

For example, here is a very simplified example of a record:
Field 1: Policy Number.
Filed 2: Policy Effective Date.
Field 3: State.
Field 4: Premium.

Typical fields for each record on the policy database:
• Policy identifier
• Risk identifier(s)
• Relevant dates
• Premium
• Exposure
• Characteristics: rating variables, underwriting variables, etc.

For use in ratemaking, records would usually be combined to create the form of the data to be used by the actuary. For example, if a policy were canceled midterm one would need to combine records to get the net premium and exposure for that policy.

[125] See Chapter 16 of Basic Ratemaking.

Claims Database:

The claims database contains detailed information on the losses and alae for policies written by an insurer.

Typical fields for each record on the claims database:[126]
• Policy identifier
• The risk identifier(s)
• Claim identifier.
• Claimant identifier.
• Relevant loss dates: the date of loss, the report date, and the transaction date.
• Claim status: open, closed, or reopened.
• Reopen date.
• Claim count.
• Paid loss.
• Event identifier: for example a catastrophe.
• Case reserve.
• Allocated loss adjustment expense.
• Salvage/subrogation.
• Characteristics.[127]

Accounting Information:

Underwriting expenses and unallocated loss adjustment expenses (ULAE) are not collected in either the policy or claims databases; they are collected in the insurer's accounting system.

[126] Some of the fields listed may not be relevant for some lines of insurance.
[127] For example, for Workers Compensation one might record the "injury kind," cause of the accident, and the body part injured.

Data from Outside Sources:

Actuaries working at an insurer use many sources of information other than data from inside the insurer. Basic Ratemaking refers to these as external data.[128]

For example, statistical agents such as ISO and NCCI collect information from many insurers within a given state. In some cases, one can obtain this data totaled across all the reporting insurers.[129] Depending on the situation, one may only be able to get premiums, exposures, losses and ALAE in total for each year, or one may be able get breakdowns by class/territory or further detail. In some cases, one could request a special compilation to serve a particular purpose.[130]

The “Fast Track Monitoring System” tracks quarterly industry data for personal lines.[131]
The purpose of Fast Track is to monitor trends over time, as it provides a good indication of how claiming and loss patterns are increasing, decreasing, or remaining stable.

Contained in Fast Track are exposure, paid loss and paid claim data, from which claim frequency, average loss (severity) and loss cost (pure premium) experience is generated. Also included are earned premiums and incurred losses used to develop loss ratio information. Auto insurance data are reported by major coverage; the frequency, average loss and loss cost data are presented for bodily injury liability, property damage liability, all comprehensive, all collision, and personal injury protection, if applicable, while the loss ratio reports provide all liability coverages combined, all comprehensive and all collision. The homeowners insurance data for frequency, severity and loss cost are shown by policy form, while the loss ratio report combines all homeowners forms.

Other sources of aggregated industry data or reports:
A.M. Best[132], Highway Loss Data Institute (HLDI)[133], Insurance Research Council (IRC)[134], the Institute for Business and Home Safety (IBHS)[135], the National Insurance Crime Bureau (NICB)[136], and the Workers Compensation Research Institute (WCRI)[137].

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[128] *The term external data is commonly used elsewhere to refer to other than insurance data, such a CPIs.*
[129] This could be referred to as “industry data” even if not all writers reported to the particular statistical agent.
[130] When I worked at the Massachusetts Workers Compensation and Automobile Rating Bureaus, we issued many regular data reports. We also fulfilled all sorts of special requests for data from member companies. An insurer could get a report with only its own data or the entire industry.
[131] Fast Track Monitoring System (Fast Track) reports are prepared quarterly by the Independent Statistical Service, Inc. (ISS), Insurance Services Office, Inc. (ISO), and National Independent Statistical Service (NISS). These are statistical reports reflecting multiyear trends in private passenger auto and homeowners insurance experience. Approximately 50 of the largest companies (fewer for any given state) representing about 70 and 50 percent of the nationwide personal auto and homeowners premium volume, respectively, participate in this voluntary report.
[132] www.ambest.com
[133] http://www.iihs.org/research/hldi/composite_intro.html
[134] www.ircweb.org
[135] www.disastersafety.org
[136] www.nicb.org/cps/rde/xchg/nicb/hs.xsl/index.htm
[137] www.wcrinet.org

Insurers can often obtain competitor's rate filings and rate manuals from insurance departments.[138]

Sometimes Consumer Price Indices (CPIs) are used to help estimate trends.[139]

The U.S. Census has lots of potentially valuable information, for example: population density, weather, thefts, and annual miles driven.[140]

Credit scores of insureds, purchased from a firm that specializing in this, can be used for classification and/or underwriting.

Additional information that may be used:[141]
• Personal automobile insurance: vehicle characteristics, department of motor vehicle records
• Homeowners insurance: distance to fire station
• Earthquake insurance: type of soil
• Medical malpractice: characteristics of hospital in which doctor practices
• Commercial general liability: type of owner (proprietor, stock)
• Workers' compensation: OSHA inspection data.

[138] *Sometimes these are obtained from recent former employees of these competitors or from insurance agents.*
[139] For example, in Appendix B of Basic Ratemaking, the expense trend is calculated based on the change in Consumer Price Indices.
[140] Geo-demographic data on the average characteristics of a particular area.
[141] See page 47 of Basic Ratemaking.

Duties of Regulators:[142]

1. **Financial solvency** and solidity of insurance companies - to ensure that the insurance companies will be able to meet their obligations to policyholders.

2. **Market conduct** - to ensure that insurers do not engage in unfair practices and that insurers fulfill the terms of their policies.

3. **Rate regulation** - to ensure that rates meet statutory standards, that is, that rates are not inadequate, excessive, or unfairly discriminatory.

Primary Data collected to carry out each of these three duties:

Financial solvency: Annual Statement.

Market conduct: Periodic market conduct examinations of individual insurers.

Rate regulation: Statistical information on premiums and losses reported to statistical agents.

Insurers may collect additional data for their own business needs.

Summary Based versus Transaction Based Statistical Plans:[143]

Summary-based Statistical Plans are less detailed and less expensive.
The premium and loss data in summary-based plans are collected on a summarized basis.

Transaction-based statistical plans are more detailed and more expensive.
Transaction level records are sent to the statistical agent on a regular basis.

Summary-based data is insufficient for many needs of actuaries.

Workers Compensation Financial Calls	Summary Based
Workers Compensation Statistical Plan Data Calls	Transaction Based

[142] See page 1 of CAS Study Note: ISO Statistical Plans, by Virginia R. Prevosto, formerly on the syllabus.
[143] See pages 3-4 of CAS Study Note: ISO Statistical Plans, by Virginia R. Prevosto, formerly on the syllabus.

Five major data collection programs at NCCI:[144]

1. **Annual Financial Call Data**: Aggregate premium and loss data (experience by state) primarily used to calculate overall loss cost and rate level changes.

2. **Workers Compensation Statistical Plan (WCSP) Data**: Individual policy loss and exposure (payroll) information collected with detail by classification, primarily used to calculate classification relativities and experience rating modifications.[145]

3. Detailed Claim Information (DCI) Data:
Sample of indemnity (mostly permanent partial disability) claims consisting of 85 data elements reflecting areas such as paid and incurred losses/claims, demographic, litigation and recovery information, used for special analyses and research.

4. Policy Issue Capture System (PICS) Data: Policy documents submitted by insurers. PICS is used to confirm proof of workers compensation coverage (and supply other information) to state accident boards and commissions. In most states, NCCI is designated as the commission's authorized agent to collect coverage information, so there is no need for insurers to submit policy information to the state commission. PICS is also used to let NCCI know when to expect a unit report.

5. Residual Market Data: Submissions from servicing carriers, including policy and calendar year premiums, losses and expenses (by state). As part of NCCI's role and responsibilities as administrator of the National Pool and other residual market pools, NCCI serves as the central clearinghouse for information on pool transactions. This information is used for pool financial reporting procedures, quarterly distribution of operating results and the determination of pool IBNR reserves.

[144] See page 1 of CAS Study Note: NCCI Data collection Calls and Statistical Plans, by Richard B. Moncher, formerly on the syllabus.
[145] This detailed information is used by workers compensation actuaries for all types of studies.

An example of data compiled from the Workers Compensation Statistical Plan:[146]

Massachusetts Workers Compensation Unit Statistical Plan Data
(Including Large Deductible Policies)
Composite Policy Year 1999/2000[147] at First Report[148]

Class Code: 5, Farm: Nursery Employees & Drivers
Industry Group: Goods & Services
Hazard Group: 2

Payroll	$12,761,866
Manual Premium	$456,751
Standard Premium	$471,494
Total Losses	$202,093

Loss Ratio to Manual Premium	44.2%
Loss Ratio to Standard Premium	42.9%
Pure Premium (per $100 payroll)	$1.58

Indemnity Losses	$133,030
Medical Losses	$69,063
Claim Count	60

Fatal: Claim Count 0
Permanent Total: Claim Count 0
Major Partial Disability:
Indemnity: $36,690 Medical: $3,335 Claim Count: 1
Minor Partial Disability:
Indemnity: $16,717 Medical: $4,986 Claim Count: 1
Temporary Total:
Indemnity: $79,623 Medical: $48,629 Claim Count: 11
Medical Only:
Losses: $12,113 Claim Count: 47

Note that data compiled for a class would usually be from several or many employers.
Detailed information would be available by employer.
Detailed information would be available by claim, for those claims of size greater than $2000.

[146] From the Massachusetts Workers Compensation Rating and Inspection Bureau.
Unit Statistical Reports similar to the those collected by the NCCI.
[147] *Policies with effective dates 7/1/99 to 6/30/00.*
[148] First report is 18 months after the policy effective date. For Statistical Plan Data, there are now a total of 10 reports, separated by 12 months.

Workers Compensation Policy Year Financial Calls:[149]

Each insurer or insurer group reports data separately by state.
Data is reported by year for each of more than 20 years at latest report.
Data from policies with Large Deductibles is excluded.[150]
In each case, the reported premiums and losses are accumulated through a given report rather than being the incremental change from the previous report.

Earned Premiums are reported in three different ways:
Standard at NCCI Designated Statistical Reporting (DSR) Level,[151]
Standard at Company Level,
Net.[152]

Losses are reported in eight different pieces:[153]
Indemnity Paid
Medical Paid
Indemnity Case Reserves
Medical Case Reserves
Indemnity Bulk Reserves
Medical Bulk Reserves
Indemnity IBNR
Medical IBNR

In addition, an incurred indemnity claim count is reported.[154]

ALAE is reported (separately) for years 1994 and subsequent.

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[149] A similar call is made for Calendar/Accident Year Data.
Calendar Year losses may be obtained from either call.
[150] Large deductibles usually have a deductible of $100,000 or more.
Experience from policies with small deductibles is included.
In some states, experience from large deductibles is collected separately in a similar format.
[151] Prior to the effect of any individual insurance company competitive pricing activity.
In many states, premium at the DSR level does not include any provision for expenses, in other words, at the level of published NCCI loss costs.
[152] Net direct earned premium is net of premium discounts, individual company deviations, and pricing programs.
It is used to reconcile the insurance company's Annual Statement to information reported on this Financial Call.
[153] See pages 2 and 3 of the call.
Outstanding, excluding IBNR = case reserves plus bulk reserves.
Some insurance companies put up bulk reserves that account for the development on known claims and additional payments on claims that will be reopened. These company's IBNR would be what is called pure IBNR.
Other companies would just include these bulk reserves in what they call IBNR.
[154] This claim count excludes medical only claims.

<u>An example of data compiled from the Workers Compensation Financial Calls</u>:[155]

Massachusetts Workers Compensation
Annual Financial Call Data
Accident Year Data[156]
Indemnity Plus Medical Losses ($ million)

Paid Plus Case Reserves

		Report			
AY	1	2	3	4	5
1993	319	404	434	446	447
1994	263	349	378	394	
1995	240	344	377		
1996	252	347			
1997	231				

1st report for AY93 is as of 12/31/93 (12 months).
2nd report for AY93 is as of 12/31/94 (24 months).
3rd report for AY93 is as of 12/31/95 (36 months).

There is similar data for older accident years.
For older accident years, one could have as many as 20 reports.

One can separate Indemnity and Medical losses.
One can separate Paid losses and Case Reserves.
One can also compile Total Incurred losses (including Bulk plus IBNR.)

ALAE is reported (separately) for years 1994 and subsequent.
One can separate data by insurer (or insurer group.)

Financial Calls would have been submitted to the rating bureau in early Spring 1998.
Consistency checks and data quality checks would be performed.
Insurers would submit corrections.
The above information would have been available for use by actuaries in late Summer 1998.

[155] From the Massachusetts Workers Compensation Rating and Inspection Bureau.
[156] As shown below, one would have similar data for Policy Years.

MASSACHUSETTS WORKERS COMP.
Annual Financial Call Data
Policy Year Data
Indemnity Plus Medical Losses ($ million)

Paid Plus Case Reserves

			Report		
PY	0	1	2	3	4
1993	172	353	397	411	415
1994	135	319	367	389	
1995	116	306	364		
1996	135	306			
1997	121				

Policy Year is just another way to break the same total losses into pieces.

1st report for PY93 is as of 12/31/94 (12 months).
2nd report for PY93 is as of 12/31/95 (24 months).
3rd report for PY93 is as of 12/31/96 (36 months).

Incomplete Policy Year:

Under the Workers Compensation Financial Calls, as of December 31, 2006, one would report Policy Year 2006. As of December 31, 2006, most of the policies written during 2006 would yet to have expired. This data would be referred to variously as: an incomplete policy year, policy year at report 0, or policy year at report 1/2.[157]

Like subsequent reports, the incomplete policy year has paid losses, case reserves, and reserves for losses that have yet to be reported. In addition, many of the losses that will be in PY 2006 have yet to occur by December 31, 2006.[158]

Thus this information is extremely immature. There is much more loss development on an incomplete policy year than on a policy year at first report. Therefore, an incomplete policy year is usually not used directly in ratemaking.[159]

[157] Policy Year 2006 as of December 31, 2007, would be referred to as Report 1.
[158] If policies are written evenly throughout the year, half of the losses that will make up Policy Year 2006, have yet to occur by 12/31/2006.
[159] However, for example, the actuary may compare the level of the most recent incomplete policy year to the previous one, in order to see if anything unusual may be going on.

Problems:

3.1. (2 points) Briefly define each of the following:
Calendar Year
Accident Year
Policy Year
Report Year

3.2. (2 points) Use the following information on three annual homeowners policies:
• Policy A is written on June 1, 2011 with an annual premium of $2,000.
The home is located in Territory 4 and has a deductible of $250.
On December 1, 2011 the insured increases the deductible to $500.
The full annual term premium after the deductible change is $1800.
• Policy B is written on May 1, 2011 with an annual premium of $1,500.
The insured home is located in Territory 15 and the insured has a $500 deductible.
The policy remains unchanged for the full term of the policy.
• Policy C is written on August 1, 2011 with an annual premium of $800.
The insured home is located in Territory 7 and the insured has a deductible of $250.
The policy is canceled on October 31, 2011.
Show the records related to these policies as they would appear in a policy database.
Display the fields: Original effective date, original termination date, transaction effective date, deductible, territory, written exposures, and written premiums.

3.3. (1 point) Ratemaking analysis is often supplemented with third-party data that is not specific to insurance. List two examples, stating in each case the relevant line of insurance.

3.4. (1.5 points) Briefly discuss the organization of a policy database for each of the following lines of insurance:
(a) Homeowners
(b) Workers Compensation
(c) Personal Automobile

∗**3.5**∗**.** (3 points) A loss occurs on August 26, 2011 and is covered under policy MA12.
It is reported to the insurer on September 4, 2011 and is labeled claim C2728.
At that time a case reserve of $25,000 is established.
The insurer makes a payment of $10,000 on November 6, 2011, and revises the case reserve to $20,000.
The insurer revises the case reserve to $40,000 on July 1, 2012.
The insurer pays $35,000 on October 8, 2012 and closes the claim.
The insurer reopens the claim on February 1, 2013 and establishes a case reserve of $30,000.
The insurer pays $15,000 on April 16, 2013 and closes the claim, this time for good.
Show the records related to this claims as they would appear in a claims database.
Display the fields: Policy, Claim Number, accident date, report date, transaction effective date, claim status, loss payment, loss case reserve.

3.6. (1 point) List 4 fields that are usually present in each record of a policy database.

3.7. (1 point) State one advantage and one disadvantage for use in ratemaking of:
Calendar Year data, Accident Year data, and Policy Year data.

3.8. (2 points) List 8 sources of data external to an insurer that may be used to help make rates for that insurer. Four should be insurance data and four should not.

3.9. (3 points) List 12 fields that are usually present in each record of a claims database.

3.10. (5, 5/15, Q. 6) (1.5 points) Discuss the appropriateness of applying each of the following data aggregation methods to the given line of business:
a. (0.5 point) Calendar Year Aggregation for Auto Physical Damage
b. (0.5 point) Policy Year Aggregation for Homeowners
c. (0.5 point) Report Year Aggregation for Medical Professional Liability

Solutions to Problems:

3.1. Calendar Year: All premiums and losses related to a given calendar year.
Accident Year: All the losses with accident dates during a given year.
Premiums are those for the corresponding Calendar year.
Policy Year: All premiums and losses related to policies with effective dates during a given year.
Report Year: All the losses on claims for which the insurer first receives notice during a given year.

3.2. Policy A with a mid-term adjustment is represented by three records. The first record includes all the information at policy inception. The second record negates the portion of the original policy that is unearned at the time of the endorsement: -0.50 exposure and -$1000 premium. The third record represents the information applicable to the portion of the policy written with the new deductible: +0.50 exposure and +$900 premium.
Policy B expired at its original expiration date and had no changes, thus the entire policy can be represented with one record.
Policy C was canceled before the policy expired. This is represented by two records. The first record for Policy C contains the information known at the inception of the policy. The second record represents an adjustment for the cancelation such that when aggregated, the two records show a result net of cancellation. Policy C was canceled 1/4 of the way through the policy period; the second record should show -0.75 exposure and written premium of: -($800)(.75) = -$600.

Policy	Original Effective Date	Original Termination Date	Transaction Effective Date	Deductible	Territory	Written Exposures	Written Premiums
A	6/1/11	5/31/12	6/1/11	$250	4	1	$2000
A	6/1/11	5/31/12	12/1/11	$250	4	-0.5	-$1000
A	6/1/11	5/31/12	12/1/11	$500	4	0.5	$900
B	5/1/11	4/30/12	5/1/11	$500	15	1	$1500
C	8/1/11	7/31/12	8/1/11	$250	7	1	$800
C	8/1/11	7/31/12	10/31/11	$250	7	-0.75	-$600

Comment: See Table 3.1 in Basic Ratemaking.

3.3. 1. Personal automobile insurance: vehicle characteristics, department of motor vehicle records
2. Homeowners insurance: distance to fire station
3. Earthquake insurance: type of soil
4. Medical malpractice: characteristics of hospital in which doctor practices
5. Commercial general liability: type of owner (proprietor, stock)
6. Workers compensation: OSHA inspection data.
Comment: Give only two examples.

3.4. In homeowners insurance, a record may be a home for an annual policy period.
In workers compensation insurance, rating is based on the payroll of relevant industry classifications so separate records are often maintained at the classification level.
In personal auto insurance, separate records are typically created for each coverage.
If multiple autos are insured on one policy, separate records also may be created for each individual auto on a policy. Moreover, separate records may be maintained for individual operators on each auto.

3.5.

Policy	Claim Number	Accident Date	Report Date	Transaction Date	Claim Status	Loss Payment	Loss Case Reserve
MA12	C2728	8/26/11	9/4/11	9/4/11	open	$0	$25,000
MA12	C2728	8/26/11	9/4/11	11/6/11	open	$10,000	$20,000
MA12	C2728	8/26/11	9/4/11	7/1/12	open	$0	$40,000
MA12	C2728	8/26/11	9/4/11	10/8/12	closed	$35,000	$0
MA12	C2728	8/26/11	9/4/11	2/1/13	reopened	$0	$30,000
MA12	C2728	8/26/11	9/4/11	4/16/13	closed	$15,000	$0

Comment: See Table 3.2 in Basic Ratemaking.
We could also have alae paid and alae case reserves.
We could have salvage and subrogation.
The reported losses (paid + case reserves) are: $25,000 @ 9/4/11, $30,000 @ 11/6/11, $50,000 @ 7/1/12, $45,000 @ 10/8/12, $75,000 @ 2/1/13, and $60,000 @ 4/16/13.
The CY paid losses are: $10,000 in CY11, $35,000 in CY12, and $15,000 in CY13.
The CY reported losses are paid plus the difference in case reserves at the end and beginning: $30,000 in CY11, $15,000 in CY12, and $15,000 in CY13.
Both the paid and reported losses by CY sum to $60,000, the ultimate reported losses.

3.6. Policy identifier, Risk identifiers, Relevant dates, Premium, Exposure, Characteristics (for example, rating variables, underwriting variables, etc.)
Comment: List only four.

3.7. Calendar Year data is available quicker than the others, but has the worst match between losses and premiums.
Accident Year data is available more quickly than Policy Year data, but less quickly than Calendar Year data.
Accident Year data has a better match of losses to premiums than Calendar Year data, but not as good a match as Policy Year data.
Policy Year data has the best match of losses to premiums, but is available less quickly than the others.

3.8. Insurance:
1. Statistical agents such as ISO and NCCI
2. The Fast Track Monitoring System
3. A.M. Best
4. Highway Loss Data Institute (HLDI)
5. Insurance Research Council (IRC)
6. Institute for Business and Home Safety (IBHS)
7. National Insurance Crime Bureau (NICB)
8. Workers Compensation Research Institute (WCRI)
9. Competitor's rate filings and rate manuals

Not Specific to Insurance:
1. Consumer Price Indices from the Bureau of Labor Statistics
2. The U.S. Census
3. Credit scores of insureds
4. Department of motor vehicle records
5. OSHA inspection data
6. State Medical Boards (information on complaints against or formal actions taken against physicians by state licensing and disciplinary boards on doctors)

<u>Comment</u>: List only four of each kind. There are other possible answers.

3.9. Policy identifier, Risk identifiers, Claim identifier, Claimant identifier, Relevant loss dates, Claim status (open, closed, or reopened), Claim count, Paid loss, Event identifier (for example a catastrophe code), Case reserve for losses, ALAE Paid, Case Reserve for ALAE, Salvage/subrogation, Claim characteristics (for example type of injury, physician information, etc.).
<u>Comment</u>: List only 8.

3.10. (a) Since most Auto Physical Damage claims are reported and settled very quickly, Calendar Year data is appropriate. Calendar Year data is available more quickly than Accident Year Data. Alternately, not appropriate as calendar year is fixed at year-end (transaction based), but Auto Physical Damage losses will still develop.
(b) The property piece of Homeowners is reported and settled quickly, while the liability piece takes somewhat longer. The majority of the expected losses are from property rather than liability. Policy Year data would take longer to be available, and therefore Accident Year data is more appropriate for Homeowners.
Alternately, Policy Year aggregation provides for a good match between premium/exposures and losses. This would be a good choice if we are trying to isolate the effects of underwriting changes such as changes to policy limits or deductibles which could be appropriate for homeowners policies.
(c) Since Medical Professional Liability claims can take a very long time to be reported and then settled, Report Year is an appropriate method of aggregation.
Comment: Werner and Modlin use Accident Years for Homeowners in Appendix B and Accident Years for Medical Malpractice in Appendix C.
In its Examiner's Report, the exam committee stated that one had to mention claims-made policies in part (c)! This seems to have been based on page 42 of Estimating Unpaid Claims Using Basic Techniques: "For some lines of insurance, such as medical malpractice, products liability, errors and omission, and directors' and officers' liability, coverage may be dependent on the date on which the claim is reported to the insurer (i.e., claims-made coverage). For these lines of business, actuaries often prefer to use *report year data* for developing estimates of unpaid claims."
In fact, report year is also useful for Medical Malpractice occurrence policies, as shown in Chapter 16 of Basic Ratemaking. Occurrence policies have a longer average time from writing to claim settlement than do similar claims-made policies; see principle 5 of claims-made ratemaking at page 314 of Basic Ratemaking. Thus, report year can also be useful for analyzing occurrence policies.
For part (c), in its Examiner's Report the exam committee listed as a sample answer:
"RY aggregation is not appropriate because it cannot be used to estimate IBNR which is very important for a long-tailed line such as this."
This answer is based on page 43 of Estimating Unpaid Claims Using Basic Techniques:
"Estimation techniques based on claims aggregated by report year only measure development on known claims and not pure IBNR; and pure IBNR is frequently the more difficult part of the total unpaid claims estimate to determine. Other methods for developing unpaid claim estimates are required to derive the pure IBNR when using report year data."
First, for claims-made policies there is no (pure) IBNR, so that would not apply if we followed the CAS Examiner's Report and focused on claims-made policies. Second, for occurrence policies, one can estimate IBNR by analyzing a sufficient number of lags for old report years. Also one can estimate IBNER by analyzing a single RY/lag combination evaluated as of different points in time.

Section 4, Exposures

Some Examples of Exposure Bases:[160]

Automobile: car-years.
Workers Compensation: $100 of payroll.[161]
Homeowners: House Years.[162]
General Liability: Sales (mercantile or manufacturing) or payroll (contracting).

Desirable Properties of an Exposure Base:

1. Proportional to Expected Loss
2. Practical
3. Historical Precedence

Proportional to Expected Loss:

Exposure base implies a uniform and continuous multiplicative relationship between the variable and the expected losses.

For homeowners insurance, homes with a larger number of occupants have a somewhat larger expected loss, all else being equal. However, the expected loss is not proportional to the number of occupants. For example, a home with 5 occupants might have expected losses 10% higher than an otherwise similar home with 2 occupants, rather than 2.5 times the expected losses. Thus, while one might use number of occupants (residents) as a rating variable or an underwriting criterion, one should not use it as an exposure base for homeowners.

The exposure base should be responsive to any change in exposure to risk.

If a contractor hires more workers, his firm is doing more work, and the possibility of a general liability claim increases.[163] His payroll would increase due to the additional workers. The exposure base of payroll is responsive to the change in his exposure to risk.

The exposure base is not a rating variable, although the dividing line between the two is somewhat arbitrary at times.

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[160] See Table 4.1 in Basic Ratemaking.
[161] Payroll is limited for officers and sole proprietors, and partners.
In at least one state, manweeks are used rather than payroll.
[162] Insurance Years ($100 or $1000 of value insured for 1 year) is also used.
[163] The example could equally well deal with the same workers working more hours per week.

For example, in automobile insurance, car years is the usual exposure base. Insure 2 similar cars and we charge you twice as much.[164] Insure 3 similar cars and we charge you three times as much as for one car.

We might use age as a rating variable for automobile insurance. However, we would not charge a 40 year old driver twice as much as a 20 year old driver.[165] If we employ business versus pleasure use as a rating variable, there is nothing inherently numerical about these discrete categories.

<u>Practical</u>:

- Objective and well defined
- Relatively easy and inexpensive to obtain
- Relatively easy and inexpensive to verify

This will allow the exposure base to be:

- Consistently measured
- Not subject to manipulation by insureds, agents, or underwriters.

For example, self-reported miles driven would be subject to manipulation. Self-reported miles driven is not a practical exposure base.[166]

For products liability insurance, the exposure to loss is approximately proportional to the number items in use. Since this is hard to track or estimate, the number of items in use is not a practical exposure base. Instead current sales are used. This can result in problems when the level of sales changes significantly over time.[167]

For example, the Acme Ladder Company sold an average of 100,000 ladders per year during the 1990s. Since then, they sold only an average of 50,000 ladders per year. Ladders remain in use for many years. Therefore, we expect some current accidents while using ladders that were sold in the 1990s. Therefore, in this case, the current lower level of sales underestimates the exposure to loss.[168]

[164] *In some situations the insured might be given a multi-car discount and for example might pay only 1.95 as much.*
[165] In fact, we would probably charge the 40 year old driver less.
[166] Using odometer readings would still be subject to some manipulation. A GPS tracking and recording system on a longhaul truck can be used to determine miles driven, resulting in a practical exposure base.
[167] This problem could be referred to as “Temporal Mismatch.”
See “Exposure Bases Revisited,” by Amy S. Bouska, PCAS 1989, formerly on the syllabus.
[168] This potential problem will exists whether occurrence or claims-made policies are sold. There are current accidents which are due to old sales. Coverage is provided based on either when the accident occurred or the claim is made, not when the ladder was sold.

Historical Precedence:

Exposure bases are rarely changed once they have been established.

Potential temporary problems caused by changing the exposure base:

- Large swings in premiums for individuals
- Changes in the rating procedures (computer programs, etc.)
- Can no longer directly use historical exposures for ratemaking

Therefore, one would only change an exposure base if one thought the new base was significantly better.

For example, the Workers Compensation exposure base used to be payroll limited to $300 per week. The switch to "unlimited payroll" made it difficult to estimate classification rates for several years.[169] [170]

New exposures increased compared to historical exposures, but classes with more high wage workers were affected more. New rates by class were estimated that would be appropriate to multiply by the unlimited payroll by class.[171]

However, these class rates were less accurate than if there had been no change in exposure base. Also individual insureds with certain mixes of classes would have had very significant changes in their premiums just due to the change in exposure base.

Therefore, when exposures bases are changed, a transition program is important. For example, during the first few years, one might have averaged premiums computed based on the new exposure base and rates with premiums computed based on the old exposure base and comparable rates.

After several years, data with the new exposure base will have been collected. At that point, one would get the gain from the new exposure base, after suffering the pain during the transition.

One would run into more severe transition problems if one switched the Workers Compensation exposure base from payroll to manweeks (hours worked).

For Automobile Insurance, one would have transition problems if one switched the exposure base from car years to miles driven.

[169] This switch was mostly made in the 1970s, on a state by state basis.
[170] Payroll is limited for officers and sole proprietors, and partners.
[171] The overall rate indication in Workers Compensation is based on loss ratios and does not use exposures.

Written Exposures:

Written Exposures are the exposures insured by a policy.

An annual automobile policy that insures one car, has one car year.
A six-month automobile policy that insures one car, has a half of a car year.
A six-month automobile policy that insures three cars, has one and a half car years.

All of the exposures written on a policy go into the calendar year, calendar quarter, or policy year in which the policy effective date falls.

Earned Exposures:

Coverage is provided under an insurance policy for a period of time. For an annual policy written December 1, at the end of the year only 1/12 of the coverage was provided. If this policy covers one car, then at the end of the year 1/12 of a car year has been earned.

Earned Exposures are the portion of exposures for which coverage has been provided by a certain date.

Exercise: An annual policy covering one home is written with effective date March 1, 2008.
What does it contribute in written and earned exposures to different Calendar Years?
[Solution: 1 written house year to Calendar Year 2008, and no written house years to Calendar Year 2009. 5/6 earned house year to Calendar Year 2008, and 1/6 earned house years to Calendar Year 2009.]

While for Calendar Years of data written and earned differ, for Policy Years of data they are the same at ultimate.

Exercise: An annual policy covering one home is written with effective date March 1, 2008.
What are the Policy Year written and earned exposures as of December 31, 2008?
What are the Policy Year written and earned exposures as of December 31, 2009?
[Solution: As of December 31, 2008, 1 written house year to Policy Year 2008.
As of December 31, 2008, 5/6 earned house year to Policy Year 2008.
As of December 31, 2009, 1 written house year to Policy Year 2008.
As of December 31, 2009, 1 earned house year to Policy Year 2008.]

Of course one can add up the contributions of many policies. For example, one might aggregate the exposures for all of an insurer's policies for a certain line of insurance in certain given state. One might aggregate the exposures separately by class and territory.

Unearned Exposures:

Unearned Exposures are the portion of exposures for which coverage has not been provided by a certain date.

Exercise: An annual policy covering one home is written with effective date March 1, 2008.
What are the unearned exposures as of December 31, 2008?
What are the unearned exposures as of December 31, 2009?
[Solution: 1/6 unearned house year as of December 31, 2008.
No unearned house years as of December 31, 2009.]

For an individual policy at any given point in time:
Written Exposures = Earned Exposures + Unearned Exposures.

Inforce Exposures:

Inforce Exposures are the number of exposures for which coverage is being provided at a given point in time.

The 19th Century Insurance Company provides homeowners insurance. On July 23, 2009, if all the homes they insure in a certain state were destroyed, they would have to pay to replace 10,000 homes. Then, as of July 23, 2009, they have 10,000 inforce exposures in this state.

This would be 10,000 inforce homes; inforce exposures have no time duration attached to them. Inforce exposures represent a snapshot of the insurer's book of business at a given point in time.

The 19th Century Insurance Company also provides private passenger automobile insurance.
On July 23, 2009, they have 50,000 semi-annual policies in effect, covering a total of 70,000 cars.
If all the cars they insure were destroyed, they would have to pay to replace 70,000 cars. As of July 23, 2009, they have 70,000 inforce exposures in this state.[172]

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[172] There are differences in definition between different insurers. According to Basic Ratemaking,
"Most companies define insured units to be the count of items exposed to loss at a given point in time. For example, if an automobile policy insures three cars, that one policy could contribute three in-force exposures at a given point in time. Alternatively, some companies may define insured unit in terms of the number of policies (the auto example above would have one in-force exposure under this definition) or the written exposures (in the auto example, there could be three in-force exposures if the term is annual, or 1.5 in-force exposures if the term is semiannual)."
In this example, some insurers would say the inforce exposures were the number of policies 50,000, the number of semi-annual policies divided by two or 25,000, or the number of cars divided by two or 35,000.

Cancelations:

An annual policy covering one car is written with effective date September 1, 2009.
If the policy is canceled on December 1, 2009, then only 3 months of coverage was provided, and there is 1/4 car year contributed to both written and earned exposures for Calendar Year 2009.

If instead the policy is canceled on March 1, 2010, then only 6 months of coverage was provided. However, at the end of 2009 we would not know that the policy would be canceled. Thus there would be 1 car year contributed to Calendar Year 2009 written exposures and -1/2 car year contributed to Calendar Year 2010 written exposures. The total written exposures add up to the correct 1/2 car year. 1/3 of a car year is contributed to the earned exposures for Calendar Year 2009, and 1/6 of a car year is contributed to the earned exposures for Calendar Year 2010, for a total of 1/2 car year.

Endorsements:

An annual policy covering one car is written with effective date September 1, 2009.

If the policy is endorsed on December 1, 2009 to add another car, then the second car will be provided with only 9 months of coverage. This policy contributes 1.75 car years to Calendar Year 2009 written exposures. From the first car, there are 4/12 car years earned in CY09 and 8/12 car years earned in CY10. From the second car, there are 1/12 car years earned in CY09 and 8/12 car years earned in CY10. Thus this policy contributes 5/12 car years to CY09 earned exposures and 16/12 car-years to CY10 earned exposures, for a total of 1.75 car years.

Car	CY09 Written	CY10 Written	CY09 Earned	CY10 Earned
1	1	0	4/12	8/12
2	0.75	0	1/12	8/12
Total	1.75	0	5/12	16/12

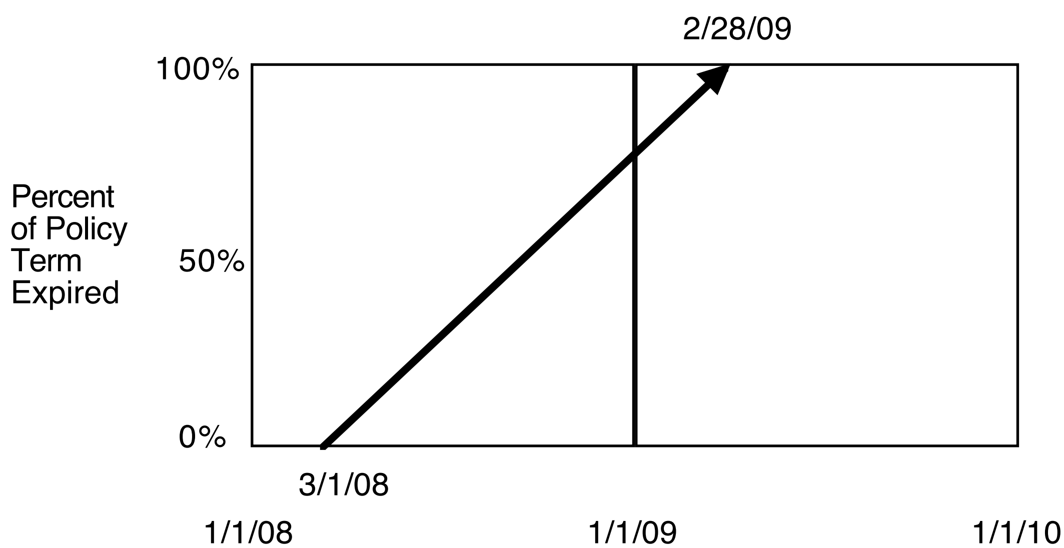
If instead the policy is endorsed on March 1, 2010 to add another car, then the second car will be provided with only 6 months of coverage. As of the end of 2009 we would not know that this policy would be endorsed. Therefore, this policy contributes 1 car year to Calendar Year 2009 written exposures and 1/2 car year to Calendar Year 2010 written exposures. From the first car, there are 4/12 car years earned in CY09 and 8/12 car years earned in CY10. From the second car, there are 6/12 car years earned in CY10. Thus this policy contributes 4/12 car years to CY09 earned exposures and 14/12 car years to CY10 earned exposures, for a total of 1.5 car years.

Car	CY09 Written	CY10 Written	CY09 Earned	CY10 Earned
1	1	0	4/12	8/12
2	0	1/2	0	6/12
Total	1	1/2	4/12	14/12

Diagrams:

One can draw diagrams to represent exposures and premium. Such diagrams are the basis of the parallelogram method of putting premiums on-level.[173] Some people will find them helpful for dealing with exposures.

For example, let us assume an annual policy is written with an effective date of March 1, 2008. Then coverage runs from 3/1/08 to 2/28/09. We can represent this in the following diagram, with time on the horizontal axis, and the percent of the policy term that has expired on the vertical axis:



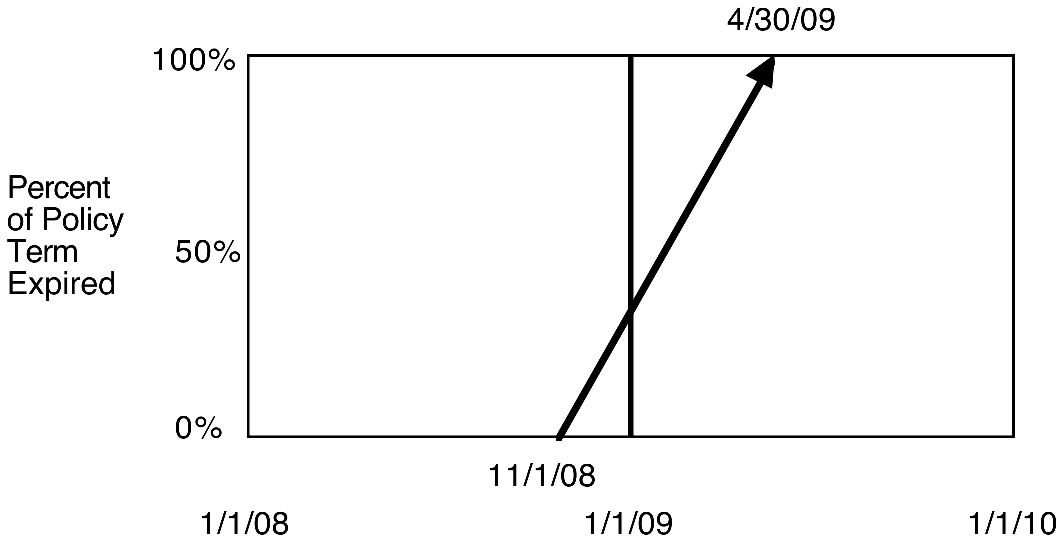
The lefthand square represents Calendar Year 2008 earned exposures, while the righthand square represents Calendar Year 2009 earned exposures. The arrow corresponds to the policy written 3/1/08. Since 5/6 of the length of the arrow is in the lefthand square, 5/6 of the exposures from this policy are earned during Calendar Year 2008. The remaining 1/6 of the exposures from this policy are earned during Calendar Year 2009.

Since this is an annual policy, the expiration date is one year after the inception date.
The slope of the line representing an annual policy is one.

We have to take into account the number of exposures written on each policy. If the 3/1/08 annual policy covered one car, then it would contribute 5/6 of a car year to Calendar Year 2008 and 1/6 of a car year to Calendar Year 2009. If instead the policy had covered two cars, then it would contribute 5/3 of a car year to Calendar Year 2008 and 1/3 of a car year to Calendar Year 2009.

[173] To be discussed in my section on Premiums.

Exercise: Draw a similar diagram representing a six-month policy written 11/1/08.
[Solution: Since this is a six month policy, the expiration date is half a year after the inception date.

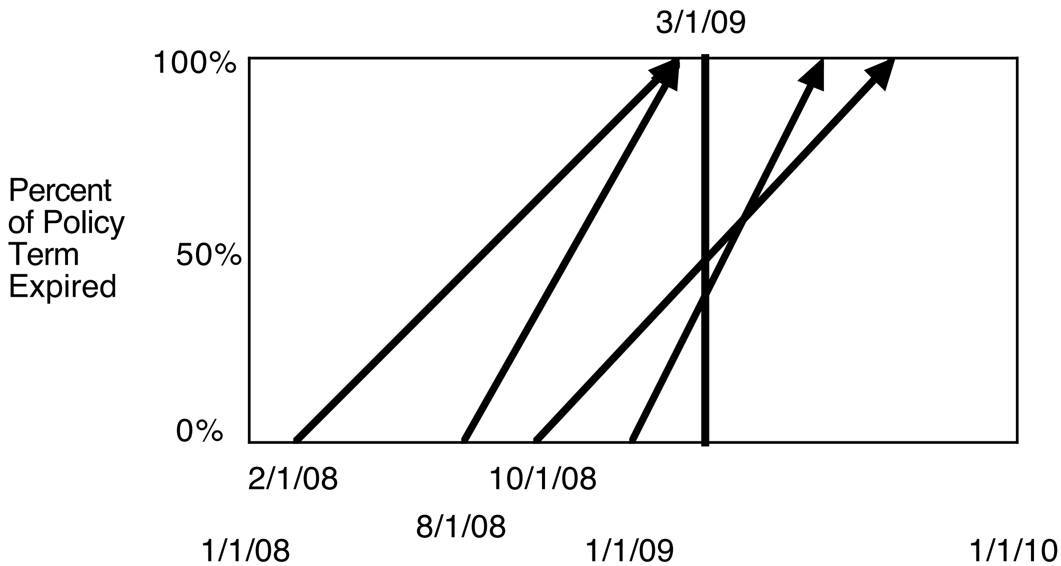


Comment: Since 1/3 of the length of the arrow is in the lefthand box, 1/3 of the exposures from this policy are earned during Calendar Year 2008. The remaining 2/3 of the exposures from this policy are earned during Calendar Year 2009.]

The slope of the line representing a six month policy is two.
In general, the slope of the line representing a policy is one over its term.

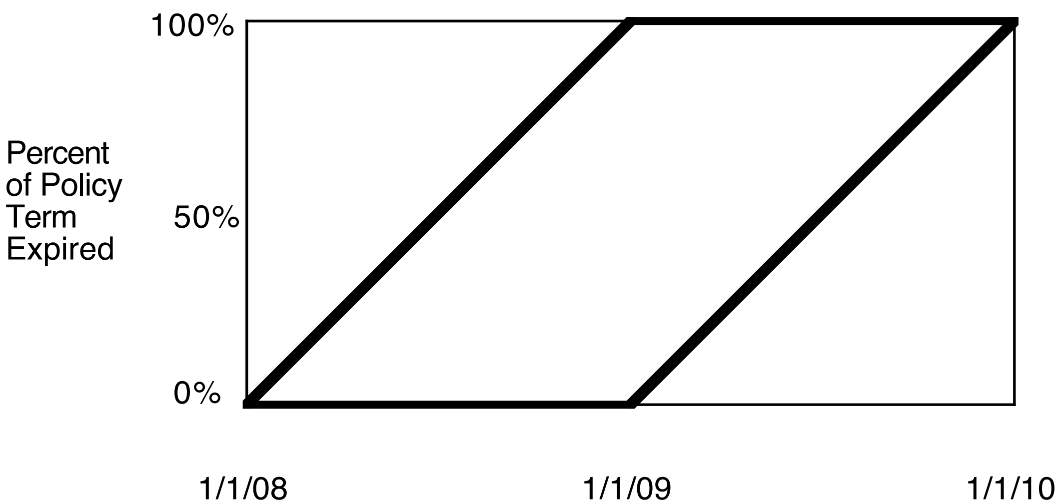
Inforce exposures can be represented by a vertical line on the given date. If the sloped line representing a policy touches the vertical line, then it is inforce on the given date.

Exercise: Annual policies are written 2/1/08 and 10/1/08.
Six-month policies are written on 8/1/08 and 1/1/09.
Draw a diagram to help determine the inforce exposures on 3/1/09.
[Solution: The 10/1/08 annual policy and 1/1/09 six month policy each contribute to the 3/1/09 inforce exposures.



Comment: If each policy covered one car, then the inforce exposures on 3/1/09 would be 2 cars.]

While a Calendar Year is represented by a square, a Policy Year is represented by a parallelogram. For example, Policy Year 2008 is the parallelogram below:



Exposure Trend:

Certain exposure bases are inflation sensitive. For example, sales or payroll are expected to increase with inflation.[174] Therefore, prior to being used to get an overall rate indication for General Liability Insurance or Workers Compensation, such exposures bases are adjusted from the past inflation level to the inflation level expected in the future. An exposure trend factor will be applied to the historical data.

For example, assume that Workers Compensation Policy Year 2010 exposures (payrolls) were \$100 million.[175] We are trying to be make new rates to be effective during Policy Year 2013. We think that exposures will increase due to inflation by 2% per year.

Then we project the reported exposures for three years of inflation:
(\$100 million) (1.02^3) = \$106.1 million.

Audits:

Certain exposure bases such as sales or payroll are usually subject to audit.[176]

For example, when written a certain Commercial General Liability policy assumed \$80 million in sales for purposes of determining the preliminary premium. Sometime after expiration of the policy the actual sales during the policy period are determined and used to calculate the final premium.[177] The actual sales will turn out to be different than \$80 million.

Such audits can have a number of effects on exposures.

A policy written in one year can contribute to the Calendar Year written exposures for the next year. For example, a policy is written March 1, 2010, with an initial estimate of sales of \$80 million. At final audit on July 1, 2011, the sales are determined to be \$82 million. Then \$2 million of written exposures are contributed to Calendar Year 2011. If instead the audited sales were \$77 million, then this policy would contribute -\$3 million of written exposures to Calendar Year 2011.

Policy Year written exposures take a while to be final; they develop.
For example, the audits of the policies written late in 2010 will not have been completed until sometime in early 2012. Thus Policy Year 2010 written exposures will not be final until then.

[174] Sales or payroll will also change as due to economic cycles and the changes in business done by particular insureds.
[175] Premiums would be put on the current rate level and developed to ultimate as discussed in my section on Premiums.
[176] In the case of Workers Compensation, the insurer would determine the payroll by state and classification.
[177] There may be audits every quarter or more frequently, in addition to the final audit.
The larger the insured, the more frequent the audits tend to be.

<u>Composite Rating</u>:[178]

Composite rating can be applied to <u>large</u> insureds. Instead of rating different lines of insurance using different exposure bases, a single exposure base is used, such as sales, payrolls, etc.

There are two different forms of composite rating. In the first form of composite rating, the composite rate is based on the manual rates for the different lines of insurance.

For example, let us assume the Winfred-Lauder Department Stores have the following data for the most recent 12 months available, extending by the current manual rates:

Line of Insurance	Exposures	Manual Premium
Auto Liability	40 vehicles	\$80,000
CGL Premises and Operations	\$500 million in sales	\$600,000
CGL Products and Completed Operations	\$500 million in sales	\$200,000
Burglary	\$300,000	\$10,000

The total manual premium is: \$890,000.
If we use the special exposure base of \$1000 of sales, then there were 500,000 units.
The composite rate per unit is: \$890,000/500,000 = \$1.78.
Thus if at audit the sales turned out to be \$600 million, the premium would be:
(600,000)(\$1.78) = \$1,068,000.[179]

In the second form of composite rating, called “loss rating”, the composite rate is based on the insureds recent experience. This will be discussed in my section on “Commercial Lines Rating Mechanisms.”

[178] See page 51 of <u>Basic Ratemaking</u> by Modlin and Werner.
[179] One could also apply experience rating, schedule rating, and/or retrospective rating.
If retrospective rating is not applied, one could apply a premium discount.

Problems:

4.1. (3 points) For each of the following lines of insurance, give a typical exposure base:
Personal Automobile
Homeowners
Workers Compensation
Commercial General Liability (list four exposure bases)
Commercial Business Property
Physician's Professional Liability
Professional Liability Personal Articles Floater

4.2. (4 points) You are given the following exposures written by quarter during 2011 and 2012.

Quarter	Written Exposures
First 2011	11,000
Second 2011	10,000
Third 2011	11,000
Fourth 2011	12,000
First 2012	15,000
Second 2012	12,000
Third 2012	13,000
Fourth 2012	14,000

Determine the number of exposures earned in Calendar Year 2012.
Show all work.
(a) (2 points) Assume that all policies are annual.
(b) (2 points) Assume that all policies are six-month.

4.3. *(1 point) Which of the following correctly identify the line of business with its common exposure base?*

Line of Business	*Exposure Base*
1 Burglary and theft	*Insured value in thousands of dollars*
2. Boiler and machinery	*Number of objects*
3. Fidelity	*Contract cost*
4. Credit	*Dollars of indebtedness*

4.4. (6 points) You are given the following caryears written by month during 2010.

Month	Exposures (caryears)
January	3000
February	600
March	600
April	1200
May	600
June	600
July	1800
August	600
September	600
October	1200
November	600
December	600

Determine the number of caryears earned in Calendar Year 2010.
Show all work.
(a) (3 points) Assume that all policies are annual.
(b) (3 points) Assume that all policies are six-month.

4.5. *(1 point) According to Bouska in "Exposures Bases Revisited", which of the following statements correctly reflect the history of the man hours versus payroll issue for the workers compensation exposure base?*

1. In the early 1980s, the exposure base for workers compensation in most states was changed from limited payroll to unlimited payroll in order to partially alleviate the inequities resulting from differing wage scales by employers in the same industry.

2. In the mid-1980s, the perceived inequity resulting from the workers compensation exposure base became a matter of national debate not only because of the union vs. nonunion wage differentials, but also because of the varying wage scales that appeared as a result of deregulation in many industries.

3. The NCCI's analysis of the Oregon data in its "Study of Premium Equity by Employer Groups" found no bias against either union or high wage paying employers among the small and medium sized employers, but it did show that high wage paying and union employers in the large sized group developed lower loss costs per premium dollar.

4.6. *(1 point) Which of the following have served as partial solutions to the perceived inequity for the high wage paying employers in workers compensation?*

1. In one state, each of several construction classes was split into two new classifications (high and low wage rates.)

2. In one state, a table of credits based on wage rates was implemented for all contracting classes.

3. In one state, a program was implemented for specific construction classes, which made the experience rating plan more responsive to the individual employer's three-year loss ratio.

4.7. *(1 point) For ISO's revised [i.e., post-1987] Commercial Lines rating plan, match the exposure bases used for premises/operations coverage with the industry groups:*

Exposure Bases: X. Gross sales Y. Payroll

Industry Groups: 1. Manufacturing 2. Contracting 3. Mercantile

4.8. (1 point) Which of the following statements is true?

1. One should review the exposure base used for a line of insurance at least every decade.
2. The widespread use of computers, now allows exposure bases to be changed without much difficulty.
3. When exposures bases are changed, a transition program is important.

∗**4.9**∗**.** (2 points) Briefly describe how the reporting of payrolls as an exposure base may be affected by the underwriting cycle.

Briefly discuss how this might affect the data used for ratemaking.

∗**4.10**∗**.** (5 points) Per 1 million widgets sold in year X, the expected number of products liability claims is by year of occurrence:

Year of occurrence:	X	X+1	X+2	X+3	X+4	X+5
Number of claims:	5	5	4	3	2	1

On average half of the claims are reported in their year of occurrence, while the other half are reported in the following year.

(a) (1/2 point) ABC Widget Company sold 10 million Widgets in 2006, and every year for the last ten years.

What is the expected number of products liability claims from ABC that will occur in 2006?

(b) (1/2 point) What is the expected number of products liability claims from ABC that will be reported in 2006?

(c) (2 points) XYZ Widget Company has the following sales figures in millions of widgets:

Year:	1997	1998	1999	2000	2001	2002	2003	2004	2005	2006
Sales:	19	18	17	16	15	14	13	12	11	10

What is the expected number of products liability claims from XYZ that will occur in each of 2005 and 2006?

(d) (1/2 point) What is the expected number of products liability claims from XYZ that will be reported in 2006?

(e) (1.5 points) Briefly discuss the choice of an exposure base for products liability in light of your answers to the previous parts of this questions.

4.11. (3 points) Given the following information:

- All policies have a six-month policy term.
- There were 100,000 car years written during calendar year 2013.
- There were 104,000 car years written during calendar year 2014.

a. (1 point) Estimate the earned car years for calendar year 2014.

b. (1 point) All policies written from January 1, 2015 onwards have an annual policy term.

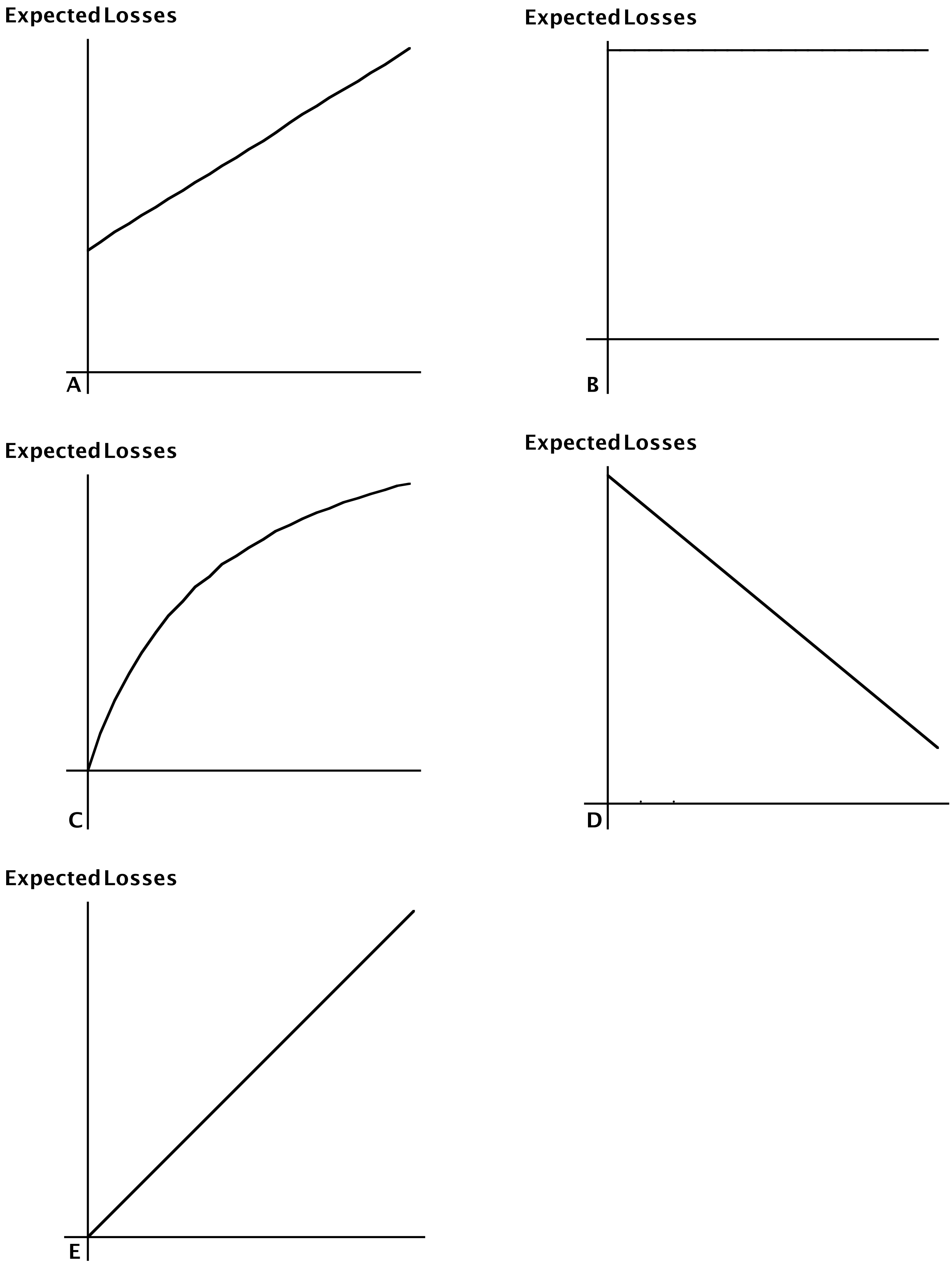
Time Period	Written Car Years
January 1 through June 30, 2015	100,000
July 1 through December 31, 2015	16,000

Estimate the earned car years for calendar year 2015.

c. (1 point) Estimate the retention ratio.

Assume no cancellations or endorsements.

<u>4.12</u>. (1 point) The expected losses as a function of 5 different quantities:



Which of these quantities would best serve as an exposure base for this line of insurance?

4.13. (3 points) The following six policies are written:

Policy Effective Date	Policy Term	Number of Automobiles Covered
September 1, 2004	6 months	1
October 1, 2004	12 months	2
January 1, 2005	12 months	3
June 1, 2005	6 months	1
August 1, 2005	6 months	2
December 1, 2005	12 months	1

As of December 31, 2005, determine the following: Calendar Year 2005 Written Exposures, Calendar Year 2005 Earned Exposures, Unearned Exposures, and the Inforce Exposures.

∗**4.14**∗**.** (2 points) Consider the following table of data regarding number of auto policies, each covering a single car, and policy effective and expiration dates:

Number of Policies	Effective Date	Expiration Date
200	7/1/2001	12/31/2001
100	7/1/2001	6/30/2002
100	10/1/2001	3/31/2002
200	1/1/2002	6/30/2002
100	1/1/2002	12/31/2002
100	4/1/2002	9/30/2002
250	7/1/2002	12/31/2002
100	7/1/2002	6/30/2003
200	10/1/2002	3/31/2003

How many exposures (car-years) are earned and written in calendar year 2002?

4.15. *(1 point) According to “Ratemaking” by McClenahan, the exposure unit used for a given type of insurance depends upon which of the following factors?*

1. Reasonableness: The exposure unit should be a reasonable measure of the exposure to loss.

2. Ease of Determination: The exposure base should be subject to accurate determination.

3. Stability: The exposure measure should be stable over time.

4. Historical Practice: Changes in the exposure base could render any prior history unusable, which may hinder current ratemaking.

4.16. (2.5 points) Each of five annual personal automobile policies covers one car.
Given the following activity on these policies:

Policy	Effective Date	Original Expiration Date	Mid-term Cancellation Date
1	March 1, 2014	February 28, 2015	N/A
2	August 1, 2014	July 31, 2015	April 30, 2015
3	January 1, 2015	December 31, 2015	N/A
4	April 1, 2015	March 31, 2016	October 30, 2015
5	October 1, 2015	September 30, 2016	July 31, 2016

The exposure base is earned car years.
a. (0.5 point) Calculate the 2015 calendar year written exposure.
b. (0.5 point) Calculate the 2015 calendar year earned exposure.
c. (0.5 point) Calculate the 2014 policy year written exposure.
d. (0.5 point) Calculate the 2015 policy year written exposure as of June 30, 2016.
e. (0.5 point) Calculate the in-force exposure as of May 1, 2015.

4.17. (2.25 points) Given the following 6-month automobile policies each of which covers one car:

Effective Date	Number of Policies
January 1, 2017	250
April 1, 2017	200
July 1, 2017	230
October 1, 2017	220
January 1, 2018	300
April 1, 2018	260
July 1, 2018	290
October 1, 2018	270

- Assume that no policies are canceled midterm.

a. (0.5 point) Calculate the written car-years for calendar year 2018.
b. (0.25 point) Calculate the in-force cars as of December 31, 2017.
c. (0.5 point) Calculate the earned car-years for calendar year 2018.
d. (0.5 point) Calculate the written car-years for the fiscal year ending June 30, 2018.
e. (0.5 point) Calculate the earned car-years for the fiscal year ending June 30, 2018.

4.18. (1 point) Briefly discuss the pros and cons of using gallons of gas as an exposure base for private passenger automobile liability insurance.

4.19. (1 points) An insurance company portfolio consists of the following:
- Each policy covers one home.
- 500 two-year policies with an effective date of June 1, 2015.
- 600 two-year policies with an effective date of June 1, 2016.
- 700 two-year policies with an effective date of June, 2017.
- 1000 one-year policies with an effective date of October 1, 2015.
- 1100 one-year policies with an effective date of October 1, 2016.
- 1200 one-year policies with an effective date of October 1, 2017.

a. (0.25 points) Calculate the written exposures for calendar year 2017.
b. (0.75 points) Calculate the earned exposures for calendar year 2017.

***4.20*.** (4.5 points) Given the following quarterly exposure information:

Calendar Year and Quarter	Written Exposures
2018 Q 1	1000
2018 Q 2	3000
2018 Q 3	4000
2018 Q 4	2000
2019 Q 1	1200
2019 Q 2	3500
2019 Q 3	5000
2019 Q 4	2500

- The insurer started writing business on January 1, 2018.
- The insurer stops writing business on December 31, 2019.
- All policies are annual.
- All policies are written on the first day of the quarter.
- There are no policy cancellations and no mid-term adjustments.
- The quarterly earnings pattern was set by analyzing historical experience across the industry and is not uniform. For each policy, regardless of in which quarter it is written, it earns 15% in Q1 of a year, 30% in Q2 of a year, 35% in Q3 of a year, and the remaining 20% of its written exposures in Q4 of a year.

(a) (1 point) Determine the earned exposures for each quarter of CY 2018.
(b) (1.5 points) Determine the earned exposures for each quarter of CY 2019.
(c) (1 point) Determine the earned exposures for each quarter of CY 2020.
(d) (0.5 point) Calculate the 2018 policy year earned exposures as of March 31, 2019.
(e) (0.5 point) Calculate the unearned exposures as of the end of 2019.

4.21. (6, 5/92, Q.53) *(3 points) In "Exposure Bases Revisited", Bouska discusses Causes and Controversy Involved in Changing Exposure Bases.*
(a) (1 point) What are the three desirable traits of an exposure base?
(b) (1.5 points) Discuss the issues surrounding Workers Compensation with regard to using hours worked versus payroll.
(c) (1/2 point) What were the findings of the special study made in Oregon regarding this issue?

4.22. (6, 5/93, Q.6) (1 point) Consider the following table of data regarding number of auto policies, each covering a single car, and policy effective and expiration dates:

Number of Policies	Effective Date	Expiration Date
100	1/1/92	6/30/92
150	4/1/92	9/30/92
200	7/1/92	12/31/92
250	10/1/92	3/31/93

How many exposures(car-years) are earned and written in calendar year 1992?

4.23. (6, 5/93, Q.36) *(3 points)*
a. (1 point) According to Bouska, "Exposure Bases Revisited," what is the distinction between a rating variable and an exposure base?
b. (2 points) Describe the three types of problems encountered in the selection of an exposure base and give an example of each type.

4.24. (6, 5/93, Q.51) (3 points) American Galactic Insurance started a new insurance product in 1993. The annual written exposures are projected to be 1000x from x = 0 to x = 3 and level thereafter with x = 0 at 1/1/93 and x = 3 at 1/1/96.
All policies are six month policies.
Calculate the projected percentage increase in earned exposures from calendar year 1994 to calendar year 1995.

4.25. (3 points) In the previous question, 6, 5/93, Q.51, determine each of the following.
(a) (1 point) Earned exposures for calendar year 1993.
(b) (1.5 points) Earned exposures for calendar year 1996.
(c) (0.5 points) Earned exposures for calendar year 1997.

4.26. (6, 5/94, Q.8) (1 point) For a new line of business:

- All policies are annual policies.
- The annual written exposure units at time x is 1000 x^2 for x = 0 to x = 1.

Determine the earned exposures in the first year for a new line of business.
Comment: I have rewritten this past exam question to match the current syllabus.

4.27. (6, 5/94, Q.11) (1 point) Which of the following are true?
1. For a given line of business more than one medium should be selected as the exposure base.
2. An exposure base should have a uniform multiplicative relationship with all expected loss costs and rates.
3. The exposure base recognizes the average situation in exposure to loss.

4.28. (6, 5/94, Q.40) *(2 points) Bouska, "Exposure Bases Revisited," describes proper selection of exposure bases.*
(a) (1 point) Describe three types of problems in the selection of an appropriate exposure base.
(b) (1 point) Identify two serious problems with exposure bases arising out of insurance company practices.

4.29. (6, 5/94, Q.49) (1.5 points) (a) (1 point) Give two reasons for using total payroll as the exposure base for Workers Compensation.
(b) (1/2 point) Give one reason against using total payroll as the exposure base for Workers Compensation medical benefits.

4.30. (6, 5/95, Q.3) (1 point) Which of the following are true?
1. Inflation-sensitive exposure bases increase the need for frequent rate filings.
2. In a steady state, rates eventually respond to a systematic underestimation of the exposures.
3. The exposure base is <u>not</u> the true exposure to loss.

4.31. (6, 5/95, Q.5) (1 point) Unique Insurance Company starts writing a new line of business. In the first year of writing the insurance, it finds that the annual rate of written exposures at time "x" is 1000x. All policies are one-year policies.
Determine the unearned exposure at the end of the first year.

4.32. (6, 5/95, Q.36) (3 points) According to McClenahan, chapter 3, “Ratemaking,” Foundations of Casualty Actuarial Science, the specific exposure unit used for a given type of insurance should depend on several factors.
(a) (2 points) List and briefly describe the four factors he discusses.
(b) (1 point) Based on the four factors in (a), discuss the use of the following exposure units for automobile ratemaking:
1) car years
2) miles driven per year

4.33. (6, 5/96, Q.27) (3 points)
(a) (2 points) List four desirable characteristics of exposure bases.
(b) (1 point) Discuss how well unlimited payroll, as an exposure base for Workers Compensation, satisfies each of the four desirable characteristics referred to in (a) above.

4.34. (6, 5/96, Q.29) (4.5 points) You are given:

- Annual rate of written exposures at time x equals 1000x.
- x = 0 at 1/1/94
- All policies are annual.

(a) (1.5 points) Determine the earned exposures for calendar year 1995, assuming annual writings continue throughout 1995.
(b) (1.5 points) Determine the earned exposures for calendar year 1995. assuming no new policies are issued after June 30, 1995.
(c) (1.5 points) Determine the earned exposures for calendar year 1995, assuming no new policies are issued after June 30, 1995 and all in-force policies are canceled June 30, 1995.
Note: I have rewritten this past exam question in order to match the current syllabus.

∗**4.35**∗**. (6, 5/97, Q.25)** (4 points) (b) (2 points) Describe the process of selecting an exposure base for a line of insurance.
(c) (2 points) The standard exposure bases are often not used for large risks. Briefly describe two alternative rating plans used for large risks that modify the usual exposure base.
Note: Part (a) of this question referred to the "Statement of Principles Regarding Property and Casualty Ratemaking."

4.36. (6, 5/98, Q.3) (1 point) Which of the following are true?
1. The factor selected as the exposure base should have a uniform multiplicative relationship with all of the expected loss costs and rates.
2. Although it makes the plan more difficult to use, the complexity of the exposure base is likely to enhance its perceived equity.
3. For policies subject to audit, systematic underestimation of the exposure may eventually lead to excessive rates.

4.37. (6, 5/98, Q.45) *(3 points) Bouska, in "Exposure Bases Revisited," discusses three types of problems that are encountered in the selection of the exposure base.*
a. (1.5 points) Briefly describe the three types of problems.
b. (1.5 points) For each of the three problems identified in part (a), select one of the following policy types and explain how that policy type highlights the problem.

- *Products Liability Insurance*
- *Pollution Liability Insurance*
- *Directors and Officers Insurance.*

4.38. (6, 5/99, Q.14) (1 point) Which of the following statements are true?
1. The factor selected as the exposure base should have a uniform multiplicative relationship with all of the expected loss costs and rates.
2. If a large risk is loss-rated, the exposure base is the risk itself.
3. Triggering products liability coverage on the date of injury, rather than the date of sale, gives rise to a temporal mismatch.

4.39. (6, 5/99, Q.43) (2 points)
a. (1 point) What is the first step in developing an exposure base for a new and unique insurance product?
b. (1 point) What are the two reasons why the uniform multiplicative relationship between the exposure base and the expected loss costs is modified in calculating the premium?

4.40. (5, 5/00, Q.43) *(3 points) Bouska, "Exposure Bases Revisited," lists 3 types of problems in selecting an exposure base that will appropriately reflect the potential for future losses. Describe and give an example of each of the 3 problems.*

4.41. (5, 5/01, Q.5) *(1 point) According to Bouska, “Exposure Bases Revisited,” which of the following reasons is given for ISO’s changing of General Liability exposure base from area to receipts?*
A. Sensitivity to inflation
B. Eliminated interpretive mismatch
C. Receipts provide a more uniform exposure base to losses than area
D. Eliminated temporal mismatch
E. Eliminated sensitivity to economic cycles

4.42. (5, 5/01, Q.6) (1 point) Which of the following is a serious problem that can arise from insurance company practices in the application of exposure bases.
A. Different insurers use different exposure bases, creating confusion in the market.
B. Exposure estimates can be and are manipulated in response to the competitive situation.
C. Exposure bases change frequently, which creates distortions in ratemaking analyses.
D. Geographic and social differences across the U.S. make exposure bases less relevant in some areas relative to others.
E. None of A, B, C, D are true.

4.43. (5, 5/02, Q.9) (1 point) Which of the following statements is true?
A. The factor selected as the exposure base should generally have a uniform multiplicative relationship with all the expected loss costs and rates.
B. The exposure base is considered a rating variable since the dividing line between the two may be somewhat arbitrary at times.
C. The failure to accurately predict the frequency and/or severity of future losses is usually a sign of a failing exposure base.
D. For a given line of business more than one medium should be selected as the exposure base.
E. None of A. B, C, D are true.

4.44. (5, 5/02, Q.38) *(1 point) Bouska, "Exposure Bases Revisited," describes a problem she defines as temporal mismatch. Give an example of temporal mismatch that occurs with medical malpractice claims-made policies.*

4.45. (5, 5/03, Q.26) (2 points) A severe mismatch between premiums and losses can be introduced by low exposure estimates.
a. (1 point) Explain when this would typically happen and why.
b. (0.5 point) What is the eventual outcome of the systematic underestimation of exposures in a steady state environment?
c. (0.5 point) What might be the result if this steady state environment abruptly changed and the low exposure estimates stopped?

4.46. (5, 5/03, Q.27) (1 point) Explain why the exposure base is a proxy for the true exposure, using automobile collision coverage as an example.

4.47. (5, 5/04, Q.30) (3.5 points)
a. (1.5 points) State three desirable criteria for an exposure base.
b. (2 points) For each of the lines of business listed below, state a reasonable exposure base and discuss the exposure base considering one of the criteria stated in part a. above.
1. Workers Compensation.
2. Boatowners Insurance.

4.48. (5, 5/05, Q.14) (1 point) A twelve-month policy providing coverage on two automobiles was written on May 1, 2004. A second twelve-month policy providing coverage on one automobile was written on November 1, 2004.
Which of the following represents the exposures as of December 31, 2004?

	Calendar Year 2004 Written Exposures	Calendar Year 2004 Earned Exposures	Inforce Exposures
A.	3.0	1.5	1.5
B.	3.0	1.5	3.0
C.	3.0	3.0	3.0
D.	6.0	1.5	3.0
E.	6.0	3.0	3.0

4.49. (5, 5/05, Q.36) (2 points)
a. (1 point) Briefly explain each of the following two statement.
1. The exposure base is not the true exposure.
2. The exposure base is not a rating variable.
b. (1 point) "Miles driven" is a potential exposure base for Personal Auto Liability Insurance.
Give one reason for and one reason against its use as an exposure base.

4.50. (5, 5/06, Q.24) (2 points) Total payroll is commonly used as an exposure base for Workers' Compensation insurance. For each of the following coverages, briefly explain why total payroll may not be the best exposure base. Suggest an alternative exposure base for each coverage and explain why it would be an improvement over total payroll.
a. (1 point) Workers' Compensation Medical Benefits Coverage
b. (1 point) Workers' Compensation Indemnity Benefits Coverage

4.51. (5, 5/07, Q.33) (1.5 points) Provide one brief argument for and one brief argument against using each of the following exposure bases for worker's compensation coverage:
a. (0.5 point) Hours-worked.
b. (0.5 point) Limited payroll.
c. (0.5 point) Unlimited payroll.

4.52. (5, 5/08, Q.12) (2.5 points) An insurance company plans to sell identity theft insurance related to unauthorized credit card use and needs to select an exposure base.
a. (1.5 points)
Identify and briefly explain three desirable characteristics of an appropriate exposure base.
b. (1.0 point) Identify a possible exposure base for identity theft insurance and evaluate this particular exposure base relative to the characteristics identified in part a. above.

4.53. (5, 5/09, Q.17) (2 points) An insurance company is considering changing the personal automobile exposure base from earned car years to number of miles driven.
a. (1 point) Identify four desirable characteristics of an exposure base.
b. (1 point) Discuss whether or not the change to a miles-driven exposure base should be made, referencing each of the four characteristics identified in part a. above.

4.54. (5, 5/10, Q.16) (2 points)
a. (1 point) Identify and briefly describe two criteria for a good exposure base.
b. (0.5 point) Evaluate "market value of the house" as an exposure base for homeowners insurance using the two criteria identified in part a. above.
c. (0.5 point) Provide two reasons why a change in exposure base may be difficult.

4.55. (5, 5/10, Q.17) (2 points)
Given the following activity on five annual personal automobile policies as of June 30, 2009:

Policy	Effective Date	Original Expiration Date	Mid-term Cancellation Date
1	July 1, 2007	June 30, 2008	N/A
2	October 1, 2007	September 30, 2008	March 31, 2008
3	January 1, 2008	December 31, 2008	N/A
4	March 1, 2008	February 28, 2009	June 30, 2008
5	July 1, 2008	June 30, 2009	N/A

The exposure base is earned car years.
a. (0.5 point) Calculate the 2008 calendar year written exposure.
b. (0.5 point) Calculate the 2008 calendar year earned exposure.
c. (0.5 point) Calculate the 2007 policy year written exposure.
d. (0.5 point) Calculate the in-force exposure as of April 1, 2008.

4.56. (5, 5/11, Q.2) (1.5 points)
An insurer is considering changing the exposure base used to price personal auto from earned car years to annual miles driven. Evaluate the merits of this change based on each of three different criteria of a good exposure base.

4.57. (5, 5/11, Q.3) (1.25 points) Given the following:

- Each policy insures only one car
- Policies are earned evenly throughout the year

Policy	Effective Date	Original Expiration Date	Cancellation Date
A	February 1, 2009	July 31, 2009	
B	May 1, 2009	October 31, 2009	
C	August 1, 2009	January 31, 2010	
D	November 1, 2009	April 30, 2010	January 31, 2010
E	January 1, 2010	June 30, 2010	
F	July 1, 2010	December 31, 2010	

a. (0.25 point) Calculate the written car years in calendar year 2010.
b. (0.25 point) Calculate the written car years in policy year 2010.
c. (0.25 point) Calculate the earned car years in calendar year 2010.
d. (0.25 point) Calculate the earned car years in policy year 2010.
e. (0.25 point) Calculate the number of in-force policies as of January 1, 2010.

4.58. (5, 5/12, Q.2) (1.5 points) An insurance company is considering changing its exposure base for workers compensation from payroll to number of employees. Evaluate the merits of this change based on each of three different criteria of a good exposure base.

4.59. (5, 5/12, Q.3) (1.5 points) Given the following information:

- An insurance company started writing business on January 1, 2011.
- All policies are one-year term.

Policy Effective Dates	Exposures
January 1 through March 31	100
April 1 through June 30	200
July 1 through September 30	300
October 1 through December 31	400

a. (1 point) Calculate the 2011 earned exposures assuming policies are written uniformly during each quarter.
b. (0.5 point) Discuss the appropriateness of the assumption in part a. above given the exposure data.

4.60. (5, 5/13, Q.1) (2 points)
Given the following information for an insurance company that writes 24-month term policies:

Policy Group	Effective Date	Expiration Date	Number of Vehicles
A	January 1, 2010	December 31, 2011	50
B	July 1, 2010	June 30, 2012	100

All policies within each group have the same effective date.
a. (0.5 point) Calculate the earned car-years for calendar year 2011.
b. (0.5 point) Calculate the earned car-years for policy year 2010 evaluated
as of December 31, 2010 and as of December 31, 2011.
c. (0.5 point) Assume Policy Group B cancels on January 1, 2011.
Calculate the 2010 policy year written car-years evaluated as of December 31, 2010 and as of December 31, 2011 for Policy Group B.
d. (0.5 point) Assume Policy Group B cancels on July 1, 2011.
Calculate the 2010 and 2011 calendar year written car-years for Policy Group B.

4.61. (5, 11/13, Q.1) (1.5 points) An insurance company is considering changing its exposure base for workers compensation from payroll to hours worked.
Evaluate the merits of this change based on three different criteria of a good exposure base.

4.62. (5, 5/14, Q.3) (2.25 points) For a single personal auto policy with an annual policy term:

- A = Calendar year 2013 written exposures as of December 31, 2013.
- B = Calendar year 2012 earned exposures
  + calendar year 2013 earned exposures as of February 1, 2013.
- C = Calendar year 2013 unearned exposures as of February 1, 2013.
- D = In-force exposures as of February 1, 2013.
- A < 0 < B < C < D.
- Exposure is earned uniformly throughout the policy term.
- This policy cancels mid-term.

a. (1 point) Provide the range of valid effective dates for this policy.
b. (0.5 point) Provide the range of valid dates of the mid-term cancellation for this policy.
c. (0.75 point) Demonstrate that it would never be possible to have A < 0 < B < C < D
if B, C, and D were as of July 1, 2013 instead of February 1, 2013.

4.63. (5, 11/14, Q.1) (2 points)
In an attempt to improve poor workers compensation underwriting results, an insurance company is considering changing its exposure base from number of employees to number of hours worked.
a. (0.5 point) Identify two criteria of a good exposure base.
b. (0.5 point) Briefly discuss whether this change in exposure base is appropriate for each of the criteria from part a. above.
c. (0.25 point) Briefly describe the impact the exposure base change could have on frequency.
d. (0.25 point) Briefly describe the impact the exposure base change could have on severity.
e. (0.5 point) Discuss an impact the exposure base change could have on the company's loss ratio.

4.64. (5, 5/15, Q.2) (2 points) An insurer is considering changing the exposure base for a commercial auto line of business to one of the following:
i. Annual fuel expense
ii. Number of miles driven
Using three relevant actuarial criteria, evaluate the effectiveness of each of these potential exposure bases and provide a recommendation for the preferred exposure base.

4.65. (5, 5/15, Q.3) (3 points) Given the following information for an insurance product:

Time Period	Written Car Years
January 1 through June 30, 2013	106.0
July 1 through December 31, 2013	107.5
January 1 through June 30, 2014	210.0
July 1 through December 31, 2014	45.0

- All policies written before January 1, 2014 had a six-month policy term.
- All policies written from January 1, 2014 onwards had an annual policy term.

a. (1.25 points) Calculate the earned car years for calendar year 2014, assuming that policies were written uniformly throughout each period.
b. (1.25 points) Assume new policies in 2014 were written uniformly over the year and the retention ratio prior to the policy term change was 77%.
Assess the effect of the policy term change on the retention ratio.
c. (0.5 point) Assess the appropriateness of the assumption of uniform writings in the calculation of calendar year 2014 earned car years.

4.66. (5, 11/15, Q.2) (1.5 points)
a. (0.75 point) Based on two relevant criteria, propose and briefly justify an appropriate exposure base for a general liability policy for a restaurant.
b. (0.75 point) Based on two relevant criteria, propose and briefly justify an appropriate exposure base for a hospital professional liability policy.

4.67. (5, 11/16, Q.1) (1.25 points)
Given the following automobile policies issued during calendar years 2013 through 2015:

Effective Date	Expiration Date	Number of Policies
April 1, 2013	September 30, 2013	100
October 1, 2013	March 31, 2014	110
April 1, 2014	September 30, 2014	105
October 1, 2014	March 31, 2015	100
April 1, 2015	September 30, 2015	110
October 1, 2015	March 31, 2016	105

• All policies have a 6-month term.
a. (0.5 point) Calculate the written car-years for calendar year 2014.
b. (0.25 point) Calculate the in-force car-years as of December 31, 2014.
c. (0.5 point) Calculate the earned car-years for calendar year 2015.

4.68. (5, 5/17, Q.1) (1.25 points) An insurance company portfolio consists of the following:
- 1,000 two-year policies with an effective date of April 1, 2015.
- 1,000 one-year policies with an effective date of July 1, 2015.

a. (0.75 point) Calculate the following for calendar year 2015:
 i. Written exposures
 ii. Earned exposures
b. (0.5 point) Calculate the earned exposures for calendar year 2016.

4.69. (5, 5/18, Q.1) (1.50 points) Given the following for an insurance company:
- The exposure base is number of occupants in the home per year.
- The company writes a one-year homeowners policy effective April 1, 2017 for a home with four occupants.
- On October 1, 2017, two occupants leave the home, and the home has only two occupants for the remainder of the policy term.

a. (0.25 points) Calculate the total written exposures for calendar year 2017.
b. (0.25 points) Calculate the total earned exposures for calendar year 2017.
c. (0.25 points)
Calculate the total policy year 2017 written exposures evaluated as of September 30, 2017.
d. (0.75 points)
Briefly evaluate the number of occupants based on the three criteria of an exposure base.

4.70. (5, 11/18, Q.1) (2.5 points) An insurer is considering changing the exposure base for boat owners line of business from boat-years to the insured value of the boat.
The insurer offers the following coverages for boat owners:
i. Liability coverage pays for damages to another boat or injuries of people not on the insured's boat.
ii. Physical damage coverage pays for damages to the insured's boat caused by common risks, such as sinking, fire, storms, theft, and collision.
Using three criteria for a good exposure base, evaluate the effectiveness of the proposed change in exposure base for both liability and physical damage coverages and provide a recommendation for the preferred exposure base.

4.71. (5, 5/19, Q.1) (2.5 points) Given the following:

Policy	Number of Vehicles	Effective Date	Expiration Date
A	2	January 1, 2018	June 30, 2018
B	3	March 1, 2018	August 31, 2018
C	1	July 1, 2018	December 31, 2018
D	2	October 1, 2018	March 31, 2019
E	1	November 1, 2018	April 30, 2019

- All policies remain in-force until their expiration date.
- An exposure is defined as one vehicle insured for one year.

a. (0.25 point) Calculate the calendar year 2018 written exposures.
b. (0.5 point) Calculate the calendar year 2018 earned exposures.
c. (0,5 point) Calculate the policy year 2018 earned exposures as of February 28, 2019.
d. (0.25 point) Calculate the in-force exposures as of October 15, 2018.
e. (1 point) Identify two criteria of an exposure base and briefly evaluate miles driven as an exposure base for personal auto insurance using those criteria.

4.72. (5, 11/19, Q.1) (1.75 points) Given the following quarterly exposure information:

Calendar Year and Quarter	Written Exposures	Earned Exposures
2017 Q 1	100	5.00
2017 Q 2	450	247.50
2017 Q 3	400	427.50
2017 Q 4	100	52.50
2018 Q 1	125	53.75
2018 Q 2	550	528.75
2018 Q 3	475	562.50
2018 Q 4	30	59.00

- The company started writing business on January 1, 2017.
- The company stops writing business on December 31, 2018.
- The quarterly earnings pattern was set by analyzing historical experience across the industry and is not uniform.
- All policies are annual.
- All policies are written on the first day of the quarter.
- There are no policy cancellations and no mid-term adjustments.

a. (0.5 point) Calculate the 2017 policy year earned exposures as of March 31, 2018.
b. (0.25 point) Calculate the in-force exposures as of May 31, 2018.
c. (0.5 point) Calculate the calendar year 2018 unearned exposures.
d. (0.5 point) Calculate the calendar year 2019 quarter 1 earned exposures.

Solutions to Problems:

4.1. Personal Automobile: Earned Car Year
Homeowners: Earned House Year
Workers Compensation: Payroll
Commercial General Liability: Sales Revenue, Payroll, Square Footage, Number of Units
Commercial Business Property: Amount of Insurance Coverage
Physician's Professional Liability: Number of Physician Years
Professional Liability: Number of Professionals (e.g., Lawyers, Actuaries, or Accountants)
Personal Articles Floater: Value of Item
Comment: See Table 4.1 in Basic Ratemaking.
For Homeowners Insurance, Amount of Insurance Years is sometimes used.
For Workers Compensation, manweeks is sometimes used.
Commercial General Liability package policy use different exposures bases for different sublines.
A personal articles floater might for example insure an expensive piece of jewelry or an expensive musical instrument.

4.2. (a) Assume each block of policies is written in the middle of its quarter.
For example, we assume the policies written in the first quarter of 2011 are all effective 2/15/11, and 3/24 of their exposures are earned in Calendar Year 2012.

Quarter	Written Expos	Assumed Effective Date	% Earned During CY 2012	Earned Expos CY 2012
First 2011	11,000	2/15/11	0.125	1,375
Second 2011	10,000	5/15/11	0.375	3,750
Third 2011	11,000	8/15/11	0.625	6,875
Fourth 2011	12,000	11/15/11	0.875	10,500
First 2012	15,000	2/15/12	0.875	13,125
Second 2012	12,000	5/15/12	0.625	7,500
Third 2012	13,000	8/15/12	0.375	4,875
Fourth 2012	14,000	11/15/12	0.125	1,750
Total	98,000			**49,750**

(b) For example, we assume the policies written in the third quarter of 2011 are all effective 8/15/11, and 3/12 = 1/4 of their exposures are earned in Calendar Year 2012.

Quarter	Written Expos	Assumed Effective Date	% Earned During CY 2012	Earned Expos CY 2012
First 2011	11,000	2/15/11	0	0
Second 2011	10,000	5/15/11	0	0
Third 2011	11,000	8/15/11	0.25	2,750
Fourth 2011	12,000	11/15/11	0.75	9,000
First 2012	15,000	2/15/12	1	15,000
Second 2012	12,000	5/15/12	1	12,000
Third 2013	13,000	8/15/12	0.75	9,750
Fourth 2014	14,000	11/15/12	0.25	3,500
Total	98,000			**52,000**

Comment: See Table 4.23 in Basic Ratemaking, which has data by month rather than by quarter.

4.3. Fidelity has the exposure base: number of persons.
Comments: Surety has the exposure base: contract costs.

4.4. (a) Assume each block of policies is written in the middle of its month.
For example, we assume the policies written in January are all effective 1/15/10, and 23/24 of their exposures are earned in Calendar Year 2010.

Month	Written Expos	Assumed Effective Date	% Earned During CY 2010	Earned Expos CY 2010
January	3,000	1/15/10	0.9583	2,875
February	600	2/15/10	0.8750	525
March	600	3/15/10	0.7917	475
April	1,200	4/15/10	0.7083	850
May	600	5/15/10	0.6250	375
June	600	6/15/10	0.5417	325
July	1800	7/15/10	0.4583	825
August	600	8/15/10	0.3750	225
September	600	9/15/10	0.2917	175
October	1,200	10/15/10	0.2083	250
November	600	11/15/10	0.1250	75
December	600	12/15/10	0.0417	25
Total	12,000			**7,000**

(b) For example, we assume the policies written in July are all effective 7/15/10, and 11/12 of their exposures are earned in Calendar Year 2010.

Month	Written Expos	Assumed Effective Date	% Earned During CY 2010	Earned Expos CY 2010
January	3,000	1/15/10	1.0000	3,000
February	600	2/15/10	1.0000	600
March	600	3/15/10	1.0000	600
April	1,200	4/15/10	1.0000	1,200
May	600	5/15/10	1.0000	600
June	600	6/15/10	1.0000	600
July	1800	7/15/10	0.9167	1,650
August	600	8/15/10	0.7500	450
September	600	9/15/10	0.5833	350
October	1,200	10/15/10	0.4167	500
November	600	11/15/10	0.2500	150
December	600	12/15/10	0.0833	50
Total	12,000			**9,750**

Comment: Uses the “15th of the month rule” as in Table 4.23 in Basic Ratemaking.
Treat part (b) as a separate question, where the given caryears just happen to be the same as in part (a). With 6-month policies, 3000 exposures corresponds to 6000 cars. For part (b) one could convert all the given values to cars, then do your work, and make sure your answer is in caryears.
If an insurer were writing 12-month policies and switched to 6-month policies, then the number of cars written in each month would change, as would the cars years written each month. However, in the long run, the annual number of car years written should stay the same as when 12-month policies were written.

4.5. The reason in statement #1 is not correct. Unlimited payroll is easier to verify, and reduces the size of rate revisions (payroll limited to for example $300 per week would increase at less than the rate of inflation.) *Also unlimited payroll requires more work at audit.*
Statement # 3 is false. There was no need to study large risks, which were (then) self-rated under the experience rating plan. *(Large Workers Compensation risks would now get a large, but less than 100% credibility. See Howard Mahler's discussion of Robin Gillam's "Parametrizing the Workers Compensation Experience Rating Plan", PCAS 1993.)* For medium size risks, they found a bias against high wage employers and union employers.

4.6. If one gives an insured larger credibility in an experience rating plan, the exposure base, rating variables, and class plan, all have less effect on the final premium.

4.7. Manufacturing and Mercantile use sales. Contracting uses payroll.

4.8. 1. Once an exposure base has been established, it has a great tendency to remain in place for many decades.
2. Even with the help of computers, changing exposure bases is expensive and difficult.
3. True. When unlimited payroll was introduced in Workers Compensation, transition programs were generally used to spread over several years the impact of the change on individual employers.

4.9. During the competitive part of the underwriting cycle, underwriters have a tendency to underestimate the payrolls for insureds. In theory, such "errors" will be corrected when the policy is audited, but that is usually eighteen months in the future (and after the renewal). Under the calendar/accident year ratemaking used for many lines, audit premiums are reported and fully earned in the calendar year of the audit, not the calendar year(s) when the policy premium was earned. Thus, even in the case of perfectly correct audits, a severe mismatch between the premiums and losses can be introduced by low exposure estimates. In the case of policy year data, while the audited payroll will be assigned to the correct policy year, the payroll/premium development factors will fluctuate with the underwriting cycle, making it difficult to estimate ultimate policy year payrolls and premiums.

4.10. (a) ABC is in a steady state situation. The expected number of claims occurring each year is: (10)(5 + 5 + 4 + 3 + 2 + 1) = 200.
(b) ABC is in a steady state situation. The expected number of claims reported each year is 200.
(c) XYZ is not in a steady state.

Year of Occur.	Year of Sale 1999	Year of Sale 2000	Year of Sale 2001	Year of Sale 2002	Year of Sale 2003	Year of Sale 2004	Year of Sale 2005	Year of Sale 2006	Total
1999	85								
2000	85	80							
2001	68	80	75						
2002	51	64	75	70					
2003	34	48	60	70	65				
2004	17	32	45	56	65	60			275
2005		16	30	42	52	60	55		**255**
2006			15	28	39	48	55	50	**235**

We expect 255 claims occurring in the year 2005.
We expect 235 claims occurring in the year 2006.
(d) Half of the claims occurring in 2005 will be reported in 2006.
Half of the claims occurring in 2006 will be reported in 2006.
We expect (255 + 235)/2 = 245 claims reported in 2006.
(e) It is common to use sales for the year of coverage as the exposure base.
Thus in this case, ABC and XYZ would have the same number of exposures for 2006.
If one is writing on an occurrence basis, we see that we expect 235 claims from XYZ compared to 200 claims from ABC.
If one is writing on a claims made basis, we see that we expect 245 claims from XYZ compared to 200 claims from ABC.
In either case, current year sales is not a precise measure of the exposure to loss.
This is an example of "temporal mismatch."
The mismatch would be worse if either the length of average time from sale to occurrence were longer or the change in sales by year were greater.
Comment: For a widget company whose sales had been increasing to 10 million in 2006, the expected number of claims occurring in 2006 would have been less than 200. XYZ has a larger legacy of old widgets sold, which continue to be involved in accidents and produce claims.
Many products would have a much longer average time between sale and occurrence of claim than was the case in this question, making the effect of a changing volume of sales much larger. This legacy of old sales is somewhat similar to the legacy costs of General Motors due to paying the health insurance costs of retired workers. General Motors current workforce is much smaller than its workforce in the past, just as XYZ's current sales are smaller than its sales in the past.

4.11. (a) 6-month policies written during January 1 through June 30, 2013 contribute none of their written exposures to CY14 earned exposures.
6-month policies written during July 1 through December 31, 2013 contribute on average half of their written exposures to CY14 earned exposures.
6-month policies written during January 1 through June 30, 2014 contribute all of their written exposures to CY14 earned exposures.
6-month policies written during July 1 through December 31, 2014 contribute on average half of their written exposures to CY14 earned exposures.
(100,000)(0 + 1/2)/2 + (104,000)(1 + 1/2)/2 = **103,000**.
(b) Since they are six-month policies, those written during the first half of 2014 contribute nothing to CY2015 earned exposures. Those written during the second half of 2014 contribute on average 1/2 of their exposures to CY2015 earned exposures. Thus, CY2014 written exposures contribute on average 1/4 of their exposures to CY15 earned exposures.
The annual policies written during the first half of 2015 contribute on average 3/4 of their exposures to CY15 earned exposures.
The annual policies written during the second half of 2015 contribute on average 1/4 of their exposures to CY15 earned exposures.
CY15 earned exposures: (1/4)(104,000) + (3/4)(100,000) + (1/4)(16,000) = **105,000**.
(c) There were 16,000 caryears written during the second half of 2015. None of these can be due to policies written during the first half of 2015, since there are no cancelations or endorsements (to add additional autos to an existing policy.) If during the first half of 2015 there were also 16,000 exposures for new policies, then we can estimate the retention ratio as:
(100 - 16) / 104 = **80.8%**.
Comment: Similar to 5, 5/15, Q. 3.
Given enough time the number of exposures written would eventually even out between the two halves of a year, although it might take decades.

As per page 10 of Basic Ratemaking: Retention Ratio = $\dfrac{\text{Number of Policies Renewed}}{\text{Number of Potential Renewal Policies}}$.

Here there is no way to determine the number of policies invited to renew or that are able to be renewed.

4.12. E. The exposure base should be (approximately) proportional to expected losses.
For example, 3 times the exposure should have (approximately) 3 times the expected losses.

4.13. The four policies written during 2005, contribute all of their exposures to the written exposures for Calendar Year 2005.
The written exposures for Calendar Year 2005 are: 3 + 1/2 + 2/2 + 1 = **5.5 cars years**.
For the first policy written Sept. 1, 2004, a 6 month policy which expires February 28, 2005, 1/3 of its exposures are during Calendar Year 2005.
For the second policy written Oct. 1, 2004, an annual policy that expires on Sept. 30, 2005, 3/4 of its exposures are during Cal. Year 2005.
For the third policy written Jan. 1, 2005, an annual policy that expires on Dec. 31, 2005, all of its exposures are during Cal. Year 2005.
For the fourth policy written June 1, 2005, a 6 month policy which expires Nov. 30, 2005, all of its exposures are during Cal. Year 2005.
For the fifth policy written Aug. 1, 2005, a 6 month policy which expires Jan. 31, 2006, 5/6 of its exposures are during Cal. Year 2005.
For the sixth policy written Dec. 1, 2005, an annual policy that expires on Nov. 30, 2006, 1/12 of its exposures are during Cal. Year 2005.
Calendar Year 2005 Earned Exposures are:
(1/3)(1/2) + (3/4)(2) + (1)(3) + (1)(1/2) + (5/6)(2/2) + (1/12)(1) = **6.083 car years**.

Effective Date	Policy Term (years)	Expiration Date	Portion Earned in CY 2005	Cars	Exposures (car years)	Exposures Earned in CY 2005
9/1/04	0.5	2/28/05	0.333	1	0.5	0.167
10/1/04	1	9/30/05	0.750	2	2	1.500
1/1/05	1	12/31/05	1.000	3	3	3.000
6/1/05	0.5	11/30/05	1.000	1	0.5	0.500
8/1/05	0.5	1/31/06	0.833	2	1	0.833
12/1/05	1	11/30/06	0.083	1	1	0.083
Sum						**6.083**

On December 31, 2005, only the last two policies have Unearned Exposures:
(1 - 5/6)(2/2) + (1 - 1/12)(1) = **1.083 car years**.
Alternately, for the four policies written in 2005, their Calendar Year 2005 earned exposures are:
(1)(3) + (1)(1/2) + (5/6)(2/2) + (1/12)(1) = 4.416 car years.
The written exposures for Calendar Year 2005 are: 3 + 1/2 + 2/2 + 1 = 5.5 car years.
Therefore, the unearned exposures at the end of 2005 are: 5.5 - 4.416 = 1.083 car years.
On December 31, 2005, the 3rd, 5th, and 6th policies are providing coverage for a total of six automobiles; the Inforce Exposures are **6 cars**.
Comment: Similar to 5, 5/05, Q.14.
Note that inforce exposures do not have any time duration associated with them.
For example, for Collision Insurance, the inforce exposures today would be the number of cars you would need to pay for if all of your insureds wrecked their cars today in one car accidents. Inforce exposures on a given date are the sum over all policies that provide coverage on that date of the cars insured on those policies. Thus inforce exposures are in cars, not caryears.

4.14.

		Car Years	Earned CY02	Written CY02
200 policies	7/1/01 to 12/31/01	100	0	0
100 policies	7/1/01 to 6/30/02	100	50	0
100 policies	10/1/01 to 3/31/02	50	25	0
200 policies	1/1/02 to 6/30/02	100	100	100
100 policies	1/1/02 to 12/31/02	100	100	100
100 policies	4/1/02 to 9/30/02	50	50	50
250 policies	7/1/02 to 12/31/02	125	125	125
100 policies	7/1/02 to 6/30/03	100	50	100
200 policies	10/1/02 to 3/31/03	100	50	100
Total			**550**	**575**

Comment: Similar to 6, 5/93, Q.6.
Remember that a six month policy covering one car represents 1/2 car-year.

4.15. Exposures measures such as payroll are responsive to changes in wage level, which for Workers Compensation insurance is a desirable rather than an undesirable feature.

4.16.

Policy	CY15 Written	CY15 Earned	PY14 Written	PY15 Written*	5/1/15 In-Force
1	0	1/6	1	0	0
2	-1/4	1/3	3/4	0	0
3	1	1	0	1	1
4	7/12	7/12	0	7/12	1
5	1	1/4	0	1	0

* As of 6/30/16.

(a) For policy #2, there would have been 1 written exposure assigned to CY14
(which is closed at the end of 2014 for accounting purposes.)
Therefore, when this policy is cancelled midterm during 2015, we need to put -1/4 written exposure in CY15, so that in total for policy #2 we have exposures of: 1 - 1/4 = 3/4.
In the case of policy #4, we know it has been cancelled by the end of 2015, so we only have 7/12 exposure contributed to CY15 written exposures.

CY15 written exposures: -1/4 + 1 + 7/12 + 1 = $2\frac{1}{3}$ car years.

(b) In each case, we take the portion of the year that the policy is in effect.

CY15 earned exposures: 1/6 + 1/3 + 1 + 7/12 + 1/4 = $2\frac{1}{3}$ car years.

(c) We only look at policies written during 2014.
PY14 written exposures: 1 + 3/4 = **1.75** car years.
(d) We only look at policies written during 2015.
Note that as of 6/30/16, we do not yet know that policy #5 will be cancelled on 7/31/16.

PY15 written exposures: 1 + 7/12 + 1 = $2\frac{7}{12}$ car years.

(e) We see whether or not the policy is in effect on 5/1/15.
In-force exposure as of May 1, 2015: 1 + 1 = **2** cars.
Comment: Similar to 5, 5/10, Q.17.

4.17. (a) Only the six-month policies written in 2018 contribute.
Each policy contributes half a caryear:
(300 + 260 + 290 + 270)/2 = **560 car-years**.
(b) Only the six-month policies written July 1, 2017 and October 1, 2017 are in force on December 31, 2017.
As of December 31, 2017 the insurer is covering: 230 + 220 = **450 cars**.
(c) The six-month policies written October 1, 2017 contribute half their exposures.
The six-month policies written January 1, 2018 contribute all of their exposures.
The six-month policies written April 1, 2018 contribute all of their exposures.
The six-month policies written July 1, 2018 contribute all of their exposures.
The six-month policies written October 1, 2018 contribute half their exposures.
(1/2) (220/2 + 300 + 260 + 290 + 270/2) = **547.5** car-years.
(d) Only the six-month policies written in July 2017, October 2017, January 2018, and April 2018 contribute. Each policy contributes half a caryear:
(230 + 220 + 300 + 260)/2 = **505 car-years**.
(e) The six-month policies written April 1, 2017 contribute half their exposures.
The six-month policies written July 1, 2017 contribute all of their exposures.
The six-month policies written October 1, 2017 contribute all of their exposures.
The six-month policies written January 1, 2018 contribute all of their exposures.
The six-month policies written April 1, 2018 contribute half their exposures.
(1/2) (200/2 + 230 + 220 + 300 + 260/2) = **490** car-years.
Comment: Similar to 5, 11/16, Q.1.

4.18. Pros: Those who use more gas, drive more miles. The more miles driven the larger the chance of a claim. Thus the exposure base varies with the hazard.
Highway miles are safer and also use less gas than city miles, thus the exposure base varies with the hazard.
It could be implemented via a pay at the pump system.
Smaller cars are less likely than larger cars to cause massive damage to other vehicles or seriously injure passengers in other cars. Thus, smaller cars are a smaller hazard for liability insurance. Smaller cars also get more miles per gallon. Thus under this system smaller cars will pay less per mile driven than larger cars. The exposure base is closer to proportional to the hazard than either car years or miles driven.
Cons: The hazard is not proportional to the gallons of gas used. A more fuel efficient car will get more miles per gallon, and therefore more expected accidents per gallon.
This exposure base is not currently in use.
It would be difficult or impractical to implement without government intervention. If it were not a mandated system, similar to the gasoline tax, it would be difficult for the insurer to determine gallons of gas used and it would be easy for the insured to manipulate.
Comment: Say just a few reasonable things that demonstrate you understand exposures.
It is not being assumed that rating variables such as age of driver, use of vehicle, territory, etc., would be eliminated, although it is unclear how this would fit into a pay at the pump system.
See for example, "Pay-at-the-Pump Auto Insurance," by J. Daniel Khazzoom.
http://www.rff.org/documents/RFF-DP-98-13-REV.pdf
See also Auto Insurance Alert, by Andrew Tobias.
See also "Pay as You Drive, Some Considerations Before You Hit the Road," by Kevin Bingham, Peter Boyd, Michael Green, and Rob Brown, January/February 2009 Contingencies.
Pay-as-you-drive insurance plans charge by the mile driven. The rate per mile can vary by where you drive, how you drive, and when you drive.

4.19. (a) The written exposures for CY17 are: (2)(700) + 1200 = **2600 house years**.
(b) Each of the two year policies written 6/1/15 contributes 5/12 of an earned exposure to CY17.
Each of the two year policies written 6/1/16 contributes 1 earned exposure to CY17.
Each of the two year policies written 6/1/17 contributes 7/12 of an earned exposure to CY17.
Each of the one year policies written 10/1/16 contributes 9/12 of an earned exposure to CY17.
Each of the one year policies written 10/1/17 contributes 3/12 of an earned exposure to CY17.
The earned exposures for CY17 are:
(5/12)(500) + (1)(600) + (7/12)(700) + (9/12)(1100) + (3/12)(1200) = **2342 house years**.
Comment: Similar to 5, 5/17, Q.1.

4.20. (a) For example, for 2018 Q3, the exposures earned in 2018 Q3 to 2019 Q2 are:
(35%)(4000) = 1400, (20%)(4000) = 800, (15%)(4000) = 600, (30%)(4000) = 1200.

Calendar Year and Quarter	Written Exposures	Earned in 2018 Q1	Earned in 2018 Q2	Earned in 2018 Q3	Earned in 2018 Q4
2018 Q 1	1000	150	300	350	200
2018 Q 2	3000		900	1050	600
2018 Q 3	4000			1400	800
2018 Q 4	2000				400
Total		**150**	**1200**	**2800**	**2000**

The exposures earned in 2018 Q4 are: (20%)(1000 + 3000 + 4000 + 2000) = 2000.
(b) We get contributions from policies written in both 2018 and 2019.

Calendar Year and Quarter	Written Exposures	Earned in 2019 Q1	Earned in 2019 Q2	Earned in 2019 Q3	Earned in 2019 Q4
2018 Q 1	1000				
2018 Q 2	3000	450			
2018 Q 3	4000	600	1200		
2018 Q 4	2000	300	600	700	
2019 Q 1	1200	180	360	420	240
2019 Q 2	3500		1050	1225	700
2019 Q 3	5000			1750	1000
2019 Q 4	2500				500
Total		**1530**	**3210**	**4095**	**2440**

The exposures earned in 2019 Q3 are: (35%)(2000 + 1200 + 3500 + 5000) = 4095.
(c) We only get contributions from policies written in 2019.

Calendar Year and Quarter	Written Exposures	Earned in 2020 Q1	Earned in 2020 Q2	Earned in 2020 Q3	Earned in 2020 Q4
2019 Q 1	1200				
2019 Q 2	3500	525			
2019 Q 3	5000	750	1500		
2019 Q 4	2500	375	750	875	
Total		**1650**	**2250**	**875**	**0**

The exposures earned in 2020 Q2 are: (30%)(5000 + 2500) = 2250.
(d) All of the exposures earned in 2018, plus the exposures earned in Q1 of 2019 from policies written in 2018: (150 + 1200 + 2800 + 2000) + (450 + 600 + 300) = **7500**.
Alternately the PY18 unearned exposures as of March 31, 2019 are:
(4000)(30%) + (2000)(30% + 35%) = 2500.
Thus the PY18 earned exposures as of March 31, 2019 are:
1000 + 3000 + 4000 + 2000 - 2500 = **7500**.

(e) Given the insurer stops writing business on December 31, 2019: 1650 + 2250 + 875 = **4775**.
Alternately, given that the insurer started writing business on January 1, 2018, we can add up all of the written exposures by quarter and subtract the sum of all of the earned exposures by quarter through the end of 2019: 22,200 - 17,425 = **4775**.
Comment: Similar to 5, 11/19, Q.1.

4.21. (a) 1. Accurately measure the exposure to loss.
2. Should be easy for the insurer to determine.
3. Should not be subject to manipulation by the insured or underwriters.
(b) For indemnity benefits (lost wages), payroll is a reasonably accurate measure of the exposure to loss. The more hours worked the more payroll and the more work related accidents one expects. However, the higher the wage rate, the higher the average weekly wage, and the higher the indemnity benefit, (within limits.) Therefore, while both payroll and hours worked would capture the frequency component, only payroll captures the severity component of indemnity losses.
Therefore, with respect to indemnity benefits, payrolls would more accurately measure this portion of the exposure to loss than would hours worked.
For medical benefits, there is no direct connection between the hourly wage and the average severity. Thus it is unclear why an employer who pays a higher hourly wage should pay more premium to cover expected medical benefits. If in fact there is little connection between hourly wages and medical losses, then hours worked would more accurately measure the medical portion of the exposure to loss than would payrolls.
Hours worked is harder for insurers to obtain than payroll.
Also payroll is harder for insureds to manipulate than hours worked.
Another advantage of payrolls is that it is inflation sensitive, and thus reduces the need for annual rate increases.
(c) Large employers were excluded from the study. No bias against either union or high wage paying employers was found among the small employers. For medium sized employers, high wage paying and union employers developed lower loss costs per premium dollar.
Comment: See also pages 13-15 of Feldblum's "Workers Compensation Ratemaking", formerly on the syllabus.
"Accuracy (Predictability): Total payroll may be more predictive of losses, for both indemnity and medical benefits, than either limited payroll or man-hours worked. Higher paid employees are more aware of Workers' Compensation benefits, seek higher quality medical treatment, and are more likely to engage attorneys to represent their claims. The statutory limits on indemnity benefits have only a partial effect on these influences."

4.22.

		Earned CY92	Written CY92
100 policies	1/1/92 to 6/30/92	50	50
150 policies	4/1/92 to 9/30/92	75	75
200 policies	7/1/92 to 12/31/92	100	100
250 policies	10/1/92 to 3/31/93	62.5	125
Total		**287.5**	**350**

Comment: Remember that a six month policy for one car represents 1/2 car-year.
For the last set of policies, half is earned during CY92 and half is earned during CY93.

4.23. a. "Once the risk's classification is known, the rate for that classification is multiplied by the number of exposure units to produce the premium. The use of a variable in the exposure base implies a uniform and continuous multiplicative relationship between the variable and the expected losses; use as a rating element implies a discrete, nonlinear relationship. For example, physician-month is an exposure base; and coverage for two physician-months costs twice as much as the coverage for one physician-month. On the other hand, age is a rating variable; and coverage for Driver A, who is twice as old as Driver B, does not (usually) cost twice as much."
b. 1. Temporal mismatch. A temporal mismatch between expected losses and an otherwise acceptable exposure base can arise. The two outstanding examples of this are claims-made policies and products liability.
2. Interpretive Mismatch. Ambiguous policy language may lead to more coverage than was intended.
3. Complexity of Hazard. The exposure base is too complex to quantify as in Directors and Officers. When there are too many factors to consider it is very difficult to pick an exposure base.

4.24. x = 0 at 1/1/93 and x = 3 at 12/31/96.

For six month policies, the portion earned during 1994 is: $\begin{cases} 0 \text{ for } 0 \le x < 0.5 \\ 2x - 1 \text{ for } 0.5 \le x < 1 \\ 1 \text{ for } 1 \le x < 1.5 \\ 4 - 2x \text{ for } 1.5 \le x \le 2 \end{cases}$

For example, for x = 3/4, 50% is earned in 1994.
For example, for x = 15/8, 25% is earned during 1994.

For six month policies, the portion earned during 1995 is: $\begin{cases} 0 \text{ for } 0 \le x < 1.5 \\ 2x - 3 \text{ for } 1.5 \le x < 2 \\ 1 \text{ for } 2 \le x < 2.5 \\ 6 - 2x \text{ for } 2.5 \le x \le 3 \end{cases}$.

Earned exposures for CY1994:

$$\int_{0.5}^{1} 1000x\,(2x - 1)\,dx + \int_{1}^{1.5} 1000x\,dx + \int_{1.5}^{2} 1000x\,(4 - 2x)\,dx = 208.33 + 625 + 416.67 = 1250.$$

Earned exposures for CY1995:

$$\int_{1.5}^{2} 1000x\,(2x - 3)\,dx + \int_{2}^{2.5} 1000x\,dx + \int_{2.5}^{3} 1000x\,(6 - 2x)\,dx = 458.33 + 1125 + 666.67 = 2250.$$

Percentage increase from CY94 to CY95 is: 2250/1250 - 1 = **80%**.
Alternately, one year later the annual rate of writing is exactly 1000 higher.
So if the earned exposures for CY94 are 1250, then we know that those for CY95 must be 2250.
Proceed as before.
Alternately, one can do a discrete approximation by quarters of a year.
In order to get the exposures written in a given quarter, we integrate the given rate, 1000 x, over the time period covered by a given quarter.

For example, for the third quarter of 1993: $\int_{0.5}^{0.75} 1000\,x\,dx = (500)(0.75^2 - 0.5^2) = 156.25.$

Then we need to get the portion of those exposures contributed to a given earned calendar year.
For example, for the third quarter of 1993, the average date of writing is very approximately the middle of quarter, at time 5/8. (Since the rate is increasing, this is only an approximation.)
From time 5/8 to 1 does not contribute to CY94 earned, while from time 1 to 9/8 does.
(Recall that we have 6 month policies.)
Thus the portion contributed to CY94 earned is: (9/8 - 1) / (1/2) = 1/4.
The whole calculation is shown in the following spreadsheet:

Quarter	Written Expos	Portion Earned CY94	Product		Quarter	Written Expos	Portion Earned CY95	Product
93Q1	31.25	0.00	0.0000		94Q1	281.25	0.00	0.0000
93Q2	93.75	0.00	0.0000		94Q2	343.75	0.00	0.0000
93Q3	156.25	0.25	39.0625		94Q3	406.25	0.25	101.5625
93Q4	218.75	0.75	164.0625		94Q4	468.75	0.75	351.5625
94Q1	281.25	1.00	281.2500		95Q1	531.25	1.00	531.2500
94Q2	343.75	1.00	343.7500		95Q2	593.75	1.00	593.7500
94Q3	406.25	0.75	304.6875		95Q3	656.25	0.75	492.1875
94Q4	468.75	0.25	117.1875		95Q4	718.75	0.25	179.6875
Sum			**1250**		Sum			**2250**

4.25. (a) x = 0 at 1/1/93 and x = 3 at 12/31/96.

For six month policies, the portion earned during 1993 is: $\begin{cases} 1 \text{ for } 0 \le x < 0.5 \\ 2 - 2x \text{ for } 0.5 \le x \le 1 \end{cases}$

Earned exposures for CY1993:

$$\int_0^{0.5} 1000x \, dx + \int_{0.5}^{1} 1000x\,(2 - 2x)\, dx = 125 + 166.67 = \mathbf{291.67}.$$

(b) For six month policies, the portion earned during 1996 is: $\begin{cases} 0 \text{ for } 2 \le x < 2.5 \\ 2x - 5 \text{ for } 2.5 \le x < 3 \\ 1 \text{ for } 3 \le x < 3.5 \\ 8 - 2x \text{ for } 3.5 \le x \le 4 \end{cases}$.

Starting at x = 3, the annual rate of writing is constant at 3000.
Earned exposures for CY1996:

$$\int_{2.5}^{3} 1000x\,(2x - 5)\, dx + \int_{3}^{3.5} 3000 \, dx + \int_{3.5}^{4} 3000\,(8 - 2x)\, dx = 708.33 + 1500 + 750 = \mathbf{2958.33}.$$

Alternately, one can do a discrete approximation by quarters of a year:

Quarter	Written Expos	Portion Earned CY93	Product		Quarter	Written Expos	Portion Earned CY96	Product
93Q1	31.25	1.00	31.2500		95Q1	531.25	0.00	0.0000
93Q2	93.75	1.00	93.7500		95Q2	593.75	0.00	0.0000
93Q3	156.25	0.75	117.1875		95Q3	656.25	0.25	164.0625
93Q4	218.75	0.25	54.6875		95Q4	718.75	0.75	539.0625
94Q1	281.25	0.00	0.0000		96Q1	750	1.00	750.0000
94Q2	343.75	0.00	0.0000		96Q2	750	1.00	750.0000
94Q3	406.25	0.00	0.0000		96Q3	750	0.75	562.5000
94Q4	468.75	0.00	0.0000		96Q4	750	0.25	187.5000
Sum			**296.875**		Sum			**2953.125**

(c) Starting at the beginning of 1996, the annual rate of writing is constant at 3000.
Thus the earned exposures for CY1997 is **3000**.

4.26. For a policy written during the first year at time x,
the portion earned during the first year is: 1 - x.
(For example, if written at time 1/4, 3/4 is earned during the first year.)

Earned exposures the first year is: $\int_0^1 (1 - x)\, 1000x^2 \, dx = 1000/12 = \mathbf{83.3}.$

4.27. 1. False. 2. True. 3. True.

4.28. a. 1. Temporal mismatch. A temporal mismatch between expected losses and an otherwise acceptable exposure base can arise. The two outstanding examples of this are claims-made policies and products liability.
2. Interpretive Mismatch. Ambiguous policy language may lead to more coverage than was intended.
3. Complexity of Hazard. The exposure base is too complex to quantify as in Directors and Officers. When there are too many factors to consider it is very difficult to pick an exposure base.
b. 1. Exposure estimates can be (and are) internally manipulated in response to the competitive situation.
2. Even when the policy premium is based on the correct exposures, the coding of the exposure information into the computer records is often poor, with whole dollars frequently switched with "per hundreds" or "per thousands."

4.29. (a) 1. Historical Precedence: payroll is currently being used.
2. Practical: payroll is already being collected for tax purposes, so the data is readily available.
3. Proportional to Expected Loss: As the manweeks worked increases, so does the payroll.
The number of expected workplace accidents (for a given classification) is approximately proportional to the manhours worked, and payroll is proportional to the manhours worked.
In addition, as the weekly wage goes up so do the indemnity benefits paid. For a give number of manweeks worked, the payroll increases in proportion to the average weekly wage.
4. Inflation Sensitive: Payroll goes up with inflation, reducing the number of rate revisions needed.
(b) Medical benefits paid do not depend on a workers average weekly wage. A company that pays its workers more per hour, will have a higher payroll, but not more in medical benefits than would an otherwise identical company that paid its workers less per hour. Thus for medical benefits, payroll is not proportional to expected medical benefits paid.
Comment: There have been some studies which have shown that the level of weekly wages and expected medical benefits are positively correlated, although not proportional.
At pages 50-51 of Basic Ratemaking: "Workers compensation has historically used payroll as an exposure base. In the 1980s, there was a lot of pressure to change the exposure base to hours worked for medical coverage in order to correct perceived inadequacies of the exposure base for union companies with higher pay scales. Although hours worked made intuitive sense, the exposure base was not changed at that time, and one of the major reasons cited was concerns regarding the transition. Instead, the rating variables and rating algorithm were adjusted to address the inequities. This debate over the choice of workers compensation exposure base continues to reemerge."

4.30. 1. False. Inflation-sensitive exposure bases, such as sales or payroll, decrease the need for frequent rate filings, since as the exposure increases, so does the collected premium even as the rates stay the same. Rates still need to be revised, but on average not as often as if the exposure base were not inflation sensitive. 2. True. 3. True.

4.31. A policy written at time x, has x exposures unearned at the end of the first year.
For example a policy written with one exposure at time .6, has .4 exposures earned during this year, and .6 exposures earned next year. Unearned exposure at the end of the first year are:

$$\int_0^1 x\,(1000x)\,dx = 333.33x^3 \Big]_{x=0}^{x=1} = \mathbf{333.33}.$$

4.32. (a) Reasonableness: The exposure unit should be a reasonable measure of the exposure to loss. Ease of Determination: The exposure unit should be accurately determinable. If an exposure base is not subject to determination, then an insurer can never be assured of receiving the proper premium for the actual exposure.
Responsiveness to Change: An exposure unit that reflects changes in the exposure to loss is preferable to one that does not.
Historical Practice: Where a significant body of historical exposure data is available, any change in the exposure base could render the prior history unusable. Since ratemaking generally depends upon the review of past statistical indications, exposure bases are rarely changed once they have been established.
(b) Car years: Reasonable to some extent. The more cars the greater the exposure to loss. However, it won't distinguish between an old Chevy and a new Rolls Royce. Easy to determine. Can be accurately measured. Somewhat responsive to change. Responsive to adding a vehicle but not to replacing with a different vehicle. Historical practice: This has been an historical measure of exposure and should remain so.
Miles driven: Reasonable to some extent. The more miles you drive, the greater the exposure to loss. However, it won't distinguish between city and country exposures. Not easy to measure accurately. Would have to rely on insured's statement of miles driven, or pay the expense of independently taking odometer readings, or use expensive GPS related technology. Very responsive to change. Generally if an insured drives more miles than last year, exposure to loss has increased. Historical practice: Used in ranges as part of classification systems and should remain so.
Comment: There is more than one right answer to part b.

4.33. (a) 1. It should be an accurate measure of the exposure to loss.
2. It should be easy for the insurer to determine.
3. It should be difficult for the insured to manipulate.
4. Require that the exposure base be immune to manipulation by underwriters.
(b) Unlimited payroll is easy to determine. Unlimited payroll is hard to be manipulated by the insured and underwriters as it is used for tax purposes. For indemnity losses, unlimited payroll is a reasonably accurate measure of the exposure, with the accuracy decreased if there are limitations on benefits. While it makes sense for the number of medical losses to be approximately proportional to the hours worked, the size of medical losses is not necessarily closely related to the worker's hourly wage. Therefore, in the case of medical losses, unlimited payroll may not reflect the exposure to loss accurately.
Comment: One could have somewhat different sensible answers to part b.

4.34. a. x = 0 at 1/1/94 and x = 2 at 12/31/95.

The portion earned during 1995 is: $\begin{cases} x, \text{ for } 0 \le x \le 1 \\ 2 - x, \text{ for } 1 < x \le 2 \end{cases}$.

For example, for x = 1/4, 3/4 is earned during 1994 while 1/4 is earned during 1995.
For x = 1.2, 80% is earned during 1995 while 20% is earned during 1996.

Earned exposures for 1995: $\int_0^1 1000x \; x \; dx + \int_1^2 1000x \; (2 - x) \; dx = (1000)(1/3 + 2/3) = 1000.$

b. Since we only write policies up to x = 1.5, the second integral only goes from 1 to 1.5.

$$\int_0^1 1000x \; x \; dx + \int_1^{1.5} 1000x \; (2 - x) \; dx = (1000)(1/3 + 11/24) = 792.$$

c. Since all in-force policies are canceled June 30, 1995, any policy written during the second half of 1994 earns half of an exposure during 1995.
Also for example a policy written on April 1, 1995 only provides 3 months of coverage before it is cancelled; all of this policy or 1/4 of an exposure is earned during 1995.

Exposures earned during 1995 is: $\begin{cases} x, \text{ for } 0 \le x \le 1/2 \\ 1/2, \text{ for } 1/2 \le x \le 1 \\ 1.5 - x, \text{ for } 1 < x \le 1.5 \end{cases}$.

$$\int_0^{1/2} 1000x \; x \; dx + \int_{1/2}^1 1000x \; (1/2) \; dx + \int_1^{1.5} 1000x \; (1.5 - x) \; dx = (1000)(1/24 + 3/16 + 7/48) = 375.$$

4.35. (b) 1. Analyze the coverage offered and the coverage trigger to determine what factors influence the expected losses. Some of these factors will not be usable in the determination of premiums.
2. Those which are usable will be divided into two groups: the first group, consisting of one factor, will be the exposure base, and the second group will be the rating variables, which influence the projected expected losses indirectly by affecting the rate.
(c) Composite rating plan: the risk's premium is calculated normally and then divided by a proxy exposure base, such as mileage or receipts for long-haul trucking firms. This gives a rate per proxy unit. When the policy expires, the firm's records are audited in order to determine the actual receipts (or mileage), and this is used to calculate the final premium.
Loss-rated: the premium is calculated directly from the risk's historical losses without any reference to the standard rating plans. In this case, it is correct to say that the exposure base is the risk itself and the rate is its expected losses.

4.36. 1. True. 2. False. The factor selected as the exposure base should be simple and have an obvious relationship to losses. In addition to making the plan easier to use, simplicity is likely to enhance its perceived equity, even if the technical accuracy is not improved.
3. True. If underreporting of exposures is related to the underwriting cycle, this can have serious impacts on the data used for ratemaking, resulting in inaccurate rates.

4.37. <u>Temporal mismatch</u>: A mismatch between the timing of expected losses and an otherwise acceptable exposure base.
Example, Products Liability: Products liability coverage is triggered by the injury, but the exposure base is sales. For products with a long lifetime, there can be many years or even decades between the sale and injury, creating the temporal mismatch.
<u>Interpretive mismatch</u>: A mismatch arises through deliberate or accidental misinterpretation of the coverage trigger.
Example, Pollution Liability: Policy language has sometimes been interpreted to provide coverage for pollution arising out of disposal of waste by the insured prior to policy inception.
<u>Complexity of Hazard</u>: The situation may be so complex, that it is extremely difficult, if not impossible, to list of all factors affecting the expected level of losses.
Example, Directors and Officers Insurance: Examples of factors affecting the expected level of losses are: the number of directors and officers, business activities, change in revenues, change in profits, change in assets, number of stockholders, number of employees, hiring/firing policies, change in overall financial condition as rated by S&P, change in stock price, attractiveness as an acquisition, responses to past acquisition offers (e.g., “poison pills”), state of domicile, response to any recent emergencies (accidents, etc.), recent changes in management, whether and how executives are granted stock options, accounting firm used, etc. Determining which factors are most important and how they are related to expected loss is a complicated task. “For example, does a company with twice as many directors have twice the exposure to loss? Probably not.”
<u>Comments</u>: As an example of temporal mismatch, Bouska also mentions claims-made policies, used for example for medical malpractice insurance.
Interpretive mismatch has also arisen with respect to coverage for exposure to asbestos.
You should not try to list all of the factors affecting D&O insurance; two or three would have been enough. *In 2000, what factors might have entered into an attempt to determine the expected loss under Enron’s future D&O insurance policy? After Enron’s bankruptcy, its Directors and Officers Policy Limits of about $350 million were paid out in various settlements.*

4.38. All of the these statements are true.

4.39. a. The first step is to analyze the coverage offered and the coverage trigger to determine what factors influence the expected losses.
b. Expected losses (and premium) do not vary only with the exposure base, but also with many other factors which are built into the rating variables. (For example, most insurers charge a private passenger automobile policy with two vehicles, less than they would charge two separate policies, in order to reflect the decreased driving per vehicle and better loss experience.) Also there may be expenses that are not proportional to expected losses. (This is reflected in Workers Compensation insurance by use of the Expense Constant and Premium Discount.)

4.40. a. Temporal mismatch: the exposure base does not accurately reflect the timing of losses. For example, in products liability, sales is often used as an exposure base. If a company goes out of business, sales are zero, but the product liability exposure still exists if the products are still being used by the consumer.
b. Interpretive mismatch: if a misinterpretation of policy language occurs, this is not typically reflected in the exposure base. This occurred with the misinterpretation of the "sudden and accidental" pollution coverage in CGL policies. Insurance companies ended up paying more losses than what was reflected in the exposure base used.
c. Complexity of the hazard: certain risks are affected by a large number of factors often there are too many factors to list or the relationship between them and the risk is difficult to quantify. An example of this is directors' and officer's coverage. Many factors affect the risk of liability for D&O and they are difficult to quantify.

4.41. A. Other reasons were: simplification of rating and sensitivity to economic cycles.

4.42. B. Another potential problem is that even when the policy premium is based on the correct exposures, the coding of the exposure information into the computer records is often poor, with whole dollars frequently switched with "per hundreds" or "per thousands."

4.43. A. While the dividing line between the two may be somewhat arbitrary at times, the exposure base is not a rating variable. A failure to accurately predict the frequency and/or severity of future losses is usually a problem with our ratemaking tools, not the sign of a failing exposure base.

4.44. Claims-made policies are triggered by the notice of a claim, but rated on the normal occurrence exposure base, a physician-month in medical malpractice, for example.
Comment: This problem can be dealt with via the incorporation of a rating step to recognize the number of years under claims-made coverage.

4.45. Competitive pressures tend to degrade the exposure data. In a very competitive (soft) insurance market, a low price can be produced in a variety of ways. In some instances, it is easier for the underwriter to "low-ball" the exposure estimate. In theory, such "errors" will be corrected when the policy is audited, but that is usually eighteen months in the future (and after the renewal). Under the calendar/accident year ratemaking used for many lines, audit premiums are reported and fully earned in the calendar year of the audit, not the calendar year(s) when the policy premium was earned. Thus, even in the case of perfectly correct audits, a severe mismatch between the premiums and losses can be introduced by low exposure estimates. In a steady state, the rates eventually respond to a systematic underestimation of the exposures; but when the insurance cycle changes quickly and the "low-balling" stops abruptly, the problem of excessive rates appears.
Comment: Actuaries use old information to make rates for the future. The actions of underwriters can affect your ratemaking data. If the actuary does not take this into account, then the resulting rates will not be appropriate. This same phenomena would affect the premium or exposure development factors for Policy Year data. *For example, assume every year there are 100 exposures (perhaps payroll) accurately estimated at the time of writing and 70 in losses. Ignoring expenses, taxes, profits, etc., the rate would be set equal to the pure premium of .70. Things go along fine in this manner for several years. Assume in "Year 10" the underwriters start to low ball the exposures and only 80 is reported. The insureds pay (0.7)(80) = 56 rather than 70 in initial premium. If we assume at audit the "extra" 20 in exposures would be picked up, then the insureds would pay an extra (20)(0.7) = 14 in premiums. So the correct premium is collected in total for the coverage provided, but some of it has been delayed, which if one considered the time value of money would be significant. This audit exposure related to policies written in Year 10 would be reported as part of Calendar Year 12. Then the ratemaking data would look like this:*

Year	*Exposures*	*Losses*	*Pure Premium*	*Will be Used to Set Rate for Year*
9	*100*	*70*	*0.70*	*11*
10	*80*	*70*	*0.875*	*12*
11	*80*	*70*	*0.875*	*13*
12	*80 + 20 = 100*	*70*	*0.70*	*14*
13	*80 + 20 = 100*	*70*	*0.70*	*15*
14	*80 + 20 = 100*	*70*	*0.70*	*16*

So we would expect the rate to jump up in year 12 to an excessive level and then settle back down in year 14 to a steady state at the assumed correct level of 0.70.
Assume that in Year 20 the underwriters return to accurately estimating the exposures as 100 at policy inception. We would still get 20 in audit exposures from policies written in Year 18 reported in Calendar Year 20. Then the ratemaking data would look like this:

Year	*Exposures*	*Losses*	*Pure Premium*	*Will be Used to Set Rate for Year*
19	*80 + 20 = 100*	*70*	*0.70*	*21*
20	*100 + 20 = 120*	*70*	*0.583*	*22*
21	*100 + 20 = 120*	*70*	*0.583*	*23*
22	*100*	*70*	*0.70*	*24*
23	*100*	*70*	*0.70*	*25*

In year 22 the rates would decline to an inadequate level, but in two years would once again return to a steady state at the assumed correct level of 0.70. All of this assumes the actuaries when making rates are not clever enough to spot what is going on and make the appropriate adjustments to compensate for the distortions in the exposure data.

4.46. "The exposure base is not the true exposure. The exposure base is a proxy for the true exposure, which we are unable to know, both because it is constantly changing and because it is generally a function of a large number of variables. For example, the collision exposure of a private passenger auto is effectively zero when it is parked in a secure garage, somewhat higher when it is being driven on an isolated highway by an alert and competent driver, and substantially higher on a crowded street with a drunk driver. The exposure base (car-month) recognizes the average situation rather than these fluctuations in the true exposure to loss." The probability of an accident, as well as the average severity of an accident if it occurs, varies with the amount the car is driven, as well as for example the time of day, the day of the week, the speed of the vehicle, the weather conditions, the road conditions, the type of road, the number and type of other drivers on the road, the mental and physical condition of the driver, the condition of the vehicle, etc. Using a car-year or car-month as the exposure base does not capture any of this variation in the exposure to loss.
Comment: *One can think of a claims intensity which varies over time as a complicated function of the interaction of many different factors.*

4.47. a. 1. should accurately reflect the overall exposure to loss.
2. should be simple to compile. 3. should not be subject to manipulation.
4. should be responsive to change. 5. should reflect historical practice.
b. For Workers Compensation one could use payroll.
Payroll is a reasonably accurate reflection of exposure to indemnity losses, where the payments depend on the injured workers weekly wage. Payroll is not as an accurate reflection of the exposure to medical losses.
Payroll is relatively simple to compile, however one must adjust to remove the effects of higher wage rates for overtime.
Payroll is not easy to manipulate since it also used for tax purposes.
Payroll reflects changes in wage rates as well as the number of manhours worked.
Payroll has been used in most states, so that one can continue to use historical data.
Alternately, for Workers Compensation one could use manhours.
Manhours reflects the exposure to indemnity losses, but not as well as payrolls, since indemnity payments depend on the injured workers weekly wage. Manhours is reasonably accurate reflection of the exposure to medical losses.
Manhours is relatively simple to compile.
Manhours is not easy to manipulate, since wage records are used for tax purposes.
However, one could underreport manweeks for insurance purposes and still report the correct wages for tax purposes.
Manhours reflects the number of employees and how long they each work; however, manhours does not reflect changes in wage rates paid.
Manhours has not been used in most states, so that one would not be able to directly use historical data.
For Boatowners Insurance one could use “boatyears” Insuring one boat for one year would be one boatyear.
A boatyear does a good job of reflecting the overall exposure to loss, assuming one has a classification system to supplement it.
A boatyear is simple to compile.
A boatyear is hard to manipulate.
A boatyear is not responsive to inflationary changes, as would for example be the value of the boat.
Boatyears has been used, so that one can continue to use historical data.
Comment: In part b, discuss only one exposure base for each line of insurance. For Boatowners, one could also use amount of insurance years. *Boatowners Insurance provides coverage for your boat and boating equipment, including unattached accessories and trailer. This insurance also includes medical payment coverage and liability protection.*

4.48. B. All of the policies are written during 2004, and therefore there are 3 car years of written exposures for Calendar Year 2004.
For the first policy written May 1, 2004, 2/3 of its exposures are during Calendar Year 2004.
For the second policy written Nov. 1, 2004, 1/6 of its exposures are during Cal. Year 2004.
Calendar Year 2004 Earned Exposures are: (2)(2/3) + (1)(1/6) = 1.5 car years.
On December 31, 2004, these policies are providing coverage for three automobiles;
the Inforce Exposures are 3 cars.

4.49. a. "The exposure base is not the true exposure. The exposure base is a proxy for the true exposure, which we are unable to know, both because it is constantly changing and because it is generally a function of a large number of variables. For example, the collision exposure of a private passenger auto is effectively zero when it is parked in a secure garage, somewhat higher when it is being driven on an isolated highway by an alert and competent driver, and substantially higher on a crowded street with a drunk driver. The exposure base (car-month) recognizes the average situation rather than these fluctuations in the true exposure to loss."
"the exposure base is not a rating variable, although the dividing line between the two is somewhat arbitrary at times. In order to determine the correct manual premium for a risk, it is first necessary to classify the risk, based on whatever the rating variables are for the risk under consideration. Once the risk's classification is known, the rate for that classification is multiplied by the number of exposure units to produce the premium. As is noted above, the use of a variable in the exposure base implies a uniform and continuous multiplicative relationship between the variable and the expected losses; use as a rating element
implies a discrete, nonlinear relationship."
For example, one would use type of business and state, to determine the rate per $100 of payroll for Workers' Compensation Insurance. However, one would then multiply by the number of exposures (payroll/$100) to get the manual premium.
b. Possible reasons for using miles driven:
The more miles driven, the larger the probability of an accident and therefore claims costs to the insurer. The exposure to loss is approximately proportional to the miles driven.
Miles driven is responsive to changes in the exposure to loss, while car years is not.
Possible reasons for not using miles driven:
Miles driven could be difficult and costly to verify.
Miles driven is subject to manipulation by either the insured or agent.
Car years are currently used, and one should avoid changing exposure bases, due to problems caused by transitioning.
<u>Comment</u>: The probability of an accident per mile driven, as well as the average severity of an accident if it occurs, varies with the time of day, the day of the week, the speed of the vehicle, the weather conditions, the road conditions, the type of road, the number and type of other drivers on the road, the mental and physical condition of the driver, the condition of the vehicle, etc. Thus even within a class/territory cell, the expected losses would not be proportional to the miles driven. If the exposure was changed from car years to miles driven, the class/territory relativities would change. For example, part of the reason senior citizens are charged less than average per car-year, is because on average they drive less. It is likely that the expected losses per mile driven, as opposed to per car-year, are higher than average for senior citizens.

4.50. a. Usually medical benefits are unlimited for work related injuries. For a given classification, it is not clear that the frequency or severity of accidents and the resulting medical benefits has very much to do with the average hourly wage. Rather the number of accidents would depend largely on the man-hours worked. An alternative exposure that better reflected the expected medical losses might be man-hours worked, which measures the exposure to work related accidents.
b. If a unionized construction company pays more per hour, using unlimited payroll this company will have a higher payroll, and therefore pay more for its workers compensation coverage. To some extent its average severity for indemnity benefits will be higher. However, most states have a maximum benefit per week, and therefore the average severity will not be proportional to the higher pay per hour compared to a nonunion contractor. Thus unlimited payroll is less than an ideal exposure base, at least for these classifications. An alternative exposure that better reflected the expected indemnity losses might be payroll limited to a certain maximum amount per week.
Comment: There may be other acceptable answers to this question.

4.51. a. Argument for: Hours worked are related to the number of accidents. A worker is more likely to be injured if he works longer hours.
Argument against: Hours worked may be difficult to verify. Also indemnity benefits depend on the worker's average weekly wage, which is not captured by hours worked. Also hours worked is not inflation sensitive.
b. Argument for: Limited payroll is related to indemnity benefits since indemnity benefits depend on the worker's average weekly wage, but are capped at the state benefit maximum.
Argument against: Limited payroll is not related to medical expenses since most states' workers compensation statutes provide for unlimited medical expenses. Also medical expenses are not directly related to weekly wages.
c. Argument for: Unlimited payroll is easy to verify through tax records.
Argument against: Unlimited payroll does not reflect the fact that the indemnity benefits are capped at the state maximum. Also medical expenses are not directly related to weekly wages.
Comment: In each case, give only one argument for and one argument against.
While in the past most states used limited payroll, all of those states have moved to unlimited payroll. I believe that Massachusetts was the last state to make this change in the early 1980s.
Workers Compensation in Washington state uses instead hours worked as the exposure base.
In part (b), we would for example limit the payroll to $1000 per week per employee.
Thus an employee who made $1500 in week, would only contribute $1000 of payroll to the exposures. In contrast, in part (c) such an employee would contribute $1500 of payroll.
Let us assume that the benefits for someone out of work in State X were 60% of his weekly wage subject to a maximum of $600 per week. Then full-time workers who made $1000, $1500, or $2000 per week would get the same maximum benefit. However, in part (c) these three workers would contribute different amounts of exposure.
Unlimited payroll versus limited payroll can have a big practical impact for classes where the weekly wages vary significantly between employers, such as in construction. Recall that manual premium is the class rate times the exposure. Thus an employer who paid more per week on average would have a higher manual premium than an otherwise similar employer who paid on average less per week.

4.52. a. 1. should accurately reflect the overall exposure to loss.
2. should be simple to compile.
3. should not be subject to manipulation.
4. The exposure base should accurately reflect differences in exposure to loss.
Alternately:
1. It should be an accurate measure of the exposure to loss.
2. It should be easy for the insurer to determine.
3. It should be difficult for the insured to manipulate.
Alternately:
1. Reasonableness - The exposure unit should be a reasonable measure of the exposure to loss. The selected measure should directly relate to loss potential to the extent possible.
2. Ease of Determination - If an exposure base is not subject to determination, then an insurer can never be assured of receiving the proper premium for the actual exposure.
3. Responsiveness to Change - An exposure unit that reflects changes in the exposure to loss is preferable to one which does not.
4. Historical Practice - Where a significant body of historical exposure data is available, any change in the exposure base could render the prior history unusable. Since ratemaking generally depends upon the review of past statistical indications, exposure bases are rarely changed once they have been established.
Alternately:
1. Verifiable – can the insurance company determine if the information provided is accurate?
2. Practical – Is the exposure base easy to estimate, or does it require costly inspections?
3. Varies with hazard – Does the exposure base increase and decrease as the actual exposure does?
b. One possible exposure base is the number of credit cards owned by the insured.
1. Verifiable through credit bureaus.
2. Practical since credit checks are fairly inexpensive.
3. Varies with hazard since more credit cards implies a larger expected loss from unauthorized use of those cards.
Comment: In part a, give three different characteristics.
There are other possible answers to part b.
Unauthorized credit card use is not the same as identity theft.
Identity thieves use personal information to impersonate a victim, stealing from bank accounts, establishing phony insurance policies, opening unauthorized credit cards or obtaining unauthorized bank loans. Identity theft coverage provides the customer reimbursement for the expenses associated with the identity and credit restoration process including phone bills, lost wages, notary and certified mailing costs, and sometimes attorney fees (with the prior consent of the insurer.) Identity theft protection and resolution service is included in some companies' homeowner and auto policies at no additional cost. The service provides the consumer with a fraud specialist to assist and guide them through the process of restoring and protecting their identity. See http://www.iii.org/individuals/other/insurance/identitytheft/

4.53. a. 1) varies with the hazard.
2) verifiable.
3) not subject to manipulation.
4) follow historical precedent.
b. 1) Miles driven varies with the hazard; the more miles driven the more likely you are to get in an accident.
2) Miles driven may not be easy to verify. Someone would have to inspect each car at the end of the year to read the odometer.
3) Miles driven is subject to manipulation. If the insured was asked how many miles driven in a year without verification, he could easily lie. Even if the miles were based on odometer readings, there are still ways to turn the numbers on an odometer back.
4) Miles driven does not follow historical precedent.
Overall, the change to miles driven should not be made due to the problems with verification and the possibility of manipulation.
Comment: There are many other possible answers.

4.54. (a) **Proportional to Expected Loss**
The exposure base chosen should be directly proportional to expected loss. In other words, all else being equal, the expected loss of a policy with two exposures should be twice the expected loss of a similar policy with one exposure.
Practical
The exposure base should be practical. In other words, the selected base should be objective and relatively easy and inexpensive to obtain and verify.
Historical Precedence
There are great advantages to continuing to use the same exposure base as used in the past, and significant problems with switching the exposure base.
(b) For homeowners insurance the expected loss is not proportional to the market value of the home, since there are many small property losses, theft losses, liability losses, etc.
The market value of a home is not practical. It is relatively expensive to assess the market value of a home. In some parts of the country, at some points in time, the market value of a home may involve a lot of judgement; it is not objective.
Market value of the home is not currently used, so this does not satisfy the criteria of historical precedence.
(c) First, any change in exposure base can lead to large premium swings for individual insureds.
Second, a change in exposure base will require a change in the rating algorithm, which depending on the unique circumstances, may require a significant effort to adjust the rating systems, manuals, etc.
Third, ratemaking analysis is normally based on several years of data. A change in exposure base may necessitate significant data adjustments for future analyses.
Comment: Only give two criteria in part (a) and give only two reasons in part (c).
There may be other full credit answers to part (b).

4.55.

Policy	CY08 Written	CY08 Earned	PY07 Written	4/1/08 In-Force
1	0	1/2	1	1
2	-1/2	1/4	1/2	0
3	1	1	0	1
4	1/3	1/3	0	1
5	1	1/2	0	0

(a) For policy #2, there would have been 1 written exposure assigned to CY07
(which is closed at the end of 2007 for accounting purposes.)
Therefore, when this policy is cancelled midterm during 2008, we need to put -1/2 written exposure in CY08, so that in total for policy #2 we have exposures of: 1 - 1/2 = 1/2.
In the case of policy #4, we know it has been cancelled by the end of 2008, so we only have 1/3 exposure contributed to CY08 written exposures.

CY08 written exposures: -1/2 + 1 + 1/3 + 1 = $\mathbf{1\frac{5}{6}}$ car years.

(b) In each case, we take the portion of the year that the policy is in effect.

CY08 earned exposures: 1/2 + 1/4 + 1 + 1/3 + 1/2 = $\mathbf{2\frac{7}{12}}$ car years.

(c) We only look at policies written during 2007.
PY07 written exposures: 1 + 1/2 = **1.5** car years.
(d) We see whether or not the policy is in effect on 4/1/08.
In-force exposure as of April 1, 2008: 1 + 1 + 1 = **3** cars.
<u>Comment</u>: Since it is not stated otherwise, assume one car per policy.
Since as of June 30, 2009 all of the policies written in 2007 have expired, in fact all five policies have expired, the PY07 written and earned exposures are equal. As of an earlier date, such as April 1, 2008, they would differ.
For PY07 evaluated as of June 30, 2009, we know that policy #2 has been canceled midterm, and thus we have only 1/2 exposure.
As of Dec. 31, 2007, we would have instead 1 exposure contributed to PY07 from policy #2.

4.56. 1. Proportional to expected losses: the more miles one drives the larger the expected losses. For a given class of driver in a given territory it is approximately proportional, so this criterion is satisfied.
2. Practical. Objective and relatively easy and inexpensive to obtain and verify:
One could fit the car with a broadcasting GPS device that would allow the insurer record the miles driven. This would be relatively easy and inexpensive. It is verifiable since the insurer would be getting real time data rather than depending on odometer readings or asking the insured.
Thus his criterion is satisfied.
3. Historical precedence; is currently being used: since we would be switching from earned car years, this criterion is <u>not</u> satisfied.
<u>Comment</u>: Different conclusions would be OK with respect to the first two criteria; the main thing is to make sure you demonstrate that you know their meaning and say something sensible.

4.57. a. Only policies E and F were written in 2010; however, policy D was cancelled in 2010.
Policies E and F contribute 1/2 car year each to CY10 written.
Policy D contributes 1/2 car year to CY09 written.
Since Policy D is cancelled after the close of CY09, we need to include a negative in CY10.
Policy D is in effect for only 3 months, so it contributes -1/4 car years to CY10
-1/4 + 1/2 + 1/2 = **0.75 car years**.
b. Only policies E and F were written in 2010.
Policies E and F contribute 1/2 car year each to PY10 written.
1/2 + 1/2 = **1 car year**.

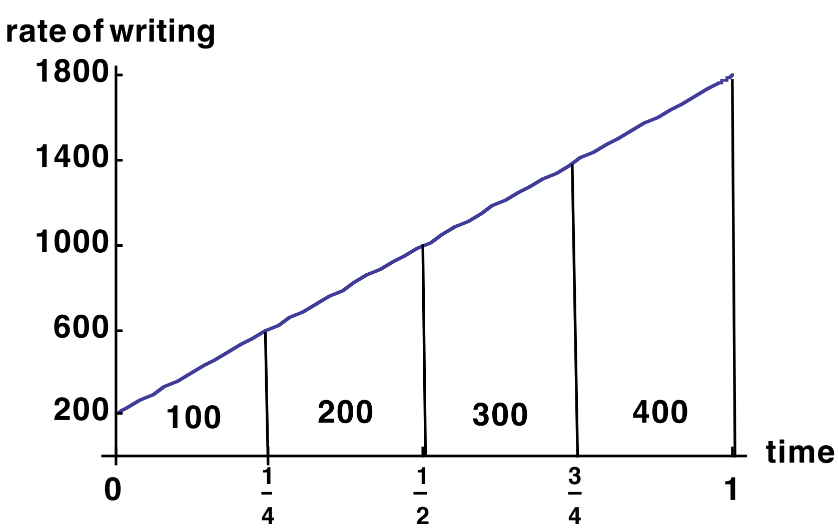
c.	Policy A: 0.	Policy B: 0.	Policy C: 1/12.
	Policy D: 1/12.	Policy E: 1/2	Policy F: 1/2

1/12 + 1/12 + 1/2 + 1/2 = **1.167 car years**.
d. Only policies E and F were written in 2010.
As of 12/31/10 or later they each contribute 1/2 car year to PY10 earned.
1/2 + 1/2 = **1 car year**
e. As of 1/1/10, Policies C, D, E are in-force. **3 in-force policies**.

4.58. The three criteria of a good exposure base:
1. Proportional to Expected Loss
2. Practical
3. Historical Precedence
An increase in either base would increase expected losses. However, payroll takes into account both how many workers are employed (in each class) but also how many hours they work. In for example construction, the number of hours worked per week can vary considerably from year to year; this can be affected by economic conditions. In addition some employees are part time rather than full time. Also payroll is proportional to the Average Weekly Wage, which affects the severity of claims, while number of employees does not. Thus I think that payroll is better based on the first criterion.
Payroll is collected for other purposes, such as taxes.
Payroll is well-defined and verifiable.
Thus payroll is practical
Number of employees varies from week to week, in some industries considerably.
Should some sort of average number of employees be used?
Also one should distinguish between full time and part time workers.
The number of employees is harder to verify than payroll.
The number of employees is easier for the employer to manipulate than payroll.
Number of employees may require additional expense to collect.
Thus, number of employees is less practical than payroll.
Payroll has been used for many years and thus is preferred based on historical precedent.
There would be significant difficulties transitioning from payroll to number of employees.
Possible difficulties include: large premium swings for individual insureds, reprogramming of the computers and other system changes, changes required in the rating algorithm, and for several years a lack of ratemaking data on the new exposure basis.
Based on all of the above, I would <u>not</u> recommended switching to number of employees.
<u>Comment</u>: Washington state uses hours worked (by class) as the exposure base.

4.59. a. Assuming uniform by quarter, the first set of policies is written on average February 15. Therefore, 7/8 of these exposures are earned in 2011.
(7/8)(100) + (5/8)(200) + (3/8)(300) + (1/8)(400) = **375** earned exposures.
b. The written exposures are increasing with time, which makes sense for a new insurer.
Thus it seems likely that for example there are more exposure written in September than in July.
However, the assumption in part (a) was that these are the same. Thus while the assumption is a good first approximation, it will lead to an overestimate of the earned exposures for 2011.
It would be more appropriate to assume written exposures increase linearly.
Comment: If one ignored the growth of exposures through the year, then half of the exposures would be assumed to be earned during 2011 or 500 exposures.
The assumption of uniform by quarter is much better than ignoring the growth all together.
If one assumed that exposures increased linearly, then for each quarter the average date of writing is further through the quarter than halfway.
For this data, a reasonable assumption would be that the annual rate at which exposures are written throughout the year increases linearly: $200 + 1600t$, $0 \le t \le 1$.
Then the exposures written during the first quarter are:

$$\int_0^{0.25} (200 + 1600t)\, dt = (200)(0.25) + (1600/2)(0.25^2) = 100.$$



Similarly, the exposures written during the last quarter are:

$$\int_{0.75}^{1} (200 + 1600t)\, dt = (200)(1 - 0.75) + (1600/2)(1^2 - 0.75^2) = 400.$$

Then the earned exposures for 2011 are:

$$\int_0^1 (1 - t)(200 + 1600t)\, dt = 200 \int_0^1 1 + 7t - 8t^2\, dt = (200)(1 + 7/2 - 8/3) = 366.67.$$

4.60. (a) Half of the exposures for A are earned in CY2011: (50)(2)/2 = 50.
Alternately, one year of A is earned in CY2011: (50)(1) = 50.
Half of the exposures for B are earned in CY2011: (100)(2)/2 = 100.
Alternately, one year of B is earned in CY2011: (100)(1) = 100.
CY2011 Earned Exposures are: 50 + 100 = **150**.
(b) Evaluated as of 12/31/2010:
Only half of A has been earned: (50)(2)/2 = 50.
Only one quarter of B has been earned: (100)(2)/4 = 50.
Total earned exposures: 50 + 50 = **100**.
Evaluated as of 12/31/2011:
All of A has been earned: (50)(2) = 100.
Only three quarters of B has been earned: (100)(2)(3/4) = 150.
Total earned exposures: 100 + 150 = **250**.
(c) Evaluated as of 12/31/2010:
B PY2010 written exposures: (100)(2) = **200**.
Evaluated as of 12/31/2011:
B has been canceled and policies were in effect for only 1/2 year rather than 2 years.
B PY2010 written exposures: (100)(1/2) = **50**.
Alternately, B has been canceled and we need to subtract 3/4 of the exposures from the written.
B written exposures: (100)(2) - (100)(2)(3/4) = **50**.
(d) CY2010 B written exposures: (100)(2) = **200**.
Due to the cancelation, the written exposures decrease by 100.
However, since the cancelation took place after the end of 2010, the change goes into CY2011.
CY2011 B written exposures : (-100)(2)/2 = **-100**.
Comment: The calendar year exposures are not changed after the end of the year.
In part (d), the total written exposures for the two calendar years total to the correct 100.

4.61. The three criteria of a good exposure base:
1. Proportional to Expected Loss
2. Practical
3. Historical Precedence

An increase in hours worked would increase the frequency of losses; but the payroll would also increase proportionally if more hours are worked. Payroll is also proportional to the Average Weekly Wage, which affects the severity of indemnity benefits, while hours worked is not. Since indemnity benefits are the majority of dollars paid, I think that payroll is better based on the first criterion.
On the other hand, the severity of medical benefits do not obviously vary with payroll, so there is some merit to hours worked.
Payroll is collected for other purposes, such as taxes.
Payroll is well-defined and verifiable.
Thus payroll is practical.
Number of hours worked is harder to verify than payroll.
Number of hours worked is easier for the employer to manipulate than payroll.
Number of hours worked may require additional expense to collect.
Thus, number of hours worked is less practical than payroll.
Payroll has been used for many years and thus is preferred based on historical precedent.
There would be significant difficulties transitioning from payroll to hours worked.
Possible difficulties include: large premium swings for individual insureds, reprogramming of the computers and other system changes, changes required in the rating algorithm, and for several years a lack of ratemaking data on the new exposure basis.
Based on all of the above, I would not recommended switching to hours worked.
Comment: Washington state uses hours worked (by class) as the exposure base.

4.62. (a) Since $D > 0$, the policy was in force on February 1, 2013; thus the policy could <u>not</u> have been written after February 1, 2013, and also must have been canceled after February 1, 2013.
$A < 0$, therefore, the policy was written in 2012 and cancelled in 2013.
Since $D > 0$ and it was an annual policy, it must have been written after February 1, 2012.
If for example, it were written on March 1, 2012, then as February 1, 2013, B would be 11 months and C would be 1 month. However, we want $B < C$, so this is no good.
If instead it were written on August 2, 2012, then as February 1, 2013, B would be just less than 6 months and C would be just more than 6 months; $B < C$.
Put another way, in order for $C > B$ since these are annual policies, we must have more than 6 months left of the policy term on February 1, 2013.
So the range of effective dates is August 2, 2012 to December 31, 2012.
(b) $A < 0$, therefore, the policy was written in 2012 and cancelled in 2013.
If it had been cancelled 2/1/2013, then $D = 0$, but we want $D > 0$.
If it had been cancelled 12/31/2013, then since it was written in 2012, the policy could not have been cancelled midterm.
Thus it must have been cancelled after February 1, 2013 and before December 31, 2013.
(c) $A < 0$, therefore, the policy was written in 2012 and cancelled in 2013.
$D > 0$ implies the policy is in-force on 7/1/2013.
(Assume for simplicity one car is insured.)
Thus since the annual policy was written in 2012, as of July 1, 2013, $B \geq 1/2$ and $C \leq 1/2$.
Thus it cannot be that $B < C$.
<u>Comment</u>: A very unusual question; challenging under exam conditions.

4.63. (a) 1. Proportional to Expected Loss
2. Practical: objective, easy to obtain, and inexpensive to verify.
3. Historical Precedence
(b) The number of hours worked should be closer to proportional to expected losses than would the number of employees; the more hours someone works the more chance for a work place accident and thus a workers compensation claim.
The number of hours should be recorded for other purposes and thus should be practical (with the possible exception of workers not paid by the number of hours worked. However, such workers usually have much smaller expected losses than those who are paid based on the number of hours worked.)
On the other hand, number of hours worked would be more subject to manipulation by the employer than would number of employees.
The number of hours worked has not been used prior, so this does not satisfy the criteria of historical precedence. There would be difficulties in transitioning to a new exposure base with:
difficulty estimating class rates, expensive changes to systems, and the likelihood of large premium swings for individual insureds.
(c) Frequency is number of claims per exposure. The number of hours worked is greater than the number of employees, and thus the frequency per exposure would decrease.
(d) Severity is dollars of loss per claim, and should be unaffected by a change in exposure base.
(e) The exposure base change should not directly affect the companies loss ratio, provided the insurer manages to create new rates by class based on the new exposure base which produce the same total premium as would the old rates based on the old exposure base.
(This is not an easy task.)
However, this insurer would now attract employers whose workers have fewer hours worked per week than average for a class, and would lose employers whose workers have more hours worked per week than average for a class. The former are expected to have fewer work place accidents on average and thus lower workers compensation costs. This favorable selection would lead to an improvement in the insurer’s loss ratio, all else being equal.
Alternately, the exposure base change could lead to wide premium swings. Thus many good customers will go to other insurers, while bad risks who cannot get coverage elsewhere remain. Therefore this insurer’s loss ratio is likely to get worse.
<u>Comment</u>: Switching to a new exposure base that is closer to proportional to expected losses has a similar affect to introducing a new useful classification variable. The insurer can attract better insureds which are currently being written by competitors who stay with the old scheme.

4.64. i. While annual fuel expense would vary somewhat with the potential for loss, it would also be affected by the price of gasoline (or diesel fuel) and the fuel efficiency of vehicles neither of which varies with the expected losses. Thus this exposure is <u>not</u> proportional to loss costs.
For example, let us assume you make rates with cost of fuel as the exposure base, based on an average price of gas of $3 a gallon. However, during the period of time the rates are in effect the average price of gas is $2.50, then the projected premiums will be inadequate when the rates are applied to the annual fuel expense. If instead the price of average price of gas during the period of time the rates are in effect is $3.50, then the premiums will be excessive.
Annual fuel expense is practical, since the insured business already keeps track of this for other purposes. (Alternately, they may <u>not</u> keep careful track for each vehicle <u>separately</u>, which would be important if there are vehicles of different types in the insured fleet.)
Annual fuel expense lacks historical precedence. Changing to a new exposure base would cause large premium swings, it would be costly to implement, and would cause extra work for information technology and actuarial staff.
I would <u>not</u> change to annual fuel expense as the exposure base since it is less proportional to losses than caryears.
ii. As the number of miles driven increases, the expected number of accidents increases, and thus the expected losses increase. While it would matter whether these are miles driven on a highway or city streets, this exposure base is more closely proportional to expected losses than is caryears.
Miles driven may be practical, depending on the technology used to determine the miles driven. (Odometer readings can be manipulated. On the other hand, a fleet of interstate trucks may already for other purposes keep careful track of the miles driven by each vehicle.)
Number of miles driven lacks historical precedence. Changing to a new exposure base would cause large premium swings, it would be costly to implement, and would cause extra work for IT and actuarial staff.
Assuming that the appropriate affordable technology (for example telematics) is available to verify the miles driven by each vehicle, I would change to number of miles driven as the exposure base, since it is more proportional to expected losses than caryears.
<u>Comment</u>: One could instead decide not to change the exposure base to either of the two choices.

4.65. (a) 6-month policies written during January 1 through June 30, 2013 contribute none of their written exposures to CY14 earned exposures.
6-month policies written during July 1 through December 31, 2013 contribute on average half of their written exposures to CY14 earned exposures.
12-month policies written during January 1 through June 30, 2014 contribute on average (1 + 1/2)/2 = 3/4 of their written exposures to CY14 earned exposures.
12-month policies written during July 1 through December 31, 2014 contribute on average (1/2 + 0)/2 = 1/4 of their written exposures to CY14 earned exposures.
(0)(106) + (1/2)(107.5) + (3/4)(210) + (1/4)(45) = **222.5**.

Time Period	Written caryears	Avg. Written Date	% earned in 2014	Earned caryears in 2014
1/1/13-6/30/13	106	4/1/13	0%	(0%)(106) = 0
7/1/13-12/31/13	107.5	10/1/13	50%	(50%)(107.5) = 53.75
1/1/14-6/30/14	210	4/1/14	75%	(75%)(210) = 157.5
7/1/14-12/31/14	45	10/1/14	25%	(25%)(45) = 11.25
Total				**222.5**

(b) Of the 6-month policies written during the second half of 2013, if 77% renew, then since they would be annual policies, they would produce (2)(77%)(107.5) = 165.55 caryears. We wrote 45 exposures during the second half of 2014, which are new policies.
If we had a similar 45 new exposures in the first half of 2014, then we would expect 166.55 + 45 = 210.55 caryears written during the first half of 2014. Since there were 210 caryears written during the first half of 2014, the retention ratio seems about the same as it was prior to the change in policy term.
We can estimate the retention ratio as: (210 - 45)/ {(2)(107.5)} = 76.7%, very close to 77%.
(c) Clearly an assumption of uniform writings each year would be inappropriate for the calculation of calendar year 2014 earned car years, since we <u>observe</u> that there are many more exposures written in the first half of 2014 than in the second half. In the absence of any additional information, an assumption of uniform writings each <u>half</u> year seems appropriate for the calculation of calendar year 2014 earned car years, as was done in part (a).
Alternately, the policies up for renewal from prior to PY2014 are 6 month which means they will be up for renewal in the first half of 2014. Since all policies are shifted from six months to annual, they will be up for renewal in the first half of 2015; the next wave of written car years will not occur until the first half of CY 2015 on these annual renewals. Thus the assumption of uniform writing during 2014 is inappropriate.
<u>Comment</u>: We have to assume no cancellations or endorsements; otherwise an endorsement can add cars to a policy, which may be some of the written exposures during the second half of CY14. Given enough time the number of exposures written would eventually even out between the two halves of a year, although it might take decades.

Retention Ratio = $\frac{\text{Number of Policies Renewed}}{\text{Number of Potential Renewal Policies}}$. Here there is no way to determine the number of policies invited to renew or that are able to be renewed.

4.66. (a) I propose **sales** as the exposure base.
Sales is proportional to expected losses. More sales corresponds to more customers, which corresponds to more expected losses. There is a uniform and continuous multiplicative relationship between sales and the expected losses. Sales is responsive to a change in the amount of business done by the restaurant and thus a change in exposure to risk. Sales also has the advantage of being inflation sensitive. During periods of high inflation, average severity will increase but so will sales.
Sales are practical. Sales are objective and well defined, relatively easy and inexpensive to obtain, and relatively easy and inexpensive to verify. Thus sales can be consistently measured, and are not subject to manipulation by insureds, agents, or underwriters.
(b) I propose a **combination of average number of occupied beds and annual number of outpatient visits**.
Average number of occupied beds is proportional to expected losses from liability related to inpatient events, while annual number of outpatient visits is proportional to expected losses from liability related to outpatient events.
These are practical. They are objective and well defined, relatively easy and inexpensive to obtain, and relatively easy and inexpensive to verify. They are each items hospitals have to keep track of anyway. They can be consistently measured, and are not subject to manipulation by insureds, agents, or underwriters.
Comment: In part (b) one could instead propose gross revenue from patients; this would have the advantage of being inflation sensitive. Other possibilities: number of patients, number of medical professionals (doctors and nurses) on staff, payroll for medical professionals,
number of physician-years (the number of physicians adjusted for those who are only on staff for part of a year in analogy to houseyears for homeowners), occupied beds.
In part (a) one could instead propose payroll.
In my solution I have not discussed the third desirable feature, Historical Precedence, since the exposure bases for these lines of insurance are not covered on the syllabus of this exam. You could do so if you are familiar with what is currently being used as an exposure base.
I believe sales is used for a general liability policy for a restaurant.
I believe a combination of average number of occupied beds and number of outpatient visits are used (by at least some insurers) for a hospital professional liability policy.

4.67. I will assume that each policy only covers one car.
(a) Only the six-month policies written in 2014 contribute: 105/2 + 100/2 = **102.5 car-years**.
(b) Only the six-month policies written October 1, 2014 are in force on December 31, 2014.
Since the question asks for in-force car-years one could answer: 100/2 = **50 car-years**.
(c) The six-month policies written October 1, 2014 contribute half their exposures.
The six-month policies written April 1, 2015 contribute all of their exposures.
The six-month policies written October 1, 2015 contribute half their exposures.
100/4 + 110/2 + 105/4 = **106.25** car-years.
Comment: In part (b), I think a better answer to a better written question would be 100 cars, since as of December 31, 2014 the insurer is covering 100 cars.

4.68. I will assume that each policy insures one exposure per year.
a. Then the written exposures for CY15 are: (2)(1000) + (1)(1000) = **3000**.
Each of the policies written 4/1/15 contributes 3/4 of an earned exposure to CY15.
Each of the policies written 7/1/15 contributes 1/2 of an earned exposure to CY15.
The earned exposures for CY15 are: (3/4)(1000) + (1/2)(1000) = **1250**.
b. Each of the policies written 4/1/15 contributes 1 earned exposure to CY16.
(Each of these policies expires at the end of June 2017.)
Each of the policies written 7/1/15 contributes 1/2 of an earned exposure to CY16.
The earned exposures for CY16 are: (1)(1000) + (1/2)(1000) = **1500**.
Comment: The question should have said for example, that each policy covers one home.

4.69. (a) CY17 written exposures: (4)(1/2) + (2)(1/2) = **3**.
(b) CY17 earned exposures: (4)(1/2) + (2)(1/4) = **2.5**.
(c) As of September 30, 2017 we do not know of the change,
so the PY17 written exposures are **4**.
(d) 1. Proportional to Expected Loss:
The expected losses varies only slightly with the number of occupants.
For most perils, the number of occupants does not affect the expected losses.
4 occupants probably increase the expected losses very slightly compared to 2 occupants due to increased number of guests and thus an increased chance of liability claims. However, the expected homeowners losses are nowhere close to twice as much for twice the number of occupants.
Thus number of occupants violates this criterion.
2. Practical:
It would be difficult and costly to keep track of and verify the number of occupants.
It would be subject to manipulation by the insured.
It would be ambiguous; what if a relative comes to stay for a visit, how long does the visit have to be before they count as an occupant.
Thus number of occupants violates this criterion.
3. Historical Precedence:
Number of occupants is not currently used for homeowners insurance; house years (or amount of insurance years) is currently used. There would be a lack of industry benchmarks. There would also be serious difficulties in ratemaking for several years while moving to this new exposure base; all of the historical data would be on the old basis. Thus number of occupants violates this criterion.
Comment: While the original policy has 4 exposures, since the change takes place during 2017 it affects the CY17 exposures, similarly to endorsing an auto policy to add or remove a vehicle.
Assuming there are no further changes, at ultimate PY17 written exposures will be 3.
Number of occupants could be used as either a rating variable or underwriting variable.

4.70. 1. Proportional to Expected Loss:
For physical damage, the more expensive the boat the more monetary damage to the boat. While expected loss from physical damage coverage may not go up exactly proportional to the insured value of the boat, it is approximately the case, and thus this criteria is satisfied.
(Based on homeowners insurance and the existence of partial losses, I would expect that expected losses to go up somewhat less than linearly with insured value.)
The higher the insured value of the boat, the larger the boat is likely to be. Larger boats are able to do more damage to other boats and more likely to seriously injure passengers of those other boats. Thus expected liability losses should increase with insured value, however, it might not be close to proportional.
Insured value sort of satisfies this criterion; more so for physical damage than liability.
While two boats have twice the expected loss of one boat, all else being equal, the expected loss is very different for boats of very different value. Thus, boat-years does <u>not</u> satisfy this criterion.
2. Practical:
For physical damage coverages, the insurer already collects and uses the insured value of the boat. Thus using insured value would be neither difficult nor costly.
It would not be particularly subject to manipulation by the insured.
Insured value satisfies this criterion.
3. Historical Precedence:
Since it is not currently used as the exposure base, insured value violates this criterion.
There would be a lack of industry benchmarks. There would also be some difficulties in ratemaking for several years while moving to this new exposure base; this would be made easier since insured value is already collected for boats that buy physical damage coverage.

I would recommend staying with boat-years as the exposure base. This will avoid the disruption of changing the exposure base, for very little if any benefit. Insured value should continue to be used to rate physical damage coverages. If insured value is not being used to help rate liability coverages, nor something related like size of boat, then it would be worthwhile to start using insured value to help rate liability coverages.
<u>Comment</u>: There are many possible full credit answers.

4.71. All policies are 6-month and written in 2018.
(a) CY 2018 written exposures: 2/2 + 3/2 + 1/2 + 2/2 + 1/2 = **4.5 caryears**.
(b) Policy D is 1/2 earned in 2018. Policy E is 1/3 earned in 2018.
CY 2018 earned exposures: 2/2 + 3/2 + 1/2 + (2/2)(1/2) + (1/2)(1/3) = **3.67 caryears**.
(c) As of February 28, 2019, only 5/6 of Policy D is earned and only 4/6 of E is earned.
PY 2018 earned exposures as of February 28, 2019:
2/2 + 3/2 + 1/2 + (2/2)(5/6) + (1/2)(4/6) = **4.167 caryears**.
(d) As of October 15, 2018, policies C and D are in force.
(Policy E has yet to become effective.)
Thus the in-force exposures as of October 15, 2018 are: 1 + 2 = **3 cars**.
Alternately, since the policies are semi-annual: (1+2)/2 = **1.5 caryears**.
(e) 1. Proportional to expected losses: Exposure base implies a uniform and continuous multiplicative relationship between the variable and the expected losses. The exposure base should be responsive to any change in exposure to risk.
As the number of miles driven increases, the expected number of accidents increases, and thus the expected losses increase. While it would matter whether these are miles driven on a highway or city streets, on balance this criterion is satisfied.
Miles driven does <u>not</u> satisfy this criteria for Other Than Collision Coverage (Comprehensive).
2. Practical. Objective and relatively easy and inexpensive to obtain and verify:
This will allow the exposure base to be consistently measured and not subject to manipulation by insureds, agents, or underwriters.
One could fit the car with a telemetric device that would allow the insurer to record the miles driven. This would be relatively easy and inexpensive. It is verifiable and not subject to manipulation. Thus this criterion is satisfied.
3. Historical precedence: is this exposure base currently being used by this insurer.
If miles driven is currently being used then this criteria is satisfied.
Since it satisfies all three criteria, I conclude that miles driven is a good exposure base to use.
<u>Comment</u>: There are other reasonable answers to part (e).
If caryears are currently being used, then switching to miles driven would incur some expense.
For several years one would not have historical exposure data useful for ratemaking. Also switching exposure bases could produce large swings in premiums for individual insureds.
The CAS accepted either of the two answers to part (d).
In part (d), I would have followed Table 4.15 in <u>Basic Ratemaking</u> and answered 3 cars, which makes sense to me.
However, according to Basic Ratemaking:
"Most companies define insured units to be the count of items exposed to loss at a given point in time. For example, if an automobile policy insures three cars, that one policy could contribute three in-force exposures at a given point in time. Alternatively, some companies may define insured unit in terms of the number of policies (the auto example above would have one in-force exposure under this definition) or the written exposures (in the auto example, there could be three in-force exposures if the term is annual, or 1.5 in-force exposures if the term is semiannual)."

4.72. (a) We can cumulate the earned exposures through March 31, 2018:
5 + 247.5 + 427.5 + 52.5 + 53.75 = 786.25.
However, this includes a contribution from policies written in the first quarter of 2018.
Based on the contribution of policies written in the first quarter of 2017 to the earned exposures in the first quarter of 2017, I estimate the similar contribution of 2018 Q1 as: (125)(5/100) = 6.25.
Thus, the 2017 policy year earned exposures as of March 31, 2018 are: 786.25 - 6.25 = **780**.
(b) This includes 2017 Q3, 2017 Q4, 2018 Q1, 2018 Q2:
400 + 100 + 125 + 550 = **1175**.
(c) Assume that they meant to ask for the unearned exposures as of the end of 2018.
Given that the insurer started writing business on January 1, 2017, we can add up all of the written exposures by quarter and subtract the sum of all of the earned exposures by quarter, through the end of 2018:
2230 - 1936.5 = **293.5**.
(d) We need to figure out how the written exposures in a quarter are earned over time.
Each annual policy (written at the beginning of a quarter) is earned over the four quarters it is in effect.
It turns out the given table is based on the following.
For each policy, regardless of in which quarter it is written, it earns 5% in Q1 of a year, 45% in Q2 of a year, 45% in Q3 of a year, and the remaining 5% of its written exposures in Q4 of a year.
There would be contributions to Q1 of 2019 earned exposures from the written exposures in each of the last three quarters of 2018, since annual policies written in these three quarters are still in effect during the first quarter of 2019. (There is no new business written in 2019.)
These policies each contribute 5% of their written exposures to Q1 of 2019 earned exposures:
(5%)(550 + 475 + 30) = **52.75**.
<u>Comment</u>: A question that covers a situation <u>not</u> discussed in any detail in the syllabus reading. In my opinion, it was totally unfair to expect students to infer what the underlying earnings pattern was under exam conditions; personally I would not have been able to.
For example, 2018 Q1: (450 + 400 + 100 + 125)(5%) = 53.75.
For example, 2018 Q2: (400 + 100 + 125 + 550)(45%) = 528.75.
This earnings pattern is based on some kind of seasonality; for this line of insurance most of the incidents that result in claims must happen in the second and third quarters of a year.
Unearned exposures is not a useful concept for a calendar year as opposed to a policy year.
Nevertheless, I would have answered part (c) for CY18:
unearned exposures = written exposures - earned exposures =
(125 + 550 + 475 + 30) - (53.75 + 528.75 = 562.50 + 59) = 1180 - 1204 = -24.
This was <u>not</u> the solution given in the CAS Examiner's Report.

Section 5, Premium

Premium is the amount the insured pays for insurance coverage.

Written premiums: those dollars of premiums on policies written during the period in question. Written premium ⇔ written exposures.

Premiums are earned as coverage is provided throughout the policy term.
Normally, premium is earned at a constant rate over the policy effective period.
Earned premium ⇔ earned exposures.

Calendar Year Data:

2006 Calendar Year Written Premium:
Premium on policies with effective dates from 1/1/06 to 12/31/06.
This would differ somewhat for lines of insurance with audited premiums, such as Workers Compensation or General Liability.[180] For example, a policy written 1/1/06 would have its final audit in early 2007. The premium that results from this final audit would be booked in 2007.

2006 Calendar Year Earned Premium: Premiums earned during 2006.
Includes for example 1/4 of the premium for an annual policy with effective date 4/1/05, and 1/2 of the premium for an annual policy with effective date 7/1/06.

Exercise: An annual policy is written with effective date October 1, 2002. The premium is $400.
What are the contributions to the Calendar Year 2002 and 2003 written and earned premiums?
[Solution: All of the $400 contributes to CY2002 written premium; none contributes to CY2003 written premium. One quarter of the $400, or $100 contributes to CY2002 earned premium; three quarters of the $400, or $300 contributes to CY2003 earned premium.
Comment: CY Written and Earned Premiums are usually not equal to each other.]

For any policy, the average date of earning is the midpoint of the period for which the policy provides coverage: the date of writing plus (policy term)/2.

For CY 2006 written premiums, the average date of writing is 7/1/06.
⇒ For CY 2006 written premiums, the average date of earning is: 7/1/06 + (policy term)/2.
For CY 2006 earned premiums, the average date of earning is 7/1/06.
⇒ For CY 2006 earned premiums, the average date of writing is: 7/1/06 - (policy term)/2.

[180] Also, as discussed subsequently, CY written premium would be affected by endorsements and cancellations.

Policy Year Data:

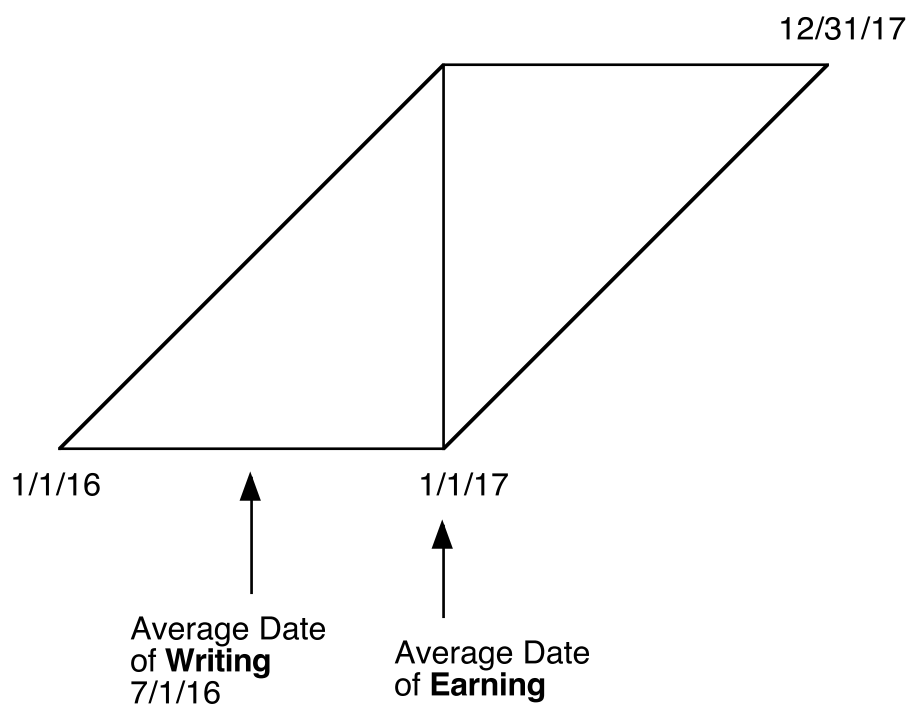
2006 Policy Year Written Premium:
Premium on policies with effective dates from 1/1/06 to 12/31/06.

The average date of writing for Policy Year 2006 is 7/1/06.

2016 Policy Year Earned Premium:
Premiums earned on policies with effective dates from 1/1/16 to 12/31/16.
As of 12/31/16, only 3/4 of the premium for an annual policy with effective date 4/1/16 has been earned.

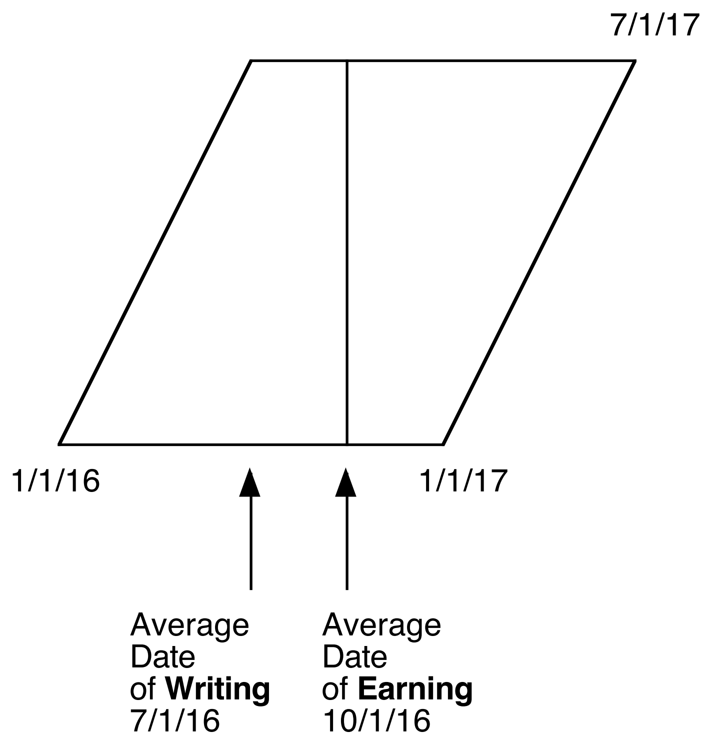
For any policy, the average date of earning is the midpoint of the period for which the policy provides coverage: the date of writing plus (policy term)/2.
⇒ For PY 2016 premiums, the average date of earning is: 7/1/16 + (policy term)/2.

For annual policies, the average date of earning for Policy 2016 is:
7/1/16 + 6 months = 1/1/17.



In contrast, for six-month policies, the average date of earning for Policy 2016 is:
7/1/16 + 3 months = 10/1/16.

In the following diagram, the slanting lines used to represent the PY, have slope two, one over the policy term. Half the area representing Policy Year 2016 is on either side of the vertical line at 10/1/16, the average date of earning.



The average date of writing remains 7/1/16, regardless of the policy term.

Premium Development:

Policy Year Earned Premiums develop as they become more mature.
At ultimate, Policy Year Earned Premiums are equal to Policy Year Written Premiums.

For example, for a line of insurance without audits, we might have for PY 2010 Premiums:[181]

	@12/31/10	@12/31/11
Written:	500	500
Earned:	250	500

As discussed previously, certain exposure bases such as sales or payroll are usually subject to audit. For example, for a large Workers Compensation policy assumed payrolls by class and state are used for purposes of determining the preliminary premium. Sometime after expiration of the policy the actual payrolls during the policy period are determined and used to calculate the final premium.[182] The actual payrolls will turn out to be different than assumed payrolls.

Therefore, Policy Year written exposures take a while to be final, and therefore so do Policy Year written premiums. Policy Year written premiums develop either upwards or downwards.

For example, the audits of the policies written late in 2010 will not have been completed until sometime in early 2012. Thus Policy Year 2010 written exposures will not be final until then.

For example, for a line of insurance with premium audits, we might have for PY 2010 Premiums:

	@12/31/10	@6/30/11	@12/31/11	@6/30/12
Written:	500	510	520	525
Earned:	250	380	520	525

Actuaries determine premium development factors based on past historical data.

[181] As of 12/31/10, either many or all of the policies making up PY10 have yet to expire, depending on the policy term.
[182] There may be audits every quarter or more frequently, in addition to the final audit.
The larger the insured, the more frequent the audits tend to be.

For example, here is a triangle of Workers Compensation Earned Premium by Policy Year:[183]

	Report 0	Report 1	Report 2	Report 3
Policy Year 1	869.550	1,412.750	1,440.185	1,444.559
Policy Year 2	738.461	1,237.551	1,263.047	1,262.312
Policy Year 3	729.324	1,266.620	1,274.723	1,273.771
Policy Year 4	612.189	1,134.360	1,145.491	1,156.457
Policy Year 5	565.457	1,066.840	1,085.465	
Policy Year 6	589.903	1,014.239		
Policy Year 7	552.572			

The observed development for Policy Year 6 from report 0 to 1 is:
1,014.239/589.903 = 1.719.
We calculate a triangle of similar ratios:

	0 to 1	1 to 2	2 to 3
Policy Year 1	1.625	1.019	1.003
Policy Year 2	1.676	1.021	0.999
Policy Year 3	1.737	1.006	0.999
Policy Year 4	1.853	1.010	1.010
Policy Year 5	1.887	1.017	
Policy Year 6	1.719		

While the pattern is somewhat stable, there is variation from year to year and over time. The actuary needs to select factors to use to develop immature years to ultimate.[184] Often this involves taking an average or weighted average of recent factors.

For illustration, let us use the average of the latest two factors.[185]

	0 to 1	1 to 2	2 to 3
2-year average	1.803	1.014	1.004

[183] Report 0 is at the end of the Policy Year. Report 1 is 12 month later, etc.
Reports beyond 3 would also be collected. There is little development beyond third report, and therefore for simplicity of the illustration these later reports are not shown. However, retrospective rating adjustments do cause Workers Compensation Premiums to change somewhat at later reports.
This data was compiled as of the end of Year 7, when the latest report for Policy Year 7 is Report 0.
Similarly, the latest report for Policy Year 6 is Report 1.
The premiums are in millions of dollars.
[184] The same type of thing will have to be done for losses and lae.
[185] Other selections would also have been reasonable. On the exam follow any instructions given. In any case, briefly state what you are doing.

We obtain the following premium development factors to ultimate:[186]

Report 0 to Ultimate: (1.803)(1.014)(1.004) = 1.836.
Report 1 to Ultimate: (1.014)(1.004) = 1.018.
Report 2 to Ultimate: 1.004.

We use these development factors to estimate the ultimate earned premiums for the immature Policy Years:

Policy Year 5: (1.004)(1,085.465) = 1,090.
Policy Year 6: (1.018)(1,014.239) = 1,032.
Policy Year 7: (1.836)(552.572) = 1015.[187]

Unearned Premiums:

Unearned Premiums are the portion of premiums for which coverage has not been provided by a certain date. Unearned Premiums ⇔ unearned exposures.

Exercise: An annual policy with $600 in premium is written with effective date May 1, 2008.
What is the unearned premium as of December 31, 2008?
What are the unearned premium as of December 31, 2009?
[Solution: $600/3 = $200 in unearned premium as of December 31, 2008.
No unearned premium as of December 31, 2009.]

For an individual policy at any given point in time:
Written Premiums = Earned Premiums + Unearned Premiums.

Calendar Year Earned Premium: CY Written Premium minus the change in unearned premium reserves during a given year.

Exercise: Premium Written in Calendar Year 2016: 400 million.
Unearned Premium as of December 31, 2015: 210 million.
Unearned Premium as of December 31, 2016: 190 million.
Determine the earned premium for Calendar Year 2016.
[Solution: 400 - (190 - 210) = 420 million.
Comment: The unearned premium reserve decreased, and thus the earned premium increased.]

[186] Treating for simplicity third report as ultimate.
[187] Since it is based on an incomplete policy year, the estimate for PY7 is subject to a lot of potential error.

Inforce Premiums:

Inforce premium is the total amount of full-term premium for all policies in effect at a given date.[188] [189] Inforce premiums ⇔ inforce exposures.

Care must be taken with the interpretation of inforce premiums.

For example, assume that ABC Insurer sells 12 month policies each with a premium of $1000. If it sold 1 million policies, its inforce premium would be $1 billion. Assume that ABC insurer decided to switch upon renewal all of its policies to 6 months each with a premium of $500. For simplicity assume that everything else stays the same. Then after sufficient time for the switch to be fully implemented, ABC's inforce premiums would now be $0.5 billion.

Even though ABC is writing the same amount of insurance, its inforce premium has changed. Thus one has to use care in interpreting inforce premiums. The same problem can arise if one is trying to compare inforce premiums between different insurers one of whom writes 6 month policies and the other writes 12 month policies.

"As in-force premium is the best estimate of the company's mix of business as of a given date, the most recent in-force premium is often used to measure the impact of a rate change on an existing portfolio of customers."

For example, if an insurer's private passenger automobile book of business in the state of Franklin has in-force premiums of $50 million, then a 10% rate increase would produce about $5 million in extra premium per policy period. If these are annual policies, that would be $5 million per year; however, if these are six-month policies, then that would be $5 million per half year, or $10 million per year.

In-force premium can be used to monitor the size of a book of business over time. However, unlike in-force exposures, in-force premiums are affected by rate changes, thus one has to use care.

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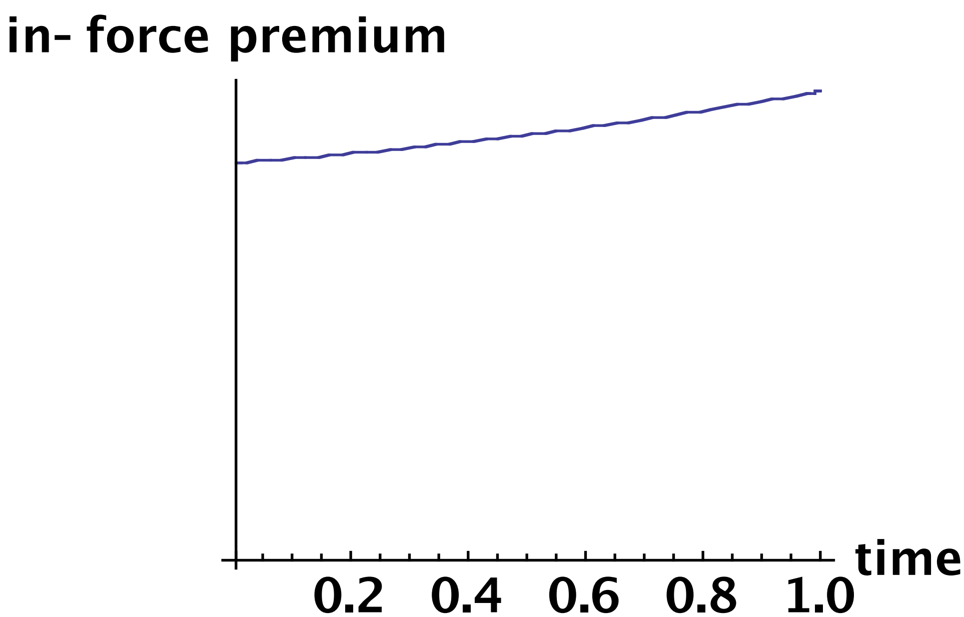
[188] Full term is 6 month if an insurer sells 6 months policies.
Full term is 12 month if an insurer sells 12 months policies.
Most insurers sell one or the other for a given line of Insurance.
In an occasional exam question, an insurer sells both types.

[189] The calculation of in-force premium is slightly more complicated in the case of a mid-term adjustment.
The in-force premium on a given date is the premium that would have been charged if the current coverage had been provided for the full policy term. At the bottom of page 70 and continuing on page 71, Basic Ratemaking has an example.

In-force Premiums versus Earned Premium:[190]

Assume an insurer writes only a single annual policy every January 1, with premium of $1 million. Then its in-force premium is a constant $1 million and its earned premium each calendar year is also $1 million.

More generally, graph the in-force premium from time zero to one:



If one has only annual policies, then the earned premium is the area under this curve. With a large book of business, the area under this curve from zero to one is approximately equal to the average of the values at times zero and one.[191] Thus the earned premium would be approximately the average of the in-force premiums at the end of the current year and prior year.

Assume instead that the insurer wrote a six-month policy each January 1, and another each July 1, each for $0.5 million. Then its in-force premium is a constant $0.5 million and its earned premium each calendar year is twice that at $1 million.

Thus if an insurer has only six-month policies, then the earned premium would be twice the area under the curve of the in-force premium. Thus the earned premium would be approximately the sum of the in-force premiums at the end of the current year and prior year.

[190] See 5, 11/91, Q.2d.
[191] This approximation would be exact if the curve of the in-force premium is a straight line.

Cancelations:

An annual policy with $600 in premium is written with effective date September 1, 2009.

If the policy is canceled on December 1, 2009, then only 3 months of coverage was provided, and there is $600/4 = $150 contributed to both written and earned premiums for Calendar Year 2009.

If instead the policy is canceled on March 1, 2010, then only 6 months of coverage was provided. However, at the end of 2009 we would not know that the policy would be canceled. Thus there would be $600 contributed to Calendar Year 2009 written premiums and -$300 contributed to Calendar Year 2010 written premiums. The total written premiums add to the correct $300. $600/3 = $200 is contributed to the earned premiums for Calendar Year 2009, and $600/6 = $100 is contributed to the earned premiums for Calendar Year 2010, for a total of $300.

Endorsements:

An annual policy covering one car is written with effective date September 1, 2009 and premium of $600.

If the policy is endorsed on December 1, 2009 to add another car, then the second car will be provided with only 9 months of coverage. Assume that the additional premium is $450.[192] This policy contributes $1050 to Calendar Year 2009 written premiums. From the first car, there is $200 earned in CY09 and $400 earned in CY10. From the second car, there are $50 earned in CY09 and $400 earned in CY10. Thus this policy contributes $250 to CY09 earned premiums and $800 to CY10 earned exposures, for a total of $1050.

Car	CY09 Written	CY10 Written	CY09 Earned	CY10 Earned
1	$600	0	$200	$400
2	$450	0	$50	$400
Total	$1050	0	$250	$800

If instead the policy is endorsed on March 1, 2010 to add another car, then the second car will be provided with only 6 months of coverage. Assume that the additional premium for this car is $300. As of the end of 2009 we would not know that this policy would be endorsed. Therefore, this policy contributes $600 to Calendar Year 2009 written premiums and $300 to Calendar Year 2010 written premiums. From the first car, there is $200 earned in CY09 and $400 earned in CY10. From the second car, there is $300 earned in CY10. Thus this policy contributes $200 to CY09 earned premiums and $700 to CY10 earned exposures, for a total of $900.

Car	CY09 Written	CY10 Written	CY09 Earned	CY10 Earned
1	$600	0	$200	$400
2	$0	$300	$0	$300
Total	$600	$300	$200	$700

This policy would contribute $600 to the inforce premiums on February 1, 2010. This policy would contribute $1200, the premium for a year of coverage, to the inforce premiums on March 1, 2010.[193]

[192] For simplicity I have assumed the second car costs as much to insure per year as the first car. However, it could cost either more or less.
[193] Inforce premiums are calculated as if all policies were full term.

Extension of Exposures:

In order to be used in a rate indication, the historical premiums must be brought to the current rate level. There are two different techniques, extension of exposures and the parallelogram method.

Using Extension of Exposures, each policy is rerated using the current rates.

Assuming one has on a computer all of the detailed information on each policy needed in order to rate it, as well as the current rate manual, extension of exposures is relatively straightforward. When it is possible to use extension of exposures, it is very accurate and thus the preferred technique.

Individual Risk Rating Plans to be discussed subsequently, particularly Schedule Rating, can complicate any attempt to put premiums on level using either technique. It may be difficult to determine what average schedule credits and debits were applied historically and/or will be applied in the future period for which rates are being made.

Parallelogram Method:

When an actuary is using data from his insurer, extension of exposures can usually be used. When the actuary is acting as a consultant or is using data for the entire insurance industry, extension of exposures may not be feasible. Where extension of exposures is not practical, the actuary will use the Parallelogram Method.

The Parallelogram Method uses approximate assumptions to calculate an "on-level factor" to be multiplied by the historical premiums for a Calendar Year or Policy Year in order to bring them on-level. It is assumed that exposures are written at a constant rate.[194]

1. Determine the timing and amount of the overall rate changes.[195]
2. Calculate the cumulative rate level index for each different rate level.
3. Calculate the weight for each group of policies written at different rate levels.
4. Calculate the average rate level index for the appropriate Calendar Year or Policy Year.
5. Calculate the on-level factor as the ratio of the current cumulative rate level index and the average cumulative rate level index for the appropriate year.

[194] This assumption can be relaxed. Even though it is beyond what is on the syllabus, some of you may benefit by looking at Frank Karlinski's short discussion of "A Refined Model For Premium Adjustment," PCAS 1977, available on the CAS webpage. http://www.casact.org/pubs/proceed/proceed77
The original paper by Miller and Davis is in PCAS 1976.

[195] This information will be given to you in the exam question. *The amounts of the overall rate changes are typically the estimates made at the time the set of rates were implemented. The actual effect probably varied somewhat from this estimate due to changes in mix of business. If there have been significant changes in the mix of business written over time, combining several past estimates of the effects of rate changes can lead to errors in the final estimate of what the premium for an older year would have been if it had been written at the current rates.*

Parallelogram Method, Calendar Years, Earned Premium:

Here is an example of how to apply the Parallelogram Method to a Calendar Year of earned premiums.[196]

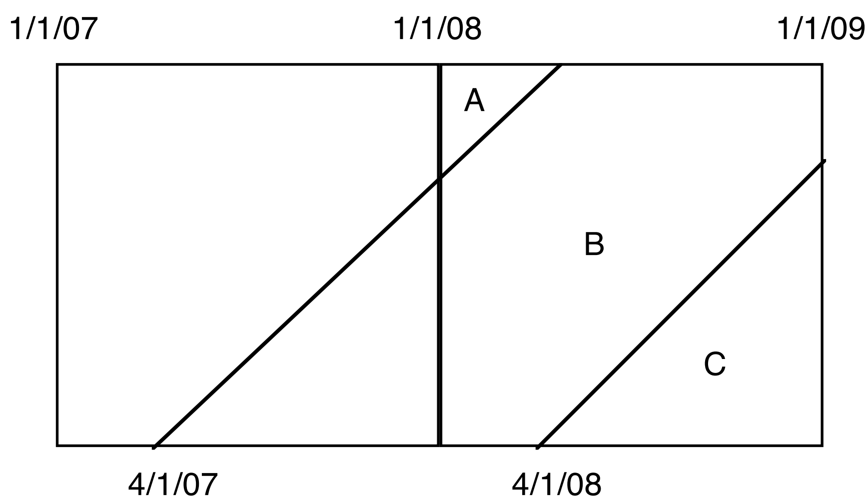
You are given the following rate change history for a level book of 12-month term policies uniformly distributed throughout the experience period.
4/1/2006 +3% 4/1/2007 +4% 4/1/2008 -10% 4/1/2009 +5%

We wish to calculate the appropriate on-level factor to apply to the 2008 Calendar Year earned premium in order to estimate earned premium at the current 4/1/2009 rate level.

We calculate a cumulative rate level index:[197] [198]

Date	Rate Level Change	Rate Level Index
Prior		1.000
4/1/06	3%	1.030
4/1/07	4%	1.071 = (1.03)(1.04)
4/1/08	-10%	0.964 = (1.03)(1.04)(0.9)
4/1/09	5%	1.012 = (1.03)(1.04)(0.9)(1.05)

Next we draw a diagram, similar to what was done when discussing exposures:



Now calculate the areas making up the righthand square corresponding to CY08.
Area A = $(1/2)(1/4)^2$ = 1/32. Area C = $(1/2)(3/4)^2$ = 9/32. Area B = 1 - 1/32 - 9/32 = 22/32.

---
[196] This is the most commonly asked case on the exam.
[197] I have set the prior to 4/1/2006 rate level equal to one.
One could have instead set the 4/1/2006 rate level equal to one, and gotten the same on-level factor at the end.
[198] All of the rate changes occur on April 1 in this example, solely for simplicity.

Therefore, CY08 earned premium was written at the following rates:
1/32 at the 4/1/06 rate, 22/32 at the 4/1/07 rate, 9/32 at the 4/1/08 rate.

Thus the average rate level for CY08 earned premium is:
(1.030)(1/32) + (1.071)(22/32) + (0.964)(9/32) = 1.040.

$$\text{On Level Factor} = \frac{\text{Current Rate Level}}{\text{Average Rate Level for CY08 Earned Premium}} = 1.012/1.040 = 0.973.$$

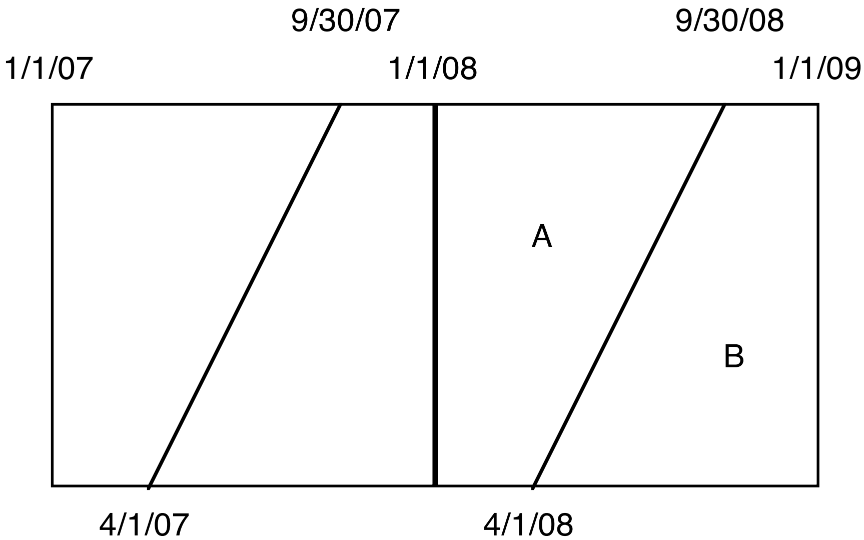
If for example, historical CY08 earned premiums were $10 million, then brought on the current rate level they would be: (0.973)($10 million) = $9.73 million. It would be the $9.73 million in earned premium which would be used in the rate indication.[199]

Each Calendar Year of earned premiums is represented by a square.

Each rate change is represented by a sloping line.
For annual policies, the slope of the line representing a rate change is one.

[199] If appropriate, this premium would also be developed to ultimate and/or trended.

Exercise: Assume that in the previous example all of the policies are six-month rather than annual. What is the on-level factor for CY08 earned premiums?
[Solution: While the rate level indices are the same, the diagram differs.
As discussed previously when dealing with exposures, the slope of the line representing a six month policy is two.



Area A = 1/2. Area B = 1/2. Thus the average rate level for CY08 earned premium is:
(1.071)(1/2) + (0.964)(1/2) = 1.018.

$$\text{On Level Factor} = \frac{\text{Current Rate Level}}{\text{Average Rate Level for CY08 Earned Premium}} = 1.012/1.018 = 0.994.$$

Comment: The policy term makes a difference!]

For six-month policies, the slope of the line representing a rate change is two.
In general, the slope of the line representing a rate change is one over the policy term.

One can also determine on-level factors for Calendar Year Written Premiums.[200] The rate level index calculation is the same. Determining what portion of the Calendar Year written at each rate level is straightforward.[201] In the previous example, 1/4 of CY08 is written at the 4/1/07 rate, while 3/4 of CY08 is written at the 4/1/08 rate.[202] Thus the average rate level for CY08 written premiums is:
(1.071)(1/4) + (0.964)(3/4) = 0.991.

$$\text{On Level Factor} = \frac{\text{Current Rate Level}}{\text{Average Rate Level for CY08 Written Premium}} = 1.012/0.991 = 1.021.$$

[200] I would not expect on-level calculations for written premiums to be asked on your exam.
[201] Most of us do not need a diagram. It does not depend on the policy term.
[202] We are still assuming a uniform rate of writing exposures.

Putting Calendar Years Written Premium On Current Rate Level:

While asked about much less often, one can instead work on written premiums for Calendar Years.

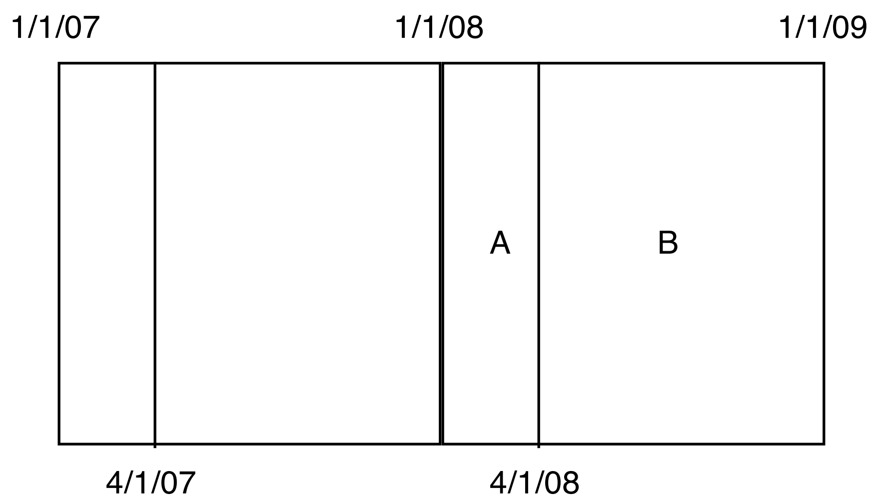
For example, assume that we wish to calculate the appropriate on-level factor to apply to the 2008 Calendar Year written premiums. Assume the previous rate changes:
4/1/2006 +3% 4/1/2007 +4% 4/1/2008 -10% 4/1/2009 +5%

As previously we calculate a cumulative rate level index:

Date	Rate Level Change	Rate Level Index
Prior		1.000
4/1/06	3%	1.030
4/1/07	4%	1.071 = (1.03)(1.04)
4/1/08	-10%	0.964 = (1.03)(1.04)(0.9)
4/1/09	5%	1.012 = (1.03)(1.04)(0.9)(1.05)

Making the usual assumption of exposures written evenly throughout the year, then 1/4 of CY08 was written at the 1.071 rate level, while the remaining 3/4 was written at the 0.964 rate level.
Thus the average rate level for CY08 written premium is: (1/4)(1.071) + (3/4)(0.964) = 0.991.
Thus the on-level factor for CY08 written premium is: 1.012/0.991 = 1.021.

One could draw a diagram, as was done when discussing earned premiums:



Area A = 1/4, while Area B = 3/4.
Note that for written premiums, each rate change is represented by a vertical line.[203]

[203] For earned premiums, the lines were slanted, with the slope depending on the policy term.

Parallelogram Method, Policy Years:

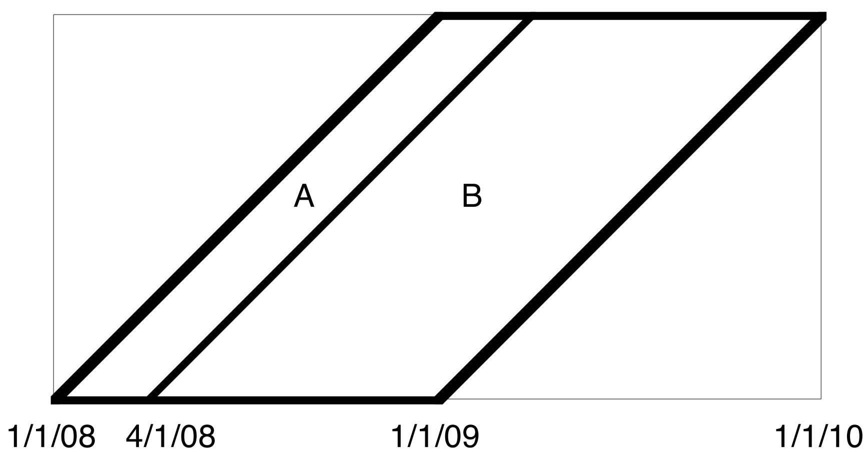
You are given the following rate change history for a level book of 12-month term policies uniformly distributed throughout the experience period.
4/1/2006 +3% 4/1/2007 +4% 4/1/2008 -10% 4/1/2009 +5%

We wish to calculate the appropriate on-level factor to apply to the 2008 Policy Year earned premium in order to estimate earned premium at the current 4/1/2009 rate level. [204]

Policies written from 1/1/08 to 3/31/08 are on the 4/1/07 level, while those written from 4/1/08 to 12/31/08 are on the 4/1/08 level.
Therefore, the average rate level for 2008 Policy Year earned premium is:
(1.071)(1/4) + (0.964)(3/4) = 0.991.

$$\text{On Level Factor} = \frac{\text{Current Rate Level}}{\text{Average Rate Level for PY08 Written Premium}} = 1.012/0.991 = 1.021.$$

If you found it helpful, you could draw a diagram:



Policy Year 2008 is represented by the parallelogram.
Area A = 1/4. Area B = 3/4.

[204] The calculation would be identical for Policy Year written premiums.

<u>Limitations of the Parallelogram Method</u>:

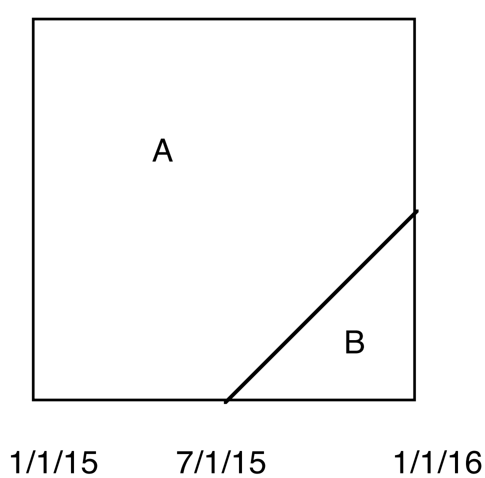
As discussed previously, the Parallelogram Method assumes exposures are written evenly throughout the year. This can create problems for lines of insurance or books of business that are expanding or contracting. One can alleviate this problem by using shorter periods of time than a year. Alternately, one can revise the weights used to take into account the varying volume of business.

Another limitation is that the Parallelogram Method is usually applied to the overall rate level. This does not result in on-level premiums by class and territory cell.[205] If one uses the loss ratio method of classification ratemaking, then extension of exposures is used to get premiums at present rates.

<u>Assumption of Constant Rate of Writing Exposures</u>:[206]

The Parallelogram Method assumes that exposures are written at a constant rate. In many practical applications this assumption is a reasonable approximation. If the rate of writing is either increasing or decreasing substantially over time, then the Parallelogram Method will be inaccurate.

Exercise: Assume a rate increase of 10% on new and renewal policies, effective July 1, 2015.
All policies are annual.
Using the Parallelogram Method determine the factor to bring CY2015 earned premiums on level.
[Solution: Area = 7/8, and Area B = $(1/2)(1/2)^2$ = 1/8.



On-level Factor is: 1.1 / {(7/8)(1) + (1/8)(1.1)} = 1.0864.]

Exercise: If instead all of the policies are written on January 1, then what is the on-level factor?
[Solution: All of the CY2015 earned premium comes from policies written January 1, 2015, all written at the lower rate. Thus the on-level factor is 1.1.]

[205] If one had each of the rate changes for each class/territory cell, one could apply the parallelogram method to each cell separately. However, in practice this is very rarely done.
[206] See for example, 5, 5/14, Q.1.

Exercise: Assume the following exposures are written in 2014 and 2015.
April 1, 2014: 100 October 1, 2014: 100 April 1, 2015: 100 October 1, 2015: 100.
Determine the factor to bring CY2015 earned premiums on level.
[Solution:

Date	Expos.	Portion Earned in CY2015	CY2015 Earned Expos.	Rate Level	Product
April 1, 2014	100	1/4	25	1.0	25
Oct. 1, 2014	100	3/4	75	1.0	75
April 1, 2015	100	3/4	75	1.0	75
Oct. 1, 2015	100	1/4	25	1.1	27.5
Total			200		202.5

Average rate level of CY2015 earned exposures: 202.5/200 = 1.0125.
On-Level Factor is: 1.1 / 1.0125 = 1.0864.
Comment: With level writings this matches the parallelogram method.]

Exercise: Assume instead the following exposures are written in 2014 and 2015.
April 1, 2014: 100 October 1, 2014: 200 April 1, 2015: 300 October 1, 2015: 400.
Determine the factor to bring CY2015 earned premiums on level.
[Solution:

Date	Expos.	Portion Earned in CY2015	CY2015 Earned Expos.	Rate Level	Product
April 1, 2014	100	1/4	25	1.0	25
Oct. 1, 2014	200	3/4	150	1.0	150
April 1, 2015	300	3/4	225	1.0	225
Oct. 1, 2015	400	1/4	100	1.1	110
Total			500		510

Average rate level of CY2015 earned exposures: 510/500 = 1.02.
On-Level Factor is: 1.1 / 1.02 = 1.0784.
Comment: With increasing writings this differs from the result of the parallelogram method.]

Exercise: Assume instead the following exposures are written in 2014 and 2015.
April 1, 2014: 400 October 1, 2014: 300 April 1, 2015: 200 October 1, 2015: 100.
Determine the factor to bring CY2015 earned premiums on level.
[Solution:

Date	Expos.	Portion Earned in CY2015	CY2015 Earned Expos.	Rate Level	Product
April 1, 2014	400	1/4	100	1.0	100
Oct. 1, 2014	300	3/4	225	1.0	225
April 1, 2015	200	3/4	150	1.0	150
Oct. 1, 2015	100	1/4	25	1.1	27.5
Total			500		502.5

Average rate level of CY2015 earned exposures: 502.5/500 = 1.005.
On-Level Factor is: 1.1 / 1.005 = 1.0945.]

Law Amendments:

When the Workers Compensation Law in a state is changed, either increasing or decreasing benefits paid to injured workers, or when the medical fee schedule is revised, actuaries estimate the average overall effect on losses.[207] Then usually a corresponding change is made to the rates based on the impact of this "law amendment." While rate changes normally apply to new and renewal business, often rate changes due to law amendments apply to all outstanding policies.

For example, new higher benefits will be paid to workers injured in workplace accidents that occur on or after 7/1/10. Therefore, Workers Compensation rates were increased by 10% on 7/1/10 in order to reflect this increase in benefits.

An annual policy written on 4/1/10, will cover accidents from 4/1/10 to 6/30/10, and from 7/1/10 to 3/31/11. The lower benefit level applies to the first group of accidents, while the new higher benefit level applies to the second group of accidents. Therefore, the rates that were in effect when this policy was written on 4/1/10 are inadequate for the coverage provided from 7/1/10 to 3/31/11.

Therefore, the rate for this policy will be increased mid-term on 7/1/10. The lower rate will apply to the first 1/4 of the policy period, and the new higher rate will apply to last 3/4 of the policy period. Thus this policy will pay (10%)(3/4) = 7.5% more due to the law amendment.[208] [209]

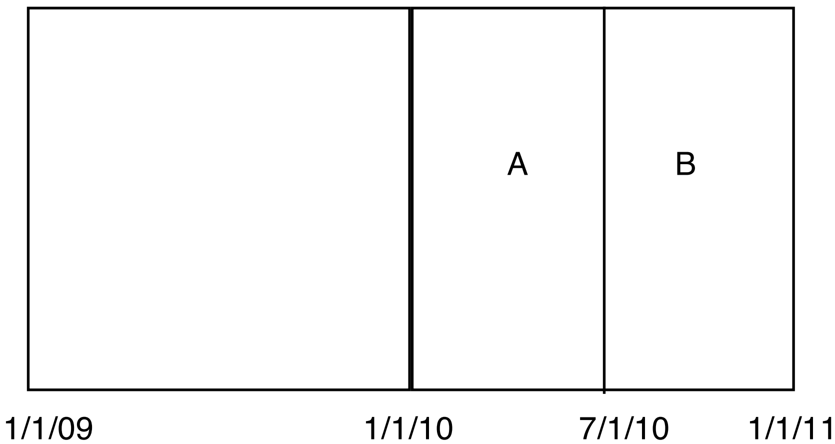
This differs from an ordinary rate change. If an insurer changed its rates after 4/1/10 for other than a law amendment, this policy would continue to use the rates in effect on 4/1/10. Therefore, determining on-level factors for rate changes due to law amendments is somewhat different.

For example, let us assume there are no rate changes other than the 7/1/10 law amendment for +10% on outstanding policies. Premium is earned as coverage is provided. Thus half of CY10 earned premiums relates to accidents from 1/1/10 to 6/30/10, while the remaining half relates to accidents from 7/1/10 to 12/31/10.

Therefore, the average rate level for CY10 earned premiums is:
(1/2)(1) + (1/2)(1.1) = 1.05.
The on-level factor for CY10 earned premiums is: 1.10/1.05 = 1.048.

[207] See my section on "Losses and LAE."
[208] If the policy is big enough to be subject to premium audits, then the payrolls for each month or quarter would have the appropriate rates apply.
[209] Some insurers may choose not to go to the effort and expense midterm of collecting the additional premiums to which they are entitled from small insureds.

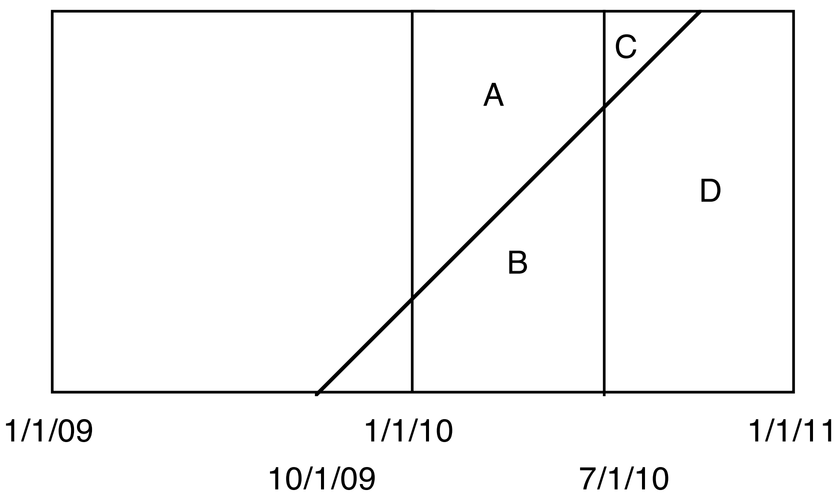
We can show this in a diagram:



CY10 earned premiums are represented by the righthand square.
Area A = 1/2. Area B = 1/2.

The law amendment rate change on all outstanding policies is represented by a vertical line.

Exercise: There is a 7/1/10 law amendment for +10% on all outstanding policies.
In addition, there was a 10/1/09 rate decrease of 7% on new and renewal policies.
Determine the on-level factor for Calendar Year 2010 earned premiums.
All policies are annual.[210]
[Solution: The 7/1/10 law amendment is represented by a vertical line.
The 10/1/09 rate change is represented by a line with slope of 1.

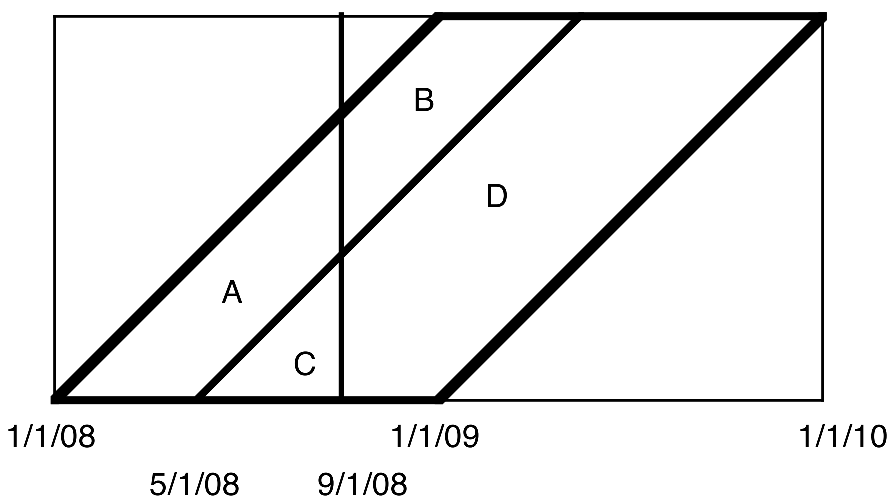


Area A = (1/2)(1/2) = 1/4. Area B = (1/2)(1/2) = 1/4.
Area C = (1/2)(1/4)(1/4) = 1/32. Area D = 1/2 - 1/32 = 15/32.
Let the rate level prior to 10/1/09 be 1.
Then Area A has a rate level of 1.
Area B has a rate level of 0.93; it is affected only by the 7% rate decrease.
Area C has a rate level of 1.10; it is affected only by the 10% law amendment.
Area D has a rate level of (1.10)(0.93) = 1.023.
Average rate level for Calendar Year 2010 earned premiums is:
(1/4)(1) + (1/4)(0.93) + (1/32)(1.1) + (15/32)(1.023) = 0.996.
The on-level factor for Calendar Year 2010 earned premiums is:
1.023/0.996 = 1.027.
<u>Comment</u>: It can get complicated when one has several rate changes, some on new and renewal policies and some on all outstanding policies!]

[210] On the exam, assume Workers Compensation policies are annual, unless stated otherwise.

One can perform similar calculations for Policy Years. As discussed previously a Policy Year is represented by a parallelogram.

Exercise: There is a 9/1/08 law amendment for +5% on all outstanding policies.
In addition, there was a 5/1/08 rate increase of 8% on new and renewal policies.
Determine the on-level factor for Policy Year 2008 premiums.
All policies are annual.
[Solution: The 9/1/08 law amendment is represented by a vertical line.
The 5/1/08 rate change is represented by a line with slope of 1.



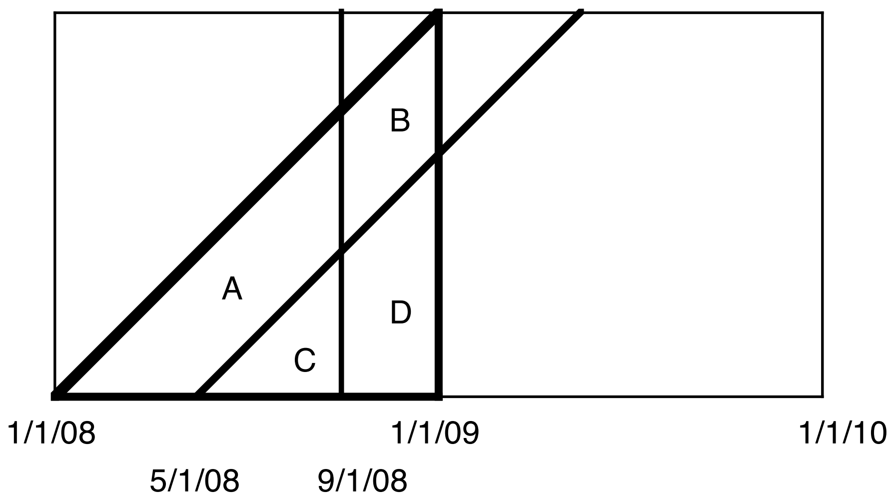
Area C = (1/2)(1/3)(1/3) = 1/18. Area A = (1/2)(2/3)(2/3) - 1/18 = 3/18.
Area B = 1/3 - 3/18 = 3/18. Area D = 2/3 - 1/18 = 11/18.
Let the rate level prior to 5/1/08 be 1.
Then Area A has a rate level of 1.
Area B has a rate level of 1.05; it is affected only by the 5% law amendment.
Area C has a rate level of 1.08; it is affected only by the 8% rate increase
Area D has a rate level of (1.05)(1.08) = 1.134.
Average rate level for Policy Year 2008 premiums is:
(3/18)(1) + (3/18)(1.05) + (1/18)(1.08) + (11/18)(1.134) = 1.095.
The on-level factor for Policy Year 2008 premiums is:
1.134/1.095 = 1.036.]

Policy Year 2008 earned premiums as of 12/31/08 would not be at ultimate, even ignoring audits. Most of the policies that make of the Policy Year have not expired by 12/31/08. In addition, many of the losses that will be in PY 2008 have yet to occur by December 31, 2008.[211] Those accidents that may have occurred may have not been reported to the insurer yet. Many known claims will have preliminary case reserves with nothing paid.

[211] If policies are written evenly throughout the year, half of the losses that will make up Policy Year 2008, have yet to occur by 12/31/2008.

Such a Policy Year of data would be referred to variously as: an incomplete policy year, policy year at report 0, or policy year at report 1/2. The information is extremely immature. There is much more loss development on an incomplete policy year than on a policy year at first report. Therefore, an incomplete policy year is usually not used directly in ratemaking.[212]

As of 12/31/08, only the portion of coverage up to that date has been earned. Therefore, the incomplete Policy of Earned Premiums consists of only that portion of the parallelogram to the left of the vertical line at 1/1/09, a triangle:



One can calculate an on-level factor in a similar manner to what was done before.[213]

Area C = (1/2)(1/3)(1/3) = 1/18. Area A = (1/2)(2/3)(2/3) - 1/18 = 3/18.
Area B = (1/3)(1/3) = 1/9. Area D = (1/3)(1/2) = 1/6.

Let the rate level prior to 5/1/08 be 1.
Then Area A has a rate level of 1.
Area B has a rate level of 1.05; it is affected only by the 5% law amendment.
Area C has a rate level of 1.08; it is affected only by the 8% rate increase
Area D has a rate level of (1.05)(1.08) = 1.134.

Average rate level for incomplete Policy Year 2008 earned premiums is:
{(3/18)(1) + (1/9)(1.05) + (1/18)(1.08) + (1/6)(1.134)} / (1/2) = 1.065.

The on-level factor for incomplete Policy Year 2008 earned premiums is:
1.134/1.065 = 1.065.

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[212] However, for example, the actuary may compare the level of the most recent incomplete policy year to the previous one, in order to see if anything unusual may be going on.
[213] I do <u>not</u> expect you to be asked to calculate an on-level factor for earned premium for an incomplete policy year.

Premium Trend:

For some lines of insurance, even if the rate manual is kept the same, premiums will increase due to inflation. For example, for Workers Compensation, even if the insured workers stay the same, the average weekly wages will increase with inflation, increasing payrolls, in turn increasing premiums. For Homeowners Insurance, even if the set of insured homes remains constant, the value of insured homes will (usually) increase with inflation, in turn increasing premiums.

Even lines of insurance not affected by inflation can have their average premiums increase. For example, basic limits premiums of Private Passenger Automobile Property Damage Liability are based on the number of insured cars. If the rate manual and the set of cars insured stay the same, and in addition the classes and territories of the insured cars do not change, then the (basic limit) premiums should not change.

However, the mix of classes and territories written by an insurer shift over time. For example, the percentage of business written in the highest rated territory of a state by an insurer might change due to: marketing decisions, competition, population changes, etc. Therefore, even if the rate manual is kept the same, the average (basic limit) premium for P.P. Auto PDL would also change over time.[214] This effect of the change in mix of business would be in addition to the effect of inflation on premiums for lines of insurance such as Workers Compensation, CGL, and Homeowners.

When computing loss ratios for use in a rate indication, we would want to adjust both the numerator, losses, and the denominator, premiums, for the same effects. The changes in losses over time will be adjusted for via loss trend.[215] The corresponding changes in premiums over time will be adjusted for via premium trend. We will discuss the “one-step” and “two-step” methods.

[214] See the Premium Trend Exhibit in Appendix A of Basic Ratemaking.
[215] See my section on “Losses and LAE.”

<u>Written Premium Trend Series Example</u>:

You are performing a rate indication, with a proposed effective date of January 1, 2008.
The proposed rates will be in effect for one year.
12 month policies are written.
You have the following data on premiums written, that is already adjusted for one-time, abrupt and measurable changes such as rate changes:[216]

Ending Date	Quarterly Average Written Premium at Current Rate Level	Annual Change
9/31/02	393	
12/31/02	395	
3/31/03	396	
6/30/03	397	
9/30/03	402	2.3%
12/31/03	405	2.5%
3/31/04	403	1.8%
6/30/04	401	1.0%
9/30/04	398	-1.0%
12/31/04	399	-1.5%
3/31/05	400	-0.7%
6/30/05	404	0.7%
9/30/05	409	2.8%
12/31/05	413	3.5%
3/31/06	416	4.0%
6/30/06	415	2.7%
9/30/06	413	1.0%
12/31/06	417	1.0%
3/31/07	416	0.0%
6/30/07	418	0.7%

You also have premiums and losses by Calendar/Accident Year:

Calendar Accident Year	Average Earned Premiums at Current Rate Level	Earned Premiums at Current Rate Level	Incurred Losses Developed to Ultimate	Loss Ratio
2002	392.11	5,234,501	4,346,582	83.0%
2003	398.72	6,528,923	4,234,733	64.9%
2004	401.04	6,030,067	4,863,410	80.7%
2005	403.37	5,810,650	3,989,632	68.7%
2006	413.93	5,620,354	3,689,457	65.6%

[216] Similar to Table 5.24 in <u>Basic Ratemaking</u>.

A 3% annual loss trend has been selected.[217]

In the simpler one-piece premium trend method, an annual premium trend is selected.
Based on the observed annual changes in the premium trend series, for example, a 1% annual premium trend is selected.

We can determine the projected loss ratio for each calendar/accident year as follows.
For policies to be written from 1/1/08 to 12/31/08, the average date of writing is 7/1/08, and since these are 12 month policies, the average accident date is 6 months later, or 1/1/09.
The loss trend factors are computed as follows:

Accident Year	Average Accident Date	Loss Trend Period	Annual Loss Trend	Loss Trend Factor
2002	7/1/02	6.5 years	3.0%	1.212
2003	7/1/03	5.5 years	3.0%	1.177
2004	7/1/04	4.5 years	3.0%	1.142
2005	7/1/05	3.5 years	3.0%	1.109
2006	7/1/06	2.5 years	3.0%	1.077

For example, $1.03^{2.5} = 1.077$.

Calendar Year 2002 earned premiums, have an average date of earning of 7/1/02, and since these are 12 month policies, an average date of writing 6 months earlier, or 1/1/02. We trend to an average date of writing of 7/1/08. The one-step premium trend factors to be applied to earned premiums are computed as follows:

Calendar Year	Average Written Date	Premium Trend Period	Annual Premium Trend	Premium Trend Factor
2002	1/1/02	6.5 years	1.0%	1.067
2003	1/1/03	5.5 years	1.0%	1.056
2004	1/1/04	4.5 years	1.0%	1.046
2005	1/1/05	3.5 years	1.0%	1.035
2006	1/1/06	2.5 years	1.0%	1.025

For example, $1.01^{6.5} = 1.067$.

The projected loss ratios are:

Calendar Accident Year	Loss Ratio	Loss Trend Factor	Premium Trend Factor	Projected Loss Ratio
2002	83.0%	1.212	1.067	**94.3%**
2003	64.9%	1.177	1.056	**72.3%**
2004	80.7%	1.142	1.046	**88.1%**
2005	68.7%	1.109	1.035	**73.6%**
2006	65.6%	1.077	1.025	**68.9%**

For example, (83.0%)(1.212)/1.067 = 94.3%.

[217] Loss trends will be discussed in my section on “Losses and LAE.”

Rather than trying to compromise on the selection of a single long-term trend, the more complicated two-step trending method, as its first step, divides the latest average written premium at current rate level by the average earned premium at current rate level for each year in the experience period.

Step 1 Premium Trend Factor =

$$\frac{\text{Latest Year Written Premium in Trend Series at Current Rate Level}}{\text{Calendar Year Earned Premium at Current Rate Level}}.$$

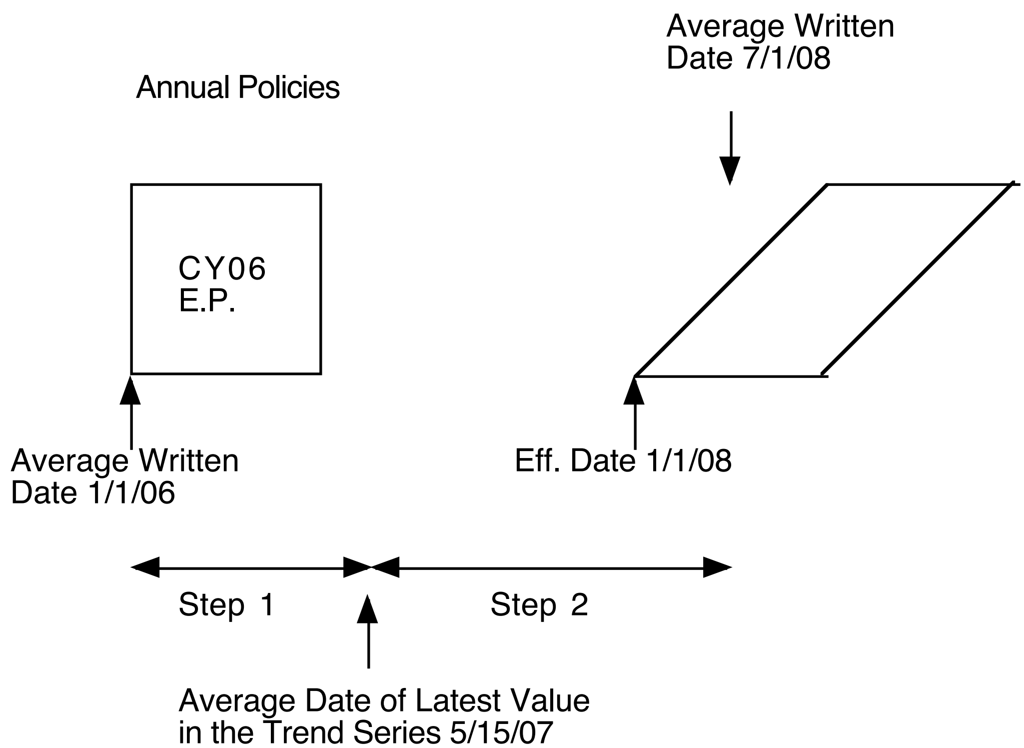
We compare the average earned premium at current rates for each calendar year to the latest point in the premium trend series. For example, for Calendar Year 2002, the premium trend factor for the first step is: 418/392.11 = 1.066.

Calendar Year	Average Earned Prem.	Latest Avg. W. P.	Step 1 Trend Factor
2002	392.11	418	1.066
2003	398.72	418	1.048
2004	401.04	418	1.042
2005	403.37	418	1.036
2006	413.93	418	1.010

In the two-piece premium trend method, for the second step an annual premium trend is selected. The second step goes from the average date of the last point in the premium trend series to the midpoint of the proposed effective period of the new rates.

Based on the observed annual changes in the premium trend series, for example, a 1% annual premium trend is selected for the second step.

The average written premium at current rate level for the quarter ending 6/30/07, the last period shown in the trend series, is given as 418.
The quarter of written premiums ending 6/30/07 has an average date of writing of 5/15/07.
For policies to be written from 1/1/08 to 12/31/08, the average date of writing is 7/1/08.



Thus the projection period is from 5/15/07 to 7/1/08, or 1.125 years.
The projection factor for premiums is: $1.01^{1.125} = 1.011$.

The projected loss ratios for each Calendar/Accident Year are:

Calendar Accident Year	Loss Ratio	Loss Trend Factor	Step 1 Premium Trend Factor	Step 2 Premium Trend Factor	Projected Loss Ratio
2002	83.0%	1.212	1.066	1.011	**93.3%**
2003	64.9%	1.177	1.048	1.011	**72.1%**
2004	80.7%	1.142	1.042	1.011	**87.5%**
2005	68.7%	1.109	1.036	1.011	**72.7%**
2006	65.6%	1.077	1.010	1.011	**69.2%**

For example: $\frac{(83.0\%)\ (1.212)}{(1.066)\ (1.011)} = 93.3\%$.

While the projected loss ratios are different using the two-piece rather than one-piece premium trend, they are similar. The direction and magnitude of the difference will vary based on the particular data.

In the two step method we are assuming that the observed ratios

$$\frac{\text{Latest Year Written Premium in Trend Series at Current Rate Level}}{\text{Calendar Year Earned Premium at Current Rate Level}}$$

are meaningful and accurate measures of what has happened in the past.
However, loss severities are subject to a lot of random fluctuation and recent years average severities include the effects of case reserves and delay in reporting claims.
Therefore, the two-step trend method is usually not applied to loss trends that are based on insurance data.[218]

[218] A two-step procedure could be applied when Consumer Price Indices form the basis of a loss trend, as is sometimes done for Homeowners Insurance and Fire Insurance.

Earned Premium Trend Series Example:

Revise the previous example, so that the trend series is earned rather than written premiums.

You are performing a rate indication, with a proposed effective date of January 1, 2008.
The proposed rates will be in effect for one year. 12 month policies are written.
You have the following data on earned premiums written, that is already adjusted for one-time, abrupt and measurable changes such as rate changes:[219]

Ending Date	Quarterly Average Earned Premium at Current Rate Level	Annual Change
9/31/02	388	
12/31/02	391	
3/31/03	393	
6/30/03	396	
9/30/03	398	2.6%
12/31/03	400	2.3%
3/31/04	402	2.3%
6/30/04	403	1.8%
9/30/04	402	1.0%
12/31/04	400	0.0%
3/31/05	400	-0.5%
6/30/05	400	-0.7%
9/30/05	403	0.2%
12/31/05	407	1.8%
3/31/06	411	2.8%
6/30/06	413	3.2%
9/30/06	414	2.7%
12/31/06	415	2.0%
3/31/07	415	1.0%
6/30/07	417	1.0%

You also have the same premiums and losses by Calendar/Accident Year as before:

Calendar Accident Year	Average Earned Premiums at Current Rate Level	Earned Premiums at Current Rate Level	Incurred Losses Developed to Ultimate	Loss Ratio
2002	392.11	5,234,501	4,346,582	83.0%
2003	398.72	6,528,923	4,234,733	64.9%
2004	401.04	6,030,067	4,863,410	80.7%
2005	403.37	5,810,650	3,989,632	68.7%
2006	413.93	5,620,354	3,689,457	65.6%

[219] Here we are ignoring seasonality, which could be a significant concern when tracking the average premium for some lines of business.

A 3% annual loss trend has been selected.

Using the one-piece premium trend method, if a 1% annual premium trend is selected, then the projected loss ratio for each calendar/accident year would be the same as before.

Exercise: Using the two-piece premium trend method, in the second step a 1% annual premium trend is selected. Determine the projected loss ratio for each calendar/accident year.
[Solution: We compare the average earned premium at current rates for each calendar year to the latest point in the premium trend series. For example, for Calendar Year 2002, the premium trend factor for the first step is: 417/392.11 = 1.063.

Calendar Year	Average Earned Prem.	Latest Avg. E. P.	Step 1 Trend Factor
2002	392.11	417	1.063
2003	398.72	417	1.046
2004	401.04	417	1.040
2005	403.37	417	1.034
2006	413.93	417	1.007

The average earned premium at current rate level for the quarter ending 6/30/07, the last period shown in the trend series, is given as 417. In the trend series of earned premiums, the quarter ending 6/30/07 have an average date of earning of 5/15/07. Since the policies are annual, the average date of writing is 6 months earlier, 11/15/06.

For policies to be written from 1/1/08 to 12/31/08, the average date of writing is 7/1/08.
Thus the projection period is from 11/15/06 to 7/1/08, or 1.625 years.[220]

The projection factor for premiums is: $1.01^{1.625}$ = 1.016.

The projected loss ratios are:

Calendar Accident Year	Loss Ratio	Loss Trend Factor	Step 1 Premium Trend Factor	Step 2 Premium Trend Factor	Projected Loss Ratio
2002	83.0%	1.212	1.063	1.016	**93.1%**
2003	64.9%	1.177	1.046	1.016	**71.9%**
2004	80.7%	1.142	1.040	1.016	**87.2%**
2005	68.7%	1.109	1.034	1.016	**72.5%**
2006	65.6%	1.077	1.007	1.016	**69.1%**

For example: (83.0%)(1.212)/{(1.063)(1.016)} = 93.1%.]

[220] You need to go from either average written date to average written date, or average earned date to average earned date. The length of the trend period and the resulting trend factor should be the same in either case.

The projected loss ratio differ slightly from those using the two-step method with a written premium trend series:

Calendar Accident Year	Projected Loss Ratio One-Piece Method	Projected Loss Ratio Two-Piece Method Written Trend Series	Projected Loss Ratio Two-Piece Method Earned Trend Series
2002	94.3%	93.3%	93.1%
2003	72.3%	72.1%	71.9%
2004	88.1%	87.5%	87.2%
2005	73.6%	72.7%	72.5%
2006	68.9%	69.2%	69.1%

Use of Written Premium vs. Earned Premium in order to compute Premium Trend:

Since these trends will apply to historical earned premium at current rate level, it makes some sense to evaluate trends based on shifts in average earned premium.

On the other hand, **written premium data is more recent**; premium for a given policy is not earned until well after it is written.

Basic Ratemaking uses in its example a series of average written premium, although average earned premium would also provide acceptable results.[221]

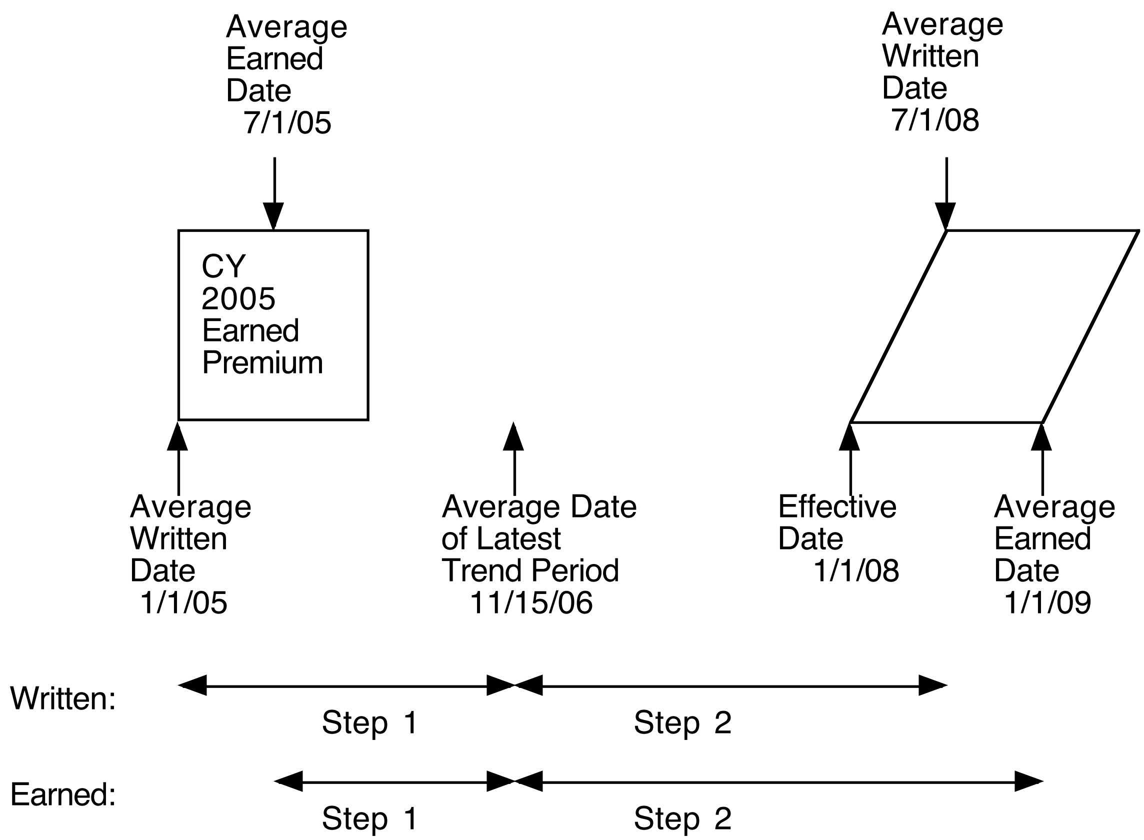
For example, in April 2004 an actuary has a series of average premiums by quarter up through the last quarter of 2003. Assuming annual homeowners policies, fourth quarter 2003 earned premiums are a mixture of policies written in the fourth quarter of 2002, first quarter of 2003, second quarter of 2003, third quarter of 2003, and fourth quarter of 2003.
The average date of writing is: 11/15/03 - 6 months = 5/15/03.

In contrast, written premiums for the fourth quarter of 2003 have an average date of 11/15/03. Therefore, written premiums reflect 6 months more of whatever changes have occurred in the insurer's book of business, than do earned premiums.

As a result, a two-step trend analysis based on average written premium will have a longer trending period for step 1 and a shorter projection period for step 2, than if based instead on average earned premium. "This confirms the intuitive appeal of using average written premium for the trend analysis in that the length of the inherently uncertain projection period is minimized."[222]

[221] For premiums set at policy inception, such as Homeowners Insurance, most actuaries prefer to use written data for premium trend. *For lines of insurance such as General Liability with premiums determined by audit, written premiums based on estimated payrolls or sales are not sufficiently accurate or stable to make them preferable to earned premiums for this purpose.*
[222] See Page 16 of "An Introduction to Premium Trend," by Burt D. Jones, formerly on the syllabus.

Here is a diagram of this two-piece trending example, where I have shown just one of the Calendar/Accident Years:[223]



Numerical Example, No Separate Trend Series:[224]

Revise the previous examples, so that there is no separate series of trend data.
You are performing a rate indication, with a proposed effective date of January 1, 2008.
The proposed rates will be in effect for one year. 12 month policies are written.
You have premiums and losses by Calendar/Accident Year:

Calendar Accident Year	Average Written Premiums at Current Rate Level	Average Earned Premiums at Current Rate Level	Earned Premiums at Current Rate Level	Incurred Losses Developed to Ultimate	Loss Ratio to Earned Premium
2002	395.06	392.11	5,234,501	4,346,582	83.0%
2003	399.61	398.72	6,528,923	4,234,733	64.9%
2004	402.39	401.04	6,030,067	4,863,410	80.7%
2005	409.80	403.37	5,810,650	3,989,632	68.7%
2006	417.25	413.93	5,620,354	3,689,457	65.6%

[223] See Figures 5.26 and 5.28 in Basic Ratemaking.
[224] See 5, 5/05, Q.37 and 5, 5/07, Q.36.

A 3% annual loss trend has been selected.

Using the one-piece premium trend method, if a 1% annual premium trend is selected, then the projected loss ratio for each calendar/accident year would be the same as before.

The loss ratios we are interested in are incurred losses over earned premiums.
We have a choice of whether or not to use the given written premiums for trending.

Exercise: Using the two-piece premium trend method, in the second step a 1% annual premium trend is selected. Use the written premiums for the first step.
Determine the projected loss ratio for each calendar/accident year.
[Solution: Using the given written premiums, we compare the average earned premium at current rates for each calendar year to the latest average written premium of 417.25.

Calendar Year	Average Earned Prem.	Latest Avg. W. P.	Step 1 Trend Factor
2002	392.11	417.25	1.064
2003	398.72	417.25	1.046
2004	401.04	417.25	1.040
2005	403.37	417.25	1.034
2006	413.93	417.25	1.008

The Calendar Year 2006 written premium has an average date of writing of 7/1/06.
For policies to be written from 1/1/08 to 12/31/08, the average date of writing is 7/1/08.
Thus the projection period is from 7/1/06 to 7/1/08, or 2 years.
The projection factor for premiums is: $1.01^2 = 1.020$.
The projected loss ratios are:

Calendar Accident Year	Loss Ratio	Loss Trend Factor	Step 1 Premium Trend Factor	Step 2 Premium Trend Factor	Projected Loss Ratio
2002	83.0%	1.212	1.064	1.020	**92.7%**
2003	64.9%	1.177	1.046	1.020	**71.6%**
2004	80.7%	1.142	1.040	1.020	**86.9%**
2005	68.7%	1.109	1.034	1.020	**72.2%**
2006	65.6%	1.077	1.008	1.020	**68.7%**

For example: (83.0%)(1.212)/{(1.064)(1.020)} = 92.7%.
Comment: Since it is the loss ratio to earned premium we are interested in, we compare calendar year earned premium to the latest point of the trend series in order to get the first step trend factor.]

Exercise: Using the two-piece premium trend method, in the second step a 1% annual premium trend is selected. Use the earned premiums for the first step.
Determine the projected loss ratio for each calendar/accident year.
[Solution: We compare the average earned premium at current rates for each calendar year to the latest average earned premium of 413.93.

Calendar Year	Average Earned Prem.	Latest Avg. Earned Prem.	Step 1 Trend Factor
2002	392.11	413.93	1.056
2003	398.72	413.93	1.038
2004	401.04	413.93	1.032
2005	403.37	413.93	1.026
2006	413.93	413.93	1.000

The Calendar Year 2006 earned premium has an average date of earning of 7/1/06.
For policies to be written from 1/1/08 to 12/31/08, the average date of writing is 7/1/08.
Since these are annual policies, the average date of earning is 6 months later or 1/1/09.
Thus the projection period is from 7/1/06 to 1/1/09, or 2.5 years.
Alternately, the Calendar Year 2006 earned premium has an average date of earning of 7/1/06.
Since these are annual policies, the average date of writing is 6 months earlier or 1/1/06.
For policies to be written from 1/1/08 to 12/31/08, the average date of writing is 7/1/08.
Thus the projection period is from 1/1/06 to 7/1/08, or 2.5 years.
In either case, the projection factor for premiums is: $1.01^{2.5} = 1.025$.
The projected loss ratios are:

Calendar Accident Year	Loss Ratio	Loss Trend Factor	Step 1 Premium Trend Factor	Step 2 Premium Trend Factor	Projected Loss Ratio
2002	83.0%	1.212	1.056	1.025	**92.9%**
2003	64.9%	1.177	1.038	1.025	**71.8%**
2004	80.7%	1.142	1.032	1.025	**87.1%**
2005	68.7%	1.109	1.026	1.025	**72.4%**
2006	65.6%	1.077	1.000	1.025	**68.9%**

For example: (83.0%)(1.212)/{(1.056)(1.025)} = 92.9%.
Comment: If asked such an exam question, clearly state whether you are using the earned or written premiums for the first step of the two-step method.]

In all of these examples, we start with an observed Accident Year Loss Ratio to Earned Premium:

$$\frac{\text{Accident Year Incurred Losses}}{\text{Calendar Year Earned Premiums}}.$$

We then apply loss trend to the numerator and premium trend to the denominator, in order to get a projected loss ratio. The same type of techniques apply to Policy Year or Calendar Year Loss Ratios. We would also develop the losses, and where appropriate premiums, to ultimate.

Adjusting Average Premiums in order to go into the Trend Series:

We need to put the average premiums in the trend series on the same level.

Some of the changes that can cause the future average premium level to differ from the past average premium level are:
1) Past rate changes.
2) Past rating plan changes.
3) The existence of rating plans which change the premium level over time.
4) Past and expected future shifts in the mix of business.

Examples of each of the above:
1. 10% increase in average rates.
2. a. Implementing a new 5% discount for a drug free workplace program.
   b. Increase in mandatory limits of liability for automobile insurance in a state.
3. Use of model year and symbols to rate collision insurance.
4. Increasing market penetration in a particular territory of a state.

These changes can have different types of effects on the average premium level:
1) One-time vs. continuous effects.
2) Measurable effects vs. effects that can only be estimated.
3) Abrupt effects vs. gradual shifts.

If a change has a one time effect on premium level, one should make a direct adjustment.
If a change occurs over time, then premium trend may be more appropriate.

1. Past Rate Changes: one-time and measurable.
2. Past Rating Plan Changes: one-time and measurable.
   a. Rating plan changes that affect the average premium level without affecting the level of coverage are essentially the same as rate changes.
   b. Rating plan changes that affect the average premium level but also include a corresponding change in the level of coverage are not really rate changes.
   Affect both premiums and losses.
3. The Use of Rating Plans that Change the Average Premium Level Over Time:
   measurable, and usually gradual and continuous.
4. Shifts in the Mix of Business: generally gradual and continuous.

In each case, you need to either make an explicit adjustment for an effect or you capture the effect in the premium trend. You have to make sure you count each effect once and only once. Do not double count an effect.

For example, let us assume the current rate level is 10% higher than the rate level in effect during the first quarter of 2008. Then prior to being used in the trend series, we would multiply the written premium for the first quarter of 2008 by 1.1. In general, the premiums are brought to the current rate level via either extension of exposures or the parallelogram method.[225]

The 19th Century Insurance Company gave a discount of 10% for Homeowners Insurance premiums for homes with a fire and burglar alarm that rings at the police, fire or other monitoring stations. The insurer decides to raise this discount to 15%, on new and renewal business.

Due to this change, the average premiums will decline. This change in discount can be treated as a rate change.

Assume that 1/4 of insureds currently qualify for the discount.[226] Then the previous "rate level" is:
$(1/4)(0.9) + (3/4)(1) = 0.975$.[227] The new "rate level" is: $(1/4)(0.85) + (3/4)(1) = 0.9625$.
The "rate change factor" is: $0.9625/0.975 = 0.987$.

We would bring premiums before the change in discount to the new level by multiplying by 0.987.

As discussed previously, the average premium will change due to shifts in the mix by class and territory written by an insurer. The premiums in the trend series would <u>not</u> be adjusted for such an impact. Rather this effect would be part of what is being measured by the premium trend.

<u>Premium Trends, Miscellaneous</u>:

By incorporating a premium trend, via either the one or two step method, we are taking into account the effect of the less measurable, gradual and continuous changes.

Similarly, the losses will also change due to shifts in the mix by class and territory written by an insurer. The losses in the trend series would <u>not</u> be adjusted for such an impact. Rather this effect would be part of what is being measured by the loss trend.

One needs to make sure that the loss and the premium trend are comparable. If they were both based on an insurer's own data, over similar periods of time, then they would both reflect whatever the change is in that insurer's mix of business.

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[225] The key is that all of the average premiums in the trend series are on the <u>same</u> rate level.
[226] If more insureds will install these fire and burglar alarm, the insurer's average premiums will go down, but there should be a corresponding decrease in losses.
[227] Ignore for simplicity whether those who qualify for the discount have different average premiums prior to the discount, than those who do not qualify.

A method that is used in some lines of business is to employ a loss ratio trend or net trend concept directly, and dispense with separate adjustments for frequency trend, severity trend, and average premium trend.

Premium trends tend to be more accurately described by an exponential rather than linear regression.[228]

Countrywide premium trends provide a good second source of information, which can be used to supplement trends based on the data from a single state.

Rate Level Indications:[229]

The indicated rate level change is the difference between the current rate level and the indicated rate level.

The indicated rate level is the rate level that achieves a balance between the expected premium income and the expected losses and expenses (including a profit provision that considers investment income) for a future policy period.

In the calculation of the indicated rate level change, one should recognize the continuous change in the frequency and severity of claims when projecting a future loss level.

There are several factors that can influence the average premium level. In the calculation of the indicated rate level change, one should recognize that the average premium per exposure can change significantly over time, even in the absence of rate changes.

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[228] You should not be asked to calculate a regression on this exam.
[229] To be discussed in my section on Overall Indications.

Problems:

5.1. (2 points) You are given on the following information:

- Final audit occurs 3 months after policy expiration.
- On average, audits result in 8% additional premium.
- Premium writings are even throughout the year.
- All policies are annual.

(a) Compute the policy year written premium development factor from 18 to 24 months.
(b) Compute the policy year earned premium development factor from 18 to 24 months.

5.2. (2 points) In most cases, the recent historical loss ratio for the same book of business makes a good starting point for estimating the expected loss ratio in a future period.
List and briefly discuss two items that can reduce the usefulness of the historical loss ratio as a predictor of the future loss ratio.

5.3. (10 points) An insurer writes 12 month policies.
There was a rate increase of 20%, effective July 1, 2001.
You have the following data on written exposures and written premiums:

Quarter	Written Exposures	Written Premium
1st of 2000	100	97,000
2nd of 2000	103	98,000
3rd of 2000	105	102,000
4th of 2000	109	105,000
1st of 2001	111	106,000
2nd of 2001	115	110,000
3rd of 2001	119	137,000
4th of 2001	120	136,000
1st of 2002	119	139,000
2nd of 2002	117	141,000
3rd of 2002	120	145,000
4th of 2002	124	148,000

The above data on written exposures and premiums will be used to estimate the premium trend.
You are performing a rate indication for this insurer, with a proposed effective date of May 1, 2003. The new rates would be in effect for six months.

(a) (2 points) For the premium trend factors to be applied to Calendar Year 2001 earned premiums, what are the trend periods using the one-step and two-step methods.

(b) (3 points) Calculate the series of average written premiums at current rate level. Show all work.

(c) (4 points) Estimate the annual premium trend, by fitting an exponential regression to this calculated series. Show all work.

(d) (1 point) In this case, would you use the one-step or two-step method? Briefly explain why.

5.4. (2 points) The Phat Insurance Company increased their general liability rates 20% effective June 1, 2010.
Phat writes 30% of its policies with an effective date of January 1, and 10% with an effective date of July 1.
The other 60% of policies are written uniformly throughout the year.
All policies have annual terms.
Calculate the on-level factor to be applied to 2010 Calendar Year Earned Premiums.
Show all work.

5.5. (4 points) You are given the following data for Workers Compensation Insurance:

Policy Year	Number of manweeks worked	Payroll ($00) Developed to Ultimate	Premiums @CRL Developed to Ultimate
2000	250,000	1,500,000	3,750,000
2001	230,000	1,425,000	3,420,000
2002	212,000	1,350,000	3,110,000

(a) (1.5 points) What series could one examine in order to estimate premium trends?

(b) (1 point) Discuss valid ways of using these estimated premium trends,
along with frequency trends and severity trends, in order to project loss ratios.

(c) (1.5 points) Give an explanation for the observed pattern of premiums at current rate level versus manweeks. What impact, if any, would this have on your trend analysis?

∗**5.6**∗**.** (2.5 points) The Simpson Insurance Company writes Homeowners insurance.

Simpson Insurance divides State X into Urban and Rural, and charges different rates in each.
Simpson Insurance has not changed its rates in State X since 2004.
You are given the following data for Simpson Insurance in State X:

Year	Exposures Urban	Exposures Rural	Average Premiums Urban	Average Premiums Rural
2006	5,000	5,000	$1000	$500
2007	5,000	6,000	$1030	$515
2008	5,000	7,000	$1060	$530
2009	5,000	8,000	$1090	$545

(a) (1 point) Estimate the annual premium trend using the data for the whole state.

(b) (1.5 points) Bartholomew is the actuary for the Simpson Insurance Company.
Bartholomew will use a premium trend as determined in part a.
Due to Simpson's small volume of data in State X, Bartholomew will use a loss trend based on the changes in pure premiums for all Homeowners insurers in State X combined.
Fully discuss why this is unlikely to produce an accurate rate indication in this situation.

5.7. (4 points) You are performing a rate indication for ABC Insurance, with a proposed effective date of November 1, 2003. The proposed rates would be in effect for one year.
ABC Insurance writes 12 month policies.
You have the following data on premiums written by ABC Insurance:

Ending Date	Quarterly Average Written Premium at Current Rate Level
6/30/99	$294
9/30/99	$295
12/31/99	$300
3/31/00	$299
6/30/00	$301
9/30/00	$304
12/31/00	$308
3/31/01	$307
6/30/01	$310
9/30/01	$312
12/31/01	$313
3/31/02	$314
6/30/02	$313
9/30/02	$316
12/31/02	$315
3/31/03	$317

You also have ABC Insurance's exposures, premiums, and losses by Calendar/Accident Year:

Calendar Accident Year	Earned House Years	Earned Premiums at Current Rate Level	Incurred Losses Developed to Ultimate	Loss Ratio
1999	5198	$1,523,450	$734,582	48.2%
2000	5517	$1,652,892	$1,130,728	68.4%
2001	5207	$1,603,006	$859,201	53.6%
2002	5047	$1,581,065	$989,632	62.6%

A 4% annual loss trend has been selected.
(a) (2 points) Using the one-piece premium trend method, a 2% annual premium trend is selected. Determine the projected loss ratio for each calendar/accident year. Show all work.
(b) (2 points) Using the two-piece premium trend method, in the second step a 1% annual premium trend is selected. Determine the projected loss ratio for each calendar/accident year. Show all work.

5.8. (8 points) Premium trend is computed based on a series of quarterly premiums from the first quarter of 1997 to the first quarter of 2004.

- Experience period is Calendar/Accident Year 2003 earned premiums and incurred losses.
- Planned effective date is December 1, 2004.
- Rates will be in effect for 1 year.

In each case, what are the beginning, end, and length of the premium trend period(s)?
(a) (1 point) The trend data is written premium. Use the one step method.
Policies have a 12-month term.
(b) (1 point) The trend data is written premium. Use the two step method.
Policies have a 12-month term.
(c) (1 point) The trend data is earned premium. Use the one step method.
Policies have a 12-month term.
(d) (1 point) The trend data is earned premium. Use the two step method.
Policies have a 12-month term.
(e) (1 point) The trend data is written premium. Use the one step method.
Policies have a 6-month term.
(f) (1 point) The trend data is written premium. Use the two step method.
Policies have a 6-month term.
(g) (1 point) The trend data is earned premium. Use the one step method.
Policies have a 6-month term.
(h) (1 point) The trend data is earned premium. Use the two step method.
Policies have a 6-month term.

∗**5.9**∗**.** (8 points) Redo the previous question, with the following change:

- Experience period is Policy Year 2002 earned premiums and incurred losses.

5.10. *(2 points) The Animalfarm Insurance Company writes Homeowners Insurance.
During the summer of 2002, all homes written by Animalfarm Insurance in a certain state are carefully inspected. Among the things the inspectors report is whether the home is insured to 80% of value or more. Briefly discuss whether the resulting effect on average premiums levels is:*
a) One-time vs. continuous.
b) Measurable vs. can only be estimated.
c) Abrupt vs. gradual.
State your reasons and any assumptions you make.

5.11. (4 points) You are given the following information:

- All policies are annual.
- The future policy period begins July 1, 2008.
- The future annual premium trend is 4% per year.
- The annual loss trend is 6% per year.
- The proposed rates will be in effect for one year.

Calendar Accident Year	Earned Exposures	Average Written Premium at Current Rate Level	Average Earned Premium at Current Rate Level	Estimated Ultimate Losses
2003	800	$2007	$1982	$1.32 million
2004	850	2165	2083	1.40 million
2005	900	2210	2191	1.65 million
2006	850	2308	2254	1.71 million

Calculate the projected loss ratio of Earned Premiums to Incurred Losses for each year, using the two-step premium trending method.
Show all work.

5.12. (3 points) You are given:

Calendar Year 2004 Effective Date	Number of Cars Written on Effective Date	Calendar Year 2005 Effective Date	Number of Cars Written on Effective Date
January 1, 2004	10	January 1, 2005	100
April 1, 2004	20	April 1, 2005	120
July 1, 2004	40	July 1, 2005	150
October 1, 2004	60	October 1, 2005	200

- Policies are effective for one year terms.
- Average written premium of $500 per car during 2004.
- There is a 20% rate increase effective January 1, 2005.

Calculate the following:
(a) (1 point) Earned exposures for calendar year 2005.
(b) (1 point) In-force exposures on January 1, 2005.
(c) (1 point) Calendar year 2005 earned premium.

∗**5.13**∗. (13 points) You are given:

Written Premiums for Calendar Year 2003: $1,000,000.
Earned Premiums for Calendar Year 2003: $900,000.
Written Premiums for Policy Year 2003: $1,100,000.
Earned Premiums for Policy Year 2003: $1,100,000.

Rate Level History

Effective Date	% Rate Change
7/1/2000	-4.0%
7/1/2001	+5.0%
1/1/2002	+3.0%
4/1/2002	+4.0%
10/1/2002	-2.0%
4/1/2003	+6.0%
7/1/2003	+2.0%
1/1/2004	-3.0%
10/1/2004	-5.0%

(a) (1 point) Assuming all policies have a term of 6 months,
calculate the on-level written premium for Calendar Year 2003.
(b) (2 points) Assuming all policies have a term of 6 months,
calculate the on-level earned premium for Calendar Year 2003.
(c) (1 point) Assuming all policies have a term of 1 year,
calculate the on-level written premium for Calendar Year 2003.
(d) (2 points) Assuming all policies have a term of 1 year,
calculate the on-level earned premium for Calendar Year 2003.
(e) (1 point) Assuming all policies have a term of 2 years,
calculate the on-level written premium for Calendar Year 2003.
(f) (3 points) Assuming all policies have a term of 2 years,
calculate the on-level earned premium for Calendar Year 2003.
(g) (1 point) Assuming all policies have a term of 6 months,
calculate the on-level premium for Policy Year 2003.
(h) (1 point) Assuming all policies have a term of 1 year,
calculate the on-level premium for Policy Year 2003.
(i) (1 point) Assuming all policies have a term of 2 years,
calculate the on-level premium for Policy Year 2003.

5.14. (1 point) A 6-month policy is written on May 1, 2006 for a premium of $1200.
As of December 31, 2006, which of the following is true?

	Calendar Year 2006 Written Premium	Calendar Year 2006 Earned Premium	Inforce Premium
A.	$1200	$1200	$1200
B.	$700	$700	$1200
C.	$1200	$700	$700
D.	$700	$700	$0
E.	$1200	$1200	$0

5.15. (1 point) An insurer writes the following policies during 2006:

Effective Date	Policy Term	Premium
June 1	6 months	$1,000
September 1	12 months	$1,600
October 1	6 months	$1,200

What is the insurer's unearned premium reserve on December 31, 2006?

5.16. (2 points) You are given the following rate change history for a level book of 12-month term policies uniformly distributed throughout the experience period.
What is the appropriate on-level factor to apply to the 2005 Calendar Year earned premium in order to produce earned premium at the 10/1/2005 rate level?
10/1/2003 +5%
10/1/2004 +20%
10/1/2005 +15%

5.17. (3 points) The Regressive Insurance Company writes six month automobile policies.
In State X, in 2004 they wrote the following car years of exposure by class and territory:

	Territory A	Territory B	Territory C	Territory D
Class 1	100	200	400	300
Class 2	50	150	250	200
Class 3	100	300	400	150

Their current class rate relativities are:

Class 1	Class 2	Class 3
1.200	0.850	1.000

Their current territory rate relativities are:

Territory A	Territory B	Territory C	Territory D
0.800	1.100	1.000	1.400

Their current base rate for one automobile on six month policies is $300.
For State X, determine Regressive's Policy Year 2004 premiums at present rates.

∗**5.18**∗**.** (2 points) The Polar Bear Insurance Company writes insurance coverage for snowmobiles.
They increased their rates 10% effective November 1, 2004.
They then increased their rates an additional 15% effective October 1, 2005.
Snowmobile policies are only written in September, October, November, and December.
Snowmobile policies are written uniformly throughout these four months.
Policies provide coverage for one year, and premiums are earned uniformly throughout the policy term.
Calculate the on-level factor to be applied to Calendar Year 2005 earned premiums.
Show all work.

∗**5.19**∗**.** (2 points) An auto insurer has in force three times as many six-month policies as annual policies. Assume that:

- There is a uniform premium volume written throughout a year.
- There is the same average premium per car-year for 6-month and annual policies.
- There is the same average number of cars per policy for 6-month and annual policies.
- There have been no recent changes in premium volume.

Determine the percentage of Calendar Year 2006 earned premium that is from each policy year.

∗**5.20**∗**.** (2 points) You have performed a rate review for your company's Motorcycle Insurance which issues 6 month policies.
You have calculated an on-level factor of 1.052 for Calendar Year 2006 Earned Premium.
The only rate change in the past few years was one that you assumed to be effective 4/1/2006.
However, upon further review, you realize that this incorrect, and that the rate change was actually implemented effective 8/1/2006.
Recalculate the on-level factor using the 8/1/2006 effective date.

5.21. (6 points) Each year, the Uniform Insurance Company writes four policies, one effective in the middle of each quarter: 2/15, 5/15, 8/15, and 11/15.
The premium for each policy is 1000.
(a) (1 point) If each policy is annual, what is the earned premium for a Calendar Year?
(b) (2 points) What is the average date of writing of the earned premium in part (a)?
(c) (1 point) If each policy is 6-month, what is the earned premium for a Calendar Year?
(d) (2 points) What is the average date of writing of the earned premium in part (c)?

∗**5.22**∗**.** (6 points) An insurance company enters a new state on January 1, 2015.

Once it starts writing, the insurer writes a uniform volume of business throughout the year.
Determine the average date of earning of Calendar Year 2015 earned premiums.
(a) (3 points) Assume annual policies.
(b) (3 points) Assume 6-month policies.

5.23. (3 points) Given the following data on annual policies:

Policy	Effective Date	Initial Policy Premium ($000)
1	November 1, 2014	200
2	May 1, 2015	120
3	October 1, 2015	260
4	February 1, 2016	180
5	September 1, 2016	300

- Six months after the policy expires, the initial policy premium on every policy increases by 5% due to the final audit.

a. Calculate calendar year 2016 written premium as of December 31, 2016.
b. Calculate calendar year 2016 earned premium as of December 31, 2016.
c. Calculate policy year 2016 written premium as of December 31, 2016.
d. Calculate policy year 2016 earned premium as of December 31, 2016.
e. Calculate policy year 2016 written premium as of December 31, 2017.
f. Calculate policy year 2016 earned premium as of December 31, 2017.
g. Calculate policy year 2016 written premium as of December 31, 2018.
h. Calculate policy year 2016 earned premium as of December 31, 2018.

5.24. (2 points) You are given the following information:

Policy	Earned Premiums ($million)			
Year	12 months	24 months	36 months	48 months
2003	22.93	42.22	42.49	42.46
2004	20.30	37.81	38.18	38.55
2005	22.62	42.67	43.42	
2006	29.50	51.95		
2007	31.03			

Estimate the premiums at ultimate for each policy year.
Assume that 48 months is ultimate.

5.25. (2 points) Discuss premium audits and their effect on ratemaking.

5.26. (2 points) An insurance company writes annual policies.
The history of rate changes is as follows:

Effective Date	Overall Rate Change
July 1, 2014	+3%
January 1, 2015	+7%
July 1, 2016	+5%

Assume that 20% of policies are written on the first day of the year and the remaining policies are written evenly throughout the year.
Calculate the on-level premium factor to current rate level for policies in-force on April 1, 2015.

5.27. (2 points) Given the following information:

- All policies have six-month terms.
- Policies are written uniformly during each six-month period and cannot be cancelled.
- The rating algorithm is: (base rate) (class factor) + (expense fee).

Effective Date of Rates	Base Rate Per Exposure	Class Factor A	Class Factor B	Expense Fee Per Exposure
January 1, 2015	$400	1.00	1.30	$40
January 1, 2016	$420	1.00	1.25	$45
January 1, 2017	$450	1.00	1.20	$50

	Written Exposures (000)	
Policy Effective Dates	Class A	Class B
January 1, 2015 - June 30, 2015	200	100
July 1, 2015 - December 31, 2015	250	150
January 1, 2016 - June 30, 2016	300	200
July 1, 2016 - December 31, 2016	400	300

Using the extension of exposures method, calculate the calendar year 2016 earned premium at current rate level.

5.28. (2 points)
Assume an auto insurer writes policies evenly throughout a year and there are no rate changes.
For Policy Year 2015, what portion of the premium is earned by September 1, 2015?
(a) Assume policies are annual.
(b) Assume policies are six months.

5.29. (3 points) An insurance company writes annual policies.
The history of rate changes is as follows:

Effective Date	Overall Rate Change
May 1, 2013	+5%
February 1, 2014	-8%
August 1, 2015	+10%

Assume that 30% of policies are written on the first day of the year and the remaining policies are written evenly throughout the year.
Calculate the on-level premium factor to current rate level for Calendar Year 2014 earned premiums.

5.30. (2 points) An insurance company writes both 6-month and 12-month private passenger automobile policies.
Given the following information:

- 12-month Policy A is written on March 1, 2017. The full-term premium is $800.
  On October 1, 2017 Policy A is canceled.
- 6-month Policy B is written on August 1, 2017. The full-term premium is $500.
- 12-month Policy C is written on September 1, 2017. The full-term premium is $600.
  On November 1, 2017 Policy C is endorsed to add the owner's son to the policy.
  The full term premium is now $1000.
- Full-term written premium represents the policy premium if policy characteristics shown were in place from original effective date to original expiration date.

a. (0.75 point) Calculate the 2017 calendar year written premium as of December 31, 2017.
b. (0.75 point) Calculate the 2017 calendar year earned premium as of December 31, 2017.
c. (0.5 point) Calculate the in-force premium as of December 1, 2017.

5.31. (1 point) Explain the purpose of premium trend adjustments.

5.32. (1.5 points) You are conducting a ratemaking analysis for rates to be effective October 1, 2017. The new rates are to be in effect for one year.
60% of the premium is written on annual policies, while the remaining 40% of the premium is written on six-month policies.
The annual premium trend is 4%.
Determine the trend factor to be applied to 2016 Calendar Year earned premiums.

5.33. (1 point) Premium trend is computed from 12-month moving averages based on a series of quarterly written premiums from the first quarter of 2011 through the third quarter of 2015.
You have been asked to trend the earned premiums for calendar/accident year 2012 using both the one-step and two-step trending methods.
Assume that the planned effective date of the next rate change is April 1, 2016 and that the rates will be in effect for 18 months. All policies are annual.
a. (0.5 points) What are the starting and ending dates of the trending period using the one-step method?
b. (0.5 points) What are the starting and ending dates of each step using the two-step method?

5.34. (1.5 points) While working on the rate indication for your company's personal inland marine book, you calculate the On-Level Earned Premium factor to be 1.1034 for calendar/accident year 2005 earned premium. You assumed that there was only one rate change for the past ten years and it took place on April 1, 2005. Upon review, however, your boss tells you that the actual date of that rate change was August 15, 2005. Assuming annual policies, calculate the percentage change in the On-Level Earned Premium factor based on the correct information. Show all work.

5.35. (2 points) You are given the following information.

Accident Year	Average Earned Premium @ Current Rate Level	Average Written Premium @ Current Rate Level
2005	$198	$200
2006	$202	$204
2007	$210	$216
2008	$214	$212

- The projected premium trend is 3%.
- The proposed effective date of new rates is September 1, 2009.
- The proposed rates will remain in effect for one year.
- All policies are annual.

Calculate the premium trend factor needed to project 2006 calendar/accident year earned premium to prospective rate levels, using the two-step trending procedure.
(a) (1 point) Use the series of written premiums.
(b) (1 point) Use the series of earned premiums.

***5.36*.** (1 point) Workers Compensation policies are written evenly over any given year.
Premiums are first audited 9 months after policy expiration.
On average the result of the audit is a 9.5% increase in premiums.
What is the policy year earned premium development factor from 24 to 36 months?

5.37. (2 points) Use the following information on Homeowners Insurance in order to construct a series appropriate to use for premium trending. Show all work.

Date	Rate Change
September 1, 2017	-6%
October 1, 2018	+10%
April 1, 2019	+5%

In addition, on January 1, 2018, the insurer increased its discount for long term customers from 5% to 10%; 70% of insureds qualify for this discount.

Time Period	Written Premium($000)	Written Exposures
First Quarter 2017	2000	4100
Second Quarter 2017	2100	4200
Third Quarter 2017	2100	4300
Fourth Quarter 2017	2200	4400
First Quarter 2018	2300	4500
Second Quarter 2018	2300	4400
Third Quarter 2018	2500	4700
Fourth Quarter 2018	2800	4700

5.38. (3.5 points) The Deerford Insurance Group sells both six month and twelve month automobile insurance policies.
In the state of Hart, Deerford Insurance Group writes its 6-month and 12-month auto policies in two different underwriting companies, Bambi Insurance and Stag Insurance.
Bambi and Stag had both been charging $750 per car year.
Bambi Insurance, which writes the 6-month policies, had a rate change of +10.0% on April 1, 2015.
Stag Insurance, which writes the 12-month policies, had a rate change of -6% on July 1, 2015.
Here is their data for the state of Hart:

Effective Date	Bambi, Autos Written	Stag, Autos Written
January 1, 2014	750	350
April 1, 2014	700	400
July 1, 2014	650	450
October 1, 2014	600	500
January 1, 2015	550	550
April 1, 2015	500	600
July 1, 2015	450	650
October 1, 2015	400	700

a. (0.5 points) How many earned car years were there in Calendar Year 2015?
b. (1 point) What is the earned premium for Calendar Year 2015?
c. (0.5 points) What is the earned premium at present rates for Calendar Year 2015 using the extension of exposures method?
d. (1.5 points) What is the on-level earned premium for Calendar Year 2015 using the parallelogram method?

5.39. (3 points) You are given the following information.
Using a two-step trending procedure, answer the questions below. Show all work.

- The premium trend series consists of quarterly values from January 1, 2010 through December 31, 2013: 1st Q 2010, 2nd Q 2010, ..., 3rd Q 2013, 4th Q 2013.
- Planned effective date is July 1, 2015.
- Rates are reviewed annually.
- Policies have a 6-month term.
- The trend will apply to calendar-accident year 2012 earned premium at current rate level.

a. (1 point) Calculate the beginning and ending dates for each of the Step 1 and Step 2 trend periods, assuming that the trend series consists of average <u>written</u> premium.
b. (1 point) Calculate the beginning and ending dates for each of the Step 1 and Step 2 trend periods, assuming that the trend series consists of average <u>earned</u> premium.
c. (1 point) Describe a situation when it may be more appropriate to use a two-step trending procedure, rather than a one-step trending procedure.

5.40. (2 points) There are two classes and two territories, with current rates of:

	Class 1	Class 2
Territory A	500	700
Territory B	800	1000

We are given the following written exposures (000):

<u>Calendar Year 2015</u>

	Class 1	Class 2
Territory A	1500	1200
Territory B	600	900

<u>Calendar Year 2016</u>

	Class 1	Class 2
Territory A	1700	1400
Territory B	600	900

<u>Calendar Year 2017</u>

	Class 1	Class 2
Territory A	1900	1600
Territory B	600	900

Based on the above historical information, determine a reasonable annual premium trend.

5.41. (1.5 points) Given the following information:
- On July 1, 2017, rates were increased by 5%.
- On July 1, 2018, rates were increased by 10%.
- There have been no subsequent rate changes.
- Policy year 2018 earned premium = $500 million.

In each case, calculate the policy year 2018 earned premium at current rate level.
a. (0.5 points) Assume the following written exposures by quarter during 2018:
1 million, 1 million, 1 million, 1 million.
b. (0.5 points) Assume the following written exposures by quarter during 2018:
0.7 million, 0.9 million, 1.1 million, 1.3 million.
c. (0.5 points) Assume the following written exposures by quarter during 2018:
1.3 million, 1.1 million, 0.9 million, 0.7 million.

5.42. (0.25 point) Briefly describe a scenario in which policy year 2018 premium is not fixed as of the end of 2019.

5.43. (2 points) The Pacific Charter Insurance Company writes Workers Compensation Insurance in only one state, and only has exposures in three classes: 8833, 8835, and 9040.

Class	Description	Current Rate
8833	HOSPITAL: PROFESSIONAL EMPLOYEES	1.13
8835	NURSING HOME - ALL EMPLOYEES	1.94
9040	HOSPITAL: ALL OTHER EMPLOYEES	2.88

Pacific Charter's recent calendar year written exposures (in units of $100 of payroll) by class:

Year	8833	8835	9040
2014	3,000,000	4,000,000	300,000
2015	3,300,000	4,100,000	330,000
2016	3,600,000	4,200,000	360,000
2017	4,000,000	4,300,000	400,000

Hardy Knox is a consulting actuary who is preparing a rate indication for Pacific Charter Insurance.
Hardy has already determined an exposure trend based on the past and future expected changes in the state average weekly wage.
Hardy will estimate an annual premium trend in addition to this exposure trend.
Based on the above historical information, determine a reasonable annual premium trend.

5.44 (1 point) Use the following information in millions:

Premium Written in Calendar Year 2017	800
Unearned Premium as of December 31, 2016	370
Unearned Premium as of December 31, 2017	390

Determine the earned premium for Calendar Year 2017.

5.45. (2 points) For a block of 6-month automobile policies:

Effective Date	Premiums ($ million)
January 1, 2017	25
April 1, 2017	20
July 1, 2017	21
October 1, 2017	22
January 1, 2018	30
April 1, 2018	24
July 1, 2018	26
October 1, 2018	28

- Assume that no policies are canceled midterm.

a. (0.5 point) Calculate the written premium for calendar year 2018.
b. (0.5 point) Calculate the earned premium for calendar year 2018.
c. (0.5 point) Calculate the written premium for the fiscal year ending June 30, 2018.
d. (0.5 point) Calculate the earned premium for the fiscal year ending June 30, 2018.

5.46 (3 points) Given the following:

Calendar Year	Earned Premium ($ million)
2018	462
2019	546

Rate Change Effective Date	Overall Rate Change
January 1, 2018	6%
July 1, 2018	11%

Quarter and Year	Average Written Premium at Current Rate Level
2Q 2017	$1,546
4Q 2017	$1,569
2Q 2018	$1,591
4Q 2018	$1,618
2Q 2019	$1,640
4Q 2019	$1,667

- No rate changes occurred in 2017 or 2019.
- Rates will be in effect for one year.
- All policies are written uniformly throughout the year.

a. (1.5 points) Calculate the trended on-level earned premium for 2018 to be used in a rate change effective August 1, 2020, assuming that all policies are semi-annual.

b. (1.5 points) Calculate the trended on-level earned premium for 2018 to be used in a rate change effective August 1, 2020, assuming instead that all policies are annual.

5.47. (6, 5/92, Q.44) (2 points) Use the following written exposure data, and the fact that a 10% rate increase was effective July 1, 1989.
Assume there are no other rate changes.
If you define the rate level in effect on June 30, 1989 as unity, what is the average rate level for Calendar Year 1990 earned premiums?

Time Period	Written Exposures
01/1/89 - 03/31/89	200
04/1/89 - 06/30/89	400
07/1/89 - 09/30/89	600
10/1/89 - 12/31/89	800
01/1/90 - 03/31/90	1000
04/1/90 - 06/30/90	1200
07/1/90 - 09/30/90	1400
10/1/90 - 12/31/90	1600

Assume the policy term is one year.

5.48. (6, 5/94, Q.1) (1 point) An insurer writes the following policies during 1992:

Effective Date	Policy Term	Premium
May 1	6 months	$6,000
August 1	12 months	$12,000
November 1	6 months	$2,400

What is the insurer's unearned premium reserve on December 31, 1992?

5.49. (6, 5/94, Q.48) (3 points)
The adjustments to rates that affect the experience period are shown below.

- Experience rate change of 10% on 7/1/92.
- Law amendment change of 2% on 1/1/93.
- Experience rate change of 15% on 7/1/93.
- Law amendment change of 3% on 1/1/94.

Premium writings are evenly distributed throughout the year.
(a) (1.5 points) What adjustment factor is needed to bring calendar year 1993 premiums to current level? (Show a diagram representing the appropriate time periods.)
(b) (1.5 points) What adjustment factor is needed to bring policy year 1993 premiums to current level? (Show a diagram representing the appropriate time periods.)

5.50. (6, 5/95, Q.30) (4 points) You are given:

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Year	Quarter	Written Exposures
1993	1st	250
	2nd	750
	3rd	1250
	4th	1750
1994	1st	2250
	2nd	2750
	3rd	3250
	4th	3750

- A 10% increase was implemented effective 9/30/93.
- All policies are annual.
- The annual rate of written exposures at time "x" is 8000x.

(Note: x = 0 at 1/1/93 and x = 2 at 12/31/94.)

(a) (3 points) Determine the percentage of 1994 earned exposures that are at the 9/30/93 (10% higher) rate level using:

1. The traditional parallelogram method .
2. An alternate method that accounts for varying exposure levels.

(b) (1 point) Based on your results, will the use of the traditional parallelogram method overstate or understate the rate level indication? Explain.

Note: I have rewritten this past exam question to match the current syllabus.

5.51. (6, 5/96, Q.31) (3 points) You are given:

Calendar Year 1996 Effective Date	Number of Cars Written on Effective Date	Calendar Year 1997 Effective Date	Number of Cars Written on Effective Date
1/1/96	10	1/1/97	100
4/1/96	20	4/1/97	120
7/1/96	40	7/1/97	150
10/1/96	60	10/1/97	200

- Policies are effective for six-month terms.
- Average written premium of $500 per car each policy period.

Calculate the following:

(a) (1 point) Earned exposures for calendar year 1997.

(b) (1 point) In-force exposures on 1/1/97.

(c) (1 point) Calendar year 1996 earned premium.

∗**5.52**∗. **(6, 5/96, Q.36)** (3 points) You are given:

Rate Change	Implementation Date	Type of Change
+8%	5/1/94	Experience
+15%	7/1/95	Law Amendment
-10%	7/1/95	Experience
+5%	4/1/96	Experience

- Policies are written uniformly throughout the year.

(a) (2 points) Calculate the premium adjustment factor to bring policy year 1995 premium to current rate level.

(b) (1 point) How are experience rate changes and law amendment rate changes different in their purpose and their effect?

5.53. (6, 5/97, Q.4) (1 point)
Given that exposures are declining during a period of rising rate levels, which of the following are true if the traditional parallelogram method is used instead of the three-dimensional method?

1. The premium adjustment factor will be overstated.
2. The adjusted loss ratio will be overstated.
3. The rate level indication will be overstated.

5.54. (6, 5/97, Q.12) (1 point) You are given for Workers Compensation Insurance:

- Full estimated policy premium is booked at inception.
- Premium develops upward by 7% at final audit, six months after the policy expires.
- All policies are written for an annual period.
- Premium is written uniformly throughout the year.

Determine the policy year premium development factor for 24 to 36 months.

5.55. (6, 5/97, Q.19) (1 point) You are given:

Effective Date	Rate Change
4/1/94	+5.0%
7/1/95	+13.0%
4/1/96	-3.0%

- All policies are 12 month policies.
- Policies are written uniformly throughout the year.

Using the parallelogram method, determine the on-level premium factor, to bring calendar year 1995 earned premium to current rate level.

5.56. (1 point) Using the information from 6, 5/97, Q.19, determine the on-level premium factor, in order to bring calendar year 1995 written premium to current rate level.

5.57. (1 point) Using the information from 6, 5/97, Q.19, except with 6-month policies, determine the on-level premium factor, in order to bring calendar year 1995 earned premium to current rate level.

5.58. (2 points) Using the information from 6, 5/97, Q.19, except with two-year policies, determine the on-level premium factor, in order to bring calendar year 1995 earned premium to current rate level. Assume there were no rate changes during 1993.

5.59. (1 point) Using the information from 6, 5/97, Q.19, determine the on-level premium factor, in order to bring Policy Year 1995 premium to the current rate level.

***5.60. (6, 5/98, Q.25)** (2 points) In Homeowners Insurance, there are various reasons for changes in replacement cost and amount of insurance in an insurer's book of business. List three reasons other than inflation.*

5.61. (6, 5/98, Q.39) (5 points) You are the actuary responsible for the 1998 rate level review.
The company began writing private passenger automobile insurance on 1/1/96, x = 0.
An overall 15% rate decrease was implemented on 1/1/97.
All policies are written for a six month term.
The function representing the rate of exposure writing at time x is as follows: f(x) = 2,500x.
Answer the following:
a. (3 points) Determine the average rate level for 1997 earned Calendar Year.
b. (1 point) Determine the average rate level for 1997 earned Calendar Year, using the traditional parallelogram method.
c. (1 point) Briefly discuss the reason for the difference between your answers in parts a and b.
What would be the impact on a rate level indication.
Note: this past exam question has been rewritten in order to match the current syllabus.

5.62. (6, 5/98, Q.41) (2 points) You are given the following information for your company's private passenger automobile line of business.

Calendar Year	Earned Premium
1994	\$1,000
1995	\$1,200
1996	\$1,400

Overall Rate Change	Effective Date
+5.0%	9/1/94
+10.0%	1/1/95
-5.0%	1/1/96
+15.0%	4/1/97

Assume all policies are semiannual and that all months have the same number of days.
Using the parallelogram method compute the calendar year 1995 earned premium at present rates.

5.63. (2 points) In the previous question, what is the calendar year 1996 earned premium at present rates?

5.64. (6, 5/98, Q.47) (2 points) Gives examples of situations in which the traditional model parallelogram method will produce an inaccurate rate indication.
Note: This past exam question has been rewritten in order to match the current syllabus.

5.65. (6, 5/99, Q.37a) (1 point) Using the information shown below, calculate the policy year premium development factor from 24 to 36 months for Workers Compensation Insurance.

- Initial estimates of policy year premium are $1 million per month from January through June and $1.1 million per month for the remainder of the year.
- Final audit occurs six months after policy expiration.
- Premium develops upward by 20% at the final audit.
- All policies are annual.

5.66. (6, 5/99, Q.38) (2 points) Based on the information shown below, calculate the average benefit as a percentage of the average wage.

Ratio to Average Wage	% of Workers	% of Wages
0.00 - 0.50	15%	6%
0.50 - 0.75	20%	12%
0.75 - 1.00	25%	21%
1.00 - 1.50	20%	24%
1.50 - 2.00	15%	26%
2.00 - 2.50	5%	11%

Minimum benefit = 0.75 of average wage
Maximum benefit = 1.50 of average wage
Compensation rate = 0.75 of pre-injury wage

5.67. (6, 5/99, Q.44) (4 points) You are an actuary employed by BMIC.
BMIC began writing products liability insurance policies on January 1, 1997.
In 1999, you have been asked to prepare a rate review for BMIC's products liability insurance.

- The products liability insurance policies are written on an annual basis.
- The number of products liability insurance policies issued have been increasing at a constant rate of 5,000 per year.
- The only rate change has been a 20% increase effective January 1, 1998.

a. (1 point) What are the total earned exposures for Calendar Year 1998?
b. (1 point) What are the earned exposures at the higher rate level in Calendar Year 1998?
c. (1/2 point) What is the average rate level for Calendar Year 1998?
d. (1/2 point) What is the appropriate premium adjustment factor for Calendar Year 1998?
e. (1 point) Would the rate level indication based on Calendar Year 1998 using the traditional parallelogram method be higher or lower than the indication using your results from part d? Why?

5.68. (6, 5/99, Q.58) (2 points) You have performed a rate review for your company's Homeowners line of business which issues annual policies. You have calculated an on-level factor of 1.080 for Calendar Year 1998 Earned Premium. The only rate change in the past few years was one that you assumed to be effective 1/1/98. However, upon further review, you realize that the effective date is incorrect and that the rate change was actually implemented effective 3/1/98. Recalculate the on-level factor using the 3/1/98 effective date.
Assume that all months have an equal number of days and that premium writings are evenly distributed through the year.

5.69. (5, 5/00, Q.38) (4 points) Based on the following data, answer the questions below.
Personal Automobile Liability Data:

Calendar Year 1997		Calendar Year 1998	
Effective Date	Number of Autos Written on Effective Date	Effective Date	Number of Autos Written on Effective Date
January 1, 1997	100	January 1, 1998	900
April 1, 1997	300	April 1, 1998	1,100
July 1, 1997	500	July 1, 1998	1,300
October 1, 1997	700	October 1, 1998	1,500

Assume:

- All policies are twelve-month policies.
- Written premium per car during calendar year 1997 is $500.
- A uniform rate increase of 15% was introduced effective July 1, 1998.

a. (1/2 point) Calculate the number of in-force exposures on January 1, 1998.
b. (1 point) Calculate the number of earned exposures for calendar year 1998.
c. (1/2 point) List the two methods that are used to adjust earned premiums to a current rate level basis.
d. (1 point) Which of the two methods listed in part c. above would be for this company's personal automobile liability business more appropriate to use?
Briefly explain why.
e. (1 point) Using your selected method from part d. above, calculate the on-level earned premium for calendar year 1998.

5.70. (3 points) Use the information in the previous question, except assume that all policies are six-month policies, which during calendar year 1997 cost $300 per car.
a. (1/2 point) Calculate the number of in-force exposures on January 1, 1998.
b. (1 point) Calculate the number of earned exposures for calendar year 1998.
c. (1.5 points) Calculate the earned premium on a current rate level basis for calendar year 1998, using each of the two methods.

5.71. (5, 5/01, Q.15) (1 point) Based on the following information, compute the policy year reported premium development factor from 12 to 24 months.

- Final audit occurs 3 months after policy expiration.
- On average, audits result in 15% additional premium.
- Premium writings are even throughout the year.
- All policies are annual.

5.72. (5, 5/01, Q.38) (2 points) Using the parallelogram method, determine the calendar year 1999 on-level earned premium. Show all work.

Calendar Year	Earned Premium	Effective Date	Rate Change
1997	$10,000	July 1, 1997	+5.2%
1998	$11,500	No Change	No Change
1999	$14,000	April 1, 1999	+7.4%

- All policies are 2-year policies.
- Policies are written uniformly throughout the year.

5.73. (5, 5/03, Q.10) (1 point) A 12-month policy is written on March 1, 2002 for a premium of $900. As of December 31, 2002, which of the following is true?

	Calendar Year 2002 Written Premium	Calendar Year 2002 Earned Premium	Inforce Premium
A.	$900	$900	$900
B.	$750	$750	$900
C.	$900	$750	$750
D.	$750	$750	$750
E.	$900	$750	$900

5.74. (5, 5/03, Q.11) (1 point)
Given the information below, determine the written premium trend period.

- Experience period is April 1, 2001 to March 31, 2002
- Planned effective date is April 1, 2003
- Policies have a 6-month term
- Rates are reviewed every 18 months
- Historical premium is earned premium

5.75. (1 point) In the previous question, 5, 5/03, Q.11, determine the written premium trend period, if instead the rates will be effective for 12 months.

5.76. (1 point) In 5, 5/03, Q.11, determine the premium trend period, if instead the series of premium trend data is earned.

5.77. (1 point) In 5, 5/03, Q.11, determine the premium trend period, if instead the policies are annual.

5.78. (5, 5/03, Q.31) (3 points) Answer the following questions.
a. (1 point) State four changes that can cause the future average premium level to differ from the past average premium level.
b. (1 point) Provide one example of each change listed in part a. above.
c. (1 point) Note whether each change has, in general, a one-time or continuous effect on the average premium level.

5.79. (5, 5/03, Q.33) (2 points)
Using the information shown below, calculate the factor needed to adjust policy year 2002 written premium to current level. Show all work.

- Policies are written uniformly throughout the year and have a term of 12 months.
- The law amendment change affects all policies in force.

Assume the following rate changes:

- Law amendment change on July 1, 2002 = +10%
- Experience rate change on October 1, 2002 = +5%
- Experience rate change on January 1, 2003 = +7%

5.80. (5, 5/04, Q.7) (1 point) Given the following data, calculate the trended loss ratio.

Number of Insureds	Earned Premium	Developed Incurred Losses
20	$50,000	$35,000

- Years of Trend = 2.5
- Annual Exposure Trend = 2.0%
- Annual Premium Trend = 2.9%
- Annual Frequency Trend = -1.0%
- Annual Severity Trend = 6.0%

5.81. (5, 5/04, Q.11) (1 point) Given the following data, calculate the policy year 2001 premium development factor from 24 to 36 months.

- Full estimated policy year premium is booked at inception, $10 million a month in 2001.
- Premium develops upward by 5% at the final audit, three months after the policy expires.
- All policies are annual.

5.82. (5, 5/04, Q.31) (4 points) Given the following information, answer the questions below. Show all work.

- Policies are written uniformly throughout the year.
- policies have a term of 12 months.
- The law amendment change affects all policies in force.

Assume the following rate changes:

- Experience rate change on October 1, 2001 = +7%
- Experience rate change on July 1, 2002 = +10%
- Law amendment change on July 1, 2003 = -5%

a. (2 points)
Calculate the factor needed to adjust calendar year 2002 earned premium to current level.
b. (2 points)
Calculate the factor needed to adjust policy year 2002 earned premium to current level.

5.83. (2 points) In 5, 5/04, Q.31, calculate the factor needed to adjust calendar year 2003 earned premium to current level.

5.84. (5, 5/04, Q.32) (3 points) You are given the following information:

- Calendar year 2002 data is being reviewed for an indicated rate change with an effective date in 2004.
- Assume 50% of the business has an effective date of January 1, with the remaining business written evenly throughout the year.

For each of the situations below, describe:
1. The adjustments that should be made to the historical premium.
2. The impact each situation has on the premium trend.

a. (1 point) A rate increase was taken in April 2002.
b. (1 point) The minimum liability limit changed in June 2003.
c. (1 point) The company has a model year rating plan, which assigns larger rating factors to more recently manufactured vehicles.

5.85. (5, 5/04, Q.35) (3 points) You are given the following information.
Using a two-step trending procedure, answer the questions below. Show all work.

- The experience period is January 1, 2001 through December 31, 2003.
- Planned effective date is July 1, 2005.
- Rates are reviewed annually.
- Policies have a 6-month term.
- The trend will apply to calendar-accident year 2002 earned premium at current rate level.

a. (1 point) Calculate the beginning and ending dates for each of the Step 1 and Step 2 trend periods, assuming the selected trend is based on average written premium.
b. (1 point) Calculate the beginning and ending dates for each of the Step 1 and Step 2 trend periods, assuming the selected trend is based on average earned premium.
c. (1 point) Describe a situation when it may be more appropriate to use a two-step trending procedure, rather than a one-step trending procedure.

5.86. (2 points) Redo parts a and b of the previous question, 5, 5/04, Q.35, assuming instead that policies have a 12-month term.

5.87. (5, 5/05, Q.12) (1 point) The average premium level should be adjusted for which of the following when determining a premium trend?
1. Rating plan changes that affect the average premium level without affecting the level of coverage.
2. Rating plan changes that affect the average premium level but also include a corresponding change in the level of coverage.
3. The use of rating plans has changed the average premium level over time, if the changes are not expected to continue.

∗**5.88**∗**. (5, 5/05, Q.37)** (4 points) Given the information below, answer the following questions.
Show all work.

Calendar/Accident Year	Average Written Premium
2002	\$1000.00
2003	\$ 933.33
2004	\$ 882.00

- The planned effective date for a rate change is January 1, 2006.
- Rates are reviewed every 18 months.
- All policies are annual, and are written uniformly throughout the year.
- A 20% rate decrease was implemented effective July 1, 2003.
- A separate analysis has determined that a shift in the limit distribution from 2002-2004 has resulted in a +3% annual premium trend.
This shift is not expected to continue past 2004.

a. (3.5 points)
Using two-step trending, determine the total premium trend factors for each year above.
b. (0.5 point) Why is two-step trending a more suitable procedure for trending premium than for trending loss frequency or severity?

5.89. (5, 5/05, Q.38) (1.5 points) The parallelogram method is used to adjust calendar year 2003 earned premium to current rate level. Given the following information, will the parallelogram method understate, overstate, or accurately state the on-level factor applied to calendar year 2003 earned premium? Explain your answer.

- There was a 10% rate increase effective on January 1, 2003.
- The written exposures grew 5% each month in 2003.

∗**5.90**∗**. (6, 11/05, Q.6)** (1 point) You are given the following information:
Policy term is 6 months with an initial premium of \$3,600 and an effective date of August 1, 2004.
An endorsement was added to the policy on October 1, 2004 for an additional premium of \$2,400.
What is the in-force premium at December 31, 2004?

5.91. (5, 5/06, Q.5) (1 points)
Which of the following is not necessary to determine the trending period for premium?
A. Anticipated effective date of prospective rate change
B. Term of the policies contributing to the experience and forecast periods
C. Historical rate changes and effective dates
D. Expected time that the projected rates will be in effect
E. Length of historical experience period

5.92. (5, 5/06, Q.26) (3.5 points) As the actuary for Company XYZ, you are performing a physical damage rate review for State X.
Use the following information to answer the questions below.

- Experience period consists of calendar year premium for 2002 through 2004.
- Current level earned premium for calendar year 2002 is $42,500,000.
- Planned effective date of rate revision is June 1, 2006.
- Anticipate annual rate revisions every 12 months.

Each year, insureds purchase newer, more expensive vehicles, resulting in upward premium drift. Historically, the premium drift has averaged 5% through 2004. However, given current trends and expectations regarding future car sales, the insurer expects a 3% premium drift in the future.
The insurer uses exponential premium trend.
a. (1.5 points)
Assume all policies have a six-month term. Use 2-step trending with average written premium to calculate the trended premium for calendar year 2002. Show all work.
b. (1.5 points)
Assume all policies have an annual term. Use 2-step trending with average written premium to calculate the trended premium for calendar year 2002. Show all work.
c. (0.5 point) Explain one advantage of using 2-step trending in this example over 1-step trending.

5.93. (5, 5/06, Q.27) (1 point)
a. (0.5 point) Explain why using average premiums is better than total premiums when analyzing premium trend.
b. (0.5 point) Give one argument for using average earned premiums in the premium trend analysis and one argument for using average written premiums.

5.94. (5, 5/06, Q.28) (3 points) Company XYZ reduced rates 8% effective May 1, 2004, which was their first rate change since January 1, 2000. Assume all policies have annual terms.
a. (1 point) Using the parallelogram method, calculate the 2005 on-level factor. Show all work.
b. (0.5 point) Assume that this change was for a boatowners line and that 50% of the policies are written uniformly throughout May and June, with the other 50% written uniformly throughout the rest of the year. Is the calculation above reasonable for this line? Explain.
c. (1.5 points) Based on the assumptions given in part b. above, calculate the 2005 on-level factor. Show all work.

5.95. (5, 5/07, Q.34) (2.0 points)
You are given the following information for four policies with annual policy terms:

Policy	Effective Date	Premium
A	January 1, 2004	$1,200
B	July 1, 2004	2,400
C	November 1, 2004	3,600
D	April 1, 2005	600

Based on these four policies, calculate:
a. (0.5 point) 2004 written premium.
b. (0.5 point) 2004 earned premium.
c. (0.5 point) 2004 policy year premium.
d. (0.5 point) Premium in-force as of March 31, 2005.
Show all work.

5.96. (2.0 points) Redo the previous question, 5, 5/07, Q.34, if instead the policies each have a term of six months, but have the same premiums as shown.

5.97. (5, 5/07, Q.35) (1.0 point)
Explain the effect of each of the following on the average premium level.
a. (0.5 point) Past rate changes.
b. (0.5 point) Model year and symbol rating plans.

5.98. (5, 5/07, Q.36) (3.0 points) You are given the following information:

- All policies are annual.
- The future policy period begins January 1, 2007.
- The future annual premium trend is 3% per year.
- The proposed rates will be in effect for one year.

Calendar Year	Earned Exposures	Average Written Premium At Current Rate Level	Average Earned Premium At Current Rate Level
2003	1,000	$3,777	$3,605
2004	1,050	3,688	3,749
2005	1,100	3,998	3,899

Calculate the trended premium for each year, using the two-step trending method. Show all work.

5.99. (5, 5/07, Q.37) (2.0 points)
Assume the following information about a worker's compensation insurer:

- All policies are annual.
- April 1, 2004: The company implemented a 10% experience rate change.
- October 1, 2004: The company implemented a 5% rate change due to a law change that impacted all in-force policies.

a. (1.0 point) Draw the diagram underlying the calculation of the current rate level factor used to adjust policy year 2004 premium to current rate level.

- Label the starting and ending dates of the historical period.
- Label the rate change and law change.
- Calculate the relative rate level of each area and label the diagram.
- Do not calculate the percentage each area represents of the year.

b. (1.0 point) Draw the diagram underlying the calculation of the current rate level factor used to adjust calendar year 2004 earned premium to current rate level.

- Label the starting and ending dates of the historical period.
- Label the rate change and law change.
- Calculate the relative rate level of each area and label the diagram.
- Do not calculate the percentage each area represents of the year.

Show all work.

5.100. (2 points) In 5, 5/07, Q.37, determine the factors to bring premium to the current rate level.
a. (1.0 point) Policy year 2004 premium.
b. (1.0 point) Calendar year 2004 earned premium.

5.101. (5, 5/08, Q.13) (2.0 points) Define the following terms.
a. Written premium
b. Earned premium
c. Unearned premium
d. In-force premium

5.102. (5, 5/08, Q.14) (2.5 points) Assume a -8% rate change was implemented effective March 1, 2005 and that all policies have annual terms.

a. (1.0 point) Calculate the on-level factors for calendar years 2005 and 2006 earned premiums using the parallelogram method.

b. (1.0 point) Calculate the on-level factors for policy years 2005 and 2006 earned premiums using the parallelogram method.

c. (0.5 point) Briefly describe the extension of exposure method and briefly explain why it may be preferable to the parallelogram method for determining on-level premiums.

5.103. (5, 5/08, Q.15) (2.0 points)

a. (0.75 point) Provide an example of a situation that can cause a one-time abrupt change in the average premium level and explain how the historical premiums should be adjusted to account for this abrupt change.

b. (1.25 points) You are given the following information.

Accident Year	Average Earned Premium @ Current Rate Level	Average Written Premium @ Current Rate Level
2004	$ 98	$100
2005	$102	$104
2006	$106	$108
2007	$110	$112

- The projected premium trend is 4%.
- The proposed effective date of new rates is January 1, 2009.
- The proposed rates will remain in effect for one year.
- All policies are semiannual.

Calculate the premium trend factor needed to project 2006 calendar/accident year earned premium to prospective rate levels, using the two-step trending procedure.

5.104. (5, 5/09, Q.18) (2 points) The following is the premium associated with five annual policies, where premium is earned uniformly throughout the year:

Policy	Effective Date	Premium
1	January 1, 2007	$750
2	April 1, 2007	$1200
3	July 1, 2007	$900
4	October 1, 2007	$800
5	January 1, 2008	$850

a. (0.5 point) Calculate the total calendar year 2007 written premium.

b. (0.5 point) Calculate the total calendar year 2008 earned premium.

c. (0.5 point) Calculate the total policy year 2007 earned premium as of March 31, 2008.

d. (0.5 point) Calculate the total in-force premium as of July 1, 2008.

5.105. (5, 5/09, Q.19) (2.5 points) Given the following information:

- All policies are semi-annual.
- A +5% rate change was implemented effective October 1, 2007.
- A benefit change of +10% was enacted affecting premium on all outstanding policies on July 1, 2008.

a. (0.75 point)
Draw and label a diagram of the parallelogram method for calendar year 2008 earned premium.
b. (1.25 points) Calculate the on-level factor for calendar year 2008 earned premium.
c. (0.5 point) Explain why the parallelogram method may not be appropriate for calculating on-level factors for snowmobile insurance.

5.106. (5, 5/09, Q.20) (1 point) a. (0.5 point) Explain why an actuary should use written premium, rather than earned premium, to analyze premium trend.
b. (0.5 point)
Explain why an actuary should use premium adjusted to current rate level to analyze premium trend.

5.107. (5, 5/09, Q.21) (1.5 points) For each of the following:
i. Explain the effect on average premium level.
ii. Briefly describe how to adjust the historical premium for use in ratemaking.
a. (0.75 point) An automobile insurer implements a new 5% discount for female drivers without offsetting base rates.
b. (0.75 point) A homeowners insurer includes an inflation guard endorsement on all policies, which automatically increases Coverage A in line with an external inflation index.

5.108. (5, 5/10, Q.18) (2 points) Given the following information:

Calendar Year	Earned Exposures	Written Exposures	On-Level Earned Premium	On-Level Written Premium
2008	1,000	1,100	$487,500	$550,000
2009	1,200	1,300	$615,000	$682,500

- All policies are annual.
- Proposed effective date is January 1, 2011.
- Rates are expected to be in effect for one year.
- Projected premium trend is 5%.

Calculate the calendar year 2008 earned premium at prospective levels using two-step trending.

5.109. (5, 5/10, Q.19) (3 points)
Given the following information for Company XYZ book of business in State X:

- All policies are semi-annual.
- A law change is effective on July 1, 2008 and applies to all in-force and future policies.
  The estimated overall premium impact of the law change is +10%.
- A 5% overall rate increase is implemented on October 1, 2008.
- 2008 calendar year earned premium is $1,000,000.

a. (1 point) Draw and fully label a diagram for calendar year 2008 earned premium reflecting the parallelogram method.
b. (1 point) Calculate the on-level factor for calendar year 2008 earned premium.
c. (1 point) Draw and fully label a diagram for policy year 2008 earned premium reflecting the parallelogram method.

5.110. (5, 5/11, Q.4) (1.5 points) Company ABC began writing annual personal automobile policies on January 1, 2010, using the following rating structure:

- Policy Premium = Base Rate x Class Factor + Policy Fee
- Base Rate = $1,000
- Policy Fee = $50

Class	Class Factor
Teens	2.00
Adults	1.00

On July 1, 2010, the company increased the base rate to $1,100 and revised the class factor for adults to 0.90.
Company ABC writes 10 policies per quarter, each with an effective date of the beginning of the quarter. The company writes an even distribution of teen and adult classes each quarter.
a. (1 point) Calculate the calendar year 2010 earned premium.
b. (0.5 point) Calculate the on-level factor that applies to the calendar year 2010 earned premium to bring premiums to current rate level.

5.111. (5, 5/11, Q.5) (2.25 points) Given the following information:

- Policy term: six months

- Proposed rates in effect: January 1, 2012, to June 30, 2013

- Selected projected premium trend: 5%

Calendar Year	Average Earned Premium at Current Rate Level	Average Written Premium at Current Rate Level
2009	\$375	\$380
2010	\$390	\$395

a. (2 points) Calculate the total premium trend factor for each of calendar years 2009 and 2010 using two-step trending.
b. (0.25 point) Briefly discuss when it is appropriate to use two-step trending.

5.112. (5, 5/12, Q.4) (2 points) Explain whether the following statements are correct or incorrect.
a. (0.5 point)
Calendar year 2011 written premium will be fixed (i.e. not change) at December 31, 2011.
b. (0.5 point)
Calendar year 2011 earned premium will be fully earned (i.e. not change) at December 31, 2011.
c. (0.5 point)
Policy year 2011 written premium will be fixed (i.e., not change) at December 31, 2011.
d. (0.5 point)
Policy year 2011 earned premium will be fully earned (i.e. not change) at December 31, 2011.

5.113. (5, 5/12, Q.5) (1 point)
a. (0.5 point) Discuss whether or not it is appropriate to perform a classification ratemaking analysis using premiums adjusted with aggregate on-level factors.
b. (0.5 point) State one advantage and one disadvantage of the parallelogram method relative to the extension of exposures method.

5.114. (5, 5/12, Q.6) (2 points) Given the following information for a Homeowners company:

- The 4th Calendar Quarter of 2011 (4Q11) Average Written Premium is $560.
- The proposed effective date of the next rate change is July 1, 2012.
- Assume a +5% prospective annual premium trend.
- Rate review is performed every 2 years.

Calendar Year Ending	Earned Exposures (House-Years)	Earned Premium at Current Rates
December 31, 2009	10,000	$5,000,000
December 31, 2010	10.000	$5,250,000
December 31, 2011	10,000	$5,512,500

a. (1 point) Use the two-step trending method to calculate the projected earned premium for the calendar year ending December 31, 2009.
b. (1 point) After completing the analysis, the actuary determines that the assumed annual increase in the amount of insurance to account for inflation was materially reduced post-January 1, 2012. Discuss any necessary adjustments to the completed analysis in part a. above.

5.115. (5, 5/13, Q.2) (2 points) Given the following information for an insurance company:

- Proposed effective date of the next rate change is January 1, 2014.
- Rates will be in effect for 1 year.
- All policies have 12-month terms and are written uniformly throughout the year.
- Calendar year 2012 earned premium at current rate level is $114,208,050.

12 Month Period Ending	Written Premium at Current Rate Level	Written Exposures
December 31, 2011	$104,500,000	110,000
June 30, 2012	$113,800,500	121,000
December 31, 2012	$123,916,100	133,100

a. (1 point) Utilizing one-step trending, calculate the calendar year 2012 projected earned premium at current rate level for use in calculating the rate change.
b. (0.25 point) Briefly discuss why a premium trend should be utilized in a rate level indication.
c. (0.25 point) Briefly discuss why it is inappropriate to use written premium at historical rate levels to determine premium trends.
d. (0.5 point) The insurance company decides to move all existing business with a $100 deductible to a $500 deductible upon renewal during calendar year 2013.
Given this new information, discuss whether the true projected earned premium will be higher, lower, or unchanged from that in part a. above.

5.116. (5, 11/13, Q.2) (2 points) Given the following information:

- All policies have six-month terms.
- Policies are written uniformly during each six-month period and cannot be cancelled.
- The rating algorithm is base rate x class factor + expense fee.
- The proposed effective date of the next rate change is July 1, 2013.
- A rate review is performed every six months.

Effective Date of Rates	Base Rate Per Exposure	Class Factor A	Class Factor B	Expense Fee
January 1, 2011	$480	1.00	0.70	$45
July 1, 2011	$488	1.00	0.70	$45
January 1, 2012	$504	1.00	0.70	$50
July 1, 2012	$500	1.00	0.75	$50
January 1, 2013	$500	1.00	0.80	$55

	Written Exposures (000)	
Policy Effective Dates	Class A	Class B
January 1, 2011 - June 30, 2011	125	50
July 1, 2011 - December 31, 2011	150	100
January 1, 2012 - June 30, 2012	175	150
July 1, 2012 - December 31, 2012	200	200

Using the extension of exposures method, calculate the calendar year 2012 earned premium at current rate level.

5.117. (SOA GIIRR, 11/13, Q.1) (3 points) All automobile insurance policies for A-Z Insurance Company sold in 2012 have premiums paid annually of 1,680. All policies cover one car. Effective January 1, 2013, there was a 20% rate increase for all new and renewal policies.
You are given the following information for three policies:

Policy Number	Effective Date	Policy Status as of 12/31/2013
14902	09/01/2012	Cancelled 02/28/2013
14903	12/16/2012	Expired 12/15/2013
14904	06/16/2013	In-force as of 12/31/2013

Assume all months count as 1/12th of a year, and approximate mid-month dates as one half of a month.

(a) (2.25 points) Calculate the total earned exposure units (caryears) and total earned premium in each of the four calendar quarters in 2013.

(b) (0.75 point) Calculate the total unearned premium at December 31, 2012 and December 31, 2013.

5.118. (5, 5/14, Q.1) (3.5 points) An insurance company writes annual policies.
The history of rate changes is as follows:

Effective Date	Overall Rate Change
January 1, 2010	+4.2%
March 1, 2011	+0.3%
January 1, 2012	-1.7%
June 1, 2013	+1.0%

a. (1 point) Calculate the on-level factor to current rate level for calendar year 2011 earned premium, assuming all policies are written uniformly throughout the year.

b. (2 points) Assume that 25% of policies are written on the first day of the year and the remaining policies are written evenly throughout the year. Calculate the on-level premium factor to current rate level for policies in-force on February 1, 2012.

c. (0.5 point) Assuming all policies are written uniformly throughout the year, and without performing additional calculations, discuss the effect on the on-level premium factor for calendar year 2011 if the policy term was 2 years instead of annual.

5.119. (SOA GIIRR, 5/14, Q.1) (3 points)
No Name Insurance Company has written the following general liability policies:

Effective Date	Term	Premium
1/1/2012	Annual	5,000
4/1/2012	Annual	1,000
7/1/2012	6-month	500
10/1/2012	2-year	5,000
1/1/2013	Annual	2,000
7/1/2013	Annual	1,500

(a) (2.25 points) Calculate earned and written premium for calendar years 2012 and 2013. No Name does not treat multi-year policies as multiple annual policies.

(b) (0.75 points) Explain how the calculation of written and earned premium might be different if No Name Insurance Company wrote motorcycle policies in a winter climate instead of general liability policies.

5.120. (SOA GIIRR, 5/14, Q.13) (3.25 points)
You are estimating on-level factors for a book of business that only has annual policies with the following historical rate changes:

Effective Date of Rate Change	Rate Change Percent
July 1, 2011	+7%
April 1, 2013	-3%

In addition, a new discount of 10% was implemented on May 1, 2012 that applied to 20% of all policies and did not affect the level of insurance coverage.
(a) (0.5 point) State the key assumption that underlies the parallelogram method.
(b) (2.25 points) Calculate the on-level factor to be used to adjust calendar year 2011 earned premium to current rates.
(c) (0.5 point) Explain how you would recognize a state-mandated change in minimum policy limits in the on-level calculation.

5.121. (5, 11/14, Q.2) (1.5 points) Given the following policy data:

Policy	Effective Date	Expiration Date	Initial Policy Premium
1	June 1, 2012	May 31, 2013	480
2	July 1, 2012	December 31, 2012	125
3	March 1, 2013	February 28, 2014	225
4	August 1, 2013	March 31, 2014	300

- Six months after the policy expires, the initial policy premium on every policy increases by 8% due to the final audit.

a. (0.5 point) Calculate calendar year 2013 earned premium as of December 31, 2013.
b. (0.5 point) Calculate calendar year 2013 written premium as of December 31, 2013.
c. (0.25 point) Calculate policy year 2013 earned premium as of December 31, 2013.
d. (0.25 point) Calculate policy year 2013 written premium as of December 31, 2014.

5.122. (5, 11/14, Q.3) (2 points) A personal auto insurer has a highly-refined classification rating plan. In the calculation of a rate level indication for this insurer, fully assess the use of the following methods to adjust premium to current rate level:
i. Parallelogram method
ii. Extension of Exposures method

5.123. (SOA GIIRR, 11/14, Q.1) (3.75 points)
You are given the following information on two 12-month policies you are analyzing:

Policy #1 was originally written on April 1, 2010 for a premium of 900, was renewed each year, and was still in force on December 31, 2013.

Policy #2 was originally written on September 1, 2011 for a premium of 1,200, was renewed on September 1, 2012, and was cancelled on March 1, 2013.

• There was a premium level increase of 5% that was effective July 1, 2012 for each policy upon its renewal.
• All rating characteristics remained the same for each policy at each renewal.

(a) (1.5 points) Calculate the 2012 and 2013 calendar year total earned premiums.
(b) (0.75 points) Calculate the 2012 calendar year total written premiums.
(c) (0.75 points) Calculate the total unearned premiums as of December 31, 2012.
(d) (0.75 points) You are adjusting premiums to current rate levels for a ratemaking analysis. Calculate the 2012 total earned premiums adjusted to current rate level.

5.124. (5, 5/15, Q.4) (2.25 points) Given the following information:

- Policies are written on an annual basis.
- Proposed rates will be in effect from January 1, 2016 to January 1, 2017.
- Calendar year 2014 earned premium = $100,000.
- Beginning with July 1, 2012 renewals, the minimum deductible was increased from $500 to $1,000.
- The premium impact of any law change is applicable to all policies, including those in-force.
- The rate change history is as follows:

Effective Date	Overall Change	Type of Change
April 1, 2014	+5%	Law
July 1, 2014	+3%	Rate
July 1, 2015	-7%	Rate

- The annual premium exponential trend fit based on data for the 12 months ending each quarter evaluated through December 31, 2014 is as follows:

Calendar Year Ending	Average Earned Premium at Current Rate and Law Level
March 2012	$510.00
June 2012	$512.50
September 2012	$499.50
December 2012	$489.00
March 2013	$481.00
June 2013	$473.00
September 2013	$477.50
December 2013	$481.50
March 2014	$487.00
June 2014	$492.50
September 2014	$496.00
December 2014	$502.00

Number of Points	Annual Exponential Trend Fit
12 point	-0.5%
8 point	2.2%
6 point	3.4%
4 point	3.1%

Calculate the trended calendar year 2014 earned premium at current rate level.
Include justification of the premium trend selection.

5.125. (5, 5/15, Q.5) (2.5 points) Given the following information for a boat owners insurer:
- On July 1, 2014, a rate change of +10% went into effect.
- 2014 earned premium = $1,000.
- Policies are written on an annual basis.

a. (0.75 point) Assuming uniform writings, calculate the calendar year 2014 on-level earned premium using the parallelogram method.

b. (1.25 points) Due to the seasonality of boat owners coverage, assume that the policy distribution of the book of business was as follows (with uniform distribution within each quarter):

Quarter 1	10%
Quarter 2	50%
Quarter 3	30%
Quarter 4	10%

Calculate the calendar year 2014 on-level earned premium, accounting for this assumed policy distribution.

c. (0.5 point) For the scenario in part b. above, describe another approach the insurer could take to calculate the on-level earned premium.

5.126. (5, 11/15, Q.1) (2.75 points) Given the following information:

Effective Date	Number of Autos Written on Effective Date
February 1, 2013	1,100
August 1, 2013	800
February 1, 2014	600
August 1, 2014	300

- All policies have six-month terms.
- The exposure base is earned car years.
- The premium per auto is $500 per six-month term for policies effective through August 31, 2014.
- A uniform rate change of -18% became effective September 1, 2014.

a. (0.75 point) Calculate the written and earned exposures for calendar year 2014.

b. (2 points) Calculate the earned premium at current rate level for calendar year 2014 using both the parallelogram method and extension of exposures method, and discuss which method is more appropriate for this situation.

5.127. (5, 11/15, Q.4) (2.25 points)
An actuary is calculating a rate change to be effective July 1, 2016.
Given the following:
- Policies are written on a semi-annual basis.
- Rates are expected to be in effect for one year.
- The exposure base is non-inflationary.
- The annual frequency and severity exponential trend fits based on data for the 12 months ending each quarter evaluated through December 31, 2014 are as follows:

Number of Points	Frequency Exponential Fit	Severity Exponential Fit
20 point	-2.9%	3.4%
16 point	-3.2%	3.0%
12 point	-2.5%	2.8%
8 point	-0.5%	2.9%
6 point	3.0%	3.1%
4 point	2.8%	3.3%

Calculate a pure premium trend factor for accident year 2012, justifying the selected trends and methodology.

5.128. (5, 5/16, Q.2) (2.5 points)
An insurance company writes both 6-month and 12-month automobile policies.
Given the following information:

Policy	Original Effective Date	Original Expiration Date	Transaction Effective Date	Territory	Full-Term Written Premium	Notes
A	January 1, 2015	December 31, 2015	January 1, 2015	1	$1,000	Start of New Policy
A	January 1, 2015	December 31, 2015	July 1, 2015	1	N/A	Policy Canceled
B	July 1, 2015	June 30, 2016	July 1, 2015	1	$500	Start of New Policy
B	July 1, 2015	June 30, 2016	September 30, 2015	2	$400	Relocated to Territory 2
C	October 1, 2015	March 31, 2016	October 1, 2015	2	$1,000	Start of New Policy

- Full-term written premium represents the policy premium if policy characteristics shown were in place from original effective date to original expiration date.

a. (0.75 point) Calculate the 2015 calendar year written premium as of December 31, 2015.
b. (0.75 point) Calculate the 2015 calendar year earned premium as of December 31, 2015.
c. (0.5 point) Calculate the in-force premium as of October 1, 2015.
d. (0.5 point) Calculate the 2015 calendar year earned exposures separately for Territory 1 and Territory 2 as of December 31, 2015.

5.129. (5, 11/16, Q.2) (1.5 points) Given the following:

Effective Date	Rate Change
September 1, 2012	-10%
September 1, 2013	-5%
September 1, 2014	-3%

- A law change mandated a rate decrease of 15% effective February 1, 2015 applicable to all in-force policies.
- All policies are annual.

a. (1 point) Calculate the on-level factor to current rate level for calendar year 2014 earned premium.
b. (0.5 point) Identify a weakness with the parallelogram method and briefly describe a solution.

5.130. (5, 11/17, Q.1) (1.5 points) Given the following information:

Calendar Year	Average Earned Premium at Current Rate Level	Average Written Premium at Current Rate Level
2014	$210	$212
2015	$220	$224
2016	$235	$240

- The projected annual premium trend = -2%.
- Fourth quarter 2016 average earned premium at current rate level = $236.
- Fourth quarter 2016 average written premium at current rate level = $242.
- The company uses a calendar-accident year aggregation of data for indications.
- All policies are annual.
- Rates are in effect for one year.
- The rate revision is planned to be effective January 1, 2018.

a. (1 point) Calculate the premium trend factor for each year using two-step trending.
b. (0.5 point) Identify two scenarios that could lead to a negative premium trend when analyzing average premium at current rate level.

5.131. (5, 11/17, Q.3) (1.5 points)
Given the following for an insurance company that writes only annual policies:

Policy	Effective Date	Annual Premium
A	July 1, 2014	$200
B	October 1, 2014	$240
C	January 1, 2015	$260
D	July 1, 2015	$280

- Policy D was cancelled March 31, 2016.

a. (0.5 point) Calculate the following for calendar year 2015:
   i. Earned premium
   ii. Written premium

b. (0.5 point) Calculate the following as of December 31, 2016:
   i. Policy year 2015 earned premium
   ii. Policy year 2015 written premium

c. (0.5 point)
Briefly describe one advantage and one disadvantage of calendar year data aggregation.

5.132. (5, 11/17, Q.16) (1.25 points)

An insurer has the following book of insurance policies as of December 31, 2016:

Policy Number	Policy Effective Date	Policy Term (in Months)	Gross Written Premium
1	January 1, 2015	12	12,000
2	June 1, 2015	6	7,000
3	August 1, 2015	12	10,000
4	February 1, 2016	12	15,000
5	May 1, 2016	6	8,000

Claim Number	Accident Date	Claim Report Date	Gross Paid Claims	Gross Case Reserves	Reinsurance Recoveries
1	September 1, 2015	October 1, 2015	2,000	0	0
2	November 1, 2015	January 1, 2016	3,000	1,000	500
3	March 1, 2016	March 1, 2016	1,000	2,000	0
4	July 1, 2016	September 1, 2016	4,000	0	1,000

a. (0.5 point) Calculate the calendar year 2016 earned premium.

b. (0.25 point) Calculate the unearned premium as of December 31, 2016.

c. (0.5 point) Calculate the reported claims net of reinsurance recoveries for accident year 2016 as of December 31, 2016.

Note: Part (c) of this past exam question is on the reserving part of the syllabus of this exam.

5.133. (SOA GIIRR, 5/18, Q.1) (4 points)
(a) (0.5 points) State one advantage and one disadvantage of using the extension of exposures method to adjust historical premiums for prior rate changes.
(b) (0.5 points) State the key assumption for the parallelogram method to be appropriate.
(c) (2 points) You are given the following information for third party auto liability premium rate changes by year. All policies are twelve-month policies.

Calendar Year	Rate Change % in Year	Effective Date of Rate Change
2015	7%	July 1, 2015
2016	11%	July 1, 2016
2017	13%	July 1, 2017

Calculate the on-level factor to use for ratemaking for calendar year 2016 earned premium using the parallelogram method.
(d) (0.5 points) The parallelogram method has shortcomings that affect its use for risk classification analysis. Describe one shortcoming.
(e) (0.5 points) Describe how you would address this shortcoming.
<u>Note</u>: I slightly reworded this past exam question.

5.134. (5, 5/18 makeup, Q.1) (1.25 points) Given the following information on policy transactions:

Policy	Transaction Date	Action	Full-Term Written Premium
A	January 1, 2015	Policy Inception	100
B	February 1, 2015	Policy Inception	250
B	August 1, 2015	Endorsement	60
B	June 5, 2017	Audit Premium	50
C	May 1, 2015	Policy Inception	150
C	October 1, 2015	Cancellation	-150
D	September 1, 2014	Policy Inception	175
D	September 1, 2015	Policy Renewal	200
D	March 1, 2016	Endorsement	80
E	April 1, 2016	Policy Inception	75

- Full-term written premium represents the policy premium if policy characteristics shown were in place from original effective date date until original expiration date.
- All policies are annual.

a. (0.5 points) Calculate the in-force premium as of September 15, 2015.
b. (0.75 points) Calculate the 2015 policy year written premium as of December 31, 2017.

5.135. (SOA GIIRR, 11/18, Q.1) (4 points)
You are given the following information on the only three policies that ABC Insurance has written:

Policy	Policy Term	Written Premium	Initial Effective Date
1	12 months	1,800	April 1, 2016
2	24 months	3,000	June 1, 2016
3	6 months	1,200	Sept 1, 2016

- ABC Insurance records written premium in the year of the effective date.
- All premiums were increased by 10% for policies written or renewed on or after March 15, 2017.
- No other rate changes have occurred since March 15, 2017.
- All policies have renewed at the end of every policy term.

(a) (1 point) Calculate the unearned premium as of December 31, 2016.
(b) (1.5 points) Calculate the calendar year 2017 earned premium.
(c) (1 point) Calculate the unearned premium as of December 31, 2017.
(d) (0.5 points) Calculate the premium on-level factor to adjust the 2017 calendar year earned premium to the current rate level.

5.136. (SOA GIIRR, 11/18, Q.14) (4 points) You are the consulting actuary for ABC Insurer and are given the following summary of historical rate changes:

Calendar Year	Rate Change % In Year	Effective Date of Rate Change
2013	6%	October 1, 2013
2014	3%	July 1, 2014
2015	-5%	October 1, 2015
2016	0%	October 1, 2016
2017	4%	October 1, 2017

- All policies are written for 6-month terms.
- Premiums are written evenly throughout the year.
- Premiums are earned evenly throughout the policy term.
- Each rate change applies to all policies written on or after the effective date of the rate change.

(a) (2 points) Calculate the premium on-level factor for calendar year 2014 used to project expected claim ratios for reserving purposes as of December 31, 2017.
(b) (0.5 points) Calculate the premium on-level factor for calendar year 2014 used to project expected claim ratios for ratemaking analysis.
(c) (1.5 points) ABC Insurer has now informed you that on April 1, 2014, a 10% discount was introduced that was applicable to 20% of all new and in-force policyholders.
Calculate the weighted average rate level for calendar year 2014 taking into account this new information.

5.137. (5, 11/18, Q.3) (1.75 points) Given the following:

Effective Date	Overall Average Rate Change	Rate Per Exposure ($)	Class Factor X	Class Factor Y	Class Factor Z	Expense Fee ($)
January 1, 2016	0.0%	1,000	1.20	0.85	1.00	120
July 1, 2017	10.0%	1,112	1.20	0.85	1.00	120
October 1, 2017	0.0%	1,175	1.10	0.75	1.00	120
April 1 , 2018	1.0%	1,175	1.10	0.75	1.00	132

- All policies are semi-annual.
- Exposures are written uniformly throughout the year.
- Expense fee is a per exposure fee that is added in the last step of the rate calculation.

a. (0.75 point) Calculate the on-level factor for calendar year 2017 earned premium using the parallelogram method.

b. (0.5 point) Calculate the on-level factor for a policy effective on April 1, 2017 within Class Y using the extension of exposure method.

c. (0.5 point) Assess the appropriateness of using the parallelogram method to calculate indicated class factors using the loss ratio method.

5.138. (SOA GIRR, 5/19, Q.1) (5 points)
XYZ Insurance acquired a book of business with 100 annual policies on November 30, 2016.

- Each policy had an annual premium of 1,500, and a renewal date of June 1.
- For both the 2017 and 2018 renewals there was a premium increase of 10% and a renewal rate of 80%.

In addition, XYZ Insurance started writing new business in 2017 with the following transactions during calendar years 2017 and 2018:

Transaction ID	Transaction Type	Policy Number	Effective Date	Policy Term	Transaction Amount
1	New business	101	May 1, 2017	12 months	1,600
2	New business	102	July 1, 2017	18 months	2,100
3	New business	201	March 1, 2018	6 months	800
4	Renewal	101	May 1, 2018	12 months	1,800
5	New business	202	June 1, 2018	24 months	2,500
6	Cancellation	102	June 30, 2018	n/a	-700

XYZ Insurance earns premium evenly throughout the year.
(a) (1.5 points) Calculate the total earned premium for calendar year 2017.
(b) (1.5 points) Calculate the total unearned premium as of June 30, 2018.
(c) (1.5 points) XYZ implemented a rate change where the premium for all policies was increased by 5% for each policy written or renewed on or after October 1, 2018.
XYZ is conducting a ratemaking analysis with new rates to be effective April 1, 2019.
Calculate the calendar year 2017 earned premium at current rate levels using the extension of exposures method.
(d) (0.5 points) State why the parallelogram approach is not appropriate to use in part (c).

5.139. (SOA GIRR, 5/19, Q.14) (4 points) You are conducting a premium trend analysis of a liability line of business for ratemaking purposes.
(a) (0.5 points) Explain an advantage of using written premiums over earned premiums for premium trend analysis.
(b) (0.5 points) Describe an alternative to using annual written premiums that will improve the responsiveness of the premium trend statistic.
(c) (1 point) The insurer sells three policy limits: 0.75, 1.00, and 1.50 million.
The following historical information is provided:

	Percent Earned Exposures by Policy Limits		
Experience Period	750,000	1,000,000	1,500,000
2015	30%	40%	30%
2016	27%	40%	33%
2017	23%	40%	37%
Current Increased Limit Factors	0.90	1.00	1.15

Interest in the higher limit is expected to continue to grow due to increasing litigation.
Calculate the annual trend due to the shift in policy limits for each year.
(d) (0.5 points) Recommend the annual trend due to the shift in policy limits to use for ratemaking.
(e) (1.5 points) You are given the following additional information:

- Rates will be effective July 1, 2019 for one year
- 50% of written premium is from six-month policies and 50% is from twelve-month policies

Calculate the trend factor to be used for 2016 earned premium using the annual trend selected in part (d).
Note: I have slightly rewritten part (e) of this past exam question.

5.140. (5, 5/19, Q.2) (2.25 points) Given the following:

Calendar Year	Earned Premium ($)
2017	3,850,000
2018	4,200,000

Rate Change Effective Date	Overall Rate Change
January 1, 2017	10%
July 1, 2017	5%

Quarter and Year	Average Written Premium at Current Rate Level
2Q 2016	$1,771
4Q 2016	$1,806
2Q 2017	$1,840
4Q 2017	$1,877
2Q 2018	$1,914
4Q 2018	$1,953

- No rate changes occurred in 2016 or 2018.
- Rates will be in effect for one year.
- All policies are semi-annual.
- All policies are written uniformly throughout the year.

a. (1.75 points) Calculate the trended on-level earned premium for 2017 to be used in a rate change effective July 1, 2019.

b. (0.5 point) Briefly describe two reasons why the 2017 trended on-level earned premium would be higher if all policies were annual rather than semi-annual.

5.141. (5, 11/19, Q.2) (1.75 points) Given the following policies for an insurance company:

Policy	Effective Date	Expiration Date	Written Premium
A	March 1, 2017	February 28, 2018	1,200
B	June 1, 2017	November 30, 2017	1,500
C	July 1, 2017	June 30, 2018	2,000
D	October 1, 2017	September 30, 2018	750
E	January 1, 2018	December 31, 2018	900
F	April 1, 2018	September 30, 2018	1,650
G	August 1, 2018	July 31, 2019	1,350

a. (0.25 point) Calculate the written premium for the fiscal year ending July 31, 2018.

b. (0.25 point) Calculate the in-force premium as of December 15, 2018.

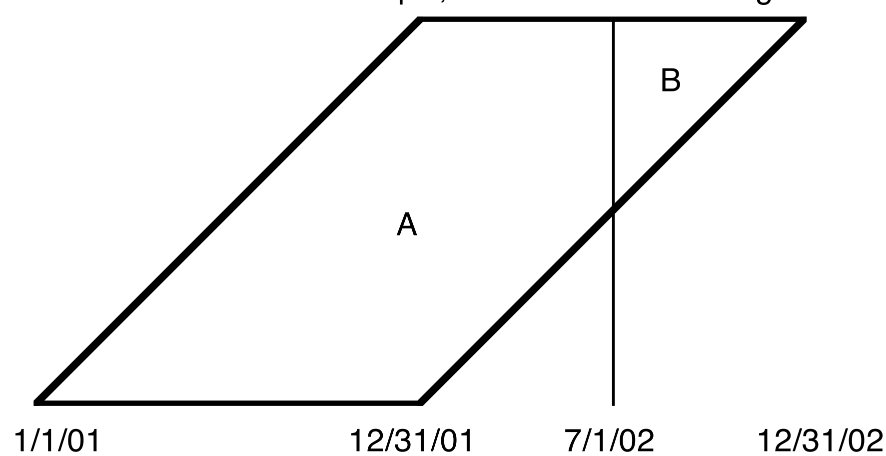
c. (0.5 point)
Calculate the 2018 calendar year written premium if Policy C is cancelled on March 31, 2018.

d. (0.5 point) Discuss if it is appropriate for this insurer to estimate earned premium for the current year by averaging the in-force premium at the end of the current year and prior year.

e. (0.25 point) Identify one potential use of in-force premium other than estimating earned premium.

Solutions to Problems:

5.1. a. At 18 months after the inception of the policy year, 1/4 of the audits have occurred.
(Policies written from 1/1 to 3/31 have been audited.)
At 24 months after the inception of the policy year, 3/4 of the audits have occurred.
(Policies written from 1/1 to 9/30 have been audited.)
Therefore, the development factor for written premiums is:
{(3/4)(1.08) + 1/4} / {(1/4)(1.08) + 3/4} = 1.06/1.02 = **1.0392**.
b. At 18 months after the inception of the policy year, only 7/8 of the premium is earned, and 1/4 of the audits have occurred. For example, assume we are dealing with Policy Year 2001.



Then Area B = (1/2)(1/2)(1/2) = 1/8. Area A = 1 - 1/8 = 7/8.
At 18 months we have: 1/4 of the policies audited, and 7/8 of the premium earned, of which 1/4 has been audited and the remaining 5/8 has not been audited.
1/8 of the premium has yet to be earned (and also not been audited).
For example, let us assume 800,000 in premium written prior to audit.
Then at 18 months prior to audits, 700,000 has been earned while 100,000 has not been earned.
200,000 written premium has been audited, (all of this has been earned) increasing the premium by: (8%)(200,000) = 16,000.
Thus at 18 months we actually have earned premium of: 700,000 + 16,000 = 716,000.
At 24 months after the inception of the policy year, all of the premium is earned, and 3/4 of the audits have occurred.
At 24 months prior to audits, 800,000 has been earned.
600,000 written premium has been audited, (all of this has been earned) increasing the premium by: (8%)(600,000) = 48,000.
Thus at 24 months we actually have earned premium of: 800,000 + 48,000 = 848,000.
848,000 / 716,000 = **1.1844**.
Alternately, the development factor for earned premiums is:

$$\frac{(3/4)(1.08) + 1/4}{(1/4)(1.08) + 5/8 + (1/8)(0)} = 1.06/0.895 = \mathbf{1.1844}.$$

5.2. 1. If the future book of business is significantly different from the past book of business, due to either cancellations, nonrenewals, or writing new business, this could affect the expected loss ratio.
2. The economic, legal, and social environments of insurance are constantly changing, affecting loss severity and loss frequency.
3. Changes in an insurer's rate levels will affect average premiums.
<u>Comment</u>: List only two items.

5.3. (a) The average earned date for CY 2001 earned premium is 7/1/01. For 12 month policies, the average date of writing is 6 months earlier, or 1/1/01. For the proposed effective period the average date of writing is 8/1/03 plus 6 months or 2/1/04.
Therefore, for the one-step method, we trend from 1/1/01 to 2/1/04, a length of 3 years and 1 month.
In the written premium data, the last point is the 4th quarter of 2002, with an average date of writing of 11/15/02.
Therefore, for the two-step method, we trend from 1/1/01 to 11/15/02, a length of 1 year and 10.5 months, and then from 11/15/02 to 2/1/04, a length of 1 year and 2.5 months.
(b)

Quarter	Written Exposures	Written Premium	W.P. @CRL	Average W.P. @CRL
1st of 2000	100	97,000	116,400	**1164**
2nd of 2000	103	98,000	117,600	**1142**
3rd of 2000	105	102,000	122,400	**1166**
4th of 2000	109	105,000	126,000	**1156**
1st of 2001	111	106,000	127,200	**1146**
2nd of 2001	115	110,000	132,000	**1148**
3rd of 2001	119	137,000	137,000	**1151**
4th of 2001	120	136,000	136,000	**1133**
1st of 2002	119	139,000	139,000	**1168**
2nd of 2002	117	141,000	141,000	**1205**
3rd of 2002	120	145,000	145,000	**1208**
4th of 2002	124	148,000	148,000	**1194**

(c) Let t be time (in quarters) and Y = ln(average written premium at current rate level).

$\bar{t}$ = (1 + 2 + 3 + ... + 12)/12 = 78/12 = 6.5.

$\overline{Y}$ = {ln(1164) + ln(1142) + ln(1166) +... + ln(1194)} / 12 = 84.7241/12 = 7.0603.

$\Sigma t^2/N$ = 650/12 = 54.1667. $\Sigma tY/N$ = 551.2246/12 = 45.9354.

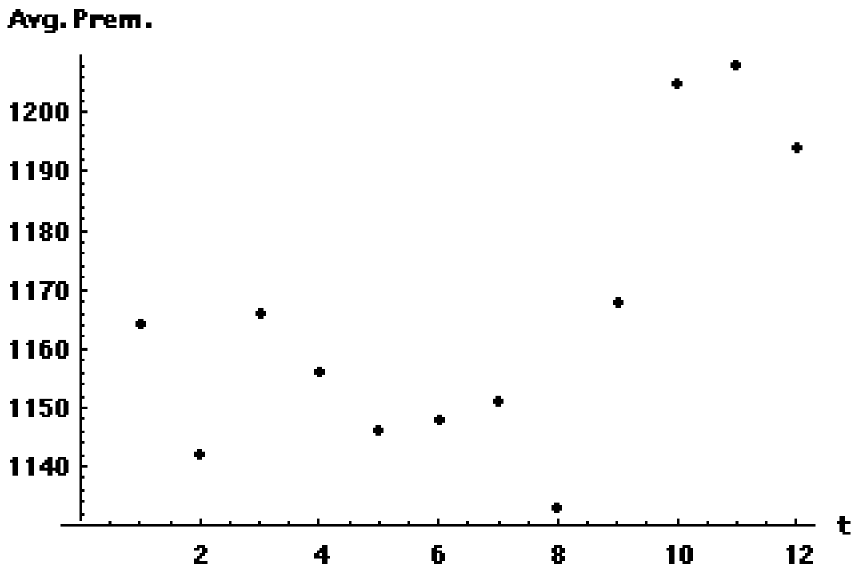
variance of t = $\Sigma t^2/N - \bar{t}^2$ = 54.1667 - 6.5^2 = 11.917.

sample covariance of t and Y = $\Sigma tY/N - \bar{t}\,\overline{Y}$ = 45.9354 - (6.5)(7.0603) = 0.04345.

slope of the regression line = 0.04345/11.917 = 0.00365.
Therefore, the exponential regression is:
average written premium at current rate level = (constant) $e^{0.00365t}$ = (constant)(1.00366^t).

Over four quarters, the trend factor is: 0.00366^4 = 1.015. ⇔ **1.5%** annual premium trend.

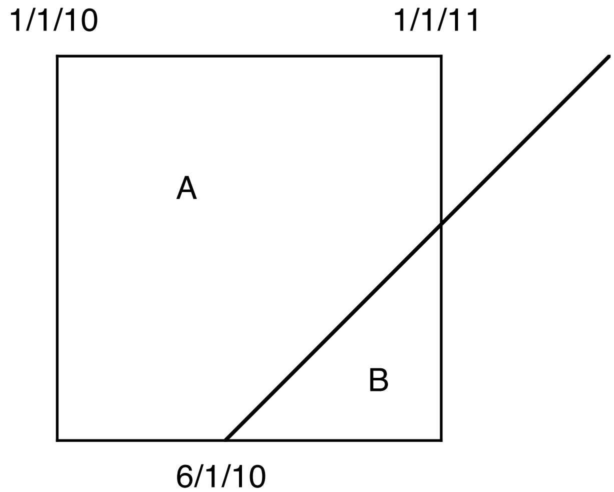
(d) During the first part of the data period the trend in average written premium is downwards, and then it is upwards:



Rather than trying to compromise on the selection of a single long-term trend, I would use the two-piece trend method.

<u>Comment</u>: We are presumably given several years of Calendar/Accident Year data including 2001, earned premiums and incurred losses. The premium trend factors are calculated from a series of written premium data, but are then applied to the Calendar Year earned premium. Once we have an estimate of the change in average premiums over time, we can apply it over the appropriate period of time to bring some year of data to the level expected during a proposed effective period. Parts (c) involves calculations that you should not be asked to do on your exam!

5.4. For the 60% of policies written uniformly throughout the year we use the parallelogram method to determine the average rate level.



Area B = (1/2)(7/12)(7/12) = 49/288. Area A = 1 - 49/288 = 239/288.
Average Rate Level for 60% of policies: (239/288)(1) + (49/288)(1.2) = 1.0340.
The January 1 policies are all written at the lower rate level of 1.
The July 1, 2009 policies are all written at the lower rate level of 1.0.
Half of their premium is earned in CY2010
The July 1, 2010 policies are all written at the higher rate level of 1.2.
Half of their premium is earned in CY2010
The average rate level for 2010 Calendar Year Earned Premiums is:
(30%)(1) + (5%)(1) + (5%)(1.2) + (60%)(1.0340) = 1.0304.
Therefore, the on-level factor is: 1.2/1.0304 = **1.165**.
Comment: Similar to 5, 5/06, Q.28.

5.5. (a) The number of manweeks seems to be decreasing at about 8% per year.
Premiums at current rates seem to be declining about 9% per year.
Payrolls seem to be declining at about 5% per year.
The average premium per $100 of payroll, or average rate,
seems to be declining at about 4% per year:

Policy Year	Payroll ($00) Developed to Ultimate	Premiums @CRL Developed to Ultimate	Average Rate
2000	1,500,000	3,750,000	2.500
2001	1,425,000	3,420,000	2.400
2002	1,350,000	3,110,000	2.304

The average premium per manweek, seems to be declining at about 1% per year:

Policy Year	number of manweeks worked	Premiums @CRL Developed to Ultimate	Premium per manweek
2000	250,000	3,750,000	15.000
2001	230,000	3,420,000	14.870
2002	212,000	3,110,000	14.670

One could estimate a premium trend from either premiums per manweek or per payroll.
(b) **The series of average premiums from which we derive the premium trend needs to be based on an exposure definition that matches the one that was used in the frequency trend analysis.**
If we calculate frequencies as number of claims per $100 of payroll, in order to estimate a frequency trend, then one needs to use a premium trend based on premiums per $100 of payroll.
If we instead calculate frequencies as number of claims per manweek, in order to estimate a frequency trend, then one needs to use a premium trend based on premiums per manweek.
(c) The average rates are declining. One explanation could be a shift in mix of business written, either long term or due to cyclic economic conditions, towards lower rated classes. (The rate for services industries is generally lower than that for Construction or Manufacturing classes.) In this case, one would expect an approximately equal corresponding reduction in loss pure premiums. This should show up in a series of frequencies per payroll. Thus using premium and frequency trends both per payroll there might be approximately no expected trend in loss ratios, due to this effect. Similarly, using premiums and frequency both per manweek, the shift in mix of business should affect both approximately equally, thus with approximately no net impact in the trend in loss ratios.
One could instead use Unit Statistical Plan Data in order to quantify the average class rate (at current rates) written by policy year, and adjust the losses (or frequency) for this effect, as well as using a premium trend based on a long term movement in average weekly wages.
Another possible explanation is that this decline in average rate may be due to changes in the average experience modifications. For example, during 2000 the average modification may have been a debit, due to worse experience than was anticipated at the time that the experience plan expected loss factors were estimated. One would have to know more details, to determine exactly how to incorporate this effect into trend. One general technique is to recalculate the premiums both at current rates and as if the experience rating plan had been in balance.
Comment: While one could use either of two somewhat different definitions of frequency, severity is always dollars of loss divided by number of claims.
Part (c) is somewhat beyond what you should be asked on your exam.

5.6. a. We get the statewide average premium as a weighted average of the urban and rural, using exposures as the weights:

Year	Exposures Urban	Exposures Rural	Average Premiums Urban	Average Premiums Rural	Average Premiums Statewide
2006	5,000	5,000	$1000	$500	$750.00
2007	5,000	6,000	$1030	$515	$749.09
2008	5,000	7,000	$1060	$530	$750.83
2009	5,000	8,000	$1090	$545	$754.62

Based on the statewide average premiums, a reasonable annual premium trend would be about: $(754.62/750.00)^{1/3} - 1 = 0.6\%$. (Other similar selections would be reasonable.)

b. We note that Urban and Rural average premiums are each increasing by about 3% per year. Yet Simpson's statewide average premiums are increasing at a much slower rate. This is due to the shift in mix of business; Simpson is writing a larger percentage of lower premium rural business in the later years. Assuming that the changes in pure premiums for all Homeowners insurers in the state combined is a reasonable estimate of the change in pure premium for an average home in the state, it would not reflect the impact of Simpson's change in mix of business.
We would expect that due to its increasing percentage of lower cost rural homes, Simpson's pure premiums would be increasing at a lower rate than those for all insurer's in the state combined.
The change in pure premiums for all Homeowners insurers in the state combined should be used together with the approximate 3% annual premium trend occurring in each territory, rather than the approximate 0.6% annual premium trend based on Simpson's statewide average premiums.
Comment: The loss and premium trends should be on a similar basis. Something that affects both, such as a change of mix of business, should be in the data used to estimate both or absent in the data to estimate both.
If the loss trend is also based on the data for his insurer, then both the premium and loss trend will be distorted in more or less the same way. To a first approximation we are probably OK.
(The loss trend will be subject to a lot of random fluctuation, but that is another issue.)
If the premium and loss trends are based on the data for all insurer, then we should also be OK.
Using premium trend based on data for his insurer and a loss trend based on all insurers, the change of mix of business will effect the premium trend but not the loss trend. This is no good.
There are only two territories in the question solely for the sake of simplicity.

5.7. (a) For policies to be written from 11/1/03 to 10/31/04, the average date of writing is 5/1/04, and the average accident date is 6 months later, or 11/1/04.
The loss trend factors are computed as follows:

Accident Year	Average Accident Date	Loss Trend Period	Annual Loss Trend	Loss Trend Factor
1999	7/1/99	5.333 years	4.0%	1.233
2000	7/1/00	4.333 years	4.0%	1.185
2001	7/1/01	3.333 years	4.0%	1.140
2002	7/1/02	2.333 years	4.0%	1.096

For example, $1.04^{2.333} = 1.096$.

Calendar Year 1999 earned premiums, have an average date of earning of 7/1/99, and an average date of writing 6 months earlier, or 1/1/99. We trend to an average date of writing of 5/1/04. The one-step premium trend factors to be applied to earned premiums are computed as follows, with for example, $1.02^{3.333} = 1.068$:

Calendar Year	Average Written Date	Premium Trend Period	Annual Premium Trend	Premium Trend Factor
1999	1/1/99	5.333 years	2.0%	1.111
2000	1/1/00	4.333 years	2.0%	1.090
2001	1/1/01	3.333 years	2.0%	1.068
2002	1/1/02	2.333 years	2.0%	1.047

The projected loss ratios for 1999 is (48.2%)(1.233)/1.111 = 53.5%.

Calendar Accident Year	Loss Ratio	Loss Trend Factor	Premium Trend Factor	Projected Loss Ratio
1999	48.2%	1.233	1.111	**53.5%**
2000	68.4%	1.185	1.090	**74.4%**
2001	53.6%	1.140	1.068	**57.2%**
2002	62.6%	1.096	1.047	**65.5%**

(b) For example, for Calendar Year 2000, the average earned premium at current rate level is 1,652,892/5517 = 299.60. We compare the average earned premium at current rates for each calendar year to the latest point in the premium trend series. For example, for Calendar Year 2000, the premium trend factor for the first step is: 317/299.60 = 1.058.

Calendar Year	Earned House Years	Earned Premiums at Current Rate Level	Average Earned Premium at Current Rate Level	Latest Average Written Prem.	Step 1 Trend Factor
1999	5198	$1,523,450	$293.08	317	1.082
2000	5517	$1,652,892	$299.60	317	1.058
2001	5207	$1,603,006	$307.84	317	1.030
2002	5047	$1,581,065	$313.25	317	1.012

The last piece of trend data, the quarter ending 3/31/03, has an average date of writing of 2/15/03.
For policies to be written from 11/1/03 to 10/31/04, the average date of writing is 5/1/04.
Thus the projection period is from 2/15/03 to 5/1/04, or 1.208 years.

The projection factor for premiums is: $1.01^{1.208} = 1.012$.

The projected loss ratios for 1999 is (48.2%)(1.233) / {(1.082)(1.012)} = 54.3%.

Calendar Accident Year	Loss Ratio	Loss Trend Factor	Step 1 Premium Trend Factor	Step 2 Premium Trend Factor	Projected Loss Ratio
1999	48.2%	1.233	1.082	1.012	**54.3%**
2000	68.4%	1.185	1.058	1.012	**75.7%**
2001	53.6%	1.140	1.030	1.012	**58.6%**
2002	62.6%	1.096	1.012	1.012	**67.0%**

Comment: A Calendar/Accident Year always means calendar year premiums and accident year losses. If one has Calendar/Accident Year earned premiums and incurred losses, then the average date of earning is equal to the average date of accident. One earns premium as one provides coverage for accidents. In this case, since our premium trend series is written premium, we want to find average dates of writing. This shifts the premium trend dates 6 months earlier, where 6 months is half of the policy term. However, the total premium trend period is equal to that for losses.

5.8. (a) Effective Period: 12/1/04 to 12/1/05, average date of writing 6/1/05.
CY03 Earned premium has an average date of earning of 7/1/03 and an average date of writing 6 months earlier or 1/1/03.
Trend period: 1/1/03 to 6/1/05, 29 months long.
(b) The last trend data point is the 1st quarter 2004 written premium, with average date of writing 2/15/04.
Step 1 of trend period: 1/1/03 to 2/15/04, 13.5 months long.
Step 2 of trend period: 2/15/04 to 6/1/05, 15.5 months long.
(c) Effective Period: 12/1/04 to 12/1/05, average date of earning 12/1/05, 6 months later than the average date of writing.
CY03 Earned premium has an average date of earning of 7/1/03.
Trend period: 7/1/03 to 12/1/05, 29 months long.
(d) The last trend data point is the earned premium from the 1st quarter 2004, with average date of earning 2/15/04.
Step 1 of trend period: 7/1/03 to 2/15/04, 7.5 months long.
Step 2 of trend period: 2/15/04 to 12/1/05, 21.5 months long.
(e) CY03 Earned premium has an average date of earning of 7/1/03 and an average date of writing 3 months earlier or 4/1/03. Trend period: 4/1/03 to 6/1/05, 26 months long.
(f) Step 1 of trend period: 4/1/03 to 2/15/04, 10.5 months long.
Step 2 of trend period: 2/15/04 to 6/1/05, 15.5 months long.
(g) Effective Period: 12/1/04 to 12/1/05, average date of earning 9/1/05, 3 months later than the average date of writing.
Trend period: 7/1/03 to 9/1/05, 26 months long.
(h) Step 1 of trend period: 7/1/03 to 2/15/04, 7.5 months long.
Step 2 of trend period: 2/15/04 to 9/1/05, 18.5 months long.
Comment: Note that the total trend period is 29 months for 12-month policies, and 26 months for 6-month policies. See 5, 5/04, Q.35.
Experience period is Calendar/Accident Year 2003 earned premiums and incurred losses.
Planned effective date is December 1, 2004. Rates will be in effect for 1 year.
Summary of the above results:

	Data	Trend Method	Policy Term	Trend Periods
(a)	Written	1-step	12	1/1/03 to 6/1/05
(b)	Written	2-step	12	1/1/03 to 2/15/04 to 6/1/05
(c)	Earned	1-step	12	7/1/03 to 12/1/05
(d)	Earned	2-step	12	7/1/03 to 2/15/04 to 12/1/05

Above each have a total trend period of 29 months.

(e)	Written	1-step	6	4/1/03 to 6/1/05
(f)	Written	2-step	6	4/1/03 to 2/15/04 to 6/1/05
(g)	Earned	1-step	6	7/1/03 to 9/1/05
(h)	Earned	2-step	6	7/1/03 to 2/15/04 to 9/1/05

Above each have a total trend period of 26 months.

5.9. (a) Effective Period: 12/1/04 to 12/1/05, average date of writing 6/1/05.
PY02 Earned premium has an average date of writing of 7/1/02.
Trend period: 7/1/02 to 6/1/05, 35 months long.
(b) The last trend data point is the 12 months of written premium from 1st quarter 2004, with average date of writing 2/15/04.
Step 1 of trend period: 7/1/02 to 2/15/04, 19.5 months long.
Step 2 of trend period: 2/15/04 to 6/1/05, 15.5 months long.
(c) Effective Period: 12/1/04 to 12/1/05, average date of earning 12/1/05, 6 months later than the average date of writing.
PY02 Earned premium has an average date of writing of 7/1/02 and an average date of earning 6 months later or 1/1/03.
Trend period: 1/1/03 to 12/1/05, 35 months long.
(d) The last trend data point is the earned premium from 1st quarter 2004, with average date of earning 2/15/04.
Step 1 of trend period: 1/1/03 to 2/15/04, 13.5 months long.
Step 2 of trend period: 2/15/04 to 12/1/05, 21.5 months long.
(e) Trend period: 7/1/02 to 6/1/05, 35 months long.
(f) Step 1 of trend period: 7/1/02 to 2/15/04, 19.5 months long.
Step 2 of trend period: 2/15/04 to 6/1/05, 15.5 months long.
(g) Effective Period: 12/1/04 to 12/1/05, average date of earning 9/1/05, 3 months later than the average date of writing.
PY02 Earned premium has an average date of writing of 7/1/02 and an average date of earning 3 months later or 10/1/02.
Trend period: 10/1/02 to 9/1/05, 35 months long.
(h) Step 1 of trend period: 10/1/02 to 2/15/04, 16.5 months long.
Step 2 of trend period: 2/15/04 to 9/1/05, 18.5 months long.
Comment: Note that the total trend period is 35 months in all cases.

5.10. a. Since all homes are carefully inspected infrequently, this is more of a one time effect.
b. Some insureds will not be renewed due to these inspections. Other insureds will be urged to increase their value insured. It is likely these effects will have to be estimated rather than measured, since it would be hard to collect data and one would never know whether these things would have happened without these inspections.
c. The effects should show up over the 12 months following the inspections, as the insureds come up for renewal (assuming annual policies.) This is like an abrupt effect, similar to a rate change on new and renewal business.
Comment: There is not a single right answer.

5.11. For the first piece of the trend compare the average earned premium at current rate level for a given calendar year to the latest available value for the average written premium at current rate level. For example, 2298/1982 = 1.159.
The second piece of the trend goes from the written premium for CY 2006 to the policy effective period, which is written on average 2.5 years later. $1.04^{2.5} = 1.103$.
The average date of writing under the new rates is 1/1/09. This corresponds to an average accident date of 7/1/09. Therefore, the loss trend period from AY06 is 3 years. $1.06^3 = 1.191$.

Year	Avg. Writ. Premium	Avg. Earn. Premium	1st Piece of Trend	2nd Piece of Trend	Trended Avg. EP	Earned Exposures	Trended Earn. Prem.
2003	$2007	$1982	1.164	1.103	$2546	800	$2,036,616
2004	$2165	$2083	1.108	1.103	$2546	850	$2,163,904
2005	$2210	$2191	1.053	1.103	$2546	900	$2,291,193
2006	$2308	$2254	1.024	1.103	$2546	850	$2,163,904
Year	Ultimate Losses	Loss Trend	Trended Losses	Trended Premium	Loss Ratio		
2003	$1,320,000	1.419	$1,872,445	$2,036,616	**91.9%**		
2004	$1,400,000	1.338	$1,873,516	$2,163,904	**86.6%**		
2005	$1,650,000	1.262	$2,083,087	$2,291,193	**90.9%**		
2006	$1,710,000	1.191	$2,036,637	$2,163,904	**94.1%**		

For example, (1982)(1.159)(1.103) ≅ 2535. (800)(2535) = 2.028 million.

(1,320,000)(1.419) ≅ 1.872 million. 1.872/2.028 = 92.3%.

Alternately, for the first piece of the trend compare the average earned premium at current rate level for a given calendar year to the latest available value for the average <u>earned</u> premium at current rate level. The average date of writing under the new rates is 1/1/09. This corresponds to an average earning date of 7/1/09. The second piece of the trend goes from the earned premium for CY 2006 to the policy effective period, which is earned on average 3 years later.

Year	Avg. Writ. Premium	Avg. Earn. Premium	1st Piece of Trend	2nd Piece of Trend	Trended Avg. EP	Earned Exposures	Trended Earn. Prem.
2003	$2007	$1982	1.137	1.125	$2535	800	$2,028,355
2004	$2165	$2083	1.082	1.125	$2535	850	$2,155,127
2005	$2210	$2191	1.029	1.125	$2535	900	$2,281,899
2006	$2308	$2254	1.000	1.125	$2535	850	$2,155,127
Year	Ultimate Losses	Loss Trend	Trended Losses	Trended Premium	Loss Ratio		
2003	$1,320,000	1.419	$1,872,445	$2,028,355	**92.3%**		
2004	$1,400,000	1.338	$1,873,516	$2,155,127	**86.9%**		
2005	$1,650,000	1.262	$2,083,087	$2,281,899	**91.3%**		
2006	$1,710,000	1.191	$2,036,637	$2,155,127	**94.5%**		

<u>Comment</u>: Similar to 5, 5/07, Q.36. Usually one would have a separate premium trend series with more frequent values than shown here.

5.12. (a) Earned exposures for calendar year 2005: **385 car years**.

Effective Date	Written Exposures	Portion Earned in CY05	Earned Exposures
1/1/2004	10	0	0
4/1/2004	20	0.25	5
7/1/2004	40	0.5	20
10/1/2004	60	0.75	45
1/1/2005	100	1	100
4/1/2005	120	0.75	90
7/1/2005	150	0.5	75
10/1/2005	200	0.25	50
Sum			**385**

(b) Since policies have a one year term, those effective 1/1/2004 expire on 12/31/2004 and are not in-force on 1/1/2005. Those effective 4/1/2004, 7/1/2004, 10/1/2004, and 1/1/2004 are in force on 1/1/2005.

In-force exposures on 1/1/2005: 20 + 40 + 60 + 100 = **220 cars**.

(c) Average written premium of: (1.2)($500) = $600 per car during 2005.

Calendar year 2005 earned premium: **$224,000**.

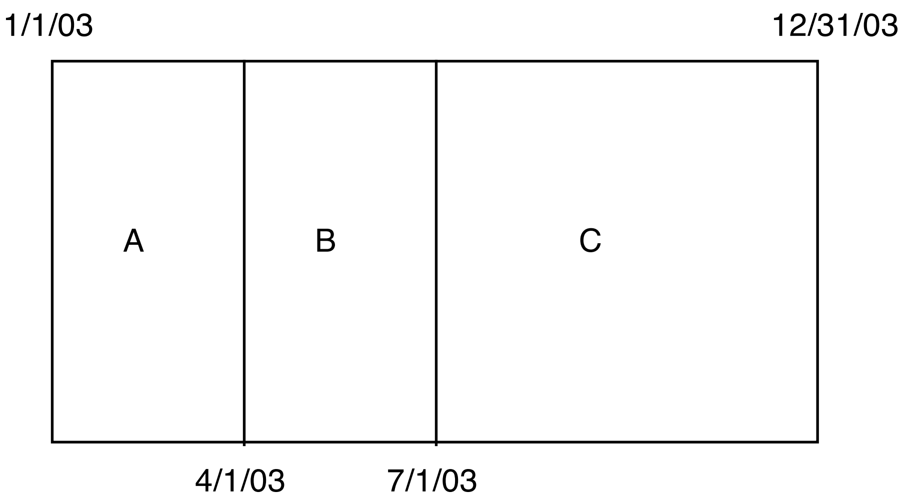
Effective Date	Cars	Average Rate per Car	Written Premium	Portion Earned in CY05	Earned Premium
1/1/2004	10	$500	$5,000	0	$0
4/1/2004	20	$500	$10,000	0.25	$2,500
7/1/2004	40	$500	$20,000	0.5	$10,000
10/1/2004	60	$500	$30,000	0.75	$22,500
1/1/2005	100	$600	$60,000	1	$60,000
4/1/2005	120	$600	$72,000	0.75	$54,000
7/1/2005	150	$600	$90,000	0.5	$45,000
10/1/2005	200	$600	$120,000	0.25	$30,000
Sum					**$224,000**

<u>Comment</u>: Similar to 6, 5/96, Q.31.

5.13. The first step is to calculate the rate level index:

Date	Rate Level Change Factor	Rate Level (7/1/00 = 1)
7/1/00		1
7/1/01	1.05	1.0500
1/1/02	1.03	1.0815
4/1/02	1.04	1.1248
10/1/02	0.98	1.1023
4/1/03	1.06	1.1684
7/1/03	1.02	1.1918
1/1/04	0.97	1.1560
10/1/04	0.95	1.0982

(a) Area A = 1/4. Area B = 1/4. Area C = 1/2.



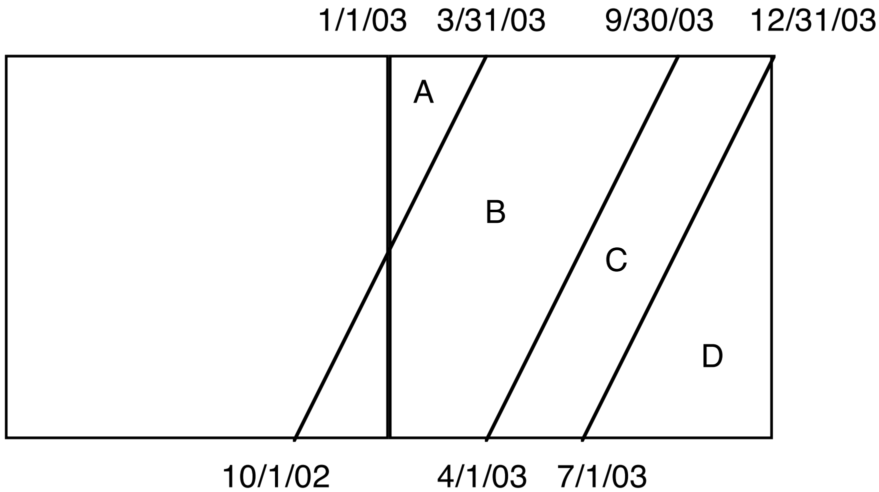
Note that for written premium for a Calendar Year, one uses vertical lines in the diagram.

Rate Level (7/1/00 = 1)	Area	Extension
1.1023	0.25	0.27557
1.1684	0.25	0.29210
1.1918	0.5	0.59588
	1	1.16355

On Level Factor: 1.0982/1.16355 = .944.

Written Premium on current rate level: (.944)(1,000,000) = **$944,000**.

(b) Area A = (1/2)(1/4)(1/2) = 1/16. Area B = (1/2)(1/4 + 3/4) - A = 7/16.
Area C = 1/2 - D = 1/4. Area D = (1/2)(1/2)(1) = 1/4.



Note that for 1/2 year policies the lines in the diagram have a slope of 2.
For example, a policy effective 4/1/03 expires 9/30/03.

Rate Level (7/1/00 = 1)	Area	Extension
1.1248	0.0625	0.07030
1.1023	0.4375	0.48224
1.1684	0.25	0.29210
1.1918	0.25	0.29794
	1	1.1426

On Level Factor: 1.0982/1.1426 = 0.961.

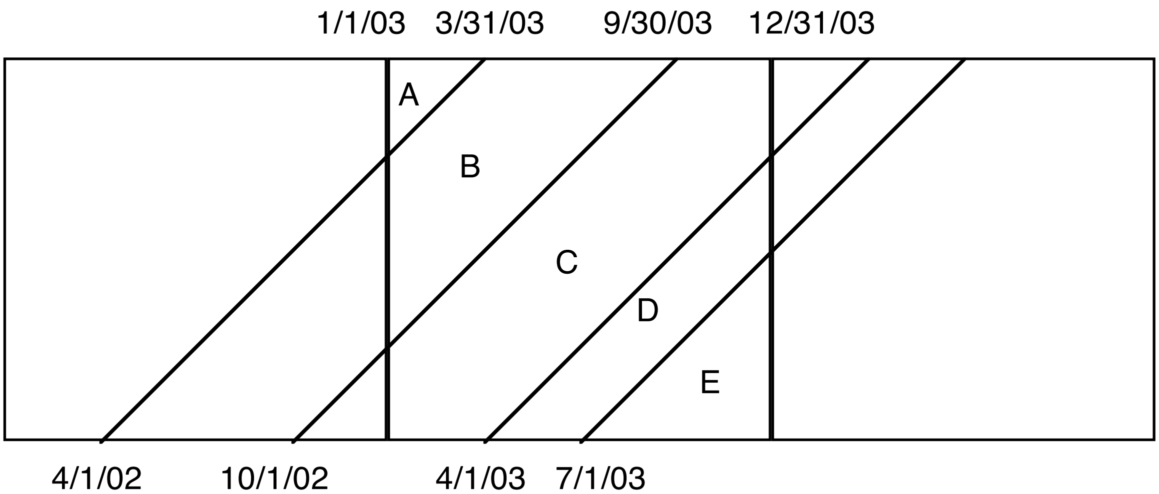
Earned Premium on current rate level: (0.961)(900,000) ≅ **$865,000**.

(c) The policy term does not affect the calculation of on-level written premiums.
Same exact answer as part (a): **$944,000**.

(d) Area A = $(1/2)(1/4)^2$ = 1/32. Area B = $(1/2)(3/4)^2$ - A = 1/4.

Area C = 1 - A - B - D - E = 7/16. Area D = $(1/2)(3/4)^2$ - E = 5/32.

Area E = $(1/2)(1/2)^2$ = 1/8.



Note that for 1 year policies the lines in the diagram have a slope of 1.
For example, a policy effective 10/1/02 expires 9/30/03.

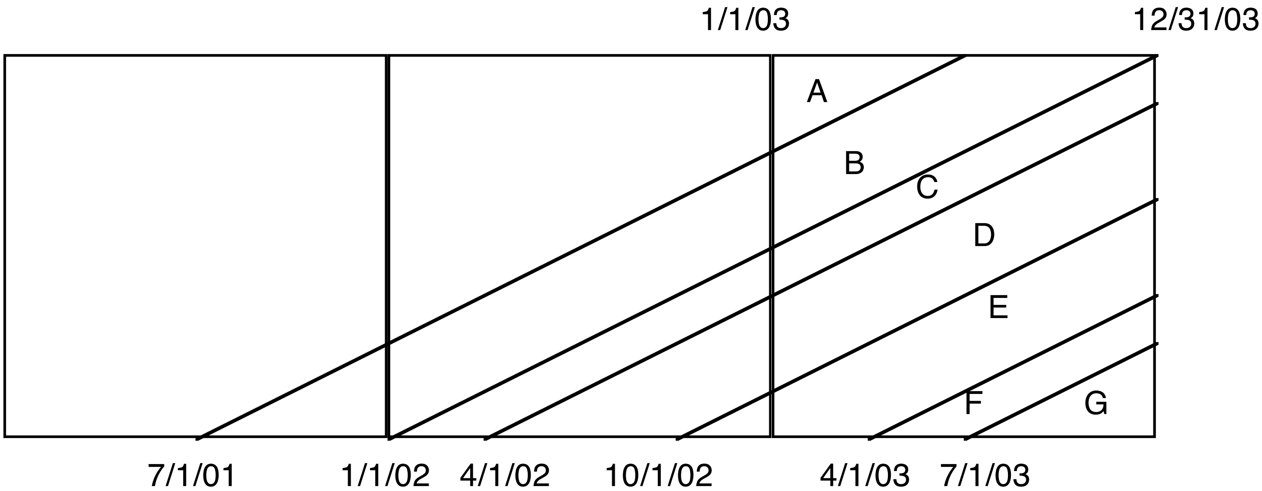
Rate Level (7/1/00 = 1)	Area	Extension
1.0815	0.03125	0.03380
1.1248	0.25	0.28119
1.1023	0.4375	0.48224
1.1684	0.15625	0.18256
1.1918	0.125	0.14897
	1	1.1288

On Level Factor: 1.0982/1.1288 = .973.

Earned Premium on current rate level: (.973)(900,000) ≅ **$876,000**.

(e) Same exact answer as part (a): **$944,000**.

(f) Area A = (1/2)(1/2)(1/4) = 1/16. Area B = (1/2)(1)(1/2) - A = 3/16.
Area C = (1/2)(5/8 + 1/8) - B - A = 1/8. Area D = (1/2)(7/8 + 3/8) - C - B - A = 1/4.
Area E = (1/2)(1/8 + 5/8) - F - G = 15/64. Area F = (1/2)(3/4)(3/8) - G = 5/64.
Area G = (1/2)(1/2)(1/4) = 1/16.



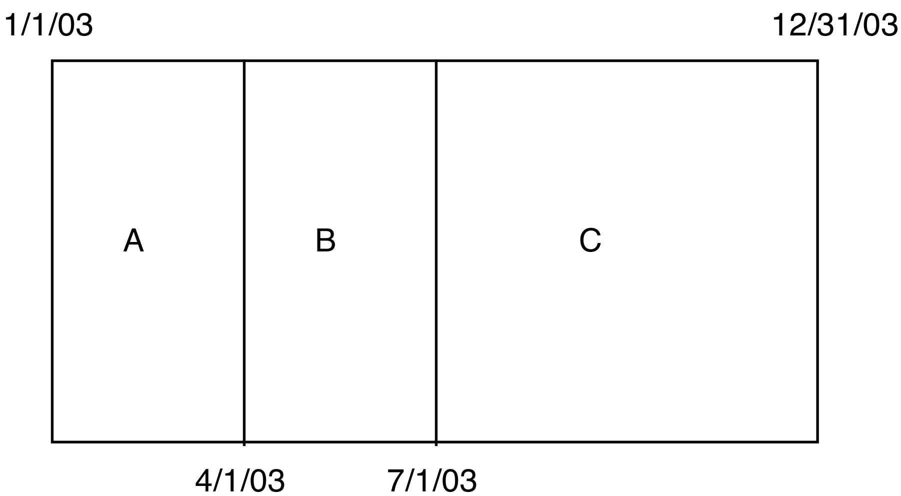
Note that for 2 year policies the lines in the diagram have a slope of 1/2.
For example, a policy effective 1/1/02 expires 12/31/03.

Rate Level (7/1/00 = 1)	Area	Extension
1	0.0625	0.06250
1.0500	0.1875	0.19687
1.0815	0.125	0.13519
1.1248	0.25	0.28119
1.1023	0.234375	0.25834
1.1684	0.078125	0.09128
1.1918	0.0625	0.07449
1.1560		
1.0982		
	1	1.0999

On Level Factor: 1.0982/1.0999 = .998.

Earned Premium on current rate level: (.998)(900,000) ≅ **$898,000**.

(g) Policy Year written and earned premiums are equal (at ultimate.)
Area A = 1/4. Area B = 1/4. Area C = 1/2.



Note that for Policy Year premium one uses vertical lines in the diagram.
Policies written in the first three months of 2003 are at the rate level of 10/1/2002.
Policies written in the next three months of 2003 are at the rate level of 4/1/2003.
Policies written in the final six months of 2003 are at the rate level of 7/1/2003.
Thus the average rate level for Policy Year 2003 is:
(3/12)(1.1023) + (3/12)(1.1684) + (6/12)(1.1918) = 1.1636
On Level Factor: 1.0982/1.1636 = .944.

Premium on current rate level: (.944)(1,100,000) ≅ **$1,038,000**.

(h) The policy term does not affect the calculation of on-level written premiums.
Same exact answer as part (g): **$1,038,000**.
(i) Same exact answer as part (g): **$1,038,000**.
Comment: I chose to set 7/1/00 equal to a rate level of 1. One could just as well have set 1/1/00 equal to a rate level of 1. Both are far enough in the past compared to the years of interest that the final results will be the same, even though the intermediate steps will differ.

5.14. E. All the premium is written in 2006. The policy expires on October 31, 2006, so all the premium is earned in 2006. As of December 31, 2006 this policy is no longer in force, so the inforce premium is zero.
Comment: Similar to 5, 5/03, Q.10.

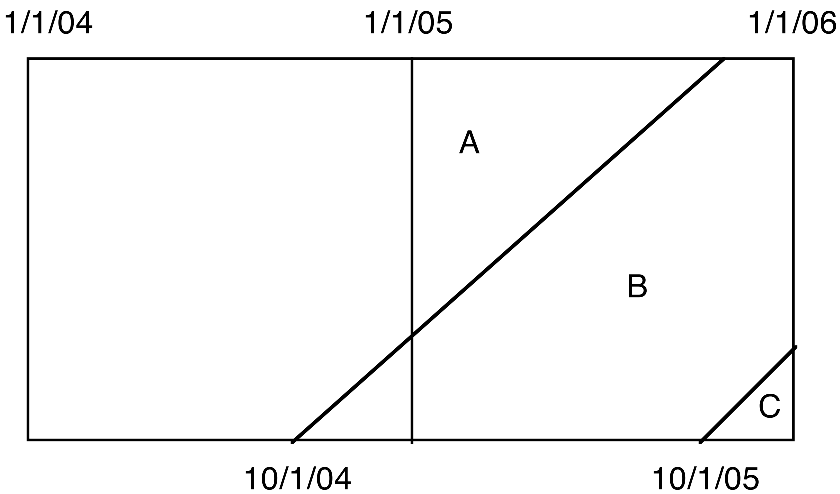
5.15.

Effective Date	Policy Term	Premium	Portion Earned During Year	Unearned Premium
June 1	6 months	$1,000	100%	0
September 1	12 months	$1,600	4/12	$1067
October 1	6 months	$1,200	1/2	$600
				$1667

Comment: Similar to 6, 5/94, Q.1.

5.16.

Date	Rate Level Change	Rate Level Index
Prior		1.000
10/1/03	5%	1.050
10/1/04	20%	1.260
10/1/05	15%	1.449



Area A = $(1/2)(3/4)^2$ = 9/32. Area C = $(1/2)(1/4)^2$ = 1/32. Area B = 1 - 9/32 - 1/32 = 22/32.

CY05 earned premium is written: 9/32 at the 10/1/03 rate, 1/32 at the 10/1/05 rate, and the remaining 22/32 at the 10/1/04 rate.

The average rate level for CY05 earned premium is:

(1.05)(9/32) + (1.260)(22/32) + (1.449)(1/32) = 1.207.

On Level Factor = Current Rate Level/ Average Rate Level = 1.449/1.207 = **1.200**.

5.17. For each class/territory cell, the current premium would have been:
(exposures)($600)(class relativity)(territory relativity).

	Rates Per Car Year				
	Terr A	Terr B	Terr C	Terr D	
Class 1	$576	$792	$720	$1008	
Class 2	$408	$561	$510	$714	
Class 3	$480	$660	$600	$840	
	Exposures in Car Years				
	Terr A	Terr B	Terr C	Terr D	
Class 1	100	200	400	300	
Class 2	50	150	250	200	
Class 3	100	300	400	150	
	Premiums at Present Rates				
	Terr A	Terr B	Terr C	Terr D	Total
Class 1	$57,600	$158,400	$288,000	$302,400	$806,400
Class 2	$20,400	$84,150	$127,500	$142,800	$374,850
Class 3	$48,000	$198,000	$240,000	$126,000	$612,000
Total	$126,000	$440,550	$655,500	$571,200	**$1,793,250**

For example, for Class 1 and Territory A, ($600)(1.2)(.8) = $576. (100)($576) = $57,600.
Comment: I have used (2)($300) = $600, since that is the current base rate for a car year of coverage. Premium = (Premium / car-year)(Number of car-years)
= (Premium / half car-year)(Number of half car-years).
On the exam, you should assume relativities are multiplicative, unless stated otherwise.

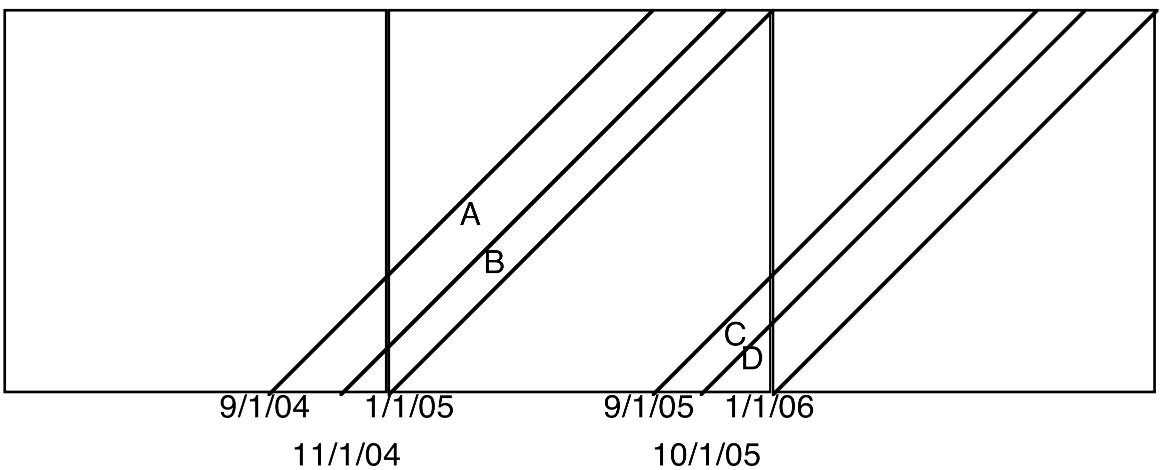
5.18. As a good approximation, assume policies are only written in the middle of each month.

Effective Date	Portion Earned in 2005	Rate Level Index	Extension
9/15/04	0.7083	1	0.7083
10/15/04	0.7917	1	0.7917
11/15/04	0.8750	1.1	0.9625
12/15/04	0.9583	1.1	1.0542
9/15/05	0.2917	1.1	0.3208
10/15/05	0.2083	1.265	0.2635
11/15/05	0.1250	1.265	0.1581
12/15/05	0.0417	1.265	0.0527
Total	4.0000		4.3119

Average rate level for Calendar Year 2005 earned premiums: 4.3119/4 = 1.078.

Current rate level: 1.265. On-level factor is: 1.265/1.078 = **1.173**.

Alternately, draw a parallelogram method diagram, where however we are only interested in areas corresponding to policies written in the four months from September to December:



Area D = $(1/2)(1/4)^2$ = 1/32. Area C = $(1/2)(1/3)^2 - (1/2)(1/4)^2$ = 7/288.

Area B = 1/6 - 1/72 = 11/72. Area A = $1/6 - \{(1/2)(1/3)^2 - (1/2)(1/6)^2\}$ = 1/6 - 1/24 = 1/8.

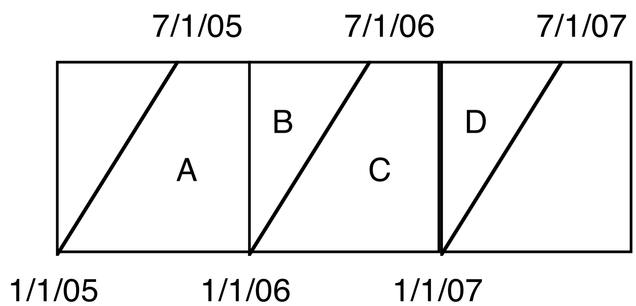
Date	Rate Level Index
Prior	1
11/1/04	1.1
10/1/05	(1.1)(1.15) = 1.265

Average rate level: $\frac{(1)(1/8) + (1.1)(11/72) + (1.1)(7/288) + (1.265)(1/32)}{1/8 + 11/72 + 7/288 + 1/32} = 1.078$.

On-level factor is: 1.265/1.078 = **1.173**.

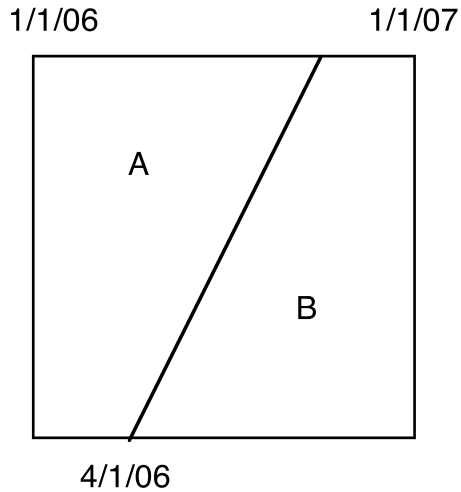
<u>Comment</u>: Somewhat similar to 5, 5/06, Q.28. *Snowmobile policies may not provide coverage for one year. If they did, premiums may not be earned uniformly throughout the policy term, since snowmobile accidents occur mostly in the Winter.*

5.19. The average 6-month policy has half of the premium of the average annual policy.
There are three times as many 6-month policies as annual policies.
Therefore, in total the 6-month policies have 3/2 of the premium of the annual policies.
Thus the six-month policies represent as a portion of the total premium: (3/2)/(3/2 + 1) = 60%.
For the six-month policies, 75% of the PY 2006 written premium is earned during CY 2006, while the remainder is earned during CY 2007.
Therefore, for the six-month policies, 75% of the CY 2006 earned premium comes from policies written in 2006 while 25% comes from policies written in 2005.
For the annual policies, 50% of the PY 2006 premium is earned during CY 2006, while the remainder is earned during CY 2007.
Therefore, for the annual policies, 50% of the CY 2006 earned premium comes from policies written in 2006 while 50% comes from policies written in 2005.
Therefore, for the insurer, the portion of CY 2006 earned premium earned that comes from **PY 2006** is: (60%)(75%) + (40%)(50%) = **65%**.
The remaining **35%** of CY 2006 earned premium comes from **PY 2005**.
Comment: Here is a diagram for the 6-month policies:



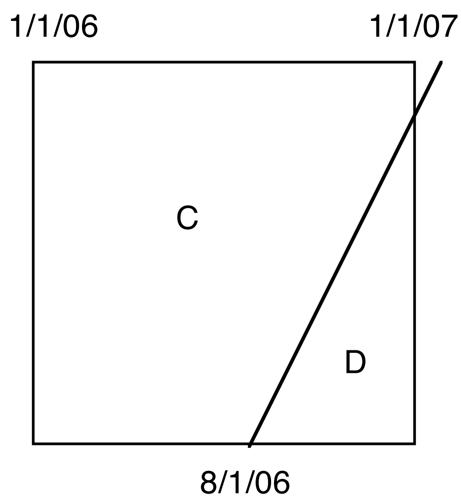
Policy Year 2005 is Area A + Area B.
Calendar Year 2006 earned premium is Area B + Area C.
Policy Year 2006 is Area C + Area D.
Area B = Area D = 1/4. Area A = Area C = 3/4.

5.20. Assume that the rate increase (you thought was on 4/1 but was actually on 8/1) was of size x. Then the current rate level is 1 + x, while that before the increase was 1.



Area A = (1/4 + 3/4)/2 = 1/2. Area B = (3/4 + 1/4)/2 = 1/2.
Assuming the increase was effective on 4/1/06, then the average rate level for CY 06 earned premiums would be: (1/2)(1) + (1/2)(1 + x) = 1 + x/2.
Then the on-level factor for CY 06 earned premiums is: (1 + x)/(1 + x/2).

$\Rightarrow$ 1.052 = (1 + x)/(1 + x/2) $\Rightarrow$ 1.052 + .526x = 1 + x. $\Rightarrow$ x = .052/.474 = .110.



Area D = (5/12)(10/12)/2 = 25/144. Area C = 1 - 25/144 = 119/144.
Therefore, given that the increase was effective 8/1/06, the average rate level for CY 06 earned premiums is: (119/144)(1) + (25/144)(1.110) = 1.019.
The on-level factor for CY 06 earned premiums is: 1.110/1.019 = **1.089**.
<u>Comment</u>: Similar to 6, 5/99, Q.58.

5.21. a. Each year the written premium is: (4)(1000) = 4000.
Since we are in a steady state, the earned premium each Calendar Year is also **4000**.
More precisely, let us take for example CY 2005.
The policy written 2/15/04 contributes 3/24 of its premium to the earned premium for CY05.

Effective Date	Portion Earned in CY05	CY05 Earned Premium
2/15/04	0.125	125
5/15/04	0.375	375
8/15/04	0.625	625
11/15/04	0.875	875
2/15/05	0.875	875
5/15/05	0.625	625
8/15/05	0.375	375
11/15/05	0.125	125
Total		**4000**

b. The contributions to the earned premium are symmetric around 1/1/2005.
Therefore, the average date of writing is 1/1/2005.
In general with policies written uniformly, **with annual policies the average date of writing of the earned premium for a Calendar Year is January 1 of that Calendar Year**.
More precisely, for CY05 earned premium, the average date of writing is a weighted average:
{(4.125)(125) + (4.375)(375) + (4.625)(625) + (4.875)(875) + (5.125)(875) + (5.375)(625) + (5.625)(375) + (5.875)(125)} / 4000 = 5.000.
c. Each year the written premium is: (4)(1000) = 4000.
Since we are in a steady state, the earned premium each Calendar Year is also **4000**.
More precisely, let us take for example CY 2005.
The policy written 8/15/04 contributes 3/12 of its premium to the earned premium for CY05.

Effective Date	Portion Earned in CY05	CY05 Earned Premium
2/15/04	0	0
5/15/04	0	0
8/15/04	0.25	250
11/15/04	0.75	750
2/15/05	1	1000
5/15/05	1	1000
8/15/05	0.75	750
11/15/05	0.25	250
Total		**4000**

d. The contributions to the earned premium are symmetric around 4/1/2005.
Therefore, the average date of writing is 4/1/2005.
In general with policies written uniformly, **with 6-month policies the average date of writing of the earned premium for a Calendar Year is April 1 of that Calendar Year**.
More precisely, for CY05 earned premium, the average date of writing is a weighted average:
{(4.625)(250) + (4.875)(750) + (5.125)(1000) + (5.375)(1000) + (5.625)(750) + (5.875)(250)} / 4000 = 5.250.

5.22. (a) Assume for simplicity that the insurer writes policies in the middle of each month.
Assume all of the premiums are 240.

Month	Premiums Earned in 2015	Average Date of These Earned Premiums
Jan.	(23/24)(240) = 230	(1/24 + 24/24)/2 = 25/48.
Feb.	(21/24)(240) = 210	(3/24 + 24/24)/2 = 27/48.
⋮		
Nov.	(3/24)(240) = 30	(21/24 + 24/24)/2 = 45/48.
Dec.	(1/24)(240) = 10	(23/24 + 24/24)/2 = 47/48.

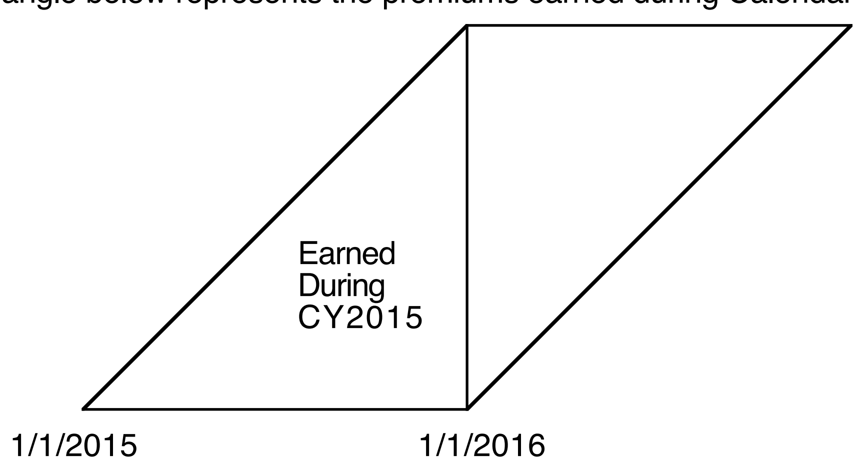
Average date of earning is:

$$\frac{(230)(25/48) + (210)(27/48) + ... + (30)(45/48) + (10)(47/48)}{230 + 210 + ... + 30 + 10} =$$

(46,120 / 48) / 1440 = 0.667.
In other words, the average of earning is two-thirds through the year, or **September 1, 2015**.
Alternately, for a policy written at time t, $0 \le t \le 1$, the portion earned in 2015 is (1- t).
The average date of this earned premium is: (t + 1)/2.
Thus the overall average date of earning is:

$$\frac{\int_0^1 (1-t)(t+1)/2\ dt}{\int_0^1 (1-t)\ dt} = \frac{(1/2)\int_0^1 1 - t^2\ dt}{1/2} = 1 - 1/3 = 2/3.$$

In other words, the average of earning is two-thirds through the year, or **September 1, 2015**.
Alternately, since we start writing 1/1/2015, we need only worry about policy year 2015.
We have annual policies.
The triangle below represents the premiums earned during Calendar Year 2015.



The average of such a right triangle is **2/3rds** of the way from its left vertex to its right side.

(b) For a policy written at time t, $0 \le t \le 1/2$, all the premium is earned during 2015 and the average date of earning is t + 1/4.
For a policy written at time t, $1/2 \le t \le 1$, the portion earned in 2015 is: 2(1- t), and the average date of this earned premium is: (t + 1)/2.
Thus the overall average date of earning is:

$$\frac{\int_0^{1/2} (t+1/4)\, dt + \int_{1/2}^{1} 2(1-t)(t+1)/2\, dt}{\int_0^{1/2} dt + \int_{1/2}^{1} 2(1-t)\, dt} = \frac{1/8 + 1/8 + \int_{1/2}^{1} 1 - t^2\, dt}{1/2 + 1/4} = \frac{1/4 + 1/2 - (1 - 1/8)/3}{3/4}$$

= 11/18. In other words, the average of earning is 11/18 of the way through the year or about 223 days through the year or about **August 11, 2015**.
Alternately, assume for simplicity that the insurer writes policies in the middle of each month.
Assume all of the premiums are 240.

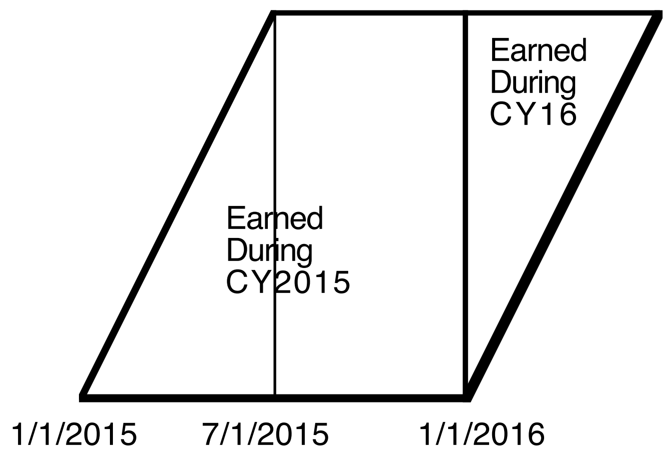
Month	Premiums Earned in 2015	Average Date of These Earned Premiums
Jan.	240	1/24 + 1/4 = 14/48.
Feb.	240	3/24 + 1/4 = 18/48.
⋮		
June	240	11/24 + 1/4 = 34/48.
July	(11/12)(240) = 220	(13/24 + 24/24)/2 = 37/48.
August	(9/12)(240) = 180	(15/24 + 24/24)/2 = 39/48.
⋮		
Nov.	(3/12)(240) = 60	(21/24 + 24/24)/2 = 45/48.
Dec.	(1/12)(240) = 20	(23/24 + 24/24)/2 = 47/48.

Average date of earning is:

$$\frac{(240)(14/48)+(240)(18/48)+...+(240)(34/48)+(220)(37/48)+ \ ...+(60)(45/48)+(20)(47/48)}{(6)(240) + 220 + 180 + 140 + 100 + 60 + 20}$$

= (63,400 / 48) / 2160 = 0.611. In other words, the average of earning is about 0.611 of the way through the year or about **August 11, 2015**.

Alternately, since we have six-month policies, the premiums earned during Calendar Year 2015 are the sum of a triangle and a rectangle:



The average of the right triangle is 2/3rds of the way from its left vertex to its right side.,
that is: (2/3)(1/2) = 1/3.
The average of the rectangle is: (1/2 + 1)/2 = 3/4.
The area of the rectangle is twice that of the triangle.
Thus the average date of the premium earned during CY2015 is:
(1/3)(1/3) + (2/3)(3/4) = 1/9 + 1/2 = **11/18**.
Comment: Beyond what you are likely ot be asked on your exam.
We are working with a percent earned versus time chart, and we are integrating time versus the density of earning in order to get the average date of earning.
Similar set up to 5, 11/14, Q. 7, which has six-month policies; the average date of accident for the first year of writing is later than the midpoint of that AY.
The average date of earning is not the usual middle of the year, due to the fact that there are no policies written in 2014 that would normally contribute to the CY2015 earned premium.
Thus in theory the trend from date would differ.
If instead the rate of writing premiums was increasing quickly throughout 2015, then the average date of earning would be even later in the year.

5.23. Policy 1 has no months in 2016. Its audit occurs on 4/30/2016.
Policy 2 has 4 out of 12 months in 2016. Its audit occurs on 10/31/16.
Policy 3 has 9 out of 12 months in 2016. Its audit occurs on 3/31/17.
Policy 4 has 11 out of 12 months in 2016. Its audit occurs on 7/31/17.
Policy 5 has 4 out of 12 months in 2016. Its audit occurs on 2/28/18.

(a)

Policy	Contribution from Initial Premium	Contribution from Audit	Total
1	0	(5%)(200)	10
2	0	(5%)(120)	6
3	0	0	0
4	180	0	180
5	300	0	300
Total			**496**

(b)

Policy	Contribution from Initial Premium	Contribution from Audit	Total
1	0	(5%)(200)	10
2	(4/12)(120)	(5%)(120)	46
3	(9/12)(260)	0	195
4	(11/12)(180)	0	165
5	(4/12)(300)	0	100
Total			**516**

(c) No contribution to PY2016 from the first three policies not written during 2016.

Policy	Contribution from Initial Premium	Contribution from Audit	Total
4	180	0	180
5	300	0	300
Total			**480**

(d)

Policy	Contribution from Initial Premium	Contribution from Audit	Total
4	(11/12)(180)	0	165
5	(4/12)(300)	0	100
Total			**265**

(e)

Policy	Contribution from Initial Premium	Contribution from Audit	Total
4	180	(5%)(180)	189
5	300	0	300
Total			**489**

(f) Same answer as part (e).

(g)

Policy	Contribution from Initial Premium	Contribution from Audit	Total
4	180	(5%)(180)	189
5	300	(5%)(300)	315
Total			**504**

(h) Same answer as part (g).

Comment: Similar to 5, 11/14, Q.2.

Audit premium is earned as soon as it is written. As of December 31, 2018, Policy Year 2016 is at ultimate, and the written and earned premiums are equal.

5.24. Premium Development Link Ratios:

Policy Year	12 to 24	24 to 36	36 to 48
2003	1.841	1.006	0.999
2004	1.863	1.010	1.010
2005	1.886	1.018	
2006	1.761		
Selected (2 year average)	1.824	1.014	1.004

Estimated Ultimate Premium for PY 2005: (43.42)(1.004) = **$43.59 million**.
Estimated Ultimate Premium for PY 2006: (51.95)(1.004)(1.014) = **$52.89 million**.
Estimated Ultimate Premium for PY 2007: (31.03)(1.004)(1.014)(1.824) = **$57.62 million**.
Comment: There are other reasonable selections of premium development factors.
For example, one could take an average of more years for the 12 to 24 link ratios, since this is subject to a lot of random fluctuation.
One could take volume weighted averages.
For example, for the 12 to 24 month link ratio: (42.67 + 51.95) / (22.62 + 29.50) = 1.815.
This data is for a line of insurance such as Workers Compensation with premium audits.

5.25. For some commercial lines of insurance the exposure is sales or payroll. A preliminary estimate of the exposures are used to determine policy premiums. The final premium is not known until after the policy expires, when the insurer determines the final value of the exposures via an audit.
Calendar Year Premiums will include the effects of audits on prior years and will not include the effects of audits on the current year. Therefore, if the exposure level is changing, or if the adequacy of preliminary exposure estimates are changing, this can distort a comparison between Calendar Year Premiums and Accident Year Losses.
Even on a line of insurance without premium audits, Policy Year premiums will develop until all of the policies have expired. When there are premium audits, there is another reason why Policy Year premiums take a while to develop to their ultimate value So as long as we either use sufficiently mature data or apply premium development factors, we can use Policy Year Premiums.
Comment: Even without audits, Policy Year written premium will develop somewhat due to midterm cancelations and endorsements. In addition, Policy Year earned premium will develop as the premium is earned during the terms of the policies making up the Policy Year.
This potential problem with the use of Calendar Year Earned Premiums can be alleviated by an appropriate adjustment such as shown in “Workers’ Compensation Ratemaking” by Sholom Feldblum, CAS Forum 1993, formerly on the syllabus.

5.26.

Effective Date	Overall Rate Change	Rate Level Index
Prior		1.0000
July 1, 2014	+3%	1.0300
January 1, 2015	+7%	1.1021
July 1, 2016	+5%	1.1572

The 20% of policies written on January 1, 2015, are all at rate level 1.1021.
For the remaining policies in-force on April 1, 2015, 3/12 are written prior to July 1, 2014 at rate level 1.0, 6/12 are written at rate level 1.03, while the remaining 3/12 are written at rate level 1.1021.
Thus the average rate level for the policies in-force on April 1, 2015 is:
(20%)(1.1021) + (80%) {(3/12)(1) + (6/12)(1.03) + (3/12)(1.1021)} = 1.0528.
Thus the on level factor is: 1.1572 / 1.0528 = **1.0992**.
Alternately, let us assume that 120 policies are written evenly throughout the year and 30 policies are written on January 1. Let us assume that the rate prior to July 1, 2014 is $100.
Then the rate on July 1, 2014 is $103, on January 1, 2015 is $110.21,
and on July 1 2016 and currently is $115.72.
Then on April 1, 2015 we have in-force 150 policies:

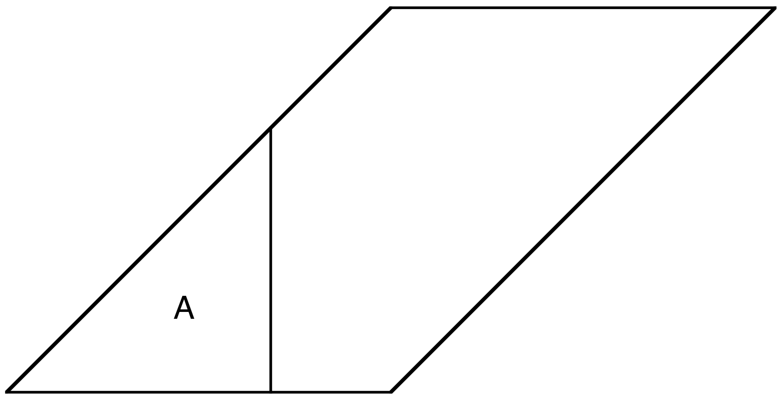
Effective Date	Number of Policies	Rate	Premium
April 2014 to June 2014	30	$100	$3000
July 2014 to Dec. 2014	60	$103	$6180
Jan. 2015 to March 2015	30 + 30 = 60	$110.21	$6612.6

Thus the on-level factor is: $\frac{(150)(\$115.72)}{\$3000 + \$6180 + \$6612.6} = \mathbf{1.0991}$.

Comment: Similar to 5, 5/14, Q.1b.

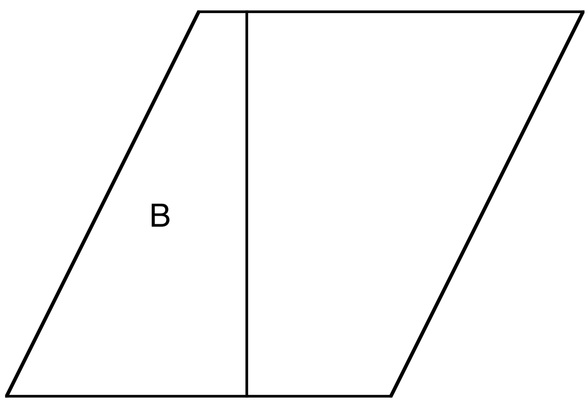
5.27. All policies are 6-month, so policies effective January 1, 2015 - June 30, 2015 contribute nothing to the calendar year 2016 earned exposures,
policies effective July 1, 2015 - December 31, 2015 contribute on average half their exposures, policies effective January 1, 2016 - June 30, 2016 contribute all of their exposures, and policies effective July 1, 2016 - December 31, 2016 contribute on average half their exposures.
At the current rate level, class A pays: ($450)(1.00) + $50 = $500,
while class B pays: ($450)(1.20) + $50 = $590.
Calendar year 2016 earned premium at current rate level is:
($500)(250/2 + 300 + 400/2)(1000) + ($590)(150/2 + 200 + 300/2)(1000) = **$563.25 million**.
Comment: Similar to 5, 11/13, Q.2.
The expense constant is earned over time, just as with any other premium.

5.28. (a) Area A is a triangle with area: $(1/2)(8/12)^2$ = **2/9**.



1/1/2015 9/1/2015 1/1/2016

(b) Area B is a trapezoid with area: (8/12 + 2/12)/2 = **5/12**.



1/1/2015 9/1/2015 1/1/2016

5.29.

Effective Date	Overall Rate Change	Rate Level Index
Prior		1.0000
May 1, 2013	+5%	1.0500
February 1, 2014	-8%	0.9660
August 1, 2015	+10%	1.0626

The January 1, 2013 policies contribute nothing to CY14 earned premium, while the January 1, 2014 policies contribute all of their premium to CY14 earned premium.

Policies written evenly from 1/1/13 to 4/30/13 contribute on average (0 + 4/12)/2 = 4/24 of their premium to CY14 earned premium.

Policies written evenly from 5/1/13 to 12/31/13 contribute on average (4/12 + 1)/2 = 16/24 of their premium to CY14 earned premium.

Policies written evenly from 1/1/14 to 1/31/14 contribute on average (1 + 11/12)/2 = 23/24 of their premium to CY14 earned premium.

Policies written evenly from 2/1/14 to 12/31/14 contribute on average (11/12 + 0)/2 = 11/24 of their premium to CY14 earned premium.

Policies written on January 1, 2014 are 30% of a year's exposures.

Policies written 1/1/13 to 4/30/13 are (4/12)(70%) = 23.33% of a year's exposures.

Thus the average rate level for CY14 earned premium is:

$$\frac{(30\%)(1)(1.05) + (23.33\%)(4/24)(1) + (46.67\%)(16/24)(1.05) + (5.83\%)(23/24)(1.05) + (64.17\%)(11/24)(0.966)}{(30\%)(1) \ + \ (23.33\%)(4/24) \ + \ (46.67\%)(16/24) \ + \ (5.83\%)(23/24) \ + \ (64.17\%)(11/24)}$$

= 1.02335 / 1 = 1.02335.

Thus the on-level factor is: 1.0626 / 1.02335 = **1.0384**.

Alternately, let us assume that 84 policies are written evenly throughout the year and 36 policies are written on January 1. Note that: 36 / (84 + 36) = 36 / 120 = 30%.

Let us assume that the rate prior to May 1, 2013 is $1000.

Then the rate on May 1, 2013 is $1050, on February 1, 2014 is $966,
and on August 1, 2015 and currently is $1062.6.

The January 1, 2013 policies contribute nothing to CY14 earned premium, while the January 1, 2014 policies contribute all of their premium of (36)(1050) = 37,800 to CY14 earned premium.

Then we add up the contributions to CY14 earned premium:

Effective Dates	Number Policies	Rate	Percent	Premium
January 1, 2014	36	$1050	1	$37,800
1/1/13 to 4/30/13	28	$1000	4/24	$4666.67
5/1/13 to 12/31/13	56	$1050	16/24	$39,200
1/1/14 to 1/31/14	7	$1050	23/24	$7043.75
2/1/14 to 12/31/14	77	$966	11/24	$34,091.75
Total				$122,802.17

At the current rate level, the CY14 earned premium would have been: (120)(1062.6) = $127,512.

Thus the on-level factor is: $127,512 / $122,802.17 = **1.0384**.

Comment: Similar to 5, 5/14, Q.1b.

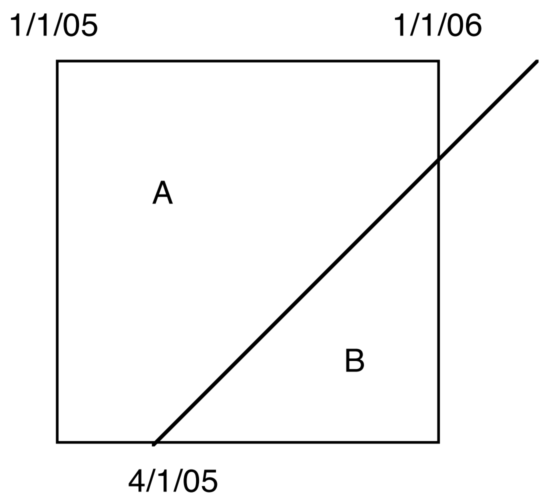
5.30. (a) Policy A is canceled before the end of 2017, so it contributes: (800)(7/12) = 467.
Policy C is rerated prior to the end of 2017, so it contributes: (2/12)(600) + (10/12)(1000) = 933.
Total CY2017 written premium is: 467 + 500 + 933 = **$1900**.
(b) Six-month Policy B has CY2017 earned premium of: (5/6)(500) = 417.
Policy C has CY2017 earned premium of: (2/12)(600) + (2/12)(1000) = 267.
Total CY2017 earned premium is: 467 + 417 + 267 = **$1151**.
(c) In-force premium as of December 1, 2017 is: 0 + 500 + 1000 = **$1500**.
Comment: Similar to 5, 5/16, Q. 2.
For Policy C, 2 months is at $600 (annual basis),
while the remaining 10 months is at $1000 (annual basis).

5.31. The purpose of premium trend adjustments is to adjust the average premiums at current rate level from historical periods to a future rating period. Adjustments account for inflation-sensitive exposures as well as changes in mix of business and rating characteristics.
Comment: Quoting from Basic Ratemaking at pages 81-82: "Since the goal of ratemaking is to determine adequate rates for the future, it is important to adjust the historical premium to the level expected during the future time period. In addition to adjusting the historical premium to the current rate level, the premium also must be adjusted to reflect any premium trend." "In addition to inflationary pressure, the average premium level can change over time due to changes in the characteristics of the policies written. These changes are referred to as distributional changes, and the resulting change in average premium level is commonly referred to as premium trend."

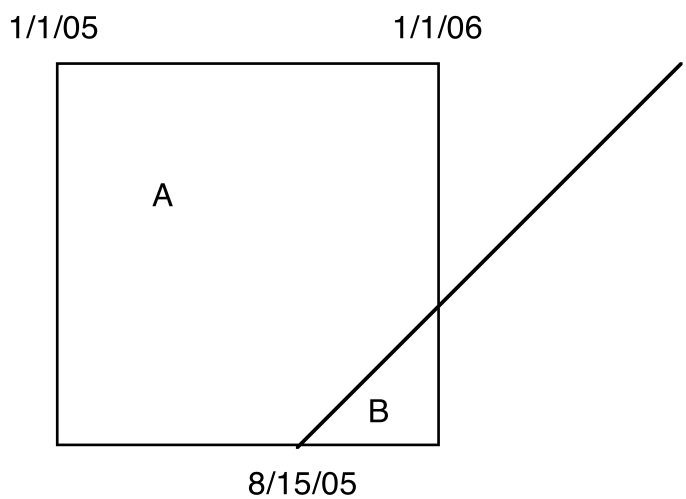
5.32. The average date of writing under the new rates is April 1, 2018.
For the annual policies, the average date of earning is 6 months after the average date of writing.
For the 6-month policies the average date of earning is 3 months after the average date of writing.
Thus overall the average date of writing is (6)(60%) + (3)(40%) = 4.8 months after the average date of writing.
July 1, 2016 to April 1, 2018 is 21 months. Thus the total trend period is: 21 + 4.8 = 25.8 months.
The premium trend factor is: $1.04^{25.8/12}$ = **1.088**.

5.33. a. The average written date for CY12 earned premiums is: 7/1/12 - 12 months / 2 = 1/1/12.
The average date of writing under the new rates is: 4/1/16 + 18 months / 2 = 1/1/17.
Trend from date is January 1, 2012.
Trend to date is January 1, 2017.
b. The last 12-month moving average includes the fourth quarter of 2014 through the third quarter of 2015. The average date of writing is 4/1/15.
Thus **the first step goes from January 1, 2012 to April 1, 2015**.
The second step goes from April 1, 2015 to January 1, 2017.

5.34. Assuming the change took place on 4/1/05, the diagram looks as follows:



Area B = $(3/4)^2/2$ = 9/32. Area A = 1 - 9/32 = 23/32.
Let x be the rate change factor on 4/1/05.
Then the average rate level for CY05 Earned Premium is:
(23/32)(1) + (9/32)x.
Thus 1.1034 = x/{(23/32)(1) + (9/32)x}.
⇒ 25.3782 + 9.9306x = 32x. ⇒ x = 25.3782/(32 - 9.9306) = 1.150.
Since the change took place on 8/15/05, the corrected diagram looks as follows:



Area B = $(9/24)^2/2$ = 0.0703125. Area A = 1 - 0.0703125 = 0.9296875.
Then the average rate level for CY05 Earned Premium is:
(0.9296875)(1) + (0.0703125)(1.150) = 1.01055.
Corrected on-level-factor is: 1.150/1.01055 = 1.1380.
Percentage change in the on-level-factor is: 1.1380/1.1034 - 1 = **3.1%**.

5.35. (a) Step one is a comparison of the 2008 written premium of 212 and the 2006 earned premium of 202. The step one factor is: 212/202 = 1.0495.
In the second step, we trend from the 2008 written premiums to the proposed rate period.
The average date of writing of the 2008 written premiums is 7/1/08.
The average date of writing under the new rates is 3/1/10.
Therefore, the second step is 20 months long. The step two factor is: $1.03^{20/12}$ = 1.0505.
Premium trend to apply to 2006 earned premiums: (1.0495)(1.0505) = **1.102**.
(b) The step one factor is: 214/202 = 1.0594.
In the second step, we trend from the 2008 earned premiums to the proposed rate period.
The average date of earning of the 2008 earned premiums is 7/1/08.
The average date of writing under the new rates is 3/1/10.
The policies are annual. Thus the average date of earning under the new rates is 9/1/10.
Therefore, the second step is 26 months long. The step two factor is: $1.03^{26/12}$ = 1.0661
Premium trend to apply to 2006 earned premiums: (1.0594)(1.0661) = **1.129**.
Comment: Similar to 5, 5/08, Q. 15b.

5.36. Assume we are looking for example at PY 2006.
Then 24 months from the start of the policy year is 12/31/07.
36 months from the start of the policy year is 12/31/08.
As of 12/31/07, all of the policies will have expired, so there is no unearned premium.
As of 12/31/07, policies that expired 3/31/07 or earlier would have been audited.
Thus 1/4 of the policies written during PY07 would have been audited by 24 months.
By 36 months all policies would have been audited.
Let us assume the total premium prior to any audits was 12,000.
Then at 24 months the premium would be: 9000 + (3000)(1.095) = 12,285.
At 36 months the premium would be: (12,000)(1.095) = 13,140.
Policy year premium development factor from 24 to 36 months is: 13,140/12,285 = **1.070**.
Comment: For a Policy Year, 24 months is 1st report, while 36 months is 2nd report.

5.37. Prior to 1/1/18. let the average premium prior to the discount be x,
then the average premium is: x{(70%)(0.95) + (30%)(1)} = 0.965x.
On 1/1/18 the average premium is: x {(70%)(0.90) + (30%)(1)} = 0.930x.
Thus, this change is equivalent to a rate change of: 0.930/0.965 = 0.9637.

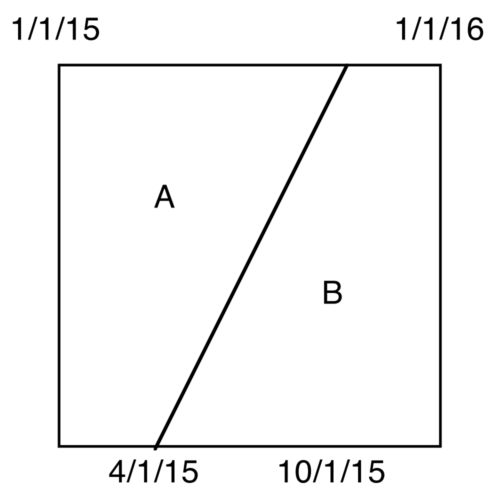
<u>Date</u>	<u>Rate Level Index</u>	<u>On-Level Factor</u>
January 1, 2017	1.000	1.0463
September 1, 2017	0.940	1.1131
January 1, 2018	(0.940)(.9637) = 0.9059	1.1550
October 1, 2018	(0.9059)(1.1) = 0.9965	1.0500
April 1, 2019	(0.9965)(1.05) = 1.0463	

The average rate level for the third quarter of 2017 is: (2/3)(1.000) + (1/3)(0.940) = 0.98.
On Level Factor for the third quarter of 2017 is: 1.0463/0.98 = 1.0677.

Quarter	Premium ($thousands)	Exposures	Average Premium	On Level Factor	On Level Average Premium
First 2017	2000	4100	$487.80	1.0463	**$510.39**
Second 2017	2100	4200	$500.00	1.0463	**$523.15**
Third 2017	2100	4300	$488.37	1.0677	**$521.43**
Fourth 2017	2200	4400	$500.00	1.1131	**$556.55**
First 2018	2300	4500	$511.11	1.1550	**$590.33**
Second 2018	2300	4400	$522.73	1.1550	**$603.75**
Third 2018	2500	4700	$531.91	1.1550	**$614.36**
Fourth 2018	2800	4700	$595.74	1.0500	**$625.53**

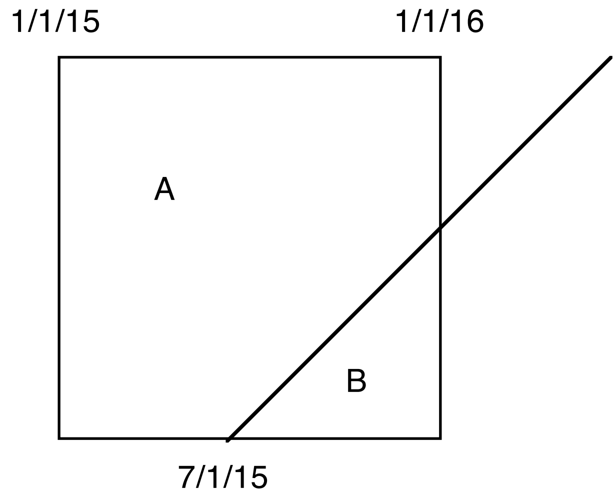
<u>Comment</u>: Alternately, one could put everything on the 10/1/08 rate level. The key is to get the average premiums on the <u>same</u> rate level.

5.38. a. For the six month autos:
{(1/2)(600) + 550 + 500 + 450 + (1/2)(400)}/2 = 1000 car years.
For the 12 month autos:
(1/4)(400) + (1/2)(450) + (3/4)(500) + 550 + (3/4)(600) + (1/2)(650) + (1/4)(700)
= 2200 car years.
A total of: 1000 + 2200 = **3200** car years.
b. For the six month autos, earned exposures at the $750 rate: {(1/2)(600) + 550}/2 = 425.
For the six month autos, earned exposures at the $825 rate: 1000 - 425 = 575.
For the 12 month autos, earned exposures at the $750 rate:
(1/4)(400) + (1/2)(450) + (3/4)(500) + 550 + (3/4)(600) = 1700.
For the 12 month autos, earned exposures at the $705 rate: 2200 - 1700 = 500.
Earned premium is: (425)($750) + (575)($825) + (1700)($750) + (500)($705) = **$2,420,625**.
c. (1000)($825) + (2200)($705) = **$2,376,000**.
d. We have to apply the parallelogram method separately to the two underwriting companies.
For Bambi Insurance, which writes the 6-month policies:



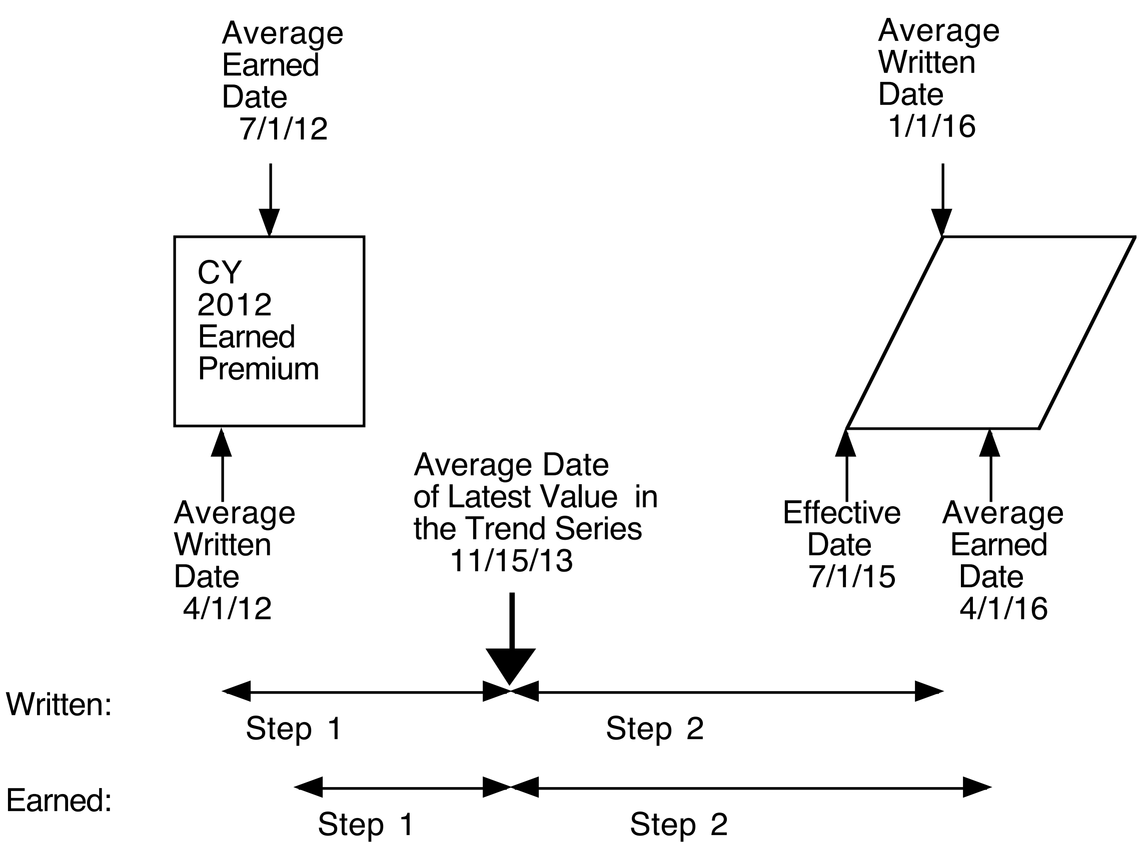
Area A = Area B = 1/2.
Average rate during 2015 is: (1/2)(1) + (1/2)(1.1) = 1.05.
On-level factor is: 1.1/1.05 = 1.0476.
Earned premium for six month autos is: (425)($750) + (575)($825) = $793,125.
On level earned premium from 6 month autos is: (1.0476)($793,125) = $830,878.

For Stag Insurance, which writes the 12-month policies:



Area B = $(1/2)(1/2)^2$ = 1/8. Area A = 1 - 1/8 = 7/8.
Average rate during 2015 is: (7/8)(1) + (1/8)(0.94) = 0.9925.
On-level factor is: 0.94/0.9925 = 0.9471.
Earned premium for 12 month autos is: (1700)($750) + (500)($705) = $1,627,500.
On level earned premium from 12 month autos is: (0.9471)($1,627,500) = $1,541,405.
Total on level earned premium is: $830,878 + $1,541,405 = **$2,372,283**.
Comment: The answers in parts c and d differ, since the premium writings are not at a constant rate throughout the year.

5.39. a. The average date of earning CY12 Premiums is 7/1/12, with an average date of writing 3 months earlier (6 month policies) or 4/1/12. The last average written premium in the series is for the quarter ending 12/31/13; this has average date of writing of 11/15/13.
Thus the first step goes from 4/1/12 to 11/15/13. The average date of writing for the effective period is six months past 7/1/15 or 1/1/16. Thus the second step goes from 11/15/13 to 1/1/16.
b. The average date of earning CY12 Premiums is 7/1/12. The last average earned premium in the series is for the quarter 12/31/13, this has average date of earning of 11/15/13. Thus the first step goes from 7/1/12 to 11/15/13. The average date of writing for the effective period is six months past 7/1/15 or 1/1/16. The average date of earning is 3 months later (6 month policies) or 4/1/16. Thus the second step goes from 11/15/13 to 4/1/16.
c. The two-step method would be more appropriate when there have been significantly different premium trends over the different periods of time involved. The advantage of
two-step trending is that it recognizes that there are situations where a single annual premium trend may not be appropriate for each year in the experience period.
Comment: Similar to 5, 5/04, Q.35. Here is a diagram, with 6-month policies as in this question:



See Figure 5.28 in Basic Ratemaking.

5.40. The average premium at current rates for 2015:
{(1500)(500) + (1200)(700) + (600)(800) + (900)(1000)} / (1500 + 1200 + 600 + 900) = 707.14.
The average premium at current rates for 2015:
{(1700)(500) + (1400)(700) + (600)(800) + (900)(1000)} / (1700 + 1400 + 600 + 900) = 697.83.
{(1900)(500) + (1600)(700) + (600)(800) + (900)(1000)} / (1900 + 1600 + 600 + 900) = 690.
697.83/707.14 - 1 = -1.317%. 690/697.83 - 1 = -1.122%.
The average of the two changes in average premiums is:
-(1.317% + 1.122%)/2 = -1.22%.
Assuming this historical pattern will continue into the future, I select an annual premium trend of **-1.2%**.
Comment: See 5, 11/17, Q.1b.
In this case, the mix of business written is changing, with a smaller percentage of exposures from the higher rated class and territory over time.

5.41. a. 2 million exposures are written at the lower rate level, while 2 million exposures are written at the higher rate level.
On level factor is: 1.1 / {(2/4)(1) + (2/4)(1.1)} = 1.0476
PY18 earned premium at current rate level: (1.0476)(500 million) = **$523.8 million**.
b. On level factor is: 1.1 / {(1.6/4)(1) + (2.4/4)(1.1)} = 1.0377
PY18 earned premium at current rate level: (1.0377)(500 million) = **$518.9 million**.
c. On level factor is: 1.1 / {(2.4/4)(1) + (1.6/4)(1.1)} = 1.0580
PY18 earned premium at current rate level: (1.0577)(500 million) = **$528.8 million**.
Comment: Similar to 5, 5/17, Q.2c.
If we assumed level writings throughout the year as in the parallelogram method, we would get the wrong answers in parts b and c.
In part b, more of the exposures are written at the higher rate, thus the on level factor and the on level premiums are lower than in part (a).
In part c, less of the exposures are written at the higher rate, thus the on level factor and the on level premiums are higher than in part (a).

5.42.
- For a line of insurance where the exposure base is sales or payroll, for example commercial general liability insurance, the final premium would depend on the final audited values.
- Retro policies would continue to have adjustments for many years after the end of the policy year.
- The insurer writes some 2-year or 3-year policies, which would not have expired by the end of 2019.

Comment: Similar to 5, 5/17, Q.2d.

5.43. As determined below, the average of the three changes in average premiums is:
-(0.69% + 0.63% + 0.82%)/3 = -0.71%.

Calendar	Exposures (million)				Premium (million)					Percent
Year	8833	8835	9040	Total	8833	8835	9040	Total	Average	Change
2014	3	4	0.3	7.3	3.39	7.76	0.864	12.014	1.646	
2015	3.3	4.1	0.33	7.73	3.729	7.954	0.9504	12.6334	1.634	-0.69%
2016	3.6	4.2	0.36	8.16	4.068	8.148	1.0368	13.2528	1.624	-0.63%
2017	4	4.3	0.4	8.7	4.52	8.342	1.152	14.014	1.611	-0.82%

Assuming this historical pattern will continue into the future,
I select an annual premium trend of **-0.7%**.
Comment: See to 5, 11/17, Q.1b.
In this case, the mix of business written is changing over time, with a smaller percentage of exposures from the Nursing Home class 8835.
If for example the average weekly wage is projected to increase at 3.0% per year, then the net effect of the selected premium trend and this exposure trend would be: (1.030)(0.993) = 1.023.

5.44. CY17 Earned Premium:
Written + Beginning Unearned Premium Reserve - Ending Unearned Premium Reserve =
800 + 370 - 390 = **780**.
Comment: The unearned premium reserve increased, which decreases the CY earned premium.

5.45. (a) Only the six-month policies written in 2018 contribute:
30 + 24 + 26 + 28 = **$108 million**.
(b) The six-month policies written October 1, 2017 contribute half their premium.
The six-month policies written January 1, 2018 contribute all of their premium.
The six-month policies written April 1, 2018 contribute all of their premium.
The six-month policies written July 1, 2018 contribute all of their premium.
The six-month policies written October 1, 2018 contribute half their premium.
22/2 + 30 + 24 + 26 + 28/2 = **$105 million**.
(c) Only the six-month policies written in July 2017, October 2017, January 2018, and April 2018 contribute: 21 + 22 + 30 + 24 = **$97 million**.
(d) The six-month policies written April 1, 2017 contribute half their premium.
The six-month policies written July 1, 2017 contribute all of their premium.
The six-month policies written October 1, 2017 contribute all of their premium.
The six-month policies written January 1, 2018 contribute all of their premium.
The six-month policies written April 1, 2018 contribute half their premium.
20/2 + 21 + 22 + 30 + 24/2 = **$95 million**.

5.46. I took the ratios of successive average written premiums:

Quarter and Year	Average Written Premium at Current Rate Level	Ratio
2Q 2017	\$1,546	
4Q 2017	\$1,569	1.0149
2Q 2018	\$1,591	1.0140
4Q 2018	\$1,618	1.0170
2Q 2019	\$1,640	1.0136
4Q 2019	\$1,667	1.0165

I select a trend of 1.5% per six months.

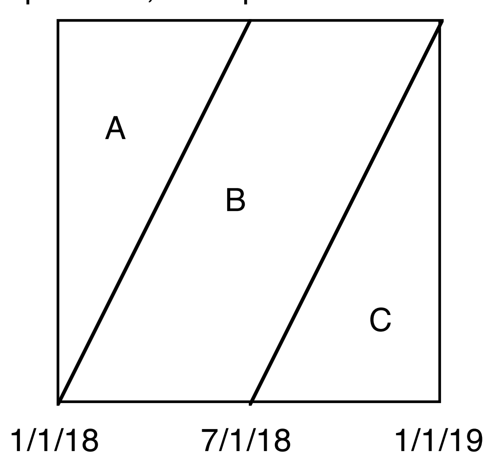
(a) Since policies are semi-annual, the average date of writing of CY2018 Earned Premium is July 1, 2018 minus 3 months = April 1, 2018.

Since new rates will be in effect for one year, the average date of writing under the new rates is August 1, 2020 plus 6 months = February 1, 2021.

Thus the trend period for CY18 is 2 years and 10 months. (This is $5\frac{2}{3}$ half-years.)

	Rate Level Index
Prior	1.00
1/1/18	1.06
7/1/18	(1.11)(1.06) = 1.1766

Draw a diagram for CY18 earned premium; since policies are semi-annual, the lines have slope 2.



Area A = (1/2)(1/2)(1) = 1/4. Area C = (1/2)(1/2)(1) = 1/4. Area B = 1 - 1/4 - 1/4 = 1/2.

Average rate level for CY18: (1/4)(1) + (1/2)(1.06) + (1/4)(1.1766) = 1.07415.

OLF = 1.1766/1.07415 = 1.0954.

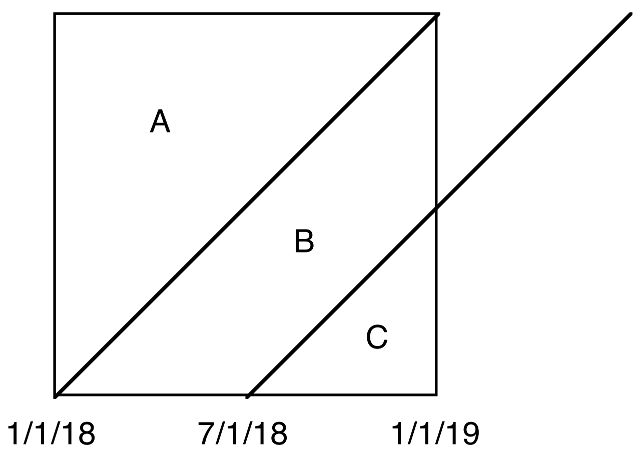
Trended on-level earned premium for CY18:

$(1.0954)(1.015^{5.667})$(\$462 million) = \$**550.6 million**.

(b) Since policies are annual, the average date of writing of CY2018 Earned Premium is July 1, 2018 minus 6 months = January 1, 2018.
Since new rates will be in effect for one year, the average date of writing under the new rates is August 1, 2020 plus 6 months = February 1, 2021.

Thus the trend period for CY18 is 3 years and 1 month. (This is $6\frac{1}{6}$ half-years.)

(The trend period is three months longer than previously.)
The diagram for CY18 has lines with slope of 1 rather than 2:



Area A = (1/2)(1)(1) = 1/2. Area C = (1/2)(1/2)(1/2) = 1/8. Area B = 1 - 1/2 - 1/8 = 3/8.
Average rate level for CY18: (1/2)(1) + (3/8)(1.06) + (1/8)(1.1766) = 1.04458.
OLF = 1.1766/1.04458 = 1.1264.
(The OLF is larger than previously; since we have rate increases, in this case more premium was earned at a lower rate level.)
Trended on-level earned premium for CY18:
$(1.1264)(1.015^{6.167})$($462 million) = $**570.4 million**.

<u>Comment</u>: Similar to 5, 5/19, Q.2.

5.47.

Time Period	Written Expos.	% Earned CY90	Earned Expos.	Rate Level
01/1/89 - 03/31/89	200	1/8	25	1.0
04/1/89 - 06/30/89	400	3/8	150	1.0
07/1/89 - 09/30/89	600	5/8	375	1.1
10/1/89 - 12/31/89	800	7/8	700	1.1
01/1/90 - 03/31/90	1000	7/8	875	1.1
04/1/90 - 06/30/90	1200	5/8	750	1.1
07/1/90 - 09/30/90	1400	3/8	525	1.1
10/1/90 - 12/31/90	1600	1/8	200	1.1

Average rate level for CY 1990 Earned premiums is:

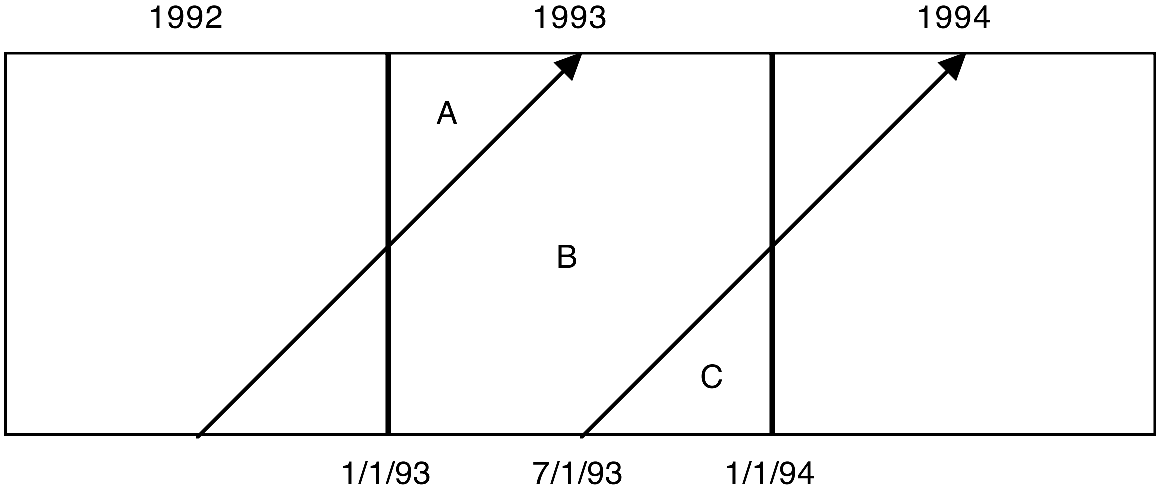
$$\frac{(25 + 150)(1) + (375 + 700 + 875 + 750 + 525 + 200)(1.1)}{25 + 150 + 375 + 700 + 875 + 750 + 525 + 200} = 3942.5 / 3600 = \mathbf{1.0951}.$$

Comment: The on-level factor is: 1.1 / 1.0951 = 1.0044.

5.48.

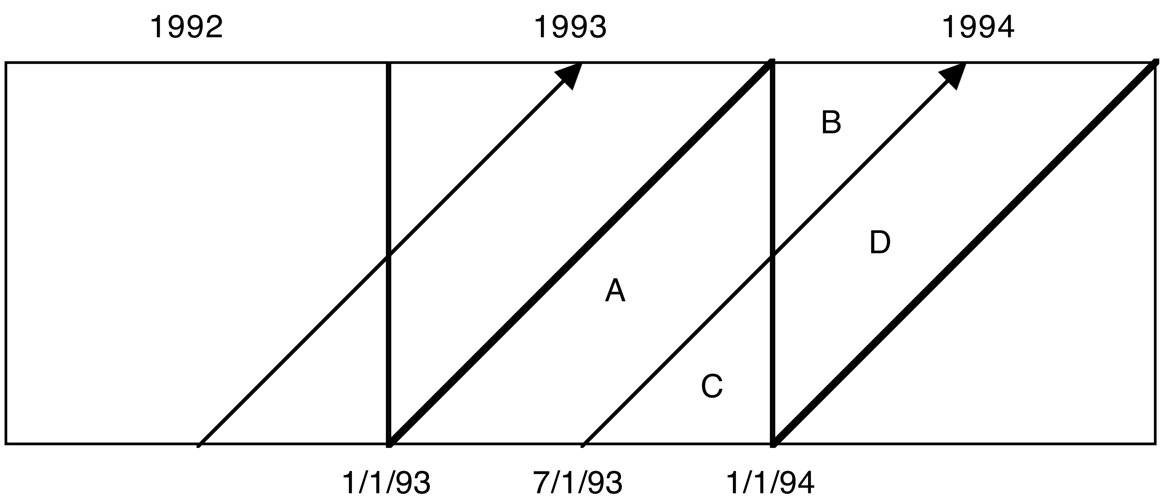
Effective Date	Policy Term	Premium	Portion Earned During Year	Unearned Premium
May 1	6 months	\$6,000	100%	0
August 1	12 months	\$12,000	5/12	\$7000
November 1	6 months	\$2,400	1/3	\$1600
				\$8600

5.49. (a) CY1993 is represented by a square:



	Area	Rate Level Index
A	1/8	1.02
B	6/8	(1.02)(1.10)
C	1/8	(1.02)(1.10)(1.15)

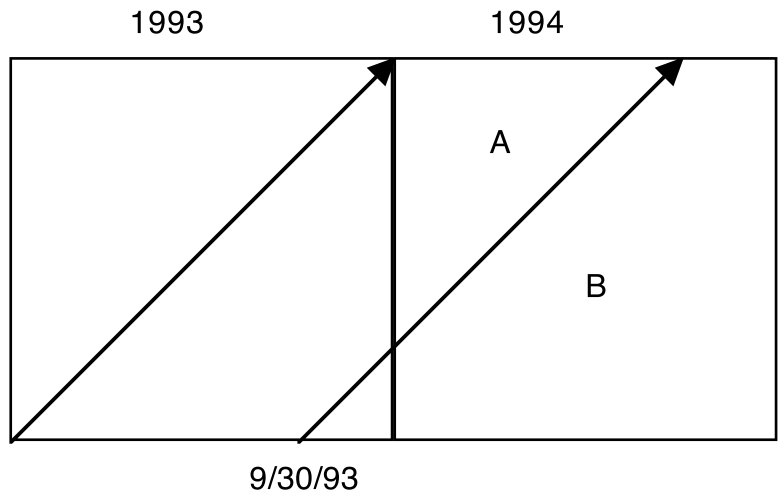
Average rate level for CY1993 is 1.1303.
On level factor is: (1.02)(1.10)(1.15)(1.03) / 1.1303 = **1.176**.
(b) The 7/1/92 experience change and 1/1/93 law change do not affect PY93, and for simplicity I will ignore them. (If you included them, the factors would cancel out in the numerator and denominator.)
PY1993 is represented by a parallelogram:



	Area	Rate Level Index
A	3/8	1.000
B	1/8	1.030
C	1/8	1.150
D	3/8	(1.150)(1.030)

Average rate level for PY1993 is 1.0917. On level factor is: (1.15)(1.03) / 1.0917 = **1.085**.

5.50. a. Using the traditional parallelogram method:



Area A = (1/2)(3/4)(3/4) = 9/32. Area B = 1 - 9/32 = 23/32.
The portion written at the 10% rate level is Area B: **23/32** = **71.88%**.
Taking into account changing exposure levels, we can use a discrete approximation.
The middle of the first quarter is x = 1/8, so only 1/8 of exposures in this quarter are earned in 1994.
Total 1994 earned exposures:
(1/8)(250) + (3/8)(750) + (5/8)(1250) + (7/8)(1750)
+ (7/8)(2250) + (5/8)(2750) + (3/8)(3250) + (1/8)(3750) = 8000.
1994 earned exposures written at the lower rate: (1/8)(250) + (3/8)(750) + (5/8)(1250) = 1093.75.
The portion written at the 10% rate level is: 1 - 1093.75/8000 = **86.33%**.
Alternately, one can integrate. x = 0 at 1/1/93 and x = 2 at 12/31/94.

The portion earned during 1994 is: $\begin{cases} x, \text{ for } 0 \le x \le 1 \\ 2 - x, \text{ for } 1 < x \le 2 \end{cases}$.

For example, for x = 3/4, 1/4 is earned during 1993 while 3/4 is earned during 1994.
For x = 1.3, 70% is earned during 1994 while 30% is earned during 1995.

Earned exposures for 1994: $\int_0^1 8000x \, x \, dx + \int_1^2 8000x\,(2 - x)\, dx = (8000)(1/3 + 2/3) = 8000.$

1994 earned exposures written at the lower rate: $\int_0^{3/4} 8000x \, x \, dx = (8000)(3/4)^3/3 = 1125.$

The portion written at the 10% rate level is: 1 - 1125/8000 = **85.94%**.
b. The parallelogram method will overstate the percentage of exposures written at the older lower rate. Thus the resulting on-level factor will be too big. Therefore, the on-level premium will be too big and the historical loss ratio will be too small. Thus, the traditional parallelogram method would <u>understate</u> the rate level indication.

5.51. (a) Since policies are six-month, none of the exposures from 7/1/96 are earned in CY97. Half of the exposures from a policy with an effective date of 10/1/96 are earned in CY97, which has a term from 10/1/96 to 3/31/97.

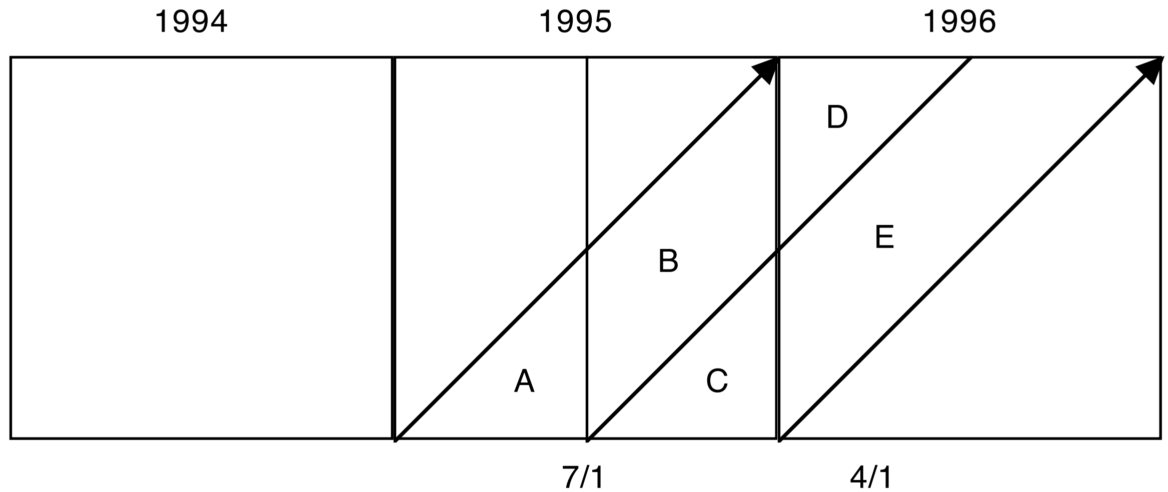
Effective Date	Cars	Term in Years	Written Exposures	Portion Earned in CY97	Earned Exposures
1/1/96	10	0.5	5	0	0
4/1/96	20	0.5	10	0	0
7/1/96	40	0.5	20	0	0
10/1/96	60	0.5	30	0.5	15
1/1/97	100	0.5	50	1	50
4/1/97	120	0.5	60	1	60
7/1/97	150	0.5	75	1	75
10/1/97	200	0.5	100	0.5	50
Sum					**250**

Earned exposures for calendar year 1997: **250 car years**.
(b) Since policies are six-month, those effective 7/1/96 expire on 12/31/96 and are not in-force on 1/1/97. Those effective 10/1/96 and 1/1/97 are in force on 1/1/97.
In-force exposures on 1/1/97: 60 + 100 = **160 cars**.
(c) Since the average rate is the same for all effective dates, the CY96 earned premium is:
(500)(10 + 20 + 40 + 60/2) = ($500 per car)(100 cars) = $**50,000**.
A more general technique that would also work when the average rates vary:

Effective Date	Cars	Average Rate per Car	Written Premium	Portion Earned in CY96	Earned Premium
1/1/96	10	$500	$5,000	1	$5,000
4/1/96	20	$500	$10,000	1	$10,000
7/1/96	40	$500	$20,000	1	$20,000
10/1/96	60	$500	$30,000	0.5	$15,000
1/1/97	100	$500	$50,000	0	$0
4/1/97	120	$500	$60,000	0	$0
7/1/97	150	$500	$75,000	0	$0
10/1/97	200	$500	$100,000	0	$0
Sum					**$50,000**

Comment: In part c we assume no policies written effective 10/1/95. If any such policies existed, half of their exposures and premiums would be earned in Calendar Year 1996.

5.52. (a) The 7/1/95 law amendment is represented by a vertical line.
The 7/1/95 experience change is represented by a 45 degree line.
Policy year 1995 is represented by a parallelogram.



Area	Size	Rate Level Index
A	(1/2)(1/2)(1/2) = 1/8	1.000
B	(1/2)(1/2) = 1/4	1.150
C	(1/2)(1/2)(1/2) = 1/8	(1.150)(0.900) = 1.035
D	(1/2)(1/2)(1/2) = 1/8	1.150
E	1/2 - 1/8 = 3/8.	(1.150)(0.900) = 1.035

Weighted average rate level index for PY95 is:
(1/8)(1) + (1/4)(1.150) + (1/8)(1.035) + (1/8)(1.150) + (3/8)(1.035) = 1.07375.
Current rate level index is: (1.150)(0.900)(1.050) = 1.08675.
Premium adjustment factor to current rate level is: 1.08675 / 1.07375 = **1.0121**.
(b) The purpose of experience rate changes is to adjust rates for changes in projected experience levels from the expected loss level. The purpose of law amendment changes is to adjust rates for statutory modifications to benefits.
Experience rate changes affect premiums only at the policy effective date. Law amendment changes usually affect premiums of all in-force policies at the date of the law amendment rate change.
Here the benefits were increased 7/1/95 and the rates were increased by 15%.
The remaining coverage provided after 7/1/95 for a policy written for example 4/1/95 will be charged at the 15% higher rate.

5.53. Since exposures are declining, using the traditional parallelogram method will overestimate the percentage of exposures written at the later higher rate level.
Thus the average historical rate level will be overestimated.
Thus **the premium adjustment factor will be understated**.
Thus the on-level premiums will be understated.
Thus **the adjusted loss ratio will be overstated**.
Thus **the rate level indication will be overstated**.

5.54. Assume for example PY97, and initial written premium per month prior to audits of x.
At December 31, 1998 policies issued between January 1 and June 30 of 1997 have completed their final audits. Policies issued between July 1 and December 31 of 1997 have not completed their final audit. The reported 1997 policy year premium at 24 months, as of December 31, 1998 is:
$(6x)(1.07) + 6x = 12.42x$.
At 36 months, as of December 31, 1999, all policies have completed their final audits.
The reported 1997 policy year premium at 36 months, December 31, 1999, is:
$(12x)(1.07) = 12.84x$.
The policy year premium development factor for 24 to 36 months is: $(12.84x) / (12.42x) =$ **1.034**.

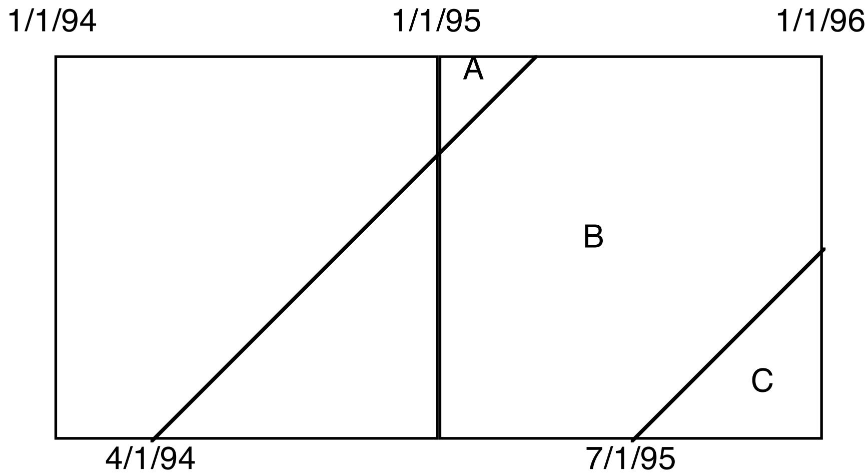
5.55.

Effective Date	Rate Level Change Factor	Rate Level (1/1/94 taken as = 1)
1/1/94		1.000
4/1/94	1.050	1.050
7/1/95	1.130	1.1865
4/1/96	0.970	1.1509

Area A = $(1/4)^2/2$ = 1/32 = 0.03125.

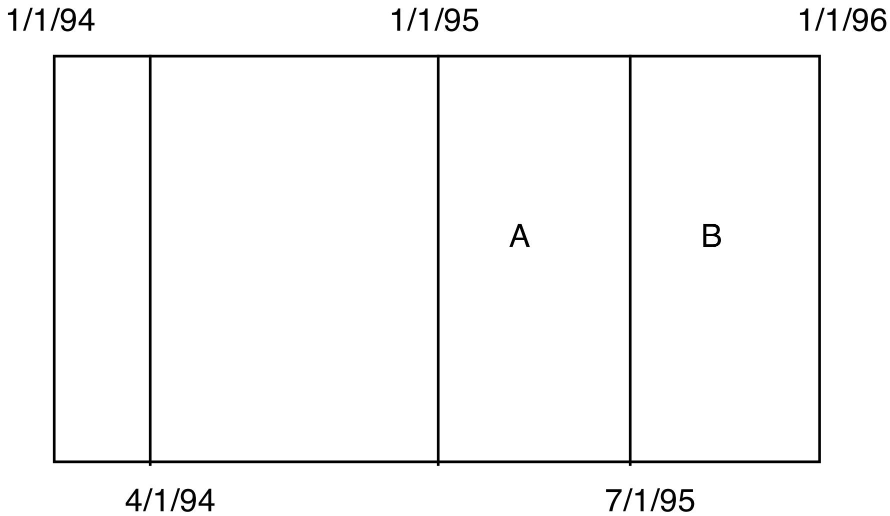
Area C = $(1/2)^2/2$ = 1/8 = 0.125.

Area B = 1 - (Area A + Area C) = 0.84375.



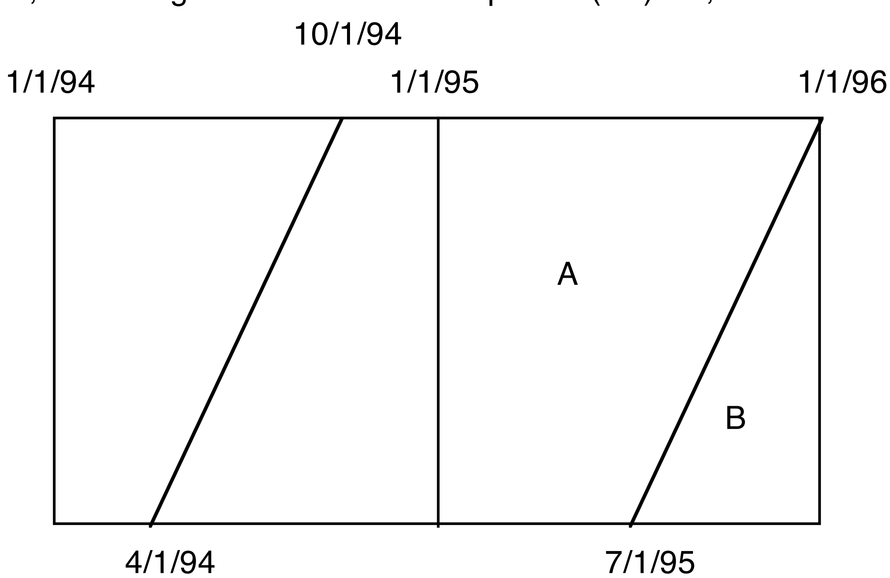
Average rate for calendar year 1995 earned premium =
(0.03125)(1.000) + (0.84375)(1.050) + (0.125)(1.1865) = 1.0655.
On level Factor = Current Rate Level / Average Rate Level CY 95 Earned Premium =
1.1509/1.0655 = **1.080**.
Comment: Basic calculation you must know how to do!

5.56. The rate level index computation is the same.
However, the diagram for written premium uses vertical lines rather than sloped lines.



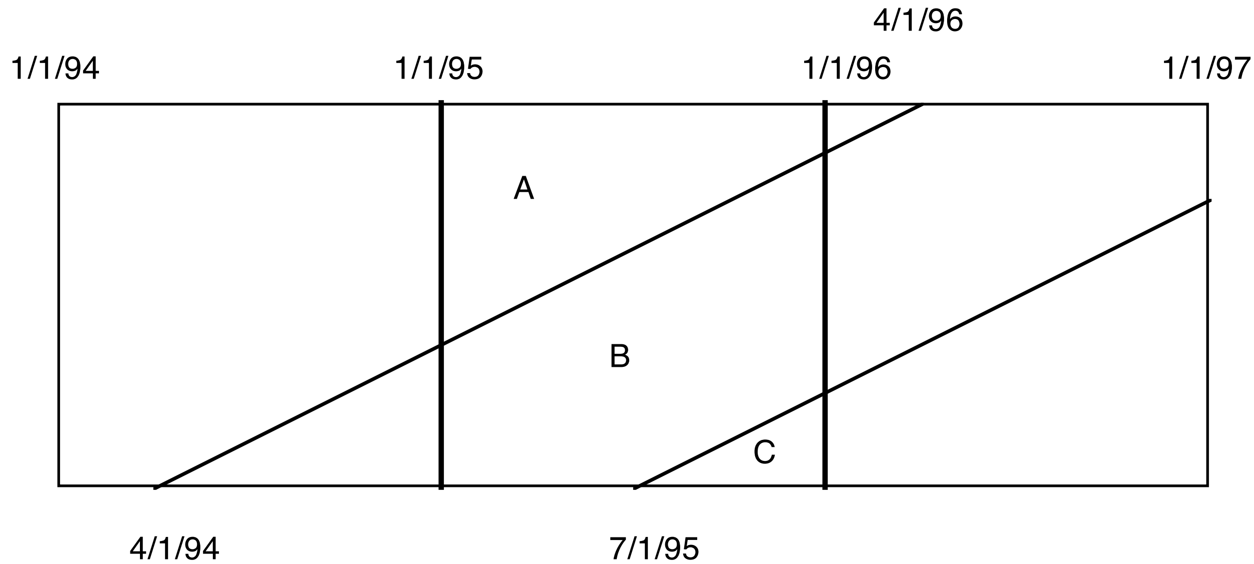
Area A = Area B = 1/2.
Average rate for calendar year 1995 written prem. = (0.5)(1.050) + (0.5)(1.1865) = 1.11825.
On level Factor = Current Rate Level / Average Rate Level CY 95 Written Premium =
1.1509/1.11825 = **1.029**.
Comment: Many of us would not have to draw a diagram. One makes no use of the policy length.

5.57. The rate level index computation is the same.
However, in the diagram the lines have slope of 1/(1/2) = 2, rather than 1.



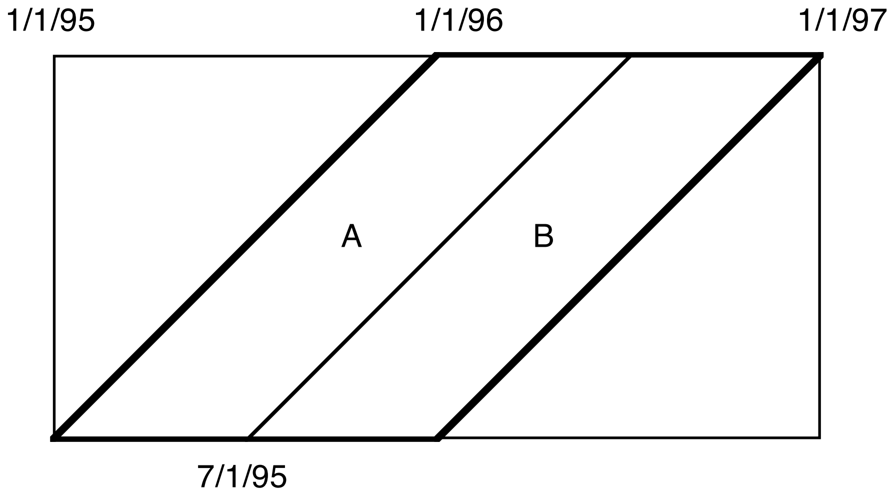
Area B = (1/2)(1/2)(1) = 1/4. Area A = 3/4.
Average rate for calendar year 1995 earned prem. = (3/4)(1.050) + (1/4)(1.1865) = 1.0841.
On level Factor = 1.1509/1.0841 = **1.062**.
Comment: The on level factor is less than when there were annual policies, since here more of the premium is earned under the new higher rates. See 6, 5/98, Q. 41.

5.58. The rate level index computation is the same.
However, in the diagram the lines have slope 1/2.



Area C = (1/2)(1/2)(1/4) = 1/16 = 0.0625.
Area A = difference of two triangles = (1/2)(5/4)(5/8) - (1/2)(1/4)(1/8) = 0.375.
Area B = 1 - 0.0625 - 0.375 = 0.5625.
Average rate for calendar year 1995 earned premium:
(0.375)(1) + (0.5625)(1.050) + (0.0625)(1.1865) = 1.0398.
On level Factor = 1.1509/1.0398 = **1.107**.
Comment: See 5, 5/01, Q. 38.

5.59. The rate level index computation is the same.
However, Policy Year 1995 (policies written during 1995) is represented by a parallelogram:



Area A = Area B = 1/2.
Average rate for Policy Year 1995 premium = (.5)(1.050) + (.5)(1.1865) = 1.11825.
On level Factor = 1.1509/1.11825 = **1.029**.
Comment: Policy Year and Calendar Year data are inherently different things. The appropriate technique to apply may differ, as it does here. Many of us would not have to draw a diagram.
For Policy Years as opposed to Calendar Years, at ultimate written and earned premiums are the same. Therefore, ignoring the issue of possible premium development, it makes no difference in this question, which deals with Policy Years, whether the premium is written or earned.

5.60. 1. Changes in mix of business.
2. Effect of new construction or property improvements.
3. Changes in selected coverage A limits above or below the level of inflation in loss exposure.
Comment: These would affect the observed and future premium trends.

5.61. a. Since we have six-month policies, for $x < 0.5$ nothing is contributed to CY97 earned.
For $0.5 \le x \le 1$, the portion contributed to CY97 earned is: $2x - 1$.
For $1 \le x \le 1.5$, the whole policy is earned in CY97.
For $1.5 \le x \le 2$, the portion contributed to CY97 earned is: $4 - 2x$.
Thus the CY97 earned exposures are:

$$2500 \left\{ \int_{0.5}^{1} (2x - 1)\, x\, dx + \int_{1}^{1.5} x\, dx + \int_{1.5}^{2} (4 - 2x)\, x\, dx \right\} = (2500)\ (5/24 + 5/8 + 5/12) = 3125.$$

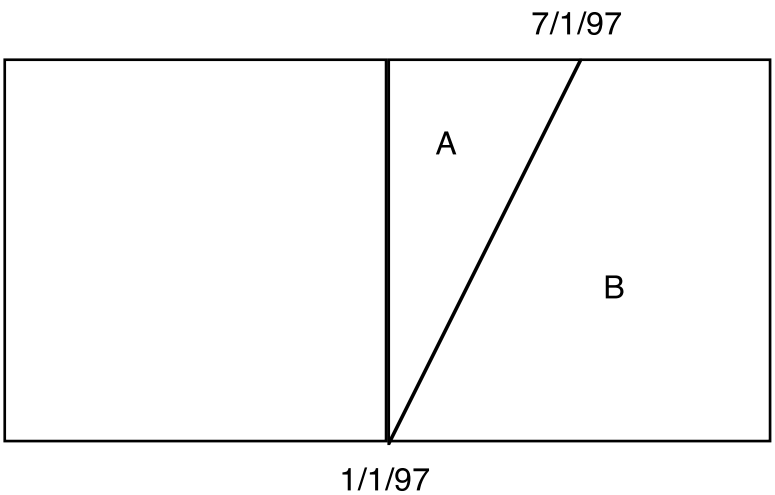
The CY97 earned exposures written at the lower rate level are:

$$2500 \int_{0.5}^{1} (2x - 1)\, x\, dx = (2500)(5/24) = 520.83.$$

Defining the 1/1/96 rate level as one, the average rate level for CY97 earned premiums is:

$$\frac{(520.83)(1) + (3125 - 520.83)(0.85)}{3125} = \mathbf{0.8750}.$$

b. Area A = (1/2)(1/2)(1) = 1/4.

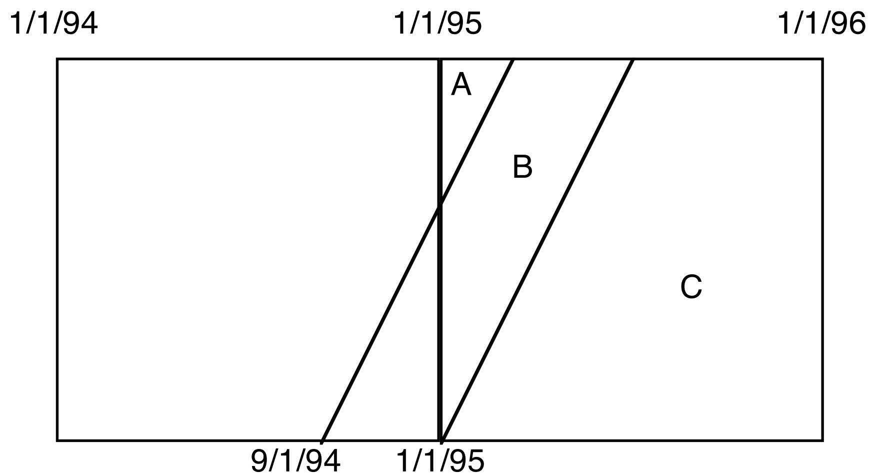


Using the traditional parallelogram method the average rate level for CY97 earned premiums is:
(1/4)(1) + (3/4)(0.85) = **0.8875**.
c. The traditional method assumes a constant rate of writing, and thus that 1/4 rather than 1/6 of the earned exposures were written at the earlier higher rate level. Thus, the traditional method overestimates the average rate level. In general, when rates are decreasing when exposures are steadily increasing, the traditional method will overstate the average rate level.
Here 0.8875 > 0.8750.
Thus using the traditional method would result in too small of an OLF: 0.85/0.8875 < 0.85/0.8750.
Thus the traditional method would result in too little on level premium, too big of a loss ratio, and too big of a rate indication.
<u>Comment</u>: In general, when rates are <u>increasing</u> when exposures are steadily increasing, the traditional method will understate the average rate level.

5.62.

Effective Date	Rate Level Change Factor	Rate Level (1/1/94 taken as = 1)
1/1/94		1.000
9/1/94	1.050	1.050
1/1/95	1.100	1.155
1/1/96	.950	1.097
4/1/97	1.150	1.262

These are semiannual policies; therefore, on the earning of premium diagram the lines have slope of 2, rather than 1. (Policies written from 7/1/94 to 12/31/95 contribute to CY 95.)



Area A has base 1/6 and height 1/3, and area: (1/2)(1/6)(1/3) = 1/36.

Area A + Area B = a triangle with base 1/2, height 1, and area 1/4. $\Rightarrow$

Area B = 1/4 - 1/36 = 2/9.

Area C = 1 - (Area A + Area B) = 3/4.

Average rate level factor for CY 95 = (1/36)(1.000) + (2/9)(1.050) + (3/4)(1.155) = 1.127.

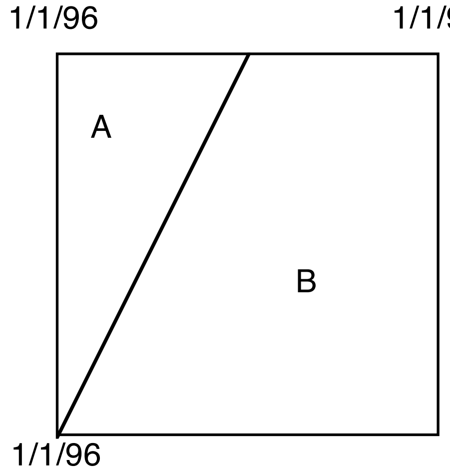
On level factor for CY 95 = 1.262/1.127 = 1.120.

CY 95 premium on current rate level = (1.120)(1200) = **1344**.

5.63.

Effective Date	Rate Level Change Factor	Rate Level (1/1/94 taken as = 1)
1/1/94		1.000
9/1/94	1.050	1.050
1/1/95	1.100	1.155
1/1/96	.950	1.097
4/1/97	1.150	1.262

These are semiannual policies; therefore, on the earning of premium diagram the lines have slope of 2, rather than 1. (Policies written from 7/1/95 to 12/31/96 contribute to CY 96.)



Area A has base 1/2 and height 1, and area: (1/2)(1)(1/2) = 1/4.
Area B = 1 - 1/4 = 3/4.
Average rate level factor for CY 96 = (1/4)(1.155) + (3/4)(1.097) = 1.112.
On level factor for CY 96 = 1.262/1.112 = 1.135.
CY 96 premium on current rate level = (1.135)(1400) = **1589**.

5.64. Increasing exposures, decreasing exposures, and irregular exposure patterns.
Also lines of insurance that exhibit significant seasonal variation.
Comment: Exposures would increase if the insurer is writing a new line of business or is penetrating a new market. Exposures would decline in some lines of business such as workers compensation insurance during a recession.

5.65. For concreteness, let us assume we are looking at PY2000.
At 24 months after the start of the policy year, 12/31/2001, policies written in the first half of 2000 have had their audits. At 36 months after the start of the policy year, 12/31/2002, all policies written in 2000 have had their audits.
As of 36 months the premium is: (1.2) ($6 million + $6.6 million) = $15.12 million.
As of 24 months the premium is: (1.2) ($6 million) + $6.6 million = $13.8 million.
Premium development is: 15.12 / 13.8 = **1.096**.

5.66. 0.75/0.75 = 1. Someone with the average wage gets the minimum benefit.
The contribution from those who get the minimum is: (15% + 20% + 25%)(0.75) = 0.45.
1.50 / 0.75 = 2. Someone with twice the average wage gets the maximum benefit.
The contribution from those who get the maximum is: (5%)(1.5) = 0.075.
The contribution from those who get between the minimum and the maximum is:
(0.75)(24% + 26%) = 0.375.
Thus, the average benefit is: 0.45 + 0.075 + 0.375 = **0.90**.
Comment: The average benefit as a percentage of the average wage =
(% at Min) (Min benefit) + (% at Max) (Max benefit)
+ (% wages not impacted by Min or Max) (compensation rate).

5.67. a. Let x = 0 correspond to 1/1/97. Then the rate of writing is 5000x.

The percent earned during CY1998 is: $\begin{cases} x, \text{ for } 0 \le x \le 1 \\ 2 - x, \text{ for } 1 \le x \le 2 \end{cases}$.

Thus the earned exposures during CY98 are:

$$\int_0^1 x \; 5000 \; x \; dx + \int_1^2 (2 - x) \; 5000 \; x \; dx = (5000)(1/3 + 3 - 7/3) = \mathbf{5000}.$$

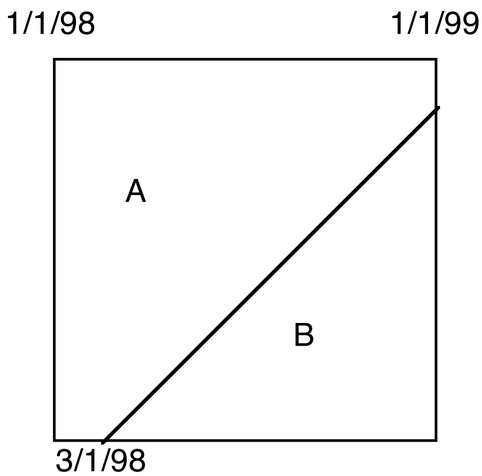
b. The earned exposures during CY98 from writings after 1/1/98 are:

$$\int_1^2 (2 - x) \; 5000 \; x \; dx = (5000)(3 - 7/3) = \mathbf{3333}.$$

c. Letting the rate level at 1/1/97 be one, the average rat level for CY98 is:
{(1) (1667) + (1.2) (3333)} / 5000 = **1.133**.
d. The appropriate premium adjustment factor for Calendar Year 1998 is: 1.2/1.133 = **1.059**.
e. Using the traditional parallelogram method would result in a lower estimated rate level for CY1998, a higher on level factor, too high premiums on level, too low of a loss ratio, and thus **too low of a rate indication**.

5.68. Assume that the rate increase (you thought was on 1/1 but was actually on 3/1) was of size x.
Then the current rate level is 1 + x, while that before the increase was 1.
Assuming the increase was effective on 1/1/98, then the average rate level for CY 98 earned premiums would be: (1/2)(1) + (1/2)(1 + x) = 1 + x/2.
Then the on-level factor for CY 98 earned premiums is: (1 + x)/(1 + x/2).

$\Rightarrow$ 1.080 = (1 + x)/(1 + x/2) $\Rightarrow$ 1.08 + 0.54x = 1 + x. $\Rightarrow$ x = 0.08/0.46 = 0.174.

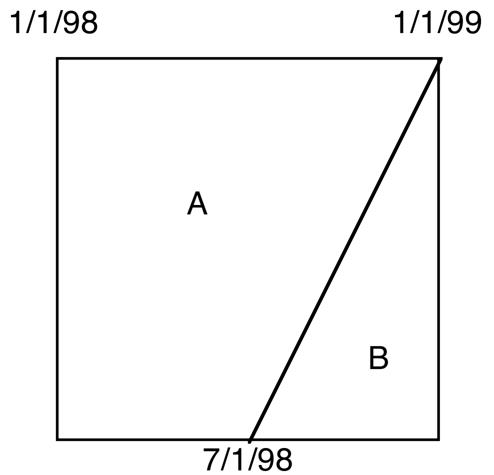


Area B = $(1/2)(5/6)^2$ = 25/72. Area A = 1 - 25/72 = 47/72.

Therefore, given that the increase was effective 3/1/98, the average rate level for CY 98 earned premiums is: (47/72)(1) + (25/72)(1.174) = 1.060.
The on-level factor for CY 98 earned premiums is: 1.174/1.060 = **1.108**.

5.69. (a) Number of in-force exposures on January 1, 1998 is: 300 + 500 + 700 + 900 = **2400**.
(b) Earned exposures for calendar year 1998 are:
(1/4)(300) + (1/2)(500) + (3/4)(700) + (1)(900) + (3/4)(1100) + (1/2)(1300) + (1/4)(1500) = **3600**.
(c) **Extension of Exposures Technique** (using premiums at present rates) and
the **Parallelogram Method** (multiplying by on-level factors).
(d) The **Extension of Exposures Technique is more appropriate**, because there are different numbers of exposures written over the course of the year. The number of exposures is increasing over time rather than constant.
(e) The present rate is: (500)(1.15) = 575. We have 3600 earned exposures, which at the current rate would be an earned premium of: (575)(3600) = **2,070,000**.
<u>Comments</u>: Only annual policies written between January 2, 1997 and January 1, 1998, inclusive, are in force on January 1, 1998.
For an annual policy written April 1, 1997, 3/4 of it is earned in CY 1997 and 1/4 in CY 1998.
The earned premium during CY 98 is:
(500){(1/4)(300) + (1/2)(500) + (3/4)(700) + (1)(900) + (3/4)(1100)} +
(575){(1/2)(1300) + (1/4)(1500)} = (500)(2575) + (575)(1025) = 1,876,875.
The on-level factor for CY 98 is: 1.15/{(7/8)(1) + (1/8)(1.15)} = 1.129.
Thus using the parallelogram method, would give an estimated CY 98 on-level earned premium of: (1.129)(1,876,875) = 2,118,992. This is too large; it is larger than the more accurate 2,070,000 from extension of exposures technique, since in fact more than 1/8 of the exposures earned in CY 98 were written at the current higher rate.

5.70. (a) Number of in-force exposures on January 1, 1998 are: 700 + 900 = **1600 cars**.
(b) Earned exposures (car years) for calendar year 1998 are:
(1/2)(700/2) + (1)(900/2) + (1)(1100/2) + (1)(1300/2) + (1/2)(1500/2) = **2200 car years**.
(c) The present rate per car year is: ($600)(1.15) = $690. We have 2200 car years, which at the current rate would be an earned premium of: ($690)(2200) = $**1,518,000**, using the extension of exposures technique. The earned premium for calendar year 1998 is:
($300){(1/2)(700) + (1)(900) + (1)(1100)} + ($345){(1)(1300) + (1/2)(1500)} = $1,412,250.



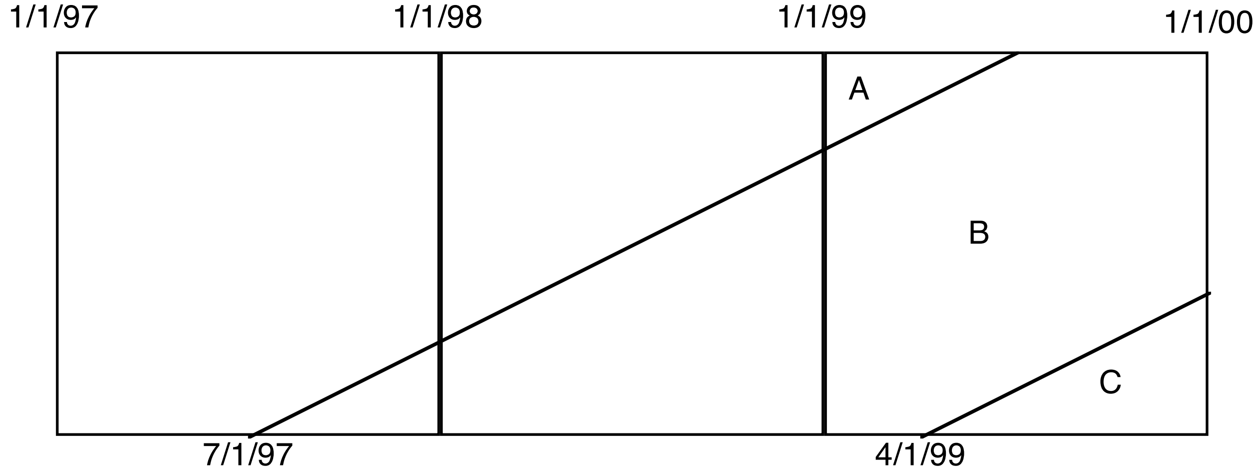
Area B = (1/2)(1/2)(1) = 1/4. Area A = 1 - Area B = 3/4.
The on-level factor for CY 98 is: 1.15 / {(3/4)(1) + (1/4)(1.15)} = 1.108.
The on-level earned premium for CY 98 is: (1.108)(1412250) = **1,564,773**, using the parallelogram method.
<u>Comment</u>: The extension of exposures technique is more accurate since the level of exposures written is not constant.
After the uniform rate increase of 15% effective July 1, 1998, the cost is $345 per car for a six month policy. This new rate applies to policies with effective dates on or after July 1, 1998.

5.71. At 12 months after the inception of the policy year, none of the audits have occurred.
At 24 months after the inception of the policy year, 3/4 of the audits have occurred.
Therefore, the development factor for written premiums is: {(3/4)(1.15) + 1/4} / 1 = **1.1125**.
<u>Comment</u>: This exam question should have specified written or earned premiums.
At 12 months after the inception of the policy year, only half of the premium is earned, and none of the audits have occurred.
At 24 months after the inception of the policy year, all of the premium is earned, and 3/4 of the audits have occurred.
Therefore, the development factor for earned premiums is: {(3/4)(1.15) + 1/4} / (1/2) = 2.225.

5.72.

Effective Date	Rate Change	Rate Level Index
January 1, 1997		1.000
July 1, 1997	+5.2%	1.052
April 1,1999	+7.4%	(1.052)(1.074) = 1.130

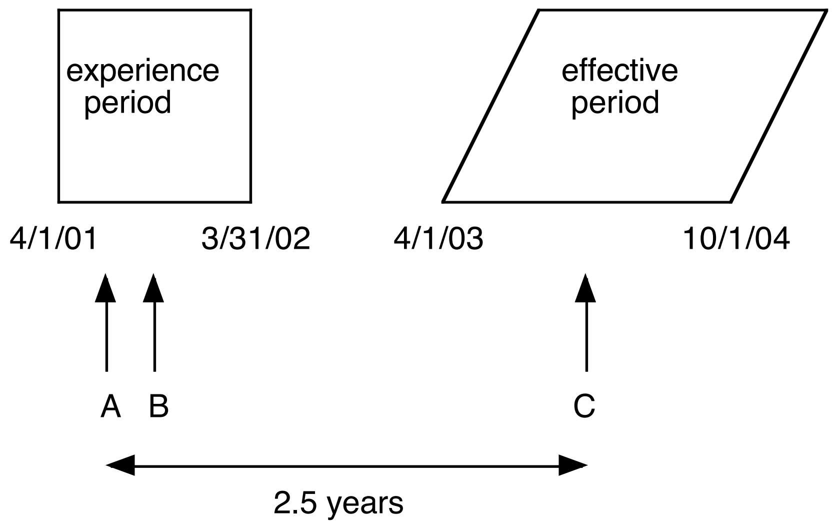
Since the policies have a 2 year term, the slopes of the lines are 1/2, rather than 1:



Area A = (1/2)(1/2)(1/4) = 1/16 = .0625. Area C = (1/2)(3/4)(3/8) = 9/64 = .1406.
Area B = 1 - Area A - Area C = .7969.
Average Rate Level CY 99 earned = (.0625)(1) + (.7969)(1.052) + (.1406)(1.130) = 1.060.
CY 99 on level factor = 1.130/1.060 = 1.066.
Calendar Year 1999 on-level earned premium is: (1.066)(14,000) = **14,924**.

5.73. E. All of written premium is assigned to the calendar year in which the policy was written, so there is $900 in CY 02 written premium. The earned premium is apportioned to calendar years based on the coverage provided. 10/12 of the 12 month effective period is during 2002, so there is: (10/12)($900) = $750 in CY 02 earned premium. At 12/31/02, the policy is still in force, so there is $900 in inforce premium.
Comment: Eventually, this policy will have: (2/12)($900) = $150 in CY 03 earned premium.

5.74. The experience period for earned premium is 4/1/01 to 3/31/02, for an average date of earning of 10/1/01. Since we have 6-month policies, this corresponds to an average date of writing 3 months earlier or 7/1/01. 7/1/01 is the trend from date. The effective date is 4/1/03 and rates will be in effect for 18 months. Thus the average date of writing for the new rates is 4/1/03 plus 9 months or 1/1/04. 1/1/04 is the trend to date.
The trend period is from 7/1/01 to 1/1/04 or **2.5 years**.
Comment: See the following diagram:



A = average date of writing (experience) = 7/1/01 = B - 6 months/2.
B = average date of earning = 10/1/01 = (4/1/01 + 3/3/02)/2.
C = average date of writing (new rates) = 1/1/04 = (4/1/03 + 10/1/04)/2.

Note that the total trend period is the same, whether one uses the one or two piece trend methods. It is easier to think of this as a one piece trend; in any case there was no mention of a specific "pivot point" to use in the two piece trend method.

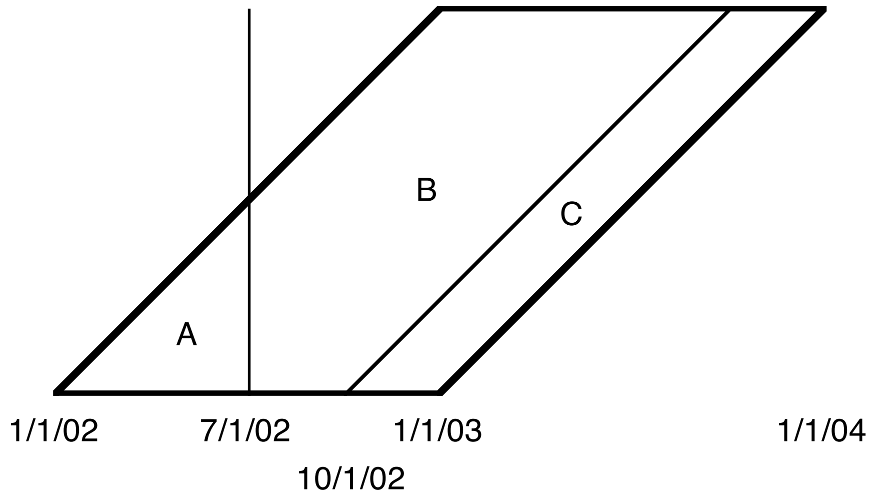
5.75. As before, 7/1/01 is the trend from date. The effective date is 4/1/03 and rates will be in effect for 12 months. Thus the average date of writing for the new rates is 4/1/03 plus 6 months or 10/1/03. 10/1/03 is the trend to date. The trend period is from 7/1/01 to 10/1/03 or **2.25 years**.

5.76. The experience period for earned premium is 4/1/01 to 3/31/02, for an average date of earning of 10/1/01. The effective date is 4/1/03 and rates will be in effect for 18 months. Thus the average date of writing for the new rates is 4/1/03 plus 9 months or 1/1/04. Since we have 6-month policies, this corresponds to an average date of earning 3 months later or 4/1/04. 4/1/04 is the trend to date. The trend period is from 10/1/01 to 4/1/04 or **2.5 years**.
Comment: The same total trend period as in 5, 5/03, Q.11.
The trend to and trend from dates are each 3 months later than in 5, 5/03, Q.11.

5.77. The experience period for earned premium is 4/1/01 to 3/31/02, for an average date of earning of 10/1/01. Since we have annual policies, this corresponds to an average date of writing 6 months earlier or 4/1/01. 4/1/01 is the trend from date. The effective date is 4/1/03 and rates will be in effect for 18 months. Thus the average date of writing for the new rates is 4/1/03 plus 9 months or 1/1/04. 1/1/04 is the trend to date.
The trend period is from 4/1/01 to 1/1/04 or **2.75 years**.

5.78. a. 1. Past rate changes.
2. Past rating plan changes.
3. Rating plans causing the premium level to change over time.
4. Change in the mix of business.
b. 1. An insurer increases rates by 10%.
2. An insurer implements a new discount program. (For example, introducing a program that gives a 5% discount for a drug free workplace for their Workers Compensation insurance or a program that gives a 5% discount for cars with air bags under Medical Payments and Personal Injury Protection Automobile Insurance.)
3. Model year and symbol rating plans in Collision and Comprehensive automobile insurance.
Using an inflation sensitive exposure base, such as payroll for Workers Compensation insurance.
Experience Rating Plans.
4. Change of optional liability limits purchased.
Change of percentage of business written by territory.
Change of percentage of business written by class.
Change of mix of deductibles purchased.
c. 1. One time change.
2. One time change.
3. Continuous change.
4. (Usually) continuous change.
<u>Comment</u>: Give only one example of each item.

5.79. Assume that the rate level prior to 7/1/02 is 1.
Policy Year 2002 written premiums:



	Area	Rate Level
Area A	(1/2)(1/2)(1/2) = 1/8	1
Area B	1 - 1/8 - 1/4 = 5/8	1.1
Area C	1/4	(1.1)(1.05) = 1.155

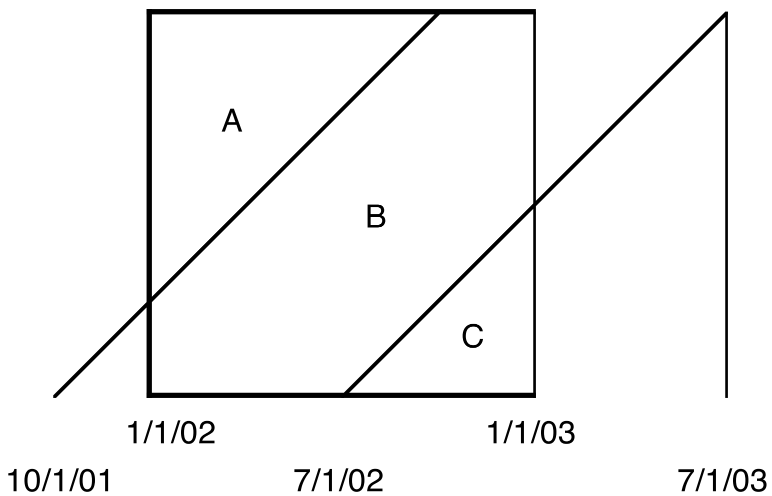
Average rate level for Policy Year 2002 written premiums:
(1)(1/8) + (1.1)(5/8) + (1.155)(1/4) = 1.101.
Current rate level: (1.1)(1.05)(1.07) = 1.236.
On-Level Factor: 1.236/1.101= **1.123**.

5.80. Loss Ratio (untrended) = 35,000/50,000 = 70%.
Net Trend Factor: $(.99^{2.5})\ (1.06^{2.5})/1.029^{2.5}$ = 1.0503.
Trended Loss Ratio: (1.0503)(70%) = **73.5%**.
Comment: Since the loss ratio has premiums in the denominator, the trended loss ratio includes the premium trend in the denominator. The given Premium Trend should account for all of the effects on premiums we expect from the experience period to the effective period, including the effects of any trend in exposures. Let us assume this is for Homeowners Insurance. Then exposures is the value of homes. While they don't specify why, in this case there is an additional 0.9% per year increase in premiums due to something other than change in the average face value of a homeowners policy. This is probably due to the fact that homeowners rates in the manual are not proportional to the face value. The best choice in this case is to use the premium trend in the denominator rather than the given exposure trend. To use both the premium and exposure trend would be double counting and clearly wrong!

5.81. PY01 at 24 months is 12/31/02. At that point all of the policies have expired, but only 3/4 of the policies have been audited.
PY01 at 36 months is 12/31/03. At that point all of the policies have been audited.
Premium Development Factor: (1.05) / {(1.05)(3/4) + (1)(1/4)} = **1.012**.

5.82. a. Assume the rate level prior to 10/1/01 is 1.

Calendar Year 2002 earned premiums:



	Area	Rate Level
Area A	(1/2)(3/4)(3/4) = 9/32	1
Area B	1 - 9/32 - 4/32 = 19/32	1.07
Area C	(1/2)(1/2)(1/2) = 1/8	(1.07)(1.1) = 1.177

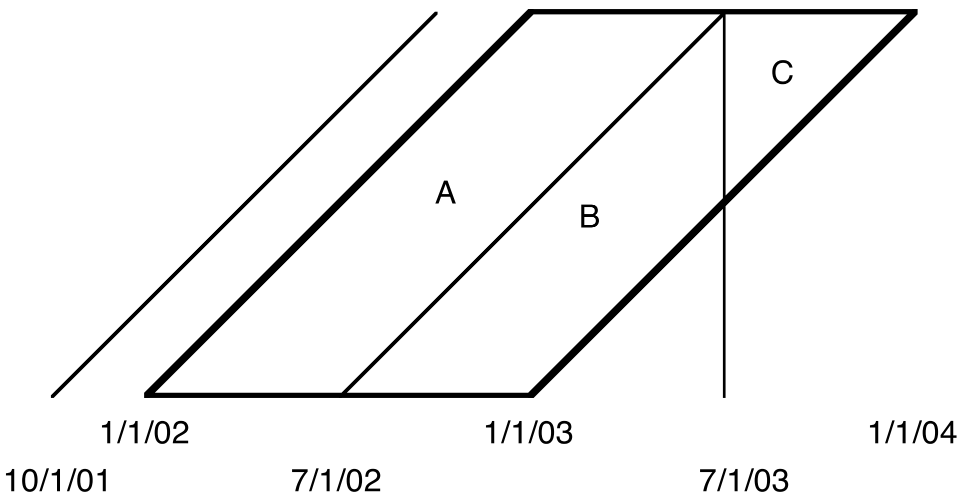
Average rate level for Calendar Year 2002 earned premiums:

(1)(9/32) + (1.07)(19/32) + (1.177)(1/8) = 1.064.

Current rate level: (1.07)(1.1)(0.95) = 1.118.

On-Level Factor: 1.118/1.064 = **1.051**.

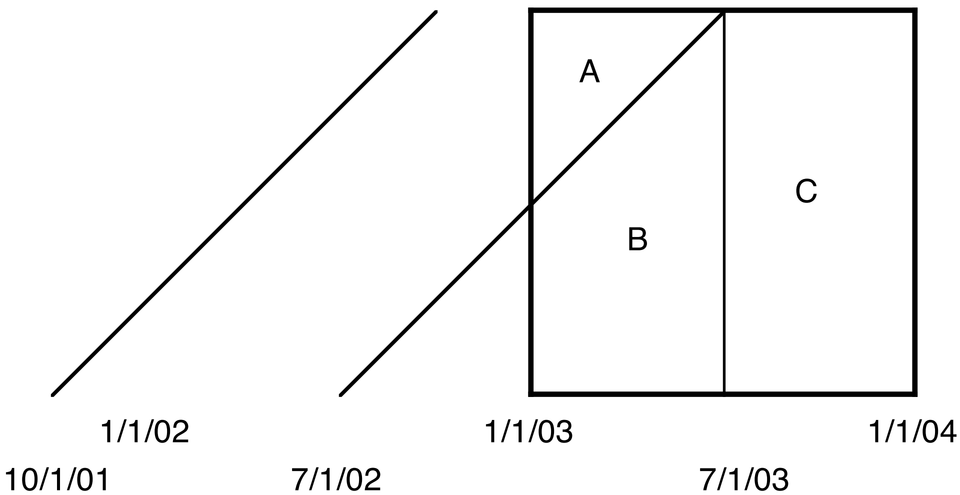
b. Assume the rate level prior to 10/1/01 is 1.
Policy Year 2002 earned premiums:



	Area	Rate Level
Area A	1/2	1.07
Area B	1/2 - 1/8 = 3/8	(1.07)(1.1) = 1.177
Area C	(1/2)(1/2)(1/2) = 1/8	(1.07)(1.1)(0.95) = 1.118

Average rate level for Policy Year 2002 earned premiums:
(1.07)(1/2) + (1.177)(3/8) + (1.118)(1/8) = 1.116.
Current rate level: (1.07)(1.1)(0.95) = 1.118.
On-Level Factor: 1.118/1.116 = **1.002**.

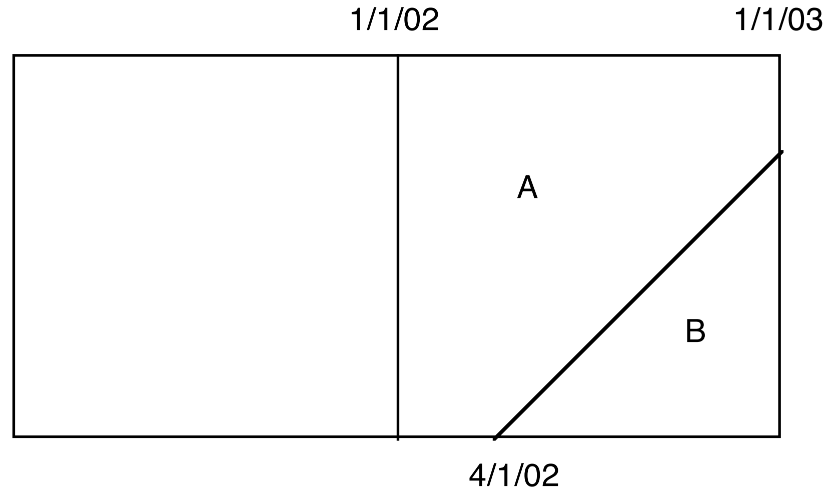
5.83. Assume the rate level prior to 10/1/01 is 1.
Calendar Year 2002 earned premiums:



	Area	Rate Level
Area A	(1/2)(1/2)(1/2) = 1/8	1.07
Area B	1/2 - 1/8 = 3/8	(1.07)(1.1) = 1.177
Area C	1/2	(1.07)(1.1)(0.95) = 1.118

Average rate level for Policy Year 2002 earned premiums:
(1.07)(1/8) + (1.177)(3/8) + (1.118)(1/2) = 1.134.
Current rate level: (1.07)(1.1)(0.95) = 1.118.
On-Level Factor: 1.118/1.134 = **0.986**.

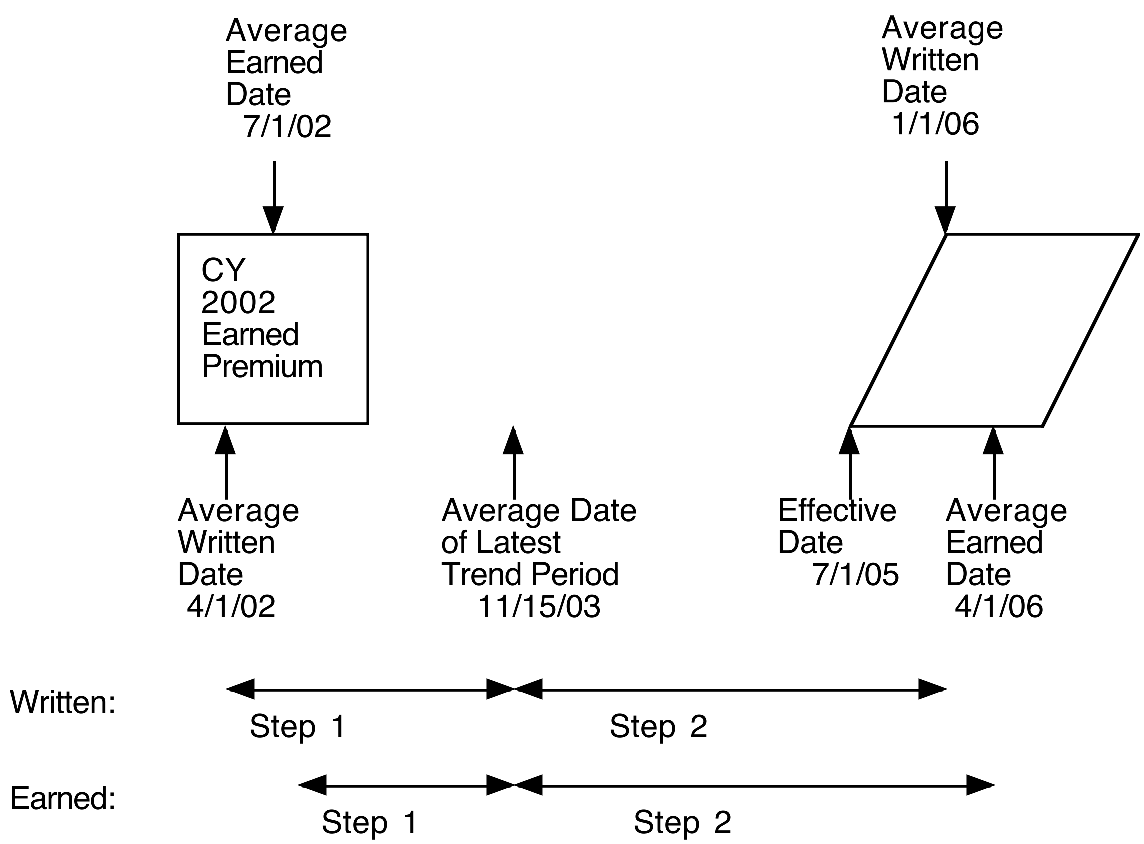
5.84. (a) 1. The Calendar Year 2002 premiums have to be adjusted to the current rate level. For example, assume it is earned premium, the rate increase is 10% effective 4/1/02, and there are annual policies. (The question does not specify whether written or earned, the size of the rate increase, nor the exact date of the rate change.) All of the exposures written 1/1/02 are earned during Calendar Year 2002, and are written at the lower rate level. Of the <u>other</u> exposures earned during Calendar Year 2002, $(1/2)(3/4)^2 = 9/32$ = Area B, are written on or after 4/1/02, at the higher rate level. The remaining 23/32 = Area A are written at the lower rate level.



Average Rate Level of Calendar Year 2002 Earned Premiums:
(50%)(1) + (50%){(23/32)(1) + (9/32)(1.1)} = 1.014. On level factor: 1.1/1.014 = 1.085.
2. The premiums in the trend series for periods before April 2002 should be multiplied by the appropriate on-level factor, in order to put them all on a common level, presumably the current rate level. For example, if the rate increase were 10% effective 4/1/02, then the written premiums for the first quarter of 2002 would be multiplied by 1.1, while those from the second quarter of 2002 would not. The premium trend will be lower after this adjustment than it would have been if this adjustment had not been made.
Alternately, if we instead assume the premiums are written, then of the remaining 50% not written on January 1, 1/4 is written at the lower rate level and 3/4 is written at the higher rate level.
Average Rate Level of Calendar Year 2002 Written Premiums:
(50%)(1) + (50%){(1/4)(1) + (3/4)(1.1)} = 1.0375. On level factor: 1.1/1.0375 = 1.060.
(b) 1. The Calendar Year 2002 premiums and losses could not be adjusted for the change in limit Then one would get an estimate of the appropriate rate change on the old basic limit level. This rate change would then be adjusted for any change in the indicated increased limit factor to go from the old to the new basic limits. The result could be applied to the current rate (for the new basic limits.)
Alternately, one could adjust both the premiums and losses to what they would have been with the new basic limits. In the case of premiums, one would multiply them by the appropriate increased limit factor to go from the old to the new basic limits.
2. If any of the data in the premium trend series is from after June 2003, then the premiums in the trend series for periods before June 2003 (presumably at the then basic limit) should be adjusted to what they would have been with the new basic limit.

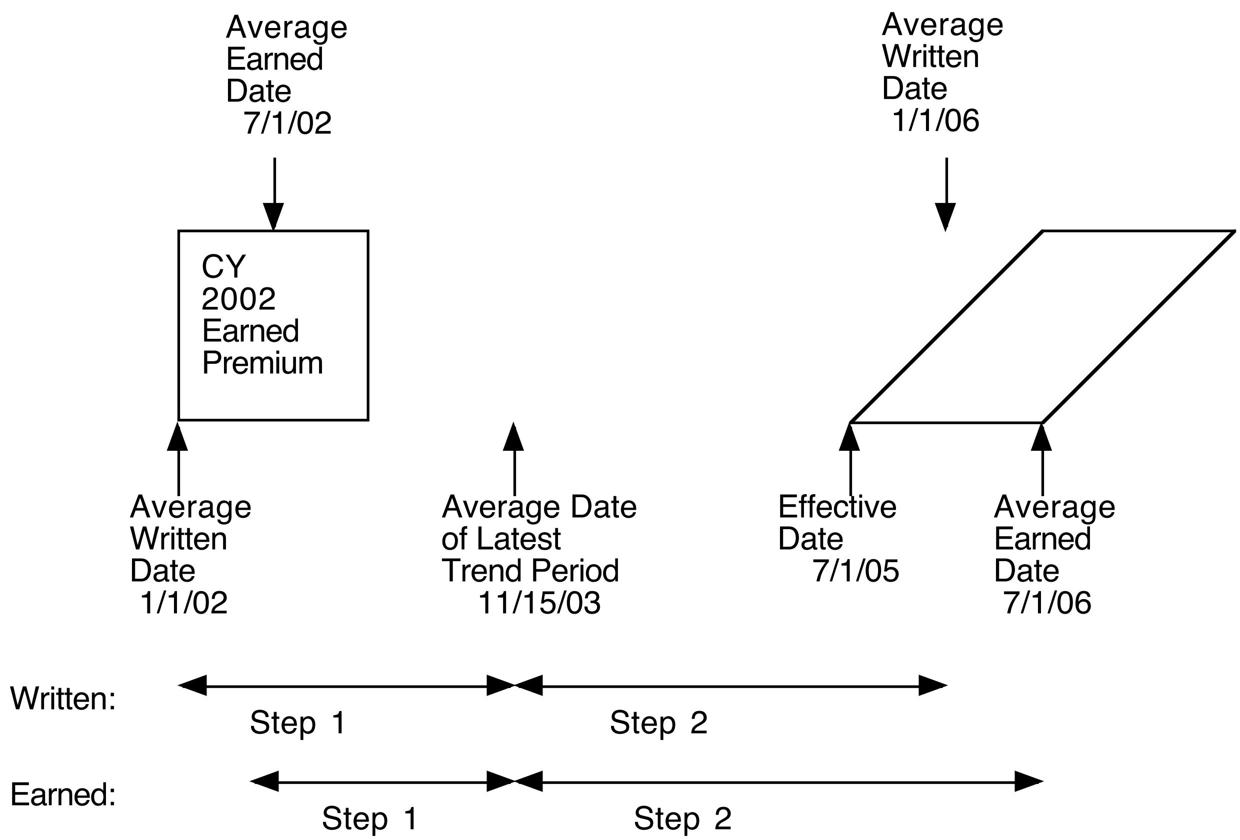
If any of the data in the premium trend series is from after June 2003, then the premium trend will be lower after this adjustment than it would have been if this adjustment had not been made.
(c) 1. Assume that the model year rating plan was in effect during Calendar Year 2002 and is expected to continue into the effective period without any major changes. Then the effect of the model year rating plan is continuous as new models enter and older models leave. There is no need to adjust the Calendar Year 2002 premiums or losses, assuming the model year relativities are reasonably accurate.
2. Assume that the model year rating plan has been in effect during the whole time of the premium trend series and is expected to continue into the effective period without any major changes. Then the effect of the model year rating plan is continuous as new models enter and older models leave. One can leave the premium trend series alone. While this will result in a larger premium trend than if an adjustment is made, this should be balanced by the corresponding loss trend, which is affected in a similar manner by the drift towards later model years.
Comment: I found parts of this question difficult, due to all of the missing information.
It was also unclear to me how detailed the answers should be. (My solution to 1a is probably more detailed than was necessary.) In part c, if the loss trend were based on CPI indices that do not reflect the impact of the drift towards later model years, then one would have to adjust the premium trend series to remove the impact of the drift towards later model years.

5.85. a. The average date of earning CY02 Premiums is 7/1/02, with an average date of writing 3 months earlier (6 month policies) or 4/1/02. Assuming the last average written premium in the series is for the quarter ending 12/31/03, this has average date of writing of 11/15/03.
Thus the first step goes from 4/1/02 to 11/15/03. The average date of writing for the effective period is six months past 7/1/05 or 1/1/06. Thus the second step goes from 11/15/03 to 1/1/06.
b. The average date of earning CY02 Premiums is 7/1/02. Assuming the last average earned premium in the series is for the quarter 12/31/03, this has average date of earning of 11/15/03. Thus the first step goes from 7/1/02 to 11/15/03. The average date of writing for the effective period is six months past 7/1/05 or 1/1/06. The average date of earning is 3 months later (6 month policies) or 4/1/06. Thus the second step goes from 11/15/03 to 4/1/06.
c. The two-step method would be more appropriate when there have been significantly different premium trends over the different periods of time involved. The advantage of two-step trending is that it recognizes that there are situations where a single annual premium trend may not be appropriate for each year in the experience period.
Comment: Here is a diagram, with 6-month policies as in this question:



See Figure 5.28 in Basic Ratemaking.
If for example you assumed instead that the trend series were monthly, then you would get somewhat different answers in parts a and b than I did.

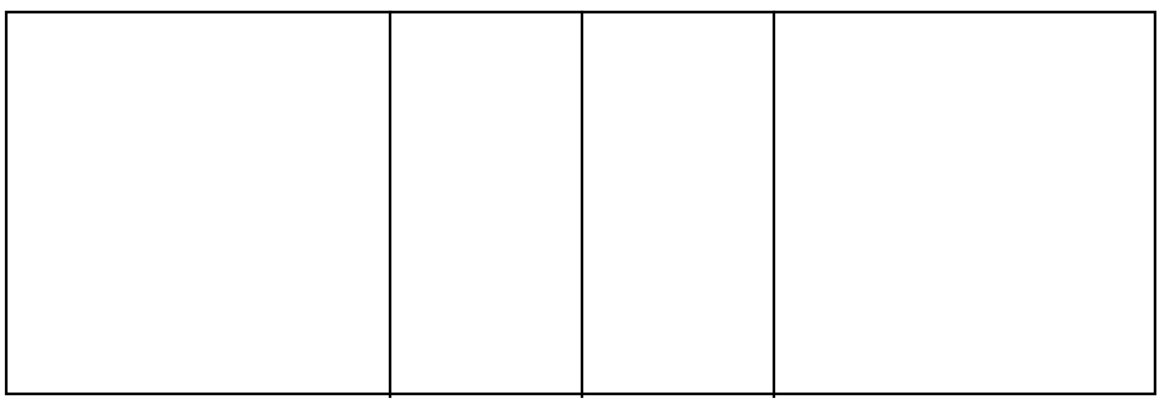
5.86. a. The average date of earning CY02 Premiums is 7/1/02, with an average date of writing 6 months earlier (12 month policies) or 1/1/02. Assuming the last average written premium in the series is for the quarter ending 12/31/03, this has average date of writing of 11/15/03.
Thus the first step goes from 1/1/02 to 11/15/03. The average date of writing for the effective period is six months past 7/1/05 or 1/1/06. Thus the second step goes from 11/15/03 to 1/1/06.
b. The average date of earning CY02 Premiums is 7/1/02. Assuming the last average earned premium in the series is for the quarter ending 12/31/03, this has average date of earning of 11/15/03. Thus the first step goes from 7/1/02 to 11/15/03. The average date of writing for the effective period is six months past 7/1/05 or 1/1/06. The average date of earning is 6 months later (12 month policies) or 7/1/06. Thus the second step goes from 11/15/03 to 7/1/06.
Comment: Here is a diagram similar to Figures 5.26 and 5.28 in Basic Ratemaking :



5.87. All of the above types of rating plan changes have a direct effect on the premium level and this effect must be reflected in the calculation of the premium trend.
<u>Comment</u>: Items #1 and #2 are intended to be one time effects; this could have been made clearer in the question. The preferred way to handle item #2 is by adjusting both the series of premiums and losses for the effect. If the effect can not be quantified, one would adjust neither the series of premiums nor losses for the effect.
In the case of item #3, since the changes are <u>not</u> expected to continue, one would adjust the premium trend series to remove the impact of the rating plan changes, but would not then need to adjust for the effect of the rating plan going forward. The more common case involves a rating plan that has changed the average premium level over time, and unlike item #3 in this question, the changes are expected to continue. In that case, the effects are usually continuous and gradual. Therefore, if the effect is measurable one would make a direct adjustment, while if it is not one would instead capture the effect in the observation of the premium data.

5.88. a. The rate decrease affects half of CY 2003 and all of CY 2004 written premium.

1/1/2002 1/1/2003 1/1/2004 1/1/2005



7/1/2003

(We draw a vertical line for the rate change, since this is Calendar Year written premium.
A diagonal line would have been drawn if this had been instead earned premium.)
If the rate level index is 1 prior to 7/1/03 and .8 on and after 7/1/03:

<u>C Y</u>	<u>Average Rate Level</u>	<u>On Level Factor</u>	<u>Premium</u>	<u>On Level Premium</u>
2002	1	0.8/1 = 0.8	1000	800
2003	(1/2)(1) + (1/2)(0.8) = 0.9	0.8/0.9 = 8/9	933.33	829.63
2004	0.8	0.8/0.8 = 1	882	882

Adjusting for the effect of the shift in the limit distribution:
CY 2002 premium at the 2004 level would be: $(800)(1.03^2) = 848.72$, and
CY 2003 premium at the 2004 level would be: (829.63)(1.03) = 854.52.
Now one must choose an annual trend to use going forward from 2004.
The two year change from 2002 to 2004 is 882/848.72 = 1.039, excluding the effect of rate changes and limit shifts.
Since the limit shift is not expected to continue beyond 2004, $\sqrt{1.039}$ = 1.019 would be not be an

unreasonable choice for an annual trend going forward.
The average date of writing for CY 2004 is 7/1/04.
The average date of writing for the new rates is: 1/1/06 + 18 months/2 = 10/1/06.
The second step trend period is from 7/1/04 to 10/1/06: 2 years and 3 months = 2.25 years.
Second step trend factor is: $1.019^{2.25}$ = 1.043.
The first step trend factor to get 2002 to the 2004 level is: 882/800 = 1.1025.
The first step trend factor to get 2003 to the 2004 level is: 882/829.63 = 1.0631.

C Y	First Step Trend Factor	Second Step Trend Factor	Total Trend Factor
2002	1.1025	1.043	**1.1499**
2003	1.0631	1.043	**1.1088**
2004	1	1.043	**1.043**

Using these trend factors, one would adjust the observed written premiums for the rate change, but not the effect of the limit shift which has been included in the step one trend, in order to get the trended, on-level written premiums to use in ratemaking:

CY	Average Written Prem.	Trend Factor	On-Level Factor	On-Level Trended Prem.
2002	$1000.00	1.1499	0.8	919.92
2003	$ 933.33	1.1088	8/9	919.89
2004	$ 882.00	1.043	1	919.93

Alternately, the first step trend factor to get 2002 to the 2004 level is: 882/848.72 = 1.0392.
The first step trend factor to get 2003 to the 2004 level is: 882/854.52 = 1.0322.

C Y	First Step Trend Factor	Second Step Trend Factor	Total Trend Factor
2002	1.0392	1.043	1.0839
2003	1.0322	1.043	1.0766
2004	1	1.043	1.043

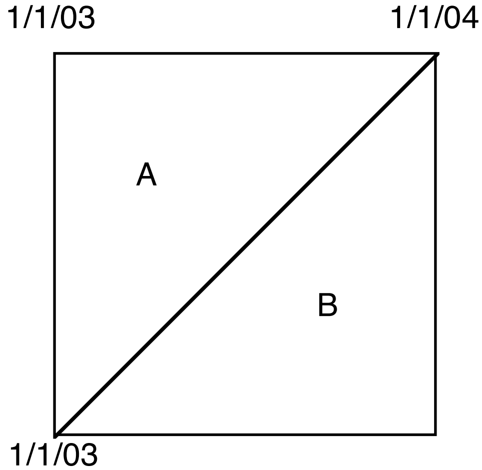
Using these trend factors, one would adjust the observed written premiums for the rate change and the effect of the limit shift:

CY	Writ. Prem.	Trend Fact.	Adj. for Limit Shift	On-Level Fact.	On-Level Trended Prem.
2002	$1000.00	1.0839	1.03^2	0.8	919.92
2003	$ 933.33	1.0766	1.03	8/9	919.97
2004	$ 882.00	1.043	1	1	919.93

b. The two step trending method rests on the assumption that the last data point of the trend series is a true number. For loss frequency or severity, this can be a dubious assumption because of random fluctuations around the true expected value. For average premium, on the other hand, the individual data points are more believable because there is not as large a random element. Average premium values are more stable than frequency or severity.

Comment: Other similar choices for the second step annual trend, such as 2%, would also be reasonable. Make sure to state your assumptions, whatever they are.
My two alternative solutions differ in their treatment of the limit shift. I think either one is okay; the results differ slightly in some cases due to intermediate rounding.

5.89. Set the rate level index prior to 1/1/03 equal to 1, and after 1/1/03 it is 1.1.
Using the parallelogram method, assuming annual policies, one would proceed as follows:



Average rate level for CY 2003 earned premium is: (0.5)(1) + (0.5)(1.1) = 1.05.
On-level factor is: 1.1/1.05 = 1.048.
However, this is based on an assumption that exposures are being written at the same rate throughout 2002 and 2003. Since written exposures are increasing during 2003, they should be given a weight greater than proportional to the area of B. Therefore, the average rate level for CY 2003 earned premiums is higher than that gotten from use of the parallelogram method. Therefore, the parallelogram method **overstates** the on-level factor applied to calendar year 2003 earned premium.
Comments: The direction of the error does not depend on the policy term, however, the magnitude does.
If one assume constant level of writing exposures during 2002 at 1, then the exposures written at time t of 2003 is 1.05^{12t}. 1 - t of this policy is earned during CY 2003.
Therefore the contribution from policies written during 2003 is:

$$\int_0^1 (1 - t)1.05^{12t}\,dt = \{1/(\ln(1.05)12)\}1.05^{12t} - \{1/(\ln(1.05)12)\}1.05^{12t}\{t - 1/(\ln(1.05)12)\}\Big]_{t=0}^{t=1}$$

$= (1.708)(1.05^{12} - 1) - (1.708)\{1.05^{12}(1 - 1.708) + 1.708\} = 0.614.$
(I do not expect you to be asked to do such integrals on your exam.)
Therefore, the average rate level for the CY 2003 earned premium is:
{(0.5)(1) + (0.614)(1.1)} / (0.5 + 0.614) = 1.055.
Correct on-level factor is: 1.1/1.055 = 1.043.
The 1.048 on-level factor from the parallelogram method is too large.
On the exam, if one needed to do a calculation in order to see the direction of the adjustment, one could have approximated by assuming policies are written at the middle of each quarter.

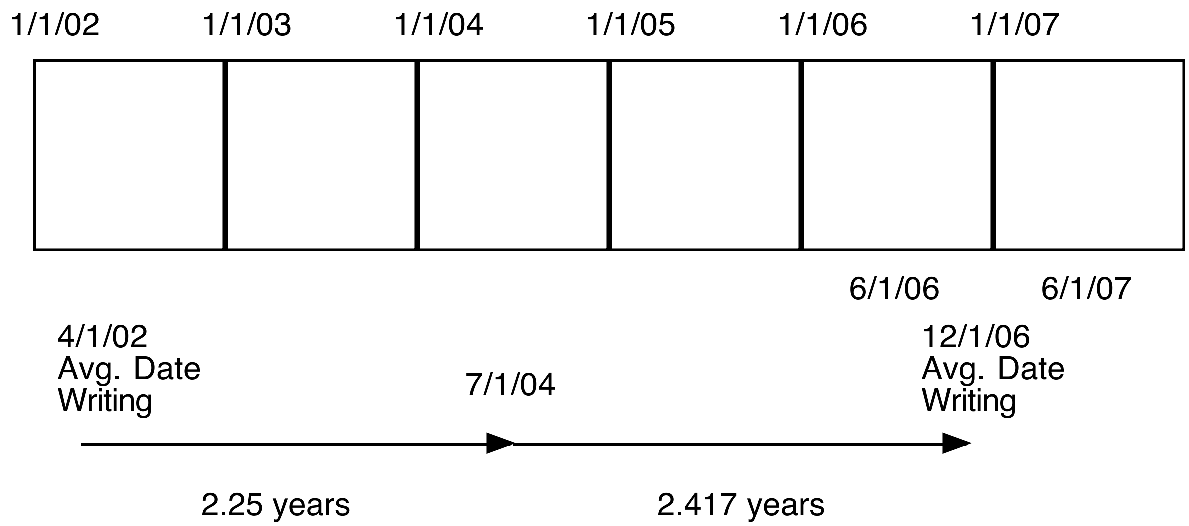
Quarter in 2003	Written Premium	Portion Earned in 2003	2003 Premium Earned
1	0.2690	0.8750	0.2354
2	0.3114	0.6250	0.1946
3	0.3605	0.3750	0.1352
4	0.4173	0.1250	0.0522
Total	1.3581		0.6173

Therefore, the average rate level for the CY 2003 earned premium is approximately:
{(0.5)(1) + (0.617)(1.1)} / (0.5 + 0.617) = 1.055.
Correct on-level factor is: 1.1/1.055 = 1.043.
The 1.048 on-level factor from the parallelogram method is too large!
If instead the written exposures had been <u>decreasing</u> during 2003, then the parallelogram method would understate the on-level factor applied to calendar year 2003 earned premium.
If instead there had been a rate <u>decrease</u> on 1/1/03, then the parallelogram method would understate the on-level factor applied to calendar year 2003 earned premium.

5.90. The endorsement was added to the policy when it only had 4 months until expiration.
We need to adjust the additional premium for the endorsement to what it would have been for 6 months rather than 4 months: (6/4)(2400) = $3600.
Thus the in-force premium at December 31, 2004 is: 3600 + 3600 = **$7200**.
<u>Comment</u>: See pages 70-71 in <u>Basic Ratemaking</u>.

5.91. C. While one would use the historical rate changes and effective dates to put the data in the premium trend series on a common rate level, one would not need them in order to determine the trending period.
<u>Comment</u>: Items B, and E will be used to determine the trend from date.
Items A, B, and D will be used to determine the trend to date.

5.92. a. The average date of earning of CY02 is 7/1/02.
The average date of writing of CY02 is 3 months earlier or 4/1/02.
The proposed effective period is 6/1/06 to 5/31/07.
The average date of writing for the effective period is 12/1/06.
It is unclear exactly what the phrase “through 2004” means.
Assume “through 2004” means written premiums for CY04, which has an average date of writing of 7/1/04.
Then the first piece of the trend is from 4/1/02 to 7/1/04 or two years and 3 months.
The second piece of the trend is from 7/1/04 to 12/1/06 or 2 years and 5 months.
(\$42.5 million)($1.05^{2.25}$)($1.03^{2.417}$) = **\$50.94 million.**

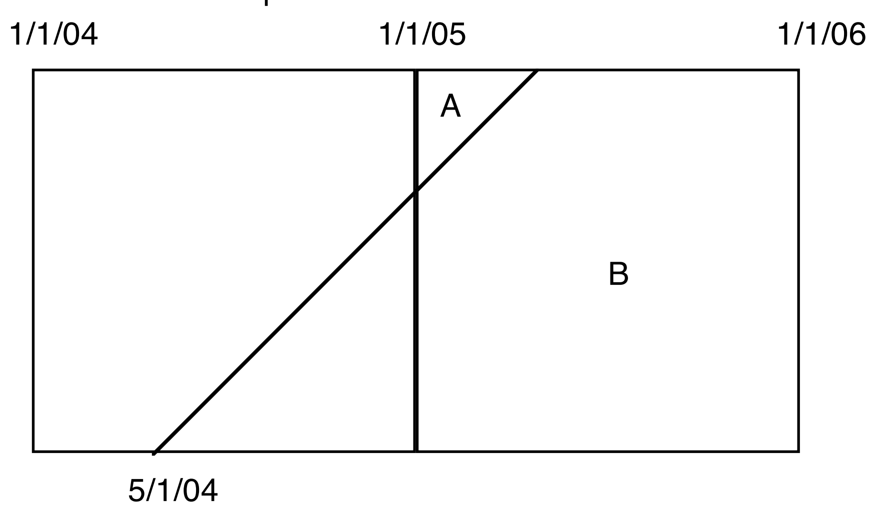


b. The average date of earning of CY02 is 7/1/02.
The average date of writing of CY02 is 6 months earlier or 1/1/02.
The average date of writing for the effective period is 12/1/06.
Then the first piece of the trend is from 1/1/02 to 7/1/04 or two years and 6 months.
The second piece of the trend is from 7/1/04 to 12/1/06 or 2 years and 5 months.
($42.5 million)$(1.05^{2.5})(1.03^{2.417})$ = **$51.57 million.**

c. The two-piece trending technique allows one to take into account more recent information on new car sales as well as expert projections of future sales, than would be reflected in insurance exposures currently available to the actuary. In contrast, the one-piece method would assume a uniform rate of change from the experience period to the effective period.
<u>Comment</u>: The cars insured during 2002 on average cost less when new (and were of older model years) than those that will be insured during the proposed effective period. If we were to run the cars insured during CY02 and the policy effective period versus the current rate manual, we would collect more average premium in the latter case since those cars are on average more expensive. The trended CY02 premium is at the higher level it would have been if the cars to be insured during the proposed effective period had been insured during 2002, instead of the cars that were actually insured during 2002. Since we were told to use “trending with average written premium,” I worked with average dates of writing in order to get the lengths of the trend periods.
Different choices as to the meaning of the phrase “through 2004” would produce different first and second trend periods, but their sum should be the same as I got. These different interpretations would lead to somewhat different final answers for the trended premium.
In actual applications of this technique, one could take the current (or proposed) vehicle series and model year relativities and average them using the earned exposures for CY02, CY03, and CY04. For example, these averages might be 0.88 for CY02, 0.95 for CY03 and 0.99 for CY04. Then the first piece trend from CY02 to CY04 would be 0.99/0.88. Similarly, the first piece trend from CY03 to CY04 would be 0.99/0.95. One could use instead written exposures, since they reflect somewhat more recent information than earned exposures.

5.93. a. Let us use Homeowners Insurance as an example. If the number of homes insured doubled, then the total premium would double. However, the total expected losses would also double. We are trying to make rates per home. Thus we are interested in the change in premium per home due to the change in the values of those homes over time. This is captured by the use of the average premium rather than the total premium.
b. We use earned premium in the overall indication and therefore the trends to be applied to them should also be based on earned premium. On the other hand, written premium data is more responsive to recent changes than is earned premium data.
The argument for earned premium: Since these trends will apply to historical earned premium at current rate level, we should evaluate trends based on shifts in average earned premium.
The argument for written premium: Even though the historical premium is earned premium, we can determine the average written date for that block of premium and then observe changes in average written premium to establish the trend. Therefore, basing the trend analysis on average written premium is a valid approach. Furthermore, average written premium has an important advantage in that it allows us to capture more recent data than average earned premium. This is because of the simple fact that the premium for a given policy is not earned until well after it is written. In fact, at any given point in time, the latest quarter's average earned premium is based on a group of policies that is a half a policy period older than the group of policies comprising the latest quarter's average written premium. Using average earned premium would unnecessarily postpone the recognition of the effects of the most recent changes in the mix of business.

5.94. a. Assume that the premium is earned and for Calendar Year 2005.



Area A = $(1/2)(4/12)^2$ = 0.056. Area B = 1 - Area A = 0.944.
Average rate level during CY05 = (1)(0.056) + (0.92)(0.944) = 0.9245.
On level factor: 0.92/0.9245 = **0.995**.
b. The calculation in part a is not reasonable for this line. More of the exposures are written at the new rate level than is assumed in the parallelogram level, which assumes a uniform distribution of writing throughout the year. Therefore, the on level factor results in too large of an adjustment to the 2005 premiums.

c. In each year May and June account for 25% of the exposures each, "with the other 50% written uniformly throughout the rest of the year." Thus each of the 10 months other than May and June have 50%/10 = 5% of the total exposures. This is the same as 60% of the exposures written evenly throughout the whole year plus 20% additional in each of May and June.
For the part written evenly throughout the year the average rate level is 0.9245, from previously. Thus the average rate level during CY05 earned premium is: (0.6)(0.9245) + (0.4)(0.92) = 0.9227. On level factor: 0.92/0.9227 = **0.997**.
More precisely, assume 24 exposures for each month other than May and June, and 5 times that or 120 exposures each for May and June. (Thus 240 exposures for May plus June, and for the other ten months a sum of another 240 exposures.) Assume the rate level was 100 and was then lowered to 92 effective May 1, 2004. January 2004 contributes 1/24 of its exposures to CY05 Earned Premiums. February 2004 contributes 3/24 of its exposures to CY05 Earned Premiums.

Month	Exposures	% Earned in CY05	Exposures Earned in CY05	Rate Level	Premiums Earned in CY05
1/04	24	0.0417	1	100	100
2/04	24	0.1250	3	100	300
3/04	24	0.2083	5	100	500
4/04	24	0.2917	7	100	700
5/04	120	0.3750	45	92	4140
6/04	120	0.4583	55	92	5060
7/04	24	0.5417	13	92	1196
8/04	24	0.6250	15	92	1380
9/04	24	0.7083	17	92	1564
10/04	24	0.7917	19	92	1748
11/04	24	0.8750	21	92	1932
12/04	24	0.9583	23	92	2116
1/05	24	0.9583	23	92	2116
2/05	24	0.8750	21	92	1932
3/05	24	0.7917	19	92	1748
4/05	24	0.7083	17	92	1564
5/05	120	0.6250	75	92	6900
6/05	120	0.5417	65	92	5980
7/05	24	0.4583	11	92	1012
8/05	24	0.3750	9	92	828
9/05	24	0.2917	7	92	644
10/05	24	0.2083	5	92	460
11/05	24	0.1250	3	92	276
12/05	24	0.0417	1	92	92
Sum			480		44288

The average rate level for CY05 earned premiums is: 44288/480 = 92.2667.
On level factor is: 92/92.2667 = **0.997**.
Alternately, of the total exposures earned during Calendar Year 2005, January through April of 2004 contribute: (5%)(.5/12) + (5%)(1.5/12) + (5%)(2.5/12) + (5%)(3.5/12) = 0.03333.
Average rate level for CY05 earned exposures is: (1)(0.03333) + (.92)(1 - 0.03333) = 0.92667.

On level factor is: 0.92/0.922667 = **0.997**.

Comment: The numerical difference in on level factor due to nonuniform writing could be more significant in other situations.

Even though it is beyond what is on the syllabus, some of you may benefit by looking at Frank Karlinski's short discussion of "A Refined Model For Premium Adjustment," PCAS 1977, available on the CAS webpage.

http://www.casact.org/pubs/proceed/proceed77

The original paper by Miller and Davis is in PCAS 1976.

5.95. a. Policies A, B, and C contribute all of their premium to CY04.

2804 written premium = 1,200 + 2,400 + 3,600 = **7,200**.

b. Policy A contributes all of its earned premium to CY04.

Policy B contributes 1/2 of its earned premium to CY04 and the other 1/2 to CY05.

Policy C contributes 1/6 of its earned premium to CY04 and the other 5/6 to CY05.

2004 (Calendar Year) earned premium = 1,200 + (2,400)(1/2) + (3,600)(1/6) = **3,000**.

c. Policies A, B, and C contribute all of their premium to PY04.

2004 PY Premium = 1,200 + 2,400 + 3,600 = **7,200**.

d. Policies B and C are in-force on 3/31/05.

In-force Premium as of 3/31/05 = 2,400 + 3,600 = **6,000**.

5.96. a. 2004 written premium = 1,200 + 2,400 + 3,600 = **7,200**.

b. 2004 (Calendar Year) earned premium = 1,200 + 2,400 + (3,600)(1/3) = **4,800**.

c. 2004 PY Premium = 1,200 + 2,400 + 3,600 = **7,200**.

d. Only policy C is in-force on 3/31/05. Inforce Premium as of 3/31/05 = **3,600**.

5.97.

a. Past rate changes directly affect the premium level, usually in a one-time and measurable way.

b. Model year and symbol rating plans affect the premium gradually and continuously. As people trade in older cars for newer and more expensive cars, the average premium level will tend to increase.

5.98. For the first piece of the trend compare the average earned premium at current rate level for a given calendar year to the latest available value for the average written premium at current rate level. For example, 3998/3605 = 1.109.
The second piece of the trend goes from the written premium for CY 2005 to the policy effective period, which is the written premium for CY 2007, a trend period of two years. $1.03^2 = 1.061$.

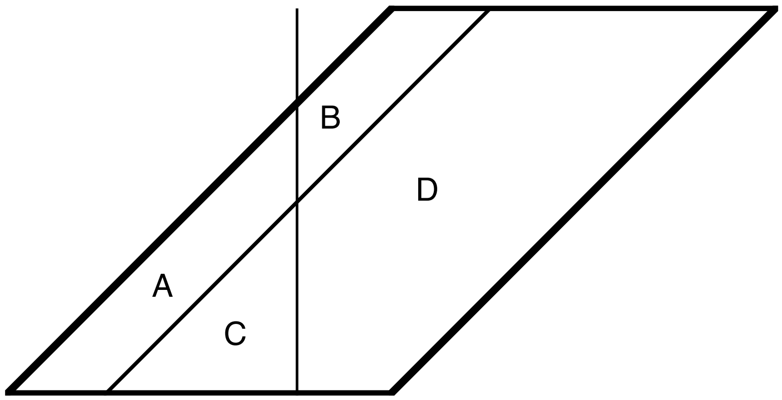
CY	Avg. Writ. Premium	Avg. Earn. Premium	1st Piece of Trend	2nd Piece of Trend	Trended Avg. EP	Earned Exposures	Trended Earn. Prem.
2003	$3777	$3605	1.109	1.061	$4241	1000	$4,241,478
2004	$3688	$3749	1.066	1.061	$4241	1050	$4,453,552
2005	$3998	$3899	1.025	1.061	$4241	1100	$4,665,626

For example, $(3605)(1.109)(1.061) \cong 4241$. $(1000)(4241) = 4.241$ million.

5.99. a. Assuming the rate level prior to April 1, 2004 is 1,

Area A: 1.00 Area B: 1.05 Area C: 1.10 Area D: (1.05)(1.10) = 1.155.

Policy Year 2004 Premium:



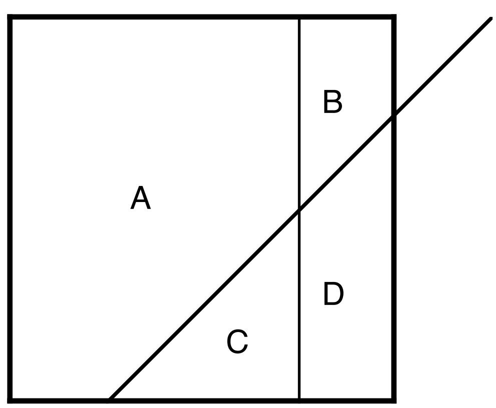
1/1/04 12/31/04 12/31/05

4/1/04 +10% Rate Change 10/1/04 +5% Law Change

b. Assuming the rate level prior to April 1, 2004 is 1,

Area A: 1.00 Area B: 1.05 Area C: 1.10 Area D: (1.05)(1.10) = 1.155.

Calendar Year 2004 Earned Premium:



1/1/04 12/31/04

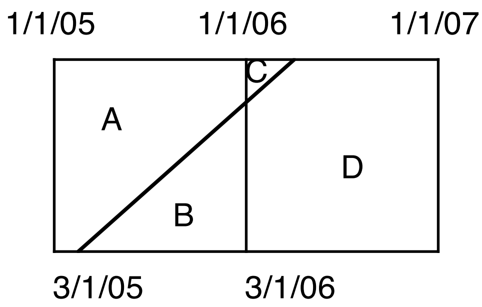
4/1/04 +10% Rate Change 10/1/04 +5% Rate Change

Comment: The letters used to label areas are arbitrary.

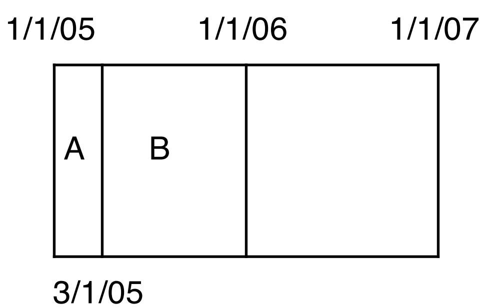
5.100. a. Area A = (1/2)(3/4)(3/4) - 1/8 = 5/32. Area B = 1/4 - 5/32 = 3/32.
Area C = (1/2)(1/2)(1/2) = 1/8. Area D = 3/4 - 1/8 = 5/8.
Average Rate Level for PY04: (5/32)(1.00) + (3/32)(1.05) + (1/8)(1.10) + (5/8)(1.155) = 1.114.
PY04 On-Level Factor is: 1.155/1.114 = **1.037**.
b. Area A = 3/4 - 1/8 = 5/8. Area B = 1/4 - 5/32 = 3/32.
Area C = (1/2)(1/2)(1/2) = 1/8. Area D = (1/2)(3/4)(3/4) - 1/8 = 5/32.
Average Rate Level for CY04 Earned Premium:
(5/8)(1.00) + (3/32)(1.05) + (1/8)(1.10) + (5/32)(1.155) = 1.041.
CY04 Earned Premium On-Level Factor is: 1.155/1.041 = **1.110**.

5.101. a. Written Premium are the dollar amounts charged by an insurer for policies written during the period in question.
b. Earned Premium is the portion of the policy premiums that have been exposed to loss during the period in question.
c. Unearned Premium is the portion of policy premium that have yet to be exposed to loss.
d. In-force Premium is the total premium of all policies that provide coverage at a specific point in time.

5.102. a. Area B is: (1/2)(5/6)(5/6) = 25/72. Area A is: 1 - 25/72 = 47/72.
Area C is: (1/2)(1/6)(1/6) = 1/72. Area D is: 1 - 1/72 = 71/72.



Average rate level during CY05 is: (1.00)(47/72) + (0.92)(25/72) = .9722.
OLF for CY05: 0.92/0.9722 = **0.9463**.
Average rate level during CY06 is: (1.00)(1/72) + (0.92)(71/72) = .9211.
OLF for CY06: 0.92/0.9211 = **0.9988**.
b. Area A is 1/6. Area B is 5/6.

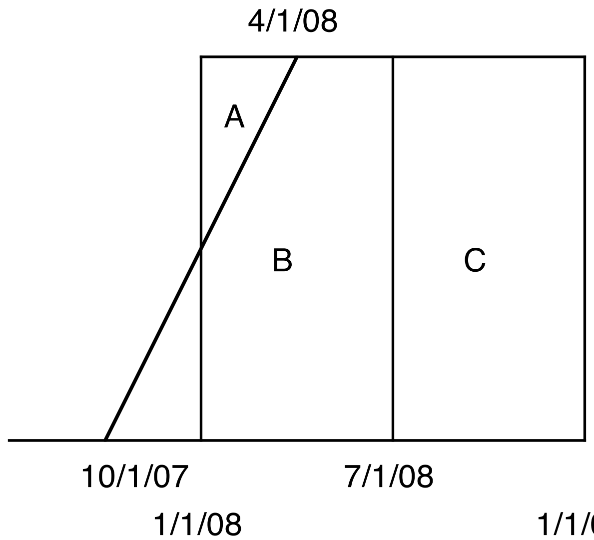


Average rate level during PY05 is: (1.00)(1/6) + (0.92)(5/6) = .9333.
OLF for PY05: 0.92/0.9333 = **0.9857**.
Average rate level during PY06 is: 0.92.
OLF for PY06: 0.92/0.92 = **1.0000**.
c. The extension of exposure method re-rates each policy using current rates.
In cases where material changes in exposure level have occurred over the period, or where there is a non-uniform pattern to the written exposures, the parallelogram method may not produce a reasonable approximation of on-level earned premium. The extension of exposure method does not share this potential problem.

5.103. a. A rate change can cause a one-time abrupt change in the average premium level.
The historical premiums should be put on a common rate level.
Alternately, a rating plan change can cause a one-time abrupt change in the average premium level.
For example, an insurer may start to give homes that have a fire and burglar alarm that rings at the police, fire or other monitoring station (security service) a 10% discount. Assuming you know the percentage of homes that will receive the new discount, this can be treated as rate decrease. Then the historical premiums should be put on a common level.
b. Assume we use Written Premiums.
Step one is a comparison of the 2007 written premium of 112 and the 2006 earned premium of 106. Step one factor is: 112/106 = 1.0566.
In the second step we trend from the 2007 written premiums to the proposed rate period.
The average date of writing of the 2007 written premiums is 7/1/07.
The average date of writing under the new rates is 7/1/09.
Therefore, the second step is two years long. The step two factor is: 1.04^2 = 1.0816.
Premium trend to apply to 2006 earned premiums: (1.0566)(1.0816) = **1.143**.
Alternately, assume we use earned premiums. Then, the step one factor is: 110/106 = 1.0377.
In the second step we trend from the 2007 earned premiums to the proposed rate period.
The average date of earning of the 2007 earned premiums is 7/1/07.
The average date of writing under the new rates is 7/1/09.
The policies are 6 months. Thus the average date of earning under the new rates is 10/1/09.
Therefore, the second step is 2.25 years long. The step two factor is: $1.04^{2.5}$ = 1.1030.
Premium trend to apply to 2006 earned premiums: (1.0377)(1.1030) = **1.145**.

5.104. a. CY 2007 written premium: 750 + 1,200 + 900 + 800 = **$3,650**.
b. CY 2008 earned premium: (1200) (3/12) + (900) (6/12) + (800) (9/12) + 850 = **$2,200**.
c. PY 2007 earned premium as of March 31, 2008:
750 + 1,200 + (900) (9/12) + (800) (6/12) = **$3,025**.
d. In-force premium as of July 1, 2008: 800 + 850 = **$1,650**.

5.105. a. Calendar Year 2008 earned premium consists of Areas A + B + C.



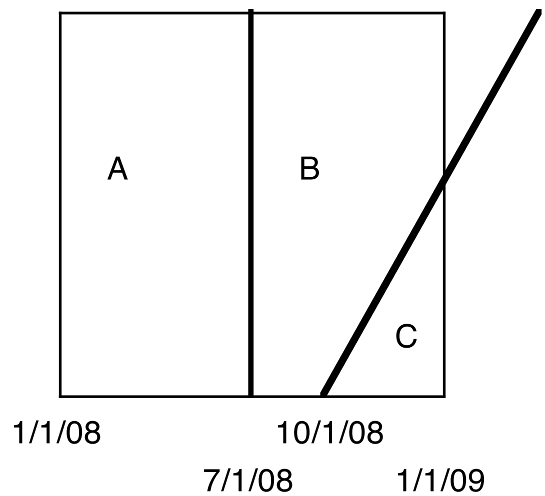
b. Area A = (1/2)(1/2)(1/4) = 1/16. Area B = 1/2 - 1/16 = 7/16. Area C = 1/2.
Rate Level for Area A = 1. Rate level for Area B = 1.05.
Rate Level for Area C = (1.05)(1.1) = 1.155.
Average rate level for Calendar Year 2008 earned premium:
(1/16)(1) + (7/16)(1.05) + (1/2)(1.155) = 1.099.
On-level factor = 1.155/1.099 = **1.051**.
c) While the parallelogram method assumes uniform earnings, snowmobile insurance is not uniformly earned throughout the year. (During the summer there are no accidents, while during the winter there can be accidents.)

5.106. a. The written premium should be used to analyze premium trend because it allows for the use of more recent data, which could reflect trends that have yet to show up in earned premium. Also, in the two-step trending procedure, the length of the uncertain projection period is shorter when using written premium than when using earned premium.
b. If premiums are not adjusted to current rate levels, then the calculated trend would reflect both the rate level changes and underlying premium trend. Rate level changes are one-time changes; however, the calculated trend would inappropriately assume that new rate level changes occur during the projection period, similar to those rate changes that occurred in the past.
Rather one should remove the effects of past rate changes from the premiums trend, and then specifically adjust the historical premiums used to determine the rate indication to put them on the current rate level.

5.107. a. i) average premium level will decrease since the new discount was not offset.
ii) Treat as a one time, measurable effect using on-level factors. In other words, treat it as a rate change. (We would need to know what percent of premiums get the new discount.)
b. i.) average premium level will increase since the Coverage A limit will increase with the inflation index and result in increased premiums.
ii) Treat as a gradual/continuous change and adjust historical premiums with a premium trend. Initially one would have to judgmentally adjust the historical premiums for the expected impact based on the changes in the external inflation index; once enough historical data was collected with the inflation guard endorsement, then its impact would be part of the premium trend calculated from the trend series.
Comment: For example, assume that the inflation guard endorsement applies to all new and renewal policies effective 1/1/2010. Then during 2010, we might be using AY2009 data as part of a rate indication for 2011. The average premiums for 2011 would include one year of the effect of the inflation guard endorsement, from 2010 to 2011, but the effect would not be reflected in (most of) the historical trend series. In contrast, by 2015 the effect of the inflation guard endorsement would be reflected in the historical trend series.

5.108. The average rate for CY08 earned is: 487,500/1000 = $487.5.
The average rate for CY09 written is: 682,500/1300 = $525.
Thus the first step of the trend is: 525/487.5.
The average date of writing for CY09 written is 7/1/09.
The average date of writing under the new rates is 7/1/11.
Thus the second step has two years of trend.
The trended CY08 earned premium is: ($487,500) (525/487.5) ($1.05^2$) = **$578,813**.
Comment: We use the average rate for CY09 written, the most recent data available, for the end of the first step and the beginning of the second step. CY09 written is more current than CY09 earned.
I have assumed that the projected premium trend is what would occur in the absence of exposure growth. Thus the trended CY08 earned premium would be on the basis of the actual number of exposures for 2008. What we are picking up is changes in premium other than due to any rate changes and exposure growth. For example, this could be due to a change in the mix of exposures by class and territory. For Homeowners, it would also be due to changes in the average values of homes insured. These should affect in a similar manner a loss trend calculated from the changes in average claim costs.

5.109. (a) The policies are semi-annual, so the line representing the rate change has a slope of 2. The law change is a vertical line.

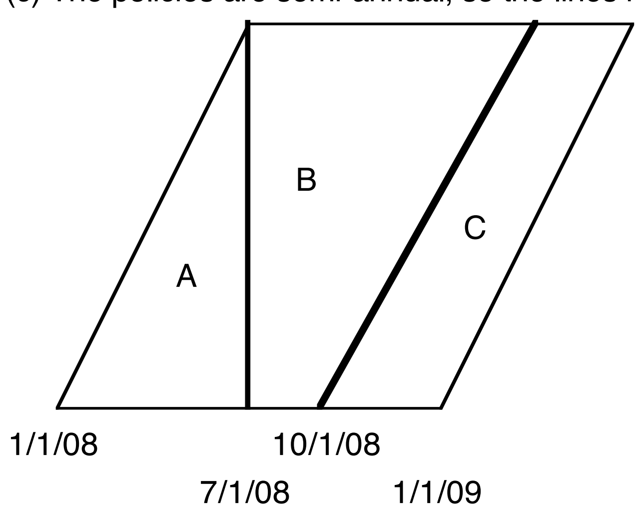


(b)	Rate Level	Area
A	1.00	1/2
B	1.10	1/2 - 1/16 = 7/16
C	(1.10)(1.05) = 1.155	(1/2)(1/4)(1/2) = 1/16

Average Rate Level = (1/2)(1) + (7/16)(1.10) + (1/16)(1.155) = 1.05344.
Current Rate Level = (1.10)(1.05) = 1.155.
On-Level Factor = 1.155 / 1.05344 = **1.0964**.

⇒ On-Level 2008 calendar year earned premium is $1.0964 million.

(c) The policies are semi-annual, so the lines representing PY08 have a slope of 2.



Comment: For the PY08 diagram, Area A = 1/4, Area C = 1/4, and thus Area B = 1/2.
Average Rate Level = (1/4)(1) + (1/2)(1.10) + (1/4)(1.155) = 1.08875.
On-Level Factor = 1.155 / 1.08875 = 1.06085.

5.110. a. At the beginning of each quarter the insurer writes 5 teens and 5 adults.
Prior to the change, the premium for teens is: (1000)(2) + 50 = 2050,
and the premium for adults is: (1000)(1) + 50 = 1050.
After to the change, the premium for teens is: (1100)(2) + 50 = 2250,
and the premium for adults is: (1100)(0.9) + 50 = 1040.
Written on 1/1/10: (5)(2050) + (5)(1050) = 15,500. All earned in CY10.
Written on 4/1/10: (5)(2050) + (5)(1050) = 15,500. 3/4 earned in CY10, due to annual policies.
Written on 7/1/10: (5)(2250) + (5)(1040) = 16,450. 2/4 earned in CY10.
Written on 10/1/10: (5)(2050) + (5)(1050) = 16,450. 1/4 earned in CY10.
(1)(15,500) + (3/4)(15,500) + (2/4)(16,450) + (1/4)(16,450) = **$39,462.5.**
b. If the current rates had been in effect, the earned premium of CY10 would have been:
(1)(16,450) + (3/4)(16,450) + (2/4)(16,450) + (1/4)(16,450) = $41,125.
The on-level factor is: 41,125 / 39,462.4 = **1.0421**.
Comment: Unlike here, usually there would be policies written in 2009 that would contribute to the CY10 earned premium. That would have made the question longer but not harder.

5.111. a. I assume the trend factors are intended to be applied to CY earned premiums.
For the first step we go from the CY earned to CY10 written.
For the second step we go from CY10 written premiums to the proposed effective period.
The average date of writing in CY10 written is 7/1/10.
The average date of writing during the effective period is 10/1/12.
So the second step is 2.25 years long.
Trend factor for CY09 earned premiums: (395/375) $1.05^{2.25}$ = **1.176**.
Trend factor for CY10 earned premiums: (395/390) $1.05^{2.25}$ = **1.130**.
b. The two-step trending method is appropriate when the last data point, in this case average written premiums, is sufficiently stable to be reliable. It would probably not be appropriate to use if we were instead trending average claim costs. The two-step trending method only has an advantage over a one-step method when we have reason to believe that the historical and prospective trends will be different.

5.112. a. True. Calendar year are accounting numbers and should not change once they are determined at the end of the year. Calendar Year 2011 numbers are based on the transaction during 2011; they are not affected by anything that happens during 2012. At the end of the calendar year, the calendar year premium is fixed.
b. True. Calendar year are accounting numbers and should not change once they are determined at the end of the year. Any premium that is earned during 2012 will not affect CY 2011. At the end of the calendar year, the calendar year premium is fixed.
c. False. Policy year 2011 written premium would change as the final premium for policies effective during 2011 was determined. There can be changes during 2012 due to cancellations and endorsements on policies written during 2011. For commercial policies the final premium may depend on audits of payroll or sales after policy expiration, which for policies written during 2011 could take place in 2012 or 2013.
d. False. Only a portion of Policy 2011 premium is earned by year end. For example an annual policy written July 1, 2011 will only be half earned at December 31, 2011. Also PY 2011 earned premium would be affected by any cancelations and endorsements during 2012 on policies written during 2011. Again for commercial policies the final premium may depend on audits.
Comment: See pages 65-66 of Basic Ratemaking.
Calendar numbers are sometimes revised (restated) if significant accounting errors are found.
Calendar 2011 numbers should be available in January 2012.

5.113. a. If for example there was an 5% average rate increase a year ago, there may have been a rate decrease of 5% in one class and a 15% rate increase in another class. Applying an on-level of 1.05 to both classes would not put each ones premiums on the current rate level. Thus using the aggregate on-level factor would not be correct,
b. The parallelogram method does not require detailed exposure data by class and territory, while the extension of exposures method does.
The extension of exposures gives a more accurate result by directly applying the current rate manual to the past book of business; the parallelogram is an approximation based on certain assumptions, such as a constant rate of writing throughout a year.
The extension of exposures produces results that can be used for classification ratemaking analysis, while the parallelogram method is generally applied at the aggregate level and thus the adjusted premium will be unacceptable for any classification ratemaking analysis.
Comment: See pages 73 and 80 of Basic Ratemaking.
If there had been no revision to class relativities during a past rate change, then it would be appropriate to adjust premiums with an aggregate on-level factor.
In some commercial lines products, underwriters can apply subjective debits and credits to manual premium. This complicates the use of the extension of exposures technique since it may be difficult to determine what debits and credits would be applied under today's schedule rating guidelines.

5.114. a. The average premium for CY09 is: $5,000,000/10,000 = $500.
Thus the first step of the premium trend is: 560/500 = 1.12.
The average date of writing for 4Q11 is 11/15/11.
Since rate review are performed every 2 years, we assume the new rates will be in effect for 2 years. Thus, the average date of writing under the new rates is one year after the proposed effective date, or 7/1/13.
The length of the second step is: one year and 7.5 months, or 1.625 years.
Second step trend factor is: $1.05^{1.625} = 1.0825$.
Projected earned premium for CY09 is: (1.12)(1.0825)(5 million) = **6.062 million**.
b. The first step should be fine. The second step assumed a +5% prospective annual premium trend based on historical data. An automatic increase in amount of insurance is one of the major reasons for the change in the average premium at current rate level per exposure. However, the assumed annual increase in the amount of insurance to account for inflation was materially reduced post-January 1, 2012. Thus the future trend of premium should be less than in the past, and the assumed 5% is significantly too high. There is not enough information given to determine what a better assumption should be, but let us assume for example that 2% would be more reasonable.
Then the second step trend factor would be instead: $1.02^{1.625} = 1.0327$.
Then the projected earned premium for CY09 would be: (1.12)(1.0327)(5 million) = 5.783 million.
Comment: An automatic increase in amount of insurance is one of the major reasons for the change in the average premium at current rate level per exposure; however, there are other reasons. If the book of business is changing, this can either increase or decrease the average premium. Also insureds may build additions or make major renovations to their houses, which should result in an increase in their amount of insurance. Also insureds may change their amount of insurance if the market value of their home changes significantly. Reinspections of insured homes can change the amount of insurance, and thus a change in the rate of reinspections will affect the rate of change in average premiums.

5.115. (a) The average premiums are: \$104,500,000 / 110,000 = \$950,
\$113,800,500 / 121,000 = \$940.50, and \$123,916,100 / 133,100 = \$931.00.
940.5/950 = 0.990. 931/945.5 = 0.990.
Thus select a semi-annual premium trend factor of 0.990.
CY2012 earned premium has an average date of earning of 7/1/2012.
Since there annual policies, the average date of writing is 6 months earlier 1/1/2012.
The average date of writing under the new rates is 7/1/2014.
Thus the trend period is 2.5 years or 5 half-years.
The calendar year 2012 projected earned premium at current rate level is:
(0.990^5) (\$114,208,050) = **\$108,610,719**.
(b) There are continuous gradual effects, for example shifts in the mix of business, that affect both premiums and losses. Assuming the loss trend is based on insurance data, then it will reflect these effects. Therefore, we need to apply premium trend reflecting these effects so that the losses and premiums are on a comparable basis.
Premium trend is particularly important for a line of insurance such as homeowners, where insured value (usually) increases over time, thus raising the average premiums in the absence of rate changes. For the same reason, premium trend is particularly important for a line of insurance that uses an inflation sensitive exposure base such as sales or payroll.
(c) For example, if there were a large rate increase during the historical period, then historical written premium levels would show a large jump, and the calculated premium trend would be large.
We would implicitly be assuming that a similar large rate increase would be occurring during the trend period; this is probably not the case. It is better to put the historical premiums on a common rate level, so that the premium trend is an estimate of what would occur in the future in the absence of rate changes. Then we can incorporate the one-time effects of actual rate changes by applying on-level factors to premiums.

(d) Less coverage. ⇔ Less Premium.

⇒ The premium for a \$500 deductible is smaller than that for a \$100 deductible.

The projected premium in part (a) does <u>not</u> take this into account.

Therefore, the true projected premium will be **lower** than in part (a).
<u>Comment</u>: It would have been helpful to me to know what line of insurance we are dealing with.
In part (d), the true trended losses will also be smaller than would be gotten by trending losses, without taking into account this planned change. Assuming the current deductible relativities are approximately correct, it would not have a significant effect on the overall rate indication, provided this proposed change were ignored with respect to <u>both</u> premiums and losses.

5.116. All policies are 6-month, so policies effective January 1, 2011 - June 30, 2011 contribute nothing to the calendar year 2012 earned exposures,
policies effective July 1, 2011 - December 31, 2011 contribute on average half their exposures,
policies effective January 1, 2012 - June 30, 2012 contribute all of their exposures, and policies effective July 1, 2012 - December 31, 2012 contribute on average half their exposures.
At current rate level, class A pays: ($500)(1.00) + $55 = $555,
while class B pays: ($500)(0.80) + $55 = $455.
Calendar year 2012 earned premium at current rate level is:
($555)(150/2 + 175 + 200/2)(1000) + ($455)(100/2 + 150 + 200/2)(1000) = **$330.75 million**.
<u>Comment</u>: The expense constant is earned over time, just as with any other premium.
I have assumed that the $55 expense constant applies per exposure; this makes sense based on the given rating algorithm and the fact that the given base rates are per exposure, but this could have been made clearer.

5.117. (a) Earned exposures for each policy by quarter:

Policy Number	Quarter 1	Quarter 2	Quarter 3	Quarter 4
14902	2/12	0	0	0
14903	3/12	3/12	3/12	2.5/12
14904	0	0.5/12	3/12	3/12
Total	5/12	3.5/12	6/12	5.5/12

The last policy has a premium of: (1.2)(1680) = 2016.
In each case, earned premium = (annual policy premium) (earned exposures).
Earned premiums for each policy by quarter:

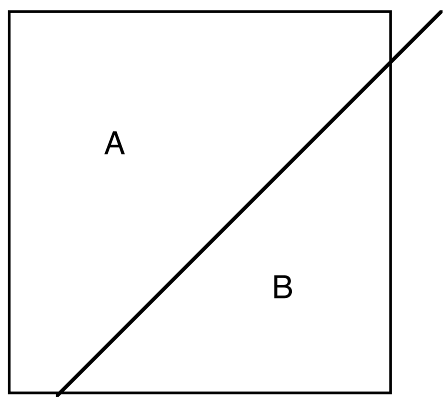
Policy Number	Quarter 1	Quarter 2	Quarter 3	Quarter 4
14902	280	0	0	0
14903	420	420	420	350
14904	0	84	504	504
Total	700	504	924	854

(b) At the end of 2012, Policy 14902 has 8/12 unearned: (1680)(8/12) = 1120.
At the end of 2012, Policy 14903 has 11.5/12 unearned: (1680)(11.5/12) = 1610.
Total unearned premium at December 31, 2012: 1120 + 1610 = **2730**.
At the end of 2013 only one policy has unearned premium; 5.5 months out of 12 are unearned.
Total unearned premium at December 31, 2013 = (2016)(5.5/12) = **924**.

5.118.

Effective Date	Overall Rate Change	Rate Level Index
		1.0000
January 1, 2010	+4.2%	1.0420
March 1, 2011	+0.3%	1.0451
January 1, 2012	-1.7%	1.0274
June 1, 2013	+1.0%	1.0376

(a) Area B = $(10/12)^2/2$ = 0.3472. Area A = 1 - 0.3472 = 0.6528.



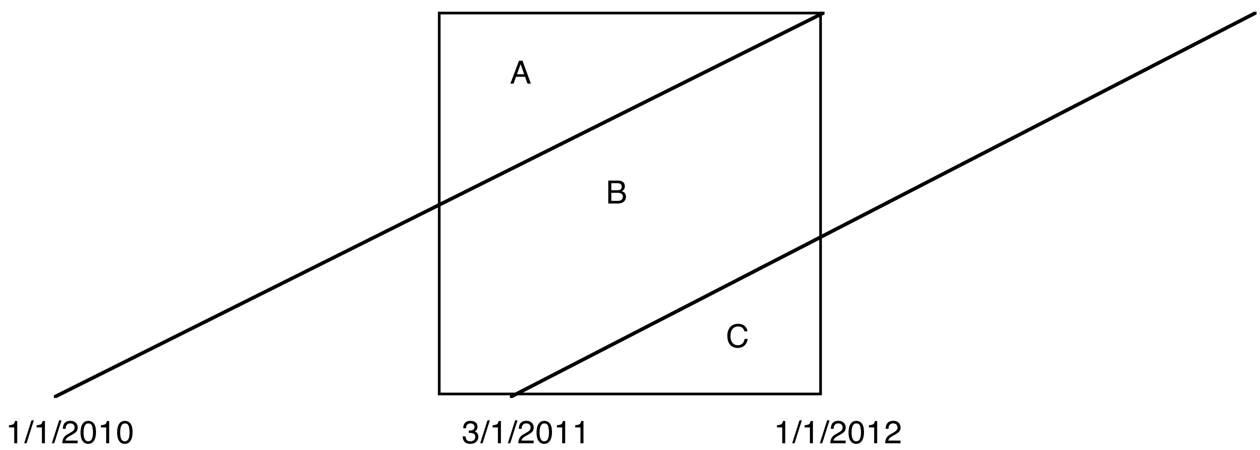
On level factor is: $\dfrac{1.0376}{(0.6528)(1.042) + (0.3472)(1.0451)}$ = **0.995**.

(b) The 25% of policies written on January 1, 2012, are all at rate level 1.0274.
For the remaining policies in-force on February 1, 2012, 1/12 are written prior to March 1, 2011 at rate level 1.0420, 1/12 are written during January 2012 at rate level 1.0274, while the remaining 10/12 are written at rate level 1.0451.
Thus the average rate level for the policies in-force on February 1, 2012 is:
(25%)(1.0274) + (75%) {(1/12)(1.0420) + (10/12)(1.0451) + (1/12)(1.0274)} = 1.0394.
Thus the on level factor is: 1.0376 / 1.0394 = **0.9983**.
Alternately, let us assume that 120 policies are written evenly throughout the year and 40 policies are written on January 1. Let us assume that the rate prior to January 1, 2010 is $100.
Then the rate on January 1, 2010 is $104.20, on March 1, 2011 is $104.51,
on January 1, 2012 is $102.74, and on June 1 2013 and currently is $103.76.
Then on February 1, 2012 we have in-force 160 policies:

Effective Date	Number of Policies	Rate	Premium
February 2011	10	$104.20	$1042
March 2011 to Dec. 2011	100	$104.51	$10,451
Jan. 2012 including Jan. 1	10 + 40 = 50	$102.74	$5137

Thus the on-level factor is: $\dfrac{(160)(\$103.76)}{\$1042 + \$10,451 + \$5137}$ = **0 .9983**.

(c) If we have two year policies, then more of the CY2011 earned premium was written at a lower rate level in effect prior to the 1/1/2010 rate increase, than if we have one year policies. Also less of the premium would be written at the higher rate level in effect after the 3/1/2011 rate increase. Thus the average rate level during CY 2011 would be less with two year policies than it was with one year policies.
Therefore, the on-level factor is **greater** when we have two year policies.
Comment: Here is the calculation when there are two year policies.



Area A = (1/2)(1)/2 = 1/4. Area C = (5/12)(10/12)/2 = 25/144. ⇒ Area B = 83/144.

On level factor is: $\frac{1.0376}{(1/4)(1.0) + (83/144)(1.042) + (25/144)(1.0451)} = 1.005.$

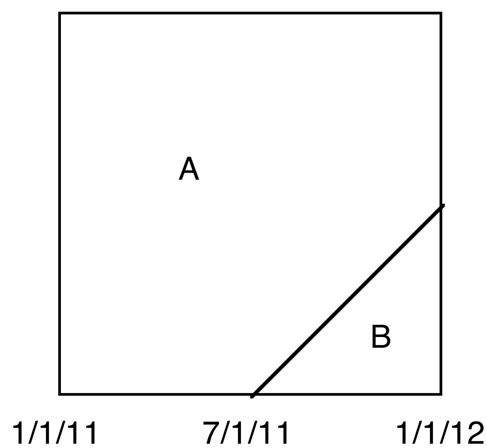
This on level factor of 1.005 is indeed greater than that with one year policies of 0.995.

5.119. (a) CY2012 written premium: 5000 + 1000 + 500 + 5000 = **11,500.**
CY2013 written premium: 2000 + 1500 = **3500**.
CY2012 earned premium: 5000 + (3/4)(1000) + 500 + (3/24)(5000) = **6875**.
CY2013 written premium: (1/4)(1000) + (12/24)(5000) + 2000 + (1/2)(1500) = **5500**.
(b) Motorcycles written in a winter climate would typically only have exposure to loss in the spring, summer and fall months. As a result, the insurer might recognize this difference by modifying the even earnings throughout the policy term. Written premium would be unaffected.
Comment: Snowmobile insurance would typically only have exposure to loss in the winter months, plus depending on the climate perhaps the early spring and late fall months.

5.120. (a) The key assumption is that exposures are uniformly distributed over time.

(b)

Date	Rate Level Index
1/1/11	1.000
7/1/11	1.070
5/1/12	(0.98)(1.070) = 1.0486
4/1/13	(0.97)(1.0486) = 1.017142



Area B = $(1/2)(1/2)^2$ = 1/8. Area A = 1 - 1/8 = 7/8.

Average rate level for CY11 earned premium: (7/8)(1.000) + (1/8)(1.070) = 1.00875.

On-level factor to be used to adjust calendar year 2011 earned premium to current rates is:
1.017142 / 1.00875 = **1.00832**.

(c) For a state-mandated increase in minimum policy limits, the average premium would increase to reflect such a change but there is also the expectation that losses would increase as well since policyholders would receive more coverage. As a result, the change should have no effect on the on-level calculation. (I have assumed that the current ILFs are appropriate.)

Alternately, one could adjust both the premiums and losses for the increase in minimum policy limits. However, this would require great care in order to get good estimates of these effects, and in any case the effects on premiums and losses would largely offset each other.

5.121. Policy 1 is a 1-year policy that has 5 out of 12 months in 2013. Its audit occurs on 11/30/13.
Policy 2 is a 6-month policy that has 6 out of 6 months in 2012. Its audit occurs on 6/30/13.
Policy 3 is a 1-year policy that has 10 out of 12 months in 2013. Its audit occurs on 8/31/14.
Policy 2 is an 8-month policy that has 5 out of 8 months in 2013. Its audit occurs on 9/30/14.

(a)

Policy	Contribution from Initial Premium	Contribution from Audit	Total
1	(5/12)(480)	(8%)(480)	238.4
2	0	(8%)(125)	10
3	(10/12)(225)	0	187.5
4	(5/8)(300)	0	187.5
Total			**623.4**

(b)

Policy	Contribution from Initial Premium	Contribution from Audit	Total
1	0	(8%)(480)	38.4
2	0	(8%)(125)	10
3	225	0	225
4	300	0	300
Total			**573.4**

(c)

Policy	Contribution from Initial Premium	Contribution from Audit	Total
1	0	0	0
2	0	0	0
3	(10/12)(225)	0	187.5
4	(5/8)(300)	0	187.5
Total			**375**

(d)

Policy	Contribution from Initial Premium	Contribution from Audit	Total
1	0	0	0
2	0	0	0
3	225	(8%)(225)	243
4	300	(8%)(300)	324
Total			**567**

Comment: No contribution to PY2013 from the first two policies not written during 2013.
Audit premium is earned as soon as it is written.

5.122. The parallelogram method requires less detailed information. However, it is based on the estimated overall impacts of past rate changes, which are in turn based on the corresponding prior estimates of portion of exposures written in each classification cell. In addition, the parallelogram method assumes a constant level of exposures are written over time. Thus the result of using the parallelogram method will be an approximation, which depending on circumstances may not be very good. The very detailed class plan decreases the likely accuracy of using the parallelogram method.
In contrast, extension of exposures requires the historical exposures by classification cell. In other words for each past policy we need to know how it would have been classified using the current classification rating plan. (Unless there have been major changes to the plan since the historical premiums have been written, this should not be a problem.) Then we just extend the past exposures by the current rates to adjust the past premiums to the current level. Extension of exposures is generally more accurate than the parallelogram method.
In summary, the parallelogram method is easier while the extension of exposures is more accurate. Assuming the necessary detailed information is available, extension of exposures would be greatly preferred to the parallelogram method, particularly for a personal auto insurer with a highly-refined classification rating plan.
In addition, the parallelogram method uses overall rate changes and thus would not work for an analysis of classification rates (assuming past rate changes varied by class) while the extension of exposures would work.

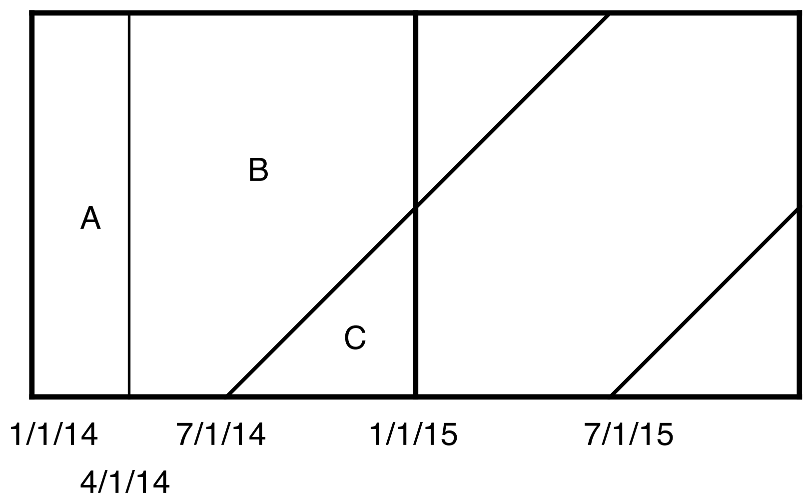
5.123. (a)

Policy 1		Policy 2	
Renewal Date	Written Premium	Renewal Date	Written Premium
April 1/10	900	Sept. 1/11	1,200
April 1/11	900	Sept. 1/12	(1,200)(1.05) = 1,260
April 1/12	900		
April 1/13	(900)(1.05) = 945		

2012 earned premium:
Policy 1 = 900
Policy 2 = (8/12)(1,200) + (4/12)(1,260) = 1,220.
Total = 900 + 1,220 = **2,120**.
2013 earned premium:
Policy 1 = (3/12)(900) + (9/12)(945) = 933.75.
Policy 2 = (2/12)(1,260) = 210.
Total = 933.75 + 210 = **1,143.75**.
(b) CY 2012 total written premiums = 900 + 1,260 = **2,160**.
(c) As of December 31, 2012, we do not know that Policy #2 will be canceled on March 1, 2013.
Total unearned premiums at December 31, 2012
= (3/12)(900) + (8/12)(1,260) = **1,065**.
(d) 2012 on level earned premiums = 945 + 1,260 = **2,205**.

5.124.

Effective Date	Overall Change	Type of Change	Rate Level Index
Prior			1.0000
April 1, 2014	+5%	Law	1.0500
July 1, 2014	+3%	Rate	1.0815
July 1, 2015	-7%	Rate	1.0058

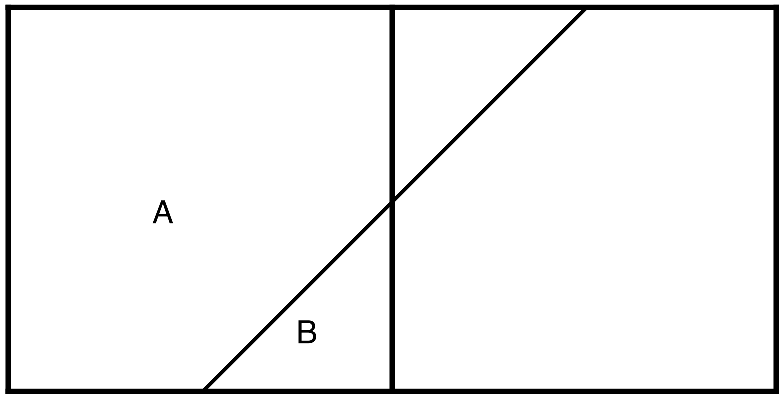


Area A = 1/4. Area C = (1/2)(1/2(1/2) = 1/8. Area B = 1 - 1/4 - 1/8 = 5/8.
Thus the on-level factor for 2014 earned premium is:

$$\frac{1.0058}{(1)(1/4)\ +\ (1.050)(5/8)\ +\ (1.0815)(1/8)} = 0.9658.$$

Increasing the minimum deductible sold will decrease the average premium.
The series of average earned premiums does <u>not</u> seem to have been adjusted for the change in the minimum deductible sold.
Since annual policies are being sold, the year ending June 2013 is the first value all on the basis of the new minimum deductible. Thus we can use the last 7 points.
I will select a premium trend of 3.4% based on the last 6 points.
(The trend based on the last 4 points of 3.1% would be similar.)
The average date of writing under the new rates is 7/1/2016.
Since annual policies are being sold, the average date of earning is 6 months later or 1/1/2107.
Thus the premium trend is from 7/1/2014 to 1/1/2017 or 2.5 years.
The trended calendar year 2014 earned premium at current rate level is:
(100,000)(0.9658)($1.034^{2.5}$) = **$105,000**.
<u>Comment</u>: Slightly different premium trend selections would also be reasonable.

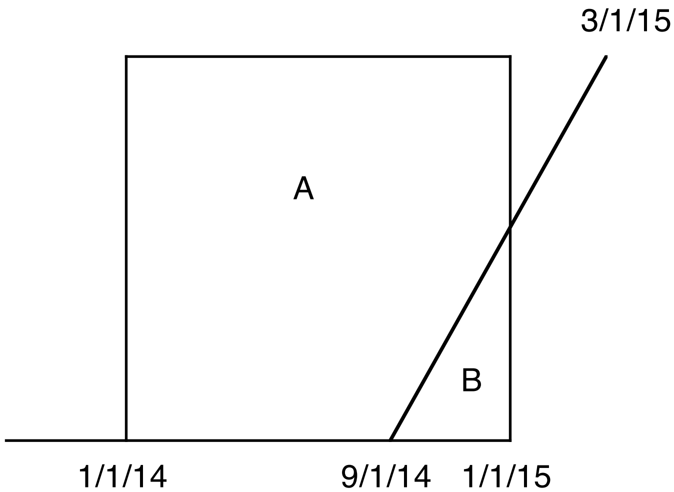
5.125. (a) Area B = (1/2)(1/2)(1/2) = 1/8. Area A = 1 - 1/8 = 7/8.



On-level factor is: $\frac{1.1}{(7/8)(1) + (1/8)(1.1)} = 1.08642.$

On-level earned premium is: (1.08642)(1000) = **$1086.42**.
(b) 1/8 of 2013 quarter 1 is earned during 2014.
3/8 of 2013 quarter 2 is earned during 2014.
5/8 of 2013 quarter 3 is earned during 2014.
7/8 of 2013 quarter 3 is earned during 2014.
(1+3/4)/2 = 7/8 of 2014 quarter 1 is earned during 2014.
5/8 of 2014 quarter 2 is earned during 2014.
3/8 of 2014 quarter 3 is earned during 2014.
1/8 of 2014 quarter 3 is earned during 2014.
Average rate level for 2014 earned premium is:
(1) {(1/8)(10%) + (3/8)(50%) + (5/8)(30%) + (7/8)(10%) + (7/8)(10%) + (5/8)(50%)}
+ (1.1) {(3/8)(30%) + (1/8)(10%)} = 0.875 + (1.1)(0.125) = 1.0125.
On-level factor is: 1.1 / 1.0125 = 1.08642.
On-level earned premium is: (1.08642)(1000) = **$1086.42**.
Alternately, assume that the given pattern is for the volume of <u>earned</u> exposures.
1/8 of the exposures earned during the 3rd quarter of 2014 is on the new rate level.
3/8 of the exposures earned during the 4th quarter of 2014 is on the new rate level.
Then the average rate level for 2014 earned premium is:
10% + 50% + (30%){7/8 + (1/8)(1.1)} + (10%){5/8 + (3/8)(1.1)} = 1.0075.
On-level factor is: 1.1 / 1.0075 = 1.09181.
On-level earned premium is: (1.09181)(1000) = **$1091.81**.
(c) The insurer could use extension of exposures, where each policy is rerated using the current rate manual, and then the amounts earned from each policy during CY2014 are aggregated.
Alternately, the insurer could use the parallelogram method on smaller time periods, such as monthly, and then aggregate the on-level premium from the smaller time periods to determine the total on-level earned amount.
<u>Comment</u>: The fact that part (a) and (b) can result in the same answer is a coincidence.

5.126. (a) CY14 written exposures: (600 + 300) / 2 = **450 caryears**.
Policy Written 8/1/13 contributes 1/6 of its exposures to CY14 earned exposures.
CY14 earned exposures: {800/6 + 600 + (300) (5/6)} / 2 = **491.67 caryears**.
(b) The current rate is: (1 - 18%)($500) = $410. However, this is for one half caryear.
Using extension of exposures, CY14 Earned premium @ current rate level:
(491.67)($410)(2) = **$403,169**.



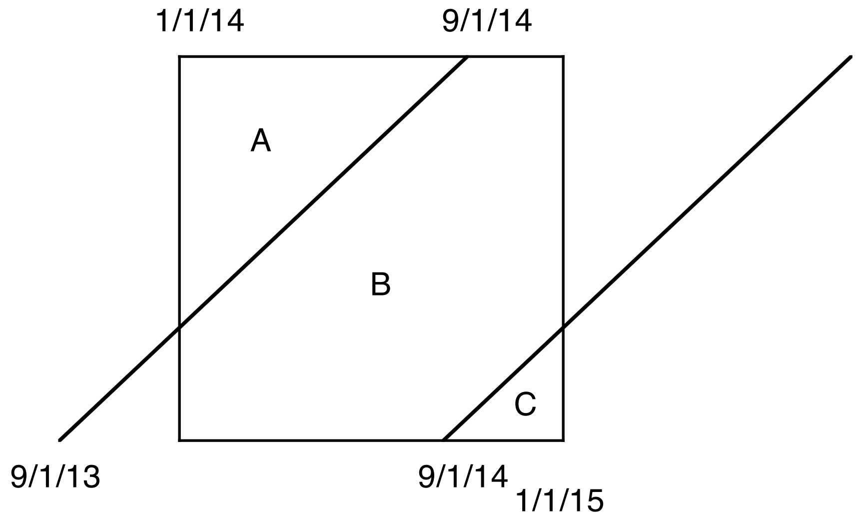
Area B is: (1/2)(4/12)(8/12) = 1/9.
CY14 earned average rate level index is: (8/9)(1) + (1/9)(0.82) = 0.980.
OLF is: 0.82 / 0.980 = 0.837.
The CY14 earned premium at actual rates is: (491.67)($500)(2) = $491,670.
Using parallelogram method, CY14 Earned premium @ current rate level:
(0.837)($491,670) = **$411,528**.
Whenever the appropriate data is available, extension of exposures is more accurate than the parallelogram method. In this case, the parallelogram method is undesirable since the exposures are neither written evenly throughout the year nor constant from year to year.

5.127. Average date of writing is 7/1/16 plus 6 months = 1/1/17.
Since there are 6 month policies, the trend to date is: 1/1/17 + 3 months = 4/1/17.
Trend from date is 7/1/2012. Trend period is 4.75 years for a one step method.
For the severity trend, I will select a 3% trend because all of the indicated trend values seem to hover around this value.
For the frequency trend, I will select a trend factor of 1%. This is because the trend value from year end 2012 (8 point trend) is negative, but there appears to be a positive trend going into the future. Therefore, I judgmentally selected a factor in between instead of doing a 2 step trend.
Thus, the one step pure premium trend factor is: $\{(1.01)(1.03)\}^{4.75}$ = **1.206**.
Alternately, the annual pure premium trends based on 12, 16, and 20 points are:
1.0040, 0.9970, 1.0023.
I will select the 16 point trend based on a compromise between stability and responsiveness.
Thus the one step pure premium trend factor is: $\{(0.968)(1.030)\}^{4.75}$ = **0.986**.
Alternately, one can use the two-step method.
Step one of the trend period is from the average date of accident for AY12 to the most recent data available in the trend series, which is the year 2014: 7/1/12 to 7/1/14.
Step two is from 7/1/14 to 4/1/17, which is the average date of accident under the new rates.
The frequency trend has changed significantly. I will therefore use a different trend for different periods. I will select -2.5% for the first step and 2.9% for the second step.
The severity trend is stable; I will select 3.1% for both steps.
Thus, the two step pure premium trend factor is: $\{(0.975)(1.031)\}^2 \{(1.029)(1.031)\}^{2.75}$ = **1.189**.
Comment: Many other selections would be reasonable.
Be sure to specify whether you are using the one-step or two-step method.

5.128. (a) Policy A is canceled before the end of 2015, so it contributes 1000/2 = 500.
Policy B is rerated prior to the end of 2015, so it contributes: (1/4)(500) + (3/4)(400) = 425.
Total CY2015 written premium is: 500 + 425 + 1000 = **$1925**.
(b) Policy B has CY2015 earned premium of: (1/4)(500) + (1/4)(400) = 225.
Six-month Policy C has CY2015 earned premium of: (1/2)(1000) = 500.
Total CY2015 earned premium is: 500 + 225 + 500 = **$1225**.
(c) In-force premium as of October 1, 2015 is: 0 + 400 + 1000 = **$1400**.
(d) I will assume that each policy covers one car.
CY2015 earned exposures for Territory 1: 1/2 + 1/4 + 0 = **3/4 car year**.
CY2015 earned exposures for Territory 2: 0 + 1/4 + 1/4 = **1/2 car year**.
Comment: For Policy B, 3 months is at $500 (annual basis),
while the remaining 9 months is at $400 (annual basis).

5.129. (a)

Effective Date	Rate Change	Rate Level Index
September 1, 2012	-10%	0.9
September 1, 2013	-5%	0.855
September 1, 2014	-3%	0.82935
February 1, 2015, outstanding	-15%	0.7049475



September 1 is 2/3 of the way through the year.

Area A = $(1/2)(2/3)^2$ = 4/18. Area C = $(1/2)(1/3)^2$ = 1/18. Area B = 1 - 4/18 - 1/18 = 13/18.

The February 1, 2015 law change has no effect on CY14 earned premiums.

The OLF is:

$$\frac{0.7049475}{(4/18)(0.9) + (13/18)(0.855) + (1/18)(0.82935)} = 0.7049475/0.863575 = \mathbf{0.8163}.$$

(b) 1. The parallelogram method assumes that policies are written evenly throughout the year.
In situations where this assumption fails to hold, one can apply the parallelogram method using more refined periods of time than a year, such as for example quarters or months.
Alternately, one can instead employ extension of exposures.
2. The parallelogram method is generally applied at the aggregate level using a series of overall average changes. So, while the overall premium may be adjusted to an approximated current rate level, the premium for certain classes will not be on-level if as is common the implemented rate changes varied by class. Consequently, the adjusted premium will likely be unacceptable for any classification ratemaking analysis.
The extension of exposures method does not share this shortcoming and can be used instead.
<u>Comment</u>: In part (b), only discuss one weakness.

5.130. (a) Since it is more current than the earned premium, I will use the fourth quarter 2016 average written premium as the pivot point for the two-step trending method.
The average date of writing under the new rates is: January 1, 2018 + 6 months = July 1, 2018.
The projection period is from average date of writing November 15, 2016 to July 1, 2018 or 19.5 months.
I am assuming we are using CY earned premiums in the rate indication.
Thus the first step is the ratio of the fourth quarter 2016 average written premium of $242 over the CY earned premium.

C Y	First Piece of Trend	Second Piece of Trend	Trend Factor for Earned Premium
2014	242/210 = 1.152	$0.98^{19.5/12} = 0.968$	(1.152)(0.968) = **1.115**
2015	242/220 = 1.100	$0.98^{19.5/12} = 0.968$	(1.100)(0.968) = **1.065**
2016	242/235 = 1.030	$0.98^{19.5/12} = 0.968$	(1.030)(0.968) = **0.997**

Alternately, since we are working on earned premiums to be used in the rate level indication, I will use the fourth quarter 2016 average earned premium as the pivot point for the two-step trending method.
The average date of writing under the new rates is: January 1, 2018 + 6 months = July 1, 2018.
Since we have annual policies, the average date of earning under the new rates is:
July 1, 2018 + 6 months = January 1, 2019.
The projection period is from average date of earning November 15, 2016 to January 1, 2019 or 25.5 months.
The first step is the ratio of the fourth quarter 2016 average earned premium of $236 over the CY earned premium.

C Y	First Piece of Trend	Second Piece of Trend	Trend Factor for Earned Premium
2014	236/210 = 1.124	$0.98^{25.5/12} = 0.958$	(1.124)(0.958) = **1.077**
2015	236/220 = 1.073	$0.98^{25.5/12} = 0.958$	(1.073)(0.958) = **1.028**
2016	236/235 = 1.004	$0.98^{25.5/12} = 0.958$	(1.004)(0.958) = **0.962**

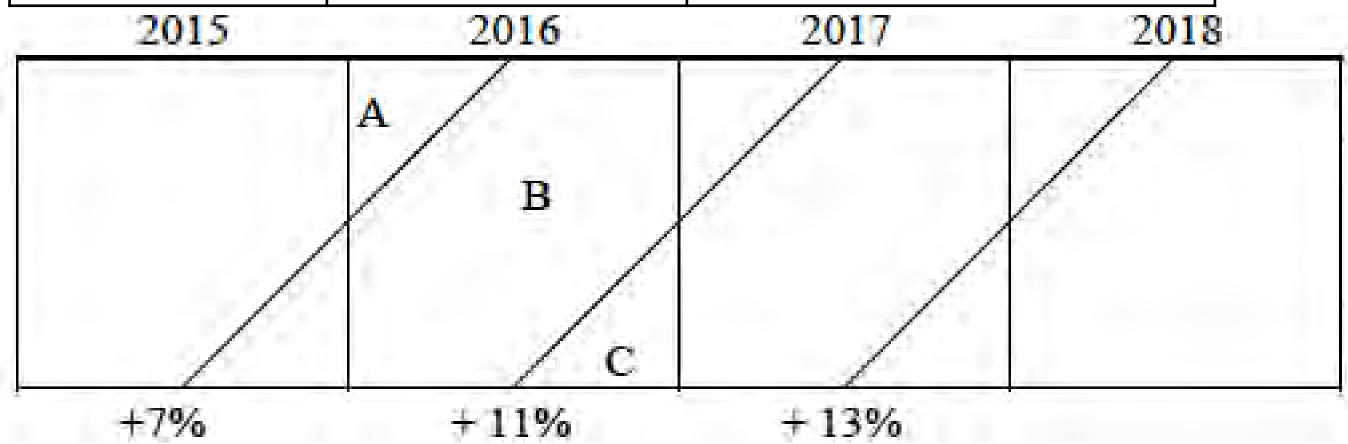
(b) 1. Over the recent past, the insurer has been writing more business in lower rated classes and territories and less business in higher rated classes and territories.
2. For Homeowners Insurance, if the average value of the homes insured has been decreasing.
(I am assuming that the average premiums are calculated per houseyear.)
3. Insureds are choosing significantly higher deductibles over time, so that the average premium is declining.
Comment: Since the observed average premiums have been increasing, while the projection is that they will decrease, in this case the two alternative methods in part (a) give significantly different results.

5.131. (a) i. CY15 Earned Premium: 200/2 + (240)(3/4) + 260 + 280/2 = **$680**.
ii. CY15 Written Premium: 260 + 280 = **$540**.
(b) i. PY15 earned premium as of the end of 2016: 260 + (3/4)(280) = **$470**.
ii. Since by the end of 2016 all of policies written during 2015 have expired, the PY15 written premium is the same as the earned: 260 + (3/4)(280) = **$470**.
(c) Advantages: Calendar Year Premium and losses are fixed at the end of the calendar year.
Once the Calendar Year is over, data is ready to be used and is thus available quickly.
Calendar Year data is easy to obtain since it is needed for accounting purposes.
Disadvantage: Calendar Year data has a poor match between premiums and losses.
Comment: Policy D written on July 1, 2015 was cancelled March 31, 2016, but it still contributes the full $280 to CY15 written premium; it would contribute -$70 to CY16 written premium.

5.132. (a) No CY18 earned premiums for policies 1 and 2
Policy 3: (10,000)(7/12) = 5,833   Policy 4: (15,000)(11/12) = 13,750
Policy 5: (8000)(100%) = 8,000   Total = **27,583**
(b) Only Policy 4: (15,000)(1/12) = **1,250**
(c) 0 for Claims 1 and 2.   Claim 3: 1000 + 2000 = 3000.   Claim 4: 4000 - 1000 = 3000.
Total = **6000**.

5.133. (a)
Advantage: It is the most precise method of adjusting historical premiums for prior rate changes.
Disadvantage: It is not viable if new rating variables have been introduced for which historical data are not available.
(b) The method assumes that premium is written evenly throughout the time period.
(c)

Region	Area	Rate Level Relative Value
A	0.125	1.0000
B	0.750	1.0700
C	0.125	1.1877

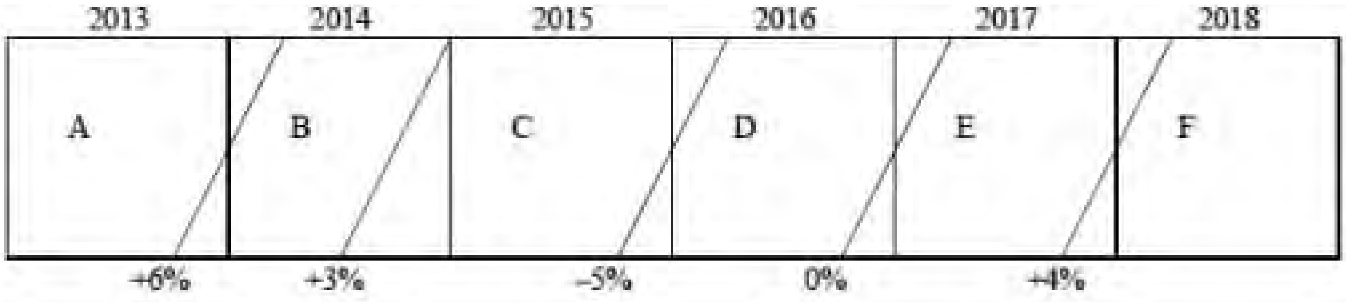


Average Rate Level CY16: (0.125)(1) + (0.750)(1.07) + (0.125)(1.1877) = 1.0760.
Current Rate Level: (1.07)(1.11)(1.13) = 1.3421.
On Level Factor: 1.3421/1.0760 = **1.2473**.
(d) Insurers make changes in the rates for rating factors for certain classes or territories which are aggregated into one estimate of the overall rate change. The premiums for specific class or territory would not be on-level if the overall average rate change is not the same as the specific class or territory rate change.
(e) I would rely on the extension of exposures method instead of the parallelogram method for classification and territory ratemaking.
Alternately, one could use the (adjusted) pure premium method rather than the loss ratio method for classification ratemaking; the pure premium method makes no use of premiums, and thus there is no need to adjust premiums to the current rate level.

5.134. (a) Policies A, B, C and D are in effect.
On September 15, 2015 policy B (endorses August 1, 2015) has full term premium of:
250 + 60 = 310.
On September 15, 2015 policy C has full term premium of 150.
On September 15, 2015 policy D (renewed September 15, 2015) has full term premium of 200.
In-force premium as of September 15, 2015: 100 + 310 + 150 + 200 = **760**
(b) Policy E is not part of Policy Year 2015, while the other policies are.
Policy B was endorsed half way through.
Policy C was cancelled after 5 months.
PY15 written premium as of December 31, 2017:
100 + (250 + 60/2 + 50) + {150 - (150)(7/12)} + (200 + 80/2) = **732.5**.
Comment: All of the given premiums are full-term.

5.135. (a) (3/12)(1800) + {(24 - 7)/24}(3000) + (2/6)(1200) = **2975**.
(b) Policy 1: (3/12)(1800) = 450. Renewal of Policy 1: (9/12)(1800)(1.1) = 1485.
Policy 2: (12/24)(3000) = 1500.
Policy 3: (2/6)(1200) = 400. Policy 3 Renewal: (6/6)(1200) = 1200.
Policy 3 Second Renewal: (4/6)(1200)(1.1) = 880.
Total = 450 + 1485 + 1500 + 400 + 1200 + 880 = **5915**.
(c) Renewal of Policy 1: (3/12)(1800)(1.1) = 495.
Policy 2 has 5 months remaining in its initial policy term: (5/24)(3,000) = 625.
Policy 3 Second Renewal: (2/6)(1200)(1.1) = 440.
Total = 495 + 625 + 440 = **1560**.
(d) Policy 1 and its renewal: (1800)(1.1) = 1980.
Policy 2: (12/24)(3000)(1.1) = 1650.
Policy 3 and its two renewals: (2)(1200)(1.1) = 2640.
Total = 1980 + 1650 + 2640 = 6270.
On-level factor = 6270/ 5915 = **1.060**.
Comment: Obviously, real P&C insurers write many more than 3 policies. If a P&C insurer did write three policies, it would be very unusual for them all to have different terms.

5.136. (a) We need to adjust CY 2014 premiums to the rate level for CY 2017.



Rate Level	Relative Rate Level	Area in CY14	Area in CY17
A	1.0000	6.25%	
B	1.0600	68.75%	
C	1.0918	25.00%	
D	1.0372		6.25%
E	1.0372		87.50%
F	1.0787		6.25%
Weighted Average		1.0642	1.0398

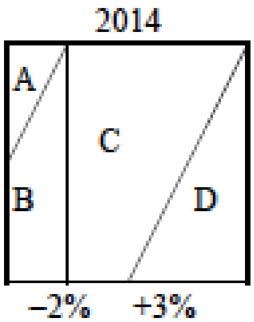
Premium on-level factor for CY2014 = 1.0398/1.0642 = **0.977**.

(b) We need to adjust CY 2014 premiums to instead the most recent rate level.

Premium on-level factor for CY2014 = 1.0787/1.0642 = **1.014**.

(c) Discount introduced on April 1, 2014 = 10%.

Percent of policyholders affected = 20%. This is equivalent to rate decrease of: (10%)(20%) = 2%.



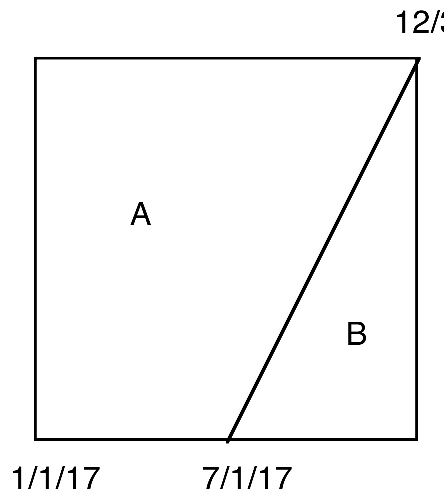
Rate Level	Rate Level Relative Value	Area in 2014
A	1.0000	6.25%
B	1.0600	18.75%
C	1.0388	50.00%
D	1.0700	25.00%
Weighted average rate level	**1.0482**	

<u>Comment</u>: For part (a), see Chapter 8 of <u>Estimating Unpaid Claims Using Basic Techniques</u> by Jacqueline Friedland.

5.137. (a)

Date	Rate Change	Rate Level Index
		1
July 1, 2017	10%	1.1
April 1, 2018	1%	1.111

Since policies are semi-annual, the line has a slope of 2:



Area B = (1/2)(1/2)(1) = 1/4. Area A = 1 - 1/4 = 3/4.
Average rate level for CY17 earned premium is: (3/4)(1) + (1/4)(1.1) = 1.025.
OLF factor for CY17 earned premium is: 1.111/1.025 = **1.0839**.
(b) The current premium per exposure for Class Y is: (1175)(0.75) + 132 = 1013.25.
The April 1, 2017 premium per exposure for Class Y is: (1000)(0.85) + 120 = 970.
The ratio is: 1013.25/970 = **1.0446**.
(c) In general, one does <u>not</u> have enough detail to use the parallelogram method to put premiums by class on level in order to calculate indicated class factors using the loss ratio method.
Therefore, it is <u>not</u> appropriate to use the parallelogram method to calculate indicated class factors using the loss ratio method.
In this particular case, since the class factors change, unless one did a separate calculation for each class, the result of the parallelogram method would <u>not</u> be appropriate.
(An extra complication is that the class factors are not applied to the expense fee.)
<u>Comment</u>: One does not usually compute an "on-level factor" when using the extension of exposure method.

5.138. (a) Of the new business, only the first two transactions contribute to CY17 earned premium.
The acquired book of business has written premiums in 2017 of: (80%) (1.1) (150,000) = 132,000.

Policy	Policy Term (months)	Months Earned in 2017	Written Premium	Earned Premium
Acquired book	12	5	150,000	62,500
Acquired book (1st renewal)	12	7	132,000	77,000
101	12	8	1,600	1,067
102	18	6	2,100	700
Total				**141,267**

For example: (5/12)(150,000) = 62,500.

(b) The acquired book of business has written premiums in 2018 of:
$(80\%)^2$ $(1.1)^2$ (150,000) = 116,160.

Policy number	Policy Term (months)	Months Unearned as of June 30, 2018	Written Premium	Unearned Premium
Acquired book (2nd renewal)	12	11	116,160	106,480
101	12	10	1,800	1,500
201	6	2	800	267
202	24	23	2,500	2,396
Total				**110,643**

For example: (11/12)(116,160) = 106,480.

(c) For example, (5/12)(190,575) = 79,406.

Policy number	Policy Term (months)	Months Earned in 2017	Written Premium at Current Rate Level	Earned Premium at Current Rate Level
Acquired book	12	5	$(150,000)(1.1^2)(1.05) =$ 190,575	79,406
Acquired book (1st renewal)	12	7	(132,000)(1.1)(1.05) = 152,460	88,935
101	12	8	(1600)(1.05) = 1680	1,120
102	18	6	(2100)(1.05) = 2205	735
Total				**170,196**

(d) The parallelogram approach is not appropriate to use for determining the earned premium at current rate levels because the exposures are <u>not</u> evenly distributed over time.

5.139.

(a) Written premiums reflect shifts in the mix of exposures more quickly than earned premiums.

(b) Either of the following are acceptable:

- Quarterly average written premiums
- 12 month moving averages of quarterly written premiums

(c) Weighted Average ILF for 2015: (30%)(0.90) + (40%)(1.00) + (30%)(1.15) = 1.0150.
Weighted Average ILF for 2016: (27%)(0.90) + (40%)(1.00) + (33%)(1.15) = 1.0225.
Weighted Average ILF for 2017: (23%)(0.90) + (40%)(1.00) + (37%)(1.15) = 1.0325.
Change between 2015 and 2016: 1.0225/1.0150 - 1 = 0.74%.
Change between 2016 and 2017: 1.0325/1.0225 - 1 = 0.98%.

(d) I recommend 0.98% based on the most recent change, due to the expected future growth in interest in higher limits.

(e) Average earned date in experience period July 1, 2016.
Average earned date in forecast period for annual policies July 1, 2020.
Average earned date in forecast period for 6-month policies April 1, 2020.
Average trending period: (48 + 45)/2 = 46.5 months.
Premium trend factor = $1.0098^{(46.5/12)}$ = **1.0385**.

<u>Comment</u>: Changes in the mix of increased limits would <u>not</u> come into play in this manner if as is common one used basic limits premiums and losses in the overall rate indication. However, similar math would apply for example to a change in the mix of business by territory.
Usually the lowest limit sold would be the base limit with an ILF of 1.00.
There are other reasonable choices in part (d).

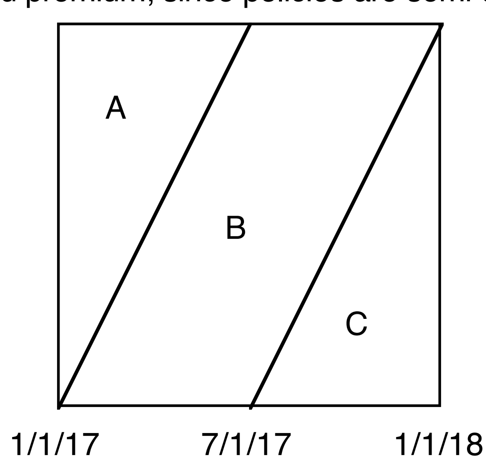
5.140. (a) I took the ratios of successive average written premiums:

Quarter and Year	Average Written Premium at Current Rate Level	Ratio
2Q 2016	$1,771	
4Q 2016	$1,806	1.0198
2Q 2017	$1,840	1.0188
4Q 2017	$1,877	1.0201
2Q 2018	$1,914	1.0197
4Q 2018	$1,953	1.0204

I select a trend of 2% per six months.
Since policies are semi-annual, the average date of writing of CY2017 Earned Premium is July 1, 2017 minus 3 months = April 1, 2017.
Since new rates will be in effect for one year, the average date of writing under the new rates is July 1, 2019 plus 6 months = January 1, 2020.
Thus the trend period for CY17 is 2 years and 9 months. (This is 5.5 half-years.)

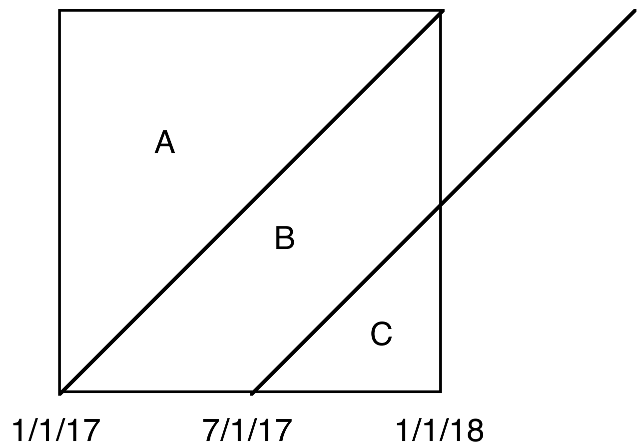
	Rate Level Index
Prior	1.00
1/1/17	1.10
7/1/17	(1.10)(1.05) = 1.155

Draw a diagram for CY17 earned premium; since policies are semi-annual, the lines have slope 2.



Area A = (1/2)(1/2)(1) = 1/4. Area C = (1/2)(1/2)(1) = 1/4. Area B = 1 - 1/4 - 1/4 = 1/2.
Average rate level for CY17: (1/4)(1) + (1/2)(1.1) + (1/4)(1.155) = 1.08875.
OLF = 1.155/1.08875 = 1.0608.
Trended on-level earned premium for CY17: (1.0608)($1.02^{5.5}$)(3,850,000) = **$4.554 million**.

(b) 1. The trend period would be three months longer.
Specifically the average date of writing of CY2017 Earned Premium would be:
July 1, 2017 minus 6 months = January 1, 2017.
The trend to written date would remain the same.
2. The on-level factor would be bigger, since more of the CY2017 Earned Premium would have been written at lower rates.
Specifically, the diagram would now have lines with slope of 1 rather than 2:



Area A = (1/2)(1)(1) = 1/2. Area C = (1/2)(1/2)(1/2) = 1/8. Area B = 1 - 1/2 - 1/8 = 3/8.
Average rate level for CY17: (1/2)(1) + (3/8)(1.1) + (1/8)(1.155) = 1.0569.
OLF = 1.155/1.0569 = 1.0928.
<u>Comment</u>: In this case, in my opinion, there is no reason to bother to use a two-piece method of premium trend.
In part (b), I believe one could give somewhat less detail than I did.
If all policies were annual rather than semi-annual, then the trended on-level EP for CY17 would be:
$(1.0928)(1.02^6)(3,850,000)$ = \$4.738 million.

5.141. (a) We are looking for policies written between August 1, 2017 and July 31, 2018: D, E, F.
750 + 900 + 1650 = **3300**.
(b) Policies E and G are in force on December 15, 2018. 900 + 1350 = **2250**.
(c) Policies E, F, and G are written in 2018, with total premium of: 900 + 1650 + 1350 = 3900.
Canceling Policy C on March 31, 2018 results in negative written premium of: 2000/4 = 500.
3900 - 500 = **3400**.
(d) If one has only annual policies and a large book of business, then the earned premium would be approximately equal to the average of the in-force premium at the end of the current year and prior year. However, since some of the policies are six-month and we have a small book of business, this approximation does not hold.
(e) “As in-force premium is the best estimate of the company’s mix of business as of a given date, the most recent in-force premium is often used to measure the impact of a rate change on an existing portfolio of customers.”
Alternately, In-force premium can be used (with caution) to monitor the size of a book of business over time.
Also from the CAS Examiner’s Report:
- Determine current potential for loss.
- In-force premium could be used to estimate how much reinsurance to purchase.

Comment: A fiscal year is a generalization of a calendar year, with a starting date other than January 1.
Basic Ratemaking does not discuss averaging in-force premiums in order to approximate the earned premium. Therefore, in my opinion, it was unfair to expect students to answer part d under exam conditions; this is the basic ratemaking exam.
Graph the in-force premium from time zero to one. If the insurer has only annual policies, then the earned premium is the area under this curve. With a large book of business, the area under this curve from zero to one is approximately equal to the average of the values at times zero and one. (This approximation is exact if the curve of the in-force premium is a straight line.)
Similarly if the insurer has only six-month policies, then the earned premium would be twice the area under this curve. Thus the earned premium would be approximately the sum of the in-force premiums at the end of the current year and prior year.